DEPARTMENT OF COMMERCE

CIRCULAR

OF THE

BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

No. 2

MEASUREMENTS OF LENGTH AND AREA, INCLUDING THERMAL EXPANSION

[5th Edition]
Issued April 30, 1915

[Superseding 3d Edition of Circular No. 2 on "Metal Tapes"]

WASHINGTON
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35. Melting Points of Chemical Elements.
36. The Testing and Properties of Electrical Condensers.
37. Electric Wire and Cable Terminology.
38. The Physical Testing of Mechanical Rubber Goods.
39. Specifications for and Measurement of Standard Sieves.
40. Sodium Oxalate as a Standard in Volumetric Analysis.
41. Testing and Properties of Textile Materials.
42. Metallographic Testing.
43. The Metric Carat.
44. Polarimetry.
45. The Testing of Materials.
46. Testing of Barometers.
47. Units of Weight and Measure; Definitions and Tables of Equivalents.
48. Standards of Gas Service.
49. Safety Rules to be Observed in the Operation and Maintenance of Electrical Equipment and Line.
50. National Standard Hose Couplings and Fittings for Public Fire Service.
51. Measurement of Time and Tests of Timepieces.
52. Regulation of Electrotyping Solutions.
53. Composition, Properties, and Testing of Printing Inks.
54. Proposed National Electrical Safety Code.

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MEASUREMENTS OF LENGTH AND AREA, INCLUDING THERMAL EXPANSION

CONTENTS

1. INTRODUCTION

This circular relates to those tests made by the Bureau involving the precise measurement of length and area. It supersedes Circular No. 2 on the Verification of Metal Tapes (third edition, issued July 1, 1909).

The Bureau will be glad to cooperate with investigators, manufacturers, and others not only in executing tests of the highest precision, but also, on request, in furnishing any information at its disposal concerning methods of measurement, the relations among and the legal status of units, the requirements that accurate standards should fulfill, and the design and construction of special apparatus.

The Bureau will not accept for tests standards and apparatus likely to change excessively or that can be checked with ordinary facilities, such as poorly divided scales or scales graduated on wood, celluloid, or paper. A certain minimum of quality will be insisted upon to admit to test.

2. CONDITIONS INFLUENCING THE DIMENSIONS OF BODIES

The dimensions of a body are determined to a great extent by such influencing conditions as temperature and mechanical stresses. Hence the complete description of any linear dimension of a body requires not only the specification of the distance between the defining points, but also all determining influences, together with the extent to which changes in these influences affect the length in question; for it is difficult, and often impossible, to reproduce the conditions under which the distance was measured with sufficient accuracy to meet all the demands of modern metrology.

In the case of bar standards of length the mechanical conditions are sufficiently defined and reproducible when it is specified that the bars are to be under atmospheric pressure and freely supported in some definite way, usually resting horizontally on two supports so placed that the bending of the bar will produce minimum change in the distance between the lines; but in the case of tapes, wires, and chains the tension as well as the mode of support must be specified. Immersion in a liquid may cause an appreciable change on account of the buoyancy supporting part of the weight. In the case of contact standards it is sometimes necessary to specify the size of the contact pieces used in making the measurements and also the pressure applied, especially if the measuring surfaces are not perfect planes, though intended to be.

The effect of temperature is taken into account by specifying both the temperature at which the standard has its assigned value and the relation that connects changes of length with changes of temperature.

The most common way of expressing the length of a body as a function of its temperature is by means of such an equation as

$$L_t = L_0 (1 + at + bt^2 + ct^3 + \cdots).$$

Here L_t is the length at t° centigrade and L_0 the length at 0°. As many of the coefficients a, b, c, etc., may be used as are sufficient to give the requisite accuracy.

When a solid has its temperature changed, especially by rapid cooling, slow changes in its dimensions continue long after it has attained the same temperature throughout its entire extent. This thermal hysteresis is often so minute as to require extremely delicate means for its detection; in the case of glass, however, it is very marked. The gradual contraction of glass thermometer bulbs has been observed to continue for over a quarter of a century, causing the zero reading to rise. The zero point of even a good thermometer may be shifted many degrees in a few minutes by heating to

several hundred degrees.[1] Most of this after-effect disappears in a few hours or days, and the disappearance is considerably accelerated by prolonged heating at a high temperature followed by slow annealing. Even precision length standards of very high grade have been found to change slightly in the course of time.

The determination of the temperature of a standard when in use and the shielding of it from sudden changes of temperature is as important as knowing its length as a function of its temperature. For accurate work the standard should be insulated as thoroughly as possible from the heat of the observer's body and of the illumination equipment; and its temperature should preferably be controlled by some constant temperature bath. The less possible it is to insulate the bar properly the more frequent must be the temperature readings and the more care must be taken that the thermometric device indicate the true temperature of the standard. If the temperature of the air surrounding a bar changes rapidly, the actual temperature of the bar may be quite different from the temperature indicated by the thermometer in contact with or near it. The X, or Tresca, and H types of cross section are designed to lessen differences between the temperature of the bar and the temperature indicated by a thermometer lying in the bar, and also to secure the maximum rigidity.

The use of a mercury-in-glass thermometer is the common and most economical method of determining the temperature, though the resistance thermometer and the thermocouple are being applied in the highest grade of precise measurements when accuracy and sensitiveness are desired. For most work at the ordinary room temperatures a thermometer having a zero mark (for frequent determination of the variation of the ice point) and graduated from 10° to 30° or 35° C, in tenths of a degree, with a Jena-glass bulb, has been found very satisfactory. The corrections of the thermometer should be known for the horizontal position, and the thermometer laid in the bar in such a position as to give the best average temperature of the interval in use.

