NBS TECHNICAL NOTE 957

U.S. DEPARTMENT OF COMMERCE / National Bureau of Standards

Threshold Photo and Electroproduction of Pions from Nuclei

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Threshold Photo and Electroproduction of Pions from Nuclei

Technical note no. 957

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Contents

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E. T. Dressler

The nonrelativistic amplitudes for photo and electroproduction of pions are derived using effective pseudoscalar and pseudovector Lagrangian densities. The results for pseudoscalar and pseudovector coupling are compared at threshold for charged and neutral pion photo and electroproduction. Cross section formulas and kinematical conditions are also presented.

Key words: Effective Lagrangian; electroproduction; impulse approximation; photoproduction; pseudoscalar and pseudovector coupling; threshold pion production.

1. Introduction

In the calculation of any nuclear cross section that involves a small perturbation to the nuclear system, the standard approach is to use the impulse approximation. That means that one assumes that an external perturbation acts on the nucleons in a nucleus as if they were free particles. Using this approximation or model, then if one knows the amplitude for the reaction off of a single nucleon, we just sum the contributions from each of the nucleons in the nucleus (a sum correlated by the nuclear wave functions). The purpose of this paper is to present the nonrelativistic amplitudes for photo and electroproduction of pions that can be used in an impulse approximation calculation for nuclear processes.

To find the amplitude from a single nucleon, one usually starts by considering which relativistic Feynman diagrams are important in the particular energy region under consideration. For the case of a purely electromagnetic interaction as in electron scattering, photo or electrodisintegration, or Compton scattering, one can use perturbation theory and quantum field theory to give the appropriate Feynman diagrams to a particular order in the coupling constant. However, in photo or electroproduction of pions, we now have electromagnetic and strong interactions, and since the coupling constant in strong interactions is large, the perturbation expansion in powers of the coupling constant will not converge. In spite of this fact, the lowest order term in the expansion using pseudovector pion-nuclear coupling, gives results which under certain conditions agree very well with experiment and with the low energy theorem derived from the theory of the PCAC (partially conserved axial current) [1]. Therefore, we will use the lowest order perturbation term as our model for the threshold photo pion production amplitude off of a nucleon.

Two of the simplest pion-nucleon couplings are pseudoscalar (ps) and pseudovector (pv) [2]. A comparison of the two couplings is made in Section 2. In order to use the amplitudes in a nuclear calculation, we must first make the nonrelativistic approximation of the relativistic Feynman amplitudes. This is done in Section 3. The nonrelativistic amplitudes are then specialized to the cases of threshold photo and electroproduction in Sections 4 and 5, respectively. In Section 6 the cross section formulas are given.

2. Pseudoscalar vs. Pseudovector Coupling

In the Lagrangian field theory approach to photo pion production, two of the simplest interaction Lagrangian densities between the pion and nuclear fields that one can choose are called the pseudoscalar and pseudovector interactions. In the pseudoscalar case, the interaction Lagrangian density for the pion-nucleon interaction is

$$\mathcal{L}_{\pi N}^{(ps)} = iG\overline{N}(x)\,\vec{\tau}\cdot\vec{\varphi}(x)\,\gamma_5 N(x) \tag{1}$$

where

$N(x)$ = nucleon field operator

$\vec{\varphi}(x)$ = pion field operator (a vector in isospin space)

$\frac{G^2}{4\pi} = 15.0$ = pion-nucleon coupling constant

$\vec{\tau}$ = nucleon isospin operator

For photo pion production, the total interaction Lagrangian density is

$$\mathcal{L}_{I}^{(ps)} = \mathcal{L}_{\pi N}^{(ps)} + \mathcal{L}_{\gamma N} + \mathcal{L}_{\gamma\pi} \tag{2}$$

where

$$\mathcal{L}_{\gamma N} = ie\overline{N}(x)\gamma_\mu A^\mu(x)N(x) \tag{3}$$

$$\mathcal{L}_{\gamma\pi} = ie\varphi^{+}(x)\left(\frac{\vec{\partial}}{\partial x_\mu} - \frac{\overleftarrow{\partial}}{\partial x_\mu}\right)\varphi(x)A^\mu(x) \tag{4}$$

$A^\mu(x)$ = electromagnetic field vector

When this interaction Lagrangian density is used in the second order term of the perturbation expansion, one finds that there are three Feynman diagrams (shown in Figs. 1-3) that can be associated with the second order term. (Solid lines are nucleons, wiggly lines are photons, and broken lines are pions. Each line is labeled by the 4-momenta of the particle.)

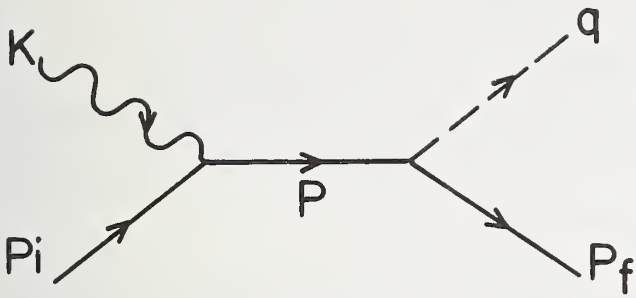


Figure 1. Nucleon pole diagram.

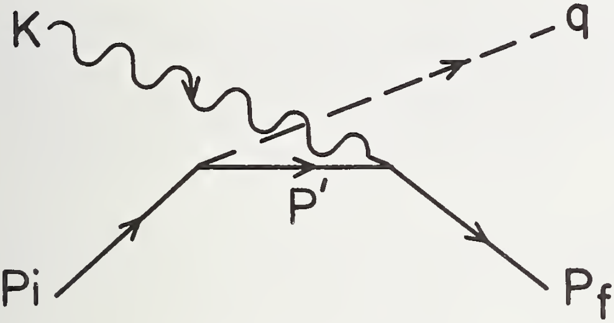


Figure 2. Crossed nucleon pole diagram.

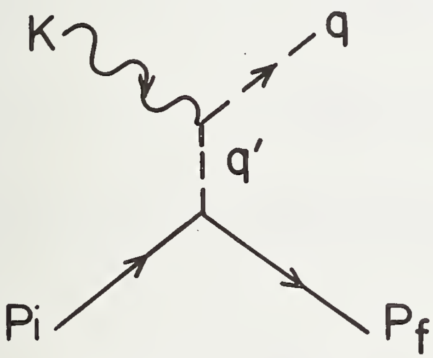


Figure 3. Pion current diagram.

Figures 1 - 2 are called the nucleon pole contributions to the Born diagrams. Figure 3 is called the pion current diagram, because it results from the interaction of the pion's electromagnetic (e-m) current with the e-m field operator given by $\mathcal{L}_{\gamma\pi}$.

For pseudovector pion-nucleon coupling, the interaction Lagrangian density for the interaction of the pion and nucleon field operators is given by

$$\mathcal{L}_{\pi N}^{(pv)} = \frac{if_\pi}{m_\pi} \bar{N}(x) \, \gamma_\mu \gamma_5 \vec{\tau} \cdot \frac{\partial \vec{\varphi}(x)}{\partial x^\mu} N(x) \tag{5}$$

where $\quad f_\pi^2 = 0.08, \left(\frac{G}{2mc^2}\right)^2 = 4\pi \left(\frac{f}{m_\pi c^2}\right)^2$

Now in order to insure gauge invariance of our total interaction Lagrangian density, $\mathcal{L}_I^{(pv)}$, in the presence of the electromagnetic field, we have to let [3]

$$\partial^\mu \rightarrow \partial^\mu + ieA^\mu(x)$$

This leads to

$$\mathcal{L}_{\pi N}^{(pv)} \rightarrow \mathcal{L}_{\pi N}^{(pv)} - e\frac{f\pi}{m_\pi}\bar{N}(x)\ \gamma_\mu\ \gamma_5\ \vec{\tau}\cdot\vec{\varphi}(x)N(x)A^\mu(x) \equiv \mathcal{L}_{\pi N}^{(pv)} + \mathcal{L}_{\pi N\gamma} \tag{6}$$

By inspection, we can see that this leads to a new term which has the interaction of the nucleon, pion, and electromagnetic fields at a point. There is no term like this in ps coupling, and inserting this into the perturbation expansion leads to an additional Feynman diagram (fig. 4) in pv theory.

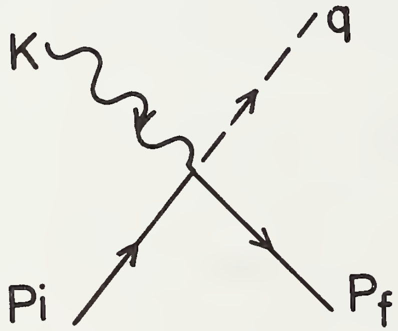
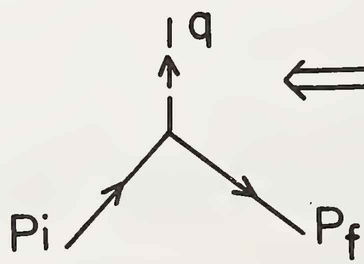


Figure 4. Point contact or "seagull" diagram.

One should note that although the diagrams in figs. 1 - 3 are also present in pv theory, they won't necessarily give the same size contribution as in the ps case, because the form of the π - N vertex is different for the two couplings (fig. 5).



$$\Longleftrightarrow \quad \begin{cases} \gamma_5 & \text{for ps} \\ \frac{1}{2m}\gamma_5 \not{q} & \text{for pv} \end{cases}$$

Figure 5. π - N vertex operators for ps and pv coupling.

Even though the size of the contribution to the production amplitude from each of diagrams 1 - 3 may be different for the ps and pv cases, the sum of the contribution from diagrams 1 - 3 using ps coupling gives essentially the same result as the sum of the contributions from diagrams 1 - 4 using pv coupling for charged pion production at threshold. However, pv and ps couplings give quite different results for π^o producrion. pv coupling gives the best agreement with experiment and is usually chosen as the "effective Lagrangian" [4]. In spite of this, we will derive our nonrelativistic amplitudes in the next section using ps coupling and show how these amplitudes are modified for the pv case.

