DEPARTMENT OF COMMERCE

CIRCULAR

OF THE

BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

No. 3

DESIGN AND TEST OF STANDARDS OF MASS

3d Edition

[Superseding 2d edition entitled "Verification of Standards of Mass," issued Jan. 10, 1907]

Issued December 23, 1918

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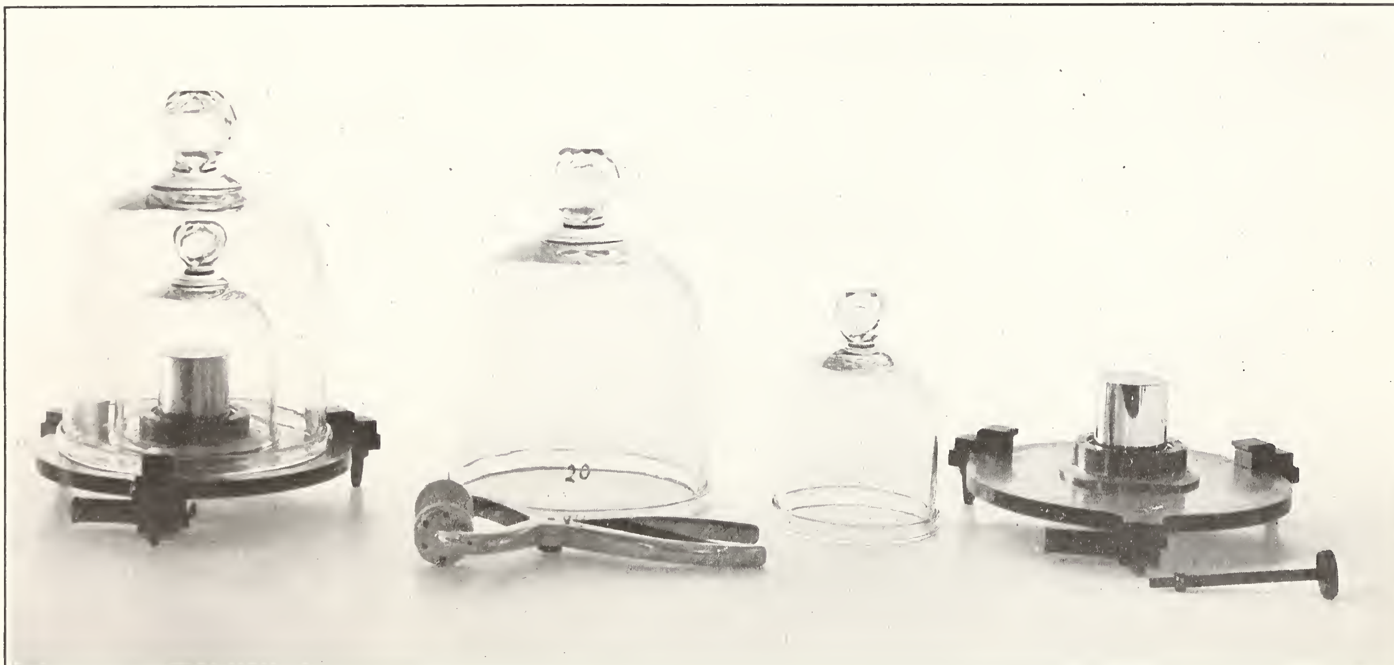

Fig. 1.—*National prototype kilograms*

From these standards are derived the values of all weights used in this country. These standards were made by an international commission. The material is an alloy of 90 per cent platinum and 10 per cent iridium, which is hard and very resistant to corrosion. It is handled by a special lifter, the faces of which are covered with very soft white plush

DESIGN AND TEST OF STANDARDS OF MASS

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I. INTRODUCTION

1. SCOPE OF CIRCULAR

This circular is intended to furnish information concerning the testing of standards of mass, and related matters that are frequently the subject of inquiries addressed to the Bureau. It will be impossible to give any but general information within the limited space available. However, any other information at the disposal of this Bureau on the subject of mass standards, balances, methods of weighing, or the design of special apparatus will be gladly furnished on receipt of specific requests.

Under regulations set forth in subsequent sections, the Bureau tests and certifies the accuracy of standards submitted, but it does not manufacture or sell such standards nor does it, except in rare instances, correct those that are not sufficiently accurate. For further details, see "Regulations governing tests," page 65.

2. FUNDAMENTAL STANDARDS

The fundamental standard of mass adopted by the United States is the international prototype kilogram.[1] This is a cylinder of platinum-iridium kept at the International Bureau of Weights and Measures near Paris in the custody of an international committee. This committee is composed of 14 eminent scientists, elected, each one from a different country, by an international conference that meets every six years. This conference is composed at present of diplomatic representatives of 26 countries, among which are all the leading nations of the world.

Authentic copies of this standard of the same form and composition have been made under the supervision of the international committee and distributed among the countries that support this international organization. Of these authentic copies the United States has two, Nos. 4 and 20. They are kept in a specially constructed vault at the National Bureau of Standards, and are used only when needed to verify the secondary standards of the Bureau.

3. BASIS OF THE CUSTOMARY UNITS

The units of the customary system were based originally on the old brass standard pound of the United States Mint; but the platinum-iridium alloy posessses a constancy far superior to brass and other base-metal alloys, and for this reason the Office of Weights and Measures (now the Bureau of Standards) was authorized in 1893 to base the pound, with its derived and related units, on the international kilogram, using the relation:

One pound avoirdupois = 0.4535924277 kg, the value determined by the joint work of the International Bureau of Weight and Measures and the British Standards Office.

4. DISTINCTION BETWEEN MASS AND WEIGHT

Mass standards are most frequently called "weights," and in accordance with this widely accepted usage, the term will be

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[1] For legal status, see B. S. Circular No. 47, p. 60.

freely used in that sense throughout this circular. But confusion often arises from the indiscriminate use of the term "weight" in two other senses; the first, to denote the mass of a body, and the second, to refer to the force with which gravity acts upon such a body. The former is really an incorrect use of the word, due to failure to separate the two ideas. Of course, it is readily understood that when we speak of a 2-ton block of iron, and a tension of 2 tons in a stretched rope, we refer in one case to a mass and in the other to a force. It should be kept in mind that, whereas the mass of a body is a property inherent in the body itself, its weight depends on the attraction of the earth, varying on that account with altitude, latitude, and other circumstances. This variation of weight with locality, though amounting only to about one-half of 1 per cent over the surface of the earth, is perfectly definite, and can be detected by means of a sensitive spring balance.

Weight is measured in units of force. As it is not feasible nor desirable for the purposes of metrology to base the unit of force on some concrete standard force, it is derived from the established units of mass and acceleration. The cgs unit of force, the dyne, is thus derived. It is common, however, to express weight in terms of the gravitational unit of force. This unit is the force of gravity that acts on a body per unit of mass (in vacuo) at any place where the acceleration of gravity is $980.665 \frac{\text{cm}}{\text{sec.}^2}\left(=32.1740 \frac{\text{ft.}}{\text{sec.}^2}\right)$, in accordance with the definition of standard weight and standard gravity by the International Committee of Weights and Measures.[2]

5. COMPARISON OF MASSES

Masses are determined by direct or indirect comparison with a standard. The most accurate means of comparing masses is the lever balance. This method involves a comparison, by means of the lever principle, of the respective forces of gravity on the bodies under consideration. Since the masses of bodies are in the same *ratio* as their weights at the same locality, the desired result is obtained. In the same way, any other dynamometer might be

[2] Procés Verbaux des Séances, Comité Internationale des Poids et Mesures, 1901, p. 173.
The unit was based, as closely as was known at the time (1888), on the intensity of gravity at mean sea level, latitude 45°. This was the specification of standard gravity from the time of the establishment of the metric system until the adoption of $980.665 \frac{\text{cm}}{\text{sec.}^2}$ as an arbitrary value. (See, for example, Delambre Base du Syst. Met. T., III, p. 590, 1806.) The value of g at sea level, latitude 45°, as computed from the absolute determination of gravity at Potsdam (1898–1906), differs from the standard value above by 1 part in 30 000.

used as the means of comparison. In all cases the effect of atmospheric buoyancy must be taken into account; for, while the force of gravity is proportional to the mass of a body, the oppositely directed buoyant force of the atmosphere is proportional to its *volume*. In accordance with Archimedes well-known principle of fluid buoyancy, the weight of a body in a fluid (liquid or gas) is diminished by an amount equal to the weight of fluid displaced.

Full details relative to the buoyant effect of the air in its relation to the actual methods used in comparing standards of mass are given in succeeding sections of this circular.

II. CLASSIFICATION OF WEIGHTS

The following classification covers the forms recommended for particular purposes, but weights not falling in any class may under certain conditions be tested. For details applying to this subject, see "Regulations governing tests," page 65.

The following two sections, 1 and 2, indicate roughly the distinctions between the different classes, but the detailed specifications must be consulted for accurate information as to the uses and specifications for each class.

Standards intended for commercial purposes should meet several requirements that are not essential to those intended for scientific work, and the converse also is true. For these reasons, it has been found necessary to divide the weights submitted for test into two general groups: Weights for commercial purposes, and weights for scientific purposes.

1. COMMERCIAL GROUP, CLASSES A, B, C, AND T

This group includes weights that range from the primary standards of the several States to those weights used in ordinary trade. The group is divided into classes A, B, C, and T. These weights have an official or legal status, and the fact that they are correct within the established tolerances will be indicated by a seal impressed on them in some conspicuous place. Weights having screw knobs or other means by which the adjustment is made accessible for alteration without destroying the seal, will not be sealed.[3]

Each of these classes covers weights that are suitable, among other things, for testing the weights of the next lower class.

[3] The verb "to seal," as here used, refers only to affixing a mark as evidence that the weight is correct; it does not mean to make the weight correct and should not be used in that sense. The word "adjust" is employed to express the latter idea.

Class T includes ordinary trade weights, which are not tested by this Bureau except there be no local authority to whom they can be submitted, or in the event of an important controversy.[4]

Class C weights are much more accurate than those of class T, the errors allowed on weights of class C being only one-tenth of those allowed on ordinary trade weights. This class includes, among others, what are commonly called test weights; but these specifications require more reliable types of construction than are sometimes used. On all but the small weights they provide for an adjusting hole closed by a soft metal cap or plug.

Classes A and B both have allowable errors about one-fifth of those of class C, but the correction for each weight of class A is accurately determined when the weight is tested, so that allowance may be made for this error in using the weight. Weights of both classes A and B are without means for easy readjustment, but class B weights may be made with a hard metal adjusting plug, which makes the first adjustment very much easier.

Class A weights are made of a single piece of metal. At present the larger weights of this class are generally made of gold-plated bronze. They are intended for such purposes as primary state standards.

2. SCIENTIFIC GROUP, CLASSES M AND S

This group includes weights used for the more precise weighings in scientific and technical laboratories. It includes such weights as primary laboratory standards, analytical weights, assay weights, etc.[5] It is divided into classes M and S.

Weights of these classes do not, as a rule, have any legal status, and will not be sealed unless they conform also to the specifications for class A, B, or C.

The small weights of this group are required to be more accurately adjusted than are corresponding weights of the commercial group, but the requirements as to form and material are more liberal.

Class S is intended to cover high-grade analytical weights and similar weights used in scientific and technical laboratories. At present, in sets of this kind the weights of 1 gram and above are universally made with the knobs merely screwed into the body of the weights, and the weights from 20 milligrams down are of aluminum.

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[4] Tolerances for these weights are given in Table No. 9, p. 72 of this circular. Complete specifications and tolerances for commercial weights and measures are given in Circular No. 61.

[5] The so-called rough laboratory weights should not be submitted to this Bureau. They can readily be checked in the laboratory or by the local sealer of weights and measures.

Class M is intended to cover weights suitable for scientific work of very high precision where a high degree of constancy is necessary. The accuracy of adjustment required of this class is no higher than that of class S because it is expected that the corrections, which will be determined to a much higher degree of precision, will be applied in using the weights.

Class M weights, like those of class A, are made of a single piece of material, without special provision for adjustment or readjustment.

III. CLASS A—HIGHEST PRECISION COMMERCIAL STANDARDS

Class A comprises commercial standards of the highest type intended for reference standards in work of exacting character. They are recommended for the use of manufacturers of class A or class B weights and as primary standards of States in which the work of inspection is well organized. While these weights are designed with particular reference to reliability and permanence, such qualities can be maintained only by using the weights with extreme care. The soft gold plating ordinarily used as surface protection for this class of weights renders them more susceptible to rapid wear with unskillful or frequent handling than even the unplated brass weights.

1. SPECIFICATIONS

(*a*) **Material.**—The material of weights of this class must be hard, nonmagnetic, must not be readily susceptible to oxidation and corrosion, and must be of a minimum porosity.

For weights down to 1 g or to 1/32 ounce the density should be between 8.2 and 8.6 g/cm³. For smaller weights there may be used platinum or other suitable material having a density between 5.3 and 22.0 g/cm³. For weights less than 5 mg or 0.1 grain aluminum may be used. All "flat" sheet-metal weights must be of a material sufficiently resistant to need no surface protection against oxidation or corrosion.

(*b*) **Structure and Form.**—The entire weight must be a single piece, except for the surface plating.

Weights down to and including 1 g or 1/32 ounce must be cylindrical in form with the diameter approximately equal to the height. The difference between the diameter and height must not be over 10 per cent of the latter. Weights of this class must be provided with a rounded knob, small in proportion to the body of the weight. The bottom must be hollowed out symmetrically, leaving a flat ring as a base upon which the weight is to rest.

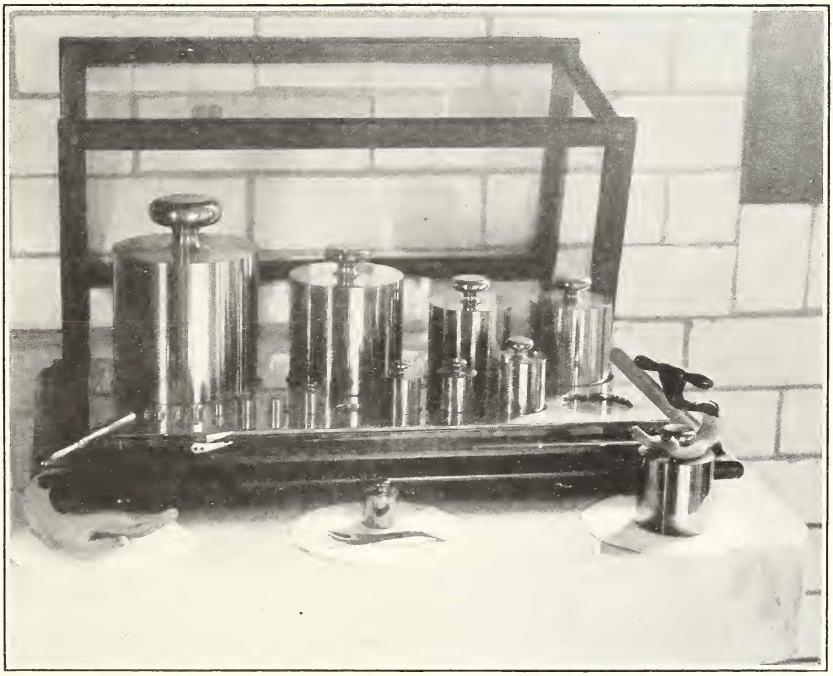

FIG. 2.—*Gold-plated, one-piece standards*

This set of metric standards from 10 kg to 1 g is one of the most important sets in use at this Bureau. It was made in 1893, so that there has been time to test its constancy by many calibrations. More recent Class A weights have the edges rounded much more than these and the knobs of the small weights are taller

The width of this ring should be from one-tenth to two-tenths of the diameter of the weight.

There must be no other cavities or depressions, except such as are required to designate the value of the weight. All edges and corners must be well rounded.

(For approved designs and dimensions, see p. 27.)

Weights less than 1 g or 1/32 ounce may be "flat," as when cut from sheet metal, with one edge or corner turned up at right angles, to faciliate handling with the forceps. The edges of these weights must be free from sharp or ragged places. Moreover, these small weights must not be cut through by the stamping of the designation, and are not to be unnecessarily thin.

(*c*) **Surface.**—The entire surface must be smooth and highly polished, and, in the case of new weights, must be free from scratches. All weights not constructed of platinum, aluminum, or metals similarly resistant to atmospheric corrosion, must be protected by a plating of gold, platinum, or similar metal. These weights must be of such material and plating that they will show no discoloration on the surface when placed in boiling water or when dried at a temperature of 110° C (230° F), as is done in preparing them for test.

Nickel plating will not be permitted on class A weights.

(*d*) **Designation of Value.**—New cylindrical weights down to and including 10 g or ¼ ounce and sheet-metal weights down to and including 100 mg or 1 grain must be plainly marked with their nominal or intended value. This must include the name of the unit or the accepted abbreviation. Smaller weights must bear the proper number, but the name of the unit may be omitted when the space available does not permit its legible inclusion.

A short table of abbreviations, recommended by this Bureau, is given in Table 8, on page 71. A much more complete list will be found in Circular No. 47.

Duplicate and triplicate weights of a set must be so marked that they can be distinguished with ease and certainty. A common and satisfactory practice is that of using, in addition to the denomination, one and two conspicuous dots on duplicates, and one, two, and three such dots on triplicates. It is preferable to have such marks on each weight rather than to have one weight entirely unmarked. Both the denomination and the distinguishing marks must be on the upper surface of the weights, and these markings must be shallow, broad, and free from burrs and sharp angles.

(*e*) **Case.**—For weights of this class a suitable dust-proof case should be provided, preferably so arranged that the weights rest upon a base of smooth, hard material such as glass or polished quartz, which can be readily kept clean.

If the case is provided with deep pockets for the cylindrical weights, these pockets must be lined with some soft material, such as plush. The fit must not be tight enough to require any appreciable force in inserting or withdrawing the weights. (See p. 67 in regard to precautions in shipment.)

(*f*) **Lifter.**—Special lifters should be provided for all weights of class A. The parts that come in contact with the weights may be of wood or ivory, or be covered with velvet or prepared chamois skin, or similar soft material, and must be smooth and free from sharp edges.

(*g*) **Tolerances.**—Weights of this class must be adjusted within the limits of error prescribed for this class on page 17. All weights must be adjusted so as to balance correct brass standards in air. (See "Buoyant force of the air," p. 55.) The constancy of the weights must be such that they will not change their values more than one-fifth of the tolerances allowed when subjected to a constancy test of three months.

2. SEALING AND CERTIFICATION

Weights meeting all the requirements will be stamped with the seal shown in Fig. 3.

FIG. 3.—*Seal for class A weights*

This seal will be impressed on standards tested and approved under this class unless such standards are so small that this is impracticable

The certificate will give the correction to be applied in using each weight. This will be based on the apparent mass as determined by comparison with brass standards in air.

TABLE 1.—Precision of Corrections, Class A

[Corrections will be given to the nearest unit in the decimal place indicated in the columns headed "To"]

Metric		Customary									
		Avoirdupois		Grains		Apothecary		Ounces troy		Pennyweights	
Denomination	To	Denomination	To	Denomination	To	Denomination	To	Denomination	To	Denomination	To
kg	g	lb.	gr.	gr.	gr.	℥	gr.	oz. t.	gr.	dwt.	gr.
20	0.1	50	1	10 000	0.1	12	0.01	1000	1	10 000	1
10	0.01	25	1	5000	0.01	10	0.01	500	1	5000	1
5	0.01	20	0.1	2000	0.01	6	0.01	400	1	4000	0.1
2	0.01	14	0.1	1000	0.01	5	0.01	300	1	3000	0.1
1	0.01	10	0.1	500	0.01	4	0.01	200	0.1	2000	0.1
g	mg	8	0.1	200	0.001	3	0.01	100	0.1	1000	0.1
500	1	7	0.1	100	0.001	2	0.01	50	0.1	500	0.1
200	1	5	0.1	50	0.001	1	0.01	40	0.1	400	0.01
100	1	4	0.1	20	0.001	ʒ		30	0.1	300	0.01
50	1	3	0.1	10	0.001	6	0.01	20	0.01	200	0.01
20	0.1	2	0.1	5	0.001	4	0.01	10	0.01	100	0.01
10	0.1	1	0.01	2	0.0001	3	0.01	5	0.01	50	0.01
5	0.1	oz.		1	0.0001	2	0.001	4	0.01	40	0.01
2	0.1	10	0.01	0.5	0.0001	1	0.001	3	0.01	30	0.01
1	0.1	8	0.01	0.2	0.0001	℈		2	0.01	20	0.01
mg		5	0.01	0.1	0.0001	2	0.001	1	0.01	10	0.01
500	0.01	4	0.01	0.05	0.0001	1	0.001	0.5	0.01	5	0.001
200	0.01	2	0.01	0.02	0.0001			0.2	0.001	4	0.001
100	0.01	1	0.01	0.01	0.0001			0.1	0.001	3	0.001
50	0.01	$\frac{1}{2}$	0.01	0.005	0.0001			0.05	0.001	2	0.001
20	0.01	$\frac{1}{4}$	0.001	0.002	0.0001			0.02	0.001	1	0.001
10	0.01	$\frac{1}{8}$	0.001	0.001	0.0001			0.01	0.001		
5	0.01	$\frac{1}{16}$	0.001					0.005	0.0001		
2	0.001	$\frac{1}{32}$	0.001					0.002	0.0001		
1	0.001	$\frac{1}{64}$	0.001					0.001	0.0001		
0.5	0.001							0.0005	0.0001		
0.2	0.001							0.0002	0.0001		
0.1	0.001							0.0001	0.0001		

IV. CLASS B.—PRECISE COMMERCIAL STANDARDS

Class B weights are intended as high-grade commercial standards to be used in cases where extreme precision is not necessary or warranted. They are suitable for primary standards for local inspectors, for manufacturers of commercial weights and measures, and for high-grade working standards such as would be needed in testing class C weights.

1. SPECIFICATIONS

(*a*) **Material.**—Weights of this class must be made of material that is not readily oxidized or corroded. The material must be moderately hard (for example, the better grades of brass or bronze) and must be nonmagnetic. Nickel, though magnetic, may be used as a plating material.

The material of the weights must be free from pits or pores and must show no effects of porosity, such as the white patches that sometimes form on porous weights electroplated with gold or nickel.

The density of the weights, including any cavity under the adjusting plug, shall be between 8.2 and 8.6 g/cm^3 except in the case of weights below 1 g or 1/32 ounce. For these smaller weights platinum or other suitable material having a density between 5.3 and 22.0 g/cm^3 shall be used. For weights below 50 mg aluminum may be used. All flat sheet-metal weights shall be of a material whose surface need not be protected against oxidation or corrosion.

(*b*) **Structure and Form.**—Metal weights shall consist of a single piece, except as described below under "Means of adjustment." Weights of this class may be made of glass or similar material filled with shot or otherwise loaded to the proper mass. In such weights the opening to the interior must be closed by fusing the material of which the shell is composed.

Weights down to and including 1 g or 1/32 ounce shall be cylindrical in form with the diameter approximately equal to the height. The difference between the diameter and height must not be over 10 per cent of the latter. Weights of this class must be provided with a rounded knob small in proportion to the body of the weight. The bottom must be hollowed out in a symmetrical manner, leaving a flat ring as a base. The width of the ring should be from one-tenth to two-tenths the diameter of the weight. There must be no other cavity or depression except as specified below. All edges and angles must be well rounded.

Smaller weights may be of flat sheet metal with an edge or corner turned up at right angles to facilitate handling with the forceps, or they may be of wire bent in shape to indicate the denomination. They must be free from sharp or ragged edges, must not be cut through in marking the designation on them, and must not be unnecessarily thin.

Adjusting Plug.—For metal weights of this class, special means of adjustment may be provided. This, if used, shall consist of a single cavity in the top of the weight, closed by a plug of brass or similar hard, tough metal securely driven into place.

This plug shall be air and water tight and shall fit so tightly as to make its removal impossible without the certainty of destroying the seal. When the plug is driven into place its top must be on a level with, or very little below, the surface of the weight, and no cracks, sharp angles, or burrs must remain. The diameter of the

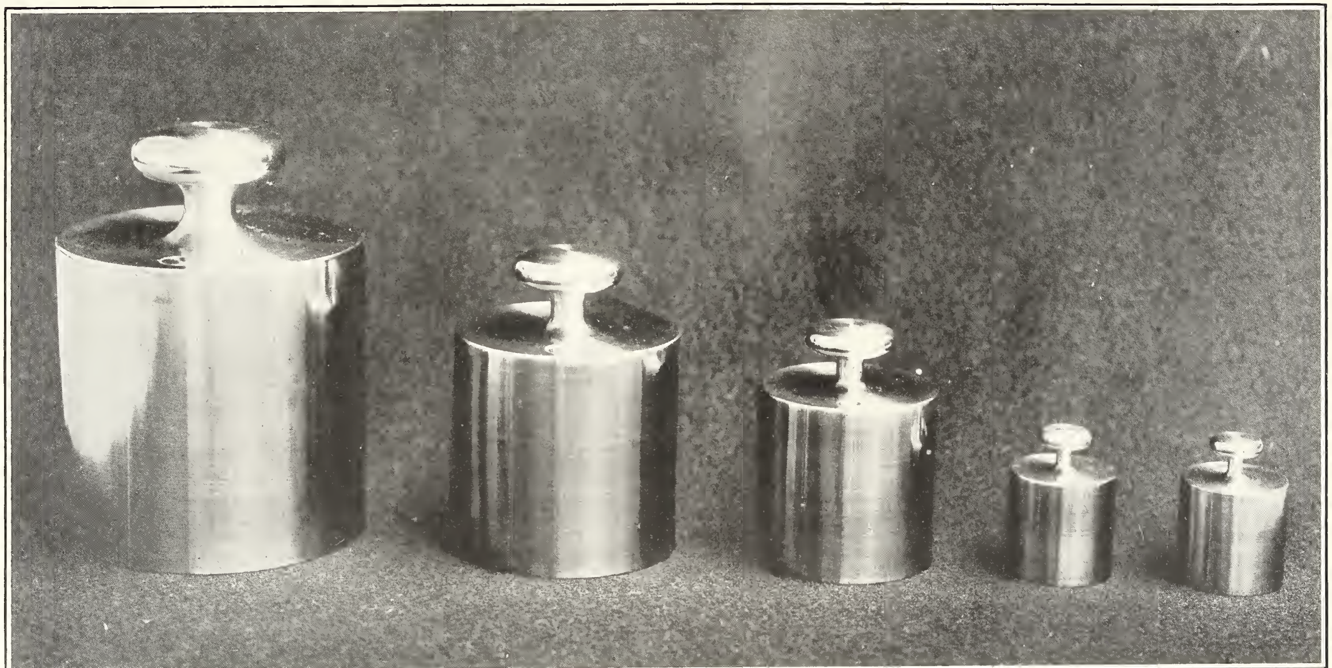

FIG. 4.—*Nickel-plated, class B weights*

These are from a set of Tobin bronze secondary standards made in 1911 and used in this Bureau in a great deal of routine testing. The knob and body of the weight are a single piece of metal. All but the small weights of the set have the typical Class B driven plug. Their constancy has been very satisfactory

plug must conform in size to the dimensions given in the following table:

	Diameter of plug
Under 500 g or 1 pound	2 mm to 5 mm or $\frac{3}{32}$ inch to $\frac{3}{16}$ inch
From 500 g to 10 kg or from 1 to 20 pounds	5 mm to 10 mm or $\frac{3}{16}$ inch to ⅜ inch
Above 10 kg or 20 pounds and less than 50 pounds	10 mm to 16 mm or ⅜ inch to ⅝ inch
50 pounds and above	16 mm to 25 mm or ⅝ inch to 1 inch

The transverse cross section of the plug must be circular. The sides of the plug and cavity must be smooth, to insure tightness against leakage of air. The plug should have a slight taper and a driving fit. When inserted in the hole and forced down by a pressure exerted by the fingers, the top of the plug must remain above the surface of the weight by an amount equal to at least one-half the diameter of the plug, so that it is necessary to drive it down into place.

In order that the plug may seat firmly enough to take the impression of the seal, its final position must be determined by a shoulder. A common and satisfactory form is indicated in Fig. 6. The form of cavity shown in Fig. 6 can be easily drilled and accurately counterbored. The fit must not be tight enough to cause the metal about the hole to bulge when the plug is driven into place. The length of the plug from its upper end to the shoulder must not be less than its diameter.

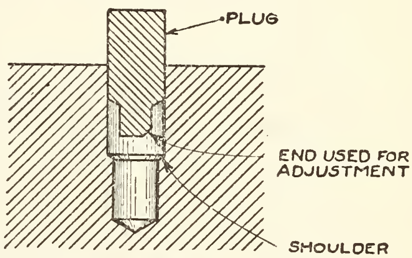

Fig. 6.—*Design of class B adjusting plug*

This plug should preferably be of the same material as the rest of the weight. It should be made with a good "driving fit," and may be tapered slightly if desired. The plug is shown in place ready to be driven.

The plug should not be driven into place in the case of new weights submitted to this Bureau for test, first, because the Bureau wishes to inspect the design and workmanship before sealing the weight; and second, because if the weight is not properly adjusted, the adjustment can be more readily corrected by the manufacturer before the plug is driven than afterwards.

The adjusting cavity should not contain any loose material. The adjusting cavities and plugs of metal weights and the volumes of filled glass weights must be so regulated as to keep the average density of the weight within the limits specified under "Material."

