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# ON THE CLIMATOLOGY OF GROUND-BASED RADIO DUCTS AND ASSOCIATED FADING REGIONS 

BY<br>E.J. DUTTON

U. S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS

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## NATIONAL BUREAU OF STANDARDS

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E. J. DUTTON


#### Abstract

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# ON THE CLIMATOLOGY OF GROUND-BASED RADIO DUCTS 

## AND ASSOCIATED FADING REGIONS

E. J. Dutton

An atmospheric duct is definedas occurring when geometrical optics indicates that a radio ray passing upwards through the atmosphere is sufficiently refracted that it travels parallel to the earth's surface. Maximum observed incidence of ducts was determined to be $13 \%, 10 \%$, and $5 \%$ by analysis of three to five years of radiosonde data for a tropical, temperate, and arctic location, respectively. Annual maxima are observed in the winter for the arctic and summer for the tropics. Arctic ducts arise from ground based temperature inversions with the ground temperature less than $-25^{\circ} \mathrm{C}$; temperate zone ducts arise from radiation inversions and accompanying humidity lapse; while tropical ducts occur with slight temperature and humidity lapses when the surface temperature is $30^{\circ} \mathrm{C}$ and greater. The mean initial elevation angle of a radio ray trapped by these ducts is found to be about 3 mr , with the maximum value about 5.8 mr . The steepest gradient of N observed is -420 N units $/ \mathrm{km}$. Observed ducts trap radio-waves of frequency $\geq 1 \mathrm{kMc}$ at all locations for at least $50 \%$ of the time.

Fading regions arising from abnormal defocussing of radio-rays passing from an elevated antenna down through the duct to a ground-based receiver are analyzed. The horizontal extent of these regions is determined for the same arctic, tropic, and temperate conditions given above.

E. J. Dutton

## Part I: Ground Based Radio Ducts

## 1. INTRODUCTION

The occurrence of atmospheric ducts places limitations upon ray tracing of VHF-UHF radio waves. Ducting is defined as occurring when a radio ray originating at the earth's surface is sufficiently refracted so that it either is bent back towards the earth's surface or travels in a path parallel to the earth's surface. Although the rigorous treatment of ducting involves consideration of the full wave equation solution [Booker and Walkinshaw, 1946] rather than a simple ray treatment, the present study will be based upon a geometrical optics definition of the limiting case in which ray tracing techniques may be used. This simple criterion is then applied to several years of radiosonde observations from stations typical of arctic, temperate, and tropical climates to derive estimates of the variation of the occurrence of radio ducts with climatic conditions.
2. METEOROLOGICAL CONDITIONS ASSOCIATED WITH RADIO REFRACTIVE INDEX PROFILES

The path followed by a radio ray in the atmosphere is dependent upon the gradient of the refractive index along that path. Of the vertical and horizontal gradient components that compose the path gradient, the horizontal gradient is normally negligibly small. Thus, the atmosphere is considered horizontally homogeneous and only the vertical gradient of the refractive index is utilized. The numerical value of the vertical gradient of the index of refraction depends on the vertical distribution of atmospheric temperature, humidity, and pressure.

Normally, temperature and humidity decrease with height in the atmosphere, since turbulence prevents any great changes in structure. However, there are periods of time in which the air becomes fairly calm, whereupon temperature inversions and humidity lapses can be built up and maintained. Temperature inversions have a two-fold importance in that (a) they can be widespread in area and persist over a relatively long period of time, and (b) they exercise a stabilizing influence on air motion such that turbulence is suppressed and strong humidity gradients may develop. Layers in which there is intense superrefraction to the point of duct formation may be formed as a result of these gradients and trapping of radio waves may follow. The inversions may start at ground level or at some greater height. The thickness of the layer can show great variability. Three processes that form temperature inversions are:

Advection: Advection is the horizontal transport of air having certain properties. Such a process is of importance in microwave propagation since it may lead to a different rate of exchange of heat and moisture between the air and the underlying ground or ocean surface, thus affecting the physical structure of the lowest layers of the atmosphere. This process results in air with different refractive index characteristics being brought into the area. The most common and important case of advection is that of dry air above a warm land surface flowing out over a cold sea. This type of advection frequently appears in the English Channel during the summer when the weather has been fine for several days.

Advective duct formation depends on two quantities: (a) the excess of the unmodified air temperature above that of water, and (b) the humidity deficit (the difference between the water vapor pressures of the modified and unmodified air). If these quantities are large, especially the humidity deficit, an intensive duct may form.

Important advective processes can also occur over land, but the conditions required for duct formation occur less frequently. However, a duct may be formed when dry, warm air flows over cold, wet ground with resultant temperature and humidity structure previously discussed.

Radiation: Differences in daytime and night-time radiation are the causes of diurnal variation in refractive conditions. A subrefractive layer may be present during the day, especially at the time of maximum surface heating. Clear skies and light surface winds at night result in considerable cooling of the earth, thus causing the formation of temperature inversions. The surface heat loss produced by nocturnal radiation is a prime factor in the formation of temperature inversions. Atmospheric stratification formed by such a combination of meteorological parameters may cause trapping. A temperature inversion is seldom strong enough to produce a duct in the middle and low latitudes, but it is of major importance in the formation of ducts in the northern latitudes. Low stratus clouds or extreme amounts of moisture (as in the tropics) tend to prevent loss by radiation which lessens the possibility of duct formation.

If the ground temperature in a nocturnal inversion falls below the dewpoint temperature, the water vapor in the lowest layers of the air condenses and the heat of condensation is released directly to the air. Under conditions of radiative fog formation, the humidity lapse tends to counteract any temperature inversion present and may cause substandard refraction if the humidity "inversion" is sufficiently strong. However, the temperature inversion may be strong enough to keep the layer standard or superrefractive.

Subsidence: Subsidence is the slow settling of air from a high pressure system. The air is heated by adiabatic compression as it descends and spreads out in a layer well above the earth's surface. This process
produces stable layers and inversions of temperature with an accompanying decrease in relative humidity. Since the air has come from a high level in the atmosphere, it is dry and may overlie a cooler, moist air mass. This type of inversion may cause the formation of an elevated superrefractive layer where the air temperature usually decreases immediately above the ground, rises through the inversion layer and decreases above the layer. This is a common occurrence which may be observed at any time. Subsidence has a tendency to destroy subrefractive layers and to intensify superrefractive layers. Although the effects of subsidence are generally observed at high levels, they are occasionally observed at lower levels, especially in the subtropics. Since subsidence frequently occurs in the lee of mountains and in the southeastern regions of northern hemisphere highs, elevated ducts may also be observed.

Conditions Inimical to Ducting: Conditions inimical to ducting are those which induce mixing of the lower atmosphere. Small scale atmospheric motions (turbulence) and consequent mixing and mass exchange result from differential surface heating and surface roughness effects. Both of these processes work to destroy stratification. They ultimately result in uniform vertical distribution of moisture through considerable depths of the lower atmosphere and the establishment of neutral temperature lapse rates. Accordingly, where the process of mechanical and convectively induced mixing are at work, the probability of the occurrence of ducting is vanishingly small. Thus, few, if any, ducts are observed over snowfree, low albedo land areas from mid-morning to late afternoon when the skies are clear, or in areas of moderate to great surface roughness when the surface winds are more than a few meters per second, irrespective of cloud conditions or the time of the day.

## Refractive Conditions Due to Local Meteorological Phenomena

Land and Sea Breeze: Land and sea breeze may produce ducts along the coastal regions since the winds are of thermal origin, resulting from temperature differences between land and sea surfaces. During the day, when the land gets warmer than the sea, the air above the land rises and is replaced by air from the sea, thus creating a circulation from the sea to the land, called a sea breeze. During the night, the land becomes colder than the sea and a circulation, called a land breeze, is set up in the opposite direction. This type of circulation is generally shallow and does not extend higher than a few hundred meters above the land or sea surface.

