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Servo Techniques in Oscillators and Measurement Systems

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SERVO TECHNIQUES IN OSCILLATORS AND MEASUREMENT SYSTEMS

Fred L. Walls & S.R. Stein

Nearly every precision oscillator includes a frequency or phase servo system. In the case of cesium standards, a crystal oscillator is frequency locked to a particular resonance line in atomic cesium; in the case of an oscillating hydrogen maser, a crystal oscillator is phase locked to the very weak signal coming from the microwave cavity. The first section of this note treats the errors and offsets of frequency-lock loops which result from background, noise, and other effects. Third harmonic lock systems and square wave frequency modulation are analyzed as possible solutions to some of these problems. The second section is a general treatment of servo system response which is applicable to both frequency-and phase-lock loops. The effects of noise on such servo systems are discussed in detail, and an example is given of how to obtain optimum performance from a pair of phase-locked oscillators. A simple circuit is suggested for phase-locking high quality oscillators, which has many advantages over previous circuits.

Key words: Atomic frequency standard; frequency discrimination; frequency domain stability; frequency-lock loop; phase-lock loop; phase noise; servo techniques; time domain stability.

I. Basic Requirements

In order to lock our local oscillator, a number of requirements must be met. We must be able to detect a reference signal at the frequency of interest, or we must be able to detect the beat frequency between the reference signal and the local oscillator. The reference signal could be the n-th harmonic or subharmonic of a stable oscillator (including the fundamental n = 0) or it might be the absorption line of methane, CH_4, at 0.88×10^{14} Hz, etc.

Next, the local oscillator must be tunable. Usually the local oscillator is voltage tunable at least over the frequency range of interest and we will simpy call it a VCO, for voltage-controlled oscillator.

Next, some means for determining if the VCO is higher or lower in frequency than the reference, i.e., frequency discrimination, must be employed. This usually means frequency modulating the VCO or using some kind of phase detector.

The type of frequency discrimination chosen determines to a large extent the details of the amplifier and feedback-servo-system which actually controls the VCO. The servo system amplifies the difference signal from the frequency or phase discrimiator and processes it to create the appropriate correction voltage for the VCO in order to steer it close to the desired reference frequency. The precision and speed to which the VCO tracks the reference is determined to a large extent by the magnitude and frequency dependence of the servo gain. [1,2]

II. Types of Frequency Discrimination Commonly Used

A. Sine Wave Modulation

The most common way to tell if the frequency of a local oscillator is above or below a reference resonance line is to frequency modulate (FM) the local oscillator and detect the phase of the resulting amplitude modulated signal.*

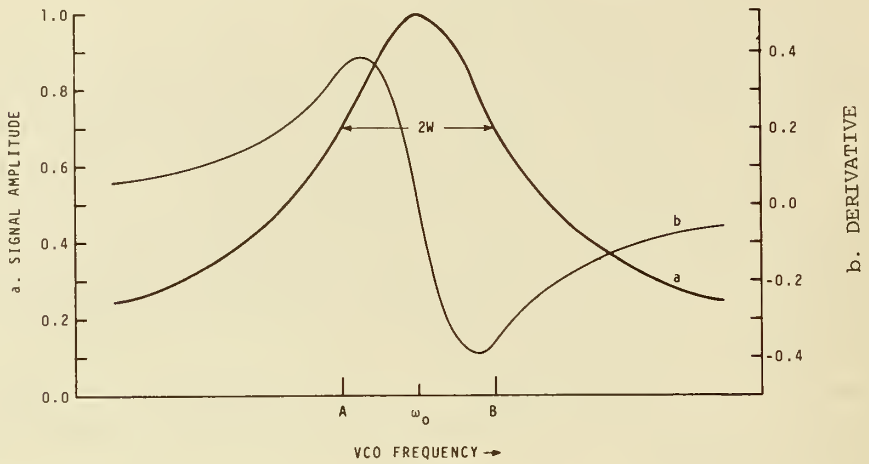

FIGURE 1 Reference line (a) and its first derivitive (b).

For the moment, let's assume that the width of the VCO output is totally symmetric and very narrow compared to the width of the reference line shown in figure 1, curve a. If the center of the VCO is at the point A, then the output signal obviously increases as the frequency of the VCO is increased, and at point B, the signal decreases as the VCO frequency increases. If the frequency of the VCO is swept back and forth, then the signal has both a dc and an ac component. If the deviation of the FM is small compared to the linewidth 2W, then the ac component of the signal, referenced to the phase of the FM as in a lock-in detector, fairly accurately reproduces the derivative of curve a as is illustrated by curve b in figure 1.

Note that in curve b, the point of zero signal which also

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* Passive frequency discriminators yielding directly an output vs. frequency similar to curve b in figure 1 exist for RF frequencies and are used in FM radio reception and other applications. Unfortunately, they usually do not have the high stabilities typically required of a frequency reference.

It is also possible to inject a low power reference signal into an oscillator and have it lock to the reference. The difficulty with this approach is that the parameters cannot be easily controlled or measured, and the stability of injection locked oscillators are typically much less than the reference. Unwanted injection locking can be a serious problem when comparing two oscillators if there is not sufficient isolation between them.

has the steepest slope nominally occurs at the center of the resonance line. This curve is referred to as a frequency discriminator curve.

This type of signal can then be used to steer the VCO because we now have a dc signal proportional to the frequency error and of opposite sign depending whether the VCO is higher or lower than the center of the resonance.

