NBS TECHNICAL NOTE 679

U.S. DEPARTMENT OF COMMERCE/ National Bureau of Standards

Frequency Domain Stability Measurements: A Tutorial Introduction

QC
100
.U5753
NO.679
1976
c.2

NATIONAL BUREAU OF STANDARDS

The National Bureau of Standards[1] was established by an act of Congress March 3, 1901. The Bureau's overall goal is to strengthen and advance the Nation's science and technology and facilitate their effective application for public benefit. To this end, the Bureau conducts research and provides: (1) a basis for the Nation's physical measurement system, (2) scientific and technological services for industry and government, (3) a technical basis for equity in trade, and (4) technical services to promote public safety. The Bureau consists of the Institute for Basic Standards, the Institute for Materials Research, the Institute for Applied Technology, the Institute for Computer Sciences and Technology, and the Office for Information Programs.

THE INSTITUTE FOR BASIC STANDARDS provides the central basis within the United States of a complete and consistent system of physical measurement; coordinates that system with measurement systems of other nations; and furnishes essential services leading to accurate and uniform physical measurements throughout the Nation's scientific community, industry, and commerce. The Institute consists of the Office of Measurement Services, the Office of Radiation Measurement and the following Center and divisions:

Applied Mathematics — Electricity — Mechanics — Heat — Optical Physics — Center for Radiation Research: Nuclear Sciences; Applied Radiation — Laboratory Astrophysics[2] — Cryogenics[2] — Electromagnetics[2] — Time and Frequency[2].

THE INSTITUTE FOR MATERIALS RESEARCH conducts materials research leading to improved methods of measurement, standards, and data on the properties of well-characterized materials needed by industry, commerce, educational institutions, and Government; provides advisory and research services to other Government agencies; and develops, produces, and distributes standard reference materials. The Institute consists of the Office of Standard Reference Materials, the Office of Air and Water Measurement, and the following divisions:

Analytical Chemistry — Polymers — Metallurgy — Inorganic Materials — Reactor Radiation — Physical Chemistry.

THE INSTITUTE FOR APPLIED TECHNOLOGY provides technical services to promote the use of available technology and to facilitate technological innovation in industry and Government; cooperates with public and private organizations leading to the development of technological standards (including mandatory safety standards), codes and methods of test; and provides technical advice and services to Government agencies upon request. The Institute consists of the following divisions and Centers:

Standards Application and Analysis — Electronic Technology — Center for Consumer Product Technology: Product Systems Analysis; Product Engineering — Center for Building Technology: Structures, Materials, and Life Safety; Building Environment; Technical Evaluation and Application — Center for Fire Research: Fire Science; Fire Safety Engineering.

THE INSTITUTE FOR COMPUTER SCIENCES AND TECHNOLOGY conducts research and provides technical services designed to aid Government agencies in improving cost effectiveness in the conduct of their programs through the selection, acquisition, and effective utilization of automatic data processing equipment; and serves as the principal focus within the executive branch for the development of Federal standards for automatic data processing equipment, techniques, and computer languages. The Institute consists of the following divisions:

Computer Services — Systems and Software — Computer Systems Engineering — Information Technology.

THE OFFICE FOR INFORMATION PROGRAMS promotes optimum dissemination and accessibility of scientific information generated within NBS and other agencies of the Federal Government; promotes the development of the National Standard Reference Data System and a system of information analysis centers dealing with the broader aspects of the National Measurement System; provides appropriate services to ensure that the NBS staff has optimum accessibility to the scientific information of the world. The Office consists of the following organizational units:

Office of Standard Reference Data — Office of Information Activities — Office of Technical Publications — Library — Office of International Relations — Office of International Standards.

---

[1] Headquarters and Laboratories at Gaithersburg, Maryland, unless otherwise noted; mailing address Washington, D.C. 20234.

[2] Located at Boulder, Colorado 80302.

NATIONAL BUREAU
OF STANDARDS
LIBRARY
MAY 12 1976
Not acc
QC100
.U5753
no.679
1976
c.2

Frequency Domain Stability Measurements: A Tutorial Introduction

Technical note, no. 679

David A. Howe

Time and Frequency Division
Institute for Basic Standards
National Bureau of Standards
Boulder, Colorado 80302

U.S. DEPARTMENT OF COMMERCE, Elliot L. Richardson, Secretary
James A. Baker, III, Under Secretary
Dr. Betsy Ancker-Johnson, Assistant Secretary for Science and Technology

NATIONAL BUREAU OF STANDARDS, Ernest Ambler, Acting Director

Issued March 1976

NATIONAL BUREAU OF STANDARDS TECHNICAL NOTE 679
Nat. Bur. Stand. (U.S.), Tech Note 679, 27 pages (Mar. 1976)

CODEN: NBTNAE

For sale by the Superintendent of Documents, U.S. Government Printing Office , Washington, D.C. 20402
(Order by SD Catalog No. C13.46:679) $0.65

FREQUENCY DOMAIN STABILITY MEASUREMENTS: A TUTORIAL INTRODUCTION

David A. Howe

This report introduces the concept of stability measurements of oscillators by spectral analysis. Development of topics does not rely heavily on mathematics. The equipment and set-up for stability measurements in the frequency domain are outlined. Examples and typical results are presented. Physical interpretations of common noise processes are discussed. The last section provides a table by which typical frequency domain stability characteristics may be translated to time domain stability characteristics.