3. Nonrelativistic Amplitudes

To calculate the amplitudes corresponding to the diagrams in figs. 1 - 3 using ps coupling, we can use the Feynman rules in appendix B of ref. [5]. The relativistic amplitudes for these diagrams are defined as

$$M_{\gamma 1}^{(ps)} = 2m\, \bar{U}_{s',\tau'}(P_f)\, \gamma_5\, \tau_\alpha \frac{(\not{P} + m)}{P^2 - m^2} \left[\gamma_\mu \frac{(1 + \tau_3)}{2} + i\frac{\sigma_{\mu\nu}}{2m} k^\nu \left(\frac{(K_p + K_n) + \tau_3(K_p - K_n)}{2}\right)\right] U_{s,\tau}(P_i)\epsilon^\mu \tag{7}$$

$$M_{\gamma 2}^{(ps)} = 2m\, \bar{U}_{s',\tau'}(P_f) \left[\gamma_\mu \frac{(1 + \tau_3)}{2} + i\frac{\sigma_{\mu\nu}}{2m} k^\nu \left(\frac{(K_p + K_n) + \tau_3(K_p - K_n)}{2}\right)\right] \frac{(\not{P}' + m)}{P'^2 - m^2}\, \gamma_5 \tau_\alpha U_{s,\tau}(P_i)\epsilon^\mu \tag{8}$$

$$M_{\gamma 3}^{(ps)} = 2m\, \bar{U}_{s',\tau'}(P_f)\, \gamma_5 \frac{[\tau_\alpha, \tau_3]}{2}\, \epsilon^\mu \frac{(q' + q)_\mu}{q'^2 - m_\pi^2}\, U_{s,\tau}(P_i) \tag{9}$$

where $\alpha = -1, +1, 0$ for π^+, π^-, and π^o production, respectively,

$$\tau_o = \tau_3,$$

$$\tau_{-1}|P\rangle = \sqrt{2}\,|n\rangle ,$$

$$[\tau_\alpha, \tau_3] = \tau_\alpha \tau_3 - \tau_3 \tau_\alpha ,$$

and

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] .$$

In order to use these amplitudes in a nonrelativistic calculation, one has to expand these matrices by going from the 4 component Dirac spinors, $U_s(P)$, to the two component Pauli spinors, χ_s. This is done by letting

$$U_{s,\tau}(P) = \sqrt{\frac{E+m}{2m}}\begin{pmatrix} I \\ \dfrac{\vec{\sigma}\cdot\vec{P}}{E+m}\end{pmatrix}\chi_{s,\tau} \approx \begin{pmatrix} I \\ \dfrac{\vec{\sigma}\cdot\vec{P}}{2m}\end{pmatrix}\chi_{s,\tau} + O\left(\frac{P^2}{m^2}\right) \tag{10}$$

where $\quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

and $\quad \chi_{s,\tau} = \chi_s\chi_\tau$, χ_s being the spin and χ_τ the isospin Pauli spinors, respectively.

Keeping only terms to $O\left(\frac{P}{m}\right)$, we denote our nonrelativistic amplitudes by M, where

$$M^{(ps)}_{\gamma 1} = \frac{1}{k_o}\left\{\epsilon_o\left[\vec{\sigma}\cdot\vec{k}\left(\frac{-k_o}{2m}\right) + \vec{\sigma}\cdot\vec{q}\left(1 + \frac{2\vec{P}_i\cdot\vec{k} + \vec{k}^2}{2m}\right)\right]\right. \tag{11}$$

$$+\left[\vec{\sigma}\cdot\vec{\epsilon}\left(-k_o + \frac{k_o^2}{2m} - \frac{\vec{k}\cdot\vec{q}}{2m}\right) + \vec{\sigma}\cdot\vec{q}\left(-\frac{\vec{P}_i\cdot\vec{\epsilon}}{m} - \frac{\vec{k}\cdot\vec{\epsilon}}{2m}\right) + \vec{\sigma}\cdot\vec{k}\left(\frac{\vec{q}\cdot\vec{\epsilon}}{2m}\right)\right.$$

$$\left.\left. + i\,\frac{\vec{\epsilon}\cdot(\vec{k}\times\vec{q})}{2m}\right]\right\}\tau_\alpha\left(\frac{1+\tau_3}{2}\right) + \frac{1}{k_o}\left\{\epsilon_o\,\vec{\sigma}\cdot\vec{k}\left(-\frac{q_o}{2m}\right)\right.$$

$$\left. + \left[\vec{\sigma}\cdot\vec{\epsilon}\left(\frac{k_o q_o}{2m} - \frac{\vec{k}\cdot\vec{q}}{2m}\right) + \vec{\sigma}\cdot\vec{k}\left(\frac{\vec{\epsilon}\cdot\vec{q}}{2m}\right) + i\,\frac{\vec{\epsilon}\cdot(\vec{k}\times\vec{q})}{2m}\right]\right\}$$

$$\tau_\alpha\left(\frac{(\kappa_p + \kappa_n) + (\kappa_p - \kappa_n)\,\tau_3}{2}\right)$$

$$M^{(ps)}_{\gamma 2} = -\frac{1}{k_o}\left\{\epsilon_o\left[\vec{\sigma}\cdot\vec{k}\left(\frac{k_o}{2m}\right) + \vec{\sigma}\cdot\vec{q}\left(1 + \frac{2\vec{P}_f\cdot\vec{k} - \vec{k}^2}{2mk_o}\right)\right]\right. \tag{12}$$

$$+\left[\vec{\sigma}\cdot\vec{\epsilon}\left(-k_o - \frac{k_o^2}{2m} + \frac{\vec{k}\cdot\vec{q}}{2m}\right) + \vec{\sigma}\cdot\vec{q}\left(-\frac{\vec{P}_f\cdot\vec{\epsilon}}{m} + \frac{\vec{k}\cdot\vec{\epsilon}}{2m}\right) + \vec{\sigma}\cdot\vec{k}\left(-\frac{\vec{q}\cdot\vec{\epsilon}}{2m}\right)\right.$$

$$\left.\left. + i\,\frac{\vec{\epsilon}\cdot(\vec{k}\times\vec{q})}{2m}\right]\right\}\frac{(1+\tau_3)}{2}\,\tau_\alpha + \frac{1}{k_o}\left\{\epsilon_o\,\vec{\sigma}\cdot\vec{k}\left(-\frac{q_o}{2m}\right) + \right.$$

$$\left[\vec{\sigma}\cdot\vec{\epsilon}\left(\frac{k_o q_o}{2m} - \frac{\vec{k}\cdot\vec{q}}{2m}\right) + \vec{\sigma}\cdot\vec{k}\left(\frac{\vec{\epsilon}\cdot\vec{q}}{2m}\right) - i\,\frac{\vec{\epsilon}\cdot(\vec{k}\times\vec{q})}{2m}\right]\Bigg\}$$

$$\left(\frac{\left(\kappa_p + \kappa_n\right) + \left(\kappa_p - \kappa_n\right)\tau_3}{2}\right)\tau_\alpha$$

$$M_{\gamma 3}^{(ps)} = -\left\{\frac{\vec{\sigma}\cdot(\vec{q} - \vec{k})}{2k\cdot q - k^2}\left[\epsilon_o\left(2\,q_o - k_o\right) - \vec{\epsilon}\cdot(2\,\vec{q} - \vec{k})\right]\right\}\frac{\left[\tau_\alpha, \tau_3\right]}{2} \tag{13}$$

For threshold pion production, we can let $\vec{q} \to 0$ which leaves

$$M_1^{(ps)} = \left[-\epsilon_o\frac{\vec{\sigma}\cdot\vec{k}}{2m} - \vec{\sigma}\cdot\vec{\epsilon}\left(1 - \frac{k_o}{2m}\right)\right]\tau_\alpha\frac{\left(1 + \tau_3\right)}{2} + \left[-\epsilon_o\frac{m_\pi}{2mk_o}\vec{\sigma}\cdot\vec{k}\right. \tag{14}$$

$$\left. + \frac{m_\pi}{2m}\vec{\sigma}\cdot\vec{\epsilon}\right]\tau_\alpha\left(\frac{\left(\kappa_p + \kappa_n\right) + \left(\kappa_p - \kappa_n\right)\tau_3}{2}\right)$$

$$M_{\gamma 2}^{(ps)} = \left[-\epsilon_o\frac{\vec{\sigma}\cdot\vec{k}}{2m} + \vec{\sigma}\cdot\vec{\epsilon}\left(1 + \frac{k_o}{2m}\right)\right]\frac{\left(1 + \tau_3\right)}{2}\tau_\alpha \tag{15}$$

$$+\left[-\epsilon_o\frac{m_\pi}{2mk_o}\vec{\sigma}\cdot\vec{k} + \frac{m_\pi}{2m}\vec{\sigma}\cdot\vec{\epsilon}\right]\left(\frac{\left(\kappa_p + \kappa_n\right) + \left(\kappa_p - \kappa_n\right)\tau_3}{2}\right)\tau_\alpha$$

$$M_{\gamma 3}^{(ps)} = \left\{\frac{\vec{\sigma}\cdot\vec{k}}{2k_o\,m_\pi - k^2}\left[\epsilon_o\left(2m_\pi - k_o\right) + \vec{\epsilon}\cdot\vec{k}\right]\right\}\frac{\left[\tau_\alpha, \tau_3\right]}{2} \tag{16}$$

By replacing $\epsilon_o \to k_o$ and $\vec{\epsilon} \to \vec{k}$ in equations 14 - 16 we see that $M_{\gamma 1}^{(ps)} + M_{\gamma 2}^{(ps)} + M_{\gamma 3}^{(ps)} \to 0$, therefore, our nonrelativistic amplitudes are exactly gauge invariant.

Now consider how these amplitudes are modified for the case of pv coupling. Using eq. 4.1 of ref. [1], the pv amplitudes are given by

$$M_{\gamma 1}^{(pv)} = - \bar{U}_{s',\tau'}(p_f)\, \not{q}\gamma_5\, \tau_\alpha \frac{(\not{p} + m)}{p^2 - m^2} \left[\gamma_\mu \frac{\left(1 + \tau_3\right)}{2} \right. \tag{17}$$

$$\left. + i \frac{\sigma_{\mu\nu}}{2m} k^\nu \left(\frac{\left(\kappa_p + \kappa_n\right) + \left(\kappa_p - \kappa_n\right)\tau_3}{2} \right) \right] U_{s,\tau}(p_i)\epsilon^\mu$$

$$M_{\gamma 2}^{(pv)} = - \bar{U}_{s',\tau'}(p_f) \left[\gamma_\mu \frac{\left(1 + \tau_3\right)}{2} + i \frac{\sigma_{\mu\nu}}{2m} k^\nu \right. \tag{18}$$

$$\left. \left(\frac{\left(\kappa_p + \kappa_n\right) + \left(\kappa_p - \kappa_n\right)\tau_3}{2} \right) \right] \frac{\not{p}' + m}{p'^2 - m^2} \not{q}\gamma_5\, \tau_\alpha\, U_{s,\tau}(p_i)\epsilon^\mu$$

$$M_{\gamma 3}^{(pv)} = \bar{U}_{s',\tau'}(p_f)\, (\not{k} - \not{q})\, \gamma_5 \frac{[\tau\alpha, \tau 3]}{2} \frac{1}{q' - m_\pi^2} \tag{19}$$

$$(q' + q)_\mu\, U_{s,\tau}(p_i)\epsilon^\mu$$

$$M_{\gamma 4}^{(pv)} = \bar{U}_{s',\tau'}(p_f) \frac{[\tau\alpha, \tau 3]}{2} \gamma_\mu\gamma_5\, U_{s,\tau}(p_i)\epsilon^\mu \tag{20}$$