Now let's examine this process in a little more detail. Assume more generally that we have a Lorentzian* line being observed on a sloped and curved background. Then:

$$\text{Signal Amp} = \sqrt{\frac{\gamma^2}{\gamma^2 + (\omega - \omega_0)^2}} + k_1(\omega - \omega_0) + k_2(\omega - \omega_0)^2 .$$

Now let $\omega - \omega_1 = B \cos \Omega t$. Then

$$\text{Signal Amp} = \sqrt{\frac{1}{1 + \frac{1}{\gamma^2}[(\omega_1 - \omega_0) + B \cos \Omega t]^2}}$$

$$+ k_1[(\omega_1 - \omega_0) + B \cos \Omega t]$$

$$+ k_2[(\omega_1 - \omega_0) + B \cos \Omega t]^2$$

where $\gamma = 2\pi W$ relates to the linewidth, ω is the instantaneous angular frequency of the VCO, ω_1 is the average frequency of the VCO, and B is the amplitude of the FM which occurs at an angular frequency Ω. For simplicity, let the offset $\omega_1 - \omega_0$ be small compared to the sweep amplitude B, and let B be small compared to γ . Then using:

$$\frac{1}{\sqrt{1+\delta}} = 1 - \frac{\delta}{2} + \frac{3}{8}\delta^2 + \ldots$$

*By this we mean a resonance which can be accurately described by the first term in the following equation.

$$\text{Signal Amp} \doteq 1 - \frac{1}{2\gamma^2}[\omega_1 - \omega_0 + B\cos\Omega t]^2$$

$$+ k_1(\omega_1 - \omega_0 + B\cos\Omega t)$$

$$+ \frac{3}{8\gamma^4}[\omega_1 - \omega_0 + B\cos\Omega t]^4$$

$$+ k_2[\omega_1 - \omega_0 + B\cos\Omega t]^2 \cdot$$

$$\cdots$$

Expanding the brackets and grouping according to the terms Ω, 2Ω, etc., yields:

$$\dot{A} = 1 - \frac{1}{2\gamma^2}(\omega_1-\omega_0)^2 + k_1(\omega_1-\omega_0) + k_2(\omega_1-\omega_0)^2$$

$$+ \frac{(\omega_1-\omega_0)^4}{4\gamma^4} \quad : \text{ "0" coefficient}$$

$$-\frac{(\omega_1-\omega_0)B}{2\gamma^2}\left[2 - \frac{2\gamma^2 k_1}{(\omega_1-\omega_0)} - 4\gamma^2 k_2 \cdot - \frac{2(\omega_1-\omega_0)^2}{\gamma^2}\right.$$

$$\left. - \frac{3B(\omega_1-\omega_0)}{2\gamma^2} \cdots \right] \cos\Omega t\text{: 1st order coefficient}$$

$$-\frac{B^2}{2\gamma^2}\left[\frac{1}{2} - \frac{3(\omega_1-\omega_0)^2}{\gamma^2} - 2\gamma^2 k_2 \cdots\right] \cos 2\Omega t \text{ : 2nd order coefficient}$$

$$+\left[\frac{3B^3(\omega-\omega_0)}{2\gamma^4} \cdots\right] \cos 3\Omega t \text{ : 3rd order coefficient .}$$

Note that a non-flat background has sizable contributions to the 1st order coefficient. This means the VCO locks to the wrong frequency. The corresponding frequency shift is found from setting to zero the first order coefficient, using only the terms linear in frequency for a suitable approximation,

$$2 - \frac{2\gamma^2 k_1}{\omega_1 - \omega_0} - 4\gamma^2 k_2 \quad \cdots = 0$$

or

$$(\omega_1 - \omega_0) \doteq \frac{\gamma^2 k_1}{1 - 2\gamma^2 k_2},$$

which is essentially the ratio of the slope of the background to the slope of the derivative. This effect can be avoided to a high degree by using the coefficient of the 3Ω term, which discriminates against the sloping background through the quadratic term, k_2.

At this point, it should be mentioned that harmonic distortion will shift the average frequency of the VCO also. This distortion can be caused by either a harmonic content in the modulation itself or by asymmetries in the resonance line. In both cases, the magnitude of the shift can be explicitly found by using the formalism we developed above. For example, if the frequency modulation were:

$$B \cos \Omega t + k_3 \cos 2\Omega t,$$

then there would be an additional term in both the 1st and 3rd order coefficients. For the first order term, it is $-Bk_3/2\gamma^2$. The frequency shift in a 1st order lock, assuming no baseline problems, is $\omega_1 - \omega_0 = -k_3/2$. The shift is equal to half the amplitude of the second harmonic modulation. This is an extremely severe condition. Take the primary cesium beam standard, for example: to achieve an accuracy of 1 part in 10^{13}, one has to keep the second harmonic of the FM, k_3 (due to harmonic content in the modulation and/or line asymmetry), below 0.002 Hz while sweeping across the line width of 30 Hz.

There is another lesson here as well. The VCO has a finite amount of noise at frequencies equal to $\omega_1 \pm \Omega$, $\omega_1 \pm 2\Omega, \ldots,$ which will mimic FM at $\Omega, 2\Omega, \ldots.$ While this won't yield an offset <u>if</u> they are symmetric about the line center, the terms Ω and 2Ω will add to the noise in the detected signal of frequency Ω and hence lead to perturbation in the locked frequency of the VCO. These effects are treated to first order in appendix A.

B. <u>Squarewave Frequency Modulation</u>

If baseline problems are not present, squarewave modulation provides an attractive alternative to sine wave modulation. For example, the second harmonic problems do not exist. The experimental problems are now: (1) how to switch the frequency without any transients and (2) keeping the power constant. Both would, of course, shift the lock point of the VCO away from line center. The optimum way to process the signal now is through an integrator which is switched synchronously with the FM. If the reference level of the integrator is the average of the two signal levels, then this system is even independent of small differences in the time spent at the two frequencies. In fact, the line can be split to higher precision with square wave than with sine wave modulation. This is because the average signal power from sine wave modulation is $\overline{A^2\cos^2\Omega t} = \frac{1}{2}A^2$ while the average from square wave is $\overline{A^2} = A^2$. This factor of two increase in power translates to a 50% increase in the resolution for picking line center; i.e., into a corresponding increase in lock accuracy and frequency stability.