(*c*) **Surface.**—The surface must be smooth and weights, if made of metal, must be carefully polished. If the weights are made of material that under ordinary conditions tarnishes to a greater extent than nickel or aluminum, the surface must be protected by a plating of gold, platinum, nickel, or other suitable metal, or by lacquer. If lacquer is used it must be hard, of only moderate thickness, transparent, and not easily chipped. Flat sheet-metal weights, however, are not to be lacquered.

(*d*) **Designation of Value.**—New cylindrical weights down to and including 10 g or ¼ ounce and sheet-metal weights down to and including 100 mg or 1 grain must be plainly marked with their nominal mass or "value." This must include the name of the unit or the accepted abbreviation. Smaller cylindrical and flat weights must bear the proper number, but where space is limited, the name of the unit may be omitted. Wire weights need not be marked but must be bent in such forms as to suggest their value.

Duplicate and triplicate weights of a set must be so marked that they can be distinguished with ease and certainty. A common and satisfactory practice is that of using in addition to the denomination, one and two conspicuous dots on duplicates, and one, two, and three such dots on triplicates. It is preferable to have such marks on each weight rather than to have one weight entirely unmarked. Both the denomination and the distinguishing marks must be on the upper surface of the weights. These markings must be shallow, broad, and free from burrs and sharp angles.

(*e*) **Case.**—The weights should be kept in a dust-proof case or box, and each weight must have a separate pocket. The pockets, if deep, must be lined with some soft material, such as velvet, or chamois skin from which all grease has been carefully removed. The fit must not be tight enough to require any appreciable force in inserting or removing the weights. (See p. 67 for precautions in shipment.)

(*f*) **Lifters.**—Special lifters must be provided. The parts that come into contact with the weight may be of wood or ivory or may be covered with velvet or prepared chamois skin, or similar soft material, and must be without sharp edges or corners.

(*g*) **Tolerances.**—Weights of this class must be adjusted within the limits of error prescribed below.

All weights must be adjusted according to the apparent mass, as this mass would be determined by comparison with brass standards in air. (See "Buoyant force of the air," p. 55.)

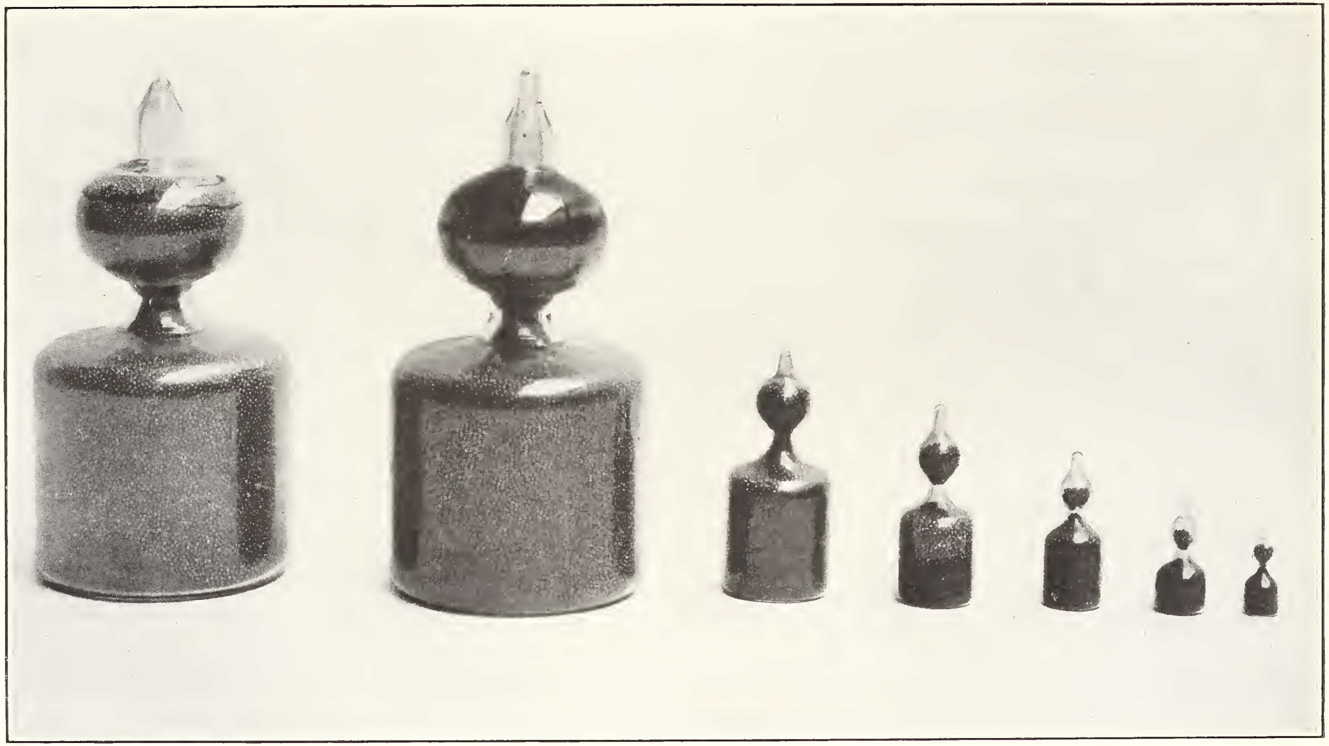

FIG. 5.—*Glass-cased weights*

These weights are made of glass shells filled with shot. The surface is easily cleaned and wears less than that of metal weights under ordinary office handling. These weights are even more reliable than the metal weights of this class because as a rule they are not injured at all or else the injury is so great that there is no doubt about it. They can easily be made strong enough to stand ordinary handling and at a low cost. They are made and used in Austria, and it is hoped that American manufacturers will take up the making of similar weights

TABLE 2.—Tolerances for Classes A and B: Precise Commercial Standards

[The maximum error allowable on each weight is given in the columns headed "Tolerance"]

Metric		Customary												Carats	
		Avoirdupois		Grains		Apothecary		Troy ounces		Pennyweights					
Denomination	Tolerance	Denomination	Tolerance	Denomination	Tolerance	Denomination	Tolerance	Denomination	Tolerance	Denomination	Tolerance			Denomination	Tolerance
kg	mg	lb.	gr.	gr.	gr.	oz. ap.	gr.	oz. t.	gr.	dwt.	gr.			c	mg
20	120	50	2.0	10 000	0.2	12	0.2	1000	2	10000	2			2500	14
10	80	25	1.2	5000	0.2	10	0.2	500	2	5000	1			2000	12
5	50	20	1.2	2000	0.1	8	0.1	400	1	4000	1			1000	8
2	30	10	0.8	1000	0.06	6	0.1	300	1	3000	0.6			500	6
1	20	8	0.6	500	0.04	5	0.1	200	1	2000	0.6			200	4
g		5	0.6	200	0.03	4	0.1	100	0.6	1000	0.4			100	2
500	14	4	0.4	100	0.02	3	0.08	50	0.4	500	0.3			50	1.5
200	8	3	0.4	50	0.01	2	0.06	40	0.3	400	0.2			20	1.0
100	6	2	0.3	20	0.006	1	0.04	30	0.3	300	0.2			10	0.6
50	4	1	0.2	10	0.004	dr. ap.		20	0.2	200	0.2			5	0.4
20	2	oz.		5	0.003	6	0.04	10	0.2	100	0.1			2	0.2
10	1.5	10	0.2	2	0.002	5	0.04	5	0.1	50	0.08			1	0.14
5	1.0	8	0.1	1	0.001	4	0.04	4	0.1	40	0.06			0.5	0.10
2	0.6	5	0.1	0.5	0.001	3	0.02	3	0.08	30	0.06			0.2	0.06
1	0.4	4	0.1	0.2	0.0005	2	0.02	2	0.06	20	0.04			0.1	0.04
mg		2	0.06	0.1	0.0004	1	0.01	1	0.04	10	0.04			0.05	0.03
500	0.3	1	0.04	0.05	0.0003	s. ap.		0.5	0.04	5	0.02			0.02	0.02
200	0.14	$\frac{1}{2}$	0.04	0.02	0.0003	2	0.01	0.4	0.03	4	0.02			0.01	0.01
100	0.10	$\frac{1}{4}$	0.02	0.01	0.0003	1	0.006	0.3	0.02	3	0.01				
50	0.07	$\frac{1}{8}$	0.01	0.005	0.0002			0.2	0.02	2	0.01				
20	0.04	$\frac{1}{16}$	0.01	0.002	0.0002			0.1	0.01	1	0.006				
10	0.03	$\frac{1}{32}$	0.01	0.001	0.0002			0.05	0.006						
5	0.02	$\frac{1}{64}$	0.004					0.04	0.005						
2	0.01							0.03	0.005						
1	0.01							0.02	0.004						
0.5	0.01							0.01	0.003						
0.2	0.01							0.005	0.002						
0.1	0.01							0.004	0.0016						
								0.003	0.0014						
								0.002	0.0010						
								0.001	0.0010						
								0.0005	0.0005						
								0.0004	0.0005						
								0.0003	0.0004						
								0.0002	0.0004						
								0.0001	0.0003						

NOTE.—Many of these denominations are not in common use and can not be recommended. They are, however, included here for the sake of completeness. The tolerance on denominations not in this list and not equivalent to any in the list will be that of the next smaller weight listed.

2. SEALING AND CERTIFICATION

Weights meeting all the requirements will be stamped with the seal shown in Fig. 7.

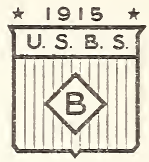

FIG. 7.—*Seal for class B weights*

This seal will be impressed on standards tested and approved under this class, unless such standards are so small that this is impracticable

The certificate will state that the weights are correct within the tolerances prescribed for this class.

V. CLASS C.—COMMERCIAL TEST WEIGHTS

Class C weights are designed for working standards for sealers of weights and measures for use in testing trade weights and weighing instruments and in work where weights of a superior grade are desired for commercial or industrial purposes.

A very important feature of these weights is that they can easily be readjusted, not, however, without destroying the seal.

1. SPECIFICATIONS

(*a*) **Material.**—Weights of class C of 5 g or 60 grains and above must be constructed of material at least as hard as brass. Steel may be used for weights of 1 pound or ½ kg, and above, and cast iron for those of 20 pounds, and above. Satisfactory materials are nickel, brass, bronze, Monel metal, the nonrusting nickel steels, and other alloys which are not readily corroded by the atmosphere or by handling with bare hands. The density of the weights as a whole shall be between 7.0 and 9.0 g/cm^3; that is to say, the volume, including the cavity and cap for adjustment, must not differ greatly from that of a solid brass or iron weight. Weights below 5 g or 60 gr. may be of the same materials or of aluminum, nickel, platinum, or similar materials, provided that the materials used for sheet-metal weights be such that their surface need not be protected against oxidation or corrosion.

(*b*) **Structure and Form.**—The construction of the weight must be such that no part of the weight can be replaced or removed without distinctly defacing the weight or destroying the seal. It is to be noted that weights having knobs or handles merely screwed into the body of the weight and those having movable rings for handles or similar loose parts are excluded from this class.[6]

No special form will be required. Manufacturers and dealers should submit to this Bureau samples or detail drawings of the types of weights that they wish to submit for test. Notice of the acceptance of any type will be sent to all manufacturers or dealers who request it, and to the heads of the weights and measures departments of each State. This provision is made in order to allow the acceptance of unusual forms that are suited to certain special lines of work, and yet can not be included in general specifications.

Means of Adjustment.—Weights above 50 g or 2 ounces must be provided with a single adjusting hole, which must open on the upper surface or side of the weight, but in no case on its bottom face.

Weights of 50 g, or 2 ounces and less, may be provided with an adjusting hole when this is desired and can be done without violating the requirements given below. The adjusting hole must be closed by a cap or plug upon which the regular seal for this class of weights can be stamped.

Sealing Caps.—The cap or plug must be so constructed that it can not be removed without destroying the seal. A screw, covered by a cap of some soft metal such as lead, in order that readjustments may be easily made, is recommended. The outer surface of this cap or plug must not project above the surface of the weight. It must be either on a level with the surface or very slightly below it.

There must be a flat space equal to at least one-third of its diameter between the edge of the cap or plug and the edge of the weight, the edge of the knob, or the projection of the knob on the upper surface of the weight. In new weights there should be an empty space in the adjusting hole large enough to contain lead equal in mass to at least five times the tolerance applying to the weight.

Samples or drawings showing the method of closing the adjusting holes, the denominations and shapes of the weights on which this method is to be used, and the size of the various parts as they

[6] Weights which have a screw knob or handle will be accepted for test, provided the knob or handle is pinned in place in such manner as to prevent its removal. The pin must fit snugly when driven in to the weight, and must project slightly above the body of the weight. The projecting part of the pin must be rounded.

will be used in weights of each of these denominations, must be submitted for approval in connection with, or as a part of, those submitted to cover the requirement stated above under "Shape."

(*c*) **Surface.**—The surface must be smooth and well finished. Weights that are constructed of material that might be badly tarnished or corroded by the air, or by handling with bare hands, must be provided with suitable surface protection. Plating with a metal such as nickel may be used, or a coating may be applied of transparent paint or lacquer which will be hard and not likely to chip, provided, however, that opaque paints may be used on weights of 20 pounds and above. Weights of rectangular design need not have their surface machined, on condition that the irregularities of the surface be slight and not of sharp contour. Neither lacquer nor paint of any kind is to be used on the sheet-metal weights.

(*d*) **Designation of Value.**—The denomination of each weight must be clearly marked upon it. This must include the number in all cases, and the name of the unit in all except cylindrical weights below 10 g or ¼ ounce, and sheet-metal weights below 100 mg or 1 gr. (For a list of abbreviations recognized by this Bureau, see p. 71.)

(*e*) **Case.**—It is recommended that weights of this class, except the rectangular-type 50-pound weights, be provided with a covered case in which they may be kept and carried. A separate pocket lined with velvet, felt, or leather should be provided for each cylindrical weight. The fit must not be tight enough to require any appreciable force in inserting or removing the weights. (See p. 67 in regard to precautions in shipment.)

(*f*) **Lifters.**—Special lifters need not be provided unless desired, because class C weights are intended to be handled with bare hands. However, such lifters, as well as other means of protecting the weights, should be provided if the weights are to be used where they would otherwise become dirty.

(*g*) **Tolerances.**—Weights of this class must be adjusted within the limits of error prescribed for this class on page 21. All weights must be adjusted according to the apparent mass, as this mass would be determined by comparison with brass standards in air. (See "Buoyant force of the air," p. 55.)

2. SEALING AND CERTIFICATION

Weights meeting all the requirements will be stamped with the seal shown in Fig. 8.

FIG. 8.—*Seal for class C weights*

This seal will be impressed on standards tested and approved under this class, unless such standards are so small that this is impracticable

The certificate will state that the weights are correct within the tolerances prescribed for this class.

TABLE 3.—Tolerances for Class C—Commercial Test Weights

[The maximum error allowable on each weight is given in the columns headed "Tolerance"]

Metric		Customary										Carats	
		Avoirdupois		Grains		Apothecary		Ounces troy		Pennyweights			
Denomination	Tolerance	Denomination	Tolerance	Denomination	Tolerance	Denomination	Tolerance	Denomination	Tolerance	Denomination	Tolerance	Denomiantion	Tolerance
kg	mg	lb.	gr.	gr.	gr.	oz. ap.	gr.	oz. t.	gr.	dwt.	gr.	c	mg
20	600	50	10	10 000	1	12	1	1000	10	10 000	10	2500	70
10	400	25	6	5000	1	10	1	500	10	5000	5	2000	60
5	250	20	6	2000	0.5	6	0.5	400	5	4000	5	1000	40
2	150	10	4	1000	0.3	5	0.5	300	5	3000	3	500	30
1	100	8	3	500	0.2	4	0.5	200	5	2000	3	200	20
g		5	3	200	0.15	3	0.4	100	3	1000	2	100	10
500	70	4	2	100	0.10	2	0.3	50	2	500	1.5	50	7
200	40	3	2	50	0.05	1	0.2	40	1.5	400	1.0	20	5
100	30	2	1.5	20	0.03	dr. ap.		30	1.5	300	1.0	10	3
50	20	1	1.0	10	0.02	6	0.2	20	1.0	200	1.0	5	2
20	10	oz		5	0.015	5	0.2	10	1.0	100	0.5	2	1
10	7	10	1.0	2	0.010	4	0.2	5	0.5	50	0.4	1	0.7
5	5	8	0.5	1	0.005	3	0.1	4	0.5	40	0.3	0.5	0.5
2	3	5	0.5	0.5	0.005	2	0.1	3	0.4	30	0.3	0.2	0.3
1	2	4	0.5	0.2	0.0025	1	0.05	2	0.3	20	0.2	0.1	0.2
mg		2	0.3	0.1	0.0020	s. ap.		1	0.2	10	0.2	0.05	0.15
500	1.5	1	0.2	0.05	0.0015	2	0.05	0.5	0.2	5	0.1	0.02	0.10
200	0.7	$\frac{1}{2}$	0.2	0.02	0.0015	1	0.03	0.4	0.15	4	0.1	0.01	0.05
100	0.5	$\frac{1}{4}$	0.1	0.01	0.0015			0.3	0.10	3	0.05		
50	0.35	$\frac{1}{8}$	0.05					0.2	0.10	2	0.05		
20	0.20	$\frac{1}{16}$	0.05					0.1	0.05	1	0.03		
10	0.15	$\frac{1}{32}$	0.05					0.05	0.03				
5	0.10	$\frac{1}{64}$	0.02					0.04	0.025				
2	0.05							0.03	0.025				
1	0.04							0.02	0.020				
								0.01	0.015				
								0.005	0.010				
								0.004	0.008				
								0.003	0.007				
								0.002	0.005				
								0.001	0.005				

VI. CLASS M—HIGH-PRECISION LABORATORY STANDARDS

This class includes weights suitable for reference standards for work of the highest precision, and for use in investigations demanding especially accurate determinations of mass.

1. SPECIFICATIONS

The material of which the weights are composed must be hard, nonmagnetic, and not readily oxidized or corroded by the atmosphere. It must show no traces of porosity. Gold, though a soft material, may be used for plating. Platinum is the material best suited for the smaller weights even as low as 1 mg, but aluminum will serve for those of 20 mg and less. All flat sheet-metal weights must be of a material whose surface need not be protected against oxidation or corrosion.

The entire weight must be a single piece, except for the plating that is permitted on weights of bronze and similar metals. The weights must be free from deep depressions, sharp angles, or other features that jeopardize their constancy. Weights of brass, bronze, or other metals that tarnish on exposure to the atmosphere must be plated with gold, platinum, or other suitable metal. There must be no darkening of the surface, and no spots of any kind may appear when the weights are placed for some time in boiling water and subsequently dried at a temperature of 110° C, as is done in preparing them for test.

The entire surface of the weight shall be smooth and highly polished. If the denomination is marked upon it, the lines of the marking must be shallow and free from burrs and sharp angles. (For list of accepted abbreviations, see Table 8, p. 71.)

Special lifters shall be provided. The parts that come in contact with the weights may be made of wood or ivory, or may be covered with velvet or similar soft material and must be smooth and without sharp edges.

Weights of this class must be adjusted within the limits of error prescribed below. They may be adjusted either according to their true mass or according to their apparent mass as determined by comparison with brass standards in air; that is, according to "weight in air against brass." The latter is generally preferable, chiefly because of the convenience in using small platinum and aluminum weights in connection with brass weights.

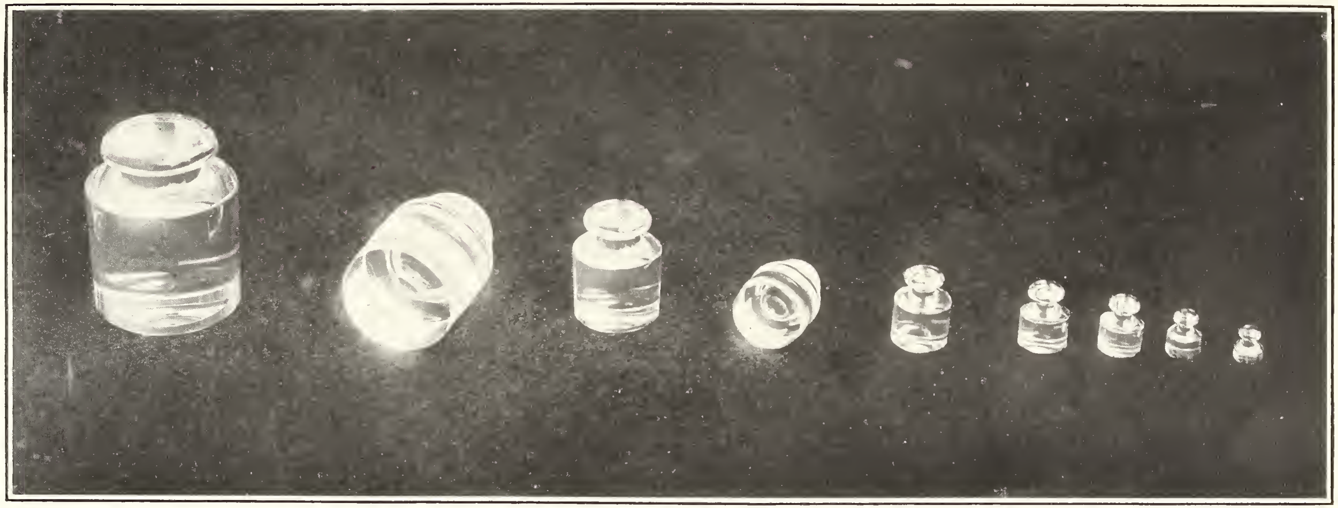

Fig. 9.—*Quartz-crystal weights*

Crystal quartz is occasionally used for weights of the grade covered by Class M. These weights are very constant, but they are easily electrified and have such large volumes that the corrections for the buoyant force of the air are very large when these weights are compared with brass or platinum standards. Platinum-iridium, platinum, or gold-plated bronze (see Figs. 1 and 2) are probably preferable in the majority of cases

2. CERTIFICATION AND SEALING

Class M weights when they incidentally conform to class A, B, or C will be sealed accordingly, when this is requested.

The Bureau will, in general, determine the volumes of all weights except the fractions of a gram. This must be done in order to determine the last figure of the corrections. The volumes, or the mean densities assumed, will be given in the certificate, as will also the atmospheric conditions under which the test was made.

TABLE 4.—Precision of Corrections and Tolerances, Class M: High-Precision Laboratory Standards

[Corrections will be given to the nearest unit in the decimal place indicated.[a] The maximum error allowable on each weight is indicated in the column headed "Tolerance"]

Denomination	Precision of corrections	Tolerance	Denomination	Precision of corrections	Tolerance
kg	g	mg	mg	mg	mg
20	0.01	100	500	0.001	0.05
10	0.01	50	200	0.001	0.05
	mg		100	0.001	0.05
5	1	30	50	0.001	0.03
2	1	10	20	0.001	0.03
1	1	5	10	0.001	0.02
g			5	0.001	0.02
500	1	3	2	0.001	0.01
200	0.1	1	1	0.001	0.01
100	0.1	0.5	0.5	0.001	0.01
50	0.1	0.3	0.2	0.001	0.01
20	0.01	0.2	0.1	0.001	0.01
10	0.01	0.15			
5	0.01	0.15			
2	0.01	0.10			
1	0.01	0.10			

[a] Weights may in certain cases be certified to a higher degree of precision than that indicated in the table, but this involves additional labor and expense. Arrangements for such work should be made by correspondence before the weights are submitted, and the request must state the character of the weight and of the work for which it is needed, and the degree of precision desired. The Bureau will not undertake to determine weights "as accurately as possible" nor to other similarly indefinite degrees of precision.

Two sets of corrections will be given on the certificate:

One of these will refer to the apparent mass as this would be found by comparison with brass standards in air. These corrections will be calculated for air whose density is 1.2 mg per milliliter. (See p. 55 for complete details.)

The apparent mass of a weight, determined as above, is of course equal to the true mass if the volume of the weight, or its density, is equal to that of the standard when the weight is tested.

In determining the other corrections, full allowance will be made for the buoyant effect of the air; that is, the observations will be "reduced to vacuo." These corrections will therefore refer to the true mass of the weights.

When the density of the weight can safely be assumed, the following values will be used. The coefficients of expansion given in the same table will be assumed in all cases.

Material	Density in grams per cubic centimeter	Coefficient of cubical expansion per degree centigrade
Brass or bronze	8.4 at 0° C	0.000054
Platinum or platinum-iridium	21.5 at 0° C	0.000026
Aluminum	2.7 at 0° C	0.000069
Quartz (crystal)	2.65 at 20° C	0.000033

VII. CLASS S—ANALYTICAL AND SIMILAR LABORATORY WEIGHTS

This class includes such weights as are commonly used in physical and chemical laboratories for most of the accurate weighing.

1. SPECIFICATIONS

(*a*) **Material.**—These weights must be made of material that is not rapidly corroded by the air. It must be hard, nonporous, and nonmagnetic. (Nickel, though magnetic, may be used for plating.) The larger weights down to and including 1 g must be of brass or bronze, or of a material having approximately the density of these metals. Below 1 g platinum may be used, and below 50 mg aluminum. All flat sheet-metal weights must be of a material whose surface need not be protected against oxidation or corrosion.

(*b*) **Structure and Form.**—These weights may have any of the common forms, provided they do not show features that might reduce their reliability. There must be no hole on the underside. Screw-knob weights may be tested under this class.

(*c*) **Surface.**—The surface must be smooth and polished. Unless the material of the weights is at least as resistant to atmospheric corrosion as nickel or aluminum, the weights must be plated with metals such as platinum, gold, or nickel, or must be lacquered.

Lacquered Weights.—Lacquer, if used, must be hard, of moderate thickness, and smooth. Lacquer must not be used, however, on "flat" sheet-metal weights. Lacquer absorbs variable amounts of moisture according to the humidity of the air. The variations due to this cause may be appreciable in weights of this class. Since the character of the lacquer varies greatly, definite figures can not be given; but for moderate ranges of humidity, say 40 to 70 per cent, a change of about 0.1 mg may be expected on a 100 g weight, and slightly more on an equivalent summation of smaller weights.

(*d*) **Designation of Value.**—The nominal mass or "value" of each weight must be plainly marked upon it, except in the case of riders and other wire weights. On the largest weight of a set, at least, and on flat sheet-metal weights down to and including 100 mg, this marking should include the name of the unit. For list of abbreviations approved by this Bureau, see page 71. On other weights the number alone will be sufficient.

The lines forming the characters must be shallow, broad, and free from burrs and sharp angles.

Duplicate and triplicate weights of a set must be so marked that they can be distinguished with ease and certainty. A common and satisfactory practice is that of using, in addition to the denomination, one and two conspicuous dots on duplicates, and one, two, and three such dots on triplicates. It is preferable to have such marks on each weight rather than to have one weight entirely unmarked.

(*e*) **Case.**—It is recommended that a dust-proof case or box be provided and that the pockets for the cylindrical weights be lined with some soft material, such as velvet. When a case is provided there must be a separate pocket for each weight and these pockets must be large enough so that no appreciable force is required in removing or inserting the weights. (See p. 67 for precautions in shipment.)

(*f*) **Lifters.**—Special lifters must be provided. The parts that come in contact with the weights may be of wood or ivory, or may be covered with velvet or similar soft material, and must be smooth and without sharp edges.

(*g*) **Tolerances.**—Weights of this class must be adjusted within the tolerances given below. They may be adjusted either according to their true mass or according to their apparent mass as determined by comparison with brass standards in air—that is, according to "weight in air against brass." The latter is generally preferable, chiefly because of the convenience in using small platinum and aluminum weights in connection with brass weights.

2. CERTIFICATION AND SEALING

Class S weights, when they incidentally conform to the requirements of class A, B, or C, will be sealed accordingly when such sealing is requested.

Two sets of corrections will be given on the certificate. One of these will refer to the apparent mass as this would be found by comparison with brass standards in air. These corrections will be calculated for air whose density is 1.2 mg per milliliter. (For details see p. 55.)

In determining the other corrections allowance will be made for the buoyant effect of the air—that is, the observations will be "reduced to vacuo." For this purpose an approximately correct density will be assumed for weights of each different material. These latter corrections will therefore refer to the true mass of the weights.

For weights of brass or of materials of the same density as brass the two sets of corrections are the same. The densities and coefficients of expansion given on page 24 will be assumed for weights of the materials there listed.

TABLE 5.—Precision of Corrections and Tolerances, Class S: Analytical and Similar Laboratory Weights

[Corrections will be certified to the degree of precision indicated in the column headed "Precision of corrections." Where the figure 1 is used the values are rounded off to the nearest unit in the decimal place indicated; where the figure 5 is used the values are rounded off to the nearest 5 or 0. The maximum error allowable on each weight is indicated in the column headed "Tolerances"]

Denomination	Precision of corrections	Tolerance	Denomination	Precision of corrections	Tolerance
kg	g	mg	mg	mg	mg
20	0. 1	100	500	0. 01	0. 05
10	0. 01	50	200	0. 01	0. 05
5	0. 01	30	100	0. 01	0. 05
2	0. 01	10	50	0. 01	0. 03
	mg		20	0. 01	0. 03
[a] 1	5 or 1	5	10	0. 01	0. 02
g			5	0. 01	0. 02
500	1	3	2	0. 01	0. 01
200	1	1	1	0. 01	0. 01
[a] 100	0. 5 or 0. 1	0. 5	0. 5	0. 01	0. 01
50	0. 1	0. 3	0. 2	0. 01	0. 01
20	0. 1	0. 2	0. 1	0. 01	0. 01
10	0. 05	0. 15			
5	0. 05	0. 15			
2	0. 05	0. 10			
[a] 1	0. 05 or 0. 01	0. 10			

[a] Single reference standards of these denominations will be certified to the higher degree of precision indicated. In this case variations in the density of the air may change the values of brass weights by a little more than 1 in the last place given in the corrections. When weights of these denominations are the largest weights of a set, they may also be certified to the higher degree of precision, but this will be done only on special request.

