## NBS TECHNICAL NOTE 725

# A Neutron Moments Computer Code, Moment I 

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[^0]A Neutron Moments Computer Code, Moment I

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A NEUTRON MOMENTS COMPUTER CODE, MOMENT I*

Charles M. Eisenhauer, George L. Simmons, and Lewis V. Spencer

In this paper we discuss a computer code for generating spatial-angular moments of neutron flux in an infinite medium. The equation for the flux moments is given and the techniques used for the solution are discussed. The structure of the computer code and of the main subroutines is also discussed. Details of the input and output data are given and the printout from a sample problem is included.

Key words: Carbon; computer code; gauss quadrature; moments; neutron transport; shielding.

[^1]
## I. INTRODUCTION

In this note we describe a computer program to calculate spatial and angular moments of the neutron flux distribution in an infinite medium. The method of moments has proved extremely accurate in the past for predicting gamma ray distributions in infinite media, but some difficulties were encountered in applying the technique to neutron problems. Principal among these difficulties were:

1. The energy range of interest in application to reactor or weapons shielding problems extends from 15 MeV to $10^{-2} \mathrm{eV}$, a range of $10^{9}$ in energy.
2. Neutron cross section data are not smooth functions of energy, but exhibit a great deal of structure. This problem manifests itself in the attempt to guarantee that the moments converge to the correct result.
3. For elastic scattering, the neutron energy $E$ is degraded to a minimum value of $E\left(\frac{A-1}{A+1}\right)^{2}$. For elements with high atomic weight $A$, therefore, the energy degradation is very small and the number of solutions required to describe the distribution of the moments as a function of energy is very large.

MOMENT I is àn attempt to overcome the difficulties discussed above, but still maintain practical computing economy. The program is written in FORTRAN IV and has been made operational on the UNIVAC 1107/1108, CDC-3600, and IBM-360/91. Our experience has indicated that the code can be converted from one machine to another with only minor modifications. These modifications involve the size of the memory available and consequently the size of the particular problem that can be solved.

## II. HISTORY OF NEUTRON MOMENTS CODE

The method of moments was first applied by Spencer and Fano ${ }^{(1)}$ to the solution of the transport equation for gamma rays. Application of this method to solution of problems in neutron transport was difficult at that time because neutron scattering cross sections could not be expressed in a simple analytic form analogous to the Klein-Nishina formula for the scattering of gamma rays. Therefore, large tabulations of neutron cross section data were required. The subsequent development of automatic computers with large storage capacity made the solution by moments feasible for neutrons. A code called NUPAK was reported by Certaine ${ }^{(2)}$ in 1954. A subsequent version called RENUPAK ${ }^{(3)}$ was reported in 1959. A history of this code along with references is given by Weisbin. (4)

A moments code for neutron transport was also developed at the National Bureau of Standards. Results of calculations made during the development of this code have appeared in several publication. Results calculated for water were contributed to the comparative study of different transport methods conducted by the American Nuclear Society in 1963. ${ }^{(5)}$ Results calculated for sodium were used by Miele. (6) Calculations of spectra from fission neutrons and 14 MeV neutrons in air were quoted in a status report by Straker.

A discussion of the NBS codes for generating moments and constructing spatial distributions was given in a 1968 unpublished report. In that report calculations for a point isotropic photofission source in carbon were presented. Since the publication of that report several improvements have been made in the code.

1. The processing of input cross section information has been simplified and speeded up.
2. The calculation of the scattering kernel has been changed in order to improve convergence of the moments.
3. General numerical procedures for interpolation and integration have been streamlined.

## III. THEORY

## A. Moments Equations

The neutron transport equation in one dimensional plane geometry has the form

$$
\begin{align*}
\cos \theta \frac{\partial \Phi}{\partial z}(E, z, \vec{w})+\Sigma(E) \Phi(E, z, \vec{w})= & \int_{E}^{\infty} \mathrm{dE} E^{\prime} \int_{4 \pi} d \vec{\Omega}^{\prime} K\left(E^{\prime} \rightarrow E, \vec{w}^{\prime} \rightarrow \vec{\omega}\right) \Phi\left(E^{\prime}, z, \vec{w}^{\prime}\right)  \tag{1}\\
& +S(E, z, \vec{w})
\end{align*}
$$

where
$\Phi(E, z, \vec{\omega}) \quad$ is the neutron $f l u x$ (neutrons $/ \mathrm{cm}^{2}-\mathrm{sec}-\mathrm{MeV}$-ster),
$\Sigma(E) \quad$ is the macroscopic total cross section $\left(\mathrm{cm}^{-1}\right)$,
$\vec{\omega} \quad$ is the neutron direction relative to the positive $z$ axis, and $\vec{\omega} \cdot \vec{k}=\cos \theta$,
$S(E, z, \vec{\omega}) \quad$ is the source strength (neutrons $/ \mathrm{cm}^{-3}-$ sec-MeV-ster),
and $K\left(E^{\prime} \rightarrow E, \vec{\omega}^{\prime} \rightarrow \vec{\omega}\right)$ is the scattering probability per unit path length (probability/cm-MeV-ster).