A land or sea breeze may modify the refractive conditions in different ways depending upon the distribution of moisture in the lower layers. Since these breezes are of a local nature and generally extend only a few miles, only coastal locations are usually affected. The very nature of sea and land breezes results in a marked refractive index pattern. With a sea breeze a duct may be formed over the water due to subsidence. The land breeze is accompanied by subsiding air over the land with resultant duct formation.

Fog: The formation of fog results in a decrease of superrefractive or ducting possibilities. When fog forms by nocturnal radiation, the water content of the air remains practically the same; however, part of the water changes from the gaseous to the liquid phase thus reducing the vapor pressure. The resulting humidity lapse rate tends to counteract the temperature inversion and cause sub-standard refraction. However, the temperature inversion may be strong enough to keep the layer standard or superrefractive. This process may also occur with advection fogs.

Nocturnal Ducts: The night-time temperature profile is a result of the interaction between nocturnal radiation, turbulence, and heat conduction. The associated refractive index profiles are such that a radar duct begins to form about the time of sunset, developing quickly during the early evening, more slowly after midnight, and dissipating rapidly after sunrise. This is mainly an inland effect resulting from large diurnal temperature variations observed in the interiors of large continents. However, a shallow body of water may have an appreciable diurnal temperature variation, as compared to the open ocean so that superrefraction may occur over such a location from time to time.

Limitations of Radiosonde Observations: It is generally recognized that radiosonde observations (RAOB's) do not have a sufficiently high degree of accuracy to be completely acceptable for use in observing changes in the degree of stratification of the very lowest layers of the atmosphere; however, until more accurate methods such as meteorological towers and refractometer measurements are more commonly used, the RAOB will continue to be used as a basis for forecasting the occurrence of superrefractive conditions.

## 3. BACKGROUND

The property of the atmosphere basic to radio ray tracing is the radio refractive index of the atmosphere, $n$, which, for VHF-UHF frequencies at standard conditions near the surface, is a number of the order of 1.0003. Although the refractive index is used in ray tracing theory, it is more convenient when evaluating refraction effects from common meteorological observations to use the refractivity [Smith and Weintraub, 1953], N , which, for the frequency range $0-30,000 \mathrm{Mc}$, is given by:

$$
\begin{equation*}
(\mathrm{n}-1) 10^{6}=\mathrm{N}=\frac{77.6}{\mathrm{~T}}\left[\mathrm{P}+\frac{4810 \mathrm{e}_{\mathrm{s}} \mathrm{RH}}{\mathrm{~T}}\right] \text {, } \tag{1}
\end{equation*}
$$

where $P$ is the station atmospheric pressure in millibars, and RH is the percent of the saturation vapor pressure, $e_{s}$, in millibars at the absolute temperature, $T$, in degrees Kelvin.

When evaluating the meteorological conditions that give rise to refractive phenomena, it is frequently instructive to examine separately the behavior of the dry component, D , and wet component, W , of N . These components are given by:

$$
\begin{equation*}
D=\frac{77.6}{T} P \text {. } \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{W}=\frac{3.73 \times 10^{5} \mathrm{e}_{\mathrm{s}} \mathrm{RH}}{\mathrm{~T}^{2}} \tag{3}
\end{equation*}
$$

The gradient of the refractivity, $\Delta N$, with respect to height may then be expressed:

$$
\begin{equation*}
\Delta N=\Delta D+\Delta W \tag{4}
\end{equation*}
$$

Average values of $\Delta N, \Delta D$ and $\Delta W$ are given for two increments between the earth's surface and one kilometer above sea level for Fairbanks, Alaska; Washington, D. C.; and Swan Island, W. I.; in Table I.

Table I
Gradient of $N, D$ and $W$ in ( $N$ units per kilometer)

| Station | Height | February |  |  | August |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | ---: | ---: |
|  | Increment | $-\Delta N$ | $-\Delta D$ | $-\Delta W$ | $-\Delta N$ | $-\Delta D$ | $-\Delta W$ |

Several general observations may be made of the data of Table I: The gradient of the dry term is relatively less variable than that of the wet term when considered as a function of season or height; the increase of $\Delta N$ from winter to summer at a particular location or from arctic to tropical-climate at a given time is most strongly reflected in $\Delta W$ rather than in $\Delta D$. The marked increase of gradient with height for Swan Island reflects the drop of refractivity across the interface of the trade wind inversion where dry subsiding air overlies the moist oceanic surface layer.

A fundamental equation used in radio ray tracing is Snell's law which, for polar coordinates, is given by:

$$
\begin{equation*}
n \mathrm{r} \cos \theta=n_{0} r_{0} \cos \theta_{0}, \tag{5}
\end{equation*}
$$

where n is the radio refractive index of the atmosphere, r is the radial distance from the center of the earth to the point under consideration,
$\theta$ is the elevation angle made by the ray at the point under consideration with the tangent to the circle of radius $r$ passing through that point. The radius to any point, $r$, is usually given as $(a+h)$ where $a$ is the radius of the earth and $h$ is the height of the point above sea level. The zero subscript refers to the value of $n, r$, or $\theta$ at the earth's surface.

For the present study, geometrical optics techniques, similar to, those considered by Bremmer [1949], are used to indicate when refraction is sufficiently great to direct a ray either back to earth or in a circular path at a constant height above the earth, i.e.:

$$
\begin{equation*}
\frac{\mathrm{n}_{\mathrm{o}} \mathrm{r}_{\mathrm{o}} \cos \theta_{\mathrm{o}}}{\mathrm{nr}} \geq 1 \tag{6}
\end{equation*}
$$

This condition then allows one to obtain the value of $\theta_{0}$ which divides the rays into two groups; those that penetrate the duct and those that are trapped within the duct. This particular value of $\theta_{o}$, called the angle of penetration and designated $\theta_{p}$, is obtained by noting those instances where

$$
\begin{equation*}
\frac{n_{o} r_{o}}{n r} \geq 1 \tag{7}
\end{equation*}
$$

and solving for the value of $\theta_{0}$ such that $(6)$ is equal to unity.
It is instructive to consider the order of magnitude of refractive index gradient needed for trapping for several commonly observed refractive index profiles. If we rewrite Snell's law,

$$
\begin{equation*}
n_{t} r_{t} \cos \theta_{t}=n_{A} r_{A} \cos \theta_{d} \tag{8}
\end{equation*}
$$

where the subscripts $t$ and A refer to the values of the variables at the transmitter height and the top of the trapping layer, then trapping occurs when

$$
\begin{equation*}
\frac{n_{t} r_{t}}{n_{A} r^{r}} \geq 1 \tag{9}
\end{equation*}
$$

and the angle of penetration at the transmitter, $\theta_{p}$, is given by setting

$$
\begin{equation*}
\frac{n_{t} r_{t}}{n_{A} r_{A}} \cos \theta_{p}=1 \tag{10}
\end{equation*}
$$

The maximum permissible $n$ gradient for a given value of $\theta_{p}$ is then given by:

$$
\begin{equation*}
\frac{\Delta n}{\Delta r}=-\frac{n_{t}-{ }^{n} A}{r_{A}-r_{t}} \tag{11}
\end{equation*}
$$

where ${ }^{n} A$ must satisfy (10), i.e.:

$$
\begin{equation*}
n_{A}=\frac{n_{t} r_{t}}{r_{A}} \cos \theta_{p} \tag{12}
\end{equation*}
$$

By designating $r_{A}=r_{t}+h_{A}$, (11) becomes

$$
\begin{equation*}
\frac{\Delta n}{\Delta r}=-\frac{n_{t}}{h_{A}}\left[1-\frac{r_{t}}{r_{t}+h_{A}} \cos \theta_{p}\right] \tag{13}
\end{equation*}
$$

By rewriting (13),

$$
\begin{equation*}
\frac{\Delta n}{\Delta r}=-\frac{n_{t}}{h_{A}}\left[1-\frac{1}{1+\frac{h_{A}}{r_{t}}} \cos \theta_{p}\right], \tag{14}
\end{equation*}
$$

and expanding $\left(1+\frac{h_{A}}{r_{t}}\right)^{-1}$ and $\cos \theta_{p}$, one obtains the expression:

$$
\begin{equation*}
\frac{\Delta n}{\Delta r}=-n_{t}\left[\frac{1}{r_{t}}+\frac{\theta^{2}}{2 h_{A}}\right] \tag{15}
\end{equation*}
$$

by neglecting terms of the order $\frac{\theta^{4}}{4!}$ and $\left(\frac{1}{r_{t}}\right)^{2}$.