Consider now the following identities:

$$- \bar{U}_{s',\tau'}(p_f)\, \not{q}\gamma_5 \frac{(\not{p} + m)}{p^2 + m^2} = \bar{U}_{s',\tau'}(p_f)\, \gamma_5 \left[\frac{2m(\not{p} + m)}{p^2 - m^2} + 1 \right] \tag{21}$$

$$- \frac{(\not{p}' + m)}{p'^2 - m^2} \not{q}\gamma_5\, U_{s,\tau}(p_i) = \left[\frac{2m(\not{p}' + m)}{p'^2 - m^2} + 1 \right] \gamma_5\, U_{s,\tau}(p_i) \tag{22}$$

By comparing eqs. 7 - 9 with eqs. 17 - 19 and using identities 21 -22, we can see the following:

$$M_{\gamma 1}^{(pv)} = M_{\gamma 1}^{(ps)} + \bar{U}_{s',\tau'}(p_f)\,\gamma_5\,\tau_\alpha \left[\gamma_\mu \frac{(1+\tau_3)}{2} + i\frac{\sigma_{\mu\nu}}{2m} k^\nu \left(\frac{(\kappa_p + \kappa_n) + (\kappa_p - \kappa_n)\tau_3}{2}\right)\right] U_{s,\tau}(p_i)\epsilon^\mu \equiv M_{\gamma 1}^{(ps)} + M'_{\gamma 1} \tag{23}$$

$$M_{\gamma 2}^{(pv)} = M_{\gamma 2}^{(ps)} + \bar{U}_{s',\tau'}(p_f) \left[\gamma_\mu \frac{(1+\tau_3)}{2} + i\frac{\sigma_{\mu\nu}}{2m} k^\nu \left(\frac{(\kappa_p + \kappa_n) + (\kappa_p - \kappa_n)\tau_3}{2}\right)\right] \tau_\alpha\,\gamma_5\, U_{s,\tau}(p_i)\epsilon^\mu \equiv M_{\gamma 2}^{(ps)} + M'_{\gamma 2} \tag{24}$$

For $M_{\gamma 3}^{(ps)}$, we use

$$\bar{U}_{s',\tau'}(p_f)\,(\not{k} - \not{q})\,\gamma_5\, U_{s,\tau}(p_i) = 2m\,\bar{U}_{s',\tau'}(p_f)\,\gamma_5\, U_{s,\tau}(p_i) \tag{25}$$

Therefore $M_{\gamma 3}^{(pv)} = M_{\gamma 3}^{(ps)}$

Taking the nonrelativistic reduction of $M'_{\gamma 1}$ and $M'_{\gamma 2}$ gives

$$M'_{\gamma 1} = \left\{ -\epsilon_o \left[\frac{\vec{\sigma}\cdot\vec{k}}{2m} - \frac{\vec{\sigma}\cdot\vec{q}}{2m} + \frac{\vec{\sigma}\cdot\vec{p}_i}{m}\right] + \vec{\sigma}\cdot\vec{\epsilon} \right\} \tau_\alpha \frac{(1+\tau_3)}{2} + \left\{ \epsilon_o \frac{\vec{\sigma}\cdot\vec{k}}{2m} + \vec{\sigma}\cdot\vec{\epsilon}\,\frac{k_o}{2m} \right\} \tau_\alpha \left(\frac{(\kappa_p + \kappa_n) + (\kappa_p - \kappa_n)\tau_3}{2}\right) \tag{26}$$

$$M'_{\gamma 2} = \left\{ \epsilon_o \left[\frac{\vec{\sigma}\cdot\vec{k}}{2m} - \frac{\vec{\sigma}\cdot\vec{q}}{2m} + \frac{\vec{\sigma}\cdot\vec{p}_i}{2m}\right] - \vec{\sigma}\cdot\vec{\epsilon} \right\} \frac{(1+\tau_3)}{2} \tau_\alpha + \left\{ \epsilon_o \frac{\vec{\sigma}\cdot\vec{k}}{2m} + \vec{\sigma}\cdot\vec{\epsilon}\,\frac{k_o}{2m} \right\} \left(\frac{(\kappa_p + \kappa_n) + (\kappa_p - \kappa_n)\tau_3}{2}\right) \tau_\alpha \tag{27}$$

Combining $M'_{\gamma 1}$ and $M'_{\gamma 2}$ with $M_{\gamma 1}^{(ps)}$ and $M_{\gamma 2}^{(ps)}$, respectively (eqs. 11, 12), we can now get the pv nonrelativistic amplitudes to $M_{\gamma 1}^{(pv)}$ and $M_{\gamma 2}^{(pv)}$.

$$M_{\gamma 1}^{(pv)} = \left\{ \epsilon_o \left[-\frac{\vec{\sigma}\cdot\vec{k}}{m} - \frac{\vec{\sigma}\cdot\vec{p}_i}{m} + \vec{\sigma}\cdot\vec{q}\left(\frac{1}{k_o} + \frac{2\vec{p}_i\cdot\vec{k} + \vec{k}^2}{2mk_o^2}\right)\right] \right. \tag{28}$$

$$+ \left[\vec{\sigma}\cdot\vec{\epsilon}\left(\frac{k_o}{2m} - \frac{\vec{k}\cdot\vec{q}}{2mk_o}\right) + \vec{\sigma}\cdot\vec{k}\,\frac{\vec{q}\cdot\vec{\epsilon}}{2mk_o} - \vec{\sigma}\cdot\vec{q}\left(\frac{\vec{k}\cdot\vec{\epsilon}}{2mk_o} + \frac{\vec{p}_i\cdot\vec{\epsilon}}{mk_o}\right) + i\,\frac{\vec{\epsilon}\cdot\left(\vec{k}\times\vec{q}\right)}{2mk_o}\right]\Bigg\}$$

$$\tau_\alpha \frac{\left(1 + \tau_3\right)}{2} + \left\{ -\epsilon_o\,\vec{\sigma}\cdot\vec{k}\left(\frac{1}{2m} - \frac{q_o}{2mk_o}\right) + \left[\vec{\sigma}\cdot\vec{\epsilon}\left(\frac{k_o}{2m} + \frac{q_o}{2m} - \frac{\vec{k}\cdot\vec{q}}{2mk_o}\right)\right.\right.$$

$$\left.\left. + \vec{\sigma}\cdot\vec{k}\,\frac{\vec{\epsilon}\cdot\vec{q}}{2mk_o} + \frac{i\vec{\epsilon}\cdot(\vec{k}\times\vec{q})}{2mk_o}\right]\right\} \tau_\alpha\left(\frac{\left(\kappa_p + \kappa_n\right) + \left(\kappa_p - \kappa_n\right)\tau_3}{2}\right)$$

$$M_{\gamma 2}^{(pv)} = -\left\{ \epsilon_o \left[-\frac{\vec{\sigma}\cdot\vec{p}_i}{2m} + \vec{\sigma}\cdot\vec{q}\left(\frac{1}{k_o} + \frac{2\vec{p}_f\cdot\vec{k} + k^2}{2mk_o^2}\right)\right]\right. \tag{29}$$

$$+ \left[\vec{\sigma}\cdot\vec{\epsilon}\left(-\frac{k_o}{2m} + \frac{\vec{k}\cdot\vec{q}}{2mk_o}\right) - \vec{\sigma}\cdot\vec{k}\,\frac{\vec{q}\cdot\vec{\epsilon}}{2mk_o} + \vec{\sigma}\cdot\vec{q}\left(\frac{\vec{k}\cdot\vec{\epsilon}}{2mk_o} - \frac{\vec{p}_f\cdot\vec{\epsilon}}{MK_o}\right)\right.$$

$$\left.\left. +i\,\frac{\vec{\epsilon}\cdot(\vec{k}\times\vec{q})}{2mk_o}\right]\right\}\frac{\left(1 + \tau_3\right)}{2}\tau_\alpha + \left\{\epsilon_o\,\vec{\sigma}\cdot\vec{k}\left(\frac{1}{2m} - \frac{q_o}{2mk_o}\right)\right.$$

$$+\left[\vec{\sigma}\,\vec{\epsilon}\left(-\frac{k_o}{2m} + \frac{q_o}{2m} - \frac{\vec{k}\cdot\vec{q}}{2mk_o}\right) + \vec{\sigma}\cdot\vec{k}\,\frac{\vec{q}\cdot\vec{\epsilon}}{2mk_o}\right.$$

$$\left.\left. -i\,\frac{\vec{\epsilon}\cdot(\vec{k}\times\vec{q})}{2mk_o}\right]\right\}\left(\frac{\left(\kappa_p + \kappa_n\right) + \left(\kappa_p - \kappa_n\right)\tau_3}{2}\right)\tau_\alpha$$

Taking the nonrelativistic reduction of $M_{\gamma 4}^{(pv)}$ gives

$$M_{\gamma 4}^{(pv)} = \left\{ \epsilon_o \left[\frac{\vec{\sigma}\cdot\vec{p}_i}{m} + \frac{\vec{\sigma}\cdot\vec{k}}{2m} - \frac{\vec{\sigma}\cdot\vec{q}}{2m} \right] - \vec{\sigma}\cdot\vec{\epsilon} \right\} \frac{[\tau_\alpha, \tau_3]}{2} \tag{30}$$

Making the substitutions $\epsilon_o \to k_o$ and $\vec{\epsilon} \to \vec{k}$, the nonrelativistic amplitude $M_{\gamma 1}^{(pv)} + M_{\gamma 2}^{(pv)} + M_{\gamma 3}^{(pv)} + M_{\gamma 4}^{(pv)}$ is also gauge invariant.