C. Direct Phase Comparisons

The technique of direct phase comparison is most commonly used to lock one oscillator to a reference signal derived from another oscillator although it can be used whenever one can directly detect the beat frequency between the reference signal and the VCO. Figure 2 shows a typical setup:

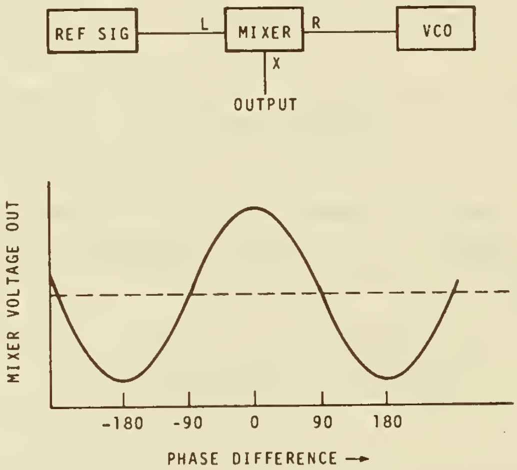

FIGURE 2 Top: A double balanced mixer used as a phase detector.
Bottom: Output voltage of phase detector vs. phase difference between the reference and the VCO.

The beat frequency is caused by the accumulation of phase $\Delta\omega t$ associated with the frequency difference $\Delta\omega$ between the two signals. If the mixer output is applied to the control of the VCO, it will cause the VCO to lock at $+90^\circ$ or -90° phase difference from the reference signal. If the phase difference is maintained constant, then the frequencies must be the same. The stability of the lock is determined by the noise at the mixer and the gain of the servo loop. This will be discussed at some length in section IV.

Part of the observed noise at the mixer is due to the mixer itself; i.e., changes in observed phase between the two signals due to thermal effects of the diode or diodes used. Amplitude changes in the signals change the observed phase difference also. For this reason, amplitude limiting is often used before the mixer.

III. Servo Systems

A. Basic Concepts

The function of the servo system is to process the error signal derived from the frequency discriminator and use it to control the frequency of the VCO. For simplicity, it will be assumed that the servo is a proportional continuous one, although digital and sampled servos are sometimes used. As long as the error signal is constantly monitored and processed, there is no theoretical reason that one kind is superior to another.

The function $G_{servo}(\omega)$ which specifies the gain around the loop as a function of angular frequency and $n(\omega)$, the servo noise as a function of frequency, fully specify the servo system. In order to determine the optimim values of $G_{servo}(\omega)$, one has to know the details of the phase shift around the loop from dc up to the desired unity gain point or $1/\tau_v$, where τ_v is the fastest correction time desired. We call it the attack time. The minimum gain at dc is set by the maximum phase error permitted and the size of the open loop phase shifts expected.

B. Example of a Phase Locked Loop

Problem: Design a servo system to lock a 5 MHz VCO to a reference oscillator with an attack time of 1 second (this is equivalent to a unity gain frequency of $1/2\pi$ Hz$\doteq$.16 Hz). Also calculate the maximum open loop frequency difference that can be tolerated between the two oscillators so that the residual phase error at the mixer doesn't exceed 10^o. The voltage control of the VCO causes a frequency change of 5×10^{-3} Hz/volt, and the phase sensitivity of the mixer is approximately .003 v per degree phase deviation from $\pm 90^o$. The solution is shown in figure 3a.

FIRST ORDER PHASE-LOCK LOOP

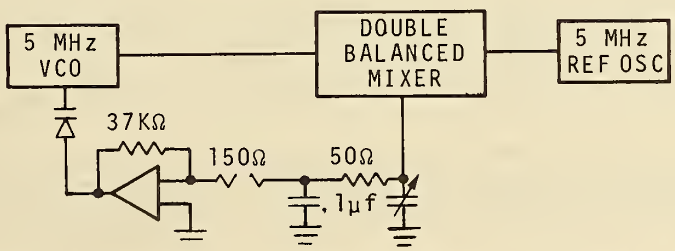

FIGURE 3a Example of a 5 MHz VCO phase-locked to a reference oscillator with a first order loop.

Ideally, the phase locked loop forces the phase difference between two oscillators to approach zero. The servo gain $G_{servo}(\omega)$ at variable time τ is:

$$G_{servo}(\omega) = \overbrace{(360)\ (5 \times 10^{-3}\ \text{Hz/volt})\ \tau}^{\phi \text{ in degrees}}\ \overbrace{(3 \times 10^{-3}/°)}^{\text{volts/degree}}\ G_{amp}(\omega)$$

where the frequency dependence of the amplifier is contained in $G_{amp}(\omega)$. Note that the phase detection yields a $G_{servo}(\omega)$ which falls off as 6 dB/octave even if the amplifier gain is independent of frequency. This means that the amplifier gain should fall off less than 6 dB/octave or the servo will be unstable. In fact, for the best stability, $G_{amp}(\omega)$ should fall between 0 and 3 dB/octave for angular frequencies near $1/\tau_v$. For simplicity, let $G_{amp}(\omega)$ be a constant. Then $G_{servo}(\omega) = 1$ at $\tau = 1$ second leads to an amplifier gain of

$$G_{amp}(\omega) = \frac{1}{(360)(5) \times 10^{-3} \qquad 3 \times 10^{-3}} = 185$$