For the case of $\theta_{p}=0$ (15) reduces to:

$$
\begin{equation*}
-\frac{\Delta \mathrm{n}}{\Delta \mathrm{r}}=\frac{\mathrm{n}}{\mathrm{a}} \sim \frac{1}{\mathrm{a}} \sim 157 \quad \mathrm{~N} \text { units/kilometer } \tag{16}
\end{equation*}
$$

It is seen from (15) that the n gradient necessary to trap a radio ray at a given value of $\theta_{p}$ is practically independent of transmitting antenna height above the earth. For example, a $\theta_{p}=0$ ray will be trapped by an $n$ gradient of -157.0 N units/kilometer at sea level, where $n_{t} \cong 1.0003$, while the necessary $n$ gradient at 3 kilometers above sea level will be -156.9 N units/kilometers for an $n_{t} \cong 1.0002$. This indicates for all practical applications, that the necessary n gradient for trapping is independent of altitude. Further, by considering the temperature and humidity gradients encountered in the troposphere, one is led to the conclusion that ducting gradients would not be expected to occur at altitudes greater than 3 kilometers. In fact, Cowan's [1953] investigation indicates that trapping gradients are nearly always confined to the first kilometer above the surface.

A consideration of (15) indicates that the magnitude of the negative gradient necessary for ducting is $1 / a$ for $\theta_{p}=0$ but increases in proportion to $\theta_{p}^{2} / 2 h$ as $\theta_{p}$ increases. The gradients necessary for
atmospheric ducts as a function of $\theta_{0}$ are given for several different profiles in figure 1. An analysis of radiosonde data indicates that gradients in excess of 0.5 N units per meter are seldom exceeded within atmospheric layers. It is interesting to note how rapidly the necessary gradients increase to the approximate upper limit of radio-sonde-observed gradients: a ground-based layer 100 meters thick attains this gradient at 8.3 milliradians while the maximum observed gradient is intercepted by the 30 -meter layer curve at $\theta_{0}=4.5$ milliradians. A third example was calculated for an elevated layer 0.5 km above the ground and 100 meters thick by assuming normal refraction between the ground and the base of the layer and solving for the necessary ducting gradient within the layer. The large values of $n$ gradient necessary for this case explains why elevated ducts were not observed. Although the preceding examples were calculated for a ground transmitter, the combinations of $\theta_{p}, \Delta n / \Delta r$, and $\Delta r$ are very nearly the same as would be obtained for any other transmitter height within the first 3 kilometers above the surface.

## 4. DESCRIPTION OF OBSERVED GROUND-BASED ATMOSPHERIC DUCTS

Approximately three years of radiosonde data typical of an arctic climate (Fairbanks, Alaska), a temperate climate (Washington, D. C.), and a tropical maritime climate (Swan Island, W. I.) were examined by means of a digital computer for the occurrence of ducts during the months of February, May, August, and November. The percentage occurrence of ducts is shown on figure 2. The maximum occurrences of $13.8 \%$ for Auguist at Swan Island and 9.2\% for Fairbanks in February are significantly greater than the values observed at other times of the year.

## REFRACTIVITY GRADIENTS NEEDED FOR RADIO DUCTS



Figure 1

## FREQUENCY OF OCCURRENCE OF GROUND BASED DUCTS



Months of the Year

Figure 2

The Washington data display a summertime maximum of $4.6 \%$. These data indicate that the temperate zone maximum incidence is about onehalf the wintertime maximum incidence in the arctic, and about one-third of the summertime tropical maximum.

The range of observed values of $\theta_{p}$ is shown in figure 3. The mean value calculated for each month as well as the maximum and minimum values of $\theta_{p}$ observed for the limiting cases are given for each month and location. The mean value of the angle of penetration under these conditions is between 2 and 3 milliradians and appears to be independent of climate. The maximum value of $\theta_{p}$ observed during ducting is 5.8 milliradians.

The refractivity gradients observed during ducting are given on figure 4. The maximum gradient of 420 N units per kilometer was observed during February at Fairbanks, Alaska. The mean values of N gradient appear to follow a slight climatic trend from a high value of 230 N units per kilometer at Fairbanks to a value of 190 N units per kilometer at Swan Island.

Another property of radio ducts is their thickness which is given in figure 5. Again there is observed a slight climatic trend as the median thickness increases from 66 meters at Fairbanks to 106 meters at Swan Island. These values of thickness correspond to the gradients given in figure 4. One can then obtain, by linear extrapolation the thickness at which the gradient is equal to $-1 / a ; i . e .$, the height corresponding to the gradient just sufficient to trap the ray at $\theta_{0}=0$. These values, shown on figure 6, display an increase in the median thickness of about $\mathbf{2 5 \%}$ for Swan Island, $100 \%$ for Washington, and $200 \%$ for Fairbanks, which results in a reversal of the climatic trend of the observed thickness between Fairbanks and Swan Island. This increase in height emphasized the preceding conclusion. Fairbanks is characterized by shallow layers with relative intense gradients.
ANGLE OF PENETRATION OF GROUNDD-BASED DUCTS


aug nov
Figure 3

REFRACTIVITY GRADIENTS OF GROUND-BASED DUCTS



ıдәшо|! $\boldsymbol{Y} / \mathrm{N} \nabla$ -
OBSERVED GROUND-BASED DUCT T.HICKNESS

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 280 |  |  |  |  |  | Washi | shington, D.C |  |  |  |  |  |  |  |  |  |
| 260 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 240 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 220 |  |  |  |  | , |  |  |  |  |  |  |  |  |  |  |  |
| 220 |  |  |  |  | V |  |  |  |  |  |  |  |  |  |  |  |
| 200 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 180 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 160 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 140 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | $\checkmark$ |  |  |  |  |  |  |  |  |  |
| 120 |  |  |  |  |  |  |  | $\cdots$ |  |  |  |  |  |  |  |  |
| $100$ |  |  |  |  |  |  |  |  |  |  |  |  | -Swan | islan |  |  |
| 80 |  |  |  |  |  |  |  |  |  |  |  |  | $\checkmark$ |  |  |  |
| 60 |  |  |  |  |  |  | Fairba | banks, A | ${ }_{\text {Alosko }}$ |  | - | $\square$ | Washin | gton, |  |  |
| 40 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 20 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Percentage of Observations That Equal or Exceed the Ordinate Value
sдəəəW u! ssəuyว!ч1 tonの pəлıəsqo

