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A THEORETICAL STUDY OF SPORADIC-E STRUCTURE IN THE LIGHT OF RADIO MEASUREMENTS

BY
KAZUHIKO TAO

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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ABSTRACT

The theoretical aspects of the mechanisms of sporadic-E reflections are described from both the standpoint of a thin layer and a scattering model. For the thin layer model, thin dielectric layers which have various distributions of electron density are considered. It is also pointed out that the scattering theory for which an autocorrelation function of the fluctuation of electron density is given by modified Bessel functions of the fourth through the seventh order is an available model for sporadic-E scatter. Moreover blobs of ionization which have a horizontal scale of the order of 200 m and a vertical scale of about 50 m are considered for sporadic-E scatter. The frequency and distance dependences of the oblique VHF propagation by means of the sporadic-E layer are discussed by comparing the theoretical results with experimental evidence.

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1. INTRODUCTION

We have seen that there are several types of sporadic-E delineated by their differing diurnal or seasonal occurrence times or latitude variation [1]. Some types of sporadic-E echoes on ionograms have the appearance of reflections from a thin layer, and other types suggest the scattering mechanism. In this paper, we shall confine our attention to sporadic-E echoes which are frequently observed in middle latitudes.

The theory concerning a thin layer which has a cloud-like shape is described in section 2 and a sporadic-E model is given in section 3 from the standpoint of scattering theory. Distance and frequency dependences of sporadic-E propagation are stated in sections 4 and 5 together with a comparison of theoretical with experimental results. In section 6 a relationship is derived between foEs and the measured field strength observed over an oblique VHF propagation path during periods of sporadic-E enhancement.

2. THIN LAYER MODEL FOR SPORADIC E

2.1 Outline of the Model

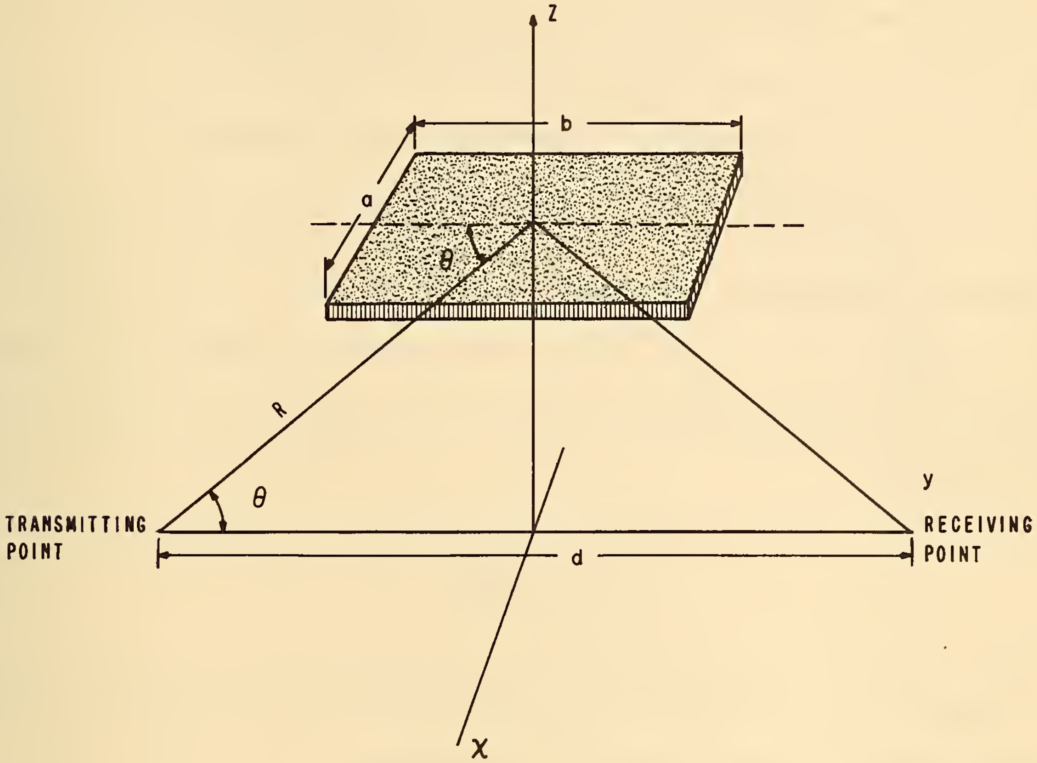
Measurement of the electron density within the sporadic-E layer has shown that relatively sharp variations in the gradients of electron density exist in both the horizontal and vertical directions. Although the theory of reflection from a thin ionized layer has been developed by Hartree [2], Booker [3], Rydbeck [4, 5] and Rawer [6], another treatment will be developed in this paper. We shall postulate many thin dielectric layers in the E region which have limited extent in the horizontal direction. The basic idea for this model was developed by Friis, Crawford and Hogg [7], in order to explain the persistent tropospheric field strengths beyond the horizon. We shall apply this concept to the problem of sporadic E.

According to diffraction theory, if a layer has a reflection coefficient ρ, the received power is given in the form,

$$P_r = P_t \frac{A_t A_r}{\lambda^2 d^2} \rho^2 \qquad (2.1)$$

where P_t is the transmitting power, A_t and A_r are the effective areas of transmitting and receiving antennas. d is the distance between the transmitter and receiver. This relation is appropriate when the following conditions are satisfied,

$$b > \frac{\sqrt{\lambda d}}{\theta}, \quad a > \sqrt{\lambda d} \qquad (2.2)$$



GEOMETRY OF A THIN LAYER MODEL

FIGURE 1

where θ is the grazing angle to the layer and a and b are horizontal dimensions as shown in Figure 1.

2.2 Calculation of the Reflection Coefficient

Let us assume that the layer is composed of many thin layers whose dielectric constants differ one from their neighbors by an increment $\Delta \epsilon$. Provided that $1 \gg \theta^2 \gg d\epsilon$, the reflection coefficient of the boundary between thin layers is given in the simpler form for both polarizations.

$$d\rho = \frac{d\epsilon}{4\theta^2} \tag{2.3}$$

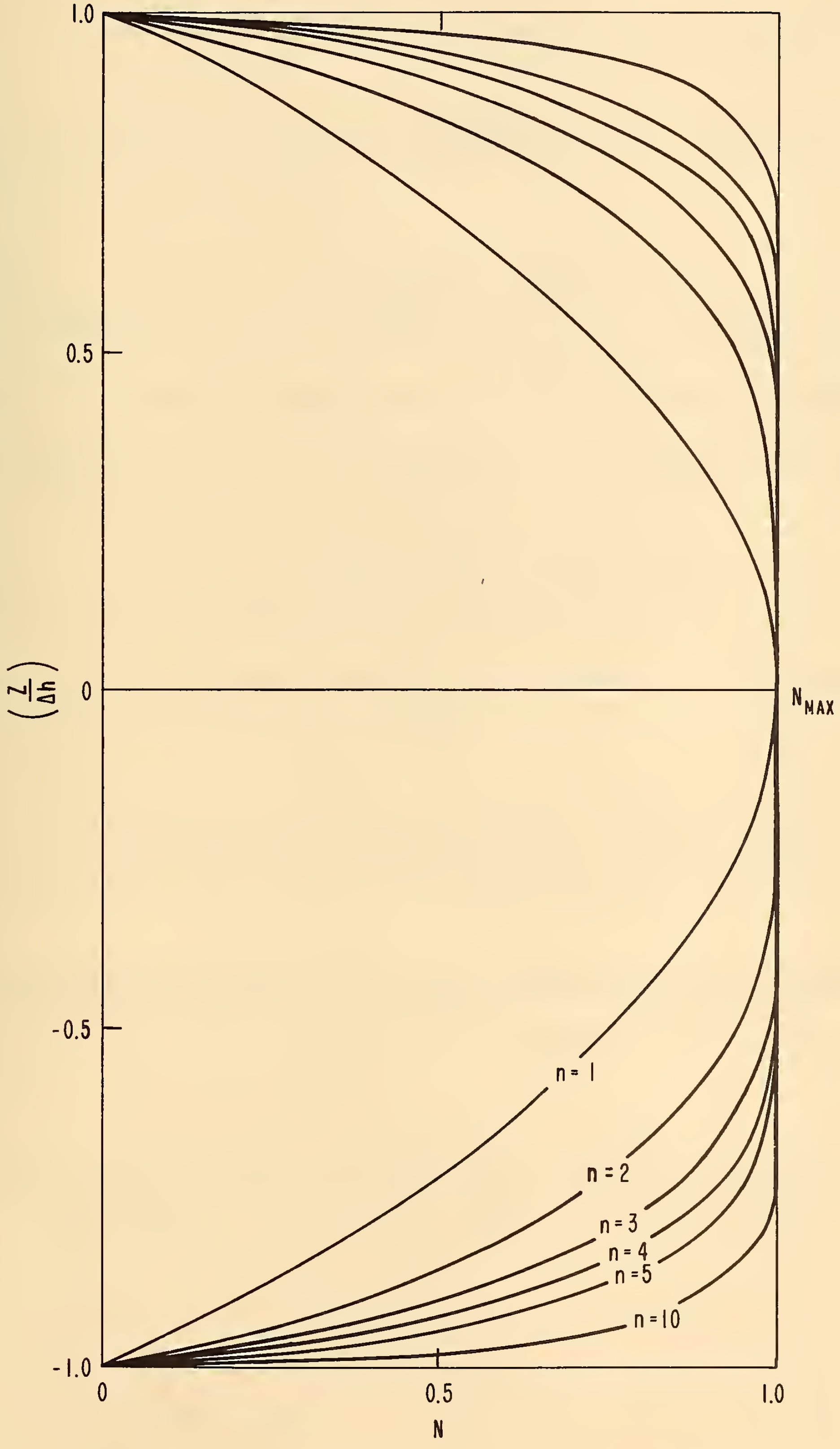
We shall next assume the vertical distribution of electron density in the sporadic-E layer to be in the following form.

$$N = N_{max} \left\{ 1 - \left(\frac{z}{\Delta h}\right)^{2n} \right\}, \quad n = 1, 2, 3 \tag{2.4}$$

where Δh is a half-thickness of the layer.