The 50 Ω resistor and the 0.1 μf capacitor terminate the 5 MHz signals at the X-port. The additional 150 Ω resistor terminates the dc signal with 200 Ω. To achieve a gain of 185 requires a feedback resistor of 37 kΩ. The capacitance from input to output is typically of the order of 20 pf so the RC time of the amplifier is $RC = 3.7 \times 10^4\ (20 \times 10^{-12}) = 74 \times 10^{-8}$ seconds. Since this is smaller than 1 second, it won't change the gain slope at frequencies less than $\frac{1}{2\pi\tau}$ = .16 Hz. Note that the equivalent frequency error gain at dc is automatically very large due to the phase detection of frequency differences. The gain at 1 hour is 3.6×10^3. The dashed line in figure 3 b illustrates the function $G_{servo}(\omega)$.

The optional variable capacitor at the mixer output is adjusted to parallel resonate at the sum frequency. This typically increases the phase sensitivity near the zero crossings by a factor of 2.*

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*suggested by Charles Stone, Austron, Inc.

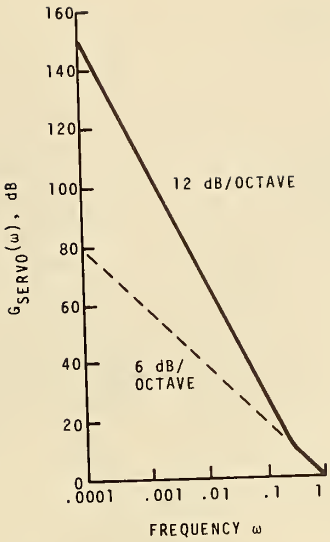

FIGURE 3b Servo gain, $G_{servo}(\omega)$, as a function of frequency for a first order (dashed line) and second order loop (solid line).

If the amplifier input or double-balanced mixer drifts 1 μV, the servo will force the VCO to drift $10^{-6}/0.003 = 0.00033$ degrees in phase relative to the reference oscillator. The maximum frequency offset that can be tolerated to keep the phase error at the mixer output less than 10° is

$$10° (3 \times 10^{-3} \text{ V}/_{o}) (185) (5 \times 10^{-3} \text{ Hz/volt}) = \text{frequency offset}$$

$$\Delta f_{max} = 2.5 \times 10^{-2} \text{ Hz or only 5 parts in } 10^{9} \text{ at 5 MHz.}$$

The frequency difference over which the phase lock loop in this example will stay locked is so small as to be very troublesome in many applications. In a phase measurement system, the phase error must be kept small ($\theta < 10°$) so that the phase sensitivity of the mixer is constant.

The following circuit increases the long term gain of the phase lock loop by several orders of magnitude, enabling one to reduce the phase error to nearly zero.

SECOND ORDER PHASE-LOCK LOOP

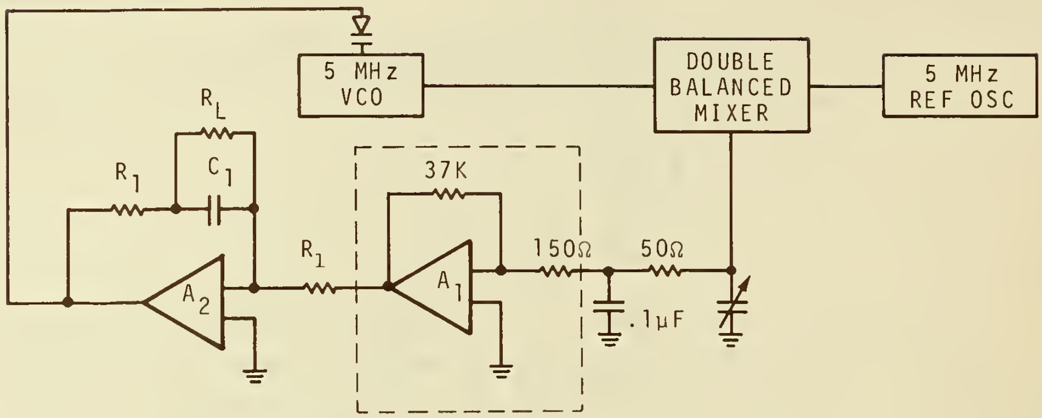

FIGURE 3c A 5 MHz VCO phase locked to a reference oscillator with a second order loop. Loop attack time is 1s.

The first stage is identical to figure 3a.

The second amplifier has unity gain for frequencies which are larger than $\frac{1}{R_1C_1}$. If R_1C_1 is set larger than the attack time of the loop then the loop is unconditionally stable. The best step response is obtained for R_1C_1 between 1 and 5 times the attack time τ_v. The loop is critically damped for $R_1C_1 \doteqdot 4\ \tau_v$. The inherent gain in long term is increased by the factor $\frac{R_L}{R_1}$ over that of figure 3a where R_L is the leakage resistance across C_1. The solid curve of figure 3b shows

$G_{servo}(\omega)$ for $\frac{RL}{R_1} = 10^4$. This increase in gain holds the phase error very close to zero and increases the frequency difference between the two oscillators that can be accommodated up to the limit imposed by the voltage swing of the second amplifier and/or the maximum tuning available in the oscillator. The solid line in figure 3b shows the total servo gain $G_{servo}(\omega)$ for figure 3c. The dashed curve of figure 3d shows the measured step response of the first order phase lock loop of figure 3a while the solid curve of figure 3d shows the measured step response of a second order loop similar to figure 3c with $R_1C_1 = \tau_v$.