Figure 5
MAXIMUM GROUND-BASED DUCT THICKNESS

Percentage of Observations That Equal or Exceed the Ordinate Value
Figure 6

These maximum duct widths may be used to estimate the maximum radio wavelengths trapped. Kerr [1951] gives the maximum wavelength, $\lambda$, trapped by given thickness, $d$ :

$$
\begin{equation*}
\lambda_{\max }=c \gamma^{1 / 2} \mathrm{~d}^{3 / 2}, \tag{17}
\end{equation*}
$$

by assuming a linear decrease of $n$ within the duct. The coefficient $c$ is a constant and $\gamma$ is a function of the $n$ gradient excess over the minimum value of $\Delta \mathrm{N} / \Delta \mathrm{H}=1 / \mathrm{a}$. Expressing $\lambda_{\text {max }}$ in centimeters, and d in meters, then

$$
\begin{gathered}
\gamma=\left(\frac{N_{t}-N_{d}}{d}-.157\right) \cdot 10^{-6}, \\
c=2.514 \times 10^{2} .
\end{gathered}
$$

The maximum wavelengths trapped during ducting conditions were estimated by (17) for the maximum duct thicknesses of figure 6. These values, given in Table II, were determined for the month with the maximum occurrence of ducts, thus allowing an estimate of the radio frequencies likely to be effected by ducting conditions. Note, for example, that the data of Table II indicate that 1000 Mc rays ( $\lambda=30 \mathrm{~cm}$ ) will be trapped by $50 \%$ of the ducts regardless of location.

The reader is cautioned that an atmospheric duct does not have the sharp boundaries of a metallic waveguide. Thus the maximum wavelengths obtained by (17) do not represent cut-off frequencies but, as Kerr is so careful to emphasize, merely suggest lower limits under the assumptions of this rudimentary theory.

Table II

Estimated Maximum Wavelength Trapped at $\theta_{0}=0$

Station

|  | $95 \%$ | $90 \%$ | $75 \%$ | $50 \%$ | $25 \%$ | $10 \%$ | $5 \%$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Fairbanks, Alaska <br> (February) | 20.5 | 22.6 | 25.5 | 34.0 | 43.5 | 61.5 | 69.0 |
| Washington, D.C. <br> (August) | 7 | 10 | 27 | 50 | 112 | 164 | 200 |
| Swan Island <br> $\quad$ (August) | 12 | 23 | 41.5 | 60 | 82.3 | 92 | 132 |

## 5. TEMPERATURE AND HUMIDITY DISTRIBUTIONS ASSOCIATED WITH DUCTS

An example of the temperature and humidity distribution typical of each station is given on figure 7. The Fairbanks duct is accompanied by a surface temperature of $-25^{\circ} \mathrm{C}$ with a strong temperature inversion and a slight humidity lapse, indicating temperature inversions associated with wintertime cooling of the air next to the ground. The Washington example appears to be typical of the temperate zone temperature inversion. The Swan Island profile shows a moderate lapse of both temperature and humidity. This apparent contradiction is explained by the strong lapse in vapor pressure associated with the moderate lapse in temperature when the initial temperature is near $30^{\circ} \mathrm{C}$. The strong vapor pressure lapse presumably arises from evaporation from the sea surface. This effect was further examined by studying the percentage of the total $N$ gradient of each duct that was contributed by the gradient of the dry and wet terms. The median contribution of the dry term gradient, summarized in Table III, displays strong seasonal and geographic differences.

Median Contribution of $\Delta D / \Delta H$ to $\Delta N / \Delta H$ for Ducting Conditions
Fairbanks Washington, D. C. Swan Island

| February | $103 \%$ | $73.0 \%$ | $9.5 \%$ |
| :--- | :--- | :--- | :--- |
| May | $40.5 \%$ | $33.5 \%$ | $2.0 \%$ |
| August | $37.0 \%$ | $26.5 \%$ | $4.5 \%$ |
| November | $62.0 \%$ | $55.0 \%$ | $6.0 \%$ |

The dry term contribution decreases from summer to winter and from arctic to tropical climates. The Swan Island ducting gradients are at least $90 \%$ due to humidity lapse, while the Fairbanks winter time maximum is due to the strong temperature inversion associated with very low surface temperatures. In fact, under these conditions, vapor pressure increases with height with the result that the dry term contributes more than $100 \%$ of the ducting gradient.

# TEMPERATURE AND HUMIDITY PROFILES FOR TYPICAL GROUND-BASED DUCTS 



Figure 7


SHADOW ZONE OCCURRENCE ON A CURVED EARTH WITH THE TRANSMITTER ABOVE THE DUCT

Figure 8

## 6. CONCLUSIONS: Part I

The results given above were derived from a consideration of radiosonde data. Although the radiosonde is not an extremely sensitive instrument, it is readily available and is the only source of climatic information involving the temperature and humidity structure of the atmosphere. It is believed that the radiosonde data will at least yield the climatic trend of $\dot{r}$ adio ducts as well as their probable temperature and humidity distributions. Further, it is evident that the choice of stations will definitely affect the percentage of ducts observed. For example, it is almost certain that a greater percentage of ducts would be observed over water in the sub-tropics than over Swan Island.

With these reservations in mind, the present study has shown:
(a) Ducts occur no more than $15 \%$ of the time.
(b) The annual cycle of the incidence of ducts is reversed for the arctic and tropical stations studied. The arctic station has a wintertime maximum and the tropic station a summertime maximum. The temperate station has a summertime maximum incidence of less than $5 \%$.
(c) The maximum initial elevation angle is limited to 5.8 mr with a mean value of about 3 mr .
(d) The steepest gradient of N observed is -420 N units per kilometer.
(e) The maximum thickness of observed ducts is such as to trap radio-waves of 1000 Mc and above at all locations for at least $50 \%$ of the observed ducts.
(f) Ducts in the arctic appear to be associated with temperature inversions at ground temperatures of $-25^{\circ} \mathrm{C}$ or less; temperate zone, with the common radiation inversion and accompanying humidity lapse; tropics, with a moderate temperature and humidity lapse for temperatures of about $30^{\circ} \mathrm{C}$.

## Part II. A Study of Fading Regions Within the Horizon Caused by A Surface Duct Below a Transmitter.

## 7. INTRODUCTION

Among the factors influencing the choice of an antenna site for a microwave receiver operating on a line-of-sight path is the desirability of locating in a region relatively free from space-wave fadeouts. Although line-of-sight propagation is normally characterized by steady, high, dependable signals, deep, prolonged space-wave fadeouts are observed from time to time. Since serious disruptions occur during such fadeouts, a systematic discussion of fading phenomena is of considerable interest to the radio circuit engineer.

This section describes the conditions under which fading occurs, fading variations, and locations that favor the occurrence of fading within the radio horizon.

Fading, or fadeout, of a radio wave may be defined as a drop in power or field strength below a specified level of intensity. For a given site, criteria may be set up by the communications engineer to establish the magnitude of drop that defines the onset of fading conditions. This magnitude usually ranges from about 5 to 30 db below the specified level of intensity. The specified level is derived from some ideal propagation condition.

Fading is also a time-dependent occurrence. It may be classified as (a) prolonged fading, which is fading of sufficient interval to cause continued communication disruptions, and (b) short-term or "roller" fading, which is only instanteously observed by a receiver. The differentiation between prolonged and short-term fading is left to the discretion of the communications engineer.

One of the main causes of deep fading of field strength within the horizon as compared with the free space field strength value is the structure of the atmosphere. When one refers to the "structure" of the atmosphere, one generally means the refractive index structure. An easier way of representing the structure of the atmosphere is by means of the radio refractivity, $N$. Theoretically, the atmosphere might be considered horizontally homogeneous and in spherical stratifications concentric with the earth, and N can be considered to decrease exponentially with increas ing height above the earth's surface. However, in reality, this picture of the atmosphere is rarely, if ever, realized, because of the synoptic meteorological conditions that are perpetually present. Stratifications caused by the synoptic meteorological pattern give rise to field strength fading within the horizon by defocusing the lobe pattern of the transmitter along a given path. Whenever the rate of change of refractivity from the surface value with height (called the gradient of $N$ with height) is less than -157 N units per kilometer, a "ducting" condition is said to exist at the surface, and, as shown in figure 1, certain rays will tend to be "trapped" or guided within the surface duct. It is this atmospheric condition of surface ducting which will be further explored herein with respect to fading within the radio horizon.