For threshold production, letting $\vec{q} \to 0$,

$$M_{\gamma 1}^{(pv)} = \left\{ - \epsilon_o \left(\frac{\vec{\sigma}\cdot\vec{k}}{m} + \frac{\vec{\sigma}\cdot\vec{p}_i}{m} \right) + \vec{\sigma}\cdot\vec{\epsilon}\, \frac{k_o}{2m} \right\} \tau_\alpha \frac{(1 + \tau_3)}{2} \tag{31}$$

$$+ \left\{ \epsilon_o\, \vec{\sigma}\cdot\vec{k} \left(\frac{1}{2m} - \frac{m_\pi}{2mk_o} \right) + \vec{\sigma}\cdot\vec{\epsilon} \left(\frac{m_\pi}{2m} - \frac{k_o}{2m} \right) \right\} \tau_\alpha \left(\frac{(\kappa_p + \kappa_n) + (\kappa_p - \kappa_n)\tau_3}{2} \right)$$

$$M_{\gamma 2}^{(pv)} = \left\{ \epsilon_o \frac{\vec{\sigma}\cdot\vec{p}_i}{2m} + \vec{\sigma}\cdot\vec{\epsilon}\, \frac{k_o}{2m} \right\} \frac{(1 + \tau_3)}{2} \tau_\alpha \tag{32}$$

$$+ \left\{ \epsilon_o\, \vec{\sigma}\cdot\vec{k} \left(\frac{1}{2m} - \frac{m_\pi}{2mk_o} \right) + \vec{\sigma}\cdot\vec{\epsilon} \left(\frac{m_\pi}{2m} - \frac{k_o}{2m} \right) \right\} \left(\frac{(\kappa_p + \kappa_n) + (\kappa_p - \kappa_n)\tau_3}{2} \right) \tau_\alpha$$

$$M_{\gamma 3}^{(pv)} = \left\{ \frac{\vec{\sigma}\cdot\vec{k}}{2k_o m_\pi - k^2} \left[\epsilon_o (2m_\pi - k_o) + \vec{\epsilon}\cdot\vec{k} \right] \right\} \frac{[\tau_\alpha, \tau_3]}{2} \tag{33}$$

$$M_{\gamma 4}^{(pv)} = \left\{ \epsilon_o \left(\frac{\vec{\sigma}\cdot\vec{k}}{2m} + \frac{\vec{\sigma}\cdot\vec{p}_i}{m} \right) - \vec{\sigma}\cdot\vec{\epsilon} \right\} \frac{[\tau_\alpha, \tau_3]}{2} \tag{34}$$

We should note at this point that in deriving these expressions, we have not used any particular gauge or mentioned any specific reference frame in order to eliminate terms. However, by stating that threshold means $\vec{q} = o$, we are by implication in the center of momentum frame where $\vec{k} = - \vec{p}_i$. In the laboratory frame, $\vec{q} \neq 0$ even at threshold.

4. Threshold Photoproduction

4.1. Kinematics and Gauge Conditions

For real photons we are free to choose the gauge where $\vec{\epsilon}\cdot\vec{k} = 0$ and $\epsilon_o = 0$. This is the transverse or Coulomb gauge. By inspection of $M_{\gamma 3}$, the contribution from the pion current diagram, we can see that this term vanishes at threshold.

Choosing to work in the center of momentum frame of the incoming photon and the target, gives

$$\vec{k} + \vec{p}_i = \vec{q} + \vec{p}_f \tag{35}$$

and at threshold

$$\vec{q} = 0 = \vec{p}_f \tag{36}$$

where $\vec{p}_i$ and $\vec{p}_f$ are the momenta of the initial and final nucleus assuming the final state is a bound nucleus.[1] Then at threshold, using

$$k_o + E_i = m_\pi + m_f \tag{37}$$

and

$$E_i = \sqrt{m_i^2 + k_o^2} \tag{38}$$

gives

$$k_o = \frac{\left(m_\pi + m_f\right)^2 - m_i^2}{2\left(m_\pi + m_f\right)} \tag{39}$$

[1] For transitions to continuum states, the kinematics are a bit more complicated.

4.2. Pseudoscalar Photoproduction

Case 1. $\gamma + p \rightarrow \pi^+ + n$

The largest contribution comes from $M_{\gamma 1}^{(ps)}$ as can be seen by considering diagram 2 which corresponds to $M_{\gamma 2}^{(ps)}$. In this diagram the proton has already emitted the π^+ and become a neutron. Since the neutron has no charge, the incoming photon can only interact with the anomalous magnetic moment, κ_n.

$$\langle M(\pi^+)\rangle = -\sqrt{2}\,\langle\vec{\sigma}\cdot\vec{\epsilon}\rangle \left[\left(1 - \frac{k_o}{2m}\right) + \frac{m_\pi}{2m}\left(\kappa_p + \kappa_n\right)\right] \tag{40}$$

$$\approx -\sqrt{2}\left(1 - \frac{k_o}{2m}\right)\langle\vec{\sigma}\cdot\vec{\epsilon}\rangle \left[1 - \frac{m_\pi}{2m}\left(\kappa_p + \kappa_n\right)\right]$$

In the cross section we have

$$|\langle M(\pi^+)\rangle|^2 \approx \frac{2\,|\langle\vec{\sigma}\cdot\vec{\epsilon}\rangle|^2}{\left(1 + \frac{m_\pi}{m}\right)} \left[1 - \frac{m_\pi}{2m}\left(\kappa_p + \kappa_n\right)\right]^2 \tag{41}$$

where we have used

$$\left(1 - \frac{k_o}{2m}\right)^2 \approx \frac{1}{\left(1 + \frac{m_\pi}{m}\right)} \tag{42}$$

in order to put the expression in a form which can be compared with the completely relativistic calculation in ref. [7].

Since $(\kappa_p + \kappa_n) = -.12$, the anomalous moment gives only a small correction and can be neglected in π^+ production.

Case 2. $\gamma + n \rightarrow \pi^- + p$

In this case, the largest contribution comes from $M_{\gamma 2}^{(ps)}$, since the isospin operators allow only contributions from the neutron's anomalous moment part of $M_{\gamma 1}^{(ps)}$. Again this can be seen intuitively by considering diagrams 1 and 2 as was done in Case 1.

$$<M(\pi^-)> = \sqrt{2}\, <\vec{\sigma}\cdot\vec{\epsilon}> \left[\left(1 + \frac{k_o}{2m}\right) + \frac{m_\pi}{2m} (\kappa_p + \kappa_n)\right] \tag{43}$$

$$\approx \sqrt{2}\, \frac{<\vec{\sigma}\cdot\vec{\epsilon}>}{\left(1 + \frac{m_\pi}{2m}\right)} \left[1 + \frac{m_\pi}{m} + \frac{m_\pi}{2m} (\kappa_p + \kappa_n)\right]$$

and

$$|<M(\pi^-)>|^2 \approx 2\, \frac{|<\vec{\sigma}\cdot\vec{\epsilon}>|^2}{\left(1 + \frac{m_\pi}{m}\right)} \left[1 + \frac{m_\pi}{m} + \frac{m_\pi}{2m} (\kappa_p + \kappa_n)\right]^2 \tag{44}$$

where again we have arranged terms to put this in the form of the relativistic results.

Case 3. $\gamma + p \rightarrow \pi^o + p$

Diagrams 1 and 2 contribute on an equal footing in this case, however, the leading terms of the $\vec{\sigma}\cdot\vec{\epsilon}$ operator cancel each other, and we are left with only higher order terms.

$$<M(p,\pi^o)> = <\vec{\sigma}\cdot\vec{\epsilon}> \left[\frac{k_o}{m} + \frac{m_\pi}{m} \kappa_p\right] \tag{45}$$

$$|<M(p,\pi^o)>|^2 \approx |<\vec{\sigma}\cdot\vec{\epsilon}>|^2 \left(\frac{m_\pi}{m}\right)^2 (1 + \kappa_p)^2 \tag{46}$$

Case 4. $\gamma + n \rightarrow \pi^o + n$

Only the anomalous magnetic moment parts of $M_{\gamma 1}^{(ps)}$ and $M_{\gamma 2}^{(ps)}$ can contribute, because $(1 + \tau_3)|n\rangle = 0$, which reflects the fact that the neutron has no charge.

$$\langle M(n,\pi^o)\rangle = - \langle\vec{\sigma}\cdot\vec{\epsilon}\rangle\left(\frac{m_\pi}{m}\right)\kappa_n \tag{47}$$

$$|\langle M(n,\pi^o)\rangle|^2 = |\langle\vec{\sigma}\cdot\vec{\epsilon}\rangle|^2\left(\frac{m_\pi}{m}\right)^2 (\kappa_n)^2 \tag{48}$$

By inspection, we can see that charged pion production is much larger than π^o production since the π^o cross sections have an overall factor of $\left(\frac{m_\pi}{m}\right)^2 \approx .02$ not present in the charged pion case. Note that the factor $\left(1 + \frac{m_\pi}{m}\right)^{-1} \approx .87$ is not present in our cross sections for π^o production although it is present in the fully relativistic cross sections in ref. [6]. This factor would come from corrections of $O\left(\frac{p^2}{m^2}\right)$ to our amplitudes which have been neglected in the spirit of our nonrelativistic approximation. As we can see, these factors are not insignificant, and obviously a nonrelativistic calculation of π^o production is not very reliable unless terms of $O\left(\frac{p^2}{m^2}\right)$ have been carefully retained in the amplitudes from the beginning.

4.3. Pseudovector Photoproduction

For pv pion-nuclear coupling, there are three terms which contribute to the cross section at threshold,

$$M_\gamma^{(pv)} = M_{\gamma 1}^{(pv)} + M_{\gamma 2}^{(pv)} + M_{\gamma 4}^{(pv)} \tag{49}$$

Within the approximations we have been using, we can see that the anomalous moment parts vanish. This does not have any significant effect for charged pion production; however, it makes the cross section for $\gamma + n \to \pi^o + n$ equal to 0.

Case 1. $\gamma + p \to \pi^+ + n$

The largest contribution now comes from diagram 4 as opposed to the ps case where diagram 1 gave the largest term

$$\langle M(\pi^+)\rangle = -\sqrt{2}\,\langle\vec{\sigma}\cdot\vec{\epsilon}\rangle\left(1 - \frac{k_o}{2m}\right) \tag{50}$$

Case 2. $\gamma + n \to \pi^- + p$

Again diagram 4 is the largest contribution with a small correction coming from diagram 2.

$$\langle M(\pi^-)\rangle\ \sqrt{2}\,\langle\vec{\sigma}\cdot\vec{\epsilon}\rangle\left(1 + \frac{k_o}{2m}\right) \tag{51}$$

Case 3. $\gamma + p \to \pi^o + p$

Diagram 4 can't contribute in this case, and $M_{\gamma 1}^{(pv)} + M_{\gamma 2}^{(pv)}$ gives

$$\langle M(p,\pi^o)\rangle = \frac{m_\pi}{m}\,\langle\vec{\sigma}\cdot\vec{\epsilon}\rangle \tag{52}$$

4.4. Discussion

In the ps case, if we neglect the very small anomalous moment corrections to the charged pion cross sections, we get exactly the same result as with pv coupling. However, for π^o production, the ps cross sections give results much closer to the experimental values (See ref. [6] for numerical values) even using our approximate amplitudes. In conclusion, we can say that for threshold photoproduction of charged pions, either eqs. 14-16 or eqs. 31-34 can be used. For π^o production, these equations can be in error by as much as 20% due to neglect of terms of $O(p^2/m^2)$.

