## A Gamma Ray Moments Computer Code, GAMM ${ }^{2} \mathrm{M}-\mathrm{I}$

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## Hipmat Duct of Standards

# 1100 <br> A Gamma Ray Moments Computer Code, GAMMØM-I 

Charles M. Eisenhauer, George L. Simmons, and<br>Lewis V. Spencer

Center for Radiation Physics
Institute for Basic Standards
National Bureau of Standards
Washington, D.C. 20234


#### Abstract

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A Gamma Ray Moments Computer Code, GAMM $\phi$ M-I*
Charles M. Eisenhauer, George L. Simmons, ${ }^{\dagger}$ and Lewis V. Spencer

In this paper we describe a computer code for generating spatial-angular moments of gamma ray energy fluence in an infinite medium. The equation for moments of the energy fluence is given and the techniques used for the solution are discussed. The structure of the code and of the main subroutines is also given. Details of the input and output data are presented and the printout from a sample problem is included.

Key words: Computer code; gamma rays; gamma ray transport; gauss quadrature; moments; shielding.

## I. Introduction

In this note we describe a computer program to calculate spatial and angular moments of the gamma ray fluence in an infinite medium. These moments can then be used to reconstruct spatial and angular distributions by techniques described elsewhere $[1,2]^{3}$.

Application of the method of moments to problems in gamma ray attenuation was first discussed by Spencer and Fano [3]. Later Goldstein and Wilkins of Nuclear Development Associates collaborated with Spencer to produce a systematic tabulation of buildup factors for gamma radiation [4]. That reference also contains an excellent discussion of the solution of the transport equation by the moments method. The data published in that report have stood for almost twenty years as the main source of information on penetration of gamma rays from point isotropic sources.

Although the data of reference [4] are sufficiently accurate for many practical applications, they have certain limitations:

1. Tabulations were restricted to energies between 0.255 and 10 MeV .
2. Tabulations were limited to depths of less than 20 mean free paths of the source radiation.

[^1]3. Results did not include the effects of annihilation or fluorescent radiation.
4. Data are available for only about six elements and water.
5. Calculations were limited to sources whose angular distribution is azimuthally symmetric with respect to a direction perpendicular to the plane of a slab.
6. The data were generated on an early computer, the SEAC, which quickly became obsolete.
An effort is in progress at the National Bureau of Standards to produce gamma ray data which are not subject to these restrictions.

The first major production of moments data at NBS was accomplished by a computer program written by Spencer and Lamkin [5] for the IBM 704 machine. Several reports $[6,7,8]$ give penetration data generated by this program in water and concrete media. The main application of these results, however, appeared in an NBS Monograph [9] on the calculation of shielding from fallout gamma radiation. The data in this Monograph form the technical basis for the Standard Method [ $10,11,12,13$ ] used by the Defense Civil Preparedness Agency (formerly Office of Civil Defense) to evaluate the protective capability of structures. This computer program extended the range of source energies down to about . 020 MeV in energy but had the disadvantage that it was written in machine language.

Accordingly, a gamma ray moments code written in FORTRAN IV language was developed with the intent of eliminating all the restrictions listed earlier. This code, called GAMMめM-I, is the subject of this report.

## II. Theory

A. Moments Equations

In order to make this report self-contained we sketch the de-
velopment of the moments equations for gamma rays. The derivation of the moments equation for gamma rays given here is very similar to that described in the documentation of MOMENT-I, a neutron moments code [14]. The main difference between the two codes is that it is convenient in the neutron moments code to solve for the collision density moments

$$
{ }_{\mathrm{n}_{\ell}}(E)=\sigma(E) \phi_{\mathrm{n}_{\ell}}(E), \text { (neutrons) }
$$

where $\Sigma(E)$ is the total cross section and $\phi_{n \ell}(E)$ are moments of the neutron fluence; whereas in the gamma ray code the quantity solved for is the energy fluence,

$$
I_{n \ell}(E)=E 力_{n \ell}(E) \quad \text { (gammas) }
$$

where $E$ is the gamma ray energy in units of the rest energy of the electron (mc ${ }^{2}$ units) and $\phi_{n^{2}}(E)$ are moments of the gamma ray fluence.

The gamma ray transport equation in one dimensional plane geometry, including fluorescent and annihilation radiation, has the form

$$
\cos \theta \frac{\partial \Phi}{\partial z}(E, z, \vec{\omega})+\mu(E) \Phi(E, z, \vec{w})=\int_{E}^{\infty} d E^{\prime} \int_{4 \pi} d_{\Omega}^{\prime} K\left(E^{\prime} \rightarrow E, \vec{w}^{\prime} \rightarrow \vec{\omega}\right) \Phi\left(E^{\prime}, z, \vec{w}\right)
$$

$$
\begin{equation*}
+\sum_{i} \delta\left(E-E_{i}\right) Y_{i} \int_{E_{i}^{\prime}}^{\infty} \mu_{i}\left(E^{\prime}\right) \Phi\left(E^{\prime}, z, \vec{w}\right) d E^{\prime}+S(E, z, \vec{w}) \tag{1}
\end{equation*}
$$

where

| $\Phi(E, z, \vec{\omega})$ | is the gamma ray fluence (photons $/\left(\mathrm{cm}^{2}\right.$-ster-mc ${ }^{2}$ unit)), |
| :---: | :---: |
| $\mu(E)$ | is the total cross section (Thomson units per electron), |
| $\vec{\omega}$ | is a unit vector in the photon direction, $\vec{k}$ is a unit vector along the positive z axis, and $\vec{\omega} \cdot \overrightarrow{\mathrm{k}}=\cos \theta$, |
| $S(E, z, \vec{w})$ | is the source strength (photons/ $\left(\mathrm{cm}^{-3}\right.$-ster$\mathrm{mc}^{2}$ unit)), |
| $K\left(E^{\prime} \rightarrow E, \vec{\omega}^{\prime} \rightarrow \vec{\omega}\right)$ | is the Klein Nishina scattering probability per unit path length (TU/electron/mc ${ }^{2}$ unit) |
| $\mathrm{E}_{\mathrm{i}}$ | is the energy of either a fluorescent photon or an annihilation photon |
| $\mu_{i}(\mathrm{E})$ | is the cross section for either photoelectric absorption or pair production (TU/electron) |
| $\mathrm{Y}_{\mathrm{i}}$ | is the yield of either fluorescent photons or annihilation photons, and |
| $E_{i}^{\prime}$ | is the threshold for photoelectric absorption or pair production (me ${ }^{2}$ units) |

The source term for fluorescent and annihilation radiation may include any process in which photons above a threshold energy $\mathrm{E}_{\mathrm{i}}$ are absorbed according to a cross section $\mu_{i}$, with the subsequent emission of photons at energy $E_{i}$. The calculations reported in reference [4] treated these processes as pure absorption (i.e. $Y_{i}=0$ ).