Thermometers used on tapes need be graduated to half degrees or whole degrees only, but should have a greater range than those used indoors. They should preferably be laid on the tape, and for this purpose should be mounted on a metal back having the width of the tape and attached to the tape by a spring clip or by tying. When, however, a standard is used for measuring an interval on metal of the same material as itself, or an interval in which a member of the same material is to fit (such as a steel beam measured with a steel tape), the determination of temperature may be omitted, on the assumption that the coefficients of expansion of the standard and of the piece being measured are the same.

When the temperature of a standard bar is secured by packing it in ice, only the purest ice should be used, preferably ice manufactured from distilled water. The ice should be ground into pieces not over half an inch

[1] Bull. Bureau of Standards, 2, p. 189; 1906.

in length. Shaved ice may be packed over the graduation to be observed, and a hole made in the ice through which to sight on the line. Some distilled water should be mixed with the ice if the melting of the ice is not sufficient to provide water. When the melting is too rapid the water should be drained off occasionally and the ice frequently packed in about the bar.

The lengths of intervals on line standards are also affected by the condition in which the graduations are maintained, inasmuch as the amount of foreign matter in a line may change the apparent width of a line or cause its appearance under a microscope to change with either the intensity or the direction of the illumination. Cleaning the graduated surface to secure better illumination and consequently better definition of a line under a microscope is likely to wear down the edges of a line irregularly and change its apparent position. Accordingly, the surfaces of precise standards should be of a metal not likely to tarnish easily; the lines should be narrow, but as deep as possible; and the bar should at all times be kept as free as possible from dust or other foreign material.

For a similar reason it is necessary that the surfaces of contact standards, plug and disk gages, etc., should be properly cared for to prevent tarnishing or rusting, especially in the case of steel standards. Steel standards usually require some protective covering while not in use, such as vaseline or oil, but this should be thoroughly removed before the use of the standards for any accurate measurements, as a film of oil or grease, especially in the case of flat measuring surfaces, may materially increase the apparent dimensions of the gage.

3. FUNDAMENTAL UNIT OF LENGTH

The fundamental unit of length of the United States is the international meter, the primary standard of which is deposited at the International Bureau of Weights and Measures near Paris, France. This is a platinum-iridium bar with three fine lines at each end; and the distance between the middle lines of each end when the bar is at the temperature of 0° C, and is supported at the two neutral points 28.5 cm each side of the center is 1 meter by definition. Two copies of this bar are in the possession of the United States and are deposited at the Bureau of Standards.

The United States yard is defined by the relation.

$$1 \text{ yard} = \frac{3600}{3937} \text{ meter.}$$

The legal equivalent of the meter for commercial purposes was fixed as 39.37 inches by the law of July 28, 1866, and experience having shown that this value was exact within the error of observation, the United States Office of Standard Weights and Measures was, by Executive order in 1893, authorized to derive the yard from the meter by the use of this relation.

4. INTERRELATION OF SECONDARY AND SPECIAL UNITS OF LENGTH AND AREA, WITH TABLES OF EQUIVALENTS [2]

The secondary units of length in the metric system are multiples and submultiples of the meter based on powers of 10, and their equivalents in meters are shown in the following table:

Table of Metric Length Units

1 kilometer	=1000	meters=10^3 meters
1 hectometer	= 100	meters=10^2 meters
1 dekameter	= 10	meters=10 meters
1 decimeter	= 0.1	meters=10^{-1} meters
1 centimeter	= 0.01	meters=10^{-2} meters
1 millimeter	= 0.001	meters=10^{-3} meters
1 micron	= 0.000 001	meters=10^{-6} meters
1 millimicron	= 0.000 000 001	meters=10^{-9} meters
1 Ångström unit	= 0.000 000 000 1	meters=10^{-10} meters

The units of area of the metric system are likewise multiples and submultiples of the square meter in powers of 10 and are shown in the table below:

Table of Metric Area Units

1 square kilometer	=1 000 000	square meters=10^6 square meters
1 hectare, or square hectometer	= 10 000	square meters=10^4 square meters
1 are, or square dekameter	= 100	square meters=10^2 square meters
1 centare	= 1	square meter = 1 square meter
1 square decimeter	= 0.01	square meter =10^{-2} square meters
1 square centimeter	= 0.0001	square meter =10^{-4} square meters
1 square millimeter	= 0.000 001	square meter =10^{-6} square meters

The secondary units of the customary or English system have been developed irregularly, and for the most part have no legal status in this country in the national law, though the interrelation of many has been specified in some of the State laws. The values of many of them are often best known in terms of one of the secondary units rather than in terms of the yard, but the values of each of the principal units in terms of the others is given in the table below, while some of the equivalents of several of the many special units follow.

Table of Customary Length Units.

	Inches	Links	Feet	Yards	Rods	Chains	Furlongs	Miles
1 inch=	1	0.126 263	0.083 333 3	0.027 777 8	0.005 050 51	0.001 262 63	0.000 126 263	0.000 015 782 8
1 link=	7.92	1	0.66	0.22	0.04	0.01	0.001	0.000 125
1 foot=	12	1.515 152	1	0.333 333	0.060 606 1	0.015 151 5	0.001 515 15	0.000 189 393 9
1 yard=	36	4.545 45	3	1	0.181 818	0.045 454 5	0.004 545 45	0.000 568 181
1 rod=	198	25	16.5	5.5	1	0.25	0.025	0.003 125
1 chain=	792	100	66	22	4	1	0.1	0.0125
1 furlong=	7920	1000	660	220	40	10	1	0.125
1 mile=	63360	8000	5280	1760	320	80	8	1

[2] The Bureau also publishes tables of equivalents (Circular No. 47, "Units of Weight and Measure—Definitions and Tables of Equivalents"), giving the relations of metric to customary units in multiples up to 10 or 1000. This circular may be had upon application.

In the above table the values printed in boldfaced type are exact relations as given, all other values having been rounded off.