This distribution of electron density is shown in Figure 2 as a parameter of various values of n. If we put n = 1, this is a parabolic distribution. In rationalized MKS units, the dielectric constant is given in the following form for the ionosphere, neglecting the effect of collisions and of the earth's magnetic field,

$$\epsilon = 1 - \frac{Ne^2}{m\epsilon_0 \omega^2}$$



ELECTRON DENSITY DISTRIBUTION IN THE SPORADIC E LAYER

FIGURE 2

so that the dielectric constant is in the form, if we put

$$f_N = \frac{N_{max}\, e^2}{4\pi m \epsilon_o},$$

$$\epsilon = 1 - \frac{f_N^2}{f^2}\left\{1 - \left(\frac{z}{\Delta h}\right)^{2n}\right\} \tag{2.5}$$

The reflected field from any incremental layer is obtained by considering phase difference

$$dE_r = d\rho E_i e^{-j\left(\frac{4\pi z}{\lambda}\right)\sin\theta}\, dz \tag{2.6}$$

Differentiating (2.5) with respect to z and inserting this into (2.6), we get

$$dE_r = \frac{n}{2}\,\frac{f_N^2}{f^2}\,\frac{z^{2n-1}}{(\Delta h)^{2n}\,\theta^2}\, E_i e^{-j\left(\frac{4\pi z}{\lambda}\right)\sin\theta}\, dz \tag{2.7}$$

The complete reflected field ($f>f_N$) is obtained by integrating the reflections from all increments within the layer

$$E_r = n\,\frac{f_N^2\, E_i}{f^2\,(\Delta h)^{2n}\,\theta^2}\int_0^{\Delta h} e^{-j\left(\frac{4\pi z}{\lambda}\right)\sin\theta}\, z^{2n-1}\, dz \tag{2.8}$$

After the calculation of integral, we get

$$E_r = n \frac{f_N^{\,2} E_i \lambda^{2n}}{f^2 (\triangle h)^{2n} \theta^2 (4\pi \sin\theta)^{2n}} \left[(2n-1) \sum_{m=1}^{n} (-1)^{m+1} \frac{(2n-2)!}{(2n-2m)!} \right.$$

$$\times \left(\frac{4\pi \sin\theta \triangle h}{\lambda} \right)^{2n-2m} \cos \left(\frac{4\pi \sin\theta \triangle h}{\lambda} \right) + \sum_{m=1}^{n} (-1)^{m+1} \frac{(2n-1)!}{(2n-2m+1)!}$$

$$\left. \times \left(\frac{4\pi \sin\theta \triangle h}{\lambda} \right)^{2n-2m+1} \sin \left(\frac{4\pi \sin\theta \triangle h}{\lambda} \right) - (2n-1)! \right] \qquad (2.9)$$

The total reflection coefficient is defined as follows

$$\rho = \frac{E_r}{E_i} \qquad (2.10)$$

In order to calculate the received power from the formula (2.1), it is convenient to use the reflection coefficient expressed as a power ratio.

If we put $L = 4\pi \sin\theta \triangle h/\lambda$ in (2.9) and let $k = n-m$, ρ^2 is expressed in simpler form

$$\rho^2 = \frac{f_N^{\,4} n^2 [(2n-1)!]^2}{f^4 \theta^4 L^{4n}} \left[(-1)^n \sum_{k=0}^{n-1} \frac{L^{2k}}{2k!} (-1)^k \left(\cos L + \frac{L \sin L}{2k+1} \right) + 1 \right]^2 \qquad (2.11)$$

Therefore the received power is obtained by combining formulae (2.1) and (2.11).

3. SCATTERING MODEL FOR SPORADIC E

3.1 Outline of this Model

Some types of sporadic E, particularly those which are frequently observed during evening hours in middle latitude, have many features of scattering phenomena. This scatter would be due to the irregularities of electron density occurring in the lower ionosphere. Recently Booker [8] has suggested that the phenomenon of sporadic E can, in some cases, be explained from the standpoint of scattering theory. The phenomenon of sporadic E, however, generally appears to have a denser concentration of electron density than the normal scattering phenomena. For this reason, it seems inappropriate to use the auto-correlation functions which are used to account for the normal scattering phenomena. Therefore, we shall at first consider an auto-correlation functions of the electron density available for the scattering model of sporadic E. We shall take the following correlation function

$$C_n(\rho) = C(o)\left[\frac{2^{1-n}}{\Gamma(n)}\right] \rho^n K_n(\rho), \quad \rho = \frac{\ell}{r} \tag{3.1}$$

where $K_n(\rho)$ is the modified Bessel function of the second kind of order n and Γ is the gamma function. $C(o)$ is the time variance $< (\Delta n)^2 >$ of the refractive index n of the atmosphere and ℓ is the scale of turbulence. This correlation function was derived by Muchmore and Gallet [9]. If we put $n = 1/2$ in the above formula, we can derive the exponential correlation function which was derived by Booker and Gordon [10]. Also the modified Bessel function of the first order is obtained in the case of $n = 1$, which is used by Norton and his collaborators [11] in order to account for tropospheric scattering. While considering $n = 1/2$ and $n = 1$, we shall pay more attention to the higher orders of n in this correlation function.

When n is an integer, $K_n(\rho)$ is defined by the equation

$$K_n(\rho) = \lim_{\epsilon \to 0} \frac{\pi}{2} \{ I_{-n-\epsilon}(\rho) - I_{n+\epsilon}(\rho) \} \cot \pi\epsilon, \tag{3.2}$$

where

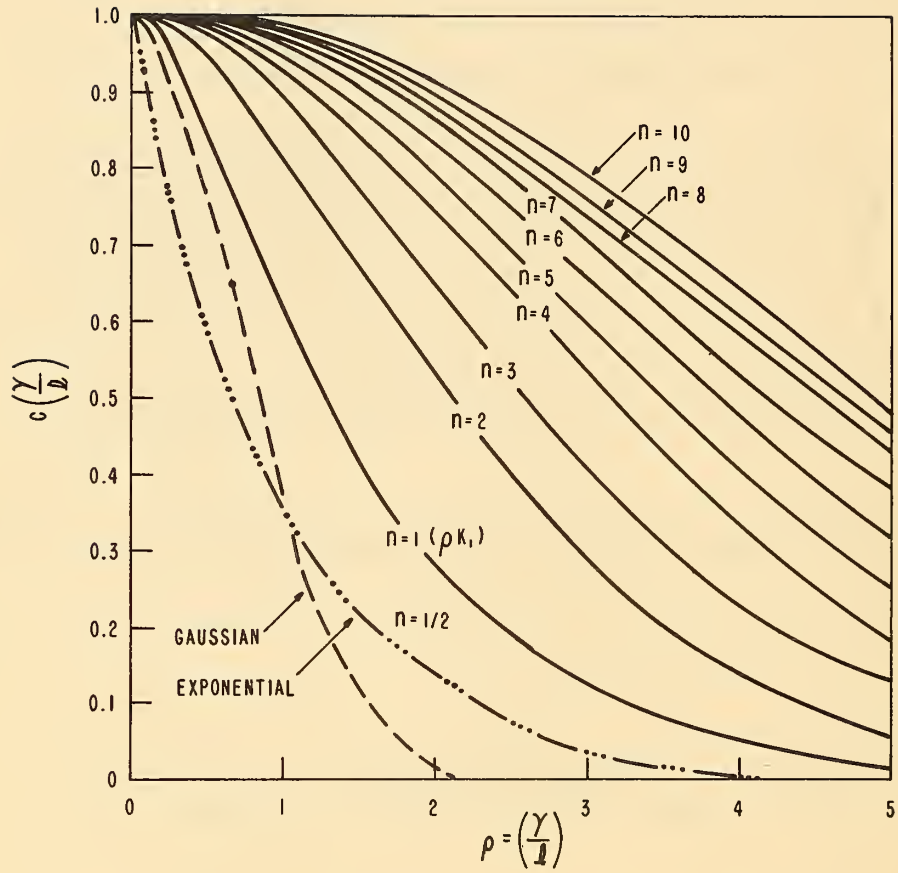
$$I_n(\rho) = \sum_{m=0}^{\infty} \frac{\rho^{n+2m}}{2^{n+2m}\, m!\, \Gamma(n+m+1)}$$

$$I_{-n}(\rho) = \sum_{m=0}^{\infty} \frac{\rho^{-n+2m}}{2^{-n+2m}\, m!\, \Gamma(-n+m+1)}$$

and which gives*

$$K_n(\rho) = 1/2 \sum_{m=0}^{n-1} \frac{(-1)^m (n-m-1)!}{m!\, (\frac{\rho}{2})^{n-2m}} + (-1)^{n+1} \sum_{m=0}^{\infty} \frac{(\frac{\rho}{2})^{n+2m}}{m!\, (n+m)!} \{ \log (\tfrac{\rho}{2}) - 1/2\, \psi(m+1) - 1/2\, \psi(n+m+1) \} \tag{3.3}$$

* cf. Watson "Bessel Function".



AUTOCORRELATION FUNCTION OF THE MODIFIED BESSEL FUNCTION

$[2^{1-n}/\Gamma(n)]\ \rho^n K_n(\rho)$

FIGURE 3

where $\psi(m+1) = -\gamma + (1 + 1/2 + 1/3 + \ldots + 1/m)$

$$\gamma = \lim_{m\to\infty} \{1 + 1/2 + 1/3 + \ldots + 1/m - \log m\} = 0.5772157\ldots.$$

(Euler's constant).