VIII. NOTES ON THE DESIGN AND CONSTRUCTION OF WEIGHTS

1. CLASSES A, B, AND M

(*a*) **Form.**—It is advantageous to fix rather rigidly the design of weights of classes A and B.

Fig. 10 shows satisfactory shapes that are already in use. In order to allow room for the tweezers or other lifters with which weights are handled, the necks of the knobs on the smaller weights

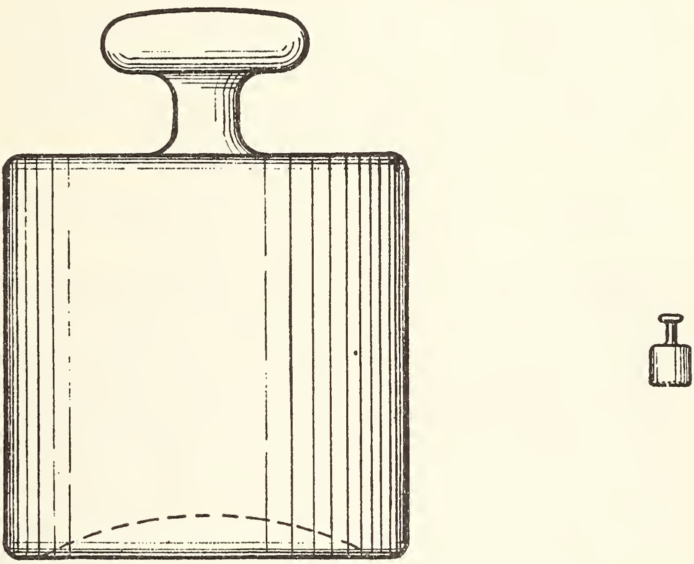

FIG. 10.—*Typical forms: Classes A, B, and M*

This shape of body and a well-rounded knob are required in classes A and B. A wide range of shape is allowed in class M; for example, that shown in Fig. 1, or that shown in Fig. 14, if the adjusting plug is omitted

must be taller in proportion to the body of the weights than on the larger ones.

In machining class A and class M weights the knob, top, and sides should be finished first, then the outer rim of the bottom, and then the central portion of the bottom is hollowed out by an amount equal approximately to the volume of the knob. The preliminary adjusting is completed in the last operation, and if the weights are to be plated they must be left sufficiently light to allow for such plating.

(*b*) **Size.**—The following table shows the value for height and diameter for the bodies of avoirdupois and metric weights of these classes, computed for metal having a density of 8.4 g/cm[3], which is an average value of the density of dense brass or Tobin bronze:

TABLE 6.—Representative Values of Height and Diameter, Classes A and B

Avoirdupois		Metric	
Denomination	Height and diameter	Denomination	Height and diameter
lb.	in.	kg	mm
50	5.94	20	144.7
20	4.38	10	114.9
10	3.48	5	91.2
5	2.76	2	67.2
2	2.03	1	53.3
1	1.61	g	
oz.		500	42.3
8	1.28	200	31.2
4	1.02	100	24.7
2	0.81	50	19.6
1	0.64	20	14.47
½	0.51	10	11.49
¼	0.40	5	9.12
⅛	0.32	2	6.72
1/16	0.25	1	5.33
1/32	0.20		

(*c*) **Plating.**—Plated weights should have a much heavier coating than is generally used for ordinary commercial plated work. There is a distinct advantage also in polishing or possibly burnishing the weights once or twice during the plating, as this tends to make the coating less porous. Serious spotting is probably due in all cases to very minute pores in the metal under the plating, and can not be avoided except by using metal that is free from pores. Tobin bronze has generally been found to be very satisfactory in this regard. Gold plating, though soft, has the advantage that it can be repeatedly polished and plated without difficulty. Besides the usual electroplating process there are the newer methods of plating—by cathode discharge in vacuum, and by the so-called "spray process," described by Schoop[7]—but these methods have not yet attained a sufficient commercial development to be definitely recommended. Such processes are, however, well worth investigation, as they avoid some of the difficulties encountered in the ordinary processes.

2. CLASS C, COMMERCIAL TEST WEIGHTS

(*a*) **Means of Adjustment.**—It is of great practical importance that the means of closing the adjusting hole be such that the weights can be readily readjusted, while permitting such

[7] M. U. Schoop, Metallurgical and Chemical Engineering, 8, pp. 404–406 (July, 1910); 11, pp. 89–91 (February, 1913). John Calder, Journal of the American Society of Mechanical Engineers, July, 1915, p. 378.

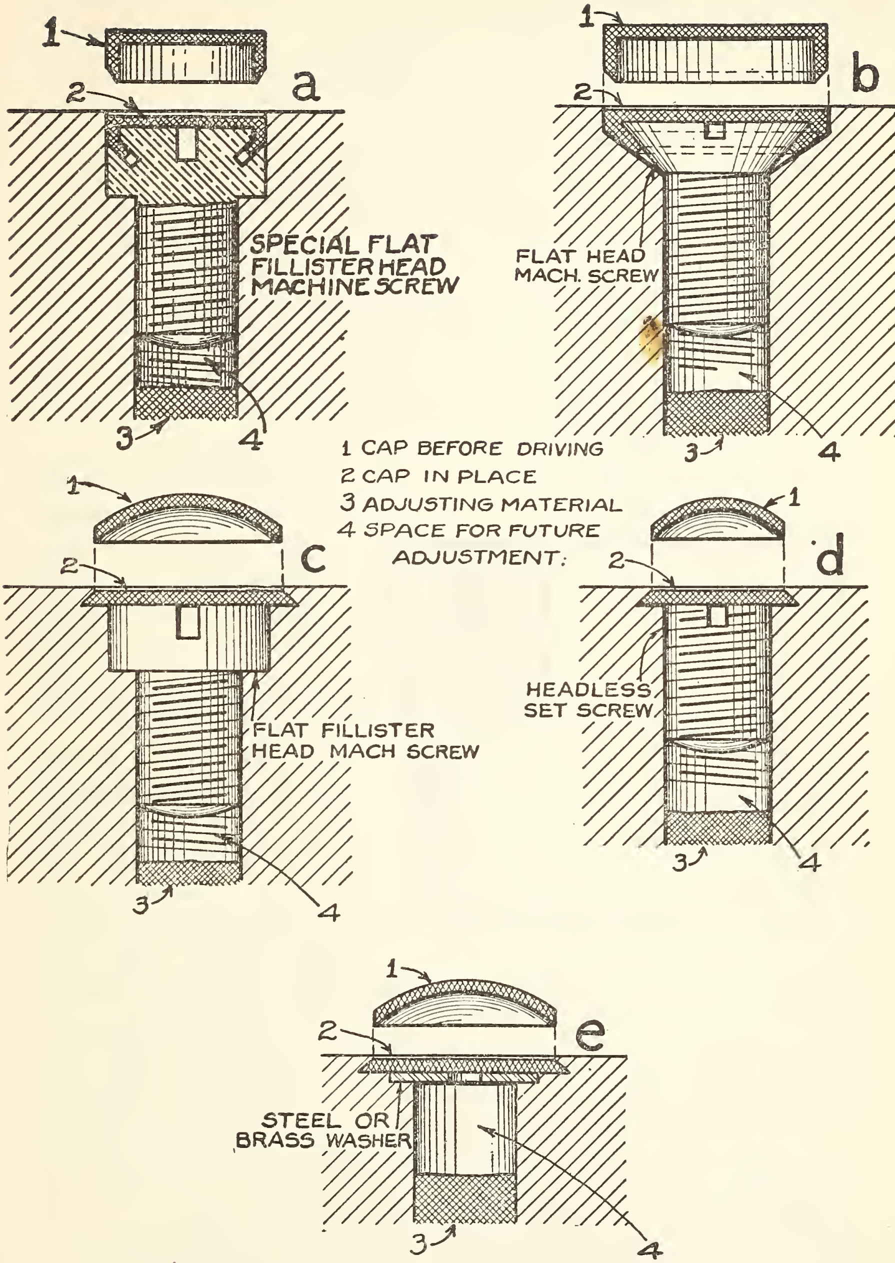

Fig. 11.—*Sealing caps for class C weights*

These are some of the satisfactory methods of closing the adjusting holes. In forms *a* and *b*, especially in *a*, care must be taken in driving the caps not to allow the material to flow up beside the punch, but the skirts of the caps must be long enough to clinch securely. If lead caps for forms *c*, *d*, and *e* are the proper size and shape, they will be secure as soon as they are driven flat readjustment only after mutilation or destruction of the seal. It is essential that the working standards of a local sealer be so constructed that they may be readjusted and sealed as often as there is need.

Five sealing devices are illustrated in Fig. 11. Type *c* is the most satisfactory of these, though the undercutting required in its construction is somewhat more difficult than the shaping of the heads of the screws of types *a* and *b*. In type *c* and in type *d* the head of the screw should be smaller than the hole at the surface of the weight by an amount sufficient to permit of the cap being withdrawn by merely turning the screw without cutting off the edge of the cap.

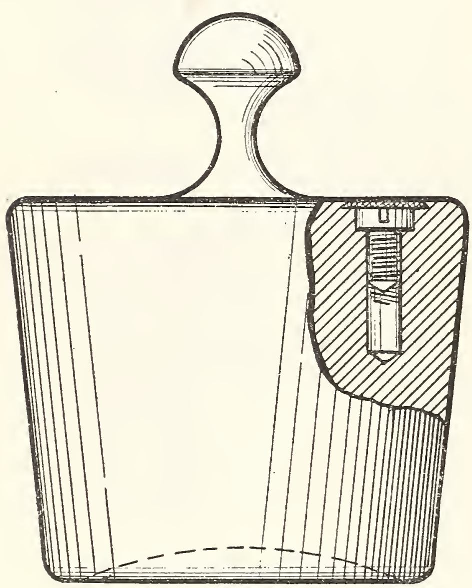

Fig. 12.—*Conical form, class C*

The small base allows weights of this form to be very quickly inserted into the pockets of the carrying base

Careful selection should be made of a design, which should be regularly adhered to, thereby facilitating the supplying at any time of new caps to those desiring to readjust their weights.

(*b*) **Acceptable Forms for the Weights.**—For class C there are a number of acceptable forms. The inverted cone form (Fig. 12) has a distinct advantage in the ease with which it can be replaced in the pocket of the carrying case. The smaller sizes must, however, be broader, resembling the form shown in Fig. 13, so as to provide the necessary stability.

Fig. 13.—*Broad form, class C*

Small weights of class C should be made broad so that they will not continually be falling over and rolling about

As a rule the knobs of class C weights should be taller than those of classes A or B in order to facilitate handling with the fingers.

The knobless cylindrical form, Fig. 14, is a good form for weights of 2 kg or 5 pounds and less. Such weights can be readily stacked, and being cylindrical in shape and having the adjusting hole in the center, they can be formed with a minimum of machine work.

The form shown in Fig. 15, having a cup-shaped depression in the top, has these two advantages: First, that weights of this form can be stacked easily; and, second, that large weights can readily be turned from dense uniform steel such as bars of cold-rolled shafting. This form is not recommended, however, except for indoor use and in places where foreign material is not likely to gather in the depression in the top.

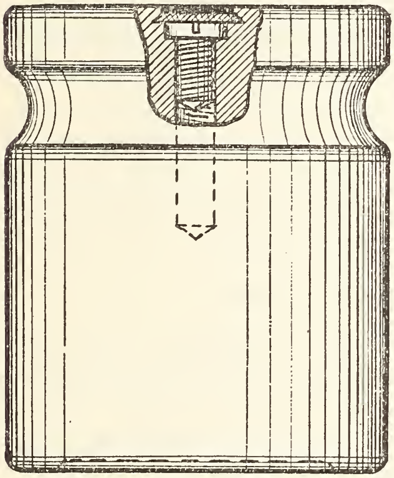

FIG. 14.—*Knobless cylindrical form, class C*

This is a very satisfactory form for weights of 5 pounds or less. It is specially recommended for reducing the cost of making weights of monel metal, high percentage nickel-steels, and other materials that are less easily worked than brass

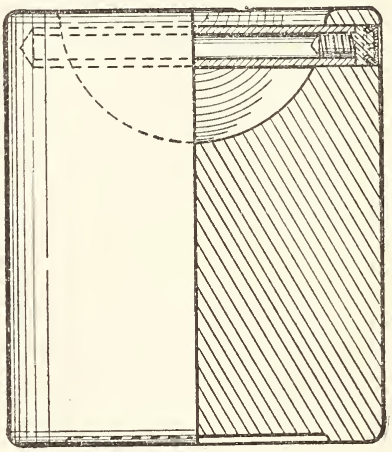

FIG. 15.—*Mint form*

The handle should not extend clear through the weight. This form is suitable for large weights for clean work. A similar form has long been used to some extent for weighing bullion in mints, assay offices, etc.

The forms here illustrated are not to be considered as especially recommended; they merely serve as examples of common and satisfactory types.

(*c*) **Pockets.**—Attention should be given to providing for ease of handling of these weights into and out of the pockets of the case. It is desirable to have the pockets slightly larger than apparently necessary, both to facilitate handling of the weights and to allow for possible shrinkage of the case.

(*d*) **Lifters.**—Special attention is called to the specifications in regard to lifters. Some forms of lifters, especially tweezers, are often well rounded where it is not needed but very sharp on edges that seize the weights. In a few cases metal tweezers with specially roughened surfaces have been noted. The tips of the tweezers used in handling the small weights can not be rounded very much, but can be rounded enough to be a great benefit. The other edges that may come in contact with the weights should be well rounded.

(*e*) **Tolerances.**—Manufacturers and dealers should bear in mind that the tolerances are the largest allowable errors, and it is to be expected that no large proportion of the weights of the set shall have errors near these limits. A properly equipped shop should ordinarily have no difficulty in working within one-half of these limits, larger errors resulting only from the accidental combination of several errors.

Care taken in adjusting, so as to keep the errors of the weights well within the tolerance limit, will reduce delays and eliminate the expense of readjusting weights having such large errors as to require rejection and return to the maker for correction.

Tables of tolerances for each class will be found under the specifications for the corresponding classes. Tolerances for trade weights, in accordance with those given in Circular 61, are given as Tables 9, 10, and 11 on pages 72, 73, and 74.

IX. TESTING OF WEIGHTS

In the small space available it will be possible to give but a brief outline of the common methods used for different grades of weighing and to describe only a few of the simpler methods of procedure for the calibration of weights by intercomparison of the weights of a set.

For the testing of ordinary trade weights by a local sealer the only practicable method is the comparison of each weight directly with a standard. For work of high precision and for testing sets of weights that come within the scientific group, some of the intercomparisons outlined below are preferable.

When this latter method of testing is used there is no gain in the purchase of complete sets of standards if this must be done, as is often the case, at the expense of quality. Probably the best denominations for reference standards for most scientific laboratories are one 100 g or 50 g weight, one 1 g weight, and one 10 mg weight.[8] The 1 g weight should be of platinum, as it is used as the starting point for the calibration of the fractions of a gram, which are usually of platinum.

1. WEIGHING

(*a*) **Zero Reading and Equilibrium Point.**—The phrase "zero point" is often used to designate the center of the scale over which the pointer of a balance moves, or, in general, the place at

[8] It is often advisable to add a smaller weight, generally 1 or 2 mg, for determining the sensitiveness of the balance; or even a small set, from 5 to 1 mg, which will also serve to measure any large differences that may exist among the larger weights.

which the pointer should come to rest without load if the instrument were perfectly constructed and balanced. It is sometimes used to designate the rest point when there is no load on the pans, whether this point is at the center of the scale or not, but this idea is better expressed by the phrase "zero reading."

The phrase "rest point" or "equilibrium point" is then used to indicate the point on the scale at which the pointer would come to rest in any particular case. In very rough testing this may be found by waiting until the pointer comes to rest and then noting the reading, but it is seldom advisable to adopt this method. In careful work the rest point must be calculated from the extreme positions to which the pointer swings on each side of this point. The number of readings taken when the rest point is to be calculated will depend upon the accuracy required and the sensitiveness of the balance. For one determination of a rest point there must be one more reading made on one side than on the other, and all of the readings used must be consecutive. Simple illustrations of the method of calculating the rest point are given in Fig. 16, in the column headed "Equilibrium Points × 2." It should be noted that the figures there given are twice the true rest point. In the later work shown in this figure the factor 2 cancels out.

(*b*) **Sensitiveness—Use of Deflections.**—In any of the following methods of weighing it is often necessary to know the sensitiveness of the balance. This may be found by adding a small weight to one of the pans and noting how far the rest point is deflected from its former position. In weighing it is often preferable, instead of adding small weights to the pans until the rest point is brought to the desired place, merely to note the deflection from this point and to calculate from the sensitiveness the weight that would be needed to bring about a reading at the appropriate rest point. The sensitiveness will, in general, be different at different loads, and as it may change with use of the balance and even with change of temperature, especially in very sensitive balances, it must be redetermined from time to time. In precision testing it is advisable to determine the sensitiveness at each weighing. On the other hand, a balance suitable for testing ordinary trade weights, being less sensitive, should not change noticeably from day to day. Illustrations showing the use of the deflection of the pointer in weighing by substitution and by transposition are given in Figs. 17 and 18.

28629°—18——3

Test .... **Date:** .... **Balance:** .... **Observer:** ....

Pans: Left	Pans: Right	Ther. Bar. Hygr.	Scale Readings: Left	Scale Readings: Right		Equilibrium Points × 2	Scale Difference × 2	Computation—Remarks	Observed Difference
(5 lb.)	B 5 lb	°	~~7.~~	10.	$\frac{10+12}{2}$ =	9 } ‡			a
		.	9.	12.		11		$\frac{3\times\frac{1}{2}}{2}$ or $3\times\frac{\frac{1}{2}\times 1}{8}$ =	— .19*
			.	.				Correction for B 5 lb	+ .23
			.	.		20.		Correction for (5 lb.)	+ .04 gr.
"	"	T. °	~~6.~~	16.		8			
		mm	8.	14.	$\frac{16+14}{2}$	15			
		.	.	.					
			.	.		23.	3.		
"	"		~~4.~~	10.	$\frac{10+8}{2}$	6			
			6.	8.		9			
	grain	Rel. Hum.	.	.					
Δ = (.)	Δ = θ(1.)	%	.	.		15.	3.		
		°	.	.					a
		.	.	.					
			.	.					
			.	.		.			
		T. °	.	12.	$\frac{7+8}{2}$ =	7.5			
		mm	7.	11.	$\frac{12+11+10}{3}$ =	11.0			
		.	8.	10.					
			.	.		18.5	.		
"	"		.	.				* Calculated to the nearest hundredth only.	
			.	.					
		Rel. Hum.	.	.				‡ This work is generally done mentally and only the final result written in.	
^ = (.)	Δ = (.)	%	.	.			.		

Fig. 16.—*Record of weighing*

This shows the calculation of equilibrium points, and the computation of the result of a transposition weighing. The factor 2 in "Equilibrium Points × 2" and in "Scale Difference × 2" cancels out in the subsequent division. The division by 2 that appears under "Computation—Remarks" enters because the transposition has made the true scale difference twice the difference between the weights

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Test .... Date .... Bal. .... Observer ....

Ther. ....°.... Hygr. .... Bar. ....mm.... at ....°....

	Pans Left	Pans Right	Scale Readings Left	Scale Readings Right	Equilib. Points × 2	Computations Observed. Difference
Transposition	Σ(50)	(50g)	6.8	10.1		a .10 × 2.5 / 2.85 =
			6.8	.		
			.	.	16.90	-.09
			.	9.0		-6.00
	C″	C″	7.8	9.0		-6.09 mg
	D 10, mg 2		.	.	16.80	
	”	”	.	11.4		
			8.3	11.3		
	D 5. mg	.	.	.	19.65	
Substitution			.	.		a 1.15 × 50. / 3.0
	C	B 5 kg.	~~7.0~~	14.1		
		Δ20 mg	7.0	14.0	21.05	+19.2
			.			+20.0
	C	(5kg)	~~6.0~~	13.9		Correction to B 5kg. = +36.0
			6.1	13.7	19.90	+75.2 mg.
	”	”	.	.		
			~~8.9~~	14.0		
	Δ50. mg	.	9.0	13.8	22.90	
			.	.		a . × .
			.	.		.
			.	.	.	.
			.	.		
			.	.		
			.	.	.	
	”	”	.	.		
			.	.		
			.	.		
	.	.	.	.	.	

11—576

Fig. 17.—*Record of weighings—condensed form*

This shows observations and computations for weighings by transposition and also by substitution. In these examples small weights were used in addition to the deflection of the pointer. Such readings as those given in the first weighing would indicate either that the zero reading of the balance was 13.43 (6.72 in actual scale divisions) or that the arms of the beam were very unequal. With the usual adjustment of a balance and the middle point of the scale taken as 10, the first rest point would indicate that after interchanging the weights, small weights would be needed on the right-hand pan instead of on the left (*c*) **Three Methods of Weighing.**—There are three distinct methods of weighing. These will be designated in the following discussion as direct weighing, weighing by substitution, and weighing by transposition.

Direct Weighing.—In direct weighing the weight to be tested is placed on one pan of the balance and the standard on the other. The pointer may then be brought to the zero reading by adding small weights to one of the pans, or by the use of a rider on the beam; or the weight necessary to bring the pointer to the zero reading may be calculated from the deflection of the pointer and the sensitiveness.

To secure satisfactory results by this method the arms of the balance must be accurately equal, if it is nominally an equal-arm balance, or accurately in the proper ratio if it is an unequal-arm balance. The latter case will not be considered here, as such balances are not extensively used for testing purposes. The lengths and ratio of the arms may change with age and use, though these lengths are not so subject to change as the sensitiveness. Therefore, the ratio of arms of a balance that is to be used for direct weighing must be tested occasionally. If two accurately equal weights are available, this test may be made by noting whether the rest point is the same with these weights applied, as when there is no load in the pans. Two weights not accurately equal may be used by noting the rest point with one on each pan, then interchanging them and again noting the rest point. The point midway between these is the true rest point, or the point where the pointer would come to rest if the weights had been equal.[9] The error due to inequality of arms is then the weight required to bring the rest point (with equal loads on the pans) back to the zero reading. This error is proportional to the load.

The direct method of weighing is so much simpler and quicker that it is worth while for a sealer to demand that his portable balance at least have its arms so nearly equal as to make interchanging of weights on the pans unnecessary in his work. For balances used in testing class C weights this equality of arms is not so important, since other methods can be used without serious inconvenience. In work of higher precision it is generally necessary to use one of the other methods of weighing.

For most, if not all, chemical analysis and for some other scientific and technical operations where accuracy of proportion

[9] This is not theoretically exact, but is sufficiently accurate for testing any balance the arms of which are approximately equal.

rather than actual quantity is needed, direct weighing may be used even when the arms of the balance are not very accurately equal, because the errors are proportional to the load, and therefore the proper proportion will be maintained in the results.

Substitution Weighing.—This is the simplest method of eliminating the effect of inequality of the arms of a balance, and it is a method applicable either to a balance having nearly equal arms or to one having one arm several times as long as the other.

The best order of procedure for testing by this method is as follows: Place the standard on one pan and counterpoise weights on the other pan sufficient to balance it approximately; note the position of the pointer on the scale; remove the standard and substitute the weight to be tested. If it is necessary to place additional weights with the weight being tested, in order to bring the rest point to its former position, then it is evident that the weight being tested is lighter than the standard by the amount of the weight added, and vice versa. If the weight is to be adjusted, this can be done at once by adding or subtracting sufficient adjusting material without again using the standard. Nevertheless, it is best to replace the standard after the adjustment is completed in order to be sure that the rest point of the balance has not changed appreciably during the work.

If the balance used is known to be very nearly constant, several weights of the same denomination may be tested before replacing the standard. How far this may be carried can be ascertained in no way except by experience with each individual balance and knowledge of the conditions under which it is being used.

Transposition Weighing.—When the arms of a balance are nearly equal, but not sufficiently equal for direct weighing, the method of transposition furnishes a more accurate comparison than that of substitution. It requires about the same amount of time for the observations, but the error of the weight being tested is not shown in quite so direct a manner.

In weighing by transposition the weight to be tested is first placed on one pan and the standard on the other. It is expedient then to bring the rest point near the center part of the scale, by adding any convenient material (an extra set of cheap weights may be used) to one of the pans; this material may be left in place and disregarded during the rest of the weighing. The rest point is then noted, and the standard and weight under test are transposed, or interchanged. Known weights may then be added to the "light" side of the balance to bring the rest point back to its former position. The difference between the standard and the weight under test is then one-half of these added weights. In work of high precision, however, it is generally impracticable to do this. In such work, after the standard and weight under test have been transposed, the new rest point is noted and the amount of added weights that would be needed is calculated from the sensitiveness obtained as mentioned in (*b*) above and illustrated in Figs. 16 and 17. Whether the weight tested is too light or too heavy can be readily ascertained from the direction of motion of the rest point.

Weighing by transposition is recommended for work of high precision, in which it is also desirable to calculate the rest point from several swings of the pointer. In other cases substitution is generally to be preferred.

Illustrations of the calculations, which are in general very simple, are given in Figs. 16 and 17.

2. RECORDING

Figs. 16 and 17 show forms used at this Bureau for recording the readings taken in weighing by transposition or substitution. Complete weighings are shown therein to illustrate the methods of recording and calculating. Both forms shown have essentially the same arrangement. The one in Fig. 17 is somewhat condensed, that in Fig. 16 being used when more room is needed for auxiliary calculations, such as buoyancy corrections. In taking readings of the swings of the pointer the center of the scale (often called the zero point) is numbered 10 or 100 in order to avoid having negative readings on the left of this point.

It should be noted in these figures that the denomination of the standard is preceded by a letter which indicates the set to which it belongs, while that of the weight being tested is inclosed in parentheses. These special designations serve not only to distinguish the standard from the weight being tested, but also to prevent them from becoming confused with the numbers used in the calculations. The corrections to the nominal values are used, instead of the total values of the weights, in order to avoid a confusing succession of nines or zeros. The plus sign for a correction indicates that the mass of a weight is greater than its nominal value, the minus sign that it is less than the nominal value. In calculations these signs may be treated in the ordinary algebraic manner.

3. TOLERANCE TESTING

When it is sufficient to know that the error of a weight is less than a prescribed amount, as is nearly always the case in commercial testing, the deflection of the pointer may in some cases be used to advantage. However, the most direct and safe method of tolerance testing is that of placing a weight equal to the tolerance on the pan with the weight being tested if the weight is too light, the standard being on the other pan, or on the pan with the standard if the weight being tested is too heavy. This shows directly whether or not the error is greater than the tolerance.

In using deflections it is not generally necessary to know the sensitiveness very closely, but merely that a certain weight is "correct" if it does not deflect the pointer beyond a certain position.

4. THE TEST OF A SINGLE WEIGHT

In testing a single weight it is generally compared with a standard of the same denomination. A weight can, however, be tested by comparing it with a standard of twice its mass, using an additional weight equal to it. The two together are compared with the standard and the correction for their sum (which is the sum of their corrections) is thus determined. The weights are then compared with each other and their difference determined. From the sum and differences of the corrections the correction for each weight can readily be computed, and proper attention to the algebraic signs will secure the proper sign for the individual corrections.

There are two distinct advantages in this latter plan. In the first place, whatever uncertainty or error there may be in the correction for the standard will be divided by 2 in calculating the correction for the weight tested. In the second place, the additional weight used may be one whose correction is known at least approximately, and the redetermination of this correction gives a valuable check on the accuracy of the work.

5. THE TEST OF SEVERAL EQUAL WEIGHTS

(*a*) **Methods and Computations.**—When a large number of equal weights are to be tested, and the highest precision of which the balance is capable is not necessary, the substitution method is greatly to be preferred for comparing each weight individually with the standard. For more accurate work, and especially in order to secure checks that will detect any serious error in recording, each weight may be compared separately with the standard, and then with each of the other weights. This is not practicable when testing a very large number of equal weights, but in that case the weights may be divided into groups and each group treated in the above manner. Although the calculation of the corrections for each weight from such a series of observations would ordinarily be a long process, it happens that even the solution by the method of least squares, which gives the most reliable results, can be put into very simple form for an intercomparison of a few weights. The formulas for two of the simplest cases are given below.