Other Units of Length

1 point=$\frac{1}{72}$ inch=0.001 157 41 foot.
1 line=$\frac{1}{12}$ inch=0.006 94 foot.
1 hand=4 inches=0.3 foot.
1 span=9 inches=$\frac{1}{8}$ fathom.
1 fathom=6 feet=0.05 cable length.
1 nautical mile
1 sea mile
1 geographical mile } U. S.=6080.20 feet=1.151 553 statute miles=1853.249 meters=1.000 033 British nautical miles.
1 degree of longitude on the equator (Clark's spheroid, 1866)=60.0679 nautical miles=69.1713 statute miles=365 224.6 feet=111 320.68 meters.
1 league=3 statute miles=15 840 feet.

Table of Customary Area Units.

	Square inches	Square links	Square feet	Square yards	Square rods	Square chains	Acres	Square miles
Sq. in.=	**1**	0.015 942 3	0.006 944 44	0.007 716 05	0.000 025 507 6	0.000 001 594 23	0.000 000 159 423	0.000 000 000 249 1
Sq. link=	**62.7264**	**1**	**0.4356**	**0.0484**	**0.0016**	**0.0001**	**0.00001**	**0.000 000 015 625**
Sq. ft.=	**144**	**2.295 684**	**1**	0.111 111 1	0.003 673 09	0.000 229 568	0.000 022 956 8	0.000 000 035 870 1
Sq. yd.=	**1296**	20.661 2	**9**	**1**	0.033 057 85	0.002 066 12	0.000 206 612	0.000 000 322 831
Sq. rod or pole=	**39204**	**625**	**272.25**	**30.25**	**1**	**0.0625**	**0.00625**	**0.000 009 765 625**
Sq. ch.=	**627 264**	**10 000**	**4356**	**484**	**16**	**1**	**0.1**	**0.000 156 25**
Acre=	**627 264 0**	**100 000**	**43 560**	**4840**	**160**	**10**	**1**	**0.001 562 5**
Sq. m. or sec.=	**4 014 489 600**	**64 000 000**	**27 878 400**	**3 097 600**	**102 400**	**6400**	**640**	**1**

In the above table the values printed in boldfaced type are exact relations as given, all other values having been rounded off.

Other Units of Area

1 square (floors and roofs)=100 square feet=9.29034 square meters.
1 township=36 square miles=93.2399 square kilometers.

5. PROTOTYPE METERS

The United States prototype meters Nos. 27 and 21 were received from the International Bureau of Weights and Measures in 1889. Their values in terms of the international meter are:

$$\text{No. } 27 = 1\text{ m} - 1.6\ \mu + 8.657\ \mu\ t + 0.00100\ \mu\ t^2$$
$$\text{No. } 21 = 1\text{ m} + 2.5\ \mu + 8.665\ \mu\ t + 0.00100\ \mu\ t^2$$

1 micron (μ) =0.000 001 meter (m). The temperature in centigrade degrees of the international hydrogen thermometer is represented by t. The probable error of comparison of the national prototypes with the international meter was found to be $\pm 0.04\ \mu$. Taking into account the uncertainties in the coefficient of expansion and other sources of error, it is estimated that the uncertainty in the lengths at temperatures between 20° and 25° C lies between ± 0.1 and $\pm 0.2\ \mu$.

The national prototype meters of all the contracting nations[3] were made at the same time under the direction of the International Committee on Weights and Measures. They are composed of an alloy of 90 per cent platinum and 10 per cent iridium, and are of the X-shaped, or Tresca, cross section. The lines are ruled at each end on well-polished areas in the neutral plane for the bar supported horizontally. The axis of each bar is defined by two parallel lines about 0.2 mm apart. At each end the axis is crossed perpendicularly by a group of three lines about 0.5 mm apart. The length represented by the bar is defined as the length of the axis intercepted between the central lines of the groups. All the lines are very fine and have sharp, smooth edges. Care was taken to make both the rulings and the polish of the surfaces so uniform that both ends appear alike when viewed under a high-power microscope. Meter No. 27, sealed in its metal case, is preserved in a fireproof vault at the Bureau of Standards. No. 21 is occasionally used to verify the secondary or working standards of the Bureau, and in special cases, where the highest accuracy is required, other meters are compared with it.

The Bureau also possesses two other platinum-iridium standards. These are known as No. 4 and No. 12. The former is divided into millimeters for its entire length, and in addition is ruled with a special line to define the yard.

6. SECONDARY STANDARDS

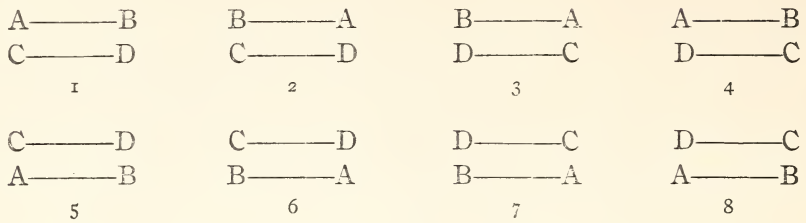
For the routine work of testing use is made of secondary or working standards whose values are carefully determined by comparison with prototype meter No. 21 from time to time to detect any possible changes. These working standards include multiples and submultiples of the meter and of the yard.