The calculated curves of this correlation function (3.1) are shown in Figure 3 by giving various values of n. In addition to these curves, the exponential and the Gaussian models are plotted in this figure. Models of the exponential, the Gaussian and the modified Bessel function of the first order have been used so far in order to explain both the ionospheric and tropospheric scattering cases. Although these correlation functions are proper for the phenomena of normal scattering, they may not be appropriate for the scattering model of sporadic E which appear to have denser concentrations of electrons. There is also another trouble that the frequency dependence of sporadic-E propagation may be difficult to explain from the correlation functions normally used in scattering theory. As will be seen from the figure, however, a correlation function of the modified Bessel function of higher order seems appropriate for the sporadic-E model.

3.2 Scale of the Sporadic-E Irregularities

It is necessary to note that the scale of turbulence which is represented in the correlation function (3.1) does not describe the scale of irregularities of the sporadic E, but expresses the usual scale of turbulence responsible for the normal scattering phenomena. So we must define the sporadic-E irregularities from the correlation curves which are found in Figure 3. We shall consider a model wherein a sporadic-E cloud which has a large horizontal dimension contains a large number of small irregularities.

Although there exist some confusion concerning the use of the word "scale" to describe the size of irregularities of electron density in the ionosphere, we shall define the "scale" of irregularities to mean the distance in which the correlation is reduced to a specified value, such as 0.5, according to Booker's definition [12].

From the correlation curves shown in Figure 3, the scale of the sporadic-E irregularities may be put in the form

$$L = m\ell \tag{3.4}$$

where ℓ is the scale of normal turbulence and m is an arbitrary multiplication factor. If we take the correlation curve of $n = 5$ as an example for the scattering model of sporadic E, the correlation function decreases to the 0.5 value at a separation of $r = L = 3.5\ \ell$. Therefore the scale of the sporadic-E irregularities has dimensions of m times ($3 \sim 4$ times) the scale of the normal turbulence. Numerical estimation for the scale of the sporadic-E irregularities will be given later.

3.3 Scattering Cross-Section

The scattering cross-section is given, in general,

$$\sigma(q) = \frac{k^4 \sin^2\chi}{16\pi^2 v} R_e \left\{ \frac{16\pi j v \ell}{q} \int_0^\infty r\, C_n(\rho)\, e^{-jq\rho} dr \right\}$$

$$= \frac{k^4 \sin^2\chi\, \ell^3}{\pi q} \int_0^\infty \rho\, C_n(\rho) \sin(q\rho)\, d\rho \qquad (3.5)$$

where

$$q = 2k\ell \sin\frac{\theta}{2} \cong k\ell\,\theta, \quad \rho = \frac{r}{\ell}$$

θ: scattering angle

Inserting (3.1) into (3.5) and remembering C (o) is set equal to $< \left(\frac{\Delta\epsilon}{\epsilon}\right)^2 >/4$, the scattering cross-section is obtained in the form (see for example references [11], [18]).

$$\sigma_n = \frac{< \left(\frac{\Delta\epsilon}{\epsilon}\right)^2 > \Gamma\left(n + \frac{3}{2}\right) \sin^2\chi \left\{1 + \left[\frac{\lambda}{4\pi\ell\sin\frac{\theta}{2}}\right]^2\right\}^{-(n+\frac{3}{2})}}{2^{2n+4}\sqrt{\pi}\,\Gamma(n)\, k^{2n-1}\, \ell^{2n} \left(\sin\frac{\theta}{2}\right)^{2n+3}} \qquad (3.6)$$

where χ is the angle between the direction of scattering and the direction of the incident electric field.

If we put $n = \frac{1}{2}$ in the above formula, $\sigma_{\frac{1}{2}}$ is obtained in the form (when the scale of turbulence is large compared with the wave length)

$$\sigma_{\frac{1}{2}} = \frac{< \left(\frac{\Delta\epsilon}{\epsilon}\right)^2 > \sin^2\chi}{32\pi\, \ell \sin^4\frac{\theta}{2}} \qquad (3.7)$$

which is the Gordon's formula [13], for the tropospheric scattering case. This was also derived by Booker for the ionospheric scattering case [14].

We have the relation of $< \left(\frac{\Delta \epsilon}{\epsilon} \right)^2 > = \left(\frac{\lambda}{\lambda_N} \right)^4 \left(\frac{\Delta N}{N} \right)^2$ in the ionosphere, so, using the scale of sporadic-E irregularities, formula (3.6) is expressed in the form

$$\sigma_n = \frac{\left(\frac{\Delta N}{N} \right)^2 \left(\frac{\lambda}{\lambda_N} \right)^4 \Gamma\left(n + \frac{3}{2}\right) (m)^{2n} \sin^2\chi \left[1 + \left\{\frac{m\lambda}{4\pi L \sin\frac{\theta}{2}}\right\}^2\right]^{-\left(n + \frac{3}{2}\right)}}{2^{2n+4} \sqrt{\pi}\, \Gamma(n) \left(\frac{2\pi}{\lambda} \right)^{2n-1} L^{2n} \left(\sin\frac{\theta}{2}\right)^{2n+3}} \tag{3.8}$$

The received power is obtained from the relation [14]

$$P_r = \frac{P_t\, b\, A}{R^2 \sin\frac{\theta}{2}} \sigma_n (\theta.\lambda) \tag{3.9}$$

where b is the layer thickness and A is the effective area of the receiving antenna.

3.4 Generalized Scattering Cross-Section in the Anisotropic Case

Although the irregularities contributed to the normal scattering phenomenon in the lower ionosphere may be considered isotropic [8], but the irregularities of sporadic E may have an anisotropic structure, such as an ellipsoidal shape, because the scale of the sporadic-E irregularities are larger than the scale of the normal turbulence. The scattering theory in an anisotropic turbulence has been developed by several researchers [15, 16, 17]. One may develop the scattering cross-section (3.8) in the case of anisotropic irregularities. The generalized scattering cross-section is given in the form

$$\sigma_n = \frac{\left(\frac{\Delta N}{N}\right)^2 \left(\frac{\lambda}{\lambda_N}\right)^4 \Gamma\left(n+\frac{3}{2}\right) (m)^{2n} \sin^2\chi \, L_1 L_2 L_3 \left(1+\frac{1}{q^2}\right)^{-\left(n+\frac{3}{2}\right)}}{2^{2n+4} \sqrt{\pi}\, \Gamma(n)\, k^{2n-1} \left(\sin\frac{\theta}{2}\right)^{2n+3} \left[(L_1^2\sin^2\gamma + L_2^2\cos^2\gamma)\sin^2\Phi + L_3^2\cos^2\Phi\right]^{n+\frac{3}{2}}} \tag{3.10}$$

where

$$q \cong \frac{2k\sin\frac{\theta}{2}}{m} \left[(L_1^2\sin^2\gamma + L_2^2\cos^2\gamma)\sin^2\Phi + L_3^2\cos^2\Phi\right]^{\frac{1}{2}}$$

and L_1, L_2, L_3 are different scales of the sporadic-E irregularities in directions parallel, normal and vertical to the mean horizontal drift (assumed to lie in the direction of maximum elongation). γ is the angle between the mean drift and the great circle plane. The angle between the plane containing the scattering blob under consideration and the great circle plane is denoted by an angle Φ. This generalized scattering cross-section has been also obtained by Norton [18] for the tropospheric scattering case.

If the sporadic-E irregularities exist near the great circle plane, we may put $\Phi \approx 0$ and in such a case the formula (3.10) becomes

$$\sigma_n = \frac{\left(\frac{\Delta N}{N}\right)^2 \left(\frac{\lambda}{\lambda_N}\right)^4 \Gamma\left(n+\frac{3}{2}\right) (m)^{2n} L_1 L_2 L_3 \left[1 + \left(\frac{m\lambda}{4\pi L_3 \sin\frac{\theta}{2}}\right)^2\right]^{-\left(n+\frac{3}{2}\right)}}{2^{2n+4}\sqrt{\pi}\,\Gamma(n)\, k^{2n-1} L_3^{2n+3} \left(\sin\frac{\theta}{2}\right)^{2n+3}} \tag{3.11}$$

Moreover we can get the scattering cross-section available for the scattering model of sporadic E, by putting n = 5 into (3.11) and letting m = 3.5 as suggested earlier.

$$\sigma_5 = 1.03 \times 10^2 \; \frac{\left(\frac{\Delta N}{N}\right)^2 \left(\frac{f_N}{f}\right)^4 L_1 L_2 \left[1 + \left(\frac{3.5\,\lambda}{4\pi L_3 \sin\frac{\theta}{2}}\right)^2\right]^{-\frac{13}{2}}}{\left(\frac{2\pi L_3}{\lambda}\right)^9 L_3^3 \left(\sin\frac{\theta}{2}\right)^{13}} \tag{3.12}$$