Following the usual practice we find it more convenient to study the energy fluence rather than the number fluence. We therefore let

$$
\begin{equation*}
I(E, z, \vec{w})=E \phi(E, z, w) \tag{2}
\end{equation*}
$$

The transport equation (1) then becomes

$$
\begin{equation*}
\cos \theta \frac{\partial I}{\partial z}(E, z, \vec{\omega})+\mu(E) I(E, z, \omega)=\int_{E} d E^{\prime} \int_{4 \pi} d \Omega^{\prime} K\left(E^{\prime} \rightarrow E, \vec{\omega}^{\prime} \rightarrow \vec{\omega}\right) I\left(E^{\prime}, z, \vec{\omega}\right) \tag{3}
\end{equation*}
$$

$$
+\sum_{i} E_{i} \delta\left(E-E_{i}\right) Y_{i} \int \frac{\mu_{i}\left(E^{\prime}\right) I\left(E^{\prime}, z, \vec{w}\right)}{E^{\prime}} d E^{\prime}+E S(E, z, \vec{w})
$$

Contrary to earlier procedure, however, we use as an independent variable the energy rather than the wavelength $\lambda$ of the photon.

$$
\begin{align*}
& \text { If one now expands the energy } f 1 \text { ux in spherical harmonics, } \\
& I(E, z, \vec{\omega})=\sum_{\ell=0}^{\infty} \sum_{\mathrm{m}=-\ell} \sqrt{\frac{2 \ell+1}{4 \pi}} I_{\ell}^{\mathrm{m}}(E, z) Y_{\ell}^{\mathrm{m}}(\theta, \varphi) \tag{4}
\end{align*}
$$

and defines spatial moments of the angular expansion coefficients as

$$
\begin{equation*}
I_{n \ell}^{m}(E)=\frac{\mu_{0}^{n+1}}{n!} \int_{-\infty}^{\infty} z^{n} I_{\ell}^{m}(E, z) d z \tag{5}
\end{equation*}
$$

the transport equation becomes a function of energy alone. However, there is an infinite set of equations for $I_{n \ell}^{m}(E)$. Here, $\mu_{0}$ is the total cross section at the source energy. The source term can be obtained by evaluating

$$
\begin{equation*}
S_{n \ell}^{m}(E)=\frac{\mu_{0}^{n+1}}{n!} \int_{-\infty}^{\infty} z^{n} d z \int_{4 \pi} \sqrt{\frac{4 \pi}{2 \ell+1}} Y_{\ell}^{m^{*}}(\theta, \varphi) \operatorname{ES}(E, z, \vec{w}) d \vec{\Omega} \tag{6}
\end{equation*}
$$

where $Y_{l}^{\mathrm{m*}}$ is the complex conjugate of $Y_{l}^{\mathrm{m}}$. The moments equation then has
the form

$$
\begin{align*}
\mu(E) I_{n \ell}^{m}(E) & =\int_{E}^{\infty} K\left(E^{\prime} \rightarrow E\right) P_{\ell}\left(1-\frac{1}{E}+\frac{1}{E^{\prime}}\right) I_{n \ell}^{m}\left(E^{\prime}\right) d E^{\prime}+ \\
& +\mu_{0} \frac{\sqrt{(\ell+1)^{2}-m^{2}}}{\left(2^{\ell}+1\right)} I_{n-1,1+1}^{m}(E)+\mu_{0} \frac{\sqrt{\ell^{2}-m^{2}}}{2 \ell+1} I_{n-1, \ell-1}^{m}+ \\
& +\mu_{0} \delta_{\ell} \sum_{i} \delta\left(E-E_{i}\right) Y_{i} \int_{E_{i}^{\prime}}^{\infty}, \mu_{i}\left(E^{\prime}\right) I_{n o}\left(E^{\prime}\right) \frac{d E^{\prime}}{E^{\prime}}+S_{n \ell}^{m}(E) \tag{7}
\end{align*}
$$

where

$$
\mathrm{P}_{\ell}(\cos \theta) \text { is the Legendre polynomial }
$$

and

$$
K\left(E^{\prime} \rightarrow E\right)=0.375 E\left[E^{\prime}+\frac{E}{E^{\prime}}-1+\cos ^{2} \theta\right] / E^{\prime 3} .
$$

For a plane source located at $z=0$, i.e. $S(E, z, \vec{\omega})=S(E, \cos \theta) \delta(z) / 4 \pi$, equation (7) reduces to

$$
\begin{align*}
\mu(E) I_{n \ell}(E) & =\int_{E}^{\infty} K\left(E^{\prime} \rightarrow E\right) P_{\ell}\left(1-\frac{1}{E}+\frac{1}{E^{\prime}}\right) I_{n \ell}\left(E^{\prime}\right) d E^{\prime}+\delta_{\ell} \sum_{k} \delta\left(E-E_{k}\right) E_{k} S_{n}^{k}  \tag{8}\\
& +\frac{\left(1-\frac{\delta}{n o}\right) \mu_{0}}{(2 \ell+1)}\left\lfloor(\ell+1) I_{n-1, \ell+1}(E)+\ell I_{n-1, \ell-1}(E)\right]+\mu_{0} E \delta_{n o}^{\delta} S_{\ell} \tag{E}
\end{align*}
$$

where

$$
\begin{equation*}
S_{\ell}(E)=\int_{-1}^{1} d(\cos \theta) P_{\ell}(\cos \theta) S(E, \cos \theta) \tag{9}
\end{equation*}
$$

and

$$
S_{n}^{k}=Y_{k} \int_{E_{k}^{\prime}} \mu_{k}^{\prime}\left(E^{\prime}\right) I_{n O}\left(E^{\prime}\right) d E^{\prime} / E^{\prime}
$$