Key Words: Fractional frequency fluctuations; frequency stability; phase fluctuations; power law noise processes; spectral density; spectrum analysis.

I. The Sine Function and Noise

A sine wave signal generator produces a voltage that changes in time in a sinusoidal way as shown in fig. 1. The signal is an oscillating signal because the sine wave repeats itself. A cycle of the oscillation is produced in one period "T". The phase is the angle "Φ" within a cycle corresponding to a particular time "t". Therefore, we have $\frac{\Phi(t)}{2\pi} = \frac{t}{T}$.

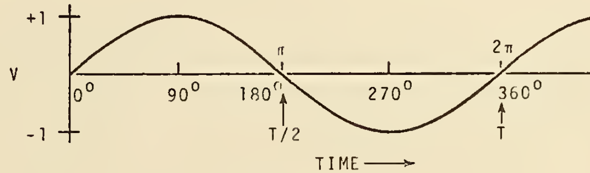


FIGURE 1

It is convenient for us to express angles in radians rather than in units of degrees, and positive zero-crossings will occur at even mulitiples of π-radians. The frequency "ν" is the number of cycles in one second, which is the reciprocal of period (seconds per cycle). The expression describing the voltage "V" out of a sine wave signal generator is given by $V(t) = V_p \sin[\Phi(t)]$ where V_p is the peak voltage amplitude. Equivalent expressions are

$$V(t) = V_p \sin\left(2\pi \frac{t}{T}\right)$$

and

$$V(t) = V_p \sin(2\pi\nu t). \quad (1)$$

Consider figure 2. Let's assume that the maximum value of "V" equals 1, hence "V_p" = 1. We say that the voltage "V(t)" is normalized to unity. If we know the frequency of a signal and the signal is a sine wave, then we can determine the incremental change in the period "T" (denoted by Δt) at a particular angle of phase.

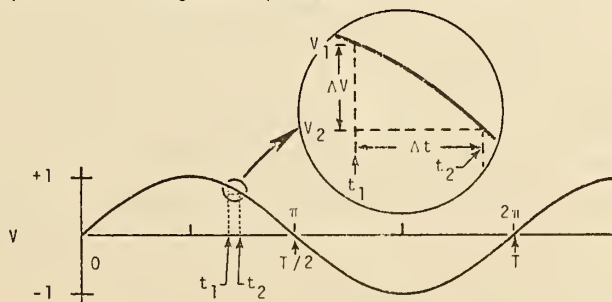


FIGURE 2

Note that no matter how big or small Δt may be, we can determine ΔV. Let us look at this from another point of view. Suppose we can measure ΔV for a particular Δt. From this there is a sine wave at a unique minimum frequency corresponding to the given ΔV and Δt. For small Δt, this frequency is called the <u>instantaneous frequency</u> at time t. The smaller the interval Δt, the better the approximation of instantaneous frequency at t.

When we speak of oscillators and the signals they produce, we recognize that an oscillator has some nominal frequency at which it operates. The "frequency stability" of an oscillator is a term used to characterize the frequency fluctuations of the oscillator signal. There is no formal definition for "frequency stability". However, one usually refers to frequency stability when comparing one oscillator with another. As we shall see later, we can define particular aspects of an oscillator's output then draw conclusions about its relative frestability stability. In general terms,

> "Frequency stability is the degree to which an oscillating signal produces the same value of frequency for any interval, Δt, throughout a specified period of time".

Let's examine the two waveforms shown in figure 3. Frequency stability depends on the amount of time involved in a measurement. Of the two oscillating signals, it is evident that "2" is more stable than "1" from time t_1 to t_3 assuming the horizontal scales are linear in time.