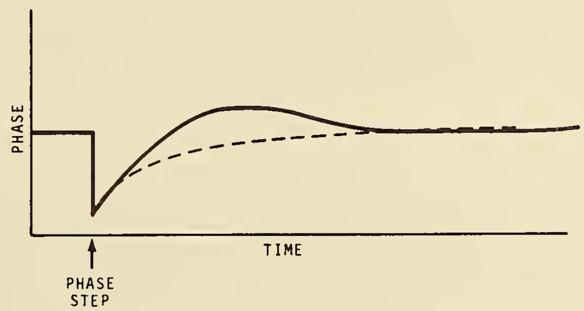

FIGURE 3d Response of circuit 3a to a step phase change (dashed line) and circuit 3c (solid line).

Second order loops such as illustrated by figure 3c can also be used to optimize frequency lock loops as well, and are very easily accomplished with present day operational amplifiers. The second order loop filter of figure 3c can be realized with a single operational amplifier. However, for clarity it was shown with two stages.

IV. Overall Estimates of Performance

We can now estimate the overall performance of the locked VCO. The frequency stability is calculated using the following formula for the spectral density:

$$S(f) = \left[\frac{G_{servo}(1/2\pi\tau)}{1 + G_{servo}(1/2\pi\tau)}\right]^2 \left[S(f, ref) + S(f, n) + \frac{S(f, VCO)}{G^2_{servo(1/2\pi\tau)}}\right],$$

where $G_{servo}(f = 1/2\pi\tau)$ is the measured servo gain, $S(f, ref)$ is the spectral density of phase fluctuations $S_\phi(f)$ for a phase lock loop or the spectral density of frequency fluctuations $S_y(f)$ for a frequency lock loop of the reference signal, $S(f, n)$ is the corresponding spectral density that would be required in the VCO to produce the servo system noise, and $S(f, VCO)$ is the corresponding spectral density of phase or frequency fluctuations of the unlocked VCO. $G_{servo}(1/2\pi\tau)$ is generally a monotonically increasing function of τ. This makes it possible to draw some general conclusions about the output spectrum of the servoed oscillator. The noise in the reference oscillator and in the control loop is low pass filtered, while the noise in the VCO is high pass filtered. This leads to the most common situation - the output spectrum is dominated by the reference oscillator at low Fourier frequencies and by the VCO at high Fourier frequencies. However, in the event that the stability of the VCO, $S(f, VCO)$, is much worse than the stability of the servo or reference, we see that $S(f)$ is $\sim[S(f, VCO]/[1 + G(1/2\pi\tau]^2$. If $G_{servo}(\frac{1}{2\pi\tau})$ has a maximum value of 10^4, for example, then the stability of the locked VCO can never be better than $S(f, VCO)/(10{,}001)^2$, even if the servo and reference are much more stable at the sample time τ.

This indicates that we want $G_{servo}(\omega)$ as large as possible. However, the maximum $G_{servo}(\omega)$ is fixed by the roll-off slope and the minimum value of τ_v which can be tolerated either by the short-term stability of the reference signal or by the additional phase shift in a first order servo loop at high frequencies which would cause the gain rolloff to exceed 9 dB/octave. If the rolloff exceeds 12 dB/octave at the unity gain point, the servo will oscillate.

The required shape of $G_{servo}(f)$ to reduce $S(f, VCO)$ below $S(f, ref)$ for Fourier frequencies below $1/2\pi\tau_v$ can be determined from the above equation. For example, to transform random walk of phase ($S_\phi \propto f^{-2}$) to white phase noise ($S_\phi \propto f^0$) requires a single integration (i.e. $|G_{servo}(f)| \propto \frac{1}{f}$. An analogous result can be derived for deterministic processes which can not be described in terms of spectral densities. For example, the unlocked VCO may exhibit frequency offset $[\phi(t) \propto t]$, frequency drift $[\phi(t) \propto t^2]$, or even frequency acceleration. If the parameter to be controlled has an open loop behavior proportional to t^p then the requirement for the closed loop system to have zero dc error is

$\lim_{f \to o} f^p G_{servo}(f) = \infty$. Thus if the oscillator in a first order phase lock loop has a frequency offset from the reference, then the closed loop system has a phase error. This problem is corrected by adding a second integration. Insufficient servo gain is even a greater problem in frequency lock systems since they lack the intrinsic integrating property of the phase-lock loop. The dc gain from a single stage servo is quite often insufficient to reduce the frequency drifts in the VCO to an acceptable level. The solution is then to use a second order loop to achieve the desired dc gain, being careful to keep the combined rolloff curves between 6 and 9 dB/octave near the unity gain point.

Figure 4 shows a typical result of insufficient dc gain in a frequency lock system.

Figure 4a shows the spectral density of phase fluctuations $S_\phi(f)$ of an oscillator typified by curve "VCO", which is locked to an oscillator typified by "ref". The servo gain is assumed to have a maximum value of 10^4 falling 6dB/octave and reaching a value of 1 at the indicated attack times.

Figure 4b shows the time domain stability resulting from the various attack times. Note that in this example the frequency stability for sample times less than 100 s is almost independent of the attack time. The noise bandwidth is assumed to be >> 1 MHz.