The computer code GAMMØM-I is designed to treat monoenergetic sources, that is, sources for which

$$
\begin{equation*}
S(E)=\delta\left(E-E_{0}\right) \tag{10}
\end{equation*}
$$

To eliminate the difficulties of the $\delta$-function we express the moments as a sum of a $\delta$-function and a continuous $f$ unction $I_{n}^{s}(E)$ :

$$
\begin{align*}
I_{n \ell}(E) & =I_{n \ell}^{s}(E)+S_{o} E_{o} \delta\left(E-E_{o}\right) C_{n \ell} \\
& +\sum_{k} \delta\left(E-E_{k}\right) E_{k}\left(\frac{\mu_{0}}{\mu_{k}}\right) \quad C_{n \ell}^{k}
\end{align*}
$$

where $C_{n \ell}$ are a set of coefficients that can be generated by recursion:

$$
\begin{equation*}
c_{n \ell}=\frac{1}{\left(2^{\ell}+1\right)}\left[(\ell+1) c_{n-1, \ell+1}+c_{n-1, \ell-1}\right] \tag{12}
\end{equation*}
$$

with

$$
C_{o l}=S_{l}(E)
$$

and the $C_{n \ell}^{k}$ are a somewhat more complicated set given by

$$
\begin{equation*}
c_{n \ell}^{k}=\frac{1}{(2 \ell+1)}\left[(\ell+1) c_{n-1, \ell+1}^{k}+\ell C_{n-1, \ell-1}^{k}\right]+\delta_{n 0} S_{n}^{k} \tag{13}
\end{equation*}
$$

The equation that is actually solved in the computer code is therefore:

$$
\begin{align*}
\mu(E) I_{n \ell}^{s}(E)= & \int_{E}^{\infty} K\left(E^{\prime} \rightarrow E\right) P_{\ell}\left(1-\frac{1}{E}+\frac{1}{E^{\prime}}\right) I_{n \ell}^{s}\left(E^{\prime}\right) d E^{\prime}+E_{0} K\left(E_{0} \rightarrow E\right) P_{\ell}\left(1-\frac{1}{E}+\frac{1}{E_{n}}\right) C_{n \ell}  \tag{14}\\
& +\delta_{\ell} \sum_{(14)} \delta\left(E-E_{k}\right) E_{k} S_{n}^{k}+\frac{\left(1-\delta_{n 0}\right)}{2^{\ell+1}} \mu_{0}\left[(\ell+1) I_{n-1, \ell+1}(E)+\ell I_{n-1, \ell-1}(E)\right]
\end{align*}
$$

where $s_{n}^{k}$ is given by

$$
S_{n}^{k}=Y_{k}\left\{S_{o} H_{k}^{\prime}\left(E_{o}\right) C_{n \ell}+\int_{E_{k}^{\prime}}^{\infty} I_{n o}^{s}\left(E^{\prime}\right) \mu_{k}^{\prime}\left(E^{\prime}\right) \frac{d E^{\prime}}{E^{\prime}}\right\}
$$

The contributions from the $\delta$-functions, which are associated with the primary and secondary sources, and are given in equation (11), is also calculated by the code.
B. Gaussian Technique Applied to Scattering Integral.

The method used by earlier workers $[2,3]$ to obtain solutions to Equation (14) was to assume a grid of energies (usually with constant
intervals of the wavelength in Compton units and proceed with the numerical solution from high to low energies. With this procedure the number of points in the numerical integration of the scattering kernel is restricted to the number of solutions previously obtained. However, the forward peak in the Klein-Nishina scattering distribution makes it desirable to obtain better definition in the region of integration for which E, $\approx E$. To obtain better definition we evaluate the integral in this region by Gaussian Quadrature as in the neutron code.

The application of Gaussian Quadrature to Equation (14) is very similar to the procedure described in Reference [12] for solution of the moments equations for neutrons. Again, to make this report self-contained we reproduce that discussion here.

This technique has the advantage that its accuracy depends only on the smoothness of the energy fluence function $I_{n}^{s}\left(E^{\prime}\right)$ in the integrand, rather than on the smoothness of the entire integrand, which is generally more rapidly varying due to the scattering kernel. We rewrite Eq. (14) in the following manner:

$$
\begin{align*}
\mu(E) I_{n \ell}^{S}(E)- & \int_{E}^{E_{1}} K\left(E^{\prime} \rightarrow E\right) P_{\ell}\left(1-\frac{1}{E}+\frac{1}{E^{\prime}}\right) I_{n \ell}^{s}\left(E^{\prime}\right) d E^{\prime}=\int_{E_{i}}^{E_{0}} K\left(E^{\prime} \rightarrow E\right) P_{\ell}\left(1-\frac{1}{E}+\frac{1}{E^{\prime}}\right) I_{n \ell}^{s}\left(E^{\prime}\right) d E^{\prime} \\
& \left.+\frac{\left(1-\delta_{n 0}\right) \mu_{0}}{(2 \ell+1)}[\ell+1) I_{n-1, \ell+1}^{s}(E)+\ell I_{n-1, \ell-1}^{s}(E)\right] \tag{15}
\end{align*}
$$

$$
+E_{0} K\left(E_{o} \rightarrow E\right) P_{\ell}\left(1-\frac{1}{E}+\frac{1}{E_{0}}\right) C_{n \ell}+\delta_{\ell} \sum_{k} \delta\left(E-E_{k}\right) E_{k} S_{n k}
$$

In this equation $E_{i}$ is the solution energy just above $E$. Letting $R$ equal the right hand side of Eq. (15) and noting that

$$
\mu(E) I^{s}(E)=\int_{E}^{\mathbf{E}_{\mathbf{i}}} \mu\left(E^{\prime}\right) I^{s}\left(E^{\prime}\right) \delta\left(E^{\prime}-E\right) d E^{\prime}
$$

we may rewrite Eq. (15),
$E_{i}$

$$
\begin{equation*}
\int_{E}\left[\mu\left(E^{\prime}\right) \delta\left(E^{\prime}-E\right)-K\left(E^{\prime} \rightarrow E\right) P_{\ell}\left(1-\frac{1}{E}+\frac{1}{E^{\prime}}\right)\right] I_{n \ell}^{S}\left(E^{\prime}\right) d E^{\prime}=R \tag{16}
\end{equation*}
$$