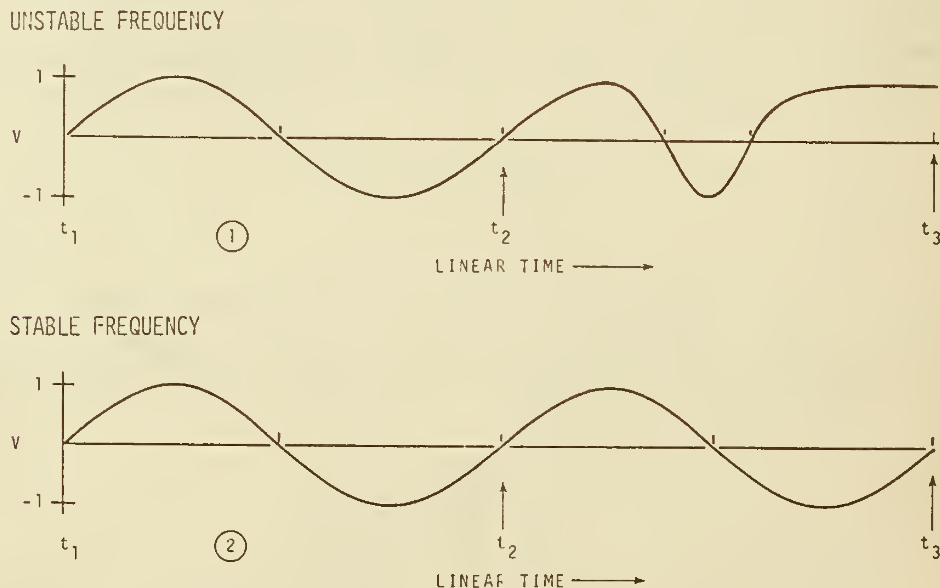


FIGURE 3

From time t_1 to t_2, there may be some question as to which of the two signals is more stable, but it's clear that from time t_2 to t_3, signal "1" is at a different frequency from that in interval t_1 to t_2.

If we want an oscillator to produce a particular frequency ν_0, then we're correct in stating that if the oscillator signal frequency deviates from ν_0 over any interval, this is a result of something which is undesirable. In the design of an oscillator, it is important to consider the sources of mechanisms which degrade the oscillator's frequency stability. All undesirable mechanisms cause noise or a noise process to exist along with the sine wave signal of the oscillator. Simply stated, noise is anything which is undesirable. To account for the noise components at the output of a sine wave signal generator, we can modify equation (1) and express the output as

$$V(t) = [V_0 + \varepsilon(t)] \sin [2\pi\nu_0 t + \phi(t)], \tag{2}$$

where V_0 ≡ nominal peak voltage amplitude,

$\varepsilon(t)$ ≡ deviation of amplitude from nominal,

ν_0 ≡ nominal fundamental frequency,

$\phi(t)$ ≡ deviation of phase from nominal.

Ideally "ε" and "ϕ" should equal zero for all time. However, in the real world there are no perfect oscillators. To determine the extent of the noise components "ε" and "ϕ", we shall turn our attention to measurement techniques.

II. Phase Spectral Density

One method of characterizing noise is by means of spectrum analysis. Let's examine the waveform shown in figure 4.

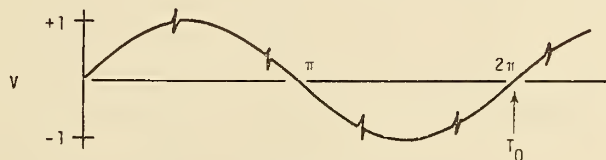


FIGURE 4

Here we have a sine wave which is perturbed for short instances by noise. Some loosely refer to these types of noises as "glitches". The waveform has a nominal frequency over one cycle which we'll call "ν_0"($\nu_0 = \frac{1}{T}$). At times, noise causes the instantaneous frequency to differ markedly from the nominal frequency. If a pure sine wave signal of frequency ν_0 is subtracted from this waveform, the remainder is the sum of the noise components. These components are of a variety of frequencies and the sum of their amplitudes is nearly zero except for the intervals during each glitch when their amplitudes momentarily reinforce each other. This is shown graphically in figure 5.

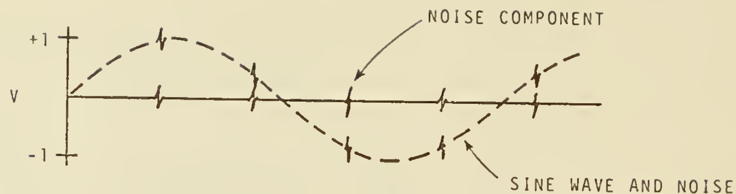


FIGURE 5

One can plot a graph showing rms power vs. frequency for a given signal. This kind of plot is called the <u>power spectrum</u>. For the waveform of figure 5, the power spectrum will have a high value at ν_0 and will have lower values for the signals produced by the glitches. Closer analysis reveals that there is a recognizable, somewhat constant repetition rate associated with the glitches. In fact, we can deduce that there is a significant amount of power in another signal whose period is the period of the glitches as shown in figure 5. Let's call the frequency of the glitches ν_s. Since this is the case, we will observe a noticeable amount of power in the spectrum at ν_s with an amplitude which is related to the characteristics of the glitches. The power spectrum shown in figure 6 has this feature. A predominant fundamental component has been depicted, and other harmonics also exist.