5. Threshold Electroproduction

5.1. Current Conservation

For electroproduction of pions where the incident photon is now virtual ($k^2 \neq 0$), we can no longer use $\vec{k}\cdot\vec{\epsilon} = 0$ to eliminate terms. This means that we now have contributions from the pion current diagram (fig. 3) even at threshold. However, we can use current conservation of the electron and nucleon currents to eliminate the ϵ_0 terms. For electroproduction, ϵ^μ now represents the electrons current and is not the polarization vector of a real photon.

We can define a nuclear current J_μ by writing our matrix elements as

$$M = \epsilon^\mu J_\mu = \epsilon_o J_o - \vec{\epsilon}\cdot\vec{J} \quad , \tag{53}$$

then current conservation gives

$$k^\mu \epsilon_\mu = k_o \epsilon_o - \vec{k}\cdot\vec{\epsilon} = 0 \tag{54}$$

$$k^\mu J_\mu = k_o J_o - \vec{k}\cdot\vec{J} = 0 \tag{55}$$

Using eqs. 54-55 to solve for ϵ_o and J_o, we can write

$$M = \frac{\vec{k}\cdot\vec{\epsilon}}{k_o}\,\frac{\vec{k}\cdot\vec{J}}{k_o} - \vec{\epsilon}\cdot\vec{J} = \vec{\epsilon}\cdot\left(\vec{k}\,\frac{\vec{k}\cdot\vec{J}}{k_o^2} - \vec{J}\right) \tag{56}$$

Therefore, we need not consider the terms in M that have ϵ_o .

5.2. Pseudoscalar Electroproduction

For electroproduction of pions the incident photon in figs. 1-3 is virtual ($k^2 \neq 0$), therefore, we have to introduce form factors at each of the vertices that a photon is attached. Eqs. 7-9 now become.

$$M_{e1}^{(ps)} = 2m\bar{U}_{s',\tau'}(p_f)\,\gamma_5\tau_\alpha\,\frac{(\not{p} + m)}{p^2 - m^2}\left[\gamma_\mu\,\frac{\left(F_1^S + \tau_3 F_1^V\right)}{2} + i\frac{\sigma_{\mu\upsilon}}{2m}\,k^\upsilon\,\frac{\left(F_2^S + \tau_3 F_2^V\right)}{2}\right]U_{s,\tau}(p_i)\,\epsilon^\mu \tag{57}$$

$$M_{e2}^{(ps)} = 2m\bar{U}_{s',\tau'}(p_f)\left[\gamma_\mu\,\frac{\left(F_1^S + \tau_3 F_1^V\right)}{2} + i\,\frac{\sigma_{\mu\upsilon}}{2m}\,k^\upsilon\frac{\left(F_2^S + \tau_3 F_2^V\right)}{2}\right] \tag{58}$$

$$\frac{(\not{p}' + m)}{p'^2 - m^2} \gamma_5 \tau_\alpha U_{s,\tau}(p_i)\epsilon^\mu$$

$$M_{e3}^{(ps)} = 2m\bar{U}_{s',\tau'}(p_f)\, \gamma_5 \frac{\left[\tau_\alpha, \tau_3\right]}{2} \epsilon^\mu \frac{\left(q' + q\right)_\mu}{q'^2 - m_\pi^2} F_\pi U_{s,\tau}(p_i) \tag{59}$$

where

$$F_1^s(k^2) = F_1^p + F_1^n \tag{60}$$

$$F_1^v(k^2) = F_1^p - F_1^n \tag{61}$$

$$F_2^s(k^2) = \kappa_p F_2^p + \kappa_n F_2^n \tag{62}$$

$$F_2^v(k^2) = \kappa_p F_2^p - \kappa_n F_2^n \tag{63}$$

$$F_1^p(o) = F_2^p(o) = F_2^n(o) = 1 \tag{64}$$

$$F_1^n(o) = 0 \tag{65}$$

Rearranging terms, we can let [1]

$$M_e^{(ps)} = M_{e1}^{(ps)} + M_{e2}^{(ps)} + M_{e3}^{(ps)} = \bar{U}_{s',\tau'}(p_f)\, \gamma_5 \Bigg\{ - \gamma_\mu F_1^v \frac{\left[\tau_\alpha, \tau_3\right]}{2} \tag{66}$$

$$+ 2m \frac{(2q - k)_\mu}{q'^2 - m_\pi^2} F_\pi \frac{\left[\tau_\alpha, \tau_3\right]}{2} - \not{q} \frac{(2p_1 + k)_\mu}{p^2 - m^2} \tau_\alpha \frac{\left(F_1^s + \tau_3 F_1^v\right)}{2}$$

$$- \not{q}\, \frac{1}{2} \frac{(\gamma_\mu \not{k} - \not{k}\gamma_\mu)}{p^2 - m^2} \tau_\alpha \left(\frac{\left(F_1^s + \tau_3 F_1^v\right)}{2} + \frac{\left(F_2^s + \tau_3 F_2^v\right)}{2} \right) +$$

$$\frac{\left(2p_2 - k\right)_\mu \not{q}}{p'^2 - m^2}\frac{\left(F_1^s + \tau_3 F_1^v\right)\tau_\alpha}{2} - \frac{1}{2}\frac{\left(\gamma_\mu \not{k} - \not{k}\gamma_\mu\right)\not{q}}{p'^2 - m^2}\left(\frac{\left(F_1^s + \tau_3 F_1^v\right)}{2}\right.$$

$$\left. + \frac{\left(F_2^s + \tau_3 F_2^v\right)}{2}\right)\tau_\alpha \Bigg\} \epsilon^\mu U_{s,\tau}(p_i)$$

If we use the test for gauge invariance on this amplitude, we find that we must let $F_1^v = F_\pi$. Since all of the form factors are assumed to be independent, an alternate way to make $M_e^{(ps)}$ gauge invariant is to make each term in eq. 66 independently gauge invariant. This can be done by letting

$$\gamma_\mu F_1^v \rightarrow \left(\gamma_\mu - \frac{\gamma \cdot k k_\mu}{k^2}\right) F_1^v \tag{67}$$

$$\frac{\left(2q - k\right)_\mu}{q'^2 - m_\pi^2} \rightarrow \left[(2q-k)_\mu - \frac{(2q-k)\cdot k k_\mu}{k^2}\right]\frac{1}{q'^2 - m_\pi^2} \tag{68}$$

$$\frac{\left(2p_1 + k\right)_\mu}{p^2 - m^2} \rightarrow \left[(2p_1 + k)_\mu - \frac{(2p_1 + k)\cdot k k_\mu}{k^2}\right]\frac{1}{p^2 - m^2} \tag{69}$$

$$\frac{(2p_2 - k)_\mu}{p'^2 - m^2} \rightarrow \left[(2p_2 - k)_\mu - \frac{(2p_2 - k)\cdot k k_\mu}{k^2}\right]\frac{1}{p'^2 - m^2} \tag{70}$$

The fourth and sixth terms in eq. 66 are already gauge invariant. Note that these "gauge corrections" only enter into the longitudinal part of the cross section since their 3 - momentum part is always in the longitudinal direction, i.e., along $\vec{k}$.

Keeping terms of O(p/m) and letting $\vec{q} \rightarrow 0$, the current operator to be used in eq. 56 is

$$\vec{J}^{(ps)} = \left(\vec{\sigma} + \frac{\vec{k}}{k^2}\ \vec{\sigma}\cdot\vec{k}\right)\frac{\left[\tau_\alpha,\tau_3\right]}{2} F_1^V - \frac{2k_o m_\pi}{\left(2k_o m_\pi - k^2\right)} \frac{\vec{k}\ \vec{\sigma}\cdot\vec{k}}{k^2} \frac{\left[\tau_\alpha,\tau_3\right]}{2} F_\pi \tag{71}$$

$$- \frac{k_o}{2m} \frac{\vec{\sigma}}{\left(1 + \frac{m_\pi}{2m}\right)} \tau_\alpha \left(\frac{\left(F_1^S + \tau_3 F_1^V\right)}{2} + \frac{\left(F_2^S + \tau_3 F_2^V\right)}{2}\right)$$

$$- \frac{m_\pi}{2m} \frac{\vec{\sigma}}{\left(1 - \frac{k^2}{2mk_o}\right)} \left(\frac{\left(F_1^S + \tau_3 F_1^V\right)}{2} + \frac{\left(F_2^S + \tau_3 F_2^V\right)}{2}\right) \tau_\alpha$$

5.3. Pseudovector Electroproduction

From eq. 5.2 of ref. [1], the amplitude for electroproduction using pseudovector coupling is

$$M_e^{(pv)} = \bar{U}_{s',\tau'}(p_f)\gamma_5 \left\{ - \gamma_\mu F_A \frac{\left[\tau_\alpha,\tau_3\right]}{2} + 2m \frac{\left(2q - k\right)_\mu}{q'^2 - m_\pi^2} F_\pi \frac{\left[\tau_\alpha,\tau_3\right]}{2} \right. \tag{72}$$

$$- \not{q} \frac{\left(2p_1 + k\right)_\mu}{p^2 - m^2} \tau_\alpha \frac{\left(F_1^S + \tau_3 F_1^V\right)}{2}$$

$$- \not{q}\ \frac{1}{2} \frac{\left(\gamma_\mu \not{k} - \not{k}\gamma_\mu\right)}{p^2 - m^2} \tau_\alpha \left(\frac{\left(F_1^S + \tau_3 F_1^V\right)}{2} + \frac{\left(F_2^S + \tau_3 F_2^V\right)}{2}\right)$$

$$+ \frac{\left(2p_2 - k\right)_\mu}{p'^2 - m^2} \not{q} \frac{\left(F_1^S + \tau_3 F_1^V\right)}{2} \tau_\alpha$$

$$- \frac{1}{2} \frac{\left(\gamma_\mu \not{k} - \not{k}\gamma_\mu\right)}{p'^2 - m^2} \not{q} \left(\frac{\left(F_1^S + \tau_3 F_1^V\right)}{2} + \frac{\left(F_2^S + \tau_3 F_2^V\right)}{2}\right) \tau_\alpha$$

$$\left. - \frac{\frac{1}{2}\left(\gamma_\mu \not{k} - \not{k}\gamma_\mu\right)}{2m} \left(F_2^S\ \tau_\alpha + F_2^V\ \frac{\left\{\tau_\alpha,\tau_3\right\}}{2}\right) \right\} \epsilon^\mu U_{s,\tau}(p_i)$$

Eq. 72 is made gauge invariant by making the substitutions in eqs. 67 - 70.

In ref. [1], Dombey and Read identify $F_A(k^2)$ with the nucleon axial vector form factor. Expressions for this form factor and the other form factors in eq. 72 are given in the appendix.