If one now expands the flux in spherical harmonics,

$$
\begin{equation*}
\Phi(E, z, \vec{w})=\sum_{\ell=0}^{\infty} \sum_{m=\ell}^{\ell} \sqrt{\frac{2 \ell+1}{4 \pi}} \Phi_{l}^{m}(E, z) Y_{l}^{m}(\odot, \varphi), \tag{2}
\end{equation*}
$$

and defines spatial moments of the angular expansion coefficients as

$$
\begin{equation*}
\Phi_{n \ell}^{m}(E)=\frac{\sum_{0}^{n+1}}{n!} \int_{-\infty}^{\infty} z^{n} \Phi_{\ell}^{m}(E, z) d z \tag{3}
\end{equation*}
$$

the transport equation becomes a function of energy alone. However, there is an infinite set of equations for $\Phi_{n l}^{m}(E)$. Here, $\Sigma_{o}$ is the total cross section at the highest energy considered. The source term can be obtained by evaluating

$$
\begin{equation*}
S_{n l}^{m}(E)=\frac{\sum_{o}^{n+1}}{n!} \int_{-\infty}^{\infty} z^{n} d z \int_{4 \pi}^{\infty} \sqrt{2 \pi} Y_{l}^{m^{*}}(\theta, \varphi) S(E, z, \vec{\omega}) d \vec{\Omega} \tag{4}
\end{equation*}
$$

where $Y_{l}^{m *}$ is the complex conjugate of $Y_{l}^{m}$. The moments equation then has the form

$$
\begin{align*}
\Sigma(E) \Phi_{n \ell}^{m}(E)= & \int_{E}^{\infty} K\left(E^{\prime} \rightarrow E\right) P_{\ell}\left(\cos ^{\oplus}\right) \Phi_{n \ell}^{m}\left(E^{\prime}\right) d E^{\prime}+ \\
& +\sum_{0} \frac{\sqrt{(\ell+1)^{2}-m^{2}}}{(2 \ell+1)} \Phi_{n-1, \ell+1}^{m}(E)+\sum_{0} \frac{\sqrt{\ell^{2}-m^{2}}}{2 \ell+1} \Phi_{n-1, \ell-1}^{m}+ \\
& +S_{n, \ell}^{m}(E) . \tag{5}
\end{align*}
$$

where $\Theta$ is the scattering angle in the laboratory system. For a plane isotropic source located at $z=0$, i.e. $S(E, z, \vec{w})=S(E) \delta(z) / 4 \pi$ :

$$
\begin{equation*}
\Sigma(E) \Phi_{n \ell}(E)=\int_{E}^{\infty} K\left(E^{\prime} \rightarrow E\right) P_{\ell}\left(\cos ^{\Theta}\right) \Phi_{n \ell}\left(E^{\prime}\right) \mathrm{dE}^{\prime}+ \tag{6}
\end{equation*}
$$

$$
\begin{aligned}
& +\frac{\left(1-\delta_{n o}\right) \Sigma_{0}}{2 \ell+1}\left[(\ell+1) \Phi_{n-1, \ell+1}(E)+\ell \Phi_{n-1, \ell-1}(E)\right]+ \\
& +\Sigma_{0} \delta_{n o} S(E) .
\end{aligned}
$$

Since the evaluation of the integral in Eq. (6) must be done numerically, we would like to make the integral as smooth as possible. In order to reduce the effect of the variations with energy associated with the integration of the scattering kernel, we take advantage of the properties of $\phi_{00}$, namely

$$
\Phi_{O O}(E) \approx \frac{S(E)}{E \Sigma(E)}
$$

and define collision density moments

$$
\begin{equation*}
\Psi_{\mathrm{nl}}(E)=\Sigma(E) \Phi_{\mathrm{nl}}(E) \tag{7}
\end{equation*}
$$

'The collision density moments equation corresponding to Eq. (6) is

$$
\begin{align*}
\psi_{n \ell}(E)= & \int_{E}^{\infty} \frac{K\left(E^{\prime} \rightarrow E\right)}{\sum\left(E^{\prime}\right)} P_{\ell}(\cos \Theta) \Psi_{n \ell}\left(E^{\prime}\right) \mathrm{dE}^{\prime}  \tag{8}\\
& +\frac{\left(1-\delta_{n o}\right) \sum_{0}}{\sum(E)(2 \ell+1)}\left[(\ell+1) \psi_{n+1, \ell+1}(E)+\ell \Psi_{n-1, \ell-1}(E)\right] \\
& +\sum_{0} \delta_{n o} S(E)
\end{align*}
$$