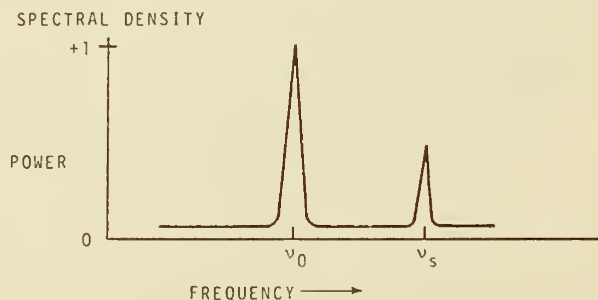


FIGURE 6

Some noise will cause the instantaneous frequency to "jitter" around ν_0, with probability of being higher or lower than ν_0. We thus usually find a "pedestal" associated with ν_0 as shown in figure 7.

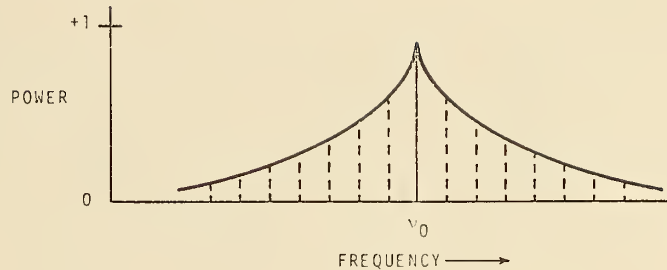


FIGURE 7

The process of breaking down a signal into all of its various components of frequency is called Fourier expansion. In other words, the addition of all the frequency components, called Fourier frequency components, produces the original signal. The value of a Fourier frequency is the difference between the frequency component and the fundamental frequency. The power spectrum can be normalized to unity such that the total area under the curve equals one. The power spectrum normalized in this way is the power spectral density.

The power spectrum, often called the RF spectrum, of V(t) is very useful in many applications. Unfortunately, if one is given the RF spectrum, it is impossible to determine whether the power at different Fourier frequencies is a result of amplitude fluctuations "$\varepsilon(t)$" or phase fluctuations "$\phi(t)$. The RF spectrum can be separated into two independent spectra, one being the spectral density of "$\varepsilon(t)$" often called the AM power spectral density and the other being the spectral density of "$\phi(t)$".

For our purposes, the phase-fluctuation components are the ones of interest. The spectral density of phase fluctuations is denoted by $S_\phi(f)$ where "f" is Fourier frequency. For the frequently encountered case where the AM power spectral density is negligibly small and the total modulation of the phase fluctuations is small (mean-square value is much less than one rad^2), the RF spectrum has approximately the same shape as the phase spectral density. However, a main difference in the representation is that the RF spectrum includes the fundamental signal (carrier), and the phase spectral density does not. Another major difference is that the RF spectrum is a power spectral density and is measured in units of watts/hertz. The phase spectral density involves no "power" measurement of the electrical signal. The units are radians^2/hertz. It is tempting to think of $S_\phi(f)$ as a "power" spectral density because in practice it is measured by passing V(t) through a phase detector and measuring the detector's output power spectrum. The measurement technique makes use of the relation that for small deviations ($\delta\phi \ll 1$ radian),

$$S_\phi(f) = \left(\frac{V_{rms}(f)}{V_s}\right)^2 \qquad (3)$$

where V_{rms} (f) is the root-mean-square noise voltage per $\sqrt{Hz}$ at a Fourier frequency "f", and V_s is the sensitivity (volts per radian) at the phase quadrature output of a phase detector which is comparing the two oscillators. In the next section, we will look at a scheme for directly measuring $S_\phi(f)$.

One question we might ask is, "How do frequency changes relate to phase fluctuations?" After all it's the frequency stability of an oscillator that is a major consideration in many applications. The frequency is equal to a rate of change in the phase of a sine wave. This tells us that fluctuations in an oscillator's output frequency are related to phase fluctuations since we must change the rate of "$\phi(t)$" to accomplish a shift in "$\nu(t)$", the frequency at time t. A rate of change of total "$\Phi_T(t)$" is denoted by "$\dot{\Phi}_T(t)$". We have then

$$2\pi\nu(t) = \dot{\Phi}_T(t) \tag{4}$$

The dot denotes the mathematical operation of differentiation on the function Φ_T with respect to its independent variable t.* From equation (4) and equation (2) we get