Let w_1 represent the first weight, w_2 the second, and s the standard. Then if two weights are to be tested, three weighings can be made, which may be represented by w_1-s, w_2-s, and w_1-w_2. The difference must be taken in the order indicated. For example, if s is heavier than w_1, its mass is greater than w_1, and therefore the difference will be negative. Let the differences found be represented by a_1, a_2, and a_3. The complete weighings may then be represented as:

$$w_1-s\ =a_1$$
$$w_2-s\ =a_2$$
$$w_1-w_2=a_3$$

Using all three weighings the best values for w_1-s and w_2-s are shown by the method of least squares to be

$$w_1-s=\tfrac{1}{3}\ (2a_1+a_2+a_3)$$
and
$$w_2-s=\tfrac{1}{3}\ (a_1+2a_2-a_3)$$

Thus, knowing the correction for s, the errors of w_1 and w_2 may be found algebraically. Similarly for three weights to be tested we have the weighings

$$w_1-s=a_1$$
$$w_2-s=a_2$$
$$w_3-s=a_3$$
$$w_1-w_2=a_4$$
$$w_2-w_3=a_5$$
$$w_1-w_3=a_6$$

and from these we find—

$$w_1-s=\tfrac{1}{4}\ (2a_1+a_2+a_3+a_4+a_6)$$
$$w_2-s=\tfrac{1}{4}\ (a_1+2a_2+a_3-a_4+a_5)$$
$$w_3-s=\tfrac{1}{4}\ (a_1+a_2+2a_3-a_5-a_6)$$

Thus, knowing the correction for s, the errors of w_1, w_2, and w_3 may be determined algebraically.

Similar solutions may be made for a larger number of weights, but the number of weighings increases so rapidly that it is seldom advisable to use this scheme for more than three weights. A larger number may be divided into groups as suggested above.

A slight modification of the above allows the use of more than one standard. This gives added assurance against possible error or change in the standards, and failure to apply the correction for the standard is shown by the residuals (see below), since the corrections for the two standards are, in general, considerably different. This modification consists in applying the correction for the standard in each weighing in which it is used; s then becomes the nominal value of the standard, and w-s the correction for w.

The following example (Fig. 18) will illustrate a convenient arrangement of the calculations for comparing three equal weights with a standard. For a discussion of the computed values and the residuals see the following paragraph:

	Observed	Computed	Residual
(1 pound)$_1$ − 1 pound$_7$	$=a_1=-0.014$ grain	−0.014	0.000
(1 pound)$_2$ − 1 pound$_7$	$=a_2=+.003$	+.003	.000
(1 pound)$_3$ − 1 pound$_7$	$=a_3=-.014$	−.015	.001
(1 pound)$_1$ − (1 pound)$_2$	$=a_4=-.019$	−.017	.002
(1 pound)$_2$ − (1 pound)$_3$	$=a_5=+.016$	+.018	.002
(1 pound)$_1$ − (1 pound)$_3$	$=a_6=+.004$	+.001	.003

(1 pound)$_1$

	+	−
$+2\ a_1=$		0.028
$+\ a_2=$	0.003	
$+\ a_3=$		.014
$+\ a_4=$		.019
$+\ a_6=$	.004	
	.007	.061
		.007
sum=		−.054

¼ sum=(1 pound)$_1$ − 1 pound$_7$ = −.014

Correction for 1 pound$_7$ = −.033

Correction for (1 pound)$_1$ = −.047 grain

(1 pound)$_2$

	+	−
+ a_1=		0.014
+2 a_2=	0.006	
+ a_3=		.014
− a_4=	.019	
+ a_5=	.016	
	.041	.028
	.028	
sum=+	.013	

¼ sum=(1 pound)$_2$−1 pound$_7$=+ .003
Correction for 1 pound$_7$ =− .033

Correction for (1 pound)$_2$ =− .030 grain

(1 pound)$_3$

	+	−
+ a_1=		0.014
+ a_2=	0.003	
+2a_3=		.028
− a_5=		.016
− a_6=		.004
	.003	.062
		.003
sum=−		.059

¼ sum=(1 pound)$_3$−1 pound$_7$=− .015
Correction for 1 pound$_7$ =− .033
Correction for (1 pound)$_3$ =− .048 grain

FIG. 18.—(*Above and preceding page*)

Arrangement of computations for comparison of four equal weights, three unknown and one standard. The standard is designated as 1 pound$_7$.

(*b*) **Residuals.**—After the final correction for each weight has been obtained the values of w_1-s, w_2-s, etc., are computed. Since the weighings can not be made with perfect accuracy, these computed values will, in general, differ slightly from the observed values a_1, a_2, etc. The differences are called the residuals and might be considered as representing the errors of the weighings if the final corrections were perfectly accurate. If any residual is larger than the error to be expected in a weighing, it is good evidence that a mistake has been made either in the weighings or in the computations. If the mistake is in a weighing, the residual for that weighing will generally be much larger than for the others, but with some of the computation methods given in this circular one or two other residuals may sometimes be made practically as large by the one mistake. If a very large number of weighings were made for a few weights, the accuracy of the weighings could be estimated from the residuals, but with the few weighings used in all of the intercomparisons shown in this circular, the residuals serve only to indicate serious mistakes. However, if most or all of the residuals are unusually large and there is no error in recording or in the computations, it is safe to assume that some condition has rendered the weighings less accurate than usual.

6. INTERCOMPARISON OF THE WEIGHTS OF A SET

(*a*) **Methods and Computations.**—When the weights to be tested form what is ordinarily called a set, then instead of comparing each weight with a standard, the largest weight or the sum of several of the weights may be compared with a standard and the weights of the set then intercompared. From these weighings the value of each individual weight can then be calculated as described later.

Especial advantages of this mode of procedure are: In the first place it does not require the use of a standard of each denomination; in the second place, it makes it possible to base the determination of the values of the small weights on a much larger standard, thus practically eliminating so far as these small weights are concerned any unknown uncertainty or error of the standard used. It is best, as a rule, to base the work on as large a standard as possible, using other standards in the comparisons as checks only. In addition, the methods given in this section have the advantage that a sufficient number of combinations are used so that the checks and residuals serve to show not only when a mistake has been made in the calculations but also when one has been made in the weighings, and in general to indicate in which weighing it has been made. (See paragraph above on "Residuals.") The results are obtained from a "least-square" solution of the observations and are therefore the best to be obtained from any given set of observations, but the use of the forms requires no knowledge of "least squares"—merely sufficient mathematical training to use the positive and negative signs properly.

The first two forms shown, 52 C and 52 B (Figs. 19 and 20), have been filled in from one of the tests made at this Bureau, as this may assist in making clear the use of the forms. The first equation on these sheets is for the value of the summation 100 g, generally indicated as Σ (100 g). This may be obtained by simply comparing it with a 100-g standard, or if the set contains a 100-g

Sheet 1.

MASS COMPUTATION

COMBINATION: 50, 20, 10, 10, 5, Σ 5

DEPARTMENT OF COMMERCE
BUREAU OF STANDARDS
Form 52 C

Original Record: Vol. ...., *pp.* .... *to* ...., *inclusive* ....

Computer: .... *Date:* .... *B. S. Test No.* ....

		Observations.
(50) + Σ (50) =	100g + .331mg	[ ]
(50) − Σ (50) =	+ .351	[ ]
(50) =	50 + .341	
Σ (50) =	50 − .010	= K. [Computed: −.010]

	Observed.	Obs. − Cal.
(20) − (10)₁ − (10)₂ + (5) − Σ (5) = a_1 =	+ .025mg	.001
(20) − (10)₁ − (10)₂ − (5) + Σ (5) = a_2 =	+ .088	.002
(20) − (10)₁ − (5) − Σ (5) = a_3 =	+ .039	.003
(20) − (10)₂ − (5) − Σ (5) = a_4 =	+ .101	.002
(10)₁ − (10)₂ + (5) − Σ (5) = a_5 =	+ .032	.002
(10)₁ − (10)₂ − (5) + Σ (5) = a_6 =	+ .093	.001
(10)₁ − (5) − Σ (5) = a_7 =	+ .047	.001
(10)₂ − (5) − Σ (5) = a_8 =	− .006	.007

	+	−		+	−		+	−
$-a_1$	.	.25	$-a_1$	.	.25	$+4a_1$	.100	.
$-a_2$	.	.88	$-a_2$	.	.88	$+4a_2$	352	.
$-a_3$	.	.39		.	.	$+4a_3$	.156	.
	.	.	$-a_4$	.	.101	$+4a_4$	404	.
$+a_5$	.32	.	$-a_5$	.	.32	$+2A_1$	.40	.
$+a_6$	.93	.	$-a_6$	.	.93	$+2A_2$	.	.690
$+a_7$	.47	.		.	.	$+A_3$	.	.305
	.	.	$+a_8$	.	6	$+A_4$	.	.57
	.172	.152		.	.345	Check	1.052 = 1.052	
	.152	.		.	.	Σ (5)		
A_1	+.020	.	A_2	.	−.345		+	−

	+	−		+	−		+	−
						$-4A_1$	.	.80
$+a_1$	.25	.	$-a_1$	.	.25	$-4A_2$	1.380	.
$-a_2$	.	88	$+a_2$	.88	.	$-3A_3$	.915	.
$-a_3$	.	39	$-a_3$	.	.39	$-3A_4$	.171	.
$-a_4$	.	.101	$-a_4$	.	.101		2466	.
$+a_5$	.32	.	$-a_5$	.	.32		.80	.
$-a_6$		.93	$+a_6$	.93	.	Sum=S	+ 2.386	
$-a_7$	.	.47	$-a_7$	.	.47	$30A_4$	− 1.710	
$-a_8$	6	.	$-a_8$	6	.	$S + 30A_4$	+ 0.676	
	.063	.368		.187	.244	$\frac{1}{240}(S + 30A_4)$	+ .003	
	.	63		.	.187	$\frac{1}{10}$ K	− .001	
A_3	.	−.0305	A_4	.	−.057	Σ (5)	+ .002	

(20)		(10)₁		(10)₂		(5)	
$\frac{1}{60}$ S	+ .0398	S	+ 2.386	S	+ 2.386	S	+ 2.386
$\frac{2}{5}$ K	− .004	$20A_1$	+ 400	$20A_2$	− 6.900	$30A_3$	− 9.150
	+ .036	Sum	+ 2.786	Sum	− 4.514	Sum	− 6.764
		$\frac{1}{120}$ Sum	+ .023	$\frac{1}{120}$ Sum	− .038	$\frac{1}{240}$ Sum	− .028
		$\frac{1}{4}$ K	− .002	$\frac{1}{4}$ K	− .002	$\frac{1}{10}$ K	− .001
			+ .021		− .040		− .029

FIG. 19.—*Computation form (52 C)*

Intercomparison of weights of a set, series 50, 20, 10, 10, 5, $\Sigma 5$. The weighings are indicated in the first two equations and in the eight equations containing a_1 to a_8. The corrections for (50) and for Σ(50) are, respectively, one-half the sum and one-half the difference (first minus second) of the results of the first two weighings; the other computations are indicated below. Numbers from a test of a set of analytical weights have been inserted as a help in the following indicated operations

Sheet 2

MASS COMPUTATION SHEET

Combination: 5, 2, 2, 1

DEPARTMENT OF COMMERCE
BUREAU OF STANDARDS
Form 52 B

Weights submitted by ....

Original Record: Vol................., *pp.* .... *to* ...., *inclusive.*

Computer: .... *Date:* .... *B. S. Test No.* ....

		Mean.	Observations.
(5) + (2)₁ + (2)₂ + (1) = 10	−		[ ]
(5) − (2)₁ − (2)₂ − (1) =	−	.	[ ]
(5) = 5	−	.	
(2)₁ + (2)₂ + (1) = 5	+	.002	= W [From Sheet (1)]

	Observed.	Obs.— Cal.	$(O.—C.)^2$
(1) − Σ(1) = a_1 =	− .083mg	.004	.
(1) − (2)₁ + Σ(1) = a_2 =	+ .104	.001	.
(1) − (2)₂ + Σ(1) = a_3 =	+ .129	.001	.
(2)₁ − (2)₂ = a_4 =	+ .015	.010	.
(1) + (2)₁ − (2)₂ − Σ(1) = a_5 =	− .048	.006	.
(1) − (2)₁ + (2)₂ − Σ(1) = a_6 =	− .107	.003	.

	+	−			
			−	.83	a_1
− a_2	.	.104	−	.155	$a_5 + a_6$
+ a_3	.129	.	−	.238	Sum
+2 a_4	.30	.	−	.79	½ Sum = S
+ 2 a_5	.	.96	−	.159	2 S
− 2 a_6	.214	.	+	.233	$a_2 + a_3$
	.373	.200	+	.74	2S + a_2 + a_3 = m
	.200	.	+	.8	4 W
Sum.	.173	.	−	.66	4 W − m = M
5 × Sum.	.865	.	+	.124	N
(5 × Sum) ÷ 7 = N	+.124	.	+	.006	$\frac{1}{10}$ (M + N) = (2)₁
W	+ .2		−	.019	$\frac{1}{10}$ (M − N) = (2)₂
m	+ .74		+	.015	= (1)
W + m = 5 (1)	+ .76		+	.2	Check: Sum = W
(1)	+ .015				
− S	+ .79				
(1) − S = Σ(1)	+ .094				

Note: Σ(1) is V1g
Check: Cr. V1g = +.100

FIG. 20.—*Computation form (52 B)*

Intercomparison of weights of a set, series 5, 2, 2, 1, Σ1. The numbers inserted in this form continue the work shown in Fig. 19. This form applies to sets in which the (2) is duplicated and in which it is not desired to divide the Σ(1) into (0.5) and Σ(0.5) as would be required by the use of the form shown in Fig. 22 (52 D). If (2) is not duplicated and (1) is duplicated or triplicated, then this form may be replaced by that shown in Fig. 21 (52 A) weight, the (100 g) and Σ (100 g) may be compared with the standard with and each other as described above for the testing of two equal weights. In these first equations it should be noted that the numbers not inclosed in parentheses are merely nominal values. In the later portion of the sheets they do not appear, and the final values calculated are the corrections to the various weights. The second equation indicates the next weighing to be made, and from these first two equations the corrections for the 100-g and 50-g weights are calculated. Special care should be taken in determining these values of (50 g) and Σ (50 g), as there are no residuals to expose immediately any mistake made. The calculation may be checked by computing Σ (100) and (50) − Σ (50) from the values found. Sometimes the work can be so arranged that the Σ (50) is involved in a preceding inter-comparison, as in the case of the Σ (5) of the next sheet (Fig. 20), the value for which was determined on the first sheet. The next weighings are indicated in the following series of equations. Each of these weighings determines the difference between two weights or groups of weights. The weights to be grouped together are not always printed together, but can readily be distinguished by the plus and minus signs. The columns headed "Obs. − Cal." or $(O.-C.)^2$ are for the residuals. The balance of the computation consists in merely following the operations indicated. Practically all of the operations can be done mentally and all results entered directly on the computation sheet.

The quantities marked "Check" or "Computed" are always computed from the final corrections, and serve to check the accuracy of the computations. The weight marked V1g on the second sheet is a known weight inserted partly in order to complete the number of required weights, but a known weight was chosen in order to serve as a further check on the latter part of the calibration.

The final results of this calibration may be summarized as follows:

Designation:	Correction, mg	Designation:	Correction, mg
(50 g)	+0.341	(5 g)	+0.029
(20 g)	+ .036	$(2\ g)_1$	+ .006
$(10\ g)_1$	+ .021	$(2\ g)_2$	− .019
$(10\ g)_2$	+ .040	(1 g)	+ .015

Each sheet is applicable to any multiple or submultiple of the denominations indicated on the sheet. The decimal multiples are most commonly used, but it is sometimes convenient to use binary multiples, especially in testing avoirdupois weights.

No arrangements have been made for the sale of these printed forms, but they can readily be copied or printed. Sample copies will be supplied to persons interested in them.

It is sometimes convenient to base the calibration on a standard substituted in place of one of the weights indicated in the equations. This is illustrated in the work inserted on Form 52*A* (Fig. 21). Here the weight corresponding to $(1)_2$ was the standard. In making the computation the ordinary procedure was followed until the unknown value of 4 *K* was needed to complete the computations. The known correction for 1 kg_7, −2.0 mg, was then entered at the bottom of the column headed 1 kg_7. The sum of the quantities above 4 *K* was then found and the value of 4 *K* necessary to give −2.0 as a correction to 1 kg_7 was easily determined and entered. This furnished the missing value of 4 *K* and 8 *K* to the other columns and the computation was completed. Another method of procedure would be to assume that the correction to 1 kg_7 was zero and enter 0.00 in place of −2.0. A further correction of − 2.0 mg per kg is then added to the corrections computed as before. For example, we have by application of the first method—

for (2 kg) a correction +1.0 mg

by the second method we would have:

for (2 kg) a correction of $+5.0+2\ (-2.0) = +1.0$ mg

The first method is generally preferable.

(*b*) **Check equations.**—As previously mentioned, mistakes usually disclose themselves by causing a large residual. The residuals, however, are not obtained until after completing the computations. By means of "check equations" mistakes in the weighings can generally be discovered before the computation is started. These equations express the relations that exist between the observations of a given series.

The check equations for the sheet 52*C* and their application to the accompanying data appear as follows:

Equations:	Particular values
$a_1-a_2=a_5-a_6$	−0.063 = −0.061
$a_1-a_3=a_5-a_7$	− .014 = − .015
$a_4-a_1=a_6+a_8$	+ .076 = + .087
$a_4-a_2=a_5+a_8$	+ .013 = + .026
$a_3-a_4=a_8-a_7$	− .062 = − .053
$a_3-a_2=a_7-a_6$	− .049 = − .046

The agreement among the particular values is sufficiently good to indicate that no mistake was made and that the weighings are

MASS COMPUTATION SHEET

Combination: 5, 2, 1, 1, 1

Department of Commerce
BUREAU OF STANDARDS
Form 52 A

Weights submitted by ....

Original Record: Vol. ...., *pp.* .... *to* ...., *inclusive.*

Computer: .... *Date:* .... *B. S. Test No.* ....

Based on a Standard, 1 kg₇, that takes the place of (1)₂ in the equations

	Mean.	Observations.
$(5\ )+(2\ )+(1\ )_1+(1\ )_2+\Sigma(1\ )=10$	– .	[ ]
$(5\ )-(2\ )-(1\ )_1-(1\ )_2-\Sigma(1\ )=$	– .	[ ]
$(5\ )=5$	– .	
$(2\ )+(1\ )_1+(1\ )_2+\Sigma(1\ )=5$	– = K	

	Observed.	Obs. – Cal.	(O. – C.)².
$(1\ )_1-(1\ )_2=a_1=$	+4.0 mg	.0	.
$(1\ )_1-\Sigma(1\ )=a_2=$	+ .7	.1	.
$(1\ )_2-\Sigma(1\ )=a_3=$	–3.5	.1	.
$(1\ )_1+(1\ )_2-(2\ )=a_4=$	–1.0	.0	.
$(1\ )_2+\Sigma(1\ )-(2\ )=a_5=$	–1.4	.2	.
$(1\ )_1+\Sigma(1\ )-(2\ )=a_6=$	+2.3	.1	.

	(1)₁ +	(1)₁ –		(1)₂ = 1 kg₇ +	(1)₂ = 1 kg₇ –		Σ(1) +	Σ(1) –
$+5a_1$	20.0	.	$-5a_1$	.	20.0	$0a_1$		
$+5a_2$	3.5	.	$0a_2$			$-5a_2$	.	3.5
$0a_3$		.	$+5a_3$	.	17.5	$-5a_3$	17.5	.
$+3a_4$	.	3.0	$+3a_4$	.	3.0	$-2a_4$	2.0	.
$-2a_5$	2.8	.	$+3a_5$	.	4.2	$+3a_5$	.	4.2
$+3a_6$	6.9	.	$-2a_6$	.	4.6	$+3a_6$	6.9	.
$+4K$	9.3	.	$+4K$	9.3		$+4K$	9.3	.
	42.5	3.0		9.3	49.3		35.7	7.7
	3.0	.		.	9.3		7.7	.
Sum	39.5	.	Sum	.	40.	Sum	28.0	.
$\frac{1}{20}$sum	2.0		$\frac{1}{20}$sum	Known →	2.0	$\frac{1}{20}$sum	1.4	

	(2) +	(2) –
$-4a_4$	4.0	.
$-4a_5$	5.6	.
$-4a_6$	.	9.2
$+8K$	18.6	.
	28.2	9.2
	9.2	.
Sum	19.0	.
$\frac{1}{20}$sum	1.0	.

Check: +9.3 ÷ 4 = +2.3 = K

$(2\ )+(1\ )_1+(1\ )_2+\Sigma(1\ )=5$ +2.4 $=K.$

Fig. 21.—*Computation form (52 A)*

Intercomparison of weights of a set, series 5, 2, 1, 1, Σ1. This form applies to sets or parts of a set in which the (100), the (10), or the (1) are duplicated instead of the (200), the (20), or the (2), and in which it is not desired to divide the (100), the (10), or the (1) into two parts as would be required in the form shown in Fig. 19 (52 C). The computations inserted in this figure illustrate a calibration based on a standard which is treated in the weighings as if it were one of the weights of the set. Any of the computation forms shown in this circular may be used in a similar manner

MASS COMPUTATION

COMBINATION: 50, 20, 20, 10, 5, Σ5

Department of Commerce
BUREAU OF STANDARDS
Form 52 D

Original Record: Vol. ...., *pp.* .... *to* ...., *inclusive* ....

Computer: .... *Date:* .... *B. S. Test No.* ....

	Mean.	Observations.
$(50\ ) + (20\ )_1 + (20\ )_2 + (10\ ) = 100 -$	.	[ ]
$(50\ ) - (20\ )_1 - (20\ )_2 - (10\ ) = -$	.	[ ]
$(50\ ) = 50 -$	.	
$(20\ )_1 + (20\ )_2 + (10\ ) = 50 -$	.	$= W.$ [Computed: — .]

	Observed.	Obs. — Cal.
$(20\ )_1 - (20\ )_2 + (10\ ) - (5\ ) - \Sigma(5\ ) = a_1 = -$	.	.
$(20\ )_1 - (20\ )_2 - (10\ ) + (5\ ) + \Sigma(5\ ) = a_2 = -$	.	.
$(20\ )_1 - (20\ )_2 + (5\ ) - \Sigma(5\ ) = a_3 = -$	.	.
$(20\ )_1 - (20\ )_2 - (5\ ) + \Sigma(5\ ) = a_4 = -$	.	.
$(20\ )_1 - (20\ )_2 = a_5 = -$	.	.
$(20\ )_1 - (10\ ) - (5\ ) - \Sigma(5\ ) = a_6 = -$	.	.
$(20\ )_2 - (10\ ) - (5\ ) - \Sigma(5\ ) = a_7 = -$	.	.
$(10\ ) - (5\ ) - \Sigma(5\ ) = a_8 = -$	.	.
$(5\ ) - \Sigma(5\ ) = a_9 = -$	.	.

	+	−
$+a_1$	.	.
$+a_2$	.	.
$+a_3$	.	.
$+a_4$	.	.
$+a_5$	.	.
	.	.
	.	.
m_1	.	.

	+	−
$-a_1$	.	.
$+a_2$	.	.
$-a_6$	.	.
$-a_7$	.	.
$-a_8$	.	.
	.	.
	.	.
m_2	.	.

	+	−
$+a_1$	.	.
$-a_2$	.	.
$-a_6$	.	.
$-a_7$	.	.
$+a_8$	.	.
	.	.
	.	.
m_3	.	.

	+	−
$+a_3$	.	.
$-a_4$	.	.
$+a_9$	.	.
	.	.
	.	.
m_4	.	.

	+	−
$+\frac{1}{10}m_2$	.	.
$-\frac{1}{6}m_4$	.	.
$+\frac{1}{15}W$	.	.
	.	.
	.	.
$\Sigma(5\ )$	.	.

$(20\ )_1$

	+	−
$+30m_1$	.	.
$+2m_2$	.	.
$-20m_3$	.	.
$+30a_6$	.	.
	.	.
	.	.
S_1	.	.
$\frac{1}{330}S_1$	.	.
$+\frac{2}{5}W$	.	.

$(20\ )_2$

	+	−
$-30m_1$	.	.
$+2m_2$	.	.
$-20m_3$	.	.
$+30a_7$	.	.
	.	.
	.	.
S_2	.	.
$\frac{1}{330}S_2$	.	.
$+\frac{2}{5}W$	.	.

$(10\ )$

	+	−
$+\frac{1}{30}m_2$	.	.
$+\frac{1}{6}m_3$	.	.
$+\frac{1}{3}W$	.	.
	.	.
	.	.

$(5\ )$

	+	−
$+\frac{1}{10}m_2$	.	.
$+\frac{1}{6}m_4$	.	.
$+\frac{1}{15}W$	.	.
	.	.
	.	.

FIG. 22.—*Computation form (52 D)*

Intercomparison of weights of a set, series 50, 20, 20, 10, 5, Σ5

28629°—18——4

SHEET.............. A
MASS COMPUTATION
COMBINATION: 4, 3, 2, 1, Σ1

Department of Commerce
BUREAU OF STANDARDS
Form 52 E

Original Record: Vol. ...., *pp.* .... *to* ...., *inclusive* ....

Computer: .... *Date:* .... *B. S. Test No.* ....

		Observations.
(10) = 10	− .	[ ]
(4) + (3) + (2) + (1) − (10) =	− .	[ ]
(4) + (3) + (2) + (1) = 10	− .	=M

	Observed.	Obs. − Cal.
(4) − (3) − (2) + (1) = a_1 =	− .	.
(4) − (3) − (2) + Σ(1) = a_2 =	− .	.
(4) − (3) − (1) = a_3 =	− .	.
(4) − (3) − Σ(1) = a_4 =	− .	.
(4) − (2) − (1) − Σ(1) = a_5 =	− .	.
(3) − (2) − (1) = a_6 =	− .	.
(3) − (2) − Σ(1) = a_7 =	− .	.
(2) − (1) − Σ(1) = a_8 =	− .	.
(1) − Σ(1) = a_9 =	− .	.

A_1	+	−	A_2	+	−	A_3	+	−
$+a_1$	.	.	$-a_1$	.	.	$-a_1$	.	.
$+a_2$	.	.	$-a_2$	.	.	$-a_2$	.	.
$+a_3$	.	.	$-a_3$	.	.		.	.
$+a_4$	.	.	$-a_4$	.	.		.	.
$+a_5$	.	.		.	.	$-a_5$	.	.
	.	.	$+a_6$	.	.	$-a_6$	.	.
	.	.	$+a_7$	.	.	$-a_7$	.	.
	.	.		.	.	$+a_8$	.	.
	.	.		.	.		.	.
	.	.		.	.		.	.
	.	.		.	.		.	.
	.	.		.	.		.	.

A_4	+	−	A_5	+	−	Check	+	−
$+a_1$	.	.		.	.	$+4A_1$	.	.
	.	.	$+a_2$	.	.	$+3A_2$	.	.
$-a_3$	.	.		.	.	$+2A_3$	.	.
	.	.	$-a_4$	.	.	$+A_4$	.	.
$-a_5$	.	.	$-a_5$	.	.	$+A_5$	.	.
$-a_6$	.	.		.	.		=	
	.	.	$-a_7$	.	.			
$-a_8$	.	.	$-a_8$	.	.			
$+a_9$	.	.	$-a_9$	.	.			
	.	.		.	.			
	.	.		.	.			
	.	.		.	.			

FIG. 23.—*Computation form (first sheet of 53 E)*

Intercomparison of weights of a set, series 10, 4, 3, 2, 1, Σ1. This form is useful in calibrating some unusual sets, mostly old avoirdupois weights containing a (4) and a (3). Sheet "A" shown here indicates the weighings to be made and the preliminary sums A_1 to A_5, which are used in the final computations on sheet "B," Fig. 24

SHEET .... B

MASS COMPUTATION

COMBINATION: 4, 3, 2, 1, Σ1

Department of Commerce
BUREAU OF STANDARDS
Form 52 E
(Continued)

B. S. Test No. ....

$A_1 = -$.
$A_2 = -$.
$A_3 = -$.
$A_4 = -$.
$A_5 = -$.
$M = -$.

Partial Check :
(Computed values)

(4)+(3)+(2)+(1)=
— . = M

(4)	+	−
$-6A_2$		
$-5A_3$		
$-6A_4$		
S_1		
$\frac{1}{75}S_1$		
$\frac{2}{5}M$		

(3)	+	−
$+7A_1$		
$+11A_2$		
$+A_3$		
$+3A_5$		
S_2		
$\frac{1}{50}S_2$		
$\frac{8}{15}M$		

(2)	+	−
$-3A_2$		
$+10A_3$		
$-3A_4$		
S_3		
$\frac{1}{75}S_3$		
$\frac{1}{5}M$		

(1)	+	−
$-A_1$		
$-3A_3$		
$+23A_4$		
$-4A_5$		
S_4		
$\frac{1}{150}S_4$		
$\frac{1}{10}M$		

Σ(1)	+	−
$-A_1$		
$-3A_3$		
$-7A_4$		
$+26A_5$		
S_5		
$\frac{1}{150}S_5$		
$\frac{1}{10}M$		

FIG. 24.—*Computation form (second sheet of 52 E)*

This sheet gives the final computation of the corrections for the weights

Department of Commerce
BUREAU OF STANDARDS
Form 52 F

MASS COMPUTATION

COMBINATION: 4, 2, 2, 1, Σ1

Original Record: Vol. ...., *pp.* .... *to* ...., *inclusive* ....

Computer: .... *Date:* .... *B. S. Test No.* ....