This integral is evaluated using the Gaussian technique by approximating the term in the square brackets by the sum of two delta functions.

$$
\begin{equation*}
\mu\left(E^{\prime}\right) \delta\left(E^{\prime}-E\right)-K\left(E^{\prime} \rightarrow E\right) P_{\ell}\left(1-\frac{1}{E}+\frac{1}{E^{\prime}}\right)=a_{1 \ell} \delta\left(E^{\prime}-E_{i}\right)+a_{2 \ell} \delta\left(E^{\prime}-E_{\ell}^{*}\right) \tag{17}
\end{equation*}
$$

Note that the function used in equation (17) could be modified by some function of $E^{\prime}$, in order to make the remaining function $I_{n}^{S}(E \prime)$ in the integrand of equation (16) more smoothly varying. We have not made any attempt to explore such modifications. Inserting the right hand side of equation (17) into (16) the integral is readily evaluated to give

$$
\begin{equation*}
a_{1 \ell} I_{n \ell}^{s}\left(E_{i}\right)+a_{2 \ell} I_{n \ell}^{s}\left(E_{\ell}^{*}\right)=R \tag{18}
\end{equation*}
$$

The three parameters $\alpha_{1 \ell}, \alpha_{2 \ell}$, and $E_{l}^{*}$ are determined by conditions on the first three moments of the scattering kernel:

$$
\begin{align*}
\mu(E) \delta_{j o}-I_{j}^{\ell}= & \int_{0}^{E_{i}-E}\left(E^{\prime}-E\right)^{j}\left[\mu\left(E^{\prime}\right) \delta\left(E^{\prime}-E\right)-K\left(E^{\prime} \rightarrow E\right) P_{\ell}\left(\cos ^{\ominus}\right)\right] d\left(E^{\prime}-E\right) \\
= & a_{1 \ell}\left(E_{i}-E\right)^{j}+a_{2 \ell}\left(E_{\ell}^{*}-E\right)^{j},  \tag{19}\\
& \text { for } j=0,1,2 \quad .
\end{align*}
$$

Solving this system of equations gives

$$
\begin{align*}
& E_{l}^{*}=E+\frac{-I_{1}^{l}\left(E_{i}-E\right)+I_{2}^{l}}{I_{1}^{l}+\left(\mu(E)-I_{o}^{l}\right)\left(E_{i}-E\right)} \\
& a_{2 l}=\frac{-I_{1}^{l}+\left(I_{0}^{l}-\mu(E)\right)\left(E_{i}-E\right)}{E_{l}^{*}-E_{i}}, \quad \text { and }  \tag{20}\\
& a_{1 \ell}=\mu(E)-I_{0}^{l}-a_{2 l} \cdot
\end{align*}
$$

In this application of the Gaussian technique to the integral in Eq. (16) the argument of one delta-function is not fixed beforehand. This quadrature is exact for any energy fluence function $I_{n}^{S}\left(E^{\prime}\right)$ which is of degree two or lower in the range between $E$ and $E_{i}$.

The energy fluence moments at $\mathrm{E}^{*}$ can be obtained by solving Eq. (12). Generally, $E^{*}$ is somewhat lower than $E$ and therefore an interpolation must be performed to obtain the value of the moments at E . Also, in evaluating the integral on the righthand side of Eq. (15), an interpolation is performed to obtain $I_{n}^{S}$ (Eノ). GAMM $\varnothing \mathrm{M}-\mathrm{I}$ uses linear interpolation for the solution at $E$ and quadratic interpolation to obtain $I_{n}^{S}(E \prime)$. Since the energy fluence is slowly varying with energy, these interpolations should introduce little error into the result.

## III. Organization and Description of GAMMøM-I

## A. General Structure

GAMM $\varnothing$ M-I is structured as a five segment overlay. The structure of this overlay is shown in Fig. 1. The function of each segment may be described briefly as follows:

Segment 1. Reads all input data except dose response coefficients.

Segment 2. Calculates discontinuity energies and solution mesh parameters.

Segment 3. Calculates scattering kernel and writes the results on Tape 8.

Segment 4. This is the main solution part of the program. The Gaussian procedure is used. Also the effect of fluorescence and annihilation is calculated in this segment. Drum storage is required.
Segment 5. This segment prints the results and calculates dose moments for the response functions that are defined by means of input data. Also the moments are written on tape 9 for later processing.


FIGURE 1. Overlay structure for GAMM $\phi$ M-I computer code

This overlay procedure is employed on the UNIVAC 1108 and should be easy to implement on other computers. Note that the interpolation and integration routines are available to each segment.

## B. Discussion of GAMMØM-I Subroutines

## GAMMA

This routine calls in the main subroutines in the following sequence:

INPUT - reads in input data related to energy grids, cross sections, and parameters related to the source angular distribution and the linkage between moments.

ENDISC - identifies the energies at which primary source energies exist and anticipates the existence of discontinuities in the scattered energy flux moments due to these sources.
STøREX - calculates Legendre coefficients of the scattering kerne1 and stores them on tape.

SECØND - calculates the source terms for fluorescent and annihilation radiation both for the continuous spectrum and the discrete lines.

COMPUTE - solves the moments equation numerically for a plane source and a fixed solution energy.

OUTPUT - writes moments on tape for each energy and calculates dose moments for plane and point sources.
INPUT
This subroutine reads in all data related to cross sections, energy grids, source characteristics, and moment linkages. A detailed description of the input data is given in section IV.

## ENDISC

This subroutine sets up the energy grid for which solutions are to be calculated. Allowance is made for double-valued solutions at energies for which discontinuities in the scattered energy fluence occur. The fineness of the grid is determined by input parameters except that 70 intervals are automatically assumed below the lowest single-scatter cutoff. The subroutine calls CMBINE which combines energy grids determined for each $\delta$-function source into a final energy grid.

This subroutine interpolates for the total cross section at each energy and writes detailed calculations of the scattering kernel on to tape. It calls EIDISC and KERNEL.

EIDISC - This subroutine sets up the energy grid for each integration over the scattering kernel. The fineness of the grid is determined by the input parameters MAXN $\varnothing$, which sets the minimum number of intervals over any single scatter region and MINN $\varnothing$ which is only used when the problem involves more than one monoenergetic source.