$$2\pi\nu(t) = \dot{\Phi}_T(t) = 2\pi\nu_0 + \dot{\phi}(t)$$

Rearranging, we have $$2\pi\nu(t) - 2\pi\nu_0 = \dot{\phi}(t)$$

or $$\nu(t) - \nu_0 = \frac{\dot{\phi}(t)}{2\pi} \tag{5}$$

The quantity $\nu(t) - \nu_0$ can be more conveniently denoted as $\delta\nu(t)$, a change in frequency at time t. Equation (5) tells us that if we differentiate the phase fluctuations $\phi(t)$ and divide by 2π, we will have calculated the frequency fluctuation $\delta\nu(t)$. Rather than specifying a frequency fluctuation in terms of a shift in frequency, it is useful to denote $\delta\nu(t)$ with respect to the nominal frequency ν_0. The quantity $\frac{\delta\nu(t)}{\nu_0}$ is called the fractional frequency fluctuation** at time t and is signified by the variable y(t). We have

$$y(t) = \frac{\delta\nu(t)}{\nu_0} = \frac{\dot{\phi}(t)}{2\pi\nu_0} \tag{6}$$

---

* As an analogy, the same operation relates the velocity of an object with its acceleration. The acceleration is equal to the rate of change of velocity.

** Some international recommendations replace "fractional" by "normalized".

The fractional frequency fluctuation $y(t)$ is a dimensionless quantity. When talking about frequency stability, its appropriateness becomes clearer if we consider the following example. Suppose in two oscillators $\delta\nu(t)$ is consistently equal to + 1 Hz and we have sampled this value for many times t. Are the two oscillators equal in their ability to produce their desired output frequencies? Not if one oscillator is operating at 10 Hz and the other at 10 MHz. In one case, the average value of the fractional frequency fluctuation is 1/10, and in the second case is 1/10,000,000 or 1×10^{-7}. The 10 MHz oscillator is then more precise. If frequencies are multiplied or divided using ideal electronics, the fractional stability is not changed.

In the frequency domain, we can measure the spectrum of frequency fluctuations $y(t)$. The spectral density of frequency fluctuations is denoted by $S_y(f)$ and is obtained by passing the signal from an oscillator through an ideal FM detector and performing spectral analysis on the resultant output voltage. $S_y(f)$ has dimensions of (fractional frequency)^2/Hz or Hz^{-1}. Differentiation of $\phi(t)$ corresponds to multiplication by $\frac{f}{\nu_0}$ in terms of spectral densities. With further calculation, one can derive that

$$S_y(f) = \left(\frac{f}{\nu_0}\right)^2 S_\phi(f) \tag{7}$$

We will address ourselves primarily to $S_\phi(f)$, that is, the spectral density of phase fluctuations. For noise-measurement purposes, $S_\phi(f)$ can be measured with a straightforward, easily duplicated equipment set-up. Whether one measures phase or frequency spectral densities is of minor importance since they bear a direct relationship. It is important, however, to make the distinction and to use equation (7) if necessary.

III. The Phase-Locked Loop

Suppose we have a noisy oscillator. We wish to measure the oscillator's phase fluctuations relative to nominal phase. One can do this by phaselocking another oscillator (called the reference oscillator) to the test oscillator and mixing the two oscillator signals 90° out of phase (phase quadrature). This is shown schematically in figure 9. The two oscillators are at the same frequency in long term as guaranteed by the phase-lock loop (PLL). A low-pass filter is used after the mixer since the difference signal is the one of interest. By holding the two signals at a relative phase difference of 90°, short-term phase fluctuations between the test and reference oscillators will appear as voltage fluctuations out of the mixer.

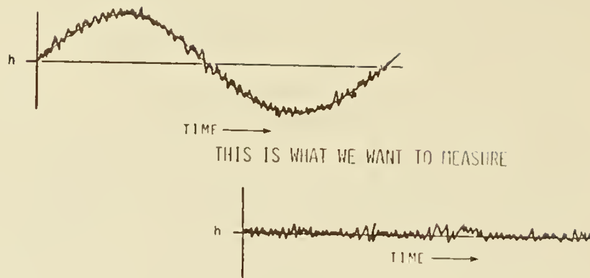


FIGURE 8

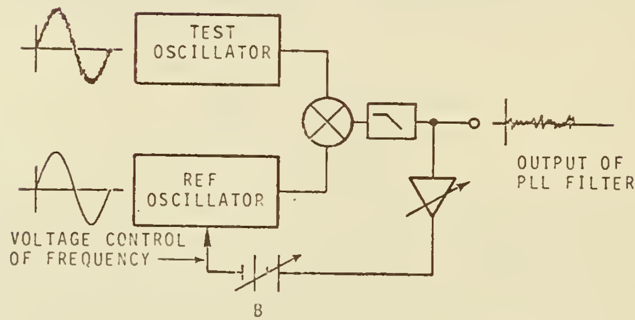


FIGURE 9

With a PLL, if we can make the servo time constant very long, then the PLL bandwidth as a filter will be small. This may be done by lowering the gain A_v of the loop amplifier. We want to translate the phase modulation spectrum to baseband spectrum so that it is easily measured on a low frequency spectrum analyzer. With a PLL filter, we must keep in mind that the reference oscillator should be as good or better than the test oscillator. This is because the output of the PLL represents the noise from both oscillators, and if not properly chosen, the reference can have noise masking the noise from the test oscillator. Often, the reference and test oscillators are of the same type and have, therefore, approximately the same noise. We can acquire a meaningful measurement by noting that the noise we measure is from two oscillators. Many times a good approximation is to assume that the noise power is twice that which is associated with one oscillator. $S_\phi(f)$ is general notation depicting spectral density on a per hertz basis. A PLL filter output necessarily yields noise from two oscillators.