(4) + (2)₁ + (1) + Σ(1) = 8 — . = K

	Observed.	Obs. — Cal.
(4) − (2)$_1$ − (2)$_2$ + (1) − Σ(1) = a_1 = —	.	.
(4) − (2)$_1$ − (2)$_2$ − (1) + Σ(1) = a_2 = —	.	.
(4) − (2)$_1$ − (1) − Σ(1) = a_3 = —	.	.
(4) − (2)$_2$ − (1) − Σ(1) = a_4 = —	.	.
(2)$_1$ − (2)$_2$ + (1) − Σ(1) = a_5 = —	.	.
(2)$_1$ − (2)$_2$ − (1) + Σ(1) = a_6 = —	.	.

(4)	+	−	(2)$_1$	+	−	(2)$_2$	+	−
$0a_1$			$-5a_1$	.	.	$-5a_1$	.	.
$0a_2$			$-5a_2$	.	.	$-5a_2$	.	.
$+3a_3$	.	.	$+a_3$	.	.	$+5a_3$	.	.
$+2a_4$	.	.	$+4a_4$	.	.	$0a_4$		
$-a_5$	.	.	$+3a_5$	.	.	$-5a_5$	.	.
$-a_6$	.	.	$+3a_6$	.	.	$-5a_6$	.	.
$+5K$	.	.	$+5K$	.	.	$+5K$	.	.
	.	.		.	.		.	.
	.	.		.	.		.	.
Sum	.	.	Sum	.	.	Sum	.	.
$\frac{1}{10}$ sum	.	.	$\frac{1}{20}$ sum	.	.	$\frac{1}{20}$ sum	.	.

(1)	+	−	Σ(1)	+	−
$+10a_1$	.	.	$0a_1$		
$0a_2$			$+10a_2$	.	.
$-7a_3$	.	.	$-7a_3$	.	.
$-8a_4$	.	.	$-8a_4$	.	.
$+4a_5$	.	.	$-6a_5$	.	.
$-6a_6$	.	.	$+4a_6$	.	.
$+5K$	.	.	$+5K$	.	.
	.	.		.	.
	.	.		.	.
Sum	.	.	Sum	.	.
$\frac{1}{30}$ sum	.	.	$\frac{1}{30}$ sum	.	.

Partial Check:
(Computed values)
(4)+(2)₁+(1)+Σ(1) =
— . = K

FIG. 25.—*Computation form (52 F)*

Intercomparison of weights of a set, series 4, 2, 2, 1, Σ1. This form may be used, by inserting an extra (2), for calibrating the ordinary sets of avoirdupois pound and ounce weights; sets in which each weight is one-half of the next larger weight. Without the insertion of some extra weights there is no very satisfactory way of intercomparing the weights of such sets.

satisfactory. Since the first three of these check equations involve all of the observations, a satisfactory agreement in these three is generally considered sufficient evidence to warrant proceeding with the computations.

The check equations for the various schemes are inserted here as they do not appear upon the respective computation sheets.

TABLE 7.—Check Equations

Sheet	Series	Check equations	Sheet	Series	Check equations
52*A*	5, 2, 1, 1, Σ1.....	$a_1+a_3=a_2$ $a_2+a_5=a_4$ $a_3+a_6=a_4$ $a_1+a_5=a_6$	52*D*	50,20,20,10,5,Σ5	$a_1-a_8=a_5$ $a_2+a_8=a_5$ $a_3-a_9=a_5$ $a_4+a_9=a_5$ $a_6-a_7=a_5$
52*B*	5, 2, 2, 1, Σ1.....	$a_3-a_2=a_4$ $a_5-a_1=a_4$ $a_1-a_6=a_4$ $\frac{a_5-a_6}{2}=a_4$	52*E*	4, 3, 2, 1, Σ1.....	$a_1-a_2=a_9$ $a_4-a_3=a_9$ $a_7-a_6=a_9$ $a_1+a_8=a_4$ $a_2+a_8=a_3$ $a_5-a_6=a_4$ $a_5-a_7=a_3$
52*C*	50,20,10,10,5,Σ5	$a_1-a_2=a_5-a_6$ $a_1-a_3=a_5-a_7$ $a_4-a_1=a_6+a_8$ $a_4-a_2=a_5+a_8$ $a_3-a_4=a_8-a_7$ $a_3-a_2=a_7-a_6$	52*F*	4, 2, 2, 1, Σ1.....	$a_4-a_3=\frac{a_5+a_6}{2}$ $a_1-a_2=a_5-a_6$

X. BUOYANT FORCE OF THE AIR

1. GENERAL DISCUSSION

As the buoyant force of the air and its effect on weighing are often not fully appreciated, it is deemed advisable to consider the relation of atmospheric buoyancy to the testing of weights. A knowledge of this subject is essential in order that the principles of buoyancy corrections in weighings may be fully understood, and that in weighings in which the buoyant force is neglected an estimate may be formed of the degree of the approximations that are obtained. For example, in weights of the commercial group either the tolerances are large enough or the densities of the materials permissible under the specifications lie within a narrow enough range that the buoyant force of the air may be neglected.

(*a*) **Cause and Amount of the Buoyant Force.**—The buoyant force of the air is exactly similar to the buoyant or floating effect familiar to all in the case of liquids, although, to be sure, in the case of the air the force is much less in amount. If pound weights, respectively, of wood, aluminum, and brass are held under water, those of aluminum and brass will appear lighter, while that of wood will actually exert an upward force, thus having, so to speak, a negative weight. Moreover, the aluminum will appear to lose more weight than the brass. Now the amount by which the weight of an object is diminished when it is submerged is exactly equal to the weight of the water displaced. A pound of aluminum has a volume of 10 cubic inches, and a mass of water having this volume weighs 0.36 pound. The buoyant effect of the water on the mass of aluminum is therefore 0.36 pound, and, when the aluminum is submerged, it will appear to weigh 0.36 pound less than its true weight; similarly the brass weight will appear to weigh 0.12 pound less than its true value.

Now assume that on the pans of an equal-arm balance which has been completely submerged in a tank of water 1-pound weights of aluminum and brass are placed, one weight being on each pan. Clearly the balance will indicate a difference in the value of the weights, the brass one appearing to weigh the more, since its volume, and therefore the buoyant force upon it, is less than that of the aluminum weight. This apparent difference in weight will amount to 0.24 pound.

Now assume that common salt is dissolved in the water of the tank. The effect of this will be to increase the density of the water, and consequently to increase the apparent difference in the values of the two weights. If we increase the salinity of the solution to the point of saturation, the difference in the apparent values of the weights will increase to 0.29 pound.

Now a similar phenomenon occurs in the case of weights weighed in air, every weight being buoyed up by an amount equal to the weight of the air which it displaces. The weight of 1 cubic yard of air is about 2½ pounds; any object, therefore, having a volume of 1 cubic yard will be buoyed up by the air with a force of 2½ pounds with the effect of reducing by that amount the weight which would be obtained for the object were it weighed in a vacuum. Since most bulky objects are porous, the size of an article gives only a rough estimate of the amount of the buoyant force of the air upon it.

The buoyant force of the air on a 1-pound weight of brass is about 1 grain; on a 50-pound weight of brass, about 50 grains. Either of these latter figures would, of course, be too large to neglect in the test or adjustment of a test weight. However, the discrepancies introduced by the buoyant force of the air usually occur merely as differences, two weights of the same mass apparently having different weights when weighed in air, due to the *difference* in the magnitudes of the forces by which the two weights are respectively buoyed up.

Moreover, as the air changes in density, which it does constantly under changing weather conditions, the difference in the buoyant forces will vary for weights of two materials being compared. In any given locality the density of the air will vary by about 10 per cent as weather conditions change; under constant weather conditions the density of the air will vary about 20 per cent, due to a difference in altitude from sea level to 6000 feet above sea level.

For example, a cast-iron 50-pound weight, if adjusted to the same apparent weight in air at sea level as a brass standard, will appear about 2 grains heavier than the brass standard at an altitude of 6000 feet, and an apparent variation of nearly 1 grain due to variations in air density may be expected at any given place.

The above changes are usually negligible in commercial work, but for some scientific work, and for the purposes of adjusting standards from which commercial weights are derived, the buoyancy correction must be carefully considered and either eliminated or properly corrected for.

These effects of changes in the buoyant force of the air can not be neglected in work of high precision, even in the case of brass weights, for brass weights of the same denomination do not have exactly the same size on account of slight differences in the metal. A 2-pound brass weight whose density would come within the limits specified for class A might appear to be 0.01 grain heavier or lighter at an altitude of 6000 feet than it does at sea level.

It is on the basis of these considerations of the effect of variation in the buoyant force of the air that in many cases the kinds of material allowable and the degree of precision to which the corrections could be certified, etc., have been determined. In a few cases extreme variations in the buoyant force of the air will change corrections of class A or class S weights by about one unit in the last place.

(*b*) **Apparent Mass and "Weight in Air Against Brass."**—Since, as has been explained, the mass of the object (or of the weight) being tested is determined by comparison of its weight with that of the standard, it is evident that the buoyant force of the air, by changing the apparent weight of both of them, prevents exact comparison of the masses unless the effect of the buoyant force is eliminated or proper corrections are made for it.

It is often convenient, however, to refer to results obtained by the ordinary comparison in air as "apparent mass," in cases where no allowance is made for the effect of this buoyant force.

As has been stated, the buoyant force of the air may be neglected on weights of the commercial group. However, it is necessary to establish the values of commercial weights on a definite basis, and to this end all corrections and tolerances apply to the apparent mass as determined in air having a density of 1.2 mg per milliliter, against (brass) standards having a density of 8.4 g per cubic centimeter at 0° C, whose coefficient of cubical expansion is 0.000054 per degree centigrade, and whose values are based on their true mass or weight in vacuo. The value of any brass weight adjusted or tested in the ordinary way is therefore practically its true mass, for all commercial purposes.

In scientific work, and especially in chemical analysis, weighings are often recorded as being made according to "weight in air against brass." Since this is generally understood as meaning the apparent mass as this would be determined against brass standards in air, it will be taken as exactly equivalent to the commercial basis, as just defined.

All weights of the commercial group (classes A, B, C, and T) are tested and certified by this Bureau on this commercial basis; and weights of classes M and S have corrections certified both on this basis and according to the true mass of the weights.

2. ELIMINATION OF BUOYANCY CORRECTIONS

The error due to the buoyancy of the air is completely eliminated in cases where the volumes of the two weights being compared are exactly equal. Since absolute equality is impossible of attainment, it is desirable to determine how nearly equal the volumes must be in order to make the difference in the buoyant forces negligible. This question is chiefly of importance in cases where the volumes are not measured but are calculated from the known or assumed densities of the material. The matter is fully discussed under "Effect of error in assumed density of the weight" on page 63.

In testing weights of the commercial group the effect of the buoyant force of the air is practically eliminated by the restrictions as to material and by the relatively large tolerances which apply.

In ordinary chemical analysis, and in scientific work of similar character, it is desirable to be able to use the platinum and aluminum fractions of a gram with the brass weights without the necessity of making continual buoyancy corrections. It is recommended that for such purposes these fractions of the gram be adjusted according to their apparent mass as determined in air against brass standards. The values of the platinum and aluminum weights on this basis may be considered as values determined from weighings in air using the density of brass as the *assumed density* of these weights. If the corrections of these weights have already been determined on an assumed density of 21.5 and 2.7, the change to the other assumed density may be readily calculated in the same manner as would be done in testing them.

Similar reductions are made in primary standards for use as reference standards for commercial weights and in some other special instances as well.

Table 12, page 75, gives the reduction terms needed to reduce the values of small platinum and aluminum weights from those based on an assumed density of 21.5 or 2.7 to those based on an assumed density of 8.4 or 21.5.

As will be explained later, the error in the assumed density for the weight would cause no discrepancies whatever in the use of the weight if the density of the air were to remain constant. A discussion of the errors arising from variations in the density of the air will be given subsequently.

3. CALCULATION OF BUOYANCY CORRECTIONS

(*a*) **General Principle — Direct Method.**—The correction for buoyancy will be the differeence between the buoyant forces acting on the two objects or weights compared.

If the weighing is expressed as

$$w_1 - w_2 = a$$

and the volumes of the weights are v_1 and v_2, respectively, and the density of the air ρ, the buoyancy correction will be

$$\rho v_1 - \rho v_2 = \rho(v_1 - v_2)$$

and if the correction is computed in the manner indicated, this correction is to be added algebraically to the apparent difference as observed on the balance.

The degree of precision to be attained in the buoyancy correction depends upon the precision with which the volume of the weights and the density of the air can be determined. The volume of the weights can be very accurately determined by hydrostatic weighing, a method familiar in its application to the determination of the density of solids. In such cases, of course, the density of the weight is not determined or used, the value of the volume sufficing for all purposes in the determination of the buoyancy corrections to the weights.

(*b*) **Density of the Air.**—The density of the atmosphere depends upon the temperature, pressure, and humidity, each of which factors must be separately determined.[10]

Determination of Temperature.—The temperature may be determined by a thermometer, and although the corrections of a good thermometer are usually small, assurance should be had that this is the case and that the differences arising from thermometer errors are negligible.

Determination of Pressure.—The pressure of the air is determined by a barometer. The readings of a mercurial barometer may need correction for instrumental errors,[11] such as residual gas pressure, capillary depression, and errors of the scale. The barometer height must be reduced to the height which would obtain at zero degree centigrade and under a standard value of gravity. The temperature reduction tables commonly given in books of physical and chemical tables are satisfactory. A brief table for this purpose is given on page 76. The value of standard gravity adopted by the International Conference on Weights and Measures is 980.665 dynes. For buoyancy corrections, however, the ordinary tables for reductions to sea level at latitude 45° are satisfactory, as extreme accuracy is not needed in this term.

Determination of Humidity.—The humidity may be determined by a hair hygrometer, a dew-point hygrometer, or a psychrometer. The various forms of the hair hygrometer are almost the only instruments that can be used inside of a closed balance case. They are not reliable under all conditions, however, and on that account must be calibrated frequently by comparison with a more reliable type of instrument.[12]

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[10] Although the composition of the air, aside from its contents of water vapor, may change very slightly, the effect of such change on the density of the air is entirely negligible even in work of high precision unless the carbon-dioxide content becomes large, as might occur when several persons are working in a poorly ventilated room.

[11] In most barometers the error due to capillary depression is eliminated to a large extent in the maker's calibration, the scale errors are negligible, and the determination of the residual gas pressure is very difficult of performance. On this account the instrumental corrections can best be determined directly by comparing the barometer with one whose corrections are accurately known.

[12] A hair hygrometer should not be tested by being placed in saturated vapor, as is often advised, for this leaves it almost worthless for some time and is likely to produce permanent changes in it.

To allow for the water vapor in the atmosphere a third reduction is made in the barometer. Representing by f, the actual pressure of the water vapor present in the air, the reduction term is $-0.378f$.

When a psychrometer is used, f is found from the usual psychrometer tables. But for the more common atmospheric conditions $0.378f$ may be found directly from the condensed table given on page 77. When the dew point is determined, f is equal to the saturation pressure at the dew point. When the relative humidity is found (as with a hair hygrometer), this per cent of the *saturation pressure* at the observed temperature gives f. A table of the pressures of saturated water vapor at different temperatures is given on page 78.

Use of Tables.—The actual density of the air may then be found by taking from a table of the densities of dry air the value corresponding to the temperature and the barometric height reduced as above. A condensed table of densities of dry air is given on page 79. The density of the air may be more conveniently, though with less precision, determined from Table 17, page 80. This table is, however, accurate enough for a great deal of precise weighing, since it gives values that are correct within a few tenths of 1 per cent.

(*c*) **Calculation Based on Density of Objects Compared: Indirect Method.**—If the volume of a weight has not been measured, it may be calculated with a fair degree of accuracy from the approximately known density of the material of which it is composed.[13]

The buoyancy corrections may be calculated directly in terms of the density of the weights and that of air. This may be done conveniently for nearly all work by using the buoyancy correction per gram of material, assuming the mass of the two weights to be the same. This is what is done in the usual approximate formula which is given below as formula No. 3.

Complete Formula.—The complete formula for calculating the buoyancy corrections from densities is derived below; in this

[13] Since the density of brass and some other materials is subject to considerable uncertainty and since, moreover, many weights have an interior cavity, care must be taken in forming a judgment of the accuracy of the calculation based on assumed densities. In the case of screw-knob weights or others having a closed cavity in the interior, the density of the material will not give an accurate value for the volume of the weight, since the cavity and the air inclosed within it are practically a part of the weight. However, if the cavity is connected by an opening or crevice with the exterior air, the weight in regard to buoyancy will act as though its volume were the net volume of metal which it contains. In the case of screw-knob weights it is practically impossible to determine whether or not the interior cavity is sealed off from the outer air, as tightness of the cavity is largely a matter of accident in the manufacture of the weight.

equation the mass is expressed directly in terms of the result of the weighing and the buoyancy corrections.

Let M_s = the *mass* of the standard,
M_n = the mass of the weight tested,
D_s = the density of the standard,[14]
D_n = the density of the weight tested,[14]
ρ = the density of the air,
g = the acceleration due to gravity;

then when the two weights are in equilibrium [15] we have

$$\left(M_s - \frac{M_s}{D_s}\rho\right)g = \left(M_n - \frac{M_n}{D_n}\rho\right)g$$

Solving for M_n, we obtain

$$M_n = M_s \frac{\left(1 - \frac{\rho}{D_s}\right)}{\left(1 - \frac{\rho}{D_n}\right)} \qquad (1)$$

Expanding, we obtain

$$M_n = M_s \left[1 + \frac{\rho}{D_n} - \frac{\rho}{D_s} + \left(\frac{\frac{\rho^2}{D_n^2} - \frac{\rho^2}{D_n D_s}}{1 - \frac{\rho}{D_n}}\right)\right] \qquad (2)$$

Approximate Formula: The Usual Method.—The term in parentheses can usually be neglected because $\frac{\rho}{D_n}$ and $\frac{\rho}{D_s}$ are small. Omitting this term we have the usual expression

$$M_n = M_s\left(1 + \frac{\rho}{D_n} - \frac{\rho}{D_s}\right) \text{ or } M_n = M_s + \rho\left(\frac{1}{D_n} - \frac{1}{D_s}\right)M_s \qquad (3)$$

The usual derivation may be seen in the second form of equation (3). $\left(\frac{1}{D_n} - \frac{1}{D_s}\right)$ is the difference in volume per unit of mass, therefore, $M_s\left(\frac{1}{D_n} - \frac{1}{D_s}\right)$ is the total difference in volume and

---

[14] When there are weights of different densities on either pan, the masses and densities must be split up into the proper values; or as can generally be done safely, the weights of one density (generally the platinum and aluminum fractions of a gram) may first be reduced to values on an assumed density equal to that of the larger weights, as is recommended for general use in the case of ordinary analytical weights.

[15] When part or all of the difference between the two weights is determined by the deflection of the pointer the weights are not in exact equilibrium. The difference determined by the deflection of the pointer must be added to the proper side of the equation and may be treated as material of the same density as that assumed for the weight by which the sensitiveness of the balance was determined. In all ordinary cases, however, this difference is so small that it may be added without regard to density.

$\rho M_s\left(\frac{1}{D_n}-\frac{1}{D_s}\right)$ is the difference in buoyant force, or the buoyancy correction.

In computing the buoyancy correction the errors of the weights may be safely neglected, as these errors cause such an exceedingly small difference in the volume of the weights that any difference arising from this small term is practically inappreciable. For all usual purposes, therefore, the nominal or designated values of the masses may be used.

In assuming, as has been done in the calculation of Table 18, "Buoyancy correction factors for materials of different densities," that the weights are of equal mass, a slight difference in volume has been neglected in addition to that arising from the neglect of the correction to the weights. This difference in volume exists on account of the fact that the weights are in equilibrium in air, and on this account the less dense weights have a slightly greater mass, as is necessary to counterbalance the greater buoyant force acting on them. In equation (2) the last term in the brackets, viz,

$$\frac{\frac{\rho^2}{D_n{}^2}-\frac{\rho^2}{D_nD_s}}{1-\frac{\rho}{D_n}} \text{ which equals } \frac{\rho^2(D_s-D_n)}{(D_n-\rho)D_nD_s}$$

appears on this account. When the value of this term is calculated, it is seen to be negligible for weights having densities lying between 2.6 and 21 mg per cubic centimeter (this range of densities including practically all weights) except for weighings requiring precision higher than 1 in 1 000 000.

Effect of Variations in the Density of the Air.—Since the value of a weight, or the corrections to it, are determined in order to permit the use of that weight in ascertaining the mass of some other object, the effect of errors in any density assumed for the weight is to be measured by the errors which arise in the use of the weight. Were the density of the air invariable, any desired value of density could be assumed for the weight in determining its value without causing error in the results obtained in using the weight, provided the same value of density were used in both cases; for, in the use of the weight in weighing, any such error originating in the determination of the mass of the weight would be automatically eliminated. On this account it is in the variations in the density of the air that we are interested in determining the effect of errors in the values of density assumed. A case of particular importance is that in which weights of platinum (generally the fractions of a gram) are adjusted or determined on the basis of an assumed density of 8.4, the value usually used for brass. This is the adjustment usually made in order that platinum weights may be used with sets of brass weights without the necessity of constantly making buoyancy corrections.

The approximate formula for the effect of variations in the density of the air in such cases is given below in equation (5) and the two following paragraphs, the "correction" being equal to the variation in the value of the weight caused by a change of $\Delta\rho$ in the density of the air.

When a weight is tested under the assumption of a certain value for its density, D, in air of a certain density, ρ, the value found under those conditions may be called $M_{D,\rho}$. This quantity, which may be termed the "effective mass," is equal to the mass of an ideal standard of the assumed density that will just balance the weight considered, in air of the same density as that in which $M_{D,\rho}$ was determined. The error caused by using one value of $M_{D,\rho}$ when the weight is being used in air of a different density may then be calculated by the necessary changes in this ideal weight.

The following formula gives the correction to be applied to one value of $M_{D,\rho}$ to reduce it to that for a different density of the air, and similarly, the error caused in using one value of $M_{D,\rho}$ for a weight that is being used as a standard in air having a density different from that in which its effective mass was determined.

Let M_s = the mass of the ideal standard of the assumed density, whose true mass is equal to the value of $M_{D,\rho}$ of the weight under discussion,

M_n = the true mass of this weight,

D_s = the density of the ideal standard, this density being equal to that assumed for the weight.

D_n = the actual density of the weight,

ρ = the density of the air in which the effective mass was determined,

$\Delta\rho$ = the change in the density of the air.

From the same equation as that used in deriving the preceding formulae, we obtain,

$$M_n\left(1-\frac{\rho}{D_n}\right)=M_s\left(1-\frac{\rho}{D_s}\right)$$

When the density of the air is changed by an amount $\Delta\rho$, a change C will be required in the mass of the ideal weight in order to produce equilibrium. We then obtain

$$M_n\left(1-\frac{\rho+\Delta\rho}{D_n}\right)=(M_s+C)\left(1-\frac{\rho+\Delta\rho}{D_s}\right)$$

and C will be the correction sought. Expanding and rearranging the terms, we have

$$M_n\left(1-\frac{\rho}{D_n}\right)-M_n\frac{\Delta\rho}{D_n}=M_s\left(1-\frac{\rho}{D_s}\right)-M_s\frac{\Delta\rho}{D_s}+C\left(1-\frac{\rho+\Delta\rho}{D_s}\right)$$

Solving for C we have

$$C=\frac{\Delta\rho\left(\frac{M_s}{D_s}-\frac{M_n}{D_n}\right)}{1-\frac{\rho+\Delta\rho}{D_s}} \tag{4}$$

Simplifying this by the introduction of approximations sufficiently accurate, we obtain

$$C=\Delta\rho M_s\left(\frac{1}{D_s}-\frac{1}{D_n}\right) \tag{5}$$

From this it is to be observed that the correction may be calculated approximately as the change in the difference between the buoyant forces acting on the two weights, considering their masses as being equal. For determining this correction quickly, Table 18, "Buoyancy correction factors for materials of different densities," may be used.

As a rough estimate of the maximum variation per gram, we may take 0.1 times the total difference in buoyancy, for weights to be retained in one locality, and twice that amount if they are to be taken from sea level to an altitude of 6000 feet.

Effect of Error in Assumed Density of the Weight.—The actual error in the mass of the weight caused by an error in the assumed density of the weight is seldom of importance, since the weight is generally used for determining the mass of other objects in air not differing greatly in density from that in which the weight was calibrated. As it may be of importance in some work to know the amount of this error, the complete formula for determining the correction is derived below.

Let M_s = the approximate mass determined,
M_n = the actual mass of the weight,
D_s = the density assumed for the weight,
D_n = the actual density of the weight,
ρ = the density of the air,
g = the acceleration due to gravity.

The weight in question may be considered as just balancing in air an ideal standard of the assumed density, whose true mass is the approximate mass found for the former. Therefore

$$\left(M_n - \frac{M_n}{D_n}\rho\right)g = \left(M_s - \frac{M_s}{D_s}\rho\right)g$$

Solving for M_n, we obtain

$$M_n = M_s \frac{1 - \frac{\rho}{D_s}}{1 - \frac{\rho}{D_n}}$$

Let $D_n = D_s + \Delta D_s$
Substituting for D_n its value, we have

$$M_n = M_s\left(1 - \frac{\rho}{D_s}\right)\left(\frac{1}{1 - \frac{\rho}{D_s + \Delta D_s}}\right)$$

$$M_n = M_s\left(1 - \frac{\rho}{D_s}\right)\left(D_s + \Delta D_s\right)\frac{1}{D_s + \Delta D_s - \rho}$$

$$M_n = M_s \frac{D_s - \rho + \Delta D_s - \frac{\rho \Delta D_s}{D_s}}{D_s - \rho + \Delta D_s}$$

$$M_n = M_s\left[1 - \frac{\rho \Delta D_s}{D_s(D_s - \rho + \Delta D_s)}\right]$$

Let ΔM_s = the correction that is to be added to the approximate mass M_s, that is, let $M_n = M_s + \Delta M_s$.

Substituting this value for M_n in the preceding equation, dividing through by M_s, and rearranging the signs of the second term, we have

$$1 + \frac{\Delta M_s}{M_s} = 1 + \frac{\rho \Delta D_s}{D_s(\rho - D_s - \Delta D_s)}$$

Putting $R=\frac{\Delta M_s}{M_s}$, which is the correction per unit of mass, we have, after subtracting 1 from each member,

$$R=\frac{\rho\Delta D_s}{D_s(\rho-D_s-\Delta D_s)} \tag{6}$$

or, solving for the difference in density ΔD_s, we have

$$\Delta D_s=\frac{RD_s(\rho-D_s)}{\rho+RD_s} \tag{7}$$

From this the allowable difference in density of the weight for a given difference in R can readily be found.

XI. REGULATIONS GOVERNING THE TESTING OF WEIGHTS

1. GENERAL REQUIREMENTS

Standards conforming to the general requirements given below will be accepted for regular test and certification. These include all weights that conform to the specifications for classes A, B, C, M, and S, and a few special weights worthy of test that do not come within the regular classification.

The Bureau reserves the right to reject any or all weights that do not conform effectively and in good faith to the foregoing specifications or following general requirements.

(*a*) Weights must be composed of material that is not readily oxidized or corroded by the atmosphere [16] or else must be protected by a suitable coating of such material.

(*b*) All but the small weights must be composed of material at least as hard as brass. Weights below 5 g, or 60 grains, may be composed of aluminum or metals of similar hardness.

(*c*) Weights provided with a cavity for making adjustments, closed with a plug or removable knob or handle, must have but one cavity, and it must be small in proportion to the weight.[17]

(*d*) There must be no hole or aperture in the bottom of weights of any form, or in the sides of weights having a cylindrical or conical body, except where such form is specifically approved in the case of class C weights.

(*e*) Weights must be regular in shape and simple in design.

(*f*) The nominal values of the weights must be plainly and permanently marked on them (see list of abbreviations on page 71),

[16] Brass or bronze oxidizes too rapidly to be used without surface protection.

[17] Such weights as metal shells filled with lead will not be tested.

except on weights so small that this is impracticable. This requirement may be waived in the case of weights of class M and of old standards sent to be retested.

(*g*) With the exception of large cast-iron weights (class C grade, 20 pounds or over, only), all metal surfaces must be smooth and sufficiently well polished to show readily scratches or other injuries. The cast-iron weights must not have deep pits or sharp projections.

(*h*) Weights submitted for test and not in conformity with the requirements applying to any of the classes A, B, C, M, and S, as outlined in this circular, must be adjusted at least within the limits of error of class C

2. REJECTION OF WEIGHTS

Weights that do not conform to the specifications for the class under which they were submitted, or that do not conform to the general requirements, will be rejected. Notice of this fact will be sent to the apparent owner of the weights; that is, in general to the person to whom the weights are to be shipped from the Bureau.

If weights are rejected for inaccuracy or other faults that can not be detected without considerable labor, the fee for testing them will be the same as though they had been submitted separately.

When weights are bought under contract to conform to the specifications of this Bureau, much time and trouble may be saved to the purchaser by notifying the Bureau of this fact and having the weights shipped to the Bureau directly from the manufacturer or dealer. Under these conditions, rejected weights will be returned directly to the manufacturer or dealer, and the bill for testing rejected weights will also be sent to him.