KERNEL - This subroutine calculates the required number of Legendre coefficients of the scattering kernel.
SECØND
This subroutine calculates the contribution of the annihilation and fluorescent secondary source radiation to the scattered energy fluence moments. This contribution is analogous to the contribution from singlescatter radiation generated by the primary source. This subroutine also calculates the contribution from photons at the discrete secondary source energies and adds them to the direct radiation from the primary source. CMPUTE

This subroutine solves equation 14 for moments of the energy fluence Solutions proceed from the highest energy downward with numerical integration of the scattering integral.

## OUTPUT

This subroutine contains options to print or write on to tape the energy fluence moments for scattered and direct radiation. Dose moments are produced by integration over the energy fluence moments, for up to ten dose response functions. Dose moments may be printed and/or puached on cards.

## IV. Input and Output Description

A. Card Input

The following cards are read in the subroutine INPUT:

Card Type 1 (I1, 12A6)
IPRøB Type of input data
$I P R \emptyset B=1$ All input data required.
IPRØB = 2 No source data required; source is assumed to be the same as in previous problem.
$I P R \emptyset B=3$ No cross section or source data required; these data are assumed to be the same as in previous problem.

IPRØB $=4$ Only data for source angular distribution is required;
all other input data are assumed to be the same as in previous
problem.
IPR $\emptyset B=5$ End of calculation (See card type 22).
TITLE Alpha numeric title for problem.
Card Type 2 (11I5)
NMUS Number of energies for which total cross sections are to be read in.
NSØRS Number of primary source energies.
LS Maximum number of Legendre coefficients calculated by code
( $l=0,1, \ldots$ LS-1) .
MINN $\varnothing$ The minimum number of intervals in the scattering kernel between the energy ranges of single scattering interactions.

MAXNØ The minimum number of intervals in the scattering kernel for any single scatter interaction.

NPXS Number of partial cross sections such as pair production or photoelectric absorption. NPXS $>0$ only if annihilation or fluorescent radiation is to be included in calculation.

IøUT Parameter determining type of output information.
IøUT = 0 Fluence moments printed and writcen on unit 9. Plane and point dose moments printed. Plane dose moments punched.

IøUT = 1 Fluence moments written on unit 9 and printed only for primary and secondary source energies. Plane and point dose moments printed. Plane dose moments punched.

IØUT = 2 Fluence moments printed. Plane and point dose moments printed.

IØUT = 3 Fluence moments printed only for primary and secondary source energies. Plane and point dose moments printed. Plane dose moments punched.

KEYIND Index which allows part of calculation to be skipped.
KEYIND $=0,1$ Do entire calculation.
KEYIND = 2 Compute moments using scattering kernel data stored previously on tape mounted on logical unit 8.

NRN Run number (arbitrary).
ICøNT Obsolete parameter (arbitrary).
NPRINT Number of solution energies for which detailed printout of solution is desired. (For diagnostic use.)

Card Type 3 (3E10.0)
ETØP Highest energy for which solutions are to be calculated. Ordinarily ETØP is equal to the highest primary source energy.
EB $\emptyset$ TM Lowest energy for which solutions are to be calculated.
SO Factor necessary to convert energy from input units to $\mathrm{mc}^{2}$ units. (Normally input energies are in MeV units and $\mathrm{S} \emptyset=1.95692$ ).
X0 Factor necessary to convert cross section from input units to Thomson units. (Normally input cross sections are $\mathrm{cm}^{2} / \mathrm{g}$ units and XO is then $(.400594 \mathrm{Z} / \mathrm{A})^{-1}$.

Card Type 4 (20I5) (Only if NPRINT > 0)
(IXEN(I), I = 1 , NPRINT) Indices of solutions for which detailed printout
is desired.
Card Type 5 ( $8 E 10.0$ ) (Only if IPR $\emptyset B \leq 2$ )
(EMUS(I), TABMU(I), I = 1 , NMUS) List of energies and tabulated cross sections.

The next 5 types of cards are required only if fluorescent or annihilation radiation is to be calculated.

Card Type 6 (20I5) (Only if IPRøB $\leq 2$ and NPXS $>0$ )
(NPART(I), $I=1$, NPXS) Number of tabulated cross sections for each reaction.
Card Type 7 (8E10.0) (Only if IPR $\varnothing$ B $\leq 2$ and NPXS $>0$ )
(YIELD(I), I = 1 , NPXS) Yield for each reaction.
Card Type 8 (8E10.0) (Only if IPR $\varnothing \mathrm{B} \leq 2$ and NPXS $>0$ )
(ECUT(I), $I=1$, NPXS) Threshold energy for each reaction.
Card Type 9 ( 8 E 10.0 ) (Only if IPR $\emptyset \mathrm{B} \leq 2$ and NPXS $>0$ )
(ESTART(I), I = 1 , NPXS) $\delta$-function energy of photon produced by each reaction.

Card Type 10 (8E10.0) (Only if IPROB $\leq 2$ and NPXS $>0$ )
(EPART(I,J), SIGMA(I,J), J = 1 , NPART(I)) List of energies and tabulated partial cross sections. (One set for each reaction.)
Card Type 11 ( 8 E 10.0 ) (Only if $\operatorname{IPR\emptyset B=1\text {)})~(I)~}$
(ESøRS(I), STS $\varnothing$ RS(I), $I=1, N S \emptyset R S)$ List of energies and strength of each primary source.
Card Type 12 (4I5)
NLØ Number of harmonic coefficients of the source angular distribution. LZR $\emptyset$ Index of source harmonic when $N L \emptyset=1$; otherwise arbitrary.
$M Z R \emptyset$ Index of azimuthal harmonic. (MZR $\varnothing=0$ unless both source distribution and detector response include azimuthal dependence).