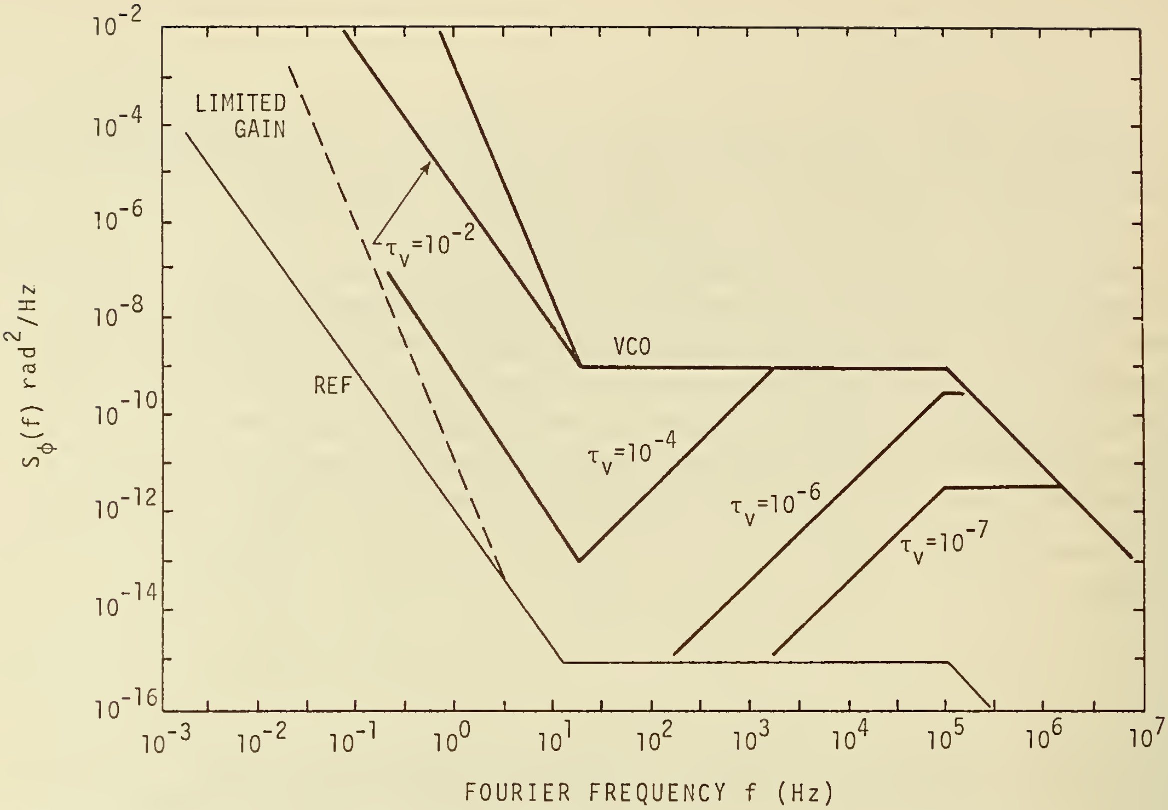

FIGURE 4a Spectral density of phase for a VCO locked to a reference with a first order loop as function of loop attack time. $G_s(\omega)$ assumed to be limited to 10^4.

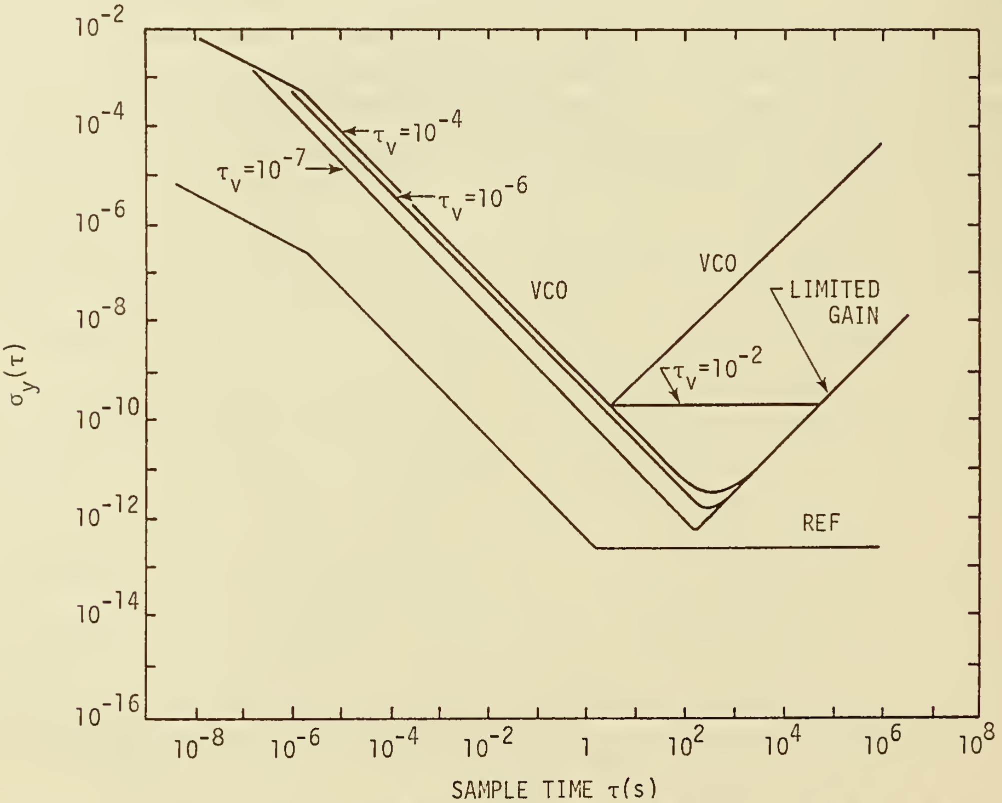

FIGURE 4b Corresponding time domain stability for the reference and VCO signals of Fig. 4a as a function of loop attack time. Noise bandwidth assumed to be > 1 MHz.