The output of the PLL filter at Fourier frequencies above the loop bandwidth is a voltage representing phase fluctuations between reference and test oscillator. It is necessary to make the time-constant of the loop long compared with the inverse of the lowest Fourier frequency we wish to measure. That is, $\tau_c > \frac{1}{2\pi\ f(\text{lowest})}$. This means that if we want to

measure $S_\phi(f)$ down to 1 Hz, the loop time-constant must be greater than $\frac{1}{2\pi}$ seconds. One can measure the time-constant by perturbing the loop (momentarily disconnecting the battery is convenient) and noting the time it takes for the control voltage to reach 70% of its final value. The signal from the mixer can then be inserted into a spectrum analyzer. A preamp may be necessary before the spectrum analyzer. The analyzer determines the mean square volts that pass through the analyzer's bandwidth centered around a pre-chosen Fourier frequency f. It is desirable to normalize results to a 1 Hz bandwidth.Assuming white phase noise (white ϕM), this can be done by dividing the mean square voltage by the analyzer bandwidth in Hz. One may have to approximate for other noise processes. (The phase noise sideband levels will usually be indicated in rms volts-per-root-Hertz on most analyzers.)

IV. Equipment for Frequency Domain Stability Measurements

(1) Low-noise mixer

This should be a high quality, double-balanced type, but single-ended types may be used. The oscillators should have well-buffered outputs to be able to isolate the coupling between the two input RF ports of the mixer. Results that are too good may be obtained if the two oscillators couple tightly via signal injection through the input ports. We want the PLL to control locking. One should read the specifications in order to prevent exceeding the maximum allowable input power to the mixer. It is best to operate near the maximum for best signal-to-noise out of the IF port of the mixer and, in some cases, it is possible to drive the mixer into saturation without burning out the device.

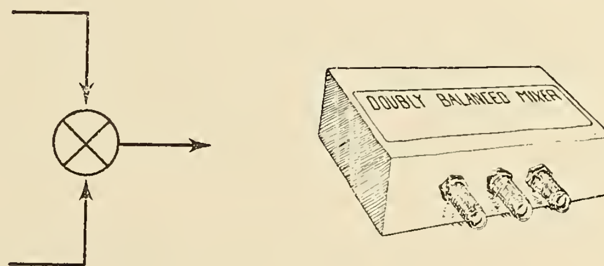


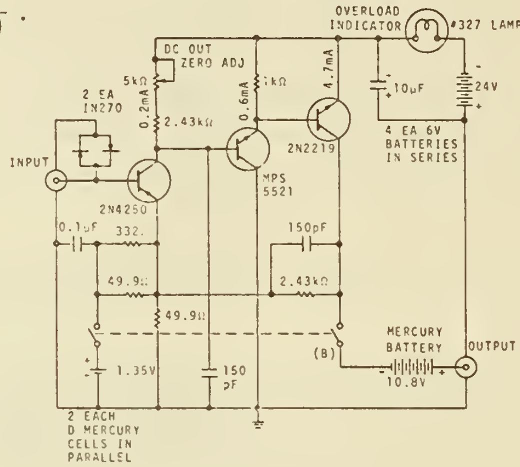
FIGURE 10

(2) Low-noise DC amplifier

The amount of gain A_V needed in the loop amplifier will depend on the amplitude of the mixer output and the degree of varactor control in the reference oscillator. We may need only a small amount of gain to acquire lock. On the other hand, it may be necessary to add as much as 80 dB of gain. Good low-noise DC amplifiers are available from a number of sources, and with cascading stages of amplification, each contributing noise, it will be the noise of the first stage which will add most significantly to the noise being measured. If a suitable low-noise first-stage amplifier is not readily available, a schematic of an amplifier with 40 dB of gain is shown

in figure 11 which will serve nicely for the first stage. The response of the amplifier should be flat from DC to the highest Fourier frequency one wishes to measure. The loop time-constant is inversely related to the gain A_v and the determination of A_v is best made by experimentation knowing that