In the nonrelativistic limit at threshold, the current operator is

$$\vec{J}^{(pv)} = \left(\vec{\sigma} + \frac{\vec{k}}{k^2}\,\vec{\sigma}\cdot\vec{k}\right)\frac{\left[\tau_\alpha,\tau_3\right]}{2}\,F_A - \frac{2k_o m_\pi}{\left(2k_o m_\pi - k^2\right)}\,\frac{\vec{k}\,\vec{\sigma}\cdot\vec{k}}{k^2}\,\frac{\left[\tau_\alpha,\tau_3\right]}{2}\,F_\pi \tag{73}$$

$$- \frac{k_o}{2m}\,\frac{\vec{\sigma}}{\left(1 + \frac{m_\pi}{2m}\right)}\,\tau_\alpha\left(\frac{\left(F_1^s + \tau_3 F_1^v\right)}{2} + \frac{\left(F_2^s + \tau_3 F_1^v\right)}{2}\right)$$

$$- \frac{m_\pi}{2m}\,\frac{\vec{\sigma}}{\left(1 - \frac{k^2}{2mk_o}\right)}\left(\frac{\left(F_1^s + \tau_3 F_1^v\right)}{2} + \frac{\left(F_2^s + \tau_3 F_2^v\right)}{2}\right)\tau_\alpha$$

$$+ \frac{k_o}{2m}\,\vec{\sigma}\left[F_2^s\,\tau_\alpha + F_2^v\,\frac{\left\{\tau_\alpha,\tau_3\right\}}{2}\right]$$

6. Cross Section Equations

6.1. Coordinate Space Representation

The amplitudes given in the previous sections are in a momentum space representation. It is convenient in order to use known nuclear wave functions to express these matrix elements in a coordinate space representation. This is done by using the fact that each of the amplitudes has a 3-momentum conserving δ-function which multiplies it. For production off of a single nucleon, we can let

$$\delta^3(\vec{p}_f + \vec{k}_\pi - \vec{p}_i - \vec{k}_\gamma)\,\chi^\tau_{s',\tau'}\,M\,\chi_{s,\tau} = \tag{74}$$

$$\int d^3x\,\frac{e^{-i\vec{p}_f\cdot\vec{x}}}{\sqrt{8\pi^3}}\,e^{-i\vec{k}_\pi\cdot\vec{x}}\,\chi^\tau_{s',\tau'}\,M\,\chi_{s,\tau}\,e^{i\vec{k}_\gamma\cdot\vec{x}}\,\frac{e^{i\vec{p}_i\cdot\vec{x}}}{\sqrt{8\pi^3}}$$

where $e^{-i\vec{p}_f\cdot\vec{x}}/\sqrt{8\pi^3}$ and $e^{i\vec{p}_i\cdot\vec{x}}/\sqrt{8\pi^3}$ are now the final and initial nucleon wave functions in coordinate space. To generalize this to a system of A nucleons, one has to replace

$\chi^{\tau}_{s',\tau'}\ \dfrac{e^{i\vec{p}_f\cdot\vec{x}}}{\sqrt{8\pi^3}}$ and $\dfrac{e^{i\vec{p}_i\cdot\vec{x}}}{\sqrt{8\pi^3}}\ \chi_{s,\tau}$ by the appropriate initial and final state nuclear wave functions and let $\int d^3x \rightarrow \sum\int d^3x_1 \ . \ . \ . \ d^3x_A$.

When we separate out the center of mass part of the initial and final state nuclear wave functions, then the integration over the center of mass coordinate gets back a 3-momentum conserving δ-function, $\delta^3(\vec{P}_f + \vec{k}_\pi - \vec{P}_i - \vec{k}_i)$, where $\vec{P}_i$ and $\vec{P}_f$ are the momenta of the center of mass of initial and final nucleus. All that is left in the coordinate space matrix element is the integration over the internal coordinates of the nucleus.

If we include interactions of the outgoing pion with the final state nucleus, then we can no longer use $e^{-i\vec{q}\cdot\vec{r}_i}$ as the pion wave function where $\vec{r}_i$ represents a relative coordinate. We let $e^{-i\vec{q}\cdot\vec{r}_i} \rightarrow \varphi_\pi(\vec{r}_i)$ where $\varphi_\pi(\vec{r}_i)$ is now distorted by Coulomb and strong final state interactions. This is called the distorted wave impulse approximation (DWIA). There is no general agreement as to how to generate these wave functions, and the investigation of this part of the problem is beyond the scope of this paper. However, an understanding of these final state interactions could be the most important and interesting aspect of photo pion production from nuclei.

6.2. Photoproduction Cross Section

Assuming the initial and final nuclear states are bound states, the photoproduction cross section can be written [8]

$$d\sigma_\gamma = \frac{1}{2\pi}\,\alpha\left(\frac{G}{2mc^2}\right)^2 \frac{(\hbar c)^4}{(2E_\pi k_o)} \frac{|<M(r)>|^2}{\left(1 - \frac{v}{c}\right)}\,\delta(E_f + E_\pi - E_i - E_\gamma) \tag{75}$$

$$\delta^3(\vec{P}_f + \vec{q} - \vec{P}_i - \vec{k})\,d^3P_f\,d^3q$$

where $|\langle M(r)\rangle|^2$ includes the sum and average over the projections of the angular momentum of the final and initial states and the coordinate space integrations.

The flux factor $\left|\frac{v}{c} - 1\right|$ can be evaluated in the center of momentum frame where

$$\vec{v} = \frac{\vec{p}_i}{m_i} \to - \frac{E_\gamma}{m_i c^2}$$

and
$$\left(1 - \frac{v}{c}\right) \to \left(1 + \frac{E_\gamma}{m_i c^2}\right)$$

The integration over d^3P_f is done using $\delta^3(\quad)$. Letting

$$d^3q = \frac{|\vec{q}|}{(\hbar c)^2} E_\pi dE_\pi d\Omega_\pi \ , \tag{76}$$

the integration over dE_π can be performed using $\delta(E_f + E_\pi - E_i - E_\gamma)$. But E_f is now a function of E_π so that we cannot use the δ-function directly

$$E_f \approx m_f + \frac{\vec{q}^{\,2}}{2m_f} = m_f + \frac{\left(E_\pi^2 - m_\pi^2\right)}{2m_f} \tag{77}$$

Then

$$\delta(E_f + E_\pi - E_i - E_\gamma)dE_\pi = \delta\left[m_f + \frac{E_\pi^2 - m_\pi^2}{2m_f} + E_\pi - E_i - E_\gamma\right]dE_\pi = \tag{78}$$

$$\frac{\delta\left(E_\pi - E_o\right)}{\left(1 + \frac{E_\pi}{m_f}\right)} dE_\pi$$

where E_o is the result one gets by setting $[\ ] = 0$ and solving for E_π. Now the dE_π integration can be done simply.

$$\frac{d\sigma_\gamma}{d\Omega_\pi} = \frac{1}{4\pi}\,\alpha\left(\frac{G}{2mc^2}\right)^2 \left|\frac{\vec{q}}{\vec{k}}\right| \frac{(\hbar c)^2\ |\langle M(r)\rangle|^2}{\left(1+\frac{E_\gamma}{m_i c^2}\right)\left(1+\frac{E_\pi}{m_f c^2}\right)} \tag{79}$$

Note that the kinematical factors in the denominator of eq. 79 were derived assuming a bound final state in a particular frame and are not necessarily present or of this form for all photoproduction processes.

6.3. Electroproduction Cross Section

From eq. 56 we saw that the electroproduction amplitudes could be written as

$$M = \vec{\epsilon}\cdot\left(\vec{k}\,\frac{\vec{k}\cdot\vec{J}}{k_o^2} - \vec{J}\right)$$

Defining a new current operator $\vec{J}'$ by

$$\vec{J}' \equiv \left(\frac{\vec{k}\ \vec{k}\cdot\vec{J}}{k_o^2} - \vec{J}\right) \quad , \tag{80}$$

the electroproduction cross section can be written [8]

$$\frac{d\sigma_e}{dE_{e'}\,d\Omega_{e'}} = \frac{\alpha^2}{16\pi^3}\left(\frac{G}{2mc^2}\right)^2\left(\frac{k_{e'}}{k_e}\right)\frac{(\hbar c)^3}{E_\pi}\,\frac{1}{k^4}\,\Bigg\{\left(\vec{k}_e\cdot\langle\vec{J}'\rangle\right)\left(\vec{k}'_e\cdot\langle\vec{J}'\rangle^*\right) \tag{81}$$

$$+\left(\vec{k}_e\cdot\langle\vec{J}'\rangle^*\right)\left(\vec{k}'_e\cdot\langle\vec{J}'\rangle\right) - \frac{k^2}{2}\langle\vec{J}'\rangle\cdot\langle\vec{J}'\rangle^*\Bigg\}\ \delta(E_f + E_\pi + E_{e'} - E_i - E_e)$$

$$\delta^3 (\vec{p}_f + \vec{q} + \vec{k}_e' - \vec{p}_i - \vec{k}_e)$$

where E_e and $E_{e'}$ are the initial and final electron energies, $\vec{k}_e$ and $\vec{k}_e'$ are the initial and final electron momenta, and

$$k^2 = (k_e - k_e')^2 = (k_{eo} - k_{eo}')^2 - (\vec{k}_e - \vec{k}_e')^2 \tag{82}$$

The longitudinal part of the current $\vec{J}'$ is given by

$$\vec{J}_L' = \frac{\vec{k}\cdot\vec{J}'}{\vec{k}^2}\,\vec{k} = -\frac{k^2}{k_o^2}\,\vec{J}_L \tag{83}$$

and the transverse current by

$$\vec{J}_T' = \vec{J}_T \tag{84}$$

From eq. 73, we can see that the contribution to $\vec{J}$ from the pion current diagram is in the $\vec{k}$ direction. Therefore, this diagram only contributes to the longitudinal current and need not be considered in $\vec{J}_T$.

Assuming the final state to be a state with one definite angular momentum,[2] the longitudinal-transverse and transverse-transverse-interference terms disappear.

---

[2]This is a safe assumption since we are considering only threshold production.