7. METHODS OF COMPARISON OF LENGTH STANDARDS

The Bureau of Standards employs various methods of making comparisons of bars sent to it for test, depending upon the character of the bar submitted, the accuracy desired, and the adaptability of the apparatus available to the bar or test piece. For precise comparison of high-grade meter and yard bars the comparison is made with our standards by transverse displacement of the two bars under two micrometer microscopes which are mounted on a rigid invar beam resting on two stone piers. To maintain a constant temperature the bars are inclosed in a double box during the comparisons. In order to eliminate as far as possible errors depending upon the relative positions of the bars, such as those produced by any inequality of illumination, microscope definition, and temperature condi-

[3] The 17 countries that signed the metric convention in 1875 are the United States, Germany, Austria, Belgium, Brazil, Argentina, Denmark, Spain, France, Italy, Peru, Portugal, Russia, Norway-Sweden, Switzerland, Turkey, and Venezuela. Since then Brazil, Venezuela, and Turkey have withdrawn, and Great Britain, Canada, Chile, Hungary, Japan, Mexico, Roumania, Servia, and Uruguay have joined the countries which contribute to the support of the International Bureau of Weights and Measures.

tions, the bars are usually compared in the eight possible combinations of positions with relation to each other as shown below:



When mercury thermometers are used to determine the temperature, they are also changed through a similar series of positions. For a more complete description of this method, see Bulletin of the Bureau of Standards, **1**, page 6 ff.; 1904–5.

Length standards requiring less accuracy of comparison are compared on apparatus of similar principle, but requiring less care and time for adjustments and readings.

Short bars are frequently compared by means of longitudinal displacement in which there is one microscope over each bar, settings being made alternately on the two ends of the interval compared. The shorter intervals, such as millimeters or less, are compared by this method or by direct run of the screw of a calibrated micrometer microscope.

Calibrations of the subdivisions of a bar are sometimes made by a similar method, or by intercomparison of each unit interval with a certain interval on the comparing machine by means of stops. For the most accurate calibrations each multiple of the unit interval is similarly compared and a least square adjustment of the results computed.[4]

Contact standards are compared on a special instrument designed and constructed at the Bureau of Standards, by which the comparison is made directly with a line standard instead of with another end standard, thus eliminating the usual intervening step of determination of the reference end standard first by comparison with a line standard.

Steel tapes are usually compared with the bench standard of the Bureau. This consists of a steel bar 50 mm by 12.5 mm (2 inches by ½ inch) cross-section and 50 meters long, with graduations ruled on inlaid platinum-iridium plugs. The tape, held at the zero end by an adjustable clamp, is stretched on the bench over the line of graduations. The zero line of the tape is placed over the zero line of the bench, the tension of the tape is adjusted to the desired amount, and comparisons between the tape and the bench are made by means of a hand glass and a small graduated scale placed on the bench at the edge of the tape.

[4] See Broch: Trav. et Mem. du Bur. Int. des Poids et Mesures, 5, 1886, for further information regarding the method of reduction of the observations.

8. CLASSIFICATION AND NATURE OF TESTS

The length measurements undertaken by this Bureau are classified under the several headings that follow. A complete test of a length standard includes the determination of the length at a known temperature and also of the expansivity, usually expressed by the average coefficient of expansion over a small range that embraces the temperatures at which the standard is likely to be used. For most work with any standard except one of the highest grade (Class A and geodetic tapes) it is sufficient to assume a coefficient of expansion derived from a knowledge of the composition of the standards. If the standard is subdivided, a calibration of the subdivisions is also necessary. The number of intervals that it is advisable to compare will depend on the character of the standard and on the use to which it is to be put. The total length and as many subdivisions as may be desired will be tested, the fee depending upon the labor involved. In many cases it is sufficient to know the correction for the full length only. For measuring distances greater or less than the full length it is unnecessary to test more than a few subdivisions. For example, any distance in even feet may be accurately measured with a 100-foot tape if the corrections are known for the entire length of the tape, for each 10-foot subdivision, and for each foot of the first 10 feet.

It should be noted that while platinum-iridium or invar may be the best material for length standards used in most scientific measurements, other materials may be better suited to the needs of manufacturers and engineers. For example, steel rules and tapes are better for determining the sizes of steel and iron machine parts and structural members, since changes in length with changes in temperature are practically the same in the steel rule or tape as in the steel or iron part being measured.

Metric standards should be graduated to be practically correct at either 0° C or 20° C, the latter temperature being used for metric tapes. Standards in the customary units of yards, feet, and inches are made to be correct at 62° F (16.67° C).

Requests for tests should generally state the use to which the results are to be applied and also the accuracy desired, in order that the tests may be adequate for the end in view, while avoiding unnecessary labor and expense. Where the highest attainable accuracy is needed, it is advisable to have the standard verified immediately before and again immediately after the important measurements in order to guard against possible changes due to undetected injury or structural alteration.

CLASS A. REFERENCE STANDARDS

[See Fee schedule 1, p. 19.]

This class includes standards of the highest type, suitable for reference standards for makers of precision apparatus and for use in the most exact scientific investigations. Perhaps the best construction is that conforming to the specifications adopted by the International Committee for the prototype meters; but less expensive constructions are sufficient for all but exceptional purposes. The bars should be of the X-shaped or of the

H-shaped cross section with all rulings in the neutral plane. The portion of the graduations to be used should be defined by two parallel longitudinal lines about 0.2 mm apart, and the graduations should be accurately perpendicular to these. Lines must be fine, with sharp, smooth edges, and ruled on surfaces that have been given a faultless mirror or dull polish, preferably the former. These surfaces should be on a material that does not oxidize or otherwise tarnish on exposure to air and moisture, and must not be protected by varnish or other such covering. Cleaning alters the character and even the position of graduations.

Next to the alloy of platinum and iridium used for the prototype meters the best material for most scientific purposes is the nickel-steel alloy known as invar; but see statement on pages 4 ff. and 11. Invar possesses a coefficient of expansion that is almost negligible at ordinary temperatures; in addition, it does not rust or tarnish readily on exposure to the atmosphere. Certain bronzes are also suitable; but since bronze tarnishes, the lines should be ruled on plugs of a non-tarnishing metal. It is important to remember that most materials undergo slight changes in the course of time, especially if subjected to considerable changes of temperature or to mechanical disturbances. Hence, whenever the highest accuracy is desired, standards should occasionally be verified by comparison with some fundamental or primary standards

B. WORKING STANDARDS

[See Fee schedule 2, p. 19.]