Card Type 13 (4I3, 4(I6, 3I3))
$N(I) \quad n$-index of the $i^{\text {th }}$ moment
$L(I) \quad l$-index of the $i^{\text {th }}$ moment
LINKH(I) i-index of higher-l linkage moment
LINKL(I) i-index of lower- $\ell$ linkage moment
Card Type 14 (8E10.0)
(CNL(I), I = 1 , NL $\varnothing$ ) Harmonic coefficients of source angular distribution.
The following cards are read in the Main Program GAMMA:
Card Type 15 (2I5) (Only if KEYIND > 1)
NENSP Number of energies at which solutions are to be obtained.
NED Number of discontinuities in solution mesh.
Card Type 16 (3(I3, IP2E11.6)) (Only if KEYIND > 1)
NEIN(K) number of integration points for $K^{\text {th }}$ solution.
ENS (K) energy corresponding to $K^{\text {th }}$ solution.
EMUN(K) total cross section corresponding to $K^{\text {th }}$ energy (TU/el)
Card Type 17 (3(I3, 1P2E11.6) (Only if KEYIND > 1)
NSW(K) number of solutions, uniformly spaced in lethargy ( $E_{i} / E_{i-1}$ constant) in $K^{\text {th }}$ solution interval.
$E D(K)$ highest energy in $K^{\text {th }}$ solution interval.
$\operatorname{DED}(\mathrm{K})$ ratio of energies of successive solutions in $K^{\text {th }}$ interval.
Subroutine OUTPUT
Card Type 18 (2I5)
NDOSE number of energies at which dose response coefficients are tabulated

NOFD number of types of detector responses to be calculated
Card Type 19 (8E10.0)
ECONS factor to convert energies in input units to energy in mc ${ }^{2}$ units (see Card Type 3)
(CONS(I), I = 1 , NOFD) factor to convert energy absorption coefficients from input units to units of TU/electron. (see Card Type 3)
Card Type 20 (8E10.0)
(EDOSE(I), I = 1 , NDOSE) energy list for dose response coefficients.
Card Type 21 (8E10.0)
(SIGMAD(I,J), $I=1$, NDOSE) energy absorption coefficient for $I^{\text {th }}$ energy and $J^{\text {th }}$ type of dose response

Card Type 22 (blank)
End of calculation
B. Scattering Kernel Tape

For the UNIVAC 1107/1108 systems, the results of the calculations of the scattering kernel are written tape unit 8. This tape must be available for all values of KEYIND. The tape is written in the binary mode. Binary records are written in the following form for each solution:
Record 1
WRITE (8) ENS(N), NIN, LSP, ND
where ENS (N) is energy for the $N^{\text {th }}$ solution
NIN is the number of integration points in the integrand
LSP is the number of Legendre coefficients LS, plus 4
ND is the number of intervals over which the integration mesh is uniformly spaced.

Record 2
WRITE (8) (NKIND (K), $K=1$, NIN)
where NKIND(K) is an index which indicates when the integration mesh changes

NKIND $>0$ : number of energies with constant integration mesh NKIND $=0$ : no change in integration mesh
Record 3
WRITE (8) (EIN(K), $\mathrm{K}=1, \mathrm{NIN})$
where $\operatorname{EIN}(K)$ is the list of energies in the integrand

Record 4
WRITE (8) (EDI(K), $K=1$, ND)
where $\operatorname{EDI}(\mathrm{K})$ is the list of energies at which the integration mesh changes

Record 5
WRITE (8) ((XKIN(K,L), L = 1, LS), $\mathrm{K}=1, \mathrm{NIN}$ )
where XKIN(K,L) is the ( $\mathrm{L}-1)^{\text {th }}$ harmonic coefficient of the cross section for the $K^{\text {th }}$ energy in the integrand
C. Moments Solution Tape

For the UNIVAC $1107 / 1108$ systems, moments are written on tape unit
9. This tape must be available for IOUT (input card 2) values less than
2. The following describes the $B C D$ records written for a single problem. Record 1 (12A6)

TITLE Problem title
Record 2 (5I5)
NENS number of solution energies
NSORS number of primary source energies
NOFD number of different detector responses
NNL number of ( $n, \ell$ ) combinations
NDOSE number of energies at which dose response coefficients are tabulated

Record 3 (16I5)
( $\mathrm{N}(\mathrm{I}), \mathrm{L}(\mathrm{I}), \mathrm{I}=1$, NNL) n and $\ell$ indices of each solution
Record 4 (1P1OE13.7)
(ENS (I), $I=1$, NENS) energy list
Record 5 (1P10E13.7)
(EMUN(I), $I=1$, NENS) 1 ist of total cross sections
Record 6 (1P1OE13.7)
(CNL(I), $I=1$, NNL) list of $C_{n \ell}$ values for each ( $n, \ell$ ) combination Record 7 thru $2 *$ NNL +6 (1P1OE13.7)
(SOLN (NENS $+\mathrm{K}, \mathrm{I}), \mathrm{K}=1$, (NSORS + NPXS)) Direct radiation moments for primary and secondary sources for the $I^{\text {th }}(\mathrm{n}, \ell)$ combination
(SOLN(K,I), $K=1$, NENS) Scattered radiation moments for the $I^{\text {th }}$ ( $\mathrm{n}, \ell$ ) combination.
D. Discussion of Output Data

The data preceded by (NPRINT > 0) are printed only for those solutions for which detailed printout is requested. These solutions are chosen by means of the input parameters NPRINT and IXEN. These data are for diagnostic use.

The printout consists of the following blocks of data:
Input data from INPUT subroutine.
Tabulation of energies and total cross sections in input units. Tabulation of energies and total cross sections in units of electron mass and Thomson units, respectively.
(NPXS $>0$ ) List of secondary source energy, threshold energy yield number of tabulated partial cross sections, list of energies and cross sections for each partial interaction.
List of source energies (primary and secondary) and source strengths (strength of secondary sources is initially set at zero).

List of normalized source energy and strength.
List of indices for each moment and values of linkage coefficients.
List of solution index, number of integration points, energy, and total cross section for each solution.

List of index numbers of solution points, energy and ratio of successive solution energies associated with each discontinuity.
Discontinuity refers to an energy below which the solution energy grid changes.
(NPXS $>0$ ) List of moment ( $n, \ell$ ) indices and contribution from integration over scattered moments.
Energy and partial cross section for each primary scource.
List of moment ( $n, \ell$ ) indices and contribution from scattered moments and primary sources.
List of moment ( $n, \ell$ ) indices and contribution from scattered moments and primary sources, weighted by radiation yield and appropriate cross section ratios.