B. Gaussian Technique Applied to Scattering Integral

The usual method of solving Eq. (8) or Eq. (6) assumes a grid of energies between some maximum energy, $\mathrm{E}_{\mathrm{O}}$, and the lowest energy of
interest and proceedswith the numerical solution of the problem, working from high to low energies. As indicated in the introduction, some compromise must be struck between covering the range of interest with an adequate number of points and being able to reach the lowest energy with a minimum number of points. This problem is aggravated at high energies by forward peaking of the elastic scattering cross section and the implication, therefore, of high probability for small energy loss collisions.

To avoid this problem, we evaluate part of the scattering integral by the technique of Gaussian Quadrature. This technique has the advantage that its accuracy depends only on the smoothness of the collision density function $\Psi_{n \ell}\left(E^{\prime}\right)$ in the integrand rather than the smoothness of the entire integrand, which is generally rapidly varying due to the scattering kernel. We rewrite Eq. (8) in the following manner,

$$
\begin{align*}
& \Psi_{n \ell}(E)-\int_{E}^{E_{i}} \frac{K\left(E^{\prime} \rightarrow E\right)}{\sum\left(E^{\prime}\right)} P_{\ell}\left(\cos ^{\Theta}\right) \Psi_{n \ell}\left(E^{\prime}\right) \mathrm{dE}^{\prime}= \\
& \int_{E_{i}}^{0} \frac{K\left(E^{\prime} \rightarrow E\right)}{\sum\left(E^{\prime}\right)} P_{\ell}\left(\cos ^{\Theta}\right) \Psi_{n \ell}\left(E^{\prime}\right) d E^{\prime}+ \\
&+\frac{\left(1-\delta_{n 0}\right) \Sigma_{0}}{(2 \ell+1) \Sigma(E)}\left[(\ell+1) \psi_{n-1, \ell+1}(E)+\ell \Psi_{n-1, \ell-1}(E)\right]+\sum_{0} \delta_{n o} S(E) . \tag{9}
\end{align*}
$$

In this equation $E_{i}$ is the solution energy just above $E$. Letting $R$ equal the right hand side of Eq. (9) and noting that

$$
\Psi(E)=\int_{i}^{E_{i}} \Psi\left(E^{\prime}\right) \delta\left(E^{\prime}-E\right) d E^{\prime},
$$

we may rewrite Eq. (9),

$$
\begin{equation*}
\int_{E}^{E_{i}^{i}}\left[\delta\left(E^{\prime}-E\right)-\frac{K\left(E^{\prime} \rightarrow E\right)}{\Sigma\left(E^{\prime}\right)} P_{\ell}(\cos \Theta)\right] \psi n \ell\left(E^{\prime}\right) d E^{\prime}=R \tag{10}
\end{equation*}
$$

This integral is evaluated using the Gaussian technique by approximating the term in the square brackets by the sum of two delta functions.

$$
\begin{equation*}
\delta\left(E^{\prime}-E\right)-\frac{K\left(E^{\prime}-E\right)}{\sum\left(E^{\prime}\right)} P_{\ell}(\cos \Theta)=a_{1 \ell} \delta\left(E^{\prime}-E_{i}\right)+a_{2 \ell} \delta\left(E^{\prime}-E_{\ell}^{*}\right) \tag{11}
\end{equation*}
$$

Clearly, Eq. (10) becomes

$$
\begin{equation*}
\alpha_{1 \ell}{ }^{\Psi} n \ell\left(E_{i}\right)+\alpha_{2 \ell} \Psi_{n \ell}\left(E_{\ell}^{*}\right)=R \tag{12}
\end{equation*}
$$

The three parameters $\alpha_{1 \ell}, \alpha_{2 \ell}$, and $E_{\ell}^{*}$ are determined by conditions on the first three moments of the scattering kernel:

$$
\begin{align*}
\delta_{j 0}-I_{j}^{\ell}= & \int_{0}^{E_{i}-E}\left(E^{\prime}-E\right)^{j}\left[\delta\left(E^{\prime}-E\right)-\frac{K\left(E^{\prime} \rightarrow E\right) P_{\ell}(\cos \Theta)}{\sum\left(E^{\prime}\right)}\right] d\left(E^{\prime}-E\right)  \tag{13}\\
= & \alpha_{1 \ell}\left(E_{i}-E\right)^{j}+\alpha_{2 \ell}\left(E_{\ell}^{*}-E\right)^{j}, \\
& \text { for } j=0,1,2 .
\end{align*}
$$