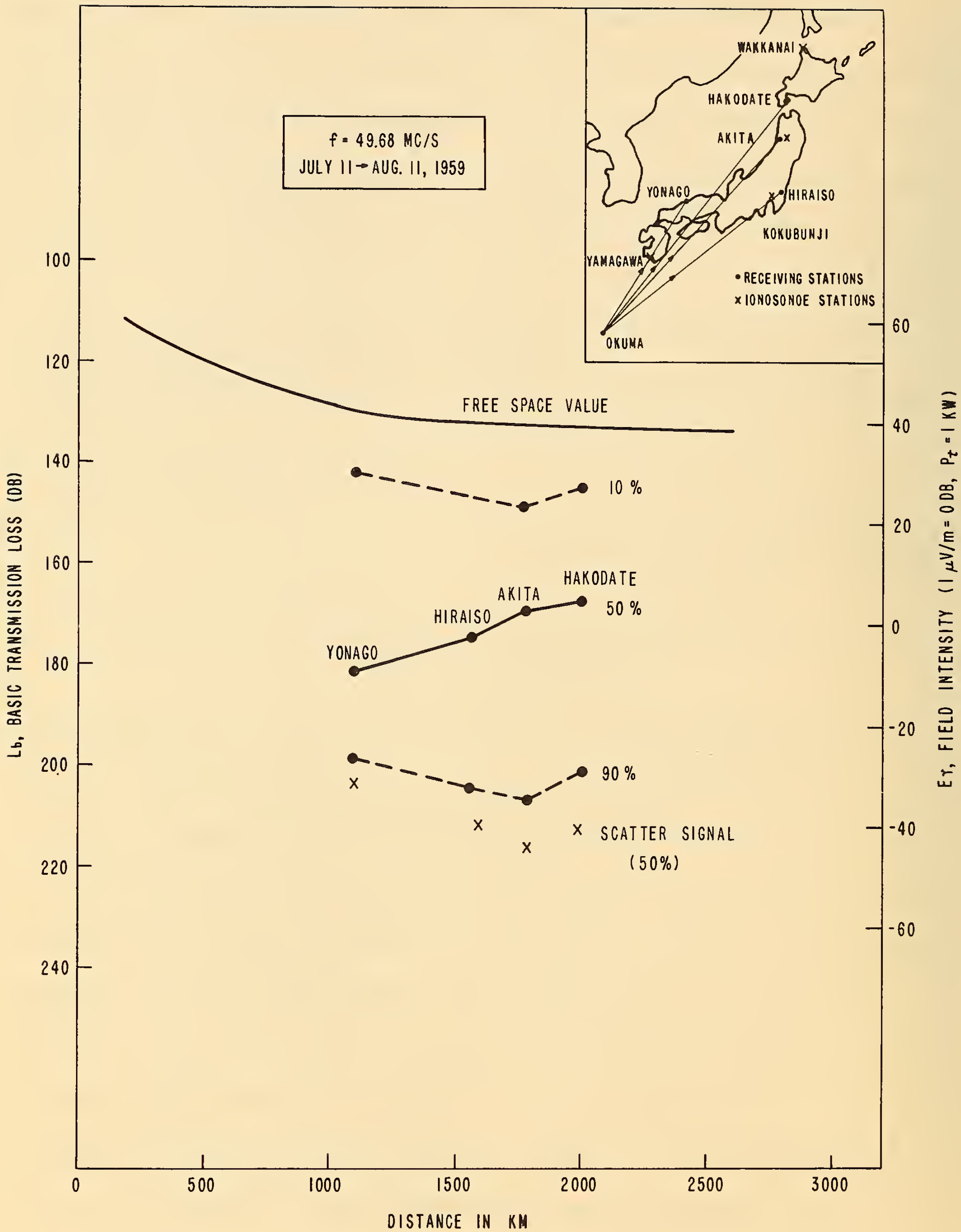
3.5 Relationship between Es Scattering and Normal Scattering in the Ionosphere

Although it has not yet been clarified as to what kind of mechanisms produce these sporadic-E irregularities, it will be possible to consider that these irregularities may have a close connection with turbulence produced by wind shear prevailing in this region. Gallet [19] proposed a mechanism of stratification of sporadic E by considering the mixing process in a turbulent region. Recently Layzer [20] has suggested that it may be necessary to consider very strong wind shears such as about 100 m/sec/km in order to maintain turbulent motion in this region and these wind shears may occur in a narrow layer of the order of 1 km in thickness. If we consider such a jet-like high shear layer, this layer may be expected to act as a partial reflector or scattering source due to fluctuations of electron density produced by turbulent mixing. Also, the recent experimental evidence has shown that wind shear as high as 140 m/sec/km exist in the 100 ~ 110 km region while, simultaneously, ionogram records show sporadic E near the level of maximum shear [21].

According to Gallet [19], the fluctuation of electron density is produced by the transport process

$$\left(\frac{\Delta N}{N} \right)^2 \cong \frac{1}{3} \ell_o^2 \left(\frac{\text{grad } N}{N} \right)^2$$

ℓ_o means the largest scale of turbulence in the layer. As the ambient daytime electron density in the E region is $N \cong 10^5$ cm^{-3} and the layer thickness of sporadic E may be considered to lie in ranges from a few kilometers down to a few hundred meters, the value of $\left(\frac{\Delta N}{N} \right)^2$ in the daytime is the order of unity to 10^{-1}. Since the sharp gradient of the electron density will decrease with disappearance of the sporadic-E clouds, the value of $\left(\frac{\Delta N}{N} \right)^2$ will also decrease to the order of 10^{-3} or 10^{-4} corresponding to normal scattering. As the sporadic-E clouds disappear or move to other places, the high concentration of electron density in this region decays to the normal state and therefore the



EXPERIMENTAL RESULTS OF THE DISTANCE DEPENDENCE OF SPORADIC E PROPAGATION

FIGURE 4

number of n (n = 5 ~ 6) in the correlation function will tend to small numbers such as n = 1. In this case, as the irregularities are considered as isotropic, we can obtain the scattering cross-section from (3.6) by putting n = 1.

$$\sigma_1 = 1.17 \times 10^{-2} \frac{\left(\frac{\Delta N}{N} \right)^2 \left(\frac{f_N}{f} \right)^4}{\left(\frac{2\pi \ell}{\lambda} \right) \ell \left(\sin \frac{\theta}{2} \right)^5} \qquad (3.13)$$

This formula represents the normal scattering case. Although there is still some ambiguity concerning the scale of turbulence in the ionosphere, we have taken a value of ℓ = 15 m as the average scale of turbulence corresponding to VHF forward scattering (f = 50 Mc/s), [22]. Booker [8] has also recently suggested the presence of irregularities of the electron density with scales in ranges from 20 to 60 meters in the lower ionosphere. Several calculated curves of the basic transmission loss for the Es scattering and the normal scattering are shown in Figure 6 by using the formulas (3.12) and (3.13).

4. DISTANCE DEPENDENCE OF SPORADIC-E PROPAGATION

An experimental determination of distance dependence is shown in Figure 4. This experiment was carried out, using the frequency of 49.68 Mc/s, in the summer of 1959 by the Radio Research Laboratories and K.D.D. (Japan's Overseas Radio and Cable System) in Japan in order to examine the distance dependence of the sporadic-E propagation with the cooperation of the NBS Central Radio Propagation Laboratory. The more detailed analysis of this experiment will be published in other papers [23, 24].

The transmitting point was Okuma, Okinawa, and the receiving points were Yonago, Akita and Hakodate in Japan. In addition to the summer (1959) data, we have added the observations at Hiraiso during the summer period in 1958. Distances between the transmitting point and each receiving point are as follows:

TABLE I

Rec. Station	Long.	Lat.	Distance	Rec. Antenna	Antenna Gain	Antenna Height Above Ground
Yonago	133°22'E	35°27'N	1100km	2-stack, 5-element Yagi	14.3db	18.4m
Hiraiso	140°37'E	36°22'N	1590km	Rhombic	20.0db	15.0m
Akita	140°32'E	39°44'N	1820km	2-stack 5-element Yagi	14.1db	30.0m
Hakodate	140°43'E	45°24'N	2020km	5-element Yagi	9.1db	6.0m*

*Approximately 280m above the sea.

The 10%, 50% and 90% values of the statistical cumulative distribution of the Es signals are plotted in Figure 4 [23]. Statistical results of foEs measured at the ionosonde stations in Japan during the same period are as follows:

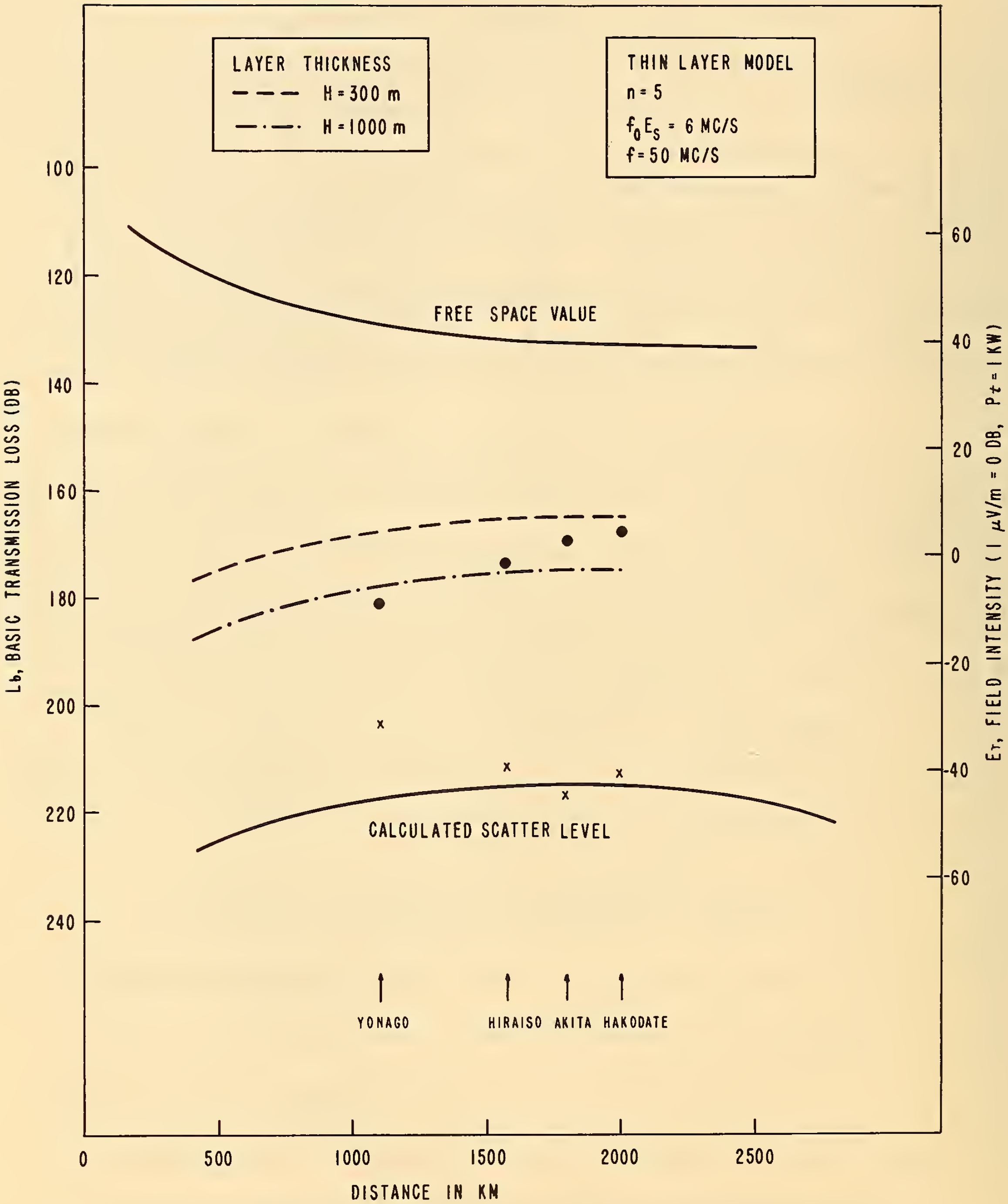
TABLE II

Ionosonde Station	foEs (10%)	foEs (50%)	foEs (90%)
Yamagawa	10.5 Mc/s	6.2 Mc/s	3.4 Mc/s
Kokubunji	10.8 Mc/s	6.4 Mc/s	4.0 Mc/s
Akita	12.8 Mc/s	7.2 Mc/s	4.3 Mc/s

We shall next examine this distance dependence from the theoretical point of view. The most appropriate theoretical curves for the experimental result are shown in Figure 5 and Figure 6 derived from the thin layer theory and the scattering theory described in sections 2 and 3 respectively. As measurements indicate that the layer thickness of sporadic E ranges from a few kilometers down to 50 meters, curves pertinent to these ranges are shown in Figure 5 and Figure 6, and seem to fit the observations reasonably well.

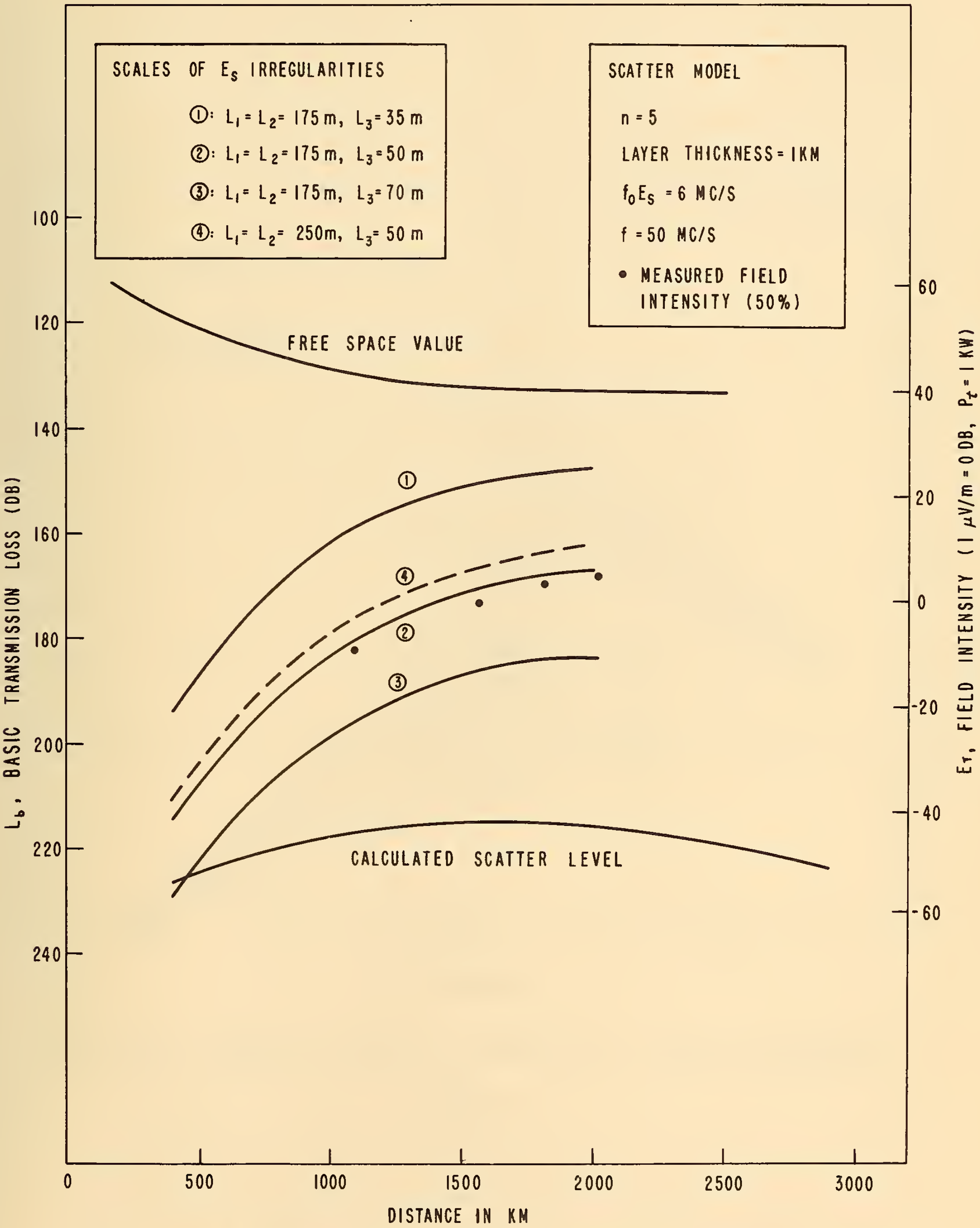
5. FREQUENCY DEPENDENCE OF SPORADIC-E PROPAGATION

It is well-known that the scattered power in normal ionospheric scatter propagation is inversely proportional to about the fifth or sixth power of frequency [8, 25, 26, 27]. Experiments show, however, that the frequency exponent increases in the case of sporadic E. It is reported by Davis, Smith and Ellyett [28] that the median value of the frequency exponent in sporadic-E propagation is 12 during summer time



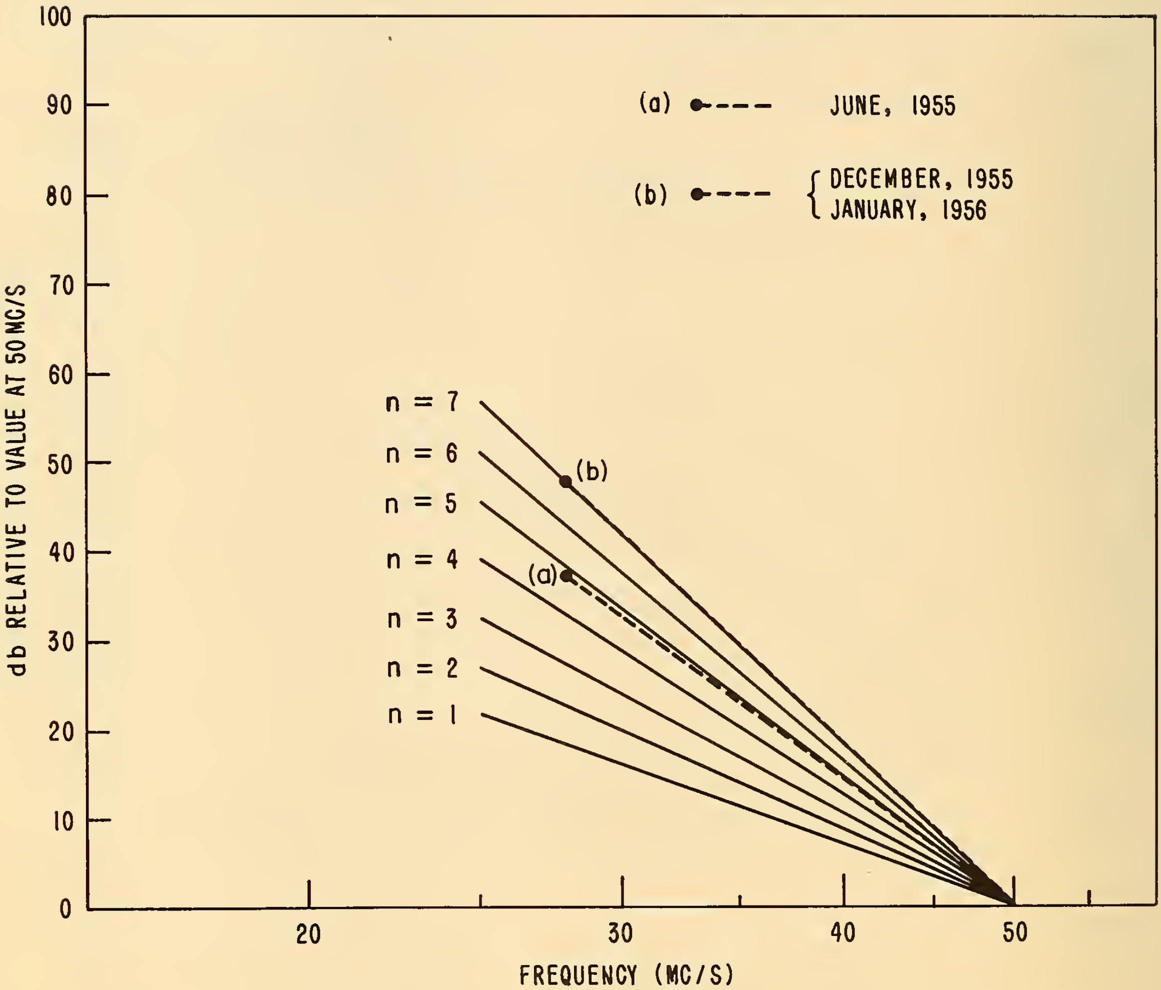
DISTANCE DEPENDENCE OF SPORADIC E PROPAGATION (THIN LAYER MODEL)