List of energies for which secondary sources may contribute to
scattered moments. (Note that the SOURCE subroutine calculates a non-zero contribution only when

$$
\frac{E_{2}}{\left(2 E_{2}+1\right)}<E<E_{2}
$$

where $E_{2}$ is the energy of the secondary source.
( $I=\operatorname{IXEN}(J)$ ) The following information is printed only for those energies for which detailed printout has been requested.
( $n, \ell$ ) index, source contribution, and source contribution at solution discontinuity.
Parameters associated with this solution energy.
Index of solution, number of integration points, integrand energy.
Legendre coefficients of the scattering cross section, for each interval of integration.
$\ell$ index, three Gauss parameters and three obsolete parameters, for each $\ell$-value.
Energy and contribution from numerical integration for each ( $n, \ell$ ) combination
Energy and moment solution for each ( $n, \ell$ ) combination.
Printout of input parameters.
List of energies and dose response coefficients.
List of energies in $\mathrm{mc}^{2}$ units and dose response coefficients in Thomson units per electron.
(IOUT $=0$ or 2) Energy, total cross section, solution index, LZRO, and MZRO, followed by a list of ( $n, \ell$ ) indices and fluence values for each ( $n, \ell$ ) combination.

Energy, total cross sections, solution index, LZRO and MZRO, followed by ( $n, \ell$ ) indices and fluence values for source energies on1y.
List of energy, interpolated dose response coefficient (normalized) fluence ( 0,0 ) moment, cumulative dose integral for each type of response.
Normalizing constant for which total energy absorbed (i.e. the zero-zero moment for total dose) is equal to unity. The deviation
of the moment from unity is a measure of inconsistency in cross sections introduced by interpolation procedures in the code. Title of calculation and index of dose response coefficient. Dose moments for a plane isotropic (PLI) source. Values are given for total, direct (including secondary sources) and scattered dose.
Dose moments for a point isotropic(PTI) source. Values are given for total, direct (including secondary sources), and scattered dose.

## Sample Problem.

TOTAL CROSS SECT1ONS ENERGY=-SIGVA
20-0660.2

$1.1180+00$



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| $1.0000+01$ | $2.3070-02$ | $8.00000+00$ |
| $2.0000+00$ | $4.4800-02$ | $1.5000+00$ |
| $4.0000-01$ | $9.5760-02$ | $3.0000-01$ |
| $6.0000-02$ | $2.7340-01$ | $5.0000-02$ |
| $1.0000-02$ | $2.6460+01$ |  |



[^2]$\begin{array}{cl}\text { NUMRER--ENERGY**S1GMA } \\ 17 & 1.0000+02 \\ & 2.1050-02 \\ & 4.0000+01 \\ & 1.6040-020 \\ & 4.0000+01 \\ & 7.6580-03 \\ & 1.0200+00 \\ 2.9190-03 \\ & 1.0000-08\end{array}$
5.0000+01 $1.7370-02$
 $\begin{array}{lll}6.0000+00 & 4.8510-03 & 1.0000+00 \\ 2.0000+00 & 6.2200-04 & 1.5000+00 \\ 1.5800-04\end{array}$ $\begin{array}{lll}N & N & M \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 \\ N & 0 & 0 \\ 0 & M & 0 \\ 0 & J & M \\ \vdots & -1 & 0\end{array}$


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| 50.0000 | $2.04200-02$ | $1.49000-02$ |
| 40.0000 | $1.96600-02$ | $1.45000-02$ |
| 30.0000 | $1.88100-02$ | $1.43000-02$ |
| 20.0000 | $1.78800-02$ | $1.41000-02$ |
| 15.0000 | $1.74900-02$ | $1.43000-02$ |
| 10.0000 | $1.74100-02$ | $1.51000-02$ |
| 8.0000 | $1.76400-02$ | $1.57000-02$ |
| 6.0000 | $1.82700-02$ | $1.68000-02$ |
| 5.0000 | $1.86600-02$ | $1.78000-02$ |
| 4.0000 | $1.97700-02$ | $1.89000-02$ |
| 3.0000 | $2.12400-02$ | $2.070000-02$ |
| 2.0000 | $2.37600-02$ | $2.36000-02$ |
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| .5000 | $2.99300-02$ | $2.97000-02$ |
| .4000 | $2.98800-02$ | $2.95000-02$ |
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| 4 | u | 8.13736414-01 | 4 | $?$ | 2.1352197-01 | 5 | 1 | 4.3274310-01 | ¢ | 0 | ย. 3 399975-01 |
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| 1) | U | 4.7213783-111 | 1 | 1 | 1.5737831-01 | 2 | 0 | 1.5738073-01 | 2 | 2 | ט.2951070-02 |
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| 0 | $1)$ | 5.2868り14-01 | 1 | 1 | 2.32464443-01 | $?$ | 0 | 6.301328f-01 | 2 | 2 | 1.0393468-01 |
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| 3 | 3 | $7.20 .33465-02$ |
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| 00 | 9.4461367-111 | 1 | 1 | 3.8429069-01 | 2 | 0 | 7.1263745-01 | 2 | 2 | 1.0000.353-01 |
| 40 | 7.2038832-01 | 4 | 2 | 2.122599?-01 | 5 | 1 | 4.2381782-01 | 6 | 0 | 7.5073150-01 |
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| 0 O | 4.7239387-01 | 1 | 1 | 1.5746,366-01 | 2 | 0 | 1.5746607-01 | 2 | 2 | 0.cy8b212-02 |
| 40 | 9.4479811-02 | 4 | $?$ | 5.3987263-02 | 5 | 1 | 6.7484385-02 | 6 | 0 | 6.7485640-02 |
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| 0 U | 4.7221981-01 | 1 | 1 | 2.2681702-01 | 2 | 0 | 5.5517139-01 | 2 | 2 | 1.0301832-01 |
| 40 | 6.2590850-01 | 4 | $?$ | 1.5827266-01 | 5 | 1 | 3.5633344-01 | 6 | 0 | -. j924588-01 |