Figure 5a shows another example where the reference (curve b) has better stability than the VCO (curve a) only at low Fourier frequencies. Curve c shows the resulting phase spectral density when the VCO is locked to the reference with a second order loop with the gain specified by figure 5b. Figure 5c shows the resulting time domain stabilities. The optimum attack time can be found by minimizing the equation for S(f) as a function of τ_v. For this example, $.1ms<\tau_v<1ms$ produce nearly identical results. If τ_v is decreased below .1ms then the higher frequency phase noise in the reference begins to cause a deterioration in $\sigma_y(\tau>\tau_v)$ and for $\tau_v>1ms$ the low frequency components of the VCO are not reduced enough so that again $\sigma_y(\tau>1ms)$ is not as low as it could be.

There are two limitations as to how much stability improvement may be obtained. In the first place, if the noise of the control loop is larger than the reference noise, then the VCO spectrum will be degraded compared to the ultimate performance. For example, excessive white noise in the detector of a frequency lock loop will result in σ_y decreasing only as $\tau^{-1/2}$. In the second place, it is not possible for σ_y to roll off faster than τ^{-1} because it is the integral of the phase fluctuations up to some high frequency cut-off.

Thus, the zero crossings of the signal will always have some rms fluctuation, not less than the integral of the high frequency phase noise. Since the frequency is phase divided by measurement time, the uncertainty in the frequency decreases as τ^{-1}. The exact performance for a given phase spectrum may be computed from the formula:

$$\sigma^2_y(\tau) = \frac{1}{\tau^2}\frac{2}{\pi^2}\frac{1}{\nu^2}\int_0^{f_h} df S_\phi(f) \sin^4(\pi f\tau).$$

As a result of this characteristic, the long term stability may be degraded if the attack time of the servo is too slow.

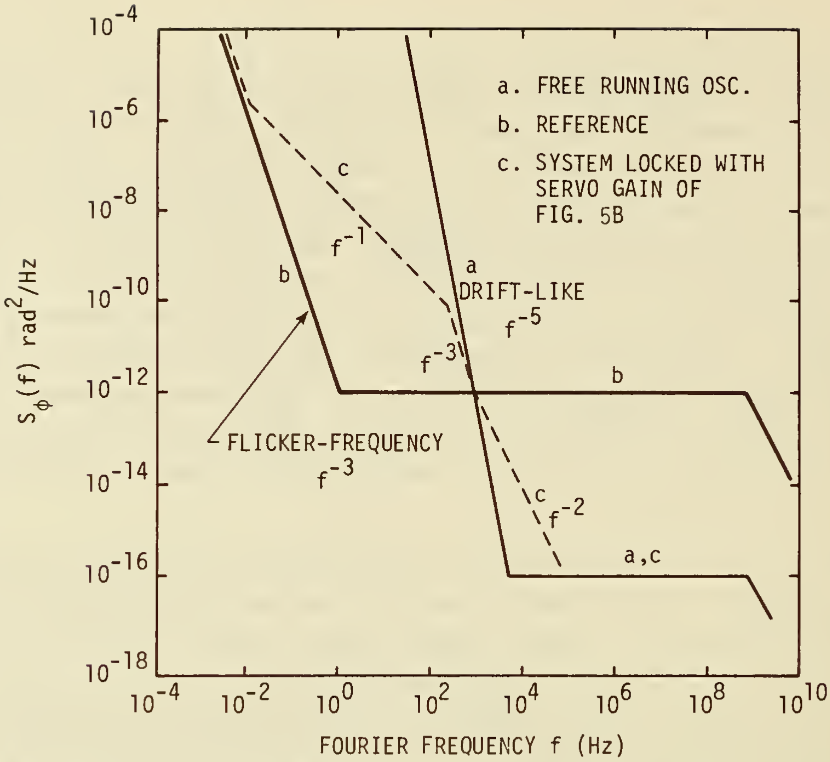

FIGURE 5a The dashed line shows the $S_\phi(f)$ which results from locking a free running oscillator typified by curve a to a reference typified by curve b, using the servo gain shown in Figure 5b.

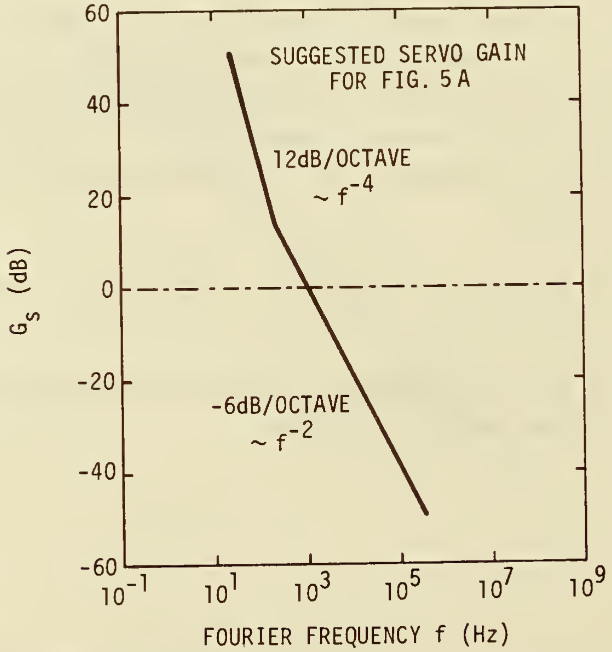

FIGURE 5b Suggested servo gain for Fig. 5a. This form of servo gain provides near optimal time domain performance for the system. See Fig. 5c.