FIGURE 5



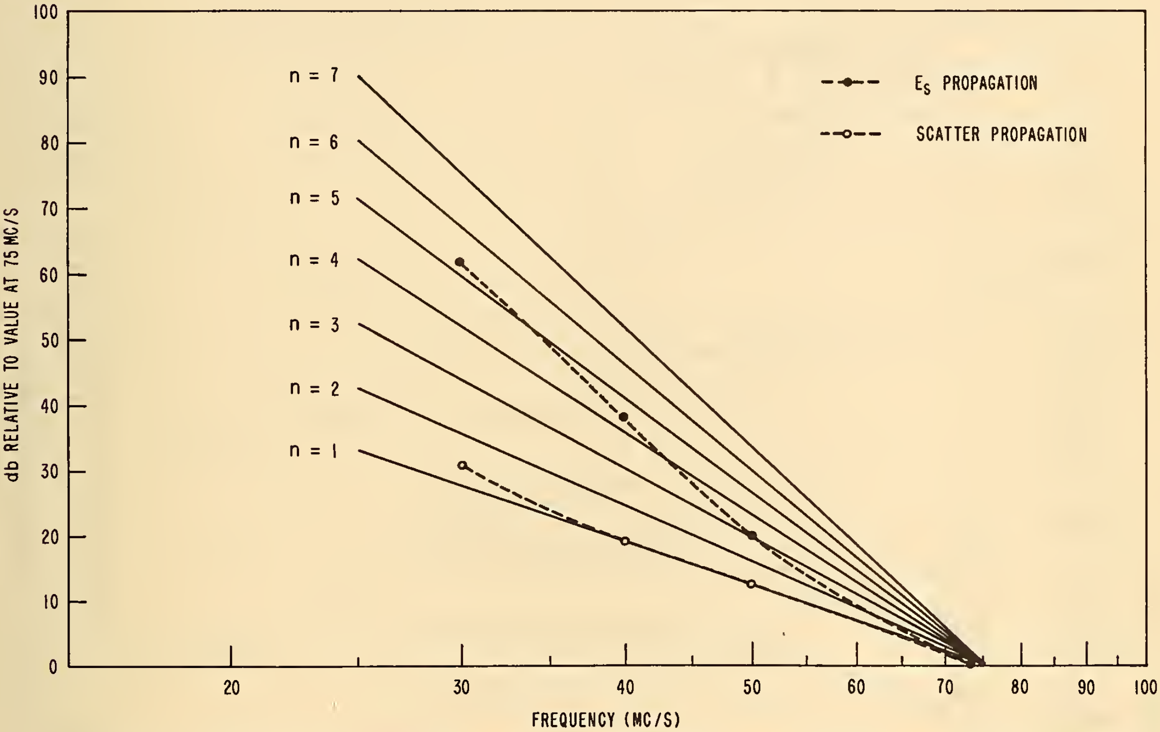
DISTANCE DEPENDENCE OF SPORADIC E PROPAGATION (SCATTER MODEL)

FIGURE 6



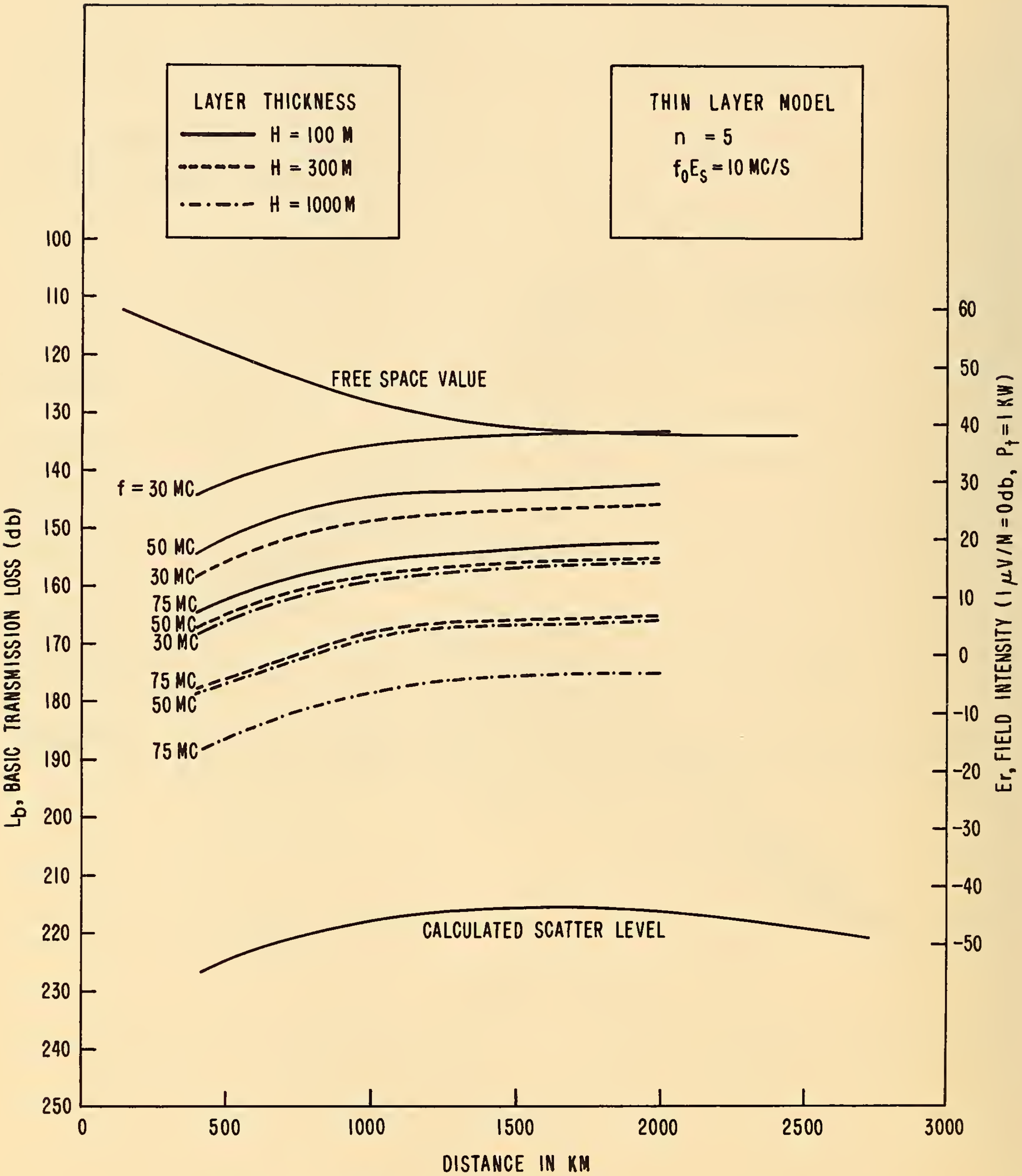
EXPERIMENTAL RESULTS OF THE FREQUENCY DEPENDENCE OF SPORADIC E PROPAGATION

FIGURE 7



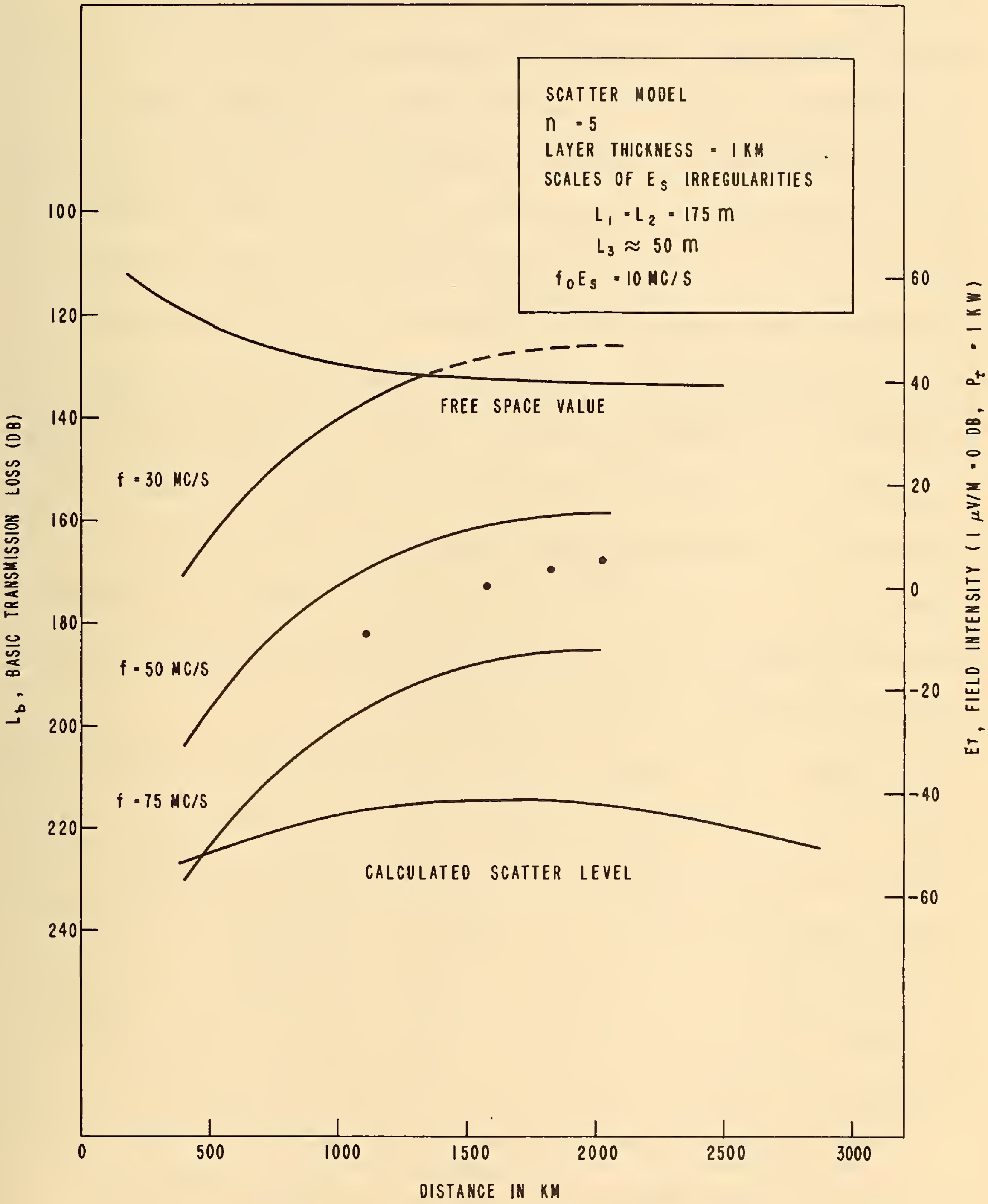
EXPERIMENTAL RESULTS OF THE FREQUENCY DEPENDENCES OF SPORADIC E PROPAGATION AND NORMAL SCATTER PROPAGATION