## 8. REGIONS AND EXTENT OF FADING WITHIN THE HORIZON IN THE PRESENCE OF SUPERREFRACTION

Serious disruptions in reception from a transmitter above a duct to a receiver within a duct can occur at particular points within the horizon. It.is these disruptions, and these locations, which are of interest to the communications engineer establishing a given transmittingreceiving path. Whenever these disruptions occur between a transmitter and receiver within the horizon, a corresponding increase in the field strength characterized by steady high, dependable signals, is usually expected beyond the horizon. The high signal strength is in keeping with the properties of a surface duct. Nevertheless, deep, prolonged fadeouts can occur in regions beyond the horizon as well as within the horizon.

Price [1948] has theoretically determined regions of deep radio fading associated with surface ducts, which he has termed "shadow zones". In the case of a transmitter above a surface duct, a representation of what occurs is shown in figure 9. It shows the location of a shadow zone above the radio horizon line in the normal interference region. In the interference region, a fadeout of signal strength due to the presence of superrefraction must be compared with the value of the field when only the interference pattern is present. Papers such as those by Norton [1941], and Kirby, Herbstreit, and Norton [1952] give methods for calculation of the normal interference field. Ikegami [1959] gives a more general method for the calculation of received power in the presence of groundbased ducts.

In the procedure that follows, a model for the determination of the location and extent of shadow zones, rather than the determinations of the actual received field strengths and powers at any particular point, will be


Figure 80

REGIONS OF FADING FROM DIFFERENT EFFECTS
OF THE RADIO RAYS
Figure 9
given for conditions typical of the arctic, temperate, and tropical climates to aid the radio circuit engineer in avoiding these troublesome interference areas.

Washington, D.C., is taken to represent an average temperate zone climate, Swan Island an average tropical zone climate, and Fairbanks, Alaska, an average arctic zone climate. Considerable work has been done in determination of various ducting conditions at these three stations as well as with the average exponential refractivity above these stations. Therefore, these three stations will be used as models for all following calculations.

The gradient of N with height determines whether or not a surface duct will exist. Therefore, surface ducting atmospheres corresponding to conditions at Washington, D. C., Swan Island, and Fairbanks, consisting of a ducting gradient up to any desired height and the $\mathrm{N}_{\mathrm{s}}=313.0,377.2$, and 313.0 exponential atmospheres of Bean and Thayer [.1959] above this height, have been chosen for this study. The ducting gradients chosen are maximum, mean, and minimum values for February, May, August, and November (representing the middle of winter, spring, summer, and fall, respectively) taken from figure 4 and given in Table IV. Also in these particular models the duct is assumed to be of uniform height throughout the region considered in the calculations.

## 9. THEORY AND RESULTS

In figure 8, although it is known that ray theory does not explain what happens to the radio ray at the point $A$ when the ray is tangent to the top of the duct, because of the apparent ray intersection, it is sufficient to define the first shadow zone as that zone between the two possible paths of the ray tangent to the duct at a point $A$ (this is the ray emitted from the transmitter with an initial angle of $\theta_{0}$ with the horizontal).

## TABLE IV

## Ranges of Surface Ducting Gradients

| Duct <br> Gradients <br> at Washington D.C. | N units/km February | N units/km May | N units/km August | N units/km November |
| :---: | :---: | :---: | :---: | :---: |
| Max | -200 | -372 | -395 | -195 |
| Mean | -185 | -230 | -240 | -170 |
| Min | -175 | -162 | -158 | -159 |
| Duct <br> Gradients <br> at SwanIsland | N units/km February | N units/km May | N units/km August | N units/km November |
| Max | -195 | -268 | -282 | -282 |
| Mean | -173 | -193 | -193 | -203 |
| Min | -160 | -158 | -158 | -170 |
| ```Ducts Gradients at Fairbanks, Alaska``` | N units/km February | N units/km May | N units/km August | N units/km November |
| Max | -420 | -159 | -286 | -222 |
| Mean | -242 | -159 | -286 | -177 |
| Min | -169 | -159 | -286 | -159 |

This is possible because it can be readily seen that no radiation will be able to enter the shadow zone, and thus, a receiver located in the shadow zone will theoretically receive no signal from the transmitter. The portion of the "split" ray that enters the duct at A will be reflected at point $B$ to point C, where it "splits" again. The distance, d, from point A to C projected along the earth's surface will be designated the "length" of the first shadow zone. If the surface duct is homogeneous throughout, the ray will be symmetrical about point $B$ and, therefore,

$$
\begin{equation*}
d_{1}=d_{2}=d / 2=d_{h \cdot \ell}, \tag{19}
\end{equation*}
$$

where $d_{1}$, and $d_{2}$ are the "half-lengths", $d_{h \ell}$, of the shadow zone.
Schulkin [1952] shows that the bending, $\tau_{1,2}$, of a radio wave between any two points in the atmosphere is given by

$$
\begin{equation*}
\tau_{1,2}=\frac{d_{1,2}}{a}+\left(\theta_{1}-\theta_{2}\right) \tag{20}
\end{equation*}
$$

where $d_{1,2}$ is the distance between the two points 1 and 2 (Seefigure 8a) projected along the earth's surface, a is the radius of the earth, and $\theta_{1}$ and $\theta_{2}$ are the angles in radians the ray makes with the horizontal at points 1 and 2 respectively. Schulkin [1952] also gives $\tau_{1,2}$ in radians as

$$
\begin{equation*}
T_{1,2}=\frac{\Delta n_{1}-\Delta n_{2}}{\theta_{m}} \text { for } 0 \leq \theta_{0} \leq 10^{\circ} \tag{21}
\end{equation*}
$$

where $\Delta n_{1}=n_{1}-1, \Delta n_{2}=n_{2}-1, \theta_{m}=\left(\theta_{1}+\theta_{2}\right) / 2$. In terms of refractivity, equation (21) becomes

$$
\begin{equation*}
\tau_{1,2}=\frac{\left(\mathrm{N}_{1}-\mathrm{N}_{2}\right) \times 10^{-6}}{\theta_{\mathrm{m}}}=\frac{\Delta \mathrm{N} \times 10^{-6}}{\theta_{\mathrm{m}}} . \tag{22}
\end{equation*}
$$

Considering now the case of the half-length, $d_{h \ell}$, equation (22) becomes

$$
\begin{equation*}
T_{1,2}=\frac{d_{h l}}{a}+\theta_{p} \tag{23}
\end{equation*}
$$

since $\theta_{2}=0$ at the top of the duct, and $\theta_{1}=\theta_{p}$ (See figure 8). Now substituting equation (22) in equation (23):

$$
\begin{equation*}
\frac{2 \Delta N \times 10^{-6}}{\theta_{p}}=\frac{d_{h l}}{a}+\theta_{p} \tag{24}
\end{equation*}
$$

$$
\begin{gather*}
\text { since here } \theta_{\mathrm{m}}=\theta_{\mathrm{p}} / 2 \text {, Rearranging equation }(24) \\
\mathrm{dh} \ell=a\left[\frac{2 \Delta \mathrm{~N} \times 10^{-6}}{\theta_{\mathrm{p}}(\mathrm{rad})}-\theta_{\mathrm{p}}(\mathrm{rad})\right] \tag{25}
\end{gather*}
$$

Since

$$
\begin{equation*}
\underset{p}{\theta_{\mathrm{p}}}=\cos ^{-1} \frac{{ }^{n} A^{r} A}{n_{0} a} \cong \sqrt{2\left[\Delta N \times 10^{-6}-\frac{\left(r_{A}-a\right)}{a}\right]}, \tag{26}
\end{equation*}
$$

where $n_{A}$ is the index of refraction at the top of the duct, $r_{A}$ is the distance from the center of the earth to the top of the duct, and $n_{0}$ is the index of refraction at the earth's surface, it is seen that $d_{h \ell}$ is uniquely determined from the surface gradient of N with height inside the duct, provided the height of the duct is known.