This class includes standards suitable for all ordinary work of precision and sufficient for the needs of college laboratories, manufacturers of the better grades of scientific apparatus, state superintendents of weights and measures, and most scientific work. Note the statement on pages 4 ff. and 11 in regard to materials. The lines of the graduations should be sharp and less than 0.03 mm wide, and they should be ruled on a good, plane surface that will not tarnish on exposure to the atmosphere, since cleaning alters the character and even the position of graduations. No varnish or other protective covering should be used. If the metal of the bar tarnishes readily, the lines should be ruled on plugs or strips of non-tarnishing metal. Some such means as a pair of parallel longitudinal lines about 0.2 mm apart should be provided for defining the portion of the graduations to be used, and the graduating lines should be accurately perpendicular to these. Standards of this class will be compared with working standards of the Bureau that are known in terms of the prototype meters, and will be certified to 0.001 mm if their quality justifies it.

C. COMMERCIAL STANDARDS

[See Fee schedule 3, p. 19.]

This class includes flat metal bars graduated along the center of a surface or along one or more edges, with lines less than 0.06 mm wide. They should be of a sufficiently good grade to meet the requirements of local sealers of weights and measures, drafting, fine machine work, etc. Such scales will be certified to 0.01 mm when the character of the graduations justifies.

D. METAL TAPES

[See Fee schedule 4, p. 20.]

This class includes steel and other metal tapes used in surveying, engineering, manufacturing, and construction work.

For the most accurate work, such as primary triangulation or extensive surveys, tapes of the alloy of nickel and steel, known as "invar," have been found to be the most satisfactory because of their low coefficient of expansion and the slowness with which they tarnish from exposure to the atmosphere. However, they require very careful handling in the field to prevent bending of the tape.

A more accurate standardization of the tape can be made if the central portion of the graduations is marked off by two longitudinal parallel lines about 1 mm apart and the parts of the graduations intercepted by these lines are employed in both the standardization and the field. If the method of making settings in the field is not suitable for this form of graduation, the graduations, which should be fine and uniform, should extend to the edge of the tape. If the graduations are on sleeves, the latter should be of invar or of some other nontarnishing metal, since polishing alters the character and even the position of lines, and in time obliterates them. The surface on which the graduations are ruled should be flat and the edges of the sleeves should be perpendicular to the graduated surface, so that the graduations will be of uniform width up to the very edge of the sleeve. Steel tapes of the same construction may also be used for precise geodetic work, but require further precautions in their use to avoid uncertainties as to the temperature of the tape. The coefficient of both invar and steel tapes should be determined when they are used for high precision work and their total length should be determined both before and after use to detect change in length due to accident.

Ordinary steel tapes to be used for engineering or construction work where an accuracy of less than 1 part in 200 000 is required may be of the construction given above or may consist of a steel ribbon with the graduations etched or ruled on the surface, or the intervals may be indicated by grooves or notches cut in sleeves securely fastened to the tape. Tapes having the intervals marked by rivets are not suitable for accurate work. Tapes having the zero mark at the edge of a ring and those having the terminal mark on a tension handle will be accepted for test, and a certificate will be given showing their errors, but they will not be marked with a Bureau of Standards identification number. (See below.)

The Bureau is equipped with two steel bench standards graduated up to 150 feet and 50 meters, with which tapes may be compared when supported either throughout their entire length or at certain intervals only. When an accuracy greater than 1 part in 200 000 is necessary, the Bureau will compare tapes of suitable length on its geodetic comparator.

The total length and as many divisions as may be desired will be tested, the fee depending upon the number of divisions compared. For most work it is sufficient to obtain the correction for the full length of the

tape only, and unless specifically requested to compare other intervals the Bureau will make comparison of the total length only. If, however, it is desired to have a tape for measuring accurately any integral number of feet it is sufficient to have every 10-foot point tested and each foot of the first or of the last 10 feet.

Tension.—Unless otherwise directed, tapes graduated into customary units will be compared under a tension of 10 pounds if not longer than 100 feet, and of 20 pounds if longer; similarly, metric tapes of 30 meters length or less at a 5-kg tension and over 30 meters at 10 kg. Tapes will also be tested at any other tension requested between 5 and 60 pounds, or between 2 and 25 kg. If desired, the Bureau will ascertain at what tension (to the nearest 0.5 pound or 0.25 kg) the tape is most nearly correct at the customary temperature and so certify it.

Spring balances submitted with tapes will be tested and the fee for the same charged, unless the Bureau is specially requested not to make this test. No test will be made of tension handles bearing the terminal mark, since they are necessarily used in determining the tape's length. Such tapes, however, will not be given a Bureau of Standards identification number. (See below.)

Supports.—The sender should clearly specify the method of support desired; otherwise tapes will be compared supported throughout their entire length. If specially requested, they will be tested either (*a*) supported at intervals of every 10 or every 25 feet or multiples of either up to 150 feet; or (*b*) for metric tapes at every 5 meters or every 12.5 meters or multiples of either up to 50 meters; or (*c*) both supported throughout and at intervals. For tapes supported at intervals only those intermediate divisions at which the tape is supported will be tested.

Temperature.—The corrections of the U. S. Bench Standard have been determined for the temperature of 62° F for the customary units and for 20° C for the metric units. Steel tapes will be compared with the U. S. Bench Standard at room temperature, usually between 21° and 30° C (70° and 86° F), and the coefficient of the tape assumed to be the same as that of the bench for reducing the lengths to the customary temperatures—62° F and 20° C—at which they are certified. The value of this coefficient of the U. S. Bench Standard as known at present is 0.000 006 45 per degree Fahrenheit, or 0.000 011 6 per degree centigrade. This rate of expansion with change of temperature when applied to a 100-foot interval of a steel tape amounts to 0.007 74 inch per degree Fahrenheit. It is accordingly necessary in all accurate work with steel tapes to know the temperature of the tape when in use, unless it is being used to measure steel structures or the space in which steel members are to fit, when it may be assumed that the steel piece and the tape expand the same amount and will have the same relative lengths at all temperatures.