Rejected weights may, upon special request, be tested and their corrections determined to the degree of precision that seems to be justified by the nature of the weights. In this case a "report" will be issued in place of the regular certificate. It will state definitely the cause of the rejection, as well as the corrections found. Weights rejected for inaccuracy will not be corrected by the Bureau unless it would involve unwarranted expense or delay to have them sent away for adjustment, nor unless it can be done without delaying the regular testing work of the Bureau. In no case will the Bureau make such corrections without specific request by the person or firm submitting the weights. Fees for this work are given in Schedule 14, page 69.

3. IDENTIFICATION OF WEIGHTS TESTED BY THE BUREAU

The shipping case or the inner wrappings will always be sealed when weights that have been tested are shipped from the Bureau and will bear the Bureau's test number.

4. SUBMITTING WEIGHTS FOR TEST

(*a*) **Application for Test.**—Written requests should be made for all tests, even when the weights are delivered in person or by messenger. The request should state the class under which the weights are to be tested, together with the other information necessary for their proper test and identification.

If weights have already been used as standards in exacting work, and it is important to know what their corrections were at the close of such work, this fact should be stated; otherwise, weights are carefully cleaned before being tested.

(*b*) **Packing.**—The weights must be carefully packed in boxes that can be used in returning them. The weights, if shipped in their regular cases, should be tightly packed in their pockets by the use of extra material of some sort. The larger weights, when they are not shipped in cases containing pockets, should have the packing next the weight bound tightly against it to avoid the possibility of abrasion, and a liberal amount of the packing should be used.

In wrapping the better grades of weights some soft but firm material should be used next to the weight. If tissue paper is used, many thicknesses of it should be firmly wrapped on, and this should then be covered with thicker, tougher material such as cotton packing or knit goods. The whole should then be wrapped securely in wrapping paper in order to exclude the dust and grit from the rough packing in which the boxes or bundles are generally packed for shipment.

In many cases the milligram weights that are packed under the glass covers of the regular cases reach the Bureau bent or otherwise damaged. For this reason greater care should be given to the packing of these weights. Sufficient extra packing should be used to hold the glass cover firmly in place.

(*c*) **Consignor's Identification.**—All packages should bear the shipper's name and address, a list of the contents, and any other marks that may be necessary to insure ready identification. The inner wrappings of each package should bear the denominations of the weight or weights inclosed.

(*d*) **Addressing of Shipment.**—Articles to be tested should be directed simply to the "Bureau of Standards, Washington, D. C." Delays may be caused by other forms of address. The covers of the shipping cases or boxes should be put on with screws, and the return or forwarding address should be on the under side of the cover. Articles are forwarded or returned "collect" except when sufficient postage has been sent to cover return by mail.

(*e*) **Amount of the Fees.**—If sets of weights are made up, as they usually are, in one or more of the following series, the testing of such a set or of a continuous portion of such a set is less expensive than the testing of the same number of weights of miscellaneous denominations. A reduction is therefore made in the fees on this account.

Series I: 10, 5, 2, 2, 1, 0.5, 0.2, 0.2, 0.1. ....
Series II: 10, 5, 2, 1, 1, 0.5, 0.2, 0.1, 0.1. ....
Series III: 10, 4, 3, 2, 1, 0.4, 0.3, 0.2, 0.1. ....
Series IV: 10, 5, 4, 3, 2, 1, 0.5, 0.4, 0.3, 0.2, 0.1. ....
Series V: 8, 4, 2, 1, ½, ¼, ⅛. ....

Series I and II are much to be preferred for simplicity and convenience both in testing and in using them. Of these the former is to be preferred unless the extra material required in Series I is of importance. Any number of weights that are consecutive in one or more of these series will be considered as a set in calculating the fee.

Schedule 11. COMMERCIAL GROUP

	Per weight
Class A and weights given a similar test:	
(*a*) Regular test, single weights.	$1.00
(*b*) Regular test, equal weights and sets made up in the series shown above.	.75
Class B and weights given a similar test:	
(*c*) Regular test, single weights.	.60
(*d*) Regular test, equal weights and sets made up in the series shown above.	.40
(*e*) For constancy test add one-half of the fee for the regular test.	
Class C and weights given a similar test:	
(*f*) Regular test, single weights.	.50
(*g*) Regular test, equal weights and sets made up in the series shown above.	.30

Schedule 12. SCIENTIFIC GROUP

	Per weight
Class M and weights given a similar test:	
(*a*) Determination of volume, single weights	$2.00
(*b*) Determination of volume, two or more weights	1.50
(*c*) Determination of corrections, single weights	.80
(*d*) Determination of corrections, equal weights and sets made up in the series shown above	.50
(*e*) For constancy test add one-half of the fee for determining the corrections.	
Class S and weights given a similar test:	
(*f*) Regular test, single weights	.60
(*g*) Regular test, equal weights and sets made up in the series shown above	.40
(*h*) For constancy test add one-half of the fee for the regular test.	

Schedule 14. SPECIAL WORK (done only in unusual circumstances, as noted on page 66)

For adjusting gold-plated one-piece weights:	
(*a*) Adjustment within the tolerances for classes A, B, and M when no plating is required, minimum fee	1.00
(*b*) Same as above, but plating required, minimum fee	2.00
For adjusting weights that have special means for adjusting as allowed in classes B, C, and S:	
(*c*) Adjustment within the tolerances for class B or C when plugs or caps have not been inserted	.50
(*d*) Adjustment within the tolerances for class S, screw-knob weights or plugged weights in which the plugs or caps have not been inserted	1.00

Special fees may be made to cover work not provided for in above schedules.

(*f*) **Remittances.**—Fees in accordance with the foregoing schedules should be sent with the request for test whenever practicable. Otherwise a bill will be rendered before the testing is begun. Certificates are not given, nor are the weights returned until the fees due thereon have been received. Remittances may be made by money order or check drawn to the order of the Secretary of Commerce.

S. W. Stratton,
Director.

Approved:
William C. Redfield,
Secretary.

APPENDIX

TABLE 8.—Short List of Abbreviations Recommended by this Bureau

Kilogram	kg
Gram	g
Centigram	cg
Milligram	mg
Pound, avoirdupois	lb.
Ounce, avoirdupois	oz.
Grain (avoirdupois and troy grains are identical)	gr.
Pound, troy	lb. t.
Ounce, troy	oz. t.
Pennyweight	dwt.
Pound, apothecaries' weight	lb. ap.
Ounce, apothecaries' weight	oz. ap. *or* ℥
Dram, apothecaries' weight	dr. ap. *or* ʒ
Scruple, apothecaries' weight	s. ap. *or* ℈
Carat (200 mg)	c

This table contains practically all of the abbreviations needed for weights, but a more complete list will be found in Bureau Circular No. 47.

TABLE 9.—Class T—Trade Weights: Tolerances, Customary System

Tolerances for trade weights as adopted by the Annual Conference on the Weights and Measures of the United States, and recommended by the Bureau of Standards for adoption by the several States. Complete specifications and tolerances for weights of this class will be found in Circular 61.

The tolerances to be allowed in excess or deficiency on commercial weights shall not be greater than the following values: Provided, however, that the manufacturers' tolerances or the tolerances to be allowed on new commercial weights shall not be greater than one-half of the values given.

Avoirdupois System

Weight	Tolerance, ordinary weights (ratio 1:1)	Tolerance, counterpoise weights for multiplying-lever scales		
		Ratio less than 100:1	Ratio 100:1 and less than 1000:1	Ratio 1000:1 and over
lbs.	gr.	gr.	gr.	gr.
50	100.0	60.0	40.0	20.0
25	60.0	36.0	24.0	12.0
20	60.0	36.0	24.0	12.0
15	40.0	24.0	16.0	8.0
10	40.0	24.0	16.0	8.0
8	30.0	18.0	12.0	6.0
5	30.0	18.0	12.0	6.0
4	20.0	12.0	8.0	4.0
3	20.0	12.0	8.0	4.0
2	15.0	9.0	6.0	3.0
1	10.0	6.0	4.0	2.0
oz.				
10	10.0	6.0	4.0	2.0
8	5.0	3.0	2.0	1.0
5	5.0	3.0	2.0	1.0
4	5.0	3.0	2.0	1.0
2	3.0	1.8	1.2	0.6
1	2.0	1.2	0.8	0.4
1/2	2.0	1.2	0.8	0.4
1/4	1.0	0.6	0.4	0.2
1/8	0.5	0.3	0.2	0.1
1/16	0.5	0.3	0.2	0.1
1/32	0.5	0.3	0.2	0.1
1/64	0.2	0.12	0.08	0.04

TABLE 10.—Class T—Trade Weights: Tolerances, Metric System

The tolerances to be allowed on commercial weights of the metric system should be equivalent to those in the avoirdupois system given above, and the following table has been computed on that basis. The manufacturers' tolerances or the tolerances to be allowed on new commercial weights shall not be greater than one-half of the values given.

Weight	Ordinary weights (ratio 1:1)	Counterpoise weights for multiplying-lever scales		
		Ratio less than 100:1	Ratio 100:1 and less than 1000:1	Ratio 1000:1 and over
kg	g	g	g	g
20	6.0	3.6	2.4	1.2
10	4.0	2.4	1.6	0.8
5	2.5	1.5	1.0	0.5
2	1.5	0.9	0.6	0.3
1	1.0	0.6	0.4	0.2
g	mg	mg	mg	mg
500	700	420	280	140
200	400	240	160	80
100	300	180	120	60
50	200	120	80	40
20	100	60	40	20
10	70	42	28	14
5	50	30	20	10
2	30	18	12	6
1	20	12	8	4
mg				
500	15	9.0	6.0	3.0
200	7	4.2	2.8	1.4
100	5	3.0	2.0	1.0

TABLE 11.—Prescription Weights: Tolerances

Tolerances adopted by the Annual Conference on the Weights and Measures of the United States, and recommended by the Bureau of Standards for adoption by the several States.

The tolerances to be allowed in excess or deficiency on apothecaries' prescription weights shall not be greater than the following values: Provided, however, That the manufacturers' tolerances or the tolerances to be allowed on new apothecaries' prescription weights shall not be greater than one-half of the values given.

Apothecaries' System

Weight	Tolerance	Weight	Tolerance
oz. ap.	gr.	s. ap.	gr.
12	4. 0	3	0. 3
10	4. 0	2	0. 25
8	3. 0	1	0. 15
5	3. 0	gr.	
4	2. 0	20	0. 15
3	2. 0	10	0. 12
2	2. 0	5	0. 08
1	1. 0	2	0. 04
dr. ap.		1	0. 03
8	1. 0	0. 5	0. 02
6	1. 0	0. 2	0. 015
4	0. 7	0. 1	0. 010
3	0. 6		
2	0. 5		
1	0. 3		
0. 5	0. 2		

Metric System

Weight	Tolerance	Weight	Tolerance
g	mg	mg	mg
500	350. 0	500	7. 0
200	200. 0	200	4. 0
100	150. 0	100	3. 0
50	100. 0	50	2. 0
20	50. 0	20	1. 0
10	40. 0	10	1. 0
5	25. 0		
2	15. 0		
1	10. 0		

TABLE 12.—Buoyancy Reduction Terms

This table gives buoyancy reduction terms for reducing corrections for weights from the basis of one assumed density to that of another assumed density. The reduction terms are for platinum, aluminum, and brass, for weighings made in air having a density of 1.2 mg per milliliter, and are based on the approximate formula

$$C = M\rho\left(\frac{1}{D_n} - \frac{1}{D_s}\right)$$

where C=the buoyancy reduction term.

ρ=the density of the air.

M=the mass of weight.

D_n=the assumed density to which the corrections are to be reduced.

D_s=the assumed from which the corrections are to be reduced.

The densities of the weights indicated in the table are the "standard densities," or the values corresponding to zero degree centigrade, but since in ordinary work the weights will usually be nearer a temperature of 20° C, the difference in volume and the buoyancy corrections are calculated for weights at 20° C.

These reduction terms are to be added algebraically to the corrections determined on the first basis. For example, if the correction for a platinum 500-mg weight is −0.054 mg, assuming its true density to be 21.5, then to obtain its apparent weight in air against brass weights, or its correction under an assumed density of 8.4, the reduction term +0.044 must be added to the former correction, giving −0.010 mg.

By using −0.010 as its correction, the weight may be considered as equivalent to a brass weight, and the errors arising from such use are only those consequent upon variations in the density of the air. Any such error will not under ordinary circumstances be more than one-tenth of the reduction term, or twice that amount if the weight is used in air having a density as low as that found at an altitude of 6000 feet.

Milligrams				Grains			
Denomination	Platinum (21.5) to brass (8.4)	Aluminum (2.7) to brass (8.4)	Aluminum (2.7) to platinum (21.5)	Denomination	Platinum (21.5) to brass (8.4)	Aluminum (2.7) to brass (8.4)	Aluminum (2.7) to platinum (21.5)
g	mg	mg	mg	gr.	gr.	gr.	gr.
1	+0.087	−0.302	−0.389	20	+0.00174	−0.00604	−0.00778
mg				10	+0.00087	−0.00302	−0.00389
500	+0.044	−0.151	−0.195	5	+0.00044	−0.00151	−0.00195
200	+0.017	−0.060	−0.078	2	+0.00017	−0.00060	−0.00078
100	+0.009	−0.030	−0.039	1	+0.00009	−0.00030	−0.00039
50	+0.004	−0.015	−0.019	0.5	+0.00004	−0.00015	−0.00019
20	+0.002	−0.006	−0.008	0.2	+0.00002	−0.00006	−0.00008
10	+0.001	−0.003	−0.004	0.1	+0.00001	−0.00003	−0.00004
5	+0.000	−0.002	−0.002	0.05	+0.00000	−0.00002	−0.00002
2	+0.000	−0.001	−0.001	0.02	+0.00000	−0.00001	−0.00001
1	+0.000	−0.000	−0.000				

TABLE 13.—Reduction of Barometric Height to Standard Temperature, Zero Degree Centigrade (Values are Subtractive and Given in Millimeters of Mercury)

This table is taken from the Smithsonian meteorological tables, Table No. 11, and is based on the formula

$$C = -B\frac{(m-l)\,t}{1+mt}$$

where C=correction for temperatures, in millimeters of mercury
B=observed height of the barometric column in millimeters of mercury
m=coefficient of expansion for mercury (0.0001818)
l=coefficient of linear expansion of brass (0.0000184)
t=temperature of the barometer indicated by an attached thermometer

Temperature of barometer	Height of mercury column in millimeters			Temperature of barometer	Height of mercury column in millimeters			Temperature of barometer	Height of mercury column in millimeters		
	740	760	780		740	760	780		740	760	780
°C				°C				°C			
15.0	1.81	1.86	1.91	20.0	2.41	2.47	2.54	25.0	3.01	3.09	3.17
.5	1.87	1.92	1.97	.5	2.47	2.54	2.60	.5	3.07	3.15	3.24
16.0	1.93	1.98	2.03	21.0	2.53	2.60	2.67	26.0	3.13	3.21	3.30
.5	1.99	2.04	2.10	.5	2.59	2.66	2.73	.5	3.19	3.28	3.36
17.0	2.05	2.10	2.16	22.0	2.65	2.72	2.79	27.0	3.25	3.34	3.42
.5	2.11	2.16	2.22	.5	2.71	2.78	2.86	.5	3.31	3.40	3.48
18.0	2.17	2.23	2.29	23.0	2.77	2.84	2.92	28.0	3.37	3.46	3.55
.5	2.23	2.29	2.35	.5	2.83	2.90	2.98	.5	3.43	3.52	3.62
19.0	2.29	2.35	2.41	24.0	2.89	2.97	3.05	29.0	3.49	3.58	3.68
.5	2.35	2.41	2.48	.5	2.95	3.03	3.11	.5	3.55	3.64	3.74

TABLE 14.—Reduction of Barometric Height for Atmospheric Humidity (Millimeters of Mercury)

This table is used for obtaining the proper reduction terms for humidity directly from the wet and dry bulb temperatures of a psychrometer, and is based on the formula

$$\text{Reduction term} = 0.378f$$

where f is the actual ("partial") pressure of the water vapor in the air computed from the wet and dry bulb temperature readings of a psychrometer having rapid, forced ventilation, at a barometric height of 755 mm of mercury. A variation of 55 mm in the barometer height produces an error in the values of this table, varying from 0 at saturation to about 0.1 or 0.2 mm at low humidities.

Temperature of air, degrees centigrade	Temparature of wet bulb, degrees centigrade																													
	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
15	0.6	0.9	1.3	1.7	2.1	2.5	3.0	3.4	3.9	4.3	4.8																				
16	0.4	0.8	1.1	1.5	1.9	2.3	2.8	3.2	3.7	4.2	4.6	5.2																			
17	0.2	0.6	0.9	1.3	1.7	2, 2	2.6	3.0	3.5	4.0	4.5	5.0	5.5																		
18		0.4	0.8	1.1	1.6	2.0	2.4	2.8	3.3	3.8	4.3	4.8	5.3	5.8																	
19		0.2	0.6	1.0	1.4	1.8	2.2	2.7	3.1	3.6	4.1	4.6	5.1	5.7	6.2																
20			0.4	0.8	1.2	1.6	2.0	2.5	2.9	3.4	3.9	4.4	4.9	5.5	6.0	6.6															
21			0.2	0.6	1.0	1.4	1.8	2.3	2.7	3.2	3.7	4.2	4.7	5.3	5.8	6.4	7.0														
22				0.4	0.8	1.2	1.6	2.1	2.5	3.0	3.5	4.0	4.5	5.1	5.7	6.2	6.9	7.5													
23				0.2	0.6	1.0	1.5	1.9	2.4	2.8	3.3	3.8	4.4	4.9	5.5	6.1	6.7	7.3	8.0												
24					0.4	0.8	1.3	1.7	2.2	2.6	3.1	3.6	4.2	4.7	5.3	5.9	6.5	7.1	7.8	8.5											
25						0.6	1.1	1.5	2.0	2.5	2.9	3.4	4.0	4.5	5.1	5.7	6.3	6.9	7.6	8.3	9.0										
26						0.5	0.9	1.3	1.8	2.3	2.8	3.3	3.8	4.3	4.9	5.5	6.1	6.7	7.4	8.1	8.8	9.5									
27						0.3	0.7	1.1	1.6	2.1	2.6	3.1	3.6	4.1	4.7	5.3	5.9	6.5	7.2	7.9	8.6	9.3	10.1								
28						0.1	0.5	1.0	1.4	1.9	2.4	2.9	3.4	4.0	4.5	5.1	5.7	6.4	7.0	7.7	8.4	9.2	9.9	10.7							
29							0.3	0.8	1.2	1.7	2.2	2.7	3.2	3.8	4.3	4.9	5.5	6.2	6.8	7.5	8.2	9.0	9.7	10.5	11.4						
30							0.1	0.6	1.0	1.5	2.0	2.5	3.0	3.6	4.1	4.7	5.3	6.0	6.6	7.3	8.0	8.8	9.5	10.3	11.2	12.0					
31								0.4	0.8	1.3	1.8	2.3	2.8	3.4	4.0	4.5	5.2	5.8	6.5	7.1	7.8	8.6	9.4	10.1	11.0	11.8	12.7				
32								0.2	0.7	1.1	1.6	2.1	2.7	3.2	3.8	4.4	5.0	5.6	6.3	6.9	7.7	8.4	9.2	10.0	10.8	11.6	12.5	13.5			
33									0.5	0.9	1.4	1.9	2.5	3.0	3.6	4.2	4.8	5.4	6.1	6.8	7.5	8.2	9.0	9.8	10.6	11.5	12.4	13.3	14.3		
34									0.3	0.7	1.2	1.8	2.3	2.8	3.4	4.0	4.6	5.2	5.9	6.6	7.3	8.0	8.8	9.6	10.4	11.3	12.2	13.1	14.1	15.1	
35									0.1	0.6	1.1	1.6	2.1	2.6	3.2	3.8	4.4	5.0	5.7	6.4	7.1	7.8	8.6	9.4	10.2	11.1	12.0	12.9	13.9	14.9	15.9

TABLE 15.—Pressure of Saturated Water Vapor (Millimeters of Mercury)

This table gives pressures of saturated water vapor for different temperatures. The values for whole degrees were calculated by F. Henning (Annalen der Physik, **22,** 1907, Table 3, p. 626) from a formula by M. Thiesen, and the values for intermediate tenths of degrees were obtained at the Bureau by interpolation from Henning's values.

The later values for the pressure of saturated water vapor obtained by Scheel and Heuse in 1910 do not differ from the values in this table by more than 1 part in 2000

Temperature, degrees centigrade	Tenths of degrees									
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	4.58	4.61	4.65	4.68	4.71	4.75	4.78	4.82	4.85	4.88
1	4.92	4.96	4.99	5.03	5.07	5.10	5.14	5.18	5.21	5.25
2	5.29	5.33	5.37	5.40	5.44	5.48	5.52	5.56	5.60	5.64
3	5.68	5.72	5.76	5.80	5.85	5.89	5.93	5.97	6.01	6.06
4	6.10	6.14	6.19	6.23	6.27	6.32	6.36	6.41	6.45	6.50
5	6.54	6.59	6.63	6.68	6.72	6.77	6.82	6.87	6.91	6.96
6	7.01	7.06	7.11	7.16	7.21	7.26	7.31	7.36	7.41	7.46
7	7.51	7.56	7.61	7.66	7.72	7.77	7.82	7.88	7.93	7.98
8	8.04	8.10	8.15	8.21	8.26	8.32	8.38	8.44	8.49	8.55
9	8.61	8.67	8.73	8.79	8.84	8.90	8.96	9.03	9.09	9.15
10	9.21	9.27	9.33	9.39	9.46	9.52	9.58	9.65	9.71	9.77
11	9.84	9.91	9.97	10.04	10.11	10.18	10.25	10.31	10.38	10.45
12	10.52	10.59	10.66	10.73	10.80	10.87	10.94	11.01	11.08	11.16
13	11.23	11.30	11.38	11.45	11.52	11.60	11.67	11.75	11.83	11.90
14	11.98	12.06	12.14	12.22	12.29	12.37	12.45	12.53	12.62	12.70
15	12.78	12.86	12.95	13.03	13.11	13.20	13.28	13.37	13.46	13.54
16	13.63	13.72	13.81	13.89	13.98	14.08	14.17	14.26	14.35	14.44
17	14.53	14.62	14.72	14.81	14.90	15.00	15.09	15.19	15.29	15.38
18	15.48	15.58	15.68	15.77	15.87	15.97	16.07	16.17	16.28	16.38
19	16.48	16.58	16.69	16.79	16.89	17.00	17.10	17.21	17.32	17.42
20	17.53	17.64	17.75	17.86	17.97	18.08	18.19	18.31	18.42	18.53
21	18.65	18.77	18.88	19.00	19.12	19.23	19.35	19.47	19.59	19.71
22	19.83	19.95	20.07	20.19	20.32	20.44	20.57	20.69	20.82	20.94
23	21.07	21.20	21.33	21.45	21.58	21.72	21.85	21.98	22.11	22.25
24	22.38	22.51	22.65	22.79	22.92	23.06	23.20	23.33	23.47	23.61
25	23.75	23.89	24.03	24.18	24.32	24.47	24.61	24.76	24.91	25.06
26	25.21	25.36	25.51	25.66	25.82	25.97	26.12	26.28	26.43	26.58
27	26.74	26.90	27.05	27.21	27.37	27.53	27.69	27.86	28.02	28.18
28	28.35	28.52	28.68	28.85	29.02	29.18	29.35	29.52	29.69	29.87
29	30.04	30.21	30.39	30.56	30.74	30.92	31.09	31.27	31.45	31.64
30	31.82	32.00	32.19	32.37	32.56	32.75	32.93	33.12	33.31	33.50
31	33.69	33.88	34.07	34.27	34.46	34.66	34.86	35.06	35.26	35.46
32	35.66	35.86	36.07	36.27	36.48	36.68	36.89	37.10	37.31	37.52
33	37.73	37.94	38.15	38.37	38.58	38.80	39.02	39.24	39.46	39.68
34	39.90	40.12	40.35	40.57	40.80	41.03	41.25	41.48	41.71	41.94
35	42.17									

TABLE 16.—Density of Dry Air (Milligrams per Milliliter)

This table gives the density at different temperatures of dry air containing about 0.04 per cent of CO_2 (which is an average value for the CO_2 content), the values being computed from the formula

$$C=\frac{1.293052}{1+0.00367\ t}\times\frac{h}{760}$$

where h is pressure in millimeters of mercury at 0° C and standard gravity, and t is temperature in degrees centigrade.

Temperature in degrees centigrade.	Pressure in millimeters of Hg (0° C, standard gravity)											
	720	725	730	735	740	745	750	755	760	765	770	775
15	1.1611	1.1691	1.1772	1.1853	1.1933	1.2014	1.2095	1.2175	1.2256	1.2336	1.2417	1.2498
16	1.1571	1.1651	1.1731	1.1812	1.1892	1.1972	1.2053	1.2133	1.2213	1.2294	1.2374	1.2454
17	1.1531	1.1611	1.1691	1.1771	1.1851	1.1931	1.2011	1.2091	1.2171	1.2251	1.2331	1.2411
18	1.1491	1.1571	1.1650	1.1730	1.1810	1.1890	1.1970	1.2049	1.2129	1.2209	1.2289	1.2369
19	1.1451	1.1531	1.1611	1.1690	1.1770	1.1849	1.1929	1.2008	1.2088	1.2167	1.2247	1.2326
20	1.1412	1.1492	1.1571	1.1650	1.1729	1.1809	1.1888	1.1967	1.2046	1.2126	1.2205	1.2284
21	1.1373	1.1452	1.1531	1.1610	1.1689	1.1768	1.1847	1.1926	1.2005	1.2084	1.2163	1.2242
22	1.1335	1.1414	1.1492	1.1571	1.1650	1.1728	1.1807	1.1886	1.1965	1.2043	1.2122	1.2201
23	1.1296	1.1375	1.1453	1.1532	1.1610	1.1689	1.1767	1.1846	1.1924	1.2002	1.2081	1.2159
24	1.1258	1.1337	1.1415	1.1493	1.1571	1.1649	1.1727	1.1806	1.1884	1.1962	1.2040	1.2118
25	1.1220	1.1298	1.1376	1.1454	1.1532	1.1610	1.1688	1.1766	1.1844	1.1922	1.2000	1.2078
26	1.1183	1.1261	1.1338	1.1416	1.1494	1.1571	1.1649	1.1727	1.1804	1.1882	1.1959	1.2037
27	1.1146	1.1223	1.1300	1.1378	1.1455	1.1533	1.1610	1.1687	1.1765	1.1842	1.1920	1.1997
28	1.1108	1.1186	1.1263	1.1340	1.1417	1.1494	1.1571	1.1648	1.1726	1.1803	1.1880	1.1957
29	1.1072	1.1149	1.1225	1.1302	1.1379	1.1456	1.1533	1.1610	1.1687	1.1764	1.1840	1.1917
30	1.1035	1.1112	1.1188	1.1265	1.1342	1.1418	1.1495	1.1571	1.1648	1.1725	1.1801	1.1878
31	1.0999	1.1075	1.1151	1.1228	1.1304	1.1381	1.1457	1.1533	1.1610	1.1686	1.1762	1.1839

TABLE 17.—Density of Air in Milligrams per Milliliter; Values Given Directly from the Temperature and the (Unreduced) Reading of the Barometer

This table is from Theorie, Konstruktion und Gebrauch der Feineren Hebelwage, by W. Felgentraeger. It is computed for air of 50 per cent relative humidity and for a place where "g" equals 981.288 cm/sec². Ordinary changes from these conditions may be expected to introduce errors of about five units in the last decimal place given. If more accurate results are required, reductions or corrections must be applied[18] as noted below the table, on page 82.