If one desires to know the distance, $d_{h}$, for a particular height, $h$, from the reflection point of the ray in the duct (point B in figure 8), where $d_{h}$ is such that

$$
\begin{equation*}
0 \leq \mathrm{d}_{\mathrm{h}} \leq \mathrm{d}_{\mathrm{h} \mathrm{\ell}}, \tag{27}
\end{equation*}
$$

then equation (25) becomes

$$
\begin{equation*}
d_{h}=a\left[\frac{2 \Delta N^{\prime} \times 10^{-6}}{\theta_{p}+\theta_{2}}-\left(\theta_{p}-\theta_{2}\right)\right] \tag{28}
\end{equation*}
$$

since $\theta_{2}$ is no longer just equal to zero, but is such that

$$
\begin{equation*}
0<\theta_{2} \leq \theta_{p} \tag{29}
\end{equation*}
$$

and $\Delta N^{\prime}$ will no longer be the same as the $\Delta N$ for the duct width.
$\theta_{2}$ may be determined from the formula:

$$
\begin{equation*}
\theta_{2} \cong \sqrt{\theta_{p}^{2}+\frac{2 h}{a}-2 \Delta N^{\prime}} \tag{30}
\end{equation*}
$$

In figures $10-18$ the shadow zone half-lengths are given as a function of duct height for the various seasons of the year. In figures 19 22 the Washington, D. C., values of shadow zone half-length are again given as a function of duct height, but this time the ducting gradientis used as a parameter. This was done specifically to point out the seasonal trend of the half-lengths. Washington is the only station plotted since it is the only station of the three considered that has an appreciable seasonal variation in weather along with sufficient data with which to plot the trend. As might be expected, a wider range of distance variations is to be found in the spring and summer months than in the fall and winter months because of the greater percentage of ducts at Washington, D. C., during the spring and summer seasons. As shown in figure 5, the $50 \%$ level, or median of observed ground-based duct thickness at Washington, D. C., is about 100 meters, at Swan Island about 108 meters, and at Fairbanks, Alaska, about 65 meters. The $50 \%$ levels of observed maximum ground-based duct thicknesses are about 200, 135, and 185 meters, respectively, for Washington, D. C., Swan Island, and Fairbanks, Alaska. In Table V, the values of $d_{h \ell}$ during the four seasons are shown for the $65,100,108,135,185$, and 200 meter surface duct heights.

As previously mentioned, the atmospheres above the assumed duct were taken to coincide with the $N_{s}=313.0,377.2$, and 313.0 exponential atmospheres of Bean and Thayer [1959] because these values most nearly correspond to the annual average $\mathrm{N}_{\mathrm{s}}$ value at Washington, D. C., Swan Island, and Fairbanks, respectively. Figures 23-26 show the distance measured along the earth's surface that a ray with an initial elevation angle


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Figure 14


Figure 16

SHADOW ZONE HALF-LENGTHS FOR
MINIMUM DUCTING CONDITIONS AT FAIRBANKS, ALASKA


SEE FIGURE 12, NOVEMBER CURVE
FOR AUGUST CURVE SEE FIGURE 17
AUGUST CURVE

$100 \quad 120 \quad 140 \quad 160 \quad 180 \quad 200$
GHT, $h$, METERS
Figure 18
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500
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$\stackrel{\circ}{\mathrm{o}} \stackrel{\circ}{\circ}$

## TABLE V

Half-lengths for Various Duct Heights and Surface Gradient Conditions

|  |  | Washington, D.C. |  | Fairbanks, Alaska |  | Swan Island |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \bar{C} D_{\text {Duct }} \\ \text { nd }_{2} h t . \end{gathered}$ | 100 m | 200 m | 65 m | 185 m | 100 m | 135 m |
| Feb. | Max | 68.21 | 96.46 | 22.08 | 37.25 | 74.00 | 98.96 |
|  | Mean | 84.50 | 119.50 | 38.63 | 65.18 | 112.80 | 147.63 |
|  | Min | 105.35 | 148.98 | 100.54 | 169.61 | 248.75 | 325.56 |
| May | Max | 30.53 | 43.17 | 231.60 | 390.71 | 43.63 | 57.13 |
|  | Mean | 52.37 | 74.05 | 231.60 | 390.71 | 75.99 | 99.46 |
|  | Min | 199.42 | 281.95 | 231.60 | 390.71 | 401.86 | 525.95 |
| Aug. | Max | 29.02 | 41.04 | 31.43 | 53.03 | 41.16 | 53.87 |
|  | Mean | 49.12 | 69.46 | 31.43 | 53.03 | 75.99 | 99.46 |
|  | Min | 440.73 | 622.53 | 31.43 | 53.03 | 401.86 | 525.95 |
| Nov. | Max | 72.56 | 102.60 | 44.10 | 74.40 | 41.16 | 53.87 |
|  | Mean | 123.93 | 175.24 | 78.53 | 132.49 | 67.38 | 88.19 |
|  | Min | 313.61 | 443.23 | 231.60 | 390.71 | 124.70 | 163.20 |

of zero will travel in the $N_{s}=313.0$ and 377.2 exponential atmospheres. The value of zero for an initial elevation angle is used because the ray can be thought of as leaving the position $A$, tangent to the duct in figure 8 and "arriving" at the transmitter, even though the reverse process is actually what is occurring. Thus, from figures $23-26$, knowing the height of the transmitter, $h_{t}$, and the height of the ground-based duct, $h_{A}$, the distance, $d_{0}$, that the radio ray travels from the transmitter to the top of the duct is given by

$$
\begin{equation*}
d_{0}=d_{\left(h_{t}-h_{A}\right)}{ }^{\prime} \tag{31}
\end{equation*}
$$

i.e., the distance obtained by using the difference of $h_{t}$ and $h_{A}$ as the value of height on the abscissa in figures $23-26$ gives the distance $d_{0}$.

The distance, $d_{h o}$, to the standard radio horizon from the trans mitter is

$$
\begin{equation*}
d_{h o}=\sqrt{2 h_{t}} \tag{32}
\end{equation*}
$$

if $h_{t}$, the transmitter height, is in feet and $d_{\text {ho }}$ is in miles. Using this fact and figures $23-26$, Table VI, which shows the ratio of $d_{0}$ to $d_{h o}$, is derived for various transmitter heights and various duct heights. This table is useful in determining the portion of the distance (measured from the transmitter) within the horizon where fading due to the presence of super refraction has no effect on transmission. Table VII shows the number (sometimes a fraction) of half-lengths of shadow zones contained within the standard radio horizon for the maximum, mean, and minimum ducting gradients during the four seasons. Table VII is for the 50 per cent level, mean, and maximum surface duct thicknesses - to represent ordinary and maximum conditions of ground-based ducts described previously.