If the above-assumed coefficient of 0.000 006 45 per degree Fahrenheit (or 0.000 011 6 per degree centigrade) is not the true coefficient of a tape, the correction given for 62° F (or 20° C), after reduction from a higher temperature on this basis, will be in error by an amount equal to the product

of the difference between the assumed and the true coefficient, the length of the interval, and the difference between 62° F (or 20° C) and the temperature at which the comparisons were made, which latter is always given on the Bureau's certificate. This error is not large, and for that reason it is generally unnecessary to determine the coefficient of steel tapes for most purposes. For example, if the true coefficient of a tape were 0.000 006 00 per degree Fahrenheit instead of 0.000 006 45, the correction of a 100-foot interval which had been compared at 82° F and reduced to 62° F would be 0.01 inch greater than the value given, or 1 part in 120,000. It is therefore necessary to determine the coefficient of expansion for only the most precise geodetic work, and, as its determination involves considerable time, labor, and expense, those who wish this done should first correspond with the Bureau as to the accuracy desired, the time available to do the work, and the fee to be charged.

Elasticity.—Young's modulus of elasticity will be determined when desired for tapes which have all the graduations on a steel ribbon of uniform cross section.

Serial Identification Number.—A Bureau of Standards serial identification number will be etched at the zero end upon the backs of tapes that have been tested subject to the general rules for such numbers on length measures (see p. 17), and under the following special conditions. It will be placed on tapes only when the initial and the terminal marks are on a single piece of metal ribbon and when the tape is not in error in the total length at the standard tension and temperature by more than 0.1 inch per 100 feet for tapes over 25 feet or by more than 0.05 inch for tapes 25 feet or less. It will not be placed on a tape on which the initial mark is a wire loop or other attachment liable to be detached or changed in shape, nor will it be placed on tapes of which the terminal mark is on a spring balance or tension handle accompanying the tape, nor on a tape which has its graduations on pieces of solder on the tape or on sleeves that are liable to come off. When a spring balance, not bearing terminal marks, accompanies a tape and is not in error by more than one-quarter of a pound, it will be marked with the same number as the tape submitted with it.

Tapes Without Reels.—When tapes submitted for test are not wound on reels, an additional charge will be made to cover the extra cost of handling. (See Fee schedule.)

E. CONTACT STANDARDS

[See Fee schedule 5, p. 20]

This class includes yard and meter end standards, end measuring rods with spherical or plane ends, reference disks, internal and external cylindrical gages, limit gages, caliper gages, thickness gages, flat-end gages, and similar objects. In the case of end measuring rods, or reference disks, when used to standardize a micrometer caliper, it is usually sufficient to determine the size at only one point. With fit or limit gages, however, or flat-end gages when used in combination it is necessary to determine the uniformity of size

throughout the gage, which is done by a systematic set of measurements of different diameters or thicknesses. Such gages when submitted for test should accordingly be accompanied by explicit directions as to the amount of work desired. Contact standards will usually be certified to 0.001 mm if the character of the defining surfaces permits.

F. PRECISION SCREWS, SCREW-THREAD GAGES, AND CALIPERS

[See Fee schedule 6, p. 20]

Besides the screws of such apparatus as dividing engines, precision lathes, micrometer microscopes, spherometers, and screw-thread gages, this class includes also micrometer calipers, vernier calipers, spherometers, paper micrometers, etc. Determinations will be made of the average value of a turn within any desired interval, as well as of the periodic and the progressive errors in the case of screws, or of the errors of readings at various points on dial-reading instruments. Persons wishing such instruments tested should specify exactly the points they wish examined, and, preferably, should first correspond with the Bureau as to the extent and accuracy of the measurements desired, the time available, and the fees.

G. AREAS AND AREA-MEASURING INSTRUMENTS

[See Fee schedule 7, p. 21]

This class includes planimeters and areometers, planimeter testing plates, and other areas or area measuring instruments. In the case of areas, such as planimeter testing plates, the accuracy attainable will depend upon the character of the defining lines. The fees for these tests will be dependent upon the nature of the test and the accuracy required.

H. THERMAL EXPANSION OF MATERIALS

[See Fee schedule 8, p. 21]

This class includes all those tests the purpose of which is to determine the linear expansivity of solids, except those made in connection with establishing the equation of a standard length. The expansion of a length standard is determined, if desired, in connection with the length test. It is important that applicants for expansion tests should communicate with the Bureau before sending specimens. They should state the general character of the material, the forms and sizes in which it can be furnished, the particular temperature range within which a knowledge of the expansion is required, the use to be made of the determinations, the accuracy needed or desirable, and any other details that might be of service in planning the test, so that it may be adequate to accomplish the end in view and at the same time avoid unnecessary labor with consequent expense and delay. Upon receipt of this information the Bureau will advise the applicant as to how far his requirements can be met, the forms and sizes in which the specimens should be submitted, and the fees to be charged.

9. CERTIFICATES AND REPORTS

When apparatus submitted fulfills the requirements for certification it will be tested and given a certificate of corrections. The certificate can indicate the corrections of the apparatus only at the time of the test, and does not guarantee the constancy of the values. When there are defects which exclude an apparatus from certification, or when in the nature of the case a certificate would be inappropriate, a report will be rendered giving such information as has been found. In such cases a special fee will be charged, depending upon the time consumed.

A Bureau of Standards serial identification number similar to that shown below—

B. S. № 591

will be engraved, stamped, or etched on apparatus, tapes, and standards which have been tested by the Bureau and found to fulfill the requirements of high-grade apparatus, to be reasonably accurate, or to conform to certain specifications, unless the article is of such a nature as to be liable to injury by the process of marking it. This number will also be given on the certificate, together with any serial numbers of the maker or owner which are noted. In the case of tapes, the placing of such a number on the tape is subject to certain special regulations, for which see page 15.