Solving this system of equations gives

$$
E_{l}^{*}=E+\frac{-I_{1}^{l}\left(E_{i}-E\right)+I_{2}^{l}}{I_{l}^{l}+\left(1-I_{o}^{l}\right)\left(E_{i}-E\right)}
$$

$$
\begin{align*}
& a_{2 \ell}=\frac{-I_{1}^{\ell}+\left(I_{0}^{\ell}-1\right)\left(E_{i}-E\right)}{E_{l}^{*}-E_{i}}, \quad \text { and }  \tag{14}\\
& a_{1 \ell}=1-I_{o}^{\ell}-a_{2 l}
\end{align*}
$$

This application of the Gaussian technique to the integral in Eq. (10) differs from the usual Gaussian Quadrature approach in that the argument of one delta-function is not fixed beforehand. This quadrature is exact for any collision density which is of degree two or lower in the range between $E$ and $E_{i}$.

The collision density moments at $E_{l}^{*}$ can be obtained by solving Eq. (12). Generally, $E_{\ell}^{*}$ is somewhat lower than $E$ and therefore an interpolation must be performed to obtain the value of the moments at E. Also, in evaluating the integral on the righthand side of Eq. (9), an interpolation is performed to obtain $\Psi_{n \ell}\left(E^{\prime}\right)$. MOMENT I has the capability of using either first or second order polynomials for these interpolations. Since the collision density (as opposed to the flux) is slowly varying with energy, these interpolations should introduce little error into the result.

## C. Neutron Scattering Laws

The scattering kernel, $K\left(E^{\prime} \rightarrow E\right)$, is the sum over all possible scattering reactions that can contribute to the integral. MOMENT I has the flexibility of considering elastic, discrete level inelastic and continuum inelastic scattering events, as well as ( $\left.n, n^{\prime} p\right),\left(n, 2 n^{\prime}\right)$, $\left(n, n^{\prime} \alpha\right)$, and $\left(n, n^{\prime} 3 \alpha\right)$ reactions. For both elastic and discrete level
inelastic scattering, the contribution to the scattering kernel may be represented as

$$
\begin{equation*}
K\left(E^{\prime} \rightarrow E\right)=\sum_{i} \sum_{j} \frac{\left(1+M_{i}\right)^{2} \Sigma_{i j}\left(E^{\prime}\right)}{2 M_{i} V_{i j} E^{\prime}} \sum_{\ell=0}^{L}\left(\ell+\frac{1}{2}\right) f_{\ell, i j}\left(E^{\prime}\right) P_{\ell}\left(\cos \Theta_{i j}\right), \tag{15}
\end{equation*}
$$

where

$$
\begin{array}{ll}
M_{i} & \text { is the atomic weight of the } i^{t h} \text { isotope (AMU), } \\
\Sigma_{i j}\left(E^{\prime}\right) \quad & \text { is the scattering cross section for the } j^{\text {th }} \text { inter- } \\
& \text { action in the } i^{\text {th }} \text { isotope }\left(\mathrm{cm}^{-1}\right), \\
f_{\ell, i j}\left(E^{\prime}\right) \quad & \text { is the Legendre polynomial expansion coefficient } \\
& \text { for the ij } \\
\\
f_{o, i j}\left(E^{\prime}\right)=1 .
\end{array}
$$

The scattering angle, $\Theta_{i j}$, in the $L A B$ system is related to the scattering angle, $\theta_{i j}$, in the center of mass system by the following relationships,

$$
\begin{align*}
& \cos \Theta_{i j}=\frac{M_{i}+1}{2} \sqrt{\frac{E}{E^{\prime}}}-\frac{1}{2} \frac{M_{i}^{2} V_{i j}^{2}\left(E^{\prime}\right)-1}{M_{i}+1} \sqrt{\frac{E^{\prime}}{E}}  \tag{16}\\
& \cos \theta_{i j}=\frac{1}{2 M_{i} V_{i j}\left(E^{\prime}\right)}\left[\left(1+M_{i}\right)^{2} \frac{E}{E^{\prime}}-M_{i}^{2} V_{i j}^{2}\left(E^{\prime}\right)+1\right], \tag{17}
\end{align*}
$$

with the ratio of velocities of the center of mass after and before a collision being given by

$$
\begin{equation*}
v_{i j}\left(E^{\prime}\right)=\sqrt{1-\frac{Q_{i j}\left(M_{i}+1\right)}{E^{\prime} M_{i}}} \tag{18}
\end{equation*}
$$

$Q_{i j}$ being the $Q$ value for the $i j^{\text {th }}$ interaction, ( MeV ).
For elastic scattering, $(Q=0), V_{i j}$ is unity, but for inelastic scattering $\mathrm{V}_{\mathbf{i j}}$ is less than unity.