## 10 SAMPLE COMPUTATIONS

It can be seen from figure 51 that

$$
\begin{equation*}
h_{1} \leq \text { loss region } \leq h_{A}+h_{2^{\prime}} \tag{33}
\end{equation*}
$$

where $h_{1}$ can be determined from figures $27-50$, which depend upon the station location and the season of the year; $h_{A}$ is the height of the duct; and $\mathrm{h}_{2}$ is determined from figures $23-26$. The following example will illus trate the determination of the shadow zone.

As an illustrative problem, assume a 500-meter transmitting antenna in the spring of the year at $40^{\circ} \mathrm{N}$ latitude, and that one desires to place the receiving antenna at (a) 40 km and (b) 90 km from the transmitter.

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TABLE
Ratio of onset point of fading zones from transmitter to distance of radio


## DISTANCE VERSUS HEIGHT ABOVE DUCT REPRESENTING AVERAGE WASHINGTON D.C. AND FAIRBANKS ALASKA ATMOSPHERE


REPRESENTING
 DUCT ATMOS ABOVE SWAN ISLAND VERSUS DISTANCE AVERAGE


7,000


| November |  |  |
| :--- | :--- | :--- |
| Max。 | Mean | Min. |
|  |  |  |
| 0.33 | 0.20 | 0.11 |
| 0.20 | 0.12 | 0.07 |
| 0.03 | 0.02 | 0.01 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |

TABLE VII
For Fairbanks, Alaska, 185 m Duct

| November |  |  |
| :--- | :--- | :--- |
| Max. | Mean | Min. |
| 0.57 | 0.32 | 0.11 |
| 0.35 | 0.20 | 0.07 |
| 0.24 | 0.05 | 0.05 |
| 0.18 | 0.10 | 0.03 |
| 0.14 | 0.08 | 0.03 |
| 0.10 | 0.06 | 0.02 |
| 0.10 | 0.056 | 0.02 |
| 0.05 | 0.03 | 0.01 |
| 0.06 | 0.03 | 0.01 |
| 0.07 | 0.037 | 0.01 |


| August |  |  |
| :--- | :--- | :--- |
| Max. | Mean | Min. |
|  |  |  |
| 0.80 | 0.80 | 0.80 |
| 0.49 | 0.49 | 0.49 |
| 0.34 | 0.34 | 0.34 |
| 0.26 | 0.26 | 0.26 |
| 0.20 | 0.20 | 0.20 |
| 0.15 | 0.15 | 0.15 |
| 0.14 | 0.14 | 0.14 |
| 0.07 | 0.07 | 0.07 |
| 0.08 | 0.08 | 0.08 |
| 0.09 | 0.09 | 0.09 |


| Min. |
| :--- |
| 0.11 |
| 0.07 |
| 0.05 |
| 0.03 |
| 0.03 |
| 0.02 |
| 0.02 |
| 0.01 |
| 0.01 |
| 0.01 |

$\frac{\text { May }}{\text { Max. Mean }}$
$\begin{array}{llllllllll}\Rightarrow & N & n & M & M & N & N & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}$
$\begin{array}{llllllllll}\Rightarrow & \hat{0} & 0 & n & n & N & N & -1 & -1 & 0 \\ \dot{0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}$

| February |  |  |
| :--- | :--- | :--- |
| $\begin{array}{l}\text { Max. } \\ \text { Duct } \\ \text { Grad. }\end{array}$ | Mean | Min. |
| 1.14 | 0.65 | 0.25 |
| 0.70 | 0.40 | 0.15 |
| 0.49 | 0.28 | 0.11 |
| 0.37 | 0.21 | 0.08 |
| 0.28 | 0.16 | 0.06 |
| 0.21 | 0.12 | 0.05 |
| 0.20 | 0.11 | 0.04 |
| 0.10 | 0.06 | 0.02 |
| 0.12 | 0.07 | 0.03 |
| 0.13 | 0.075 | 0.03 | | Hgt. of |
| :--- |
| Trans. |
| (Mtrs.) | | $\circ$ | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 |  |  |
|  | in | $\infty$ | 0 |  | 1500 2000 3000 5000 7000

\[

\]

\[

\]

\[

\]

TABLE VII
For Swan Island, 135 m Duct

| November |  |  |
| :--- | :--- | :--- |
| Max. | Mean | Min. |
|  |  |  |
| 0.38 | 0.23 | 0.13 |
| 0.22 | 0.12 | 0.02 |
| 0.096 | 0.06 | 0.03 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |


| May |  |  |  | August |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Max. | Mean | Min. |  | Max. | Mean |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 0.36 | 0.21 | 0.04 | 0.38 | 0.21 | 0.04 |  |
| 0.20 | 0.12 | 0.02 | 0.22 | 0.12 | 0.02 |  |
| 0.09 | 0.05 | 0.01 | 0.096 | 0.05 | 0.01 |  |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  |


| February |  |  |
| :--- | :--- | :--- |
| Max. <br> Duct <br> Grad. | Mean | Min. |
| 0.21 | 0.14 | 0.06 |
| 0.12 | 0.08 | 0.035 |
| 0.05 | 0.035 | 0.016 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |
| 0.00 | 0.00 | 0.00 |



How high should the receiving antenna be to be free from fading loss caused by a ground-based superrefractive layer?

Since both of the distances required are within the standard horizon distance, $d_{\text {ho }}$, which, for a 500 meter transmitter is

$$
d_{h o}=92.181 \mathrm{~km},
$$

this is a case of within-the-horizon propagation.
Since the $40^{\circ} \mathrm{N}$ latitude position is within the temperate zone, use will be made of the Washington, D. C., model ducting atmosphere. As previously mentioned, 100 meters is the median mean duct thickness at Washington, D. C., and 200 meters is the median maximum duct thickness observed at Washington. These are the two duct thicknesses to use in this calculation. Also, since it is the spring of the year, figures $30-32$ must be used to determine $h_{1}$.

For the 40 kilometer distance (case (a)), the subscript of equation (31) becomes.

$$
h_{t}-h_{A}=500-100=400 \text { meters, }
$$

which is then entered on the abscissa of figure 23 , where,

$$
d_{0}=d_{400}=83 \mathrm{~km} .
$$

For a 200 meter thick duct,

$$
h_{t}-h_{A}=300 \text { meters. }
$$

Therefore, in this case,

$$
\mathrm{d}_{\mathrm{o}}=\mathrm{d}_{300}=72 \mathrm{~km} .
$$

In either case, as can be seen from figure 8, $d_{o}$ is greater than 40 kilometers. Therefore, any height of the receiving antenna will be
satisfactory at this point, since the first shadow zone has not yet been reached.

For the 90 -kilometer distance (case (b)), it can be seen that this distance is beyond the onset point of the first shadow zone, and figures 30-32 must be used here.

All of figures 27-50 are used diffèrently depending upon which side of the reflection point $B$ of figure 51 one desires to locate an antenna. If on the left-hand side of point $B$, one must subtract the desired distance from the distance from $B$ to the top of the duct traversed by the ray and use this new distance to determine height, $h$, in figures 27 - 50. This procedure must be followed every time the ray heads downward. If on the right-hand side of point $B$ of figure 51 (or any other reflection point), one simply uses the distance from the reflection point direetly. One must keep in mind that any previous half-lengths must be subtracted from the desired distance from the transmitter, as well as subtracting $d_{0}$, in order to obtain the distance used in figures 27-50.