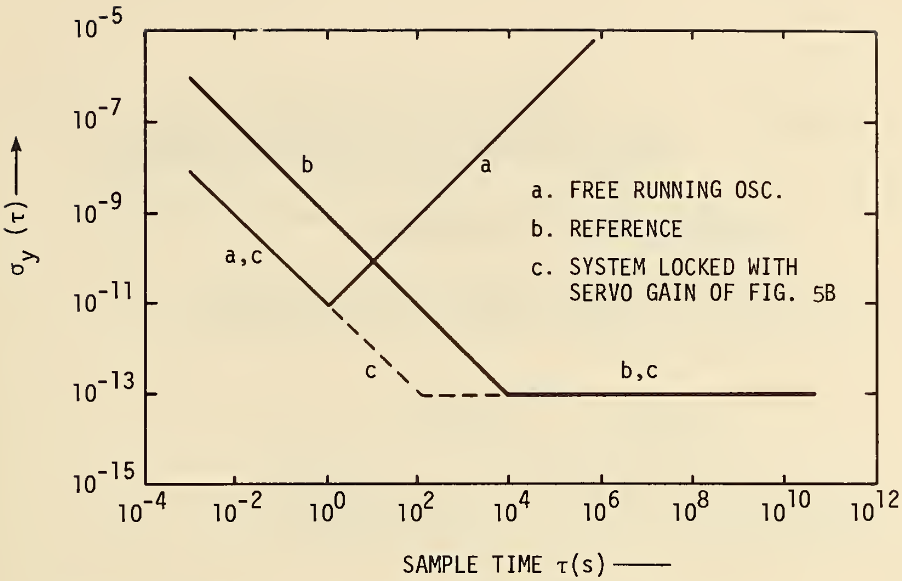

FIGURE 5c Time domain stability calculated from $S_\phi(f)$ shown in Fig. 5a. Note that the time domain stability of the system is much better than either component between approximately 0.5 and 10^4 seconds. Noise bandwidth $f_h = \infty$.

APPENDIX A: The Effect of Frequency Noise at Ω and 2Ω Away from the VCO Carrier on Stability

A. Noise at Fourier Frequency Ω

When the VCO is modulated at frequency Ω, then the detected 1st derivative signal is to first order given by:

$$\text{Sig Amp} = \frac{(\omega_1 - \omega_0)\ B}{\gamma^2}$$

The RMS amplitude of the frequency excursion of the VCO at Fourier frequency Ω is equal to

$$\omega_1 \sqrt{S_y(\Omega/2\pi)} = b\ ,$$

which we will set equal to b. $S_y(f)$ is the spectral density of frequency fluctuations. The effective FM amplitude is now given by:

$$B' = \sqrt{\left(B + \frac{b}{\sqrt{2}}\right)^2 + \left(\frac{b}{\sqrt{2}}\right)^2} \doteq B\left(1 + \frac{1}{\sqrt{2}}\ \frac{b}{B}\right)\ ,$$

where B is the initially applied FM amplitude and b/B is assumed to be less than 1.

The signal is now

$$\text{Sig Amp} = \frac{B}{\gamma^2}\ \left(\overline{(\omega_1 - \omega_0)} + \delta\omega_1(\tau_v)\right)\ \left(1 + \frac{1}{\sqrt{2}}\ \frac{b}{B}\right)\ ,$$

where $\overline{\omega_1 - \omega_0}$ is the average frequency offset, and $\delta\omega_1$ is the absolute frequency fluctuation at the servo attack time τ_v. We see that as long as

$$b = \omega_1\ \sqrt{S_y(\Omega/2\pi)}$$

is less than the amplitude of the applied FM that the precision in picking line center is not seriously impaired.

B. Noise at Fourier Frequency 2Ω

Neglecting everything but the extra contribution due to noise at a Fourier frequency 2Ω removed from the carrier, the detected 1st derivative signal is given by:

$$\text{Sig Amp} = \frac{1}{2\gamma^2}\left[2B(\omega_1 - \omega_0) + Bk_3\right]$$

$$= \frac{B}{2\gamma^2}\left(\overline{\omega_1 - \omega_0} + \delta\omega_1 + k_3\right),$$

where $k_3 = \omega_1 \sqrt{S_y(2\Omega/2\pi)}$.

We see that the requirement here is that k_3 must be small against $\delta\omega_1 = \omega_1\sigma_y(\tau_v)$ in order not to introduce any additional noise. To make this comparison requires a knowledge of the dependence of $S_y(f)$ on f; that is, the character of the noise. If $S_y(f)$ is proportional to f or f^2; i.e., if flicker of phase or white phase noise dominates at times of order τ_v to $1/2\Omega$, then the noise at Fourier frequencies 2Ω probably is the major term.

If $S_y(f)$ is proportional to f^{-2}, f^{-1}, or f^0, then the noise at 2Ω is probably a minor term. The exact values of course can be found by substituting in the measured values for $S_y(\Omega/\pi)$ and $\sigma_y(\tau_v)$.

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[2] Kesner, D., Take the Guesswork out of Phase Locked Loop Design, EDN, 5, pp. 54-60, January 1973.

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16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

Nearly every precision oscillator includes a frequency or phase servo system. In the case of cesium standards, a crystal oscillator is frequency locked to a particular resonance line in atomic cesium; in the case of an oscillating hydrogen maser, a crystal oscillator is phase locked to the very weak signal coming from the microwave cavity. The first section of this note treats the errors and offsets of frequency-lock loops which result from background, noise and other effects. Third harmonic lock systems and square wave frequency modulation are analyzed as possible solutions to some of these problems. The second section is a general treatment of servo system response which is applicable to both frequency and phase-lock loops. The effects of noise on such servo systems are discussed in detail, and an example is given of how to obtain optimum performance from a pair of phase-locked oscillators. A simple circuit is suggested for phase-locking high quality oscillators, which has many advantages over previous circuits.

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Atomic frequency standard; frequency discrimination; frequency domain stability; frequency-lock loop; phase-lock loop; phase noise; servo techniques; time domain stability.

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