The remaining interactions are treated by assuming that the reactions are isotropic in the $L A B$ system and that the secondary neutron distribution is a Maxwellian. The kernel can then be written

$$
\begin{equation*}
K_{i j}\left(E^{\prime} \rightarrow E\right)=\Sigma_{i j}\left(E^{\prime}\right) \frac{E}{T_{i}^{2}} \exp \left(-E / T_{i}\right) \tag{19}
\end{equation*}
$$

with the nuclear temperature, $T_{i}$, being given by the prescription

$$
\begin{equation*}
T_{i}\left(E^{\prime}\right)=A_{i j} \sqrt{\frac{M_{i}}{E^{\prime}}} \tag{20}
\end{equation*}
$$

with $A_{i j}$ being characteristic of the particular nuclei and particle considered. This procedure has been adopted primarily because the tabulation of continuum type reaction data is simplified, without loss of accuracy, by the use of an analytical expression.

## D. Integration of the Scattering Kerne1

In Fig. I, we indicate the $\ell=0$ component of the scattering
kernel for a particular solution energy from energies for which solutions have been previously calculated. The contributions from various interactions are clearly indicated; the discontinuities are recognized by the code and for each cross section contribution an integration mesh, independent of the solution mesh, is determined. Typically, the integration over each individual contribution is done by using a prespecified
number of points. Provisions are made for allowing overlay of interactions with the resulting discontinuities. This procedure results in an integration scheme which treats each interval and discontinuity correctly, without resorting to smoothing a discontinuous function.
IV. ORGANIZATION AND DESCRIPTION OF MOMENT-I

## A. General Structure

MOMENT-I is written in such a way that it may be easily overlayed in order to maximize the amount of core available for data storage. In Fig. 2, the structure of the code is shown with each level suitable for overlaying and the subroutines associated with each level.

On CDC 3600 and UNIVAC 1107/1108 computers this overlay
structure can be used to expand the effective size of the available core. The following is a brief description of each subroutine.
B. Discussion of Neutron Subroutines

## MOMENT I

This routine calls in the main subroutines. Tapes are identified by numbers appropriate to the computer installation. The subroutine REMTIM, which prints out the CPU and input-output times elapsed, is called at several points. The following subroutines are called in a linear sequence:

INPUTA - reads in input data related to cross sections and energy grids.

INPUTB - reads in input data related to angular and linkage parameters.

SIGMAX - generates partial cross sections for material of
interest from input cross section data.

CMPUTN - computes moments for a given energy.
OUTNUT - processes moments for output on tape, cards and/or printer.

INPUT A
This subroutine reads in data related to cross section and energy grids. It reads the cross section tape (LIB) and selects appropriate materials. A description of the input data is given in Section V-A.

INPUT B
This subroutine reads in data describing the angular characteristics of the source. A description of these input data is given in Section V-A.

SIGMAX

This subroutine organizes the writing of scattering kernel information on the tape LIBCX. It calls NEIMSH AND NKERNL. NEIMSH

This subroutine determines the integration spacing for the scattering kerne1. Energies at which each partial interaction turns on and off are determined. This list is merged with a list of energies related to the solution mesh to produce a list of energies at which potential discontinuities in the integrand may occur. Between each of these energies an integration mesh is generated, determined by input parameters such as MAXN $\emptyset$ and MINN $\varnothing$. NKERNL

This subroutine calculates the contribution from each partial
interaction to the scattering kernel. Contributions for elastic scattering and inelastic scattering by discrete levels are calculated by calling NCRØSS. Inelastic scattering via the continuum is calculated by calling CøNTIN. The scattering kernel is written on tape LIBCX. GAUSSN is called to calculate the appropriate Gaussian integration parameters.

## NCRQSS

This subroutine calculates the scattering cross section for fixed initial and final energies and for a fixed excitation energy $Q$. CØNTIN

This subroutine calculates the inelastic scattering cross section for fixed initial and final energies according to the formula

$$
K\left(E^{\prime} \rightarrow E\right)=\Sigma\left(E^{\prime}\right) \frac{E}{T^{2}} e^{-E / T},
$$

where

$$
T=A \cdot \sqrt{M / E}
$$

and $A$ is a normalization constant. CMPUTN

This subroutine solves the moments equation, namely,

$$
\begin{aligned}
\Psi_{n \ell}(E)= & \int_{E}^{E} K \frac{\left(E^{\prime} \rightarrow E\right)}{\sum\left(E^{\prime}\right)} P_{\ell}(\cos \Theta) \Psi_{n \ell}\left(E^{\prime}\right) d E^{\prime}+S_{n \ell}(E) \\
& +\frac{\sum_{0}}{\sum(E)} \frac{1}{(2 \ell+1)}\left[(\ell+1) \Psi_{n-1, \ell+1}+\ell, \Psi_{n-1, \ell-1}\right]
\end{aligned}
$$

where

$$
\Psi_{n \ell}(E)=\Sigma(E) \Phi_{n \ell}(E)
$$

are moments of the collision density.