$$\tau_c < \frac{1}{2\pi f\ (\text{lowest})} .$$



LOW NOISE AMPLIFIER

FIGURE 11

(3) Voltage-controlled reference quartz oscillator

This oscillator should be a good one with specifications available on its frequency domain stability. The reference should be no worse than the test oscillator. The varactor control should be sufficient to maintain phase-lock of the reference. In general, low quality test oscillators may have varactor control of as much as 1×10^{-6} fractional frequency change per volt. High quality test oscillators may have only 1×10^{-10} fractional frequency change per volt. Some provision should be available on the reference oscillator for tuning the mean frequency over a frequency range that will enable phase-lock. Many factors enter into the choice of the reference oscillator, and often it is convenient to simply use two test oscillators phase-locked together. In this way, one can assume that the noise out of the PLL filter is no worse than 3 dB greater then the noise from each oscillator. If it is uncertain that both oscillators are contributing approximately equal noise, then one should perform measurements on three oscillators taking two at a time. The noisier-than-average oscillator will reveal itself.

(4) Spectrum analyzer

The signal analyzer typically should be capable of measuring the noise in rms volts in a narrow bandwidth from near 1 Hz to the highest Fourier frequency of interest. This may be 50 kHz for carrier frequencies of 10 MHz or lower. For voltage measuring analyzers, it is typical to use units of "volts per Hz". The spectrum analyzer and any associated input amplifier will exhibit high-frequency rolloff. The Fourier frequency at which the voltage has dropped by 3 dB is the measurement system bandwidth f_h, or $\omega_h = 2\pi f_h$. This can be measured directly with a variable signal generator.

Rather than measure the spectral density of phase fluctuations between two oscillators, it is possible to measure the phase fluctuations introduced by a device such as an active filter or amplifier. Only a slight modification of the existing PLL filter equipment set up is needed. The scheme is shown in Figure 12.

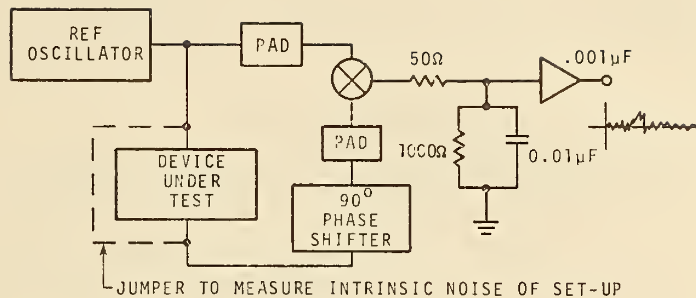


FIGURE 12

Figure 12 is a differential phase noise measurement set-up. The output of the reference oscillator is split so that part of the signal passes through the device under test. We want the two signals going to the mixer to be 90° out of phase, thus, phase fluctuations between the two input ports cause voltage fluctuations at the output. The voltage fluctuations then can be measured at various Fourier frequencies on a spectrum analyzer.

To estimate the noise inherent in the test set-up, one can in principle bypass the device under test and compensate for any change in amplitude and phase at the mixer. The PLL filter technique must be converted to a differential phase noise technique in order to measure inherent test equipment noise. It is a good practice to measure the system noise before proceeding to measurement of device noise.

A frequency domain measurement set-up is shown schematically in figure 13. The component values for the low-pass filter out of the mixer are suitable for oscillators operating at around 5 MHz.

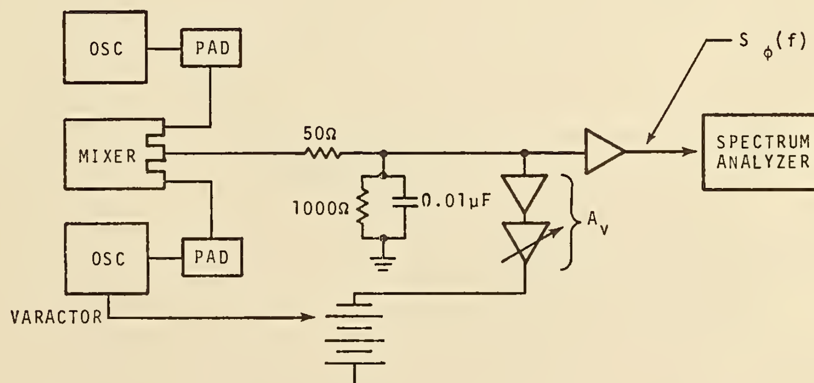


FIGURE 13

The active gain element (A_V) of the loop is a DC amplifier with flat frequency response. One may replace this element by an integrator to achieve high gain near DC and hence, maintain better lock of the reference oscillator in long term. Otherwise long-term drift between the reference and test oscillators might require manual re-adjustment of the frequency of one or the other oscillator.