The cross section can now be written as

$$\frac{d\sigma}{dE_{e'}d\Omega_{e'}} = \frac{\alpha^2}{8\pi^3}\left(\frac{G}{2mc^2}\right)^2\left(\frac{k'_e}{k_e}\right)\frac{(\hbar c)^3}{k^4}\frac{d^3q}{E_\pi}\,d^3P_f\quad\delta(\mathcal{E}_f - \mathcal{E}_i)\,\delta^3(\vec{P}_f - \vec{P}_i) \tag{85}$$

$$\left\{\frac{k^4}{\vec{k}^2}\frac{1}{k_o^4}\left[2\left(\vec{k}_e\cdot\vec{k}\right)\left(\vec{k}'_e\cdot\vec{k}\right) - \frac{1}{2}k^2\vec{k}^2\right]\frac{|\vec{k}\cdot\langle\vec{J}_L\rangle|^2}{\vec{k}^2} + \left[\frac{2k_e^2k_e'^2\sin^2\theta_e}{\vec{k}^2} - \frac{k^2}{2}\right]\right.$$

$$\left.\left[|\hat{\epsilon}_+\cdot\langle\vec{J}_T\rangle|^2 + |\hat{\epsilon}_-\cdot\langle\vec{J}_T\rangle|^2\right]\right\}$$

where $\hat{\epsilon}_\pm = \dfrac{\hat{\epsilon}_x \pm i\hat{\epsilon}_y}{\sqrt{2}}$

and θ_e is the angle between $\vec{k}_e$ and $\vec{k}'_e$. Neglecting the electron mass, we can write for the coefficient of the longitudinal current

$$\frac{k^4}{\vec{k}^2}\frac{1}{k_o^4}\left[2\left(\vec{k}_e\cdot\vec{k}\right)\left(\vec{k}'_e\cdot\vec{k}\right) - \frac{1}{2}k^2\vec{k}^2\right] = \frac{k^4}{k_o^2}\quad\frac{\epsilon}{(1-\epsilon)} \tag{86}$$

and for the coefficient of the transverse current

$$\left[\frac{2\vec{k}_e^2\vec{k}_e'^2\sin^2\theta_e}{\vec{k}^2} - \frac{k^2}{2}\right] = -\frac{k^2}{2}\frac{1}{(1-\epsilon)} \tag{87}$$

Dividing through by k^2, the cross section can now be written

$$\frac{d\sigma}{dE_{e'}d\Omega_{e'}} = \frac{\alpha^2}{8\pi^3}\left(\frac{G}{2mc^2}\right)^2\left(\frac{k'_e}{k_e}\right)\frac{(\hbar c)^3}{|k^2|}\frac{d^3q}{E_\pi}\,d^3P_f\quad\frac{1}{(1-\epsilon)} \tag{88}$$

$$\delta^4(\wp_f - \wp_i)\left\{\frac{\left[|\vec{\epsilon}_+\cdot<\vec{J}_T>|^2 + |\vec{\epsilon}_-\cdot<\vec{J}_T>|^2\right]}{2} - \epsilon\,\frac{k^2}{k_o^2}\,\frac{|\vec{k}\cdot<\vec{J}_L>|^2}{\vec{k}^2}\right\}$$

where $$\epsilon = \left(1 - 2\frac{\vec{k}^2}{k^2}\tan^2\frac{\theta_e}{2}\right)^{-1} \tag{89}$$

It is useful for the purpose of comparing with photoproduction data to express parts of the cross section in center of mass variables (variables in the photon-target center of mass system will be denoted by a $*$) [9]. The rest of the cross section will be evaluated in the lab system.

It is known that

$$\frac{d^3q}{E_\pi}\,\frac{d^3P_f}{E_f}\,\delta^4(\wp_f - \wp_i) = \frac{d^3q^*}{E_\pi^*}\,\frac{d^3P_f^*}{E_f^*}\,\delta^4\left(\wp_f^* - \wp_i^*\right) \tag{90}$$

If we transform to a system where $\vec{k}$ is in the z- direction, then we know the transverse terms won't change and

$$|\hat{\epsilon}_+\cdot<\vec{J}_T>|^2 + |\hat{\epsilon}_-\cdot<\vec{J}_T>|^2 = |\hat{\epsilon}_+\cdot<\vec{J}_T>^*|^2 + |\hat{\epsilon}_-\cdot<\vec{J}_T>^*|^2 \tag{91}$$

The longitudinal current transforms by

$$\frac{|\vec{k}\cdot<\vec{J}_L>|^2}{\vec{k}^2} = \left(\frac{k_o}{k_o^*}\right)^2\frac{|\vec{k}\cdot<\vec{J}_L>^*|^2}{\vec{k}^2} \tag{92}$$

If we assume the electron and pion are detected, the cross section is

$$\frac{d\sigma}{dE_e'\,d\Omega_e'\,d\Omega_\pi^*} = \frac{\alpha^2}{8\pi^3}\left(\frac{G}{2mc^2}\right)^2\left(\frac{k_e'}{k_e}\right)\frac{(\hbar c|\vec{q}^*|)}{|k^2|}\,\frac{1}{(1-\epsilon)}\,\frac{1}{\left(1 + \frac{E_\pi^*}{m_f c^2}\right)} \tag{93}$$

$$\left\{ \frac{\left[|\hat{\epsilon}_{+}\cdot\langle\vec{J}_T\rangle^{*}|^2 + |\hat{\epsilon}_{-}\cdot\langle\vec{J}_T\rangle^{*}|^2 \right]}{2} - \epsilon \frac{k^2}{k_o^{*2}} \frac{|\vec{k}\cdot\langle\vec{J}_L\rangle^{*}|^2}{\vec{k}^2} \right\} \tag{93 (cont.)}$$

where

$$k_o^{*} = \frac{k_o\left(m_i + k_o\right) - \vec{k}^2}{\left[\left(m_i + k_o\right)^2 - \vec{k}^2\right]^{\frac{1}{2}}} \tag{94}$$

$$\vec{k}^{*} = \frac{\vec{k}\, m_i}{\left[\left(m_i + k_o\right)^2 - \vec{k}^2\right]^{\frac{1}{2}}} \tag{95}$$

Let

$$\Gamma_t = \frac{\alpha}{2\pi^2}\left(\frac{k_e'}{k_e}\right)\frac{K_L}{|k^2|}\frac{1}{(1-\epsilon)}\frac{1}{(\hbar c)}$$

Then

$$\frac{d\sigma}{dE_{e'}\, d\Omega_{e'}\, d\Omega_\pi^{*}} = \Gamma_t \frac{\alpha}{4\pi}\left(\frac{G}{2mc^2}\right)^2 \frac{(\hbar c)^2 |\vec{q}^{\,*}|}{K_L} \frac{1}{\left(1 + \frac{E_\pi^{*}}{m_f c^2}\right)} \tag{96}$$

$$\left\{ \frac{|\hat{\epsilon}_{+}\cdot\langle\vec{J}_T\rangle^{*}|^2 + |\hat{\epsilon}_{-}\cdot\langle\vec{J}_T\rangle^{*}|^2}{2} - \epsilon \frac{k^2}{k_o^{*2}} \frac{|\vec{k}\cdot\langle\vec{J}_L\rangle^{*}|^2}{\vec{k}^2} \right\} = \frac{\Gamma_t}{4\pi}\left(\sigma_T + \epsilon\sigma_L\right)$$

where K_L is the equivalent real photon energy in the lab system.

where

$$\sigma_T = \alpha \left(\frac{G}{2mc^2}\right)^2 (\hbar c)^2 \left|\frac{\vec{q}^{\,*}}{K_L}\right| \left(1 + \frac{E_\pi^*}{m_f c^2}\right)^{-1} \tag{97}$$

$$\left\{\frac{\left[|\hat{\epsilon}_+ \cdot \langle \vec{J}_T \rangle^*|^2 + |\hat{\epsilon}_- \cdot \langle \vec{J}_T \rangle^*|^2\right]}{2}\right\}$$

$$\sigma_L = \alpha \left(\frac{G}{2mc^2}\right)^2 (\hbar c)^2 \left|\frac{\vec{q}^{\,*}}{K_L}\right| \left(1 + \frac{E_\pi^*}{m_f c^2}\right)^{-1} \left\{- \frac{k^2}{k_o^{*2}} \frac{|\vec{k} \cdot \langle \vec{J}_L \rangle^*|^2}{\vec{k}^2}\right\} \tag{98}$$

7. Discussion of Results

In order to see where the nonrelativistic current operators are no longer valid, the electroproduction cross section off of a proton producing a π^+ is calculated using the fully relativistic operator in eq. 72 and compared with the nonrelativistic results from eq. 73. In fig. 6, we see that the nonrelativistic results are not very good beyond $k^2 = -.1(\mathrm{GeV}/c)^2$. In figs. 7 and 8 are plotted the transverse and longitudinal cross sections.

In order to get better agreement between the two results, one would have to include terms which depend on the initial and final 3 - momentum of the nucleon. This is no problem for pion production off of a single nucleon, because $\vec{P}_i = -\vec{k}$ and $\vec{P}_f = 0$. But for pion production off a nucleon in a nucleus $\vec{P}_i \neq -\vec{k}$ and $\vec{P}_f \neq 0$ and these terms greatly complicate the calculation.

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8. Appendix

From ref. [1] and [7], the form factors to be used in eq. (71) and (73) are

$$F_A = \frac{1}{1 - \frac{k^2}{m_A^2}} \quad , \quad m_A = 1.06 \text{ GeV}$$

$$G_M^p = \frac{(1 + \kappa_p)}{1 - \frac{k^2}{.71}}$$

$$G_E^p = \frac{1}{1 - \frac{k^2}{.71}}$$

$$G_M^n = \frac{\kappa_n}{1 - \frac{k^2}{.71}}$$

$$G_E^n = \frac{-\tau G_M^n}{(1 + 4\tau)} \quad , \quad \tau = -\frac{k^2}{4m^2}$$

The pion form factor is

$$F_\pi = \frac{1}{1 - \frac{k^2}{m_\rho^2}} \quad , \quad m_\rho = 0.765 \text{ GeV}$$