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| 00 | 9.4461367-111 | 1 | 1 | 3.8429069-01 | 2 | 0 | 7.1263745-01 | 2 | 2 | 1.0000.353-01 |
| 40 | 7.2038832-01 | 4 | 2 | 2.122599?-01 | 5 | 1 | 4.2381782-01 | 6 | 0 | 7.5073150-01 |
| UIRECT DOSE CONTIRIBUTION |  |  |  |  |  |  |  |  |  |  |
| 0 O | 4.7239387-01 | 1 | 1 | 1.5746,366-01 | 2 | 0 | 1.5746607-01 | 2 | 2 | 0.cy8b212-02 |
| 40 | 9.4479811-02 | 4 | $?$ | 5.3987263-02 | 5 | 1 | 6.7484385-02 | 6 | 0 | 6.7485640-02 |
| SCATTERED DOSE CUNTRIHUTION |  |  |  |  |  |  |  |  |  |  |
| 0 U | 4.7221981-01 | 1 | 1 | 2.2681702-01 | 2 | 0 | 5.5517139-01 | 2 | 2 | 1.0301832-01 |
| 40 | 6.2590850-01 | 4 | $?$ | 1.5827266-01 | 5 | 1 | 3.5633344-01 | 6 | 0 | -. j924588-01 |


| TOTAL | DOSE |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00 | 9.4461367-111 | 1 | 1 | 3.8429069-01 | 2 | 0 | 7.1263745-01 | 2 | 2 | 1.0000.353-01 |
| 40 | 7.2038832-01 | 4 | 2 | 2.122599?-01 | 5 | 1 | 4.2381782-01 | 6 | 0 | 7.5073150-01 |
| UIRECT DOSE CONTIRIBUTION |  |  |  |  |  |  |  |  |  |  |
| 0 O | 4.7239387-01 | 1 | 1 | 1.5746,366-01 | 2 | 0 | 1.5746607-01 | 2 | 2 | 0.cy8b212-02 |
| 40 | 9.4479811-02 | 4 | $?$ | 5.3987263-02 | 5 | 1 | 6.7484385-02 | 6 | 0 | 6.7485640-02 |
| SCATTERED DOSE CUNTRIHUTION |  |  |  |  |  |  |  |  |  |  |
| 0 U | 4.7221981-01 | 1 | 1 | 2.2681702-01 | 2 | 0 | 5.5517139-01 | 2 | 2 | 1.0301832-01 |
| 40 | 6.2590850-01 | 4 | $?$ | 1.5827266-01 | 5 | 1 | 3.5633344-01 | 6 | 0 | -. j924588-01 |

PLANE DOSE RATE INTEGRALS FOR THF 2 RESPONSE


$-1^{1},-72$
1.25 VEV CONCRETF
$E O=$
$P O 1$ IT
$\begin{array}{lll}3 & 3 & 1.2605857+00 \\ 3 & 3 & 4.72 .28785-01\end{array}$

| 3 | 1 | $1.9920978+00$ |
| :--- | :--- | :--- |
| 3 | 1 | $4.7239118-01$ |
| 3 | 1 | $1.5197066+00$ |

$1.2450265+00$
$5.2971207+00$
$4.1238909-01$
$4.7239950-01$
$7.7263740-01$
$4.8247213+00$
No No no
POINT DUSE RATE INTEGRALS FOR THE 2 RESPONSE

| 0 | 0 | $9.4461367-01$ | 1 | 1 | $1.1528421+00$ | 2 | 0 | $2.1379124+0 n$ | 2 |
| :---: | :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | 0 | $3.6019416+00$ | 4 | 2 | $1.8572744+00$ | 5 | 1 | $2.9667249+00$ | 6 |
| OIRECT DOSE CONTRIRUTION |  |  |  |  |  |  |  |  |  |
| 0 | 0 | $4.7239387-01$ | 1 | 1 | $4.7239099-01$ | 2 | 0 | $4.7239822-01$ | 2 |
| 4 | 0 | $4.7239907-01$ | 4 | 2 | $4.7238857-01$ | 5 | 1 | $4.723907 P-01$ | 6 |

[1] U. Fano, L. V. Spencer, and M. J. Berger, "Penetration and Diffusion of X-rays," Handbuch der Physik, XXXVIII (Springer-Verlag, Berlin, 1959).
[2] L. V. Spencer, "Plural Series Representation of Functions, J. Res. Nat. Bur. Standards (to be published).
[3] L. V. Spencer and U. Fano, "Penetration and Diffusion of X-Rays. Calculation of the Spatial Distributions by Polynomial Expansion," J. Res. Nat. Bur. Standards 46, 446 (1951).
[4] H. Goldstein and J. Ernest Wilkins, Jr., "Calculations of the Penetration of Gamma Rays," AEC Report NY0-3075, June 30, 1954.
[5] L. V. Spencer and J. C. Lamkin (unpublished).
[6] L. V. Spencer and J. C. Lamkin, "Slant Penetration of $\gamma$-Rays in $\mathrm{H}_{2} \mathrm{O},{ }^{12}$ NBS' Report 5944, unpublished.
[7] L. V. Spencer and J. C. Lamkin, "Slant Penetration of $\gamma$-Rays: Mixed Radiation Sources," NBS Report 6322, unpublished.
[8] L. V. Spencer and J. C. Lamkin, "Slant Penetration of $\gamma$-Rays in Concrete," NBS Report 6591, unpublished.
[9] L. V. Spencer, "Structure Shielding Against Fallout Radiation from Nuclear Weapons," NBS Monograph 42, U.S. Gov't Printing Office, June, 1962.
[10] C. Eisenhauer, "An Engineering Method for Calculating Protection Afforded by Structures Against Fallout Radiation, " NBS Monograph 76, U.S. Gov't Printing Office, July, 1964.
[11] "Design and Review of Structure for Protection from Fallout Gamma Radiation," PM-100-1, Office of Civil Defense, Feb., 1965.
[12] E. T. Bramlitt and C. M. Eisenhauer, "A Discussion and Tabulation of the OCD Standard Method for Fallout Gamma Radiation Shielding Analysis," TR-70, Office of Civil Defense, December, 1971.
[13] "Shelter Design and Analysis, Volume 1, Office of Civil Defense, June, 1968 (Rev: 1969, 1970, 1971).
[14] C. Eisenhauer, G. L. Simmons and L. V. Spencer, "A Neutron Moments Computer Code, Moment I.," NBS Technical Note 725, June, 1972.

16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

In this paper we describe a computer code for generating spatial-angular moments of gamma ray energy fluence in an infinite medium. The equation for moments of the energy fluence is given and the techniques used for the solution are discussed. The structure of the code and of the main subroutines is also given. Details of the input and output data are presented and the printout from a sample problem is included.
17. KEY WORDS (Alphabetical order, separated by semicolons)

Computer code; gamma rays; gamma ray transport; gauss quatrature; moments; shielding.
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