10. GENERAL INSTRUCTIONS TO APPLICANTS FOR TESTS

(*a*) *Application for Test.*—All articles submitted for test should be accompanied by a written request. This request should enumerate the articles, giving the identification marks of each, and should state explicitly the nature of the test desired. It is suggested that a prior application be made from two weeks to a month preceding the shipment of the apparatus if it is desired that the test be made promptly when the apparatus is received, inasmuch as regular tests are made in the order in which the applications are received, except as this practice may be varied by grouping similar tests together. This will facilitate the work of the Bureau as well as the prompt return of the apparatus. When the test is one regularly provided for in the appended schedules, the fee may be computed in advance, and should be sent at the time the apparatus is shipped.

(*b*) *Nature of Test.*—The application should state clearly the nature of the test desired, viz, the points at which test is to be made and the temperature or any other conditions. The most accurate results can be obtained and delays are avoided only when such full information is given. It is also desirable that the conditions under which the apparatus is used and the character of the work for which it is employed be stated. The classification of tests in this circular should be followed, and the schedule numbers below should be used to indicate the test desired. When apparatus is sent simply for test without definite instructions the Bureau will, when practicable, decide upon the nature of the test without correspondence.

(*c*) *Special Tests.*—The Bureau will gladly cooperate with scientific investigators, manufacturers of apparatus, and others who need higher precision than is provided in the regular tests, as far as the regular work of the Bureau will permit. Kinds of tests not at present provided for may be undertaken if the work is important and the facilities and time are available. Approved tests not provided for in the regular schedules will be considered special, and a special fee will be charged for them. The test should be arranged for by correspondence before shipment of the apparatus. The application should state fully the purpose for which the apparatus has been used or is to be used in the future, the need for the test, and the precision desired. The special fee charged will depend chiefly upon the time consumed and the amount of alteration required in the regular testing apparatus. An estimate of the fee will be given when possible.

(*d*) *Condition of Apparatus.*—Before submitting apparatus for test the applicant should ascertain that it fully satisfies the requirements for the test desired. All apparatus must be in good working condition. No repair work will be done at the Bureau; if repairs are needed, they should either be made by the applicant, or the apparatus should be sent to the maker before it is submitted for test. When defects are found after a test has been begun, which exclude an apparatus from receiving the usual certificate, a report will be rendered giving such information as has been found. In such cases a fee will be charged, depending upon the time consumed. All possible care will be taken in handling apparatus, but the risk of injury or breakage in shipment or under test must be borne by the applicant.

(*e*) *Identification Marks.*—All packages should be plainly marked with the shipper's name and address, and when convenient with a list of the contents. Each separate piece of apparatus or sample of material should be provided with an identification mark or number. The identification mark should be given in the application for the test.

(*f*) *Shipping Directions.*—Apparatus or test specimens should be securely packed in cases or packages which will not be broken in transportation and which may be used in returning them to the owner. The shipment in both directions is at the applicant's risk. It is recommended that shipment be made by express. Great care should be taken in packing. Clean, dry excelsior is a suitable packing material in most cases. Each instrument should also be wrapped in strong paper or other covering to prevent dust and excelsior from getting into it. The tops of boxes should be put on with screws, as the jar due to nailing and the subsequent opening is liable to cause damage. The tops of the shipping boxes should have the return or forwarding address on the underside. Transportation charges are payable by the party requesting the test. The charges for shipment to the Bureau must be prepaid, and, unless otherwise arranged, articles will be returned or forwarded by express "collect."

(*g*) *Address.*—Apparatus submitted for test, as well as all correspondence, should be addressed simply "Bureau of Standards, Washington, D. C."

(*h*) *Remittances.*—Fees in accordance with the appended schedules should be sent when the apparatus is shipped, or promptly upon receipt of bill. Certificates are not given, nor is apparatus returned, until the fees due thereon have been received. Remittances may be made by money order or by check drawn to the order of the "Secretary of Commerce."

11. SCHEDULES OF FEES

SCHEDULE 1.—REFERENCE STANDARDS

(See sec. 8, class A, p. 11, for specifications of standards included in this class)

(*a*) Determination of the total length of a yard or meter by comparison with the primary working standards of the Bureau at room temperature with a probable error of about 0.3 μ	$6.00
(*b*) Determination of the total length at an additional temperature to obtain the coefficient of expansion	5.00
(*c*) Determination of 12 equal submultiples of a length	12.00
(*d*) Determination of 10 equal submultiples of a length	10.00
(*e*) Determination of from 9 to 5 equal submultiples of a length	7.00
(*f*) Determination of less than 5 equal submultiples of a length	4.00
(*g*) Determination of any other single interval	3.00
(*h*) For determination of total length of a standard of length other than a yard or meter, the fee will be dependent upon the length of the bar, modification of apparatus necessary, and time required.	

SCHEDULE 2.—WORKING STANDARDS

(See sec. 8, class B, p. 12, for specifications of standards included in this class)

(*a*) Determination of the total length of a yard or meter at room temperature to an accuracy of 0.001 mm if the character of the graduation justifies	$4.00
(*b*) Determination of the total length at an additional lower temperature to obtain the coefficient of expansion	3.50
(*c*) Determination of 12 equal submultiples of a length	6.00
(*d*) Determination of 10 equal submultiples of a length	5.00
(*e*) Determination of from 9 to 5 equal submultiples of a length	3.50
(*f*) Determination of less than 5 equal submultiples of a length	2.50
(*g*) Determination of any other single interval	2.00
(*h*) For determination of total length of a standard of length other than a yard or meter, the fee will be dependent upon the length of the bar, time required, etc.	