## V. INPUT INSTRUCTIONS AND DESCRIPTION OF INPUT AND OUTPUT DATA

A. Card Input

SUBROUTINE INPUTA
Card Type 1 (20A4)
TITLE Alphanumeric title for problem.

## Card Type 2 (16I5)

KEYIND. Index which allows part of calculation to be skipped.
KEYIND $=0$ Do entire calculation. KEYIND $=1$ Generate partial cross sections and store on LIBCX tape for later use . KEYIND $=2$ Compute moments using partial cross sections stored previously on tape LIBCX.

NW Number of elements in mixture.
LS Maximum number of Legendre harmonic coefficients calculated by code ( $\ell=0,1, \ldots$ LS-1).

MAXN $\varnothing$ The minimum number of intervals in the scattering kernel for any partial interaction

MINN $\varnothing$ The minimum number of intervals in the scattering kernel between the energy ranges of partial interactions.

NIS $\varnothing$ Number of isotopes on the cross section library tape.
NSPEC Number of energies at which source spectrum is tabulated. If NSPEC $=0$, a U-235 fission spectrum is used.

NPRINT Number of solution energies for which detailed printout of solutions is desired. (For diagnostic use).

IPX Print index. IPX $=0$, bypass input cross section printout; IPX $>0$, printout complete cross section input.

NED Number of energies at which solution grid changes (NED $\geq 2$ ). NSIG Number of energies at which grid of total cross sections tabulated in the program changes (NSIG $\geq 2$ ).

Card Type 3 (16I5) (Only if NPRINT $>0$ )
(INDXPR(I), $I=1$, NPRINT) Indices of solutions for which detailed printout is desired.

Card Type 4 (I10.2E10.4) (One card for each element in the problem).
IDENTP (I) Identification number for each element in mixture.
CONC(I) Concentration of each element in atoms/barn-cm.
W1 (I) Ratio of mass of each element to the neutron mass.
Card Type 5 (6E12.5) (Not required for NSPEC $=0$ )
(ESPEC(I), I = 1, NSPEC) Energies at which source spectrum is tabulated.
(SSPEC(I), I = 1, NSPEC) Number spectrum of source.
Card Type $6(4($ E10.0, I10))
ED(I) Energy at which grid of solution energies changes

$$
(E D(1)=E T \emptyset P)
$$

$N Q D(I) \quad$ Number of equally spaced lethargy intervals between $E D(I)$ and $\operatorname{ED}(I+1)$.

Card Type 7 (4(E10.0, I10))
ERATE(I) Energy at which grid of total cross section tabulation changes.

NREP(I) Number of intervals between ERATE(I) and ERATE(I+1). SUBROUTINE INPUTB

Card Type 8 (5I5)
NL $\emptyset$ Number of harmonic coefficients of the source angular distribution.

LZR $\varnothing$ Index of source harmonic when $N L \varnothing=1$; otherwise arbitrary.
MZR $\varnothing$ Index of azimuthal harmonic. MZR $\varnothing \neq 0$ only when neither source nor detector response is azimuthally symmetric.

NNL Number of ( $n, \ell$ ) combinations for which solutions are desired.
KP Input-Output index.

| KP | PRINT | PUNCH | TAPE |
| :---: | :--- | :--- | :--- |
| 0 | $\ell=0$ only | $\ell=0$ only | none |
| 1 | $\ell=0$ only | $\ell=0$ only | all $\ell$ |
| 2 | all $\ell$ | $\ell=0$ only | all $\ell$ |
| 3 | all $\ell$ | all $\ell$ | all $\ell$ |

Card Type $9(4 I 3,4(I 6,3 I 3))$
$N(I) \quad n$-index of $i^{\text {th }}$ moment.
$L(I) \quad \ell$-index of $i^{\text {th }}$ moment.
LINKH(I) i-index of higher- $\ell$ linkage moment.
LINKL(I) i-index of lower-l linkage moment.
Card Type 10 (5EI5.6)
(CNL(I), I = 1 , NL $\varnothing$ ) Harmonic coefficients of source angular distribution. The subroutine INPUTB then calculates the linkage coefficients and the $C_{n \ell}$ values.

For the CDC- 3600 and UNIVAC $1107 / 1108$ systems, LIB is tape unit 7 and for the IBM 360/75-91, LIB is tape unit 1 . LIB must be available for KEYIND < 2. The following is a description of the $B C D$ tape format for a particular isotope.