Applied to the problem where figures $30-32$ are used, for the maximum midspring ducting conditions (figure 30) and the 100 -meterthick duct, recalling $d_{0}=83$ kilometers, the antenna is to be located

$$
90-83=7 \mathrm{~km}
$$

beyond the onset point of the shadow zones. For the $200-$ meter duct the antenna is located

$$
90-72=18 \mathrm{~km}
$$

beyond point A of figure 51. From figure 10, the half-length traveled by a ray in a 100 -meter duct under maximum midspring ducting conditions is 30.5 kilometers. Thus, as described above, the distance used in figure 30 is

$$
30.5-7.0=23.5 \mathrm{~km}
$$

to find $h_{1}$, because at 25 kilometers the ray is still sloping downward and hence is on the left-hand side of point $B$ of figure 51 . From figure 10 , $d_{h \ell}=43$ kilometers for a 200 meter duct; therefore, the distance used in figure 30 is

$$
43-18=25 \mathrm{~km}
$$

Similarly, utilizing figures 11 and 12 for mean ducting and minimum ducting conditions for a 100-meter and 200-meter duct:

Figure 31 distance to be used for the $100-$ meter duct $=45 \mathrm{~km}$. Figure 31 distance to be used for the 200 -meter duct $=56 \mathrm{~km}$. Figure 32 distance to be used for the 100 -meter duct $=191 \mathrm{~km}$. Figure 32 distance to be used for the 200 -meter duct $=262 \mathrm{~km}$ and hence
$h_{1}$ for a 100 -meter duct under max. spring ducting conditions $=92.3 \mathrm{~m}$. $h_{1}$ for a $200-$ meter duct under max. spring ducting conditions $=166.0 \mathrm{~m}$. $h_{1}$ for a 100 -meter duct under mean spring ducting conditions $=96.0 \mathrm{~m}$. $\mathrm{h}_{1}$ for a $200-$ meter duct under mean spring ducting conditions $=187.0 \mathrm{~m}$. $h_{1}$ for a $100-$ meter duct under min. spring ducting conditions $=99.0 \mathrm{~m}$. $h_{1}$ for a 200-meter duct under min. spring ducting conditions $=199.0 \mathrm{~m}$.

Figures 23 and 24 will be used to determine $h_{2}$ from the total dis tance, $d$. The distance to be used is obtained by subtracting the distance, $\mathrm{d}_{\mathrm{o}}$, from the desired distance. For the 100 -meter duct, $\mathrm{d}_{\mathrm{o}}=83$ kilometers and

$$
\mathrm{d}=90-83=7 \mathrm{~km}
$$

Likewise for the 200 -meter duct, $d_{o}=72 \mathrm{~km}$ and

$$
\mathrm{d}=90-72=18 \mathrm{~km}
$$

Thus it is found that for the model assumed:
$h_{2}$ for the 100 -meter duct height case will always be 3 meters,
$h_{2}$ for the 200 -meter duct height case will always be 18 meters.

## distance versus height above duct representing AVERAGE SWAN ISLAND ATMOSPHERE




HEIGHT,h, IN METERS
Figure 28

WASHINGTON, D.C. FOR
DUCTING CONDITIONS

HEIGHT,h, IN METERS

## HEIGHT VERSUS <br> MAXIMUM MIDSPRING (MAY)

HEIGHT VERSUS DISTANCE AT WASHINGTON,D.C. FOR
MEAN MIDSPRING
(MAY) DUCTING CONDITIONS

HEIGHT, h, IN METERS
Figure 31

HEIGHT, h, IN METERS

## Figure 32

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HEIGHT VERSUS DISTANCE AT WASHINGTON,D.C. FOR
MEAN MIDSUMMER (AUGUST) DUCTING CONDITIONS


HEIGHT VERSUS DISTANCE AT WASHINGTON，D．C．FOR
MINIMUM MIDSUMMER（AUGUST）DUCTING CONDITIONS

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HEIGHT，h，IN METER

## Figure 35


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## FOR DUCTING CONDITIONS


HEIGHT,h, IN METERS

## Figure 37

HEIGHT VERSUS DISTANCE AT WASHINGTON,D.C. FOR
MINIMUM MIDFALL (NOVEMBER) DUCTING CONDITIONS

HEIGHT,h, IN METERS
Figure 38
my NI ‘p‘ $\exists$ ONV

HEIGHT VERSUS DISTANCE AT SWAN ISLAND FOR
MEAN MIDWINTER (FEBRUARY) DUCTING CONDITIONS


HEIGHT,h, IN METERS
Figure 39

HEIGHT,h, IN METERS
Figure 40
my NI ‘p‘ヨONVLSIO
FOR
CONDITIONS

HEIGHT,h, IN METERS
Figure 41
my NI'p‘ヨONV

HEIGHT,h, IN METERS

## Figure 42


HEIGHT, h, IN METERS

## Figure 43

FOR
CONDITIONS
 HEIGHT,h, IN METERS
Figure 44


шу NI ‘p‘ヨONVISIO


## HEIGHT,h, IN METERS

Figure 46
my NI ‘p‘ヨONVASIO
HEIGHT VERSUS DISTANCE AT FAIRBANKS FOR
MINIMUM MIDWINTER (FEBRUARY) DUCTING CONDITIONS

HEIGHT, h, IN METERS
Figure 47

HEIGHT, h, IN METERS

## Figure 48

mY NI ‘p‘ヨONVISIO


HEIGHT,h, IN METERS

## 50 <br> Figure


my NI ‘p‘ヨONVISIO

SHADOW ZONE LOSS REGION
Figure 51

Since from equation (33)

$$
h_{1} \leq \text { loss region } \leq h_{A}+h_{2}
$$

Therefore, in answer to part (b) of the problem, for maximum ducting conditions with a 100-meter thick duct:

$$
92.3 \mathrm{~m} . \leq \text { loss region } \leq 3+100 \mathrm{~m}
$$

$$
92.3 \mathrm{~m} . \leq \text { loss region } \leq 103 \mathrm{~m}
$$

with a 200-meter thick duct:

$$
166 \mathrm{~m} . \leq \text { loss region } \leq 18+200 \mathrm{~m}
$$

$$
166 \mathrm{~m} . \leq \text { loss region } \leq 218 \mathrm{~m}
$$

For mean ducting conditions with a l00-meter thick duct:

$$
96 \mathrm{~m} . \leq 10 \mathrm{ss} \text { region } \leq 103 \mathrm{~m}
$$

with a 200-meter thick duct:

$$
187 \mathrm{~m} . \leq \text { loss region } \leq 218 \mathrm{~m}
$$

For minimum ducting conditions with a 100 -meter thick duct:

$$
99 \mathrm{~m} . \leq \text { loss region } \leq 103 \mathrm{~m}
$$

with a 200-meter thick duct:

$$
199.3 \mathrm{~m} . \leq \text { loss region } \leq 218 \mathrm{~m}
$$

It should be pointed out that this method does not guarantee prevention of all fadeouts due to superrefraction, but merely affords an estimate. It is therefore recommended that if RAOB equipment is available, actual computations using equations (25) - (30) be made over a period of time to completely determine the ducting fadeout regions at a particular point.

## 11. VARIATIONS IN FADING

It should be remembered that the three-station model is representative of the arctic, temperate, and tropical zones only, and that great variations could be found within specific subdivisions of these areas. Furthermore, temporal variations are to be considered with respect to formation and decay of ducts. One must also consider the waveguide effect of ducts, in that they have a particular "cut-in" frequency, and will not respond to all frequencies as discussed in section 4. Thus, a fadeout may appear at one frequency and not at another. The fact that the atmos phere is never ideally homogeneous should also be considered. To obtain precise results, one should try to have RAOB or similar readings as close to the transmission path as possible when actually trying to run shadow zone calculations of his own. One should also avail himself of as many daily profiles for the purpose of determining $N$ as possible, so as to get a closer picture of the hours during which super refractive conditions can be anticipated, and also to obtain a picture of how the duct height varies between the times of its buildup and decay.

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[^0]:    * NBS Group, Joint Institute for Laboratory Astrophysics at the University of Colorado.
    ** Located at Boulder, Colorado.