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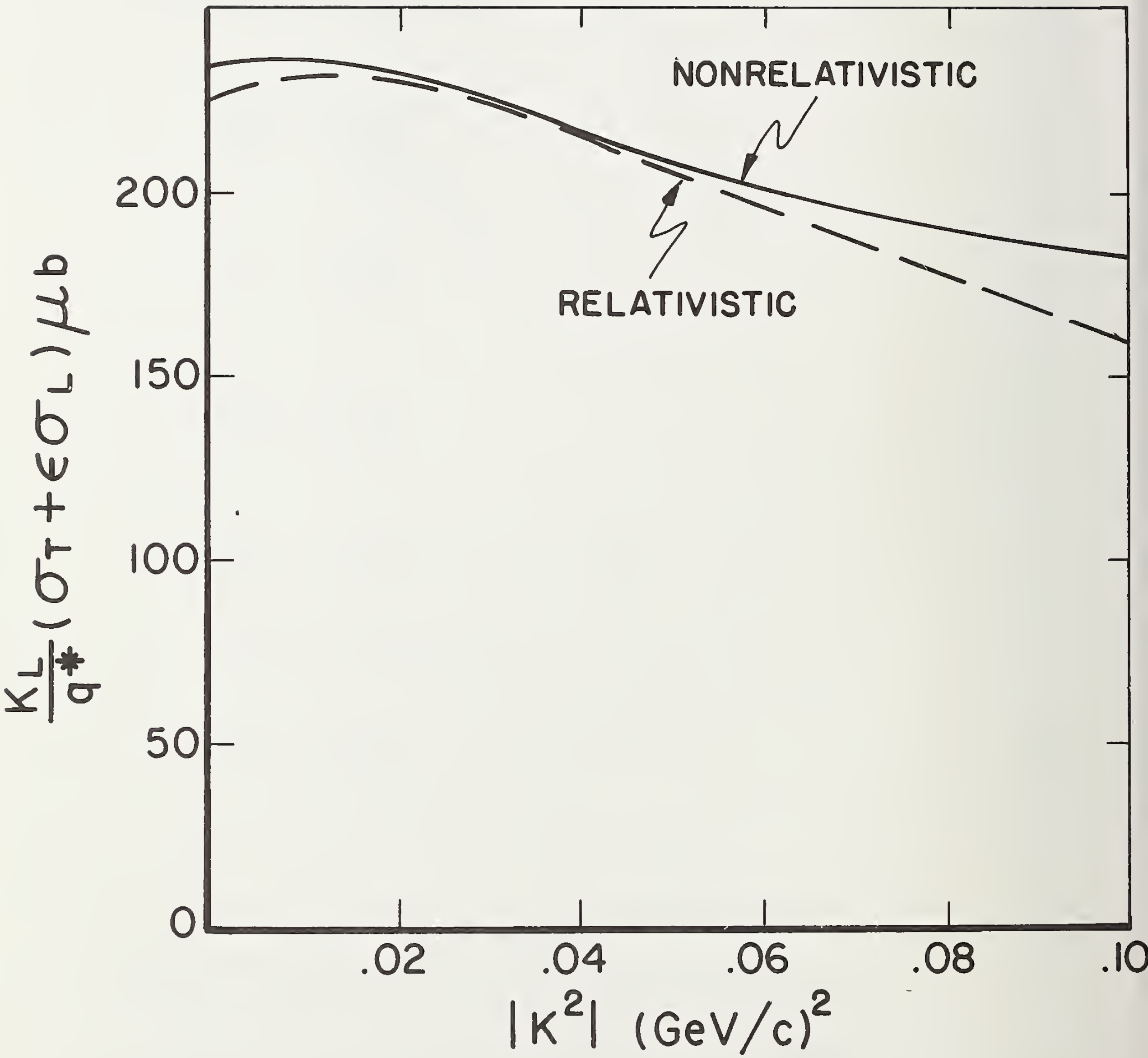

Figure 6. Comparison of cross section slope for $e(p,e'\pi^+)n$ using fully relativistic and nonrelativistic amplitudes.

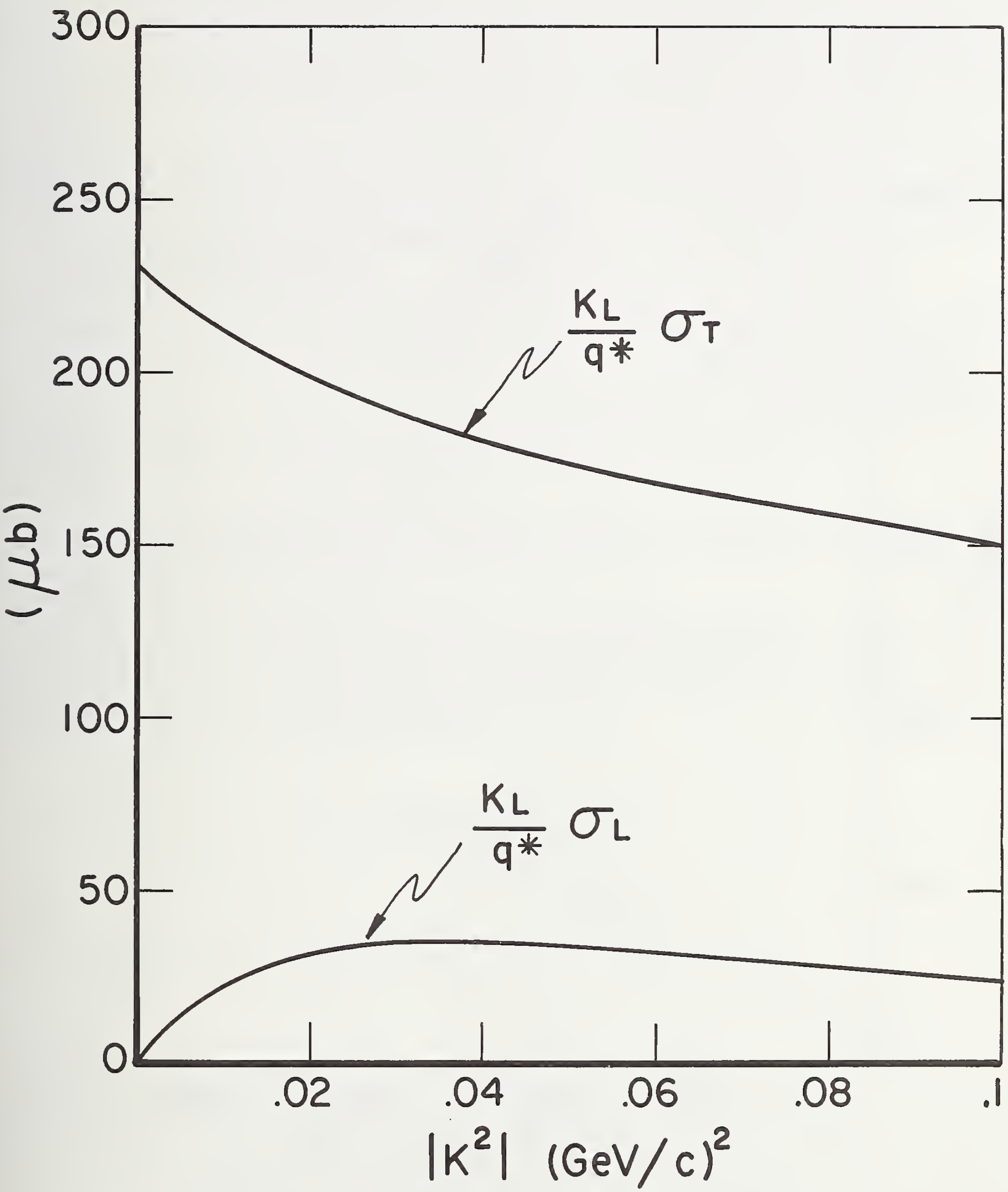

Figure 7. Slope of the transverse and longitudinal cross sections using nonrelativistic amplitudes.

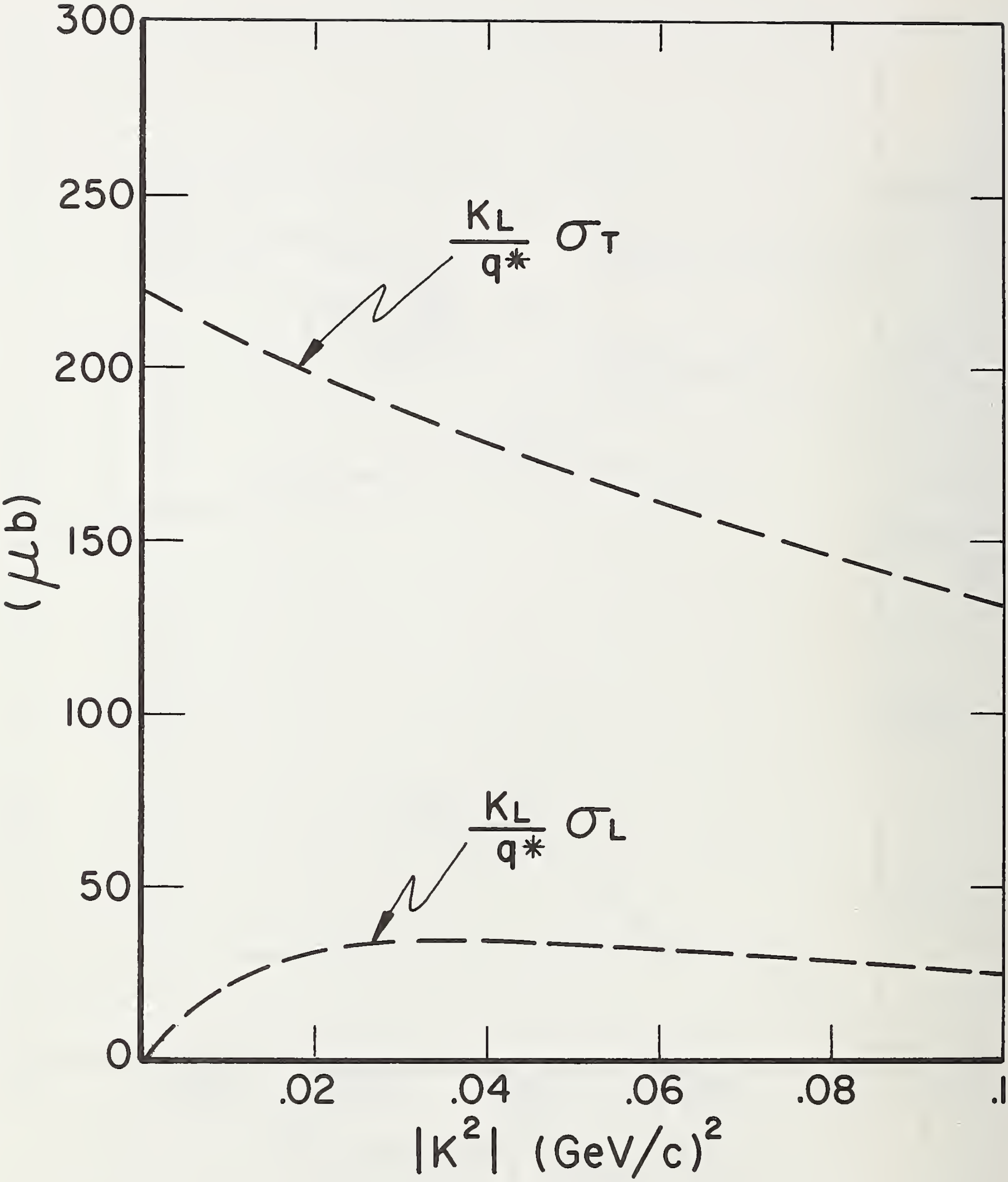

Figure 8. Slope of the transverse and longitudinal cross sections using fully relativistic amplitudes.

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The nonrelativistic amplitudes for photo and electroproduction of pions are derived using effective pseudoscalar and pseudovector Lagrangian densities. The results for pseudscalar and pseudovector coupling are compared at threshold for charged and neutral pion photo and electroproduction. Cross section formulas and kinematical conditions are also presented.

17. KEY WORDS (*six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons*)

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