Record 1 (3X, 5I5, 13A4)
IDENT Isotope identification number (it corresponds to IDENTP in the INPUTA Data).

NX Number of partial cross sections.
NTO Number of energies for which the total cross section is tabulated.

NA Number of energies for which the absorption cross section is tabulated.

NFL Number of energies at which angular harmonics for which elastic scattering are tabulated.
(A1(I), $I=1,13)$ Alphanumeric data which describes the cross section. Record 2 ( $3 \mathrm{X}, 16 \mathrm{I} 5$ )
(NT(I), I=1,NX) Each entry indicates the number of energies for which each partial cross section is tabulated.

Record 3 (3X,5E15.7)
(Q1(I) , $I=1, N X) \quad Q$ value for the $I^{t h}$ partial cross section. For continuum type secondary distributions, the following prescription is used:

$$
\begin{aligned}
& \text { Continuum } \mathrm{Q}=100+\mathrm{A} \\
& (\mathrm{n}, 2 \mathrm{n}) \quad \mathrm{Q}=200+\mathrm{A}
\end{aligned}
$$

$$
\begin{aligned}
& \qquad\left(n, n^{\prime} p\right) \quad Q=300+A \\
& \left(n, N^{\prime} \alpha\right),\left(n, n^{\prime} 3 \alpha\right) \quad Q=400+A \\
& \text { Here } A \text { is the parameter in Eq. }
\end{aligned}
$$

Record 4 (3X,5E15.7)
(ELO(I), $I=1, N X)$ Lowest energy for cross section tabulation of the $I^{\text {th }}$ partial cross section.
(EHI(I), I=1,NX) Highest energy for cross section tabulation of the $I^{\text {th }}$ partial cross section.
(EISO(I), I=l,NX) Energy below which the $I^{\text {th }}$ interaction is isotropic.

In the following records, the energy lists must be tabulated in descending order.

Record 5 (3X,5E15.7)
(ET(I), I=1,NTO) Energy list for total cross section
Record 6 (3X,5E15.7)

```
(ST(I),I=l,NTO) Total cross section corresponding to each energy
ET(I).
```

Record 7 (3X,5E15.7)
(EZ(I), I=1,NA) Energy list for the absorption cross section
Record 8 (3X,5E15.7)
(SA(I), I=1,NA) Absorption cross section corresponding to each energy, EA(I).

Record 9 (3X,5E15.7)
(EEL(I), I=l,NT(I)) Energy list for elastic scattering.
Record $10(3 \mathrm{X}, 5 \mathrm{E} 15.7)$
(SEL(I), I=1,NT(1)) Elastic scattering cross section corresponding to each energy EEL(I).

```
(EFL(I),I=1,NFL) Energy list for elastic scattering harmonic
coefficient.
```


## Record 12 (3X,8F9.6)

(F1(I),F2(I),...,F8(I),I=1,NFL) Angular expansion coefficients, $f_{\ell}$, for each energy EFL(I).

Each partial cross section is now tabulated and requires $2^{*} N X-2$ records, one record for the energy list and another for the cross section list. The format is (3X,5E15.7).
( $\mathrm{E}(\mathrm{I}), \mathrm{I}=1, \mathrm{NT}(\mathrm{J})$ ) Energy list for the $\mathrm{J}^{\text {th }}$ partial cross section. (S(I),I=1,NT(J)) Cross section list for the $J^{\text {th }}$ partial cross section.

Following these records an end of isotope record must be written; it has the form
(3H99,75X) End of isotope record.

## C. Scattering Kernel Tape

For CDC-3600 and UNIVAC $1107 / 1108$ systems, LIBCX is tape unit 8 and for the IBM 360/75-91 it is unit 2. LIBCX must be available for all values of KEYIND. The tape is written in BINARY mode and with the exception of the first and second solution energies, has the following form:

For the $I^{\text {th }}$ solution energy, there are NEIS(I) records corresponding to each integration region calculated by EIMESH. The record is written in the form

WRITE (LIBCX) E(I), S(I), NINTS, (EI(J), WATES(J), (XKIN(J,M), $\mathrm{M}=1, \mathrm{~T} S), \mathrm{J}=1, \mathrm{NINTS}$ )
where $E(I)$ is the energy of the $I^{\text {th }}$ solution,
$S(I)$ is the total cross section at energy $E(I)$,
NINTS is the number of integration points,
$E I(J)$ is the integration energy,
WATES(J) is the Simpsons rule integration weights, and
$\operatorname{XKIN}(J, M)$ is the scattering kernel value at energy $\operatorname{EI}(J)$ and M-1 harmonic coefficient.