FIGURE 8



FREQUENCY AND DISTANCE DEPENDENCES OF SPORADIC E PROPAGATION BY MEANS OF THE THIN LAYER MODEL

FIGURE 9



DISTANCE AND FREQUENCY DEPENDENCES OF SPORADIC E
PROPAGATION BY MEANS OF THE SCATTER MODEL

FIGURE 10

in 1955 and 17 during winter time. This experiment was carried out between Cedar Rapids and Sterling and signal intensities were recorded simultaneously at 27.775 Mc/s and 49.8 Mc/s. These experimental results are plotted with dotted curves in Figure 7. In a recent experiment which has been carried out by the Central Radio Propagation Laboratory in order to examine the frequency dependence of scatter propagation using five different frequencies, that is, approximately 30, 40, 50, 74 and 108 Mc/s. Blair [29] reported on the frequency dependence of sporadic-E propagation besides that of the normal scatter propagation. This experimental evidence is shown with a dashed curve in Figure 8. An additional dashed curve representing normal scatter propagation is also plotted in the same figure. The curve shows that the signal enhancement due to sporadic E diminishes rapidly with frequency when compared with normal scattering.

We shall try to check this frequency dependence from the theoretical point of view. In Figure 9 are plotted the theoretical curves derived from the thin layer theory with frequencies (30 Mc/s, 50 Mc/s, 75 Mc/s) as a parameter and several layer thicknesses (100 m, 300 m, 1000 m) with fixed value of the critical frequency of Es (foEs = 10 Mc/s) and an exponent of the electron distribution (n = 5). The corresponding calculation is given in Figure 10 by using the scattering model.

Concerning the frequency dependence, the thin layer model which is described in this paper does not seem to be a good fit. The scattering model which has an auto-correlation function of the modified Bessel function of a high order seems to be preferable as will be seen from the formula (3.12) or Figure 7, 8 and 10. From the scattering theory for sporadic E [i.e. equations (3.8) and (3.9)], we get

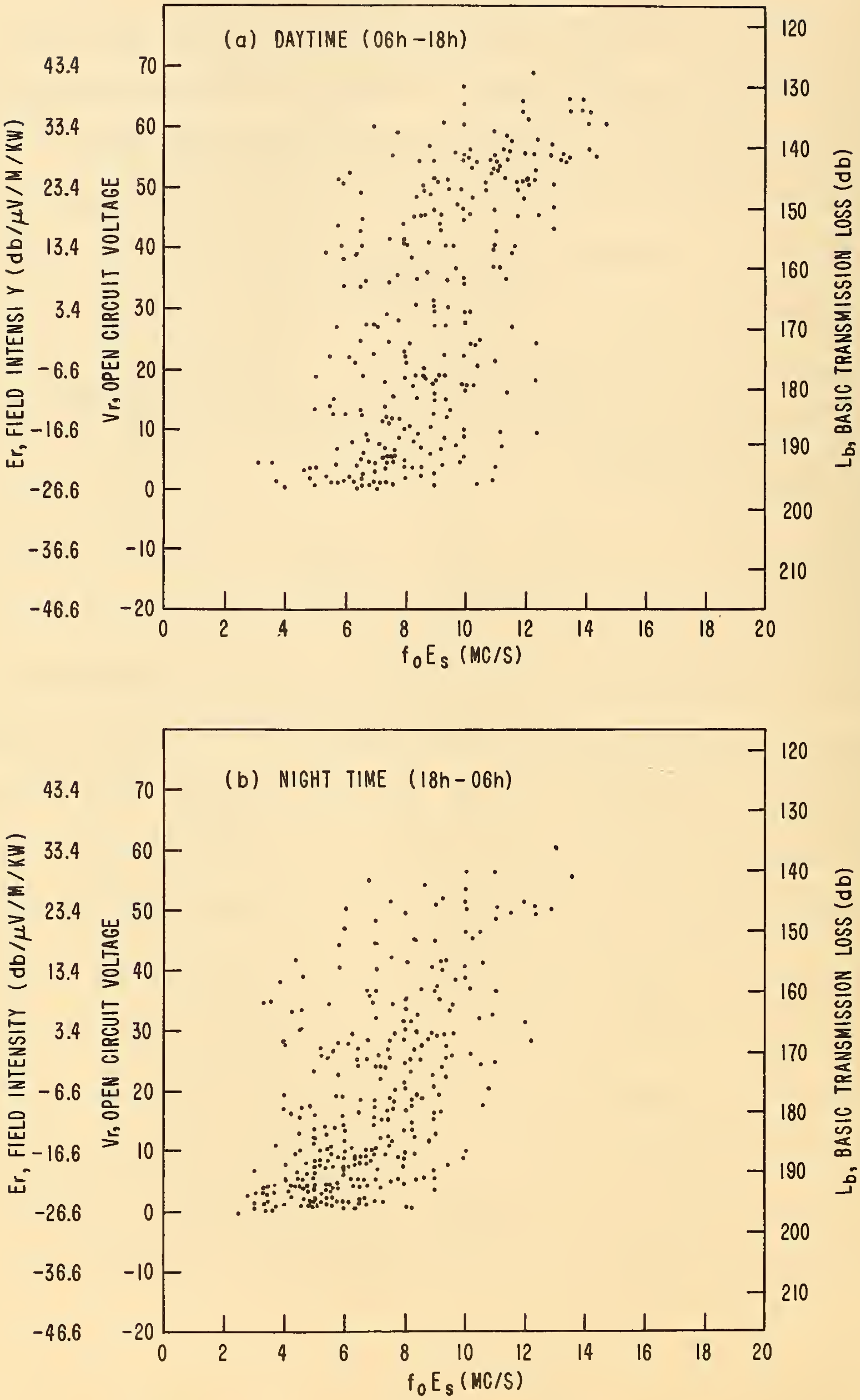
$$P_r \propto \frac{1}{f^{(2n+5)}}$$

If we take a high order value of n in the correlation function, that is, $n = 4 \sim n = 7$, we obtain