The barometer readings should be corrected for instrumental errors such as errors of the scale or residual gas pressure if these errors are not neglible. The reductions for temperature and for gravity (latitude and elevation) are included in the computation of the table. The temperature used must be that at the balance case; the temperature of the barometer should not differ from this by more than 5°.

b \ T	16°	17°	18°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°
690	1.103	099	094	090	086	082	077	073	069	065	061	057	052
1	104	100	096	092	087	083	079	075	071	066	062	058	054
2	106	102	097	093	089	085	081	076	072	068	064	060	055
3	108	103	099	095	091	086	082	078	074	070	065	061	057
4	109	105	101	096	092	088	084	080	075	071	067	063	059
5	111	106	102	098	094	089	085	081	077	073	068	064	060
6	112	108	104	099	095	091	087	083	078	074	070	066	062
7	114	110	105	101	097	093	088	084	080	076	072	067	063
8	116	111	107	103	098	094	090	086	082	077	073	069	065
9	117	113	109	104	100	096	092	087	083	079	075	070	066
700	1.119	115	110	106	102	097	093	089	085	080	076	072	068
1	120	116	112	107	103	099	095	090	086	082	078	074	069
2	122	118	113	109	105	100	096	092	088	084	079	075	071
3	124	119	115	111	106	102	098	094	089	085	081	077	072
4	125	121	117	112	108	104	099	095	091	087	082	078	074
5	127	122	118	114	110	105	101	097	092	088	084	080	076
6	128	124	120	115	111	107	103	098	094	090	085	081	077
7	130	126	121	117	113	108	104	100	096	091	087	083	078
8	132	127	123	118	114	110	106	101	097	093	089	084	080
9	133	129	125	120	116	112	107	103	099	094	090	086	082
710	1.135	130	126	122	117	113	109	105	100	096	092	087	083
1	136	132	128	123	119	115	110	106	102	097	093	089	085
2	138	134	129	125	121	116	112	108	103	099	095	090	086
3	140	135	131	127	122	118	114	109	105	101	096	092	088
4	141	137	132	128	124	119	115	111	106	102	098	094	089
5	143	138	134	130	125	121	117	112	108	104	099	095	091
6	144	140	136	131	127	123	118	114	110	105	101	097	092
7	146	142	137	133	128	124	120	115	111	107	102	098	094
8	148	143	139	134	130	126	121	117	113	108	104	100	095
9	149	144	140	136	132	127	123	119	114	110	106	101	097
720	1.151	146	142	138	133	129	124	120	116	111	107	103	099
1	152	148	144	139	135	130	126	122	117	113	109	104	100
2	154	150	145	141	136	132	128	123	119	114	110	106	102
3	156	151	147	142	138	134	129	125	120	116	112	107	103
4	157	153	148	144	140	135	131	126	122	118	113	109	105
5	159	154	150	145	141	137	132	128	124	119	115	111	106
6	160	156	151	147	143	138	134	130	125	121	116	112	108
7	162	158	153	149	144	140	136	131	127	122	118	114	109
8	164	159	155	150	146	141	137	133	128	124	120	115	111
9	165	161	156	152	147	143	139	134	130	125	121	117	112
730	1.167	162	158	153	149	145	140	136	131	127	123	118	114
1	169	164	160	155	151	146	142	137	133	129	124	120	115
2	170	166	161	157	152	148	143	139	135	130	126	121	117
3	172	167	163	158	154	149	145	140	136	132	127	123	119
4	173	169	164	160	155	151	147	142	138	133	129	124	120
5	175	170	166	161	157	153	148	144	139	135	130	126	122
6	177	172	168	163	159	154	150	145	141	136	132	128	123
7	178	174	169	165	160	156	151	147	142	138	133	129	125
8	180	175	171	166	162	157	153	148	144	139	135	131	126
9	181	177	172	168	163	159	154	150	145	141	136	132	128
740	1.183	178	174	169	165	160	156	151	147	142	138	134	129

[18] When a large number of corrections or reductions must be introduced it is generally as easy and often preferable for other reasons to determine the density from other tables.

Interpolation Table

Δb \ Δτ	0.0°	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
mm										
0.0	0	− 0	− 1	− 1	− 2	− 2	− 3	− 3	− 4	− 4
0.1	+ 0	− 0	− 1	− 1	− 2	− 2	− 3	− 3	− 4	− 4
0.2	+ 0	− 0	− 1	− 1	− 2	− 2	− 2	− 3	− 4	− 4
0.3	+ 0	+ 0	− 0	− 1	− 1	− 2	− 2	− 3	− 3	− 4
0.4	+ 1	+ 0	− 0	− 1	− 1	− 2	− 2	− 3	− 3	− 4
0.5	+ 1	+ 0	− 0	− 1	− 1	− 2	− 2	− 2	− 3	− 3
0.6	+ 1	+ 0	+ 0	− 0	− 1	− 1	− 2	− 2	− 3	− 3
0.7	+ 1	+ 1	+ 0	− 0	− 1	− 1	− 2	− 2	− 3	− 3
0.8	+ 1	+ 1	+ 0	− 0	− 1	− 1	− 2	− 2	− 2	− 3
0.9	+ 1	+ 1	+ 0	+ 0	− 0	− 1	− 2	− 1	− 2	− 3

b \ τ	16°	17°	18°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°
740	1.183	178	174	169	165	160	156	151	147	142	138	134	129
1	185	180	176	171	166	162	157	153	148	144	140	135	131
2	186	182	177	173	168	164	159	155	150	146	141	137	132
3	188	183	179	174	170	165	161	156	152	147	143	138	134
4	189	185	180	176	171	167	162	158	153	149	144	140	136
5	191	186	182	177	173	168	164	159	155	150	146	141	137
6	193	188	183	179	174	170	165	161	156	152	147	143	139
7	194	190	185	180	176	171	167	162	158	153	149	144	140
8	196	191	187	182	177	173	168	164	159	155	150	146	142
9	197	193	188	184	179	175	170	166	161	157	152	148	143
750	1.199	194	190	185	181	176	172	167	163	158	154	149	145
1	201	196	191	187	182	178	173	169	164	160	155	151	146
2	202	198	193	188	184	179	175	170	166	161	157	152	148
3	204	199	195	190	185	181	176	172	167	163	158	154	149
4	205	201	196	192	187	182	178	173	169	164	160	155	151
5	207	202	198	193	189	184	179	175	170	166	161	157	152
6	209	204	199	195	190	186	181	177	172	167	163	158	154
7	210	206	201	196	192	187	183	178	174	169	164	160	155
8	212	207	203	198	193	189	184	180	175	171	166	161	157
9	213	209	204	200	195	190	186	181	177	172	168	163	158
760	1.215	210	206	201	196	192	187	183	178	174	169	165	160
1	217	212	207	203	198	193	189	184	180	175	171	166	162
2	218	214	209	204	200	195	190	186	181	177	172	168	163
3	220	215	211	206	201	197	192	187	183	178	174	169	165
4	221	217	212	208	203	198	194	189	184	180	175	171	166
5	223	218	214	209	204	200	195	191	186	181	177	172	168
6	225	220	215	211	206	201	197	192	188	183	178	174	169
7	226	222	217	212	208	203	198	194	189	184	180	175	171
8	228	223	219	214	209	204	200	195	191	186	181	177	172
9	230	225	220	215	211	206	201	197	192	188	183	178	174
770	1.231	226	222	217	212	208	203	198	194	189	185	180	175
1	233	228	223	219	214	209	205	200	195	191	186	182	177
2	234	230	225	220	215	211	206	201	197	192	188	183	178
3	236	231	227	222	217	212	208	203	198	193	189	185	180
4	238	233	228	223	219	214	209	205	200	195	191	186	182
5	239	234	230	225	220	216	211	206	202	197	192	188	183
6	241	236	231	227	222	217	212	208	203	198	194	189	185
7	242	238	233	228	223	219	214	209	205	200	195	191	186
8	244	239	234	230	225	220	216	211	206	202	197	192	188
9	246	241	236	231	227	222	217	213	208	203	199	194	189
780	247	242	238	233	228	223	219	214	209	205	200	195	191

Humidity Correction

t \ Per cent	10	20	30	40	50	60	70	80	90	100
°C										
10	+2	+2	+1	+1	0	−1	−1	−2	−2	−3
20	+4	+3	+2	+1	0	−1	−2	−3	−4	−5
30	+7	+6	+4	+2	0	−2	−4	−6	−7	−9

28629°—18——6

To insure the accuracy of the last figure given in the table, greater attention must be given to the temperature of the barometer, to the reduction of the barometer for gravity, and to the relative humidity of the air.

The temperature of the barometer must be within 2° of the temperature of the balance case, which is the temperature to be used; otherwise the barometer height may be reduced to what it would be at that temperature.

If the local value of "g" differs from 981.288 cm/sec^2 by 1.000 unit, then the usual reduction for gravity may be made but −0.5 mm must be added to this reduction term in order to eliminate the reduction that is included in the table. As this gravity reduction term is a constant for any one locality, it need be calculated only once for any one place.

Corrections for variations from 50 per cent relative humidity may be found in the appended "Humidity correction table."

TABLE 18.—Buoyancy Correction Factors for Materials of Different Densities,[19] Milligrams per Gram

In this table, above the diagonal line are given the buoyancy corrections per gram for material of several different densities, the proper correction being found at the intersection of the vertical column under the density of one of the materials, with the horizontal row corresponding to the density of the other material. The density of air is taken as 1.2 mg per milliliter. The differences in volume are also given below the diagonal line for use where 1.2 mg per milliliter is not a sufficiently close approximation to the density of the air.

Density grams per milliliter	Difference in volume, milliliter	21.5	21.1	8.4	8.3	7.8	7.4	2.7	2.6	Buoyancy correction, milligrams per gram
21.5			0.00108	0.0870	0.0888	0.0980	0.1063	0.389	0.406	
21.1		0.0009		0.0859	0.0877	0.0970	0.1052	0.388	0.405	
8.4		0.0725	0.0716		0.0018	0.0110	0.0193	0.302	0.319	
8.3		0.0740	0.0731	0.0015		0.0092	0.0175	0.300	0.317	
7.8		0.0817	0.0808	0.0092	0.0077		0.0083	0.291	0.308	
7.4		0.0886	0.0877	0.0161	0.0146	0.0069		0.282	0.299	
2.7		0.3239	0.3230	0.2514	0.2499	0.2422	0.2353		0.0170	
2.6		0.3381	0.3372	0.2656	0.2641	0.2564	0.2495	0.0142		

[19] On account of differences in the coefficients of expansion of different materials, the difference in volume observed or computed will not be constant, but will vary with the temperature. For example, if a platinum and a brass weight are changed in temperature from 0° to 20° C the difference in volume of the weights will change by 0.1 ml per kilogram and consequently the buoyancy correction will be altered by a little more than 0.1 mg per kilogram. Errors from this source, however, are negligible for the range of accuracy for which this table is suitable.

TABLE 19.—Definitions of Units of Mass

FUNDAMENTAL UNITS

A KILOGRAM (kg) is a unit of mass equivalent to the mass of the international prototype kilogram at the International Bureau of Weights and Measures.

An AVOIRDUPOIS POUND (lb. av.) is a unit of mass equivalent to 0.453 592 427 7 kilogram.

A GRAM (g) is a unit of mass equivalent to one-thousandth of the mass of the international prototype kilogram at the International Bureau of Weights and Measures.

A TROY POUND (lb. t.) is a unit of mass equivalent to $\frac{5760}{7000}$ of that of the avoirdupois pound.

MULTIPLES AND SUBMULTIPLES

1 metric ton (t) =1000 kilograms.
1 hectogram (hg) =100 grams=0.1 kilogram.
1 dekagram (dkg) =10 grams=0.01 kilogram.
1 decigram (dg) =0.1 gram.
1 centigram (cg) =0.01 gram.
1 milligram (mg) =0.001 gram.
1 avoirdupois ounce (oz. av.) =$\frac{1}{16}$ avoirdupois pound.
1 avoirdupois dram (dr. av.) =$\frac{1}{256}$ avoirdupois pound=$\frac{1}{16}$ avoirdupois ounce.
1 grain (gr.) =$\frac{1}{7000}$ avoirdupois pound =$\frac{10}{4375}$ avoirdupois ounce =$\frac{1}{5760}$ troy pound.
1 apothecaries' pound (lb. ap.) = 1 troy pound =$\frac{5760}{7000}$ avoirdupois pound.
1 apothecaries' or troy ounce (oz. ap. *or* ℥, *or* oz. t.) = $\frac{1}{12}$ troy pound =$\frac{480}{7000}$ avoirdupois pound =480 grains.
1 apothecaries' dram (dr. ap. *or* ʒ) = $\frac{1}{96}$ apothecaries' pound =⅛ apothecaries' ounce =60 grains.
1 pennyweight (dwt.) =$\frac{1}{20}$ troy ounce =24 grains.
1 apothecaries' scruple (s. ap. *or* ℈) =⅓ apothecaries' dram =20 grains.
1 metric carat[20] (c) =200 milligrams =0.2 gram.
1 short hundredweight (sh. cwt.) =100 avoirdupois pounds.
1 long hundredweight (l. cwt.) =112 avoirdupois pounds.
1 short ton (sh. tn.) =2000 avoirdupois pounds.
1 long ton (l. tn.) =2240 avoirdupois pounds.

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[20] See Bureau of Standards Circular No. 43, The Metric Carat, for a discussion of the metric carat and tables of the relations of the metric carat to the old carat of 205.3 mg in use in this country previous to July 1, 1913.

TABLE 20.—Conversion Factors for Units of Mass

UNITS OF MASS LESS THAN POUNDS AND KILOGRAMS

Units	Grains	Apothecaries' scruples	Pennyweights	Avoirdupois drams
1 grain =	1	0.05	0.041 666 67	0.036 571 43
1 apoth. scruple =	20	1	0.833 333 3	0.731 428 6
1 pennyweight =	24	1.2	1	0.877 714 3
1 avoir. dram =	27.343 75	1.367 187 5	1.139 323	1
1 apoth. dram =	60	3	2.5	2.194 286
1 avoir. ounce =	437.5	21.875	18.229 17	16
1 apoth. or troy ounce=	480	24	20	17.554 28
1 apoth. or troy pound=	5760	288	240	210.6514
1 avoir. pound =	7000	350	291.6667	256
1 milligram =	0.015 432 356	0.000 771 618	0.000 643 014 8	0.000 564 383 3
1 gram =	15.432 356	0.771 618	0.643 014 85	0.564 383 3
1 kilogram =	15 432.356	771.6178	643.014 85	564.383 32

Units	Apothecaries' drams	Avoirdupois ounces	Apothecaries' or troy ounces	Apothecaries' or troy pounds
1 grain =	0.016 666 7	0.002 285 71	0.002 083 33	0.000 173 611 1
1 apoth. scruple =	0.333 333	0.045 714 3	0.041 666 7	0.003 472 222
1 pennyweight =	0.4	0.054 857.1	0.05	0.004 166 667
1 avoir. dram =	0.455 729 2	0.0625	0.056 966 146	0.004 747 178 8
1 apoth. dram =	1	0.137 142 9	0.125	0.010 416 667
1 avoir. ounce =	7.291 66	1	0.911 458 3	0.075 954 861
1 apoth. or troy ounce =	8	1.097 142 9	1	0.083 333 33
1 apoth. or troy pound=	96	13.165 714	12	1
1 avoir. pound =	116.6667	16	14.583 333	1.215 277 8
1 milligram =	0.000 257 205 9	0.000 035 273 96	0.000 032 150 74	0.000 002 679 23
1 gram =	0.257 205 9	0.035 273 96	0.032 150 74	0.002 679 23
1 kilogram =	257.205 94	35.273 96	32.150 742	2.679 228 5

Units	Avoirdupois pounds	Milligrams	Grams	Kilograms
1 grain =	0.000 142 857 1	64.798 918	0.064 798 918	0.000 064 798 9
1 apoth. scruple =	0.002 857 143	1295.9784	1.295 978 4	0.001 295 978
1 pennyweight =	0.003 428 571	1555.1740	1.555 174 0	0.001 555 174
1 avoir. dram =	0.003 906 25	1771.8454	1.771 845 4	0.001 771 845
1 apoth. dram =	0.008 571 429	3887.9351	3.887 935 1	0.003 887 935
1 avoir. ounce =	0.0625	28 349.527	28.349 527	0.028 349.53
1 apoth. or troy ounce =	0.068 571 43	31 103.481	31.103 481	0.031 103 48
1 apoth. or troy pound=	0.822 857.1	373 241.77	373.241 77	0.373 241 77
1 avoir. pound =	1	453 592.4277	453.592 427 7	0.453 592 427 7
1 milligram =	0.000 002 204 62	1	0.001	0.000 001
1 gram =	0.002 204 62	1000	1	0.001
1 kilogram =	2.204 622 341	1 000 000	1000	1

UNITS OF MASS GREATER THAN AVOIRDUPOIS OUNCES

Units	Avoirdupois ounces	Avoirdupois pounds	Short hundred-weights	Short tons
1 avoirdupois ounce =	1	0.0625	0.000 625	0.000 031 25
1 avoirdupois pound =	16	1	0.01	0.0005
1 short hundredweight=	1600	100	1	0.05
1 short ton =	32 000	2000	20	1
1 long ton =	35 840	2240	22.4	1.12
1 kilogram =	35.273 957	2.204 622 34	0.022 046 223	0.001 102 311 2
1 metric ton =	35 273.957	2204.622 34	22.046 223	1.102 311 2

Units	Long tons	Kilograms	Metric tons
1 avoirdupois ounce =	0.000 027 901 79	0.028 349 53	0.000 028 349 53
1 avoirdupois pound =	0.000 446 428 6	0.453 592 427 7	0.000 453 592 43
1 short hundredweight=	0.044 642 86	45.359 243	0.045 359 243
1 short ton =	0.892 857 1	907.184 86	0.907 184 86
1 long ton =	1	1016.047 04	1.016 047 04
1 kilogram =	0.000 984 206 4	1	0.001
1 metric ton =	0.984 206 40	1000	1

TABLE 21.—Comparison of the Various Tons and Pounds in Use in the United States (from 1 to 9 Units)

Troy pounds	Avoirdupois pounds	Kilograms	Short tons	Long tons	Metric tons
1	0. 822 857	0. 373 24	0. 000 411 43	0. 000 367 35	0. 000 373 24
2	1. 645 71	0. 746 48	0. 000 822 86	0. 000 734 69	0. 000 746 48
3	2. 468 57	1. 119 73	0. 001 234 29	0. 001 102 04	0. 001 119 73
4	3. 291 43	1. 492 97	0. 001 645 71	0. 001 469 39	0. 001 492 97
5	4. 114 29	1. 866 21	0. 002 057 14	0. 001 836 73	0. 001 866 21
6	4. 937 14	2. 239 45	0. 002 468 57	0. 002 204 08	0. 002 239 45
7	5. 760 00	2. 612 69	0. 002 880 00	0. 002 571 43	0. 002 612 69
8	6. 582 86	2. 985 93	0. 003 291 43	0. 002 938 78	0. 002 985 93
9	7. 405 71	3. 359 18	0. 003 702 86	0. 003 306 12	0. 003 359 18
1. 215 28	1	0. 453 59	0. 0005	0. 000 446 43	0. 000 453 59
2. 430 56	2	0. 907 18	0. 0010	0. 000 892 86	0. 000 907 18
3. 645 83	3	1. 360 78	0. 0015	0. 001 339 29	0. 001 360 78
4. 861 11	4	1. 814 37	0. 0020	0. 001 785 71	0. 001 814 37
6. 076 39	5	2. 267 96	0. 0025	0. 002 232 14	0. 002 267 96
7. 291 67	6	2. 721 55	0. 0030	0. 002 678 57	0. 002 721 55
8. 506 94	7	3. 175 15	0. 0035	0. 003 125 00	0. 003 175 15
9. 722 22	8	3. 628 74	0. 0040	0. 003 571 43	0. 003 628 74
10. 937 50	9	4. 082 33	0. 0045	0. 004 017 86	0. 004 082 33
2. 679 23	2. 204 62	1	0. 001 102 31	0. 000 984 21	0. 001
5. 358 46	4. 409 24	2	0. 002 204 62	0. 001 968 41	0. 002
8. 037 69	6. 613 87	3	0. 003 306 93	0. 002 952 62	0. 003
10. 716 91	8. 818 49	4	0. 004 409 24	0. 003 936 83	0. 004
13, 937 50	11. 023 11	5	0. 005 511 56	0. 004 921 03	0. 005
16. 075 37	13. 227 73	6	0. 006 613 87	0. 005 905 24	0. 006
18. 754 60	15. 432 36	7	0. 007 716 18	0. 006 889 44	0. 007
21. 433 83	17. 636 98	8	0. 008 818 49	0. 007 873 65	0. 008
24. 113 06	19. 841 60	9	0. 009 920 80	0. 008 857 86	0. 009
2430. 56	2000	907. 18	1	0. 892 87	0. 907 18
4861. 11	4000	1814. 37	2	1. 785 71	1. 814 37
7291. 67	6000	2721. 55	3	2. 678 57	2. 721 55
9722. 22	8000	3628. 74	4	3. 571 43	3. 628 74
12 152. 78	10 000	4535. 92	5	4. 464 29	4. 535 92
14 583. 33	12 000	5443. 11	6	5. 357 14	5. 443 11
17 013. 89	14 000	6350. 29	7	6. 250 00	6. 350 29
19 444. 44	16 000	7257. 48	8	7. 142 86	7. 257 48
21 875. 00	18 000	8164. 66	9	8. 035 71	8. 164 66
2722. 22	2240	1016. 05	1. 12	1	1. 016 05
5444. 44	4480	2032. 09	2. 24	2	2. 032 09
8166. 67	6720	3048. 14	3. 36	3	3. 048 14
10 888. 89	8960	4064. 19	4. 48	4	4. 064 19
13 611. 11	11 200	5080. 24	5. 60	5	5. 080 24
16 333. 33	13 440	6096. 28	6. 72	6	6. 096 28
19 055. 56	15 680	7112. 32	7. 84	7	7. 112 32
21 777. 78	17 920	8128. 38	8. 96	8	8. 128 38
24 500. 00	20 160	9144. 42	10. 08	9	9. 144 42
2679. 23	2204. 62	1000	1. 102 31	0. 984 21	1
5358. 48	4409. 24	2000	2. 204 62	1. 968 41	2
8037. 69	6613. 87	3000	3. 306 93	2. 952 62	3
10 716. 91	8818. 49	4000	4. 409 24	3. 936 83	4
13 937. 50	11 023. 11	5000	5. 511 56	4. 921 03	5
16 075. 37	13 227. 73	6000	6. 613 87	5. 905 24	6
18 754. 60	15 432. 36	7000	7. 716 18	6. 889 44	7
21 433. 83	17 636. 98	8000	8. 818 49	7. 873 65	8
24 113. 06	19 841. 60	9000	9. 920 80	8. 857 86	9

TABLE 22.—Equivalents in Kilograms of 1 to 999 Avoirdupois Pounds

[Reduction factor: 1 avoirdupois pound=0.4535924277 kilogram]

Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos
0		100	45.35924	200	90.71849	300	136.07773	400	181.43697	500	226.79621	600	272.15546	700	317.51470	800	362.87394	900	408.23318
1	0.45359	1	45.81284	1	91.17208	1	136.53132	1	181.89056	1	227.24981	1	272.60905	1	317.96829	1	363.32753	1	408.68678
2	.90718	2	46.26643	2	91.62567	2	136.98491	2	182.34416	2	227.70340	2	273.06264	2	318.42188	2	363.78113	2	409.14037
3	1.36078	3	46.72002	3	92.07926	3	137.43851	3	182.79775	3	228.15699	3	273.51623	3	318.87548	3	364.23472	3	409.59396
4	1.81437	4	47.17361	4	92.53286	4	137.89210	4	183.25134	4	228.61058	4	273.96983	4	319.32907	4	364.68831	4	410.04755
5	2.26796	5	47.62720	5	92.98645	5	138.34569	5	183.70493	5	229.06418	5	274.42342	5	319.78266	5	365.14190	5	410.50115
6	2.72155	6	48.08080	6	93.44004	6	138.79928	6	184.15853	6	229.51777	6	274.87701	6	320.23625	6	365.59550	6	410.95474
7	3.17515	7	48.53439	7	93.89363	7	139.25288	7	184.61212	7	229.97136	7	275.33060	7	320.68985	7	366.04909	7	411.40833
8	3.62874	8	48.98798	8	94.34722	8	139.70647	8	185.06571	8	230.42495	8	275.78420	8	321.14344	8	366.50268	8	411.86192
9	4.08233	9	49.44157	9	94.80082	9	140.16006	9	185.51930	9	230.87855	9	276.23779	9	321.59703	9	366.95627	9	412.31552
10	4.53592	110	49.89517	210	95.25441	310	140.61365	410	185.97290	510	231.33214	610	276.69138	710	322.05062	810	367.40987	910	412.76911
1	4.98952	1	50.34876	1	95.70800	1	141.06725	1	186.42649	1	231.78573	1	277.14497	1	322.50422	1	367.86346	1	413.22270
2	5.44311	2	50.80235	2	96.16159	2	141.52084	2	186.88008	2	232.23932	2	277.59857	2	322.95781	2	368.31705	2	413.67629
3	5.89670	3	51.25594	3	96.61519	3	141.97443	3	187.33367	3	232.69292	3	278.05216	3	323.41140	3	368.77064	3	414.12989
4	6.35029	4	51.70954	4	97.06878	4	142.42802	4	187.78727	4	233.14651	4	278.50575	4	323.86499	4	369.22424	4	414.58348
5	6.80389	5	52.16313	5	97.52237	5	142.88161	5	188.24086	5	233.60010	5	278.95934	5	324.31859	5	369.67783	5	415.03707
6	7.25748	6	52.61672	6	97.97596	6	143.33521	6	188.69445	6	234.05369	6	279.41294	6	324.77218	6	370.13142	6	415.49066
7	7.71107	7	53.07031	7	98.42956	7	143.78880	7	189.14804	7	234.50729	7	279.86653	7	325.22577	7	370.58501	7	415.94426
8	8.16466	8	53.52391	8	98.88315	8	144.24239	8	189.60163	8	234.96088	8	280.32012	8	325.67936	8	371.03861	8	416.39785
9	8.61826	9	53.97750	9	99.33674	9	144.69598	9	190.05523	9	235.41447	9	280.77371	9	326.13296	9	371.49220	9	416.85144
20	9.07185	120	54.43109	220	99.79033	320	145.14958	420	190.50882	520	235.86806	620	281.22731	720	326.58655	820	371.94579	920	417.30503
1	9.52544	1	54.88468	1	100.24393	1	145.60317	1	190.96241	1	236.32165	1	281.68090	1	327.04014	1	372.39938	1	417.75863
2	9.97903	2	55.33828	2	100.69752	2	146.05676	2	191.41600	2	236.77525	2	282.13449	2	327.49373	2	372.85298	2	418.21222
3	10.43263	3	55.79187	3	101.15111	3	146.51035	3	191.86960	3	237.22884	3	282.58808	3	327.94733	3	373.30657	3	418.66581
4	10.88622	4	56.24546	4	101.60470	4	146.96395	4	192.32319	4	237.68243	4	283.04167	4	328.40092	4	373.76016	4	419.11940
5	11.33981	5	56.69905	5	102.05830	5	147.41754	5	192.77678	5	238.13602	5	283.49527	5	328.85451	5	374.21375	5	419.57300
6	11.79340	6	57.15265	6	102.51189	6	147.87113	6	193.23037	6	238.58962	6	283.94886	6	329.30810	6	374.66735	6	420.02659
7	12.24700	7	57.60624	7	102.96548	7	148.32472	7	193.68397	7	239.04321	7	284.40245	7	329.76169	7	375.12094	7	420.48018
8	12.70059	8	58.05983	8	103.41907	8	148.77832	8	194.13756	8	239.49680	8	284.85604	8	330.21529	8	375.57453	8	420.93377
9	13.15418	9	58.51342	9	103.87267	9	149.23191	9	194.59115	9	239.95039	9	285.30964	9	330.66888	9	376.02812	9	421.38737
30	13.60777	130	58.96702	230	104.32626	330	149.68550	430	195.04474	530	240.40399	630	285.76323	730	331.12247	830	376.48171	930	421.84096
1	14.06137	1	59.42061	1	104.77985	1	150.13909	1	195.49834	1	240.85758	1	286.21682	1	331.57606	1	376.93531	1	422.29455
2	14.51496	2	59.87420	2	105.23344	2	150.59269	2	195.95193	2	241.31117	2	286.67041	2	332.02966	2	377.38890	2	422.74814
3	14.96855	3	60.32779	3	105.68704	3	151.04628	3	196.40552	3	241.76476	3	287.12401	3	332.48325	3	377.84249	3	423.20174
4	15.42214	4	60.78139	4	106.14063	4	151.49987	4	196.85911	4	242.21836	4	287.57760	4	332.93684	4	378.29608	4	423.65533
5	15.87573	5	61.23498	5	106.59422	5	151.95346	5	197.31271	5	242.67195	5	288.03119	5	333.39043	5	378.74968	5	424.10892
6	16.32933	6	61.68857	6	107.04781	6	152.40706	6	197.76630	6	243.12554	6	288.48478	6	333.84403	6	379.20327	6	424.56251
7	16.78292	7	62.14216	7	107.50141	7	152.86065	7	198.21989	7	243.57913	7	288.93838	7	334.29762	7	379.65686	7	425.01610
8	17.23651	8	62.59576	8	107.95500	8	153.31424	8	198.67348	8	244.03273	8	289.39197	8	334.75121	8	380.11045	8	425.46970
9	17.69010	9	63.04935	9	108.40859	9	153.76783	9	199.12708	9	244.48632	9	289.84556	9	335.20480	9	380.56405	9	425.92329
40	18.14370	140	63.50294	240	108.86218	340	154.22143	440	199.58067	540	244.93991	640	290.29915	740	335.65840	840	381.01764	940	426.37688
1	18.59729	1	63.95653	1	109.31578	1	154.67502	1	200.03426	1	245.39350	1	290.75275	1	336.11199	1	381.47123	1	426.83047
2	19.05088	2	64.41012	2	109.76937	2	155.12861	2	200.48785	2	245.84710	2	291.20634	2	336.56558	2	381.92482	2	427.28407
3	19.50447	3	64.86372	3	110.22296	3	155.58220	3	200.94145	3	246.30069	3	291.65993	3	337.01917	3	382.37842	3	427.73766
4	19.95807	4	65.31731	4	110.67655	4	156.03580	4	201.39504	4	246.75428	4	292.11352	4	337.47277	4	382.83201	4	428.19125
5	20.41166	5	65.77090	5	111.13014	5	156.48939	5	201.84863	5	247.20787	5	292.56712	5	337.92636	5	383.28560	5	428.64484
6	20.86525	6	66.22449	6	111.58374	6	156.94298	6	202.30222	6	247.66147	6	293.02071	6	338.37995	6	383.73919	6	429.09844
7	21.31884	7	66.67809	7	112.03733	7	157.39657	7	202.75582	7	248.11506	7	293.47430	7	338.83354	7	384.19279	7	429.55203
8	21.77244	8	67.13168	8	112.49092	8	157.85016	8	203.20941	8	248.56865	8	293.92789	8	339.28714	8	384.64638	8	430.00562
9	22.22603	9	67.58527	9	112.94451	9	158.30376	9	203.66300	9	249.02224	9	294.38149	9	339.74073	9	385.09997	9	430.45921