SCHEDULE 3.—COMMERCIAL STANDARDS

(See sec. 8, class C, p. 12, for specifications of standards included in this class)

(*a*) Determination of the total length of a yard or meter at room temperature to an accuracy of 0.01 mm if the character of the graduation justifies	$2.00
(*b*) Determination of the corrections of 12 equal submultiples of a length	4.00
(*c*) Determination of the corrections of 10 equal submultiples of a length	3.00
(*d*) Determination of the corrections of from 9 to 5 equal submultiples of a length	2.00
(*e*) Determination of the corrections of less than 5 equal submultiples of a length	1.50
(*f*) Determination of any other single length	1.00
(*g*) Examination of intervening graduations of a bar and certification of their degree of accuracy to not more than 0.001 inch or 0.1 mm without giving the individual corrections of the points examined, for each 10 points examined	.50
(*h*) For determination of total length of a standard of length other than a yard or meter the fee will be dependent upon the length of the bar, time required, etc.	

SCHEDULE 4.—METAL TAPES

(See sec. 8, class D, p. 13, for regulations regarding the testing of tapes)

(*a*) For total length not greater than 100 feet or 50 meters, either supported throughout or at intervals $0.75
(*b*) For each additional 100-foot or 50-meter interval ..... .50
(*c*) For each 100-foot or 50-meter interval on the back of any tape compared ..... .50
(*d*) For comparing total length both supported throughout and at intervals for length of 100 feet or 50 meters 1.25
(*e*) Same for each additional 100 feet or 50 meters 1.00
(*f*) For each subdivision compared ..... .10
(*g*) For determination of length at an additional tension, or with an additional number of points of support when being tested supported at intervals, for each 100-foot or 50-meter interval ..... .25
(*h*) For determining the tension to the nearest 0.5 pound or 0.25 kilogram at which the tape is most nearly correct at the standard temperature, there will be an additional charge for each 100-foot or 50-meter interval of ..... .50
(*i*) For determination of Young's modulus of elasticity for each 100-foot or 50-meter interval. .75
(*j*) For determining the weight of a tape per foot or per meter ..... .50
(*k*) For testing spring balances accompanying tapes ..... .25
(*l*) For graduating tapes, each line, including the zero (this does not include the comparison of the lines) ..... .75
(*m*) For tapes not sent on a reel there will be an additional charge for each 100-foot or 50-meter length or fraction thereof of ..... .25
(*n*) A discount of 20 per cent will be allowed on the above fees when tapes are submitted in lots of 5 or more.
(*o*) Comparison of a 50-meter tape on the geodetic comparator 25.00
(*p*) Comparison of 2 or more 50-meter tapes on the geodetic comparator, each 15.00
(*q*) Comparison and determination of coefficient of expansion of a 50-meter tape on the geodetic comparator 40.00
(*r*) Comparison and determination of coefficient of expansion of two or more 50-meter tapes on the geodetic comparator, each 25.00
(*s*) Comparison of 50-meter tapes on the geodetic comparator at an additional tension or method of support 5.00

SCHEDULE 5.—CONTACT STANDARDS

(See sec. 8, Class E, p. 15, for standards included in this class)

(*a*) Comparison of end standard yard or meter $4.00
(*b*) Comparison of spherical end standards not more than 500 mm long 1.50
(*c*) Comparison of end standards having flat ends and not more than 500 mm long, including examination of the variation of the length between different parts of the contact surfaces 3.00
(*d*) Determination of a single diameter of a disk gage 1.50
(*e*) Each additional diameter of a disk gage for determining its eccentricity ..... .50
(*f*) Internal or external cylindrical gages, including investigation of eccentricity and taper. 6.00
(*g*) Cylindrical limit gages, including investigation of eccentricity and taper of both parts .. 10.00
(*h*) Examination of correctness of dimensions of products of manufacture, for a single point measured 1.50
(*i*) Each additional point measured under (*h*) ..... .50
(*j*) The above determinations under (*a*) to (*i*) being to an accuracy of about 0.001 mm when the character of the surfaces justify it, the same measurements will be made to an accuracy between 0.01 mm and 0.005 mm at half the above prices.
(*k*) A discount of 25 per cent will be made when two or more articles of the same size are submitted at the same time.

SCHEDULE 6.—PRECISION SCREWS AND CALIPERS

(See sec. 8, Class F, p. 16, for apparatus included in this class)

(*a*) Average screw value of a micrometer microscope screw over any desired interval of the screw $2.00
(*b*) Examination of periodic and progressive errors of the same 5.00
(*c*) Test of micrometer or venier caliper at five points 2.00
(*d*) Test of a paper micrometer at 5 points 2.00
(*e*) Each additional point under (*c*) or (*d*) ..... .25
(*f*) The fee for precision screws and screw-thread gages will depend upon the size of the screw or gage, the accuracy desired, and the time required.

SCHEDULE 7.—AREAS AND AREA-MEASURING INSTRUMENTS

(See sec. 8, Class G, p. 16, for tests included in this class)

(*a*) Measurement of four diameters of a planimeter-testing disk..... $2. 00
(*b*) Measurement of other areas, fee dependent upon size, number of measurements required, etc.
(*c*) Testing a planimeter or areometer..... 3. 00

SCHEDULE 8.—THERMAL EXPANSION OF MATERIALS

(See sec. 8, Class H, p. 16, for details of tests included in this class)

(*a*) The fees for this test will depend upon the range of temperature over which the expansivity is desired, the number of points at which readings are to be made, the size of samples available, the number of similar samples submitted at one time, etc. It will therefore be necessary for parties to correspond with the Bureau before submitting materials for test.

For educational and scientific institutions and societies a discount of 50 per cent will be allowed on all tests under the above schedules. Government or State authorities entitled to tests free of charge under the law must make application in writing for each test in order to avail themselves of the privilege.

S. W. Stratton,
Director.

Approved:
E. F. Sweet,
Acting Secretary.