$$P_r \propto \frac{1}{f^{13}} \sim \frac{1}{f^{19}} \,.$$

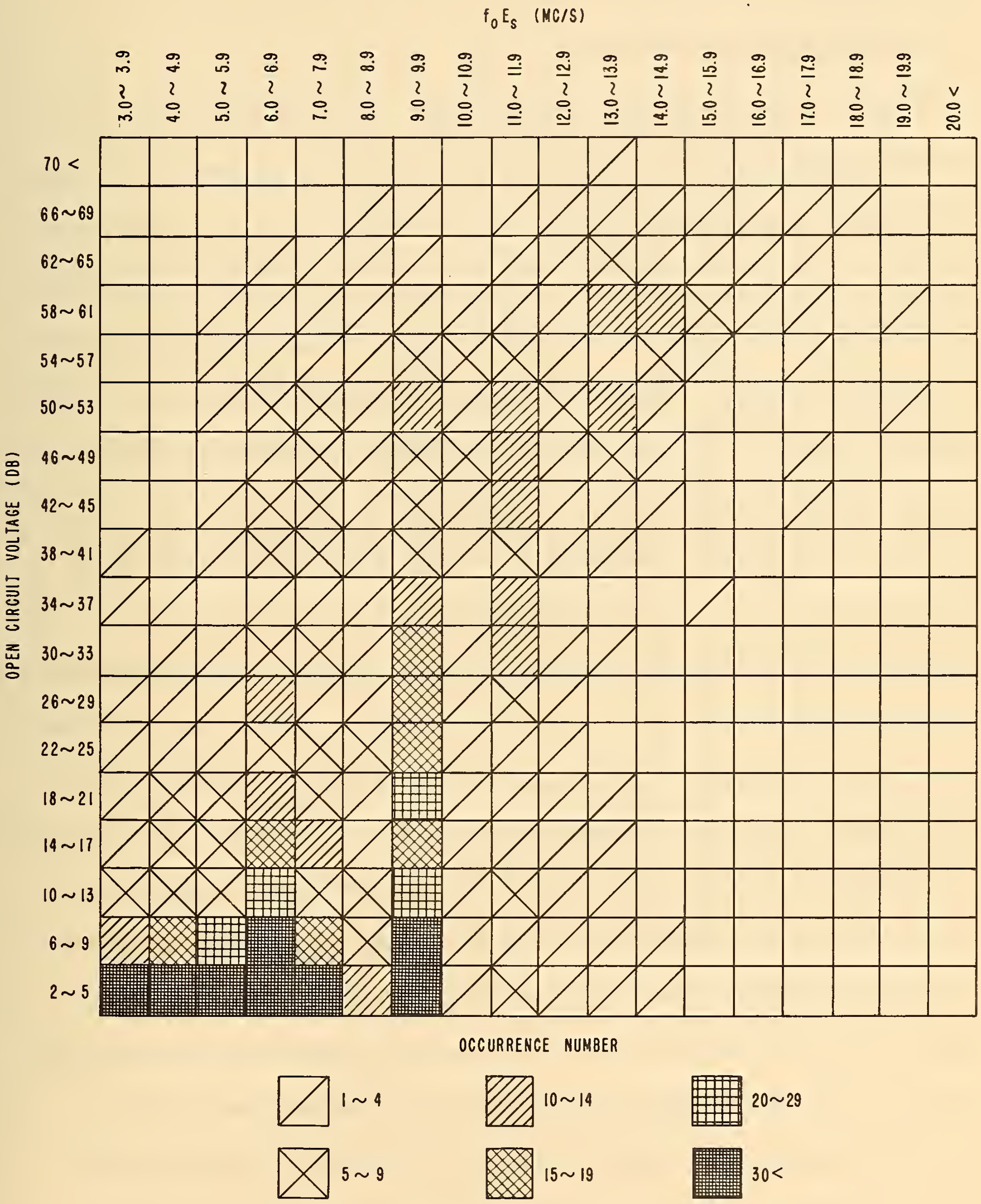
Therefore we can explain the frequency dependence of sporadic E by considering an auto-correlation function which has a very slow decrease with $\rho \; (= \frac{r}{l})$. In Figures 7 and 8, are drawn families of curves with various values of n in the modified Bessel function of n order. From these figures, the auto-correlations of the modified Bessel function with $n = 4 \sim n = 7$ seem to be reasonable for the scattering model of sporadic E. It is also of interest to note that the curve of $n = 1$ fits the normal ionospheric scatter propagation measurements. Although Norton [30] has stated in his recent paper that this $\rho K_1 (\rho)$ correlation function has definite advantages over the exponential model for describing the turbulence of refractivity in the troposphere and

E_s INTENSITY – YONAGO f_0E_s–YAMAGAWA JAN–DEC, 1958



CORRELATION BETWEEN f_0E_S AND FIELD STRENGTH OF SPORADIC E

FIGURE 11



CORRELATION BETWEEN f_0E_s AND THE FIELD STRENGTH OF SPORADIC E

FIGURE 12

stratosphere, also it seems to be quite good for the normal turbulence situation in the ionosphere.

6. CORRELATION BETWEEN foEs AND FIELD STRENGTH

In this section, we shall briefly consider a correlation between foEs and the received Es field strength. Figure 11 shows some correlograms between these parameters. The normalized Es field strengths are the hourly median values observed at Yonago during the period January to December in 1958. As the ionosonde station at Yamagawa is situated almost exactly at the midpoint between Okuma (transmitting point) and Yonago (receiving point), it is appropriate to use values of foEs observed at Yamagawa to investigate a relationship between the field strength and foEs. Since the field strength used in this analysis is the hourly median value, the corresponding value of foEs is interpolated from the measured values at each hour. A more rigorous analysis has been made by Yamaoka [31] by using the instantaneous value of field strength to the value of foEs observed every 15 minutes. This is shown in Figure 12 and data during periods of July and August in 1959 are used. From these figures, it is interesting to note that the field strength increases abruptly in the interval from foEs = 8 Mc/s to foEs = 10 Mc/s. Although the cause of this increase is not yet clear, it may be related to the fact that this interval is near to the plasma frequency for normal reflection in this transmission circuit (i.e. 49.68 Mc/sec $\theta = \sim 12$ Mc). This seems to suggest that there are two modes of sporadic-E propagation, that is, the high field strength corresponding to high values of foEs seems to be due to the reflection mechanism and the relatively low field strength is due to the scattering mechanism. Theoretical curves calculated from the thin layer and scattering models are compared with some measured field intensities for various values of foEs. From Figure 13 it can be seen that a fairly high field intensity of sporadic-E enhancement corresponding to a comparatively high plasma frequency may be explained by reflection from a thin ionized layer whereas a relatively low sporadic-E field intensity may be produced by the scattering mechanism.

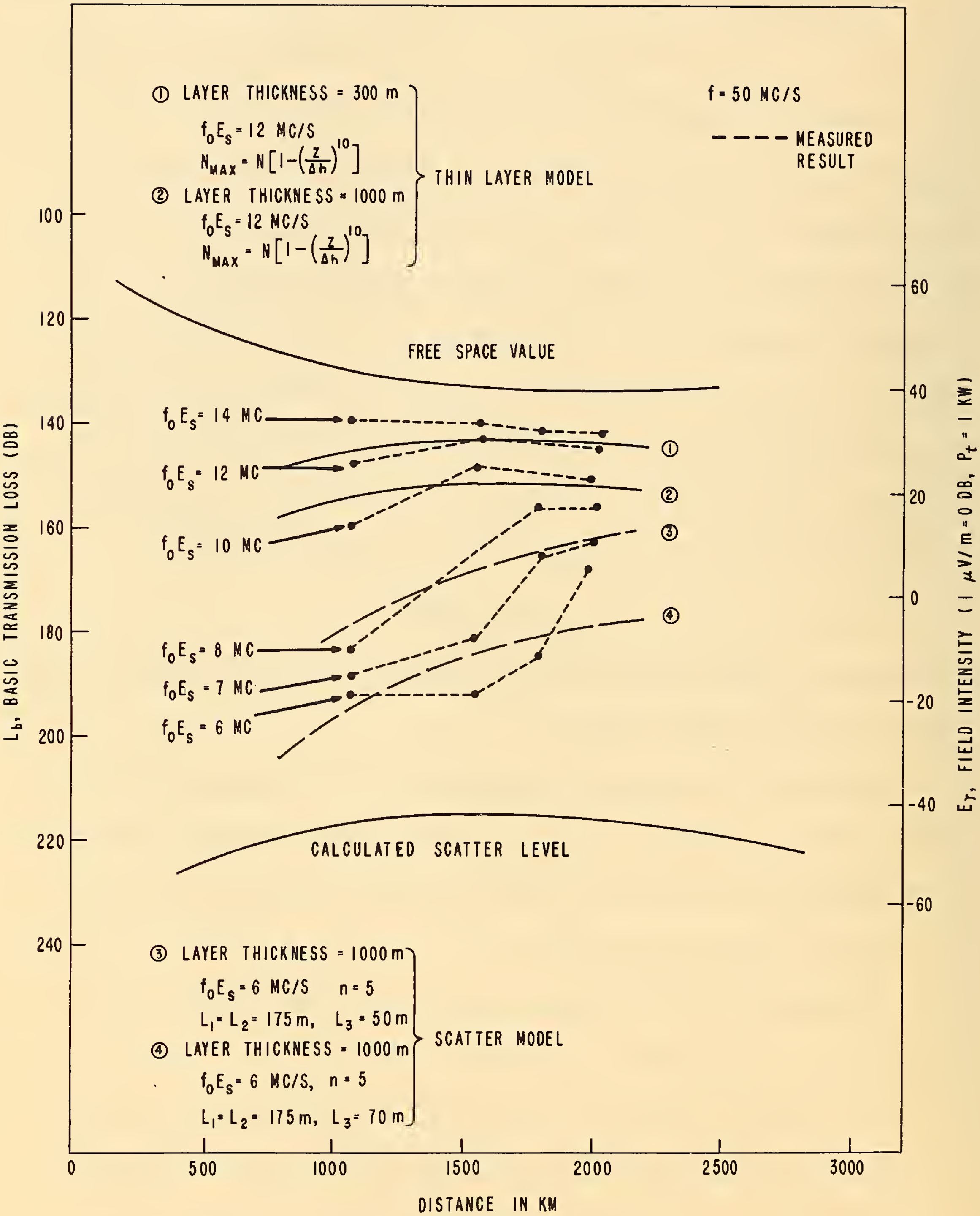
7. CONCLUSIONS

From the above theoretical investigation for the sporadic-E model, some conclusions may be drawn.

i) High field strength of sporadic-E enhancements may be explained by reflection from a thin layer with electron density approaching the necessary plasma density (i.e. 2×10^6 e/cc for the Okuma-to-Yonago circuit).

ii) Relatively low field strength of sporadic E may be explained by the scattering model.

iii) Scattering model for sporadic E is expressed by using an auto-correlation function of the modified Bessel function of higher order and each blob of ionization may be considered to



THEORETICAL EXPLANATION FOR A RELATIONSHIP BETWEEN f_oE_s AND E_s FIELD INTENSITY

FIGURE 13

have a horizontal scale of the order of 200 m and a vertical scale of 50 m.

iv) Concerning the frequency dependence of sporadic E, the thin layer theory developed in this paper does not seem to be appropriate, however, this frequency dependence can be explained by the scattering model.

v) If we assume a layer thickness of sporadic E of 300 m ~ 1000 m, we can explain the distance dependence of the median field strength from both the thin layer and the scattering theories.

8. ACKNOWLEDGMENT

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