50	22.67962	**150**	68.03886	**250**	113.39811	**350**	158.75735	**450**	204.11659	**550**	249.47584	**650**	294.83508	**750**	340.19432	**850**	385.55356	**950**	430.91281
1	23.13321	1	68.49246	1	113.85170	1	159.21094	1	204.57018	1	249.92943	1	295.28867	1	340.64791	1	386.00716	1	431.36640
2	23.58681	2	68.94605	2	114.30529	2	159.66453	2	205.02378	2	250.38302	2	295.74226	2	341.10151	2	386.46075	2	431.81999
3	24.04040	3	69.39964	3	114.75888	3	160.11813	3	205.47737	3	250.83661	3	296.19586	3	341.55510	3	386.91434	3	432.27358
4	24.49399	4	69.85323	4	115.21248	4	160.57172	4	205.93096	4	251.29020	4	296.64945	4	342.00869	4	387.36793	4	432.72718
5	24.94758	5	70.30683	5	115.66607	5	161.02531	5	206.38455	5	251.74380	5	297.10304	5	342.46228	5	387.82153	5	433.18077
6	25.40118	6	70.76042	6	116.11966	6	161.47890	6	206.83815	6	252.19739	6	297.55663	6	342.91588	6	388.27512	6	433.63436
7	25.85477	7	71.21401	7	116.57325	7	161.93250	7	207.29174	7	252.65098	7	298.01022	7	343.36947	7	388.72871	7	434.08795
8	26.30836	8	71.66760	8	117.02685	8	162.38609	8	207.74533	8	253.10457	8	298.46382	8	343.82306	8	389.18230	8	434.54155
9	26.76195	9	72.12120	9	117.48044	9	162.83968	9	208.19892	9	253.55817	9	298.91741	9	344.27665	9	389.63590	9	434.99514
60	27.21555	**160**	72.57479	**260**	117.93403	**360**	163.29327	**460**	208.65252	**560**	254.01176	**660**	299.37100	**760**	344.73025	**860**	390.08949	**960**	435.44873
1	27.66914	1	73.02838	1	118.38762	1	163.74687	1	209.10611	1	254.46535	1	299.82459	1	345.18384	1	390.54308	1	435.90232
2	28.12273	2	73.48197	2	118.84122	2	164.20046	2	209.55970	2	254.91894	2	300.27819	2	345.63743	2	390.99667	2	436.35592
3	28.57632	3	73.93557	3	119.29481	3	164.65405	3	210.01329	3	255.37254	3	300.73178	3	346.09102	3	391.45027	3	436.80951
4	29.02992	4	74.38916	4	119.74840	4	165.10764	4	210.46689	4	255.82613	4	301.18537	4	346.54461	4	391.90386	4	437.26310
5	29.48351	5	74.84275	5	120.20199	5	165.56124	5	210.92048	5	256.27972	5	301.63896	5	346.99821	5	392.35745	5	437.71669
6	29.93710	6	75.29634	6	120.65559	6	166.01483	6	211.37407	6	256.73331	6	302.09256	6	347.45180	6	392.81104	6	438.17029
7	30.39069	7	75.74994	7	121.10918	7	166.46842	7	211.82766	7	257.18691	7	302.54615	7	347.90539	7	393.26463	7	438.62388
8	30.84429	8	76.20353	8	121.56277	8	166.92201	8	212.28126	8	257.64050	8	302.99974	8	348.35898	8	393.71823	8	439.07747
9	31.29788	9	76.65712	9	122.01636	9	167.37561	9	212.73485	9	258.09409	9	303.45333	9	348.81258	9	394.17182	9	439.53106
70	31.75147	**170**	77.11071	**270**	122.46996	**370**	167.82920	**470**	213.18844	**570**	258.54768	**670**	303.90693	**770**	349.26617	**870**	394.62541	**970**	439.98465
1	32.20506	1	77.56431	1	122.92355	1	168.28279	1	213.64203	1	259.00128	1	304.36052	1	349.71976	1	395.07900	1	440.43825
2	32.65865	2	78.01790	2	123.37714	2	168.73638	2	214.09563	2	259.45487	2	304.81411	2	350.17335	2	395.53260	2	440.89184
3	33.11225	3	78.47149	3	123.83073	3	169.18998	3	214.54922	3	259.90846	3	305.26770	3	350.62695	3	395.98619	3	441.34543
4	33.56584	4	78.92509	4	124.28433	4	169.64357	4	215.00281	4	260.36205	4	305.72130	4	351.08054	4	396.43978	4	441.79902
5	34.01943	5	79.37867	5	124.73792	5	170.09716	5	215.45640	5	260.81565	5	306.17489	5	351.53413	5	396.89337	5	442.25262
6	34.47302	6	79.83227	6	125.19151	6	170.55075	6	215.91000	6	261.26924	6	306.62848	6	351.98772	6	397.34697	6	442.70621
7	34.92662	7	80.28586	7	125.64510	7	171.00435	7	216.36359	7	261.72283	7	307.08207	7	352.44132	7	397.80056	7	443.15980
8	35.38021	8	80.73945	8	126.09869	8	171.45794	8	216.81718	8	262.17642	8	307.53567	8	352.89491	8	398.25415	8	443.61339
9	35.83380	9	81.19304	9	126.55229	9	171.91153	9	217.27077	9	262.63002	9	307.98926	9	353.34850	9	398.70774	9	444.06699
80	36.28739	**180**	81.64664	**280**	127.00588	**380**	172.36512	**480**	217.72437	**580**	263.08361	**680**	308.44285	**780**	353.80209	**880**	399.16134	**980**	444.52058
1	36.74099	1	82.10023	1	127.45947	1	172.81871	1	218.17796	1	263.53720	1	308.89644	1	354.25569	1	399.61493	1	444.97417
2	37.19458	2	82.55382	2	127.91306	2	173.27231	2	218.63155	2	263.99079	2	309.35004	2	354.70928	2	400.06852	2	445.42776
3	37.64817	3	83.00741	3	128.36666	3	173.72590	3	219.08514	3	264.44439	3	309.80363	3	355.16287	3	400.52211	3	445.88136
4	38.10176	4	83.46101	4	128.82025	4	174.17949	4	219.53874	4	264.89798	4	310.25722	4	355.61646	4	400.97571	4	446.33495
5	38.55536	5	83.91460	5	129.27384	5	174.63308	5	219.99233	5	265.35157	5	310.71081	5	356.07006	5	401.42930	5	446.78854
6	39.00895	6	84.36819	6	129.72743	6	175.08668	6	220.44592	6	265.80516	6	311.16441	6	356.52365	6	401.88289	6	447.24213
7	39.46254	7	84.82178	7	130.18103	7	175.54027	7	220.89951	7	266.25876	7	311.61800	7	356.97724	7	402.33648	7	447.69573
8	39.91613	8	85.27538	8	130.63462	8	175.99386	8	221.35310	8	266.71235	8	312.07159	8	357.43083	8	402.79008	8	448.14932
9	40.36973	9	85.72897	9	131.08821	9	176.44745	9	221.80670	9	267.16594	9	312.52518	9	357.88443	9	403.24367	9	448.60291
90	40.82332	**190**	86.18256	**290**	131.54180	**390**	176.90105	**490**	222.26029	**590**	267.61953	**690**	312.97878	**790**	358.33802	**890**	403.69726	**990**	449.05650
1	41.27691	1	86.63615	1	131.99540	1	177.35464	1	222.71388	1	268.07312	1	313.43237	1	358.79161	1	404.15085	1	449.51010
2	41.73050	2	87.08975	2	132.44899	2	177.80823	2	223.16747	2	268.52672	2	313.88596	2	359.24520	2	404.60445	2	449.96369
3	42.18410	3	87.54334	3	132.90258	3	178.26182	3	223.62107	3	268.98031	3	314.33955	3	359.69880	3	405.05804	3	450.41728
4	42.63769	4	87.99693	4	133.35617	4	178.71542	4	224.07466	4	269.43390	4	314.79314	4	360.15239	4	405.51163	4	450.87087
5	43.09128	5	88.45052	5	133.80977	5	179.16901	5	224.52825	5	269.88749	5	315.24674	5	360.60598	5	405.96522	5	451.32447
6	43.54487	6	88.90412	6	134.26336	6	179.62260	6	224.98184	6	270.34109	6	315.70033	6	361.05957	6	406.41882	6	451.77806
7	43.99847	7	89.35771	7	134.71695	7	180.07619	7	225.43544	7	270.79468	7	316.15392	7	361.51316	7	406.87241	7	452.23165
8	44.45206	8	89.81130	8	135.17054	8	180.52979	8	225.88903	8	271.24827	8	316.60751	8	361.96676	8	407.32600	8	452.68524
9	44.90565	9	90.26489	9	135.62414	9	180.98338	9	226.34262	9	271.70186	9	317.06111	9	362.42035	9	407.77959	9	453.13884

TABLE 23.—Equivalents in Avoirdupois Pounds of 1 to 999 Kilograms

[Reduction factor: 1 kilogram=2.204622341 avoirdupois pounds]

Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds	Kilos	Pounds
0		100	220.4622	200	440.9245	300	661.3867	400	881.8489	500	1,102.3112	600	1,322.7734	700	1,543.2356	800	1,763.6979	900	1,984.1601
1	2.2046	1	222.6669	1	443.1291	1	663.5913	1	884.0536	1	1,104.5158	1	1,324.9780	1	1,545.4403	1	1,765.9025	1	1,986.3647
2	4.4092	2	224.8715	2	445.3337	2	665.7959	2	886.2582	2	1,106.7204	2	1,327.1826	2	1,547.6449	2	1,768.1071	2	1,988.5694
3	6.6139	3	227.0761	3	447.5383	3	668.0006	3	888.4628	3	1,108.9250	3	1,329.3873	3	1,549.8495	3	1,770.3117	3	1,990.7740
4	8.8185	4	229.2807	4	449.7430	4	670.2052	4	890.6674	4	1,111.1297	4	1,331.5919	4	1,552.0541	4	1,172.5164	4	1,992.9786
5	11.0231	5	231.4853	5	451.9476	5	672.4098	5	892.8720	5	1,113.3343	5	1,333.7965	5	1,554.2588	5	1,774.7210	5	1,995.1832
6	13.2277	6	233.6900	6	454.1522	6	674.6144	6	895.0767	6	1,115.5389	6	1,336.0011	6	1,556.4634	6	1,776.9256	6	1,997.3878
7	15.4324	7	235.8946	7	456.3568	7	676.8191	7	897.2813	7	1,117.7435	7	1,338.2058	7	1,558.6680	7	1,779.1302	7	1,999.5925
8	17.6370	8	238.0992	8	458.5614	8	679.0237	8	899.4859	8	1,119.9481	8	1,340.4104	8	1,560.8726	8	1,781.3349	8	2,001.7971
9	19.8416	9	240.3038	9	460.7661	9	681.2283	9	901.6905	9	1,122.1528	9	1,342.6150	9	1,563.0772	9	1,783.5395	9	2,004.0017
10	22.0462	110	242.5085	210	462.9707	310	683.4329	410	903.8952	510	1,124.3574	610	1,344.8196	710	1,565.2819	810	1,785.7441	910	2,006.2063
1	24.2508	1	244.7131	1	465.1753	1	685.6375	1	906.0998	1	1,126.5620	1	1,347.0243	1	1,567.4865	1	1,787.9487	1	2,008.4110
2	26.4555	2	246.9177	2	467.3799	2	687.8422	2	908.3044	2	1,128.7666	2	1,349.2289	2	1,569.6911	2	1,790.1533	2	2,010.6156
3	28.6601	3	249.1223	3	469.5846	3	690.0468	3	910.5090	3	1,130.9713	3	1,351.4335	3	1,571.8957	3	1,792.3580	3	2,012.8202
4	30.8647	4	251.3269	4	471.7892	4	692.2514	4	912.7136	4	1,133.1759	4	1,353.6381	4	1,574.1004	4	1,794.5626	4	2,015.0248
5	33.0693	5	253.5316	5	473.9938	5	694.4560	5	914.9183	5	1,135.3805	5	1,355.8427	5	1,576.3050	5	1,796.7672	5	2,017.2294
6	35.2740	6	255.7362	6	476.1984	6	696.6607	6	917.1229	6	1,137.5851	6	1,358.0474	6	1,578.5096	6	1,798.9718	6	2,019.4341
7	37.4786	7	257.9408	7	478.4030	7	698.8653	7	919.3275	7	1,139.7898	7	1,360.2520	7	1,580.7142	7	1,801.1765	7	2,021.6387
8	39.6832	8	260.1454	8	480.6077	8	701.0699	8	921.5321	8	1,141.9944	8	1,362.4566	8	1,582.9188	8	1,803.3811	8	2,023.8433
9	41.8878	9	262.3501	9	482.8123	9	703.2745	9	923.7368	9	1,144.1990	9	1,364.6612	9	1,585.1235	9	1,805.5857	9	2,026.0479
20	44.0924	120	264.5547	220	485.0169	320	705.4791	420	925.9414	520	1,146.4036	620	1,366.8659	720	1,587.3281	820	1,807.7903	920	2,028.2526
1	46.2971	1	266.7593	1	487.2215	1	707.6838	1	928.1460	1	1,148.6082	1	1,369.0705	1	1,589.5327	1	1,809.9949	1	2,030.4572
2	48.5017	2	268.9639	2	489.4262	2	709.8884	2	930.3506	2	1,150.8129	2	1,371.2751	2	1,591.7373	2	1,812.1996	2	2,032.6618
3	50.7063	3	271.1685	3	491.6308	3	712.0930	3	932.5553	3	1,153.0175	3	1,373.4797	3	1,593.9420	3	1,814.4042	3	2,034.8664
4	52.9109	4	273.3732	4	493.8354	4	714.2976	4	934.7599	4	1,155.2221	4	1,375.6843	4	1,596.1466	4	1,816.6088	4	2,037.0710
5	55.1156	5	275.5778	5	496.0400	5	716.5023	5	936.9645	5	1,157.4267	5	1,377.8890	5	1,598.3512	5	1,818.8134	5	2,039.2757
6	57.3202	6	277.7824	6	498.2446	6	718.7069	6	939.1691	6	1,159.6314	6	1,380.0936	6	1,600.5558	6	1,821.0181	6	2,041.4803
7	59.5248	7	279.9870	7	500.4493	7	720.9115	7	941.3737	7	1,161.8360	7	1,382.2982	7	1,602.7604	7	1,823.2227	7	2,043.6849
8	61.7294	8	282.1917	8	502.6539	8	723.1161	8	943.5784	8	1,164.0406	8	1,384.5028	8	1,604.9651	8	1,825.4273	8	2,045.8895
9	63.9340	9	284.3963	9	504.8585	9	725.3208	9	945.7830	9	1,166.2452	9	1,386.7075	9	1,607.1697	9	1,827.6319	9	2,048.0942
30	66.1387	130	286.6009	230	507.0631	330	727.5254	430	947.9876	530	1,168.4498	630	1,388.9121	730	1,609.3743	830	1,829.8365	930	2,050.2988
1	68.3433	1	288.8055	1	509.2678	1	729.7300	1	950.1922	1	1,170.6545	1	1,391.1167	1	1,611.5789	1	1,832.0412	1	2,052.5034
2	70.5479	2	291.0101	2	511.4724	2	731.9346	2	952.3969	2	1,172.8591	2	1,393.3213	2	1,613.7836	2	1,834.2458	2	2,054.7080
3	72.7525	3	293.2148	3	513.6770	3	734.1392	3	954.6015	3	1,175.0637	3	1,395.5259	3	1,615.9882	3	1,836.4504	3	2,056.9126
4	74.9572	4	295.4194	4	515.8816	4	736.3439	4	956.8061	4	1,177.2683	4	1,397.7306	4	1,618.1928	4	1,838.6550	4	2,059.1173
5	77.1618	5	297.6240	5	518.0863	5	738.5485	5	959.0107	5	1,179.4730	5	1,399.9352	5	1,620.3974	5	1,840.8597	5	2,061.3219
6	79.3664	6	299.8286	6	520.2909	6	740.7531	6	961.2153	6	1,181.6776	6	1,402.1398	6	1,622.6020	6	1,843.0643	6	2,063.5265
7	81.5710	7	302.0333	7	522.4955	7	742.9577	7	963.4200	7	1,183.8822	7	1,404.3444	7	1,624.8067	7	1,845.2689	7	2,065.7311
8	83.7756	8	304.2379	8	524.7001	8	745.1624	8	965.6246	8	1,186.0868	8	1,406.5491	8	1,627.0113	8	1,847.4735	8	2,067.9358
9	85.9803	9	306.4425	9	526.9047	9	747.3670	9	967.8292	9	1,188.2914	9	1,408.7537	9	1,629.2159	9	1,849.6781	9	2,070.1404
40	88.1849	140	308.6471	240	529.1094	340	749.5716	440	970.0338	540	1,190.4961	640	1,410.9583	740	1,631.4205	840	1,851.8828	940	2,072.3450
1	90.3895	1	310.8518	1	531.3140	1	751.7762	1	972.2385	1	1,192.7007	1	1,413.1629	1	1,633.6252	1	1,854.0874	1	2,074.5496
2	92.5941	2	313.0564	2	533.5186	2	753.9808	2	974.4431	2	1,194.9053	2	1,415.3675	2	1,635.8298	2	1,856.2920	2	2,076.7542
3	94.7988	3	315.2610	3	535.7232	3	756.1855	3	976.6477	3	1,197.1099	3	1,417.5722	3	1,638.0344	3	1,858.4966	3	2,078.9589
4	97.0034	4	317.4656	4	537.9279	4	758.3901	4	978.8523	4	1,199.3146	4	1,419.7768	4	1,640.2390	4	1,860.7013	4	2,081.1635
5	99.2080	5	319.6702	5	540.1325	5	760.5947	5	981.0569	5	1,201.5192	5	1,421.9814	5	1,642.4436	5	1,862.9059	5	2,083.3681
6	101.4126	6	321.8749	6	542.3371	6	762.7993	6	983.2616	6	1,203.7238	6	1,424.1860	6	1,644.6483	6	1,865.1105	6	2,085.5727
7	103.6173	7	324.0795	7	544.5417	7	765.0040	7	985.4662	7	1,205.9284	7	1,426.3907	7	1,646.8529	7	1,867.3151	7	2,087.7774
8	105.8219	8	326.2841	8	546.7463	8	767.2086	8	987.6708	8	1,208.1330	8	1,428.5953	8	1,649.0575	8	1,869.5197	8	2,089.9820
9	108.0265	9	328.4887	9	548.9510	9	769.4132	9	989.8754	9	1,210.3377	9	1,430.7999	9	1,651.2621	9	1,871.7244	9	2,092.1866

50	110.2311	**150**	330.6934	**250**	551.1556	**350**	771.6178	**450**	992.0801	**550**	1,212.5423	**650**	1,433.0045	**750**	1,653.4668	**850**	1,873.9290	**950**	2,094.3912
1	112.4357	1	332.8980	1	553.3602	1	773.8224	1	994.2847	1	1,214.7469	1	1,435.2091	1	1,655.6714	1	1,876.1336	1	2,096.5958
2	114.6404	2	335.1026	2	555.5648	2	776.0271	2	996.4893	2	1,216.9515	2	1,437.4138	2	1,657.8760	2	1,878.3382	2	2,098.8005
3	116.8450	3	337.3072	3	557.7695	3	778.2317	3	998.6939	3	1,219.1562	3	1,439.6184	3	1,660.0806	3	1,880.5429	3	2,101.0051
4	119.0496	4	339.5118	4	559.9741	4	780.4363	4	1,000.8985	4	1,221.3608	4	1,441.8230	4	1,662.2852	4	1,882.7475	4	2,103.2097
5	121.2542	5	341.7165	5	562.1787	5	782.6409	5	1,003.1032	5	1,223.5654	5	1,444.0276	5	1,664.4899	5	1,884.9521	5	2,105.4143
6	123.4589	6	343.9211	6	564.3833	6	784.8456	6	1,005.3078	6	1,225.7700	6	1,446.2323	6	1,666.6945	6	1,887.1567	6	2,107.6190
7	125.6635	7	346.1257	7	566.5879	7	787.0502	7	1,007.5124	7	1,227.9746	7	1,448.4369	7	1,668.8991	7	1,889.3613	7	2,109.8236
8	127.8681	8	348.3303	8	568.7926	8	789.2548	8	1,009.7170	8	1,230.1793	8	1,450.6415	8	1,671.1037	8	1,891.5660	8	2,112.0282
9	130.0727	9	350.5350	9	570.9972	9	791.4594	9	1,011.9217	9	1,232.3839	9	1,452.8461	9	1,673.3084	9	1,893.7706	9	2,114.2328
60	132.2773	**160**	352.7396	**260**	573.2018	**360**	793.6640	**460**	1,014.1263	**560**	1,234.5885	**660**	1,455.0507	**760**	1,675.5130	**860**	1,895.9752	**960**	2,116.4374
1	134.4820	1	354.9442	1	575.4064	1	795.8687	1	1,016.3309	1	1,236.7931	1	1,457.2554	1	1,677.7176	1	1,898.1798	1	2,118.6421
2	136.6866	2	357.1488	2	577.6111	2	798.0733	2	1,018.5355	2	1,238.9978	2	1,459.4600	2	1,679.9222	2	1,900.3845	2	2,120.8467
3	138.8912	3	359.3534	3	579.8157	3	800.2779	3	1,020.7401	3	1,241.2024	3	1,461.6646	3	1,682.1268	3	1,902.5891	3	2,123.0513
4	141.0958	4	361.5581	4	582.0203	4	802.4825	4	1,022.9448	4	1,243.4070	4	1,463.8692	4	1,684.3315	4	1,904.7937	4	2,125.2559
5	143.3005	5	363.7627	5	584.2249	5	804.6872	5	1,025.1494	5	1,245.6116	5	1,466.0739	5	1,686.5361	5	1,906.9983	5	2,127.4606
6	145.5051	6	365.9673	6	586.4295	6	806.8918	6	1,027.3540	6	1,247.8162	6	1,468.2785	6	1,688.7407	6	1,909.2029	6	2,129.6652
7	147.7097	7	368.1719	7	588.6342	7	809.0964	7	1,029.5586	7	1,250.0209	7	1,470.4831	7	1,690.9453	7	1,911.4076	7	2,131.8698
8	149.9143	8	370.3766	8	590.8388	8	811.3010	8	1,031.7633	8	1,252.2255	8	1,472.6877	8	1,693.1500	8	1,913.6122	8	2,134.0744
9	152.1189	9	372.5812	9	593.0434	9	813.5056	9	1,033.9679	9	1,254.4301	9	1,474.8923	9	1,695.3546	9	1,915.8168	9	2,136.2790
70	154.3236	**170**	374.7858	**270**	595.2480	**370**	815.7103	**470**	1,036.1725	**570**	1,256.6347	**670**	1,477.0970	**770**	1,697.5592	**870**	1,918.0214	**970**	2,138.4837
1	156.5282	1	376.9904	1	597.4527	1	817.9149	1	1,038.3771	1	1,258.8394	1	1,479.3016	1	1,699.7638	1	1,920.2261	1	2,140.6883
2	158.7328	2	379.1950	2	599.6573	2	820.1195	2	1,040.5817	2	1,261.0440	2	1,481.5062	2	1,701.9684	2	1,922.4307	2	2,142.8929
3	160.9374	3	381.3997	3	601.8619	3	822.3241	3	1,042.7864	3	1,263.2486	3	1,483.7108	3	1,704.1731	3	1,924.6353	3	2,145.0975
4	163.1421	4	383.6043	4	604.0665	4	824.5288	4	1,044.9910	4	1,265.4532	4	1,485.9155	4	1,706.3777	4	1,926.8399	4	2,147.3022
5	165.3467	5	385.8089	5	606.2711	5	826.7334	5	1,047.1956	5	1,267.6578	5	1,488.1201	5	1,708.5823	5	1,929.0445	5	2,149.5068
6	167.5513	6	388.0135	6	608.4758	6	828.9380	6	1,049.4002	6	1,269.8625	6	1,490.3247	6	1,710.7869	6	1,931.2492	6	2,151.7114
7	169.7559	7	390.2182	7	610.6804	7	831.1426	7	1,051.6049	7	1,272.0671	7	1,492.5293	7	1,712.9916	7	1,933.4538	7	2,153.9160
8	171.9605	8	392.4228	8	612.8850	8	833.3472	8	1,053.8095	8	1,274.2717	8	1,494.7339	8	1,715.1962	8	1,935.6584	8	2,156.1206
9	174.1652	9	394.6274	9	615.0896	9	835.5519	9	1,056.0141	9	1,276.4763	9	1,496.9386	9	1,717.4008	9	1,937.8630	9	2,158.3253
80	176.3698	**180**	396.8320	**280**	617.2943	**380**	837.7565	**480**	1,058.2187	**580**	1,278.6810	**680**	1,499.1432	**780**	1,719.6054	**880**	1,940.0677	**980**	2,160.5299
1	178.5744	1	399.0366	1	619.4989	1	839.9611	1	1,060.4233	1	1,280.8856	1	1,501.3478	1	1,721.8100	1	1,942.2723	1	2,162.7345
2	180.7790	2	401.2413	2	621.7035	2	842.1657	2	1,062.6280	2	1,283.0902	2	1,503.5524	2	1,724.0147	2	1,944.4769	2	2,164.9391
3	182.9837	3	403.4459	3	623.9081	3	844.3704	3	1,064.8326	3	1,285.2948	3	1,505.7571	3	1,726.2193	3	1,946.6815	3	2,167.1438
4	185.1883	4	405.6505	4	626.1127	4	846.5750	4	1,067.0372	4	1,287.4994	4	1,507.9617	4	1,728.4239	4	1,948.8861	4	2,169.3484
5	187.3929	5	407.8551	5	628.3174	5	848.7796	5	1,069.2418	5	1,289.7041	5	1,510.1663	5	1,730.6285	5	1,951.0908	5	2,171.5530
6	189.5975	6	410.0598	6	630.5220	6	850.9842	6	1,071.4465	6	1,291.9087	6	1,512.3709	6	1,732.8332	6	1,953.2954	6	2,173.7576
7	191.8021	7	412.2644	7	632.7266	7	853.1888	7	1,073.6511	7	1,294.1133	7	1,514.5755	7	1,735.0378	7	1,955.5000	7	2,175.9623
8	194.0068	8	414.4690	8	634.9312	8	855.3935	8	1,075.8557	8	1,296.3179	8	1,516.7802	8	1,737.2424	8	1,957.7046	8	2,178.1669
9	196.2114	9	416.6736	9	637.1359	9	857.5981	9	1,078.0603	9	1,298.5226	9	1,518.9848	9	1,739.4470	9	1,959.9093	9	2,180.3715
90	198.4160	**190**	418.8782	**290**	639.3405	**390**	859.8027	**490**	1,080.2649	**590**	1,300.7272	**690**	1,521.1894	**790**	1,741.6516	**890**	1,962.1139	**990**	2,182.5761
1	200.6206	1	421.0829	1	641.5451	1	862.0073	1	1,082.4696	1	1,302.9318	1	1,523.3940	1	1,743.8563	1	1,964.3185	1	2,184.7807
2	202.8253	2	423.2875	2	643.7497	2	864.2120	2	1,084.6742	2	1,305.1364	2	1,525.5987	2	1,746.0609	2	1,966.5231	2	2,186.9854
3	205.0299	3	425.4921	3	645.9543	3	866.4166	3	1,086.8788	3	1,307.3410	3	1,527.8033	3	1,748.2655	3	1,968.7278	3	2,189.1900
4	207.2345	4	427.6967	4	648.1590	4	868.6212	4	1,089.0834	4	1,309.5457	4	1,530.0079	4	1,750.4701	4	1,970.9324	4	2,191.3946
5	209.4391	5	429.9014	5	650.3636	5	870.8258	5	1,091.2881	5	1,311.7503	5	1,532.2125	5	1,752.6748	5	1,973.1370	5	2,193.5992
6	211.6437	6	432.1060	6	652.5682	6	873.0304	6	1,093.4927	6	1,313.9549	6	1,534.4171	6	1,754.8794	6	1,975.3416	6	2,195.8039
7	213.8484	7	434.3106	7	654.7728	7	875.2351	7	1,095.6973	7	1,316.1595	7	1,536.6218	7	1,757.0840	7	1,977.5462	7	2,198.0085
8	216.0530	8	436.5152	8	656.9775	8	877.4397	8	1,097.9019	8	1,318.3642	8	1,538.8264	8	1,759.2886	8	1,979.7509	8	2,200.2131
9	218.2576	9	438.7198	9	659.1821	9	879.6443	9	1,100.1065	9	1,320.5688	9	1,541.0310	9	1,761.4933	9	1,981.9555	9	2,202.4177