After all NEIS(I) integration records have been written on the tape, the Gaussian weights are written in the form:

WRITE (LIBCX) (A1 (J), A2(J), E2(J), J = I, LS)
The first energy solution does not require either of these records. The second solution energy requires only the Gaussian weights and, therefore, the first record on the tape is the Gaussian Weights for the second solution energy.
D. Moments Solution Tape

In all versions of MOMENT-I, LSAVE is tape unit 10. This tape must be available for $K P(i n p u t$ card 8 ) values greater than zero (0). The following describes the form of this BCD tape that would be written for a single problem.

Record 1 (20A4)
(TITLE (I), $\mathrm{I}=1,20$ )
Problem title
Record 2 (2I5)

NENS
NNL

Number of solution energies
Number of ( $n, \ell$ ) combinations

Record 3 (16I5)

$$
((N(I), L(I), I=1, N N L) \quad(n, \ell) \text { array for the NNL combinations }
$$

Record 4 (IP5E15.7)
(S(I), I=1,NENS) Total cross sections
Record 5 (IP5E15.7)
(E(I), Ix1,NENS) Solution energy list
Record 6 (IP5E15.7)
(SORS(I), I=1,NENS) Source strength at each energy
Record 7 (IP5E15.7)
(CNL (I) , $I=1, N N L) \quad C_{n l}$ for each ( $n, \ell$ ) combination
Record 8 thru NNL+7 (IP5E15.7)
(SOLN(I,J), I=I,NENS) The solutions for the $N(J), L(J)$ moment

## E. Discussion of Output Data

The data preceded by (IPX $>0$ ) are printed only when the input parameter IPX is equal to a positive integer. They are useful only when one wants to examine the details of the input cross section data.

The data preceded by (NPRINT $>0$ ) are printed only for those solutions for which detailed printout is requested. These solutions are chosen by means of the input parameters NPRINT and INDXPR. These data are useful for trying to find out why the code is not working properly.

Printout consists of the following blocks of data:
Input data from INPUTA subroutine.
Label for each element for which data is read from the LIB
cross section tape.
(IPX $>0$ ) List of energies and total microscopic cross sections read from LIB tape.

Information on partial cross sections.
Solution energies and interpolated macroscopic total cross sections.

Tabulated energies and total macroscopic cross sections.
(IPX $>0$ ) List of energies and macroscopic partial cross sections for each interaction.
(IPX $>0$ ) List of energies and Legendre coefficients $(\ell=1,8)$ for elastic scatter.

Input data from INPUTB subroutine.
List of linkage coefficients.
(NPRINT >0) Parameters generated in EIMESH subroutine.
(NPRINT $>0$ ) List of energies, weights and scattering kernels for the numerical integration term.
(NPRINT $>0$ ) Parameters generated in GAUSSN subroutine.
List of solution indices, number of partial interactions contributing to the scattering integral, indices of the previous solution, energies, and total cross sections. This list is printed as LIBCX tape is being written.

List similar to the previous list, written after LIBCX tape has been generated.
(NPRINT >0) Source terms.
(NPRINT >0) Numerical integration data.
(NPRINT >0) Gauss parameters.
(NPRINT >0) List of energies and solutions at the GAUSS energy before final interpolation on solutions.

List of indices and energies printed as each solution is generated.

Message indicating that an input energy parameter has been reached and that the energy mesh for solutions may change.

Title.
List of total flux moments for each energy and each ( $n, \ell$ ) combination. Plane source.

Title。
List of scattered flux moments. Plane source.
Last data card.

## VI. SAMPLE PROBLEM

This section contains an example of a typical printout for a sample problem. In this problem the parameters IPX and NPRINT were set equal to zero. The printout does not therefore include data obtained by setting these indices to some positive value.

The sample problem gives results for the penetration of neutrons from a photo-fission source in carbon. Results for this type of source have been discussed in References 4 and 8. Ordinarily, at least six spatial moments would be calculated for $\ell=0$ harmonic coefficients in order to construct spatial distributions of the flux. However, for purposes of illustration, only three spatial moments are included in the sample problem. A comparison of moments calculated for this problem with earlier results calculated by Weisbin ${ }^{(4)}$ is given in Reference 9.




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FIGURE 2. Over1ay Structure for MOMENT-I Code.


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| C. Eisenhauer, G. L. Simmons, and L. V. Spencer 8. |  |  |  |
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15. SUPPLEMENTARY NOTES
16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

In this paper we discuss a computer code for generating spatial-angular moments of neutron flux in an infinite medium. The equation for the flux moments is given and the techniques used for the solution are discussed. The structure of the computer code and of the main subroutines is also discussed. Details of the input and output data are given and the printout from a sample problem is included.
17. KEY WORDS (Alphabetical order, separated by semicolons)

Carbon; computer code; gauss quadrature; moments; neutron transport; shielding.
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