V. Procedure and Example

At the input to the spectrum analyzer, the voltage varies as the phase fluctuations in short-term

$$S_\phi(f) = \left(\frac{V_{rms}(f)}{V_s}\right)^2$$

V_s is the phase sensitivity of the mixer in volts per radian. Using the previously described equipment set-up, V_s can be measured by disconnecting the feedback loop to the varactor of the reference oscillator. The peak voltage swing is equal to V_s in units of volts/rad if the resultant beat note is a sine wave.*

The value for the measured $S_\phi(f)$ in decibels is given by:

$$S_\phi(f) = 20 \log \frac{V_{rms} \text{Voltage at } f}{V_s \text{ full-scale } \phi\text{-detector voltage*}}$$

Example: *Given a PLL with two oscillators such that, at the mixer output:*

$V_s = 1$ *volt/rad*

$V_{rms}(45\ Hz) = 100$ *nV per root hertz*

Solve for $S_\phi(45\ Hz)$.

$$S_\phi(45\ Hz) = \left[\frac{100\ nV\ (Hz^{-1/2})}{1\ V/rad}\right]^2 = \left[\frac{10^{-7}}{1}\right]^2 rad^2\ Hz^{-1} = 10^{-14}\ \frac{rad^2}{Hz}$$

In decibels,

$$S_\phi(45\ Hz) = 20 \log \frac{100\ nV}{1\ V} = 20 \log \frac{10^{-7}}{10^0}$$

$$= 20\ (-7) = -140\ dB \text{ at } 45\ Hz$$

---

*This may not be the case for state-of-the-art $S_\phi(f)$ measurements where one must drive the mixer very hard to achieve low mixer noise levels. Hence, the output will not be a sine wave.

VI. Power-Law Noise Processes

Power-law noise processes are models of precision oscillator noise that produce a particular slope on the spectral density plot. We often classify these noise processes into one of five categories. For plots of $S_\phi(f)$, they are:

1. Random walk FM (random walk of frequency), S_ϕ plot goes down as $1/f^4$.
2. Flicker FM (flicker of frequency), S_ϕ plot goes down as $1/f^3$.
3. White FM (white of frequency), S_ϕ plot goes down as $1/f^2$.
4. Flicker ϕM (flicker of phase), S_ϕ plot goes down as $1/f$.
5. White ϕM (white of phase), S_ϕ plot is flat.

Power law noise processes are characterized by their functional dependence on Fourier frequency.

The spectral density plot of a typical oscillator's output usually is a combination of different power-law noise processes. It is very useful and meaningful to categorize the noise processes. The first job in evaluating a spectral density plot is to determine which type of noise exists for a particular range of Fourier frequencies. It is possible to have all five noise processes being generated from a single oscillator, but, in general, only two or three noise processes are dominant. Figure 14 is a graph of $S_\phi(f)$ showing the five noise processes on a log-log scale. Figure 15 shows the spectral density of phase fluctuations for a typical high-quality oscillator.

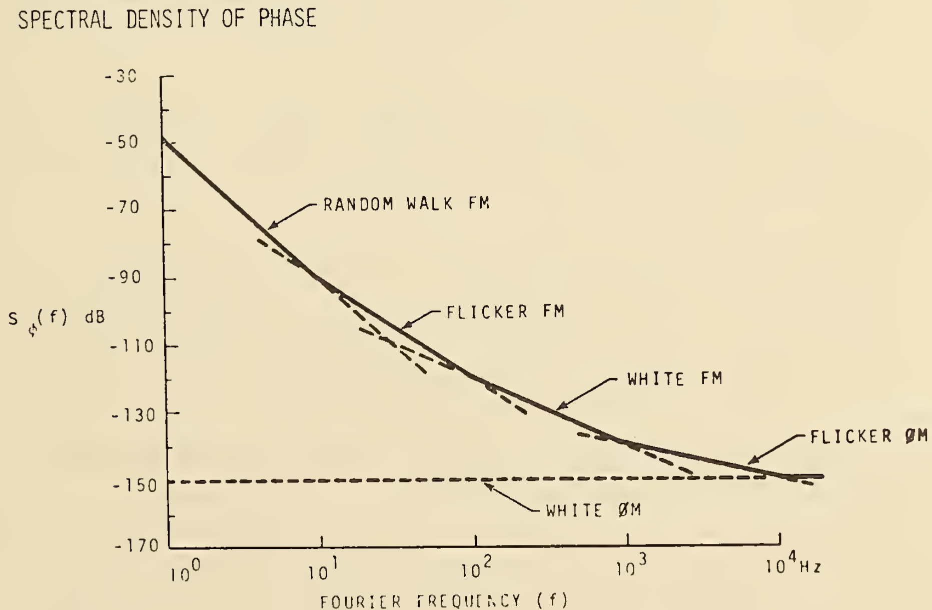


FIGURE 14

SPECTRAL DENSITY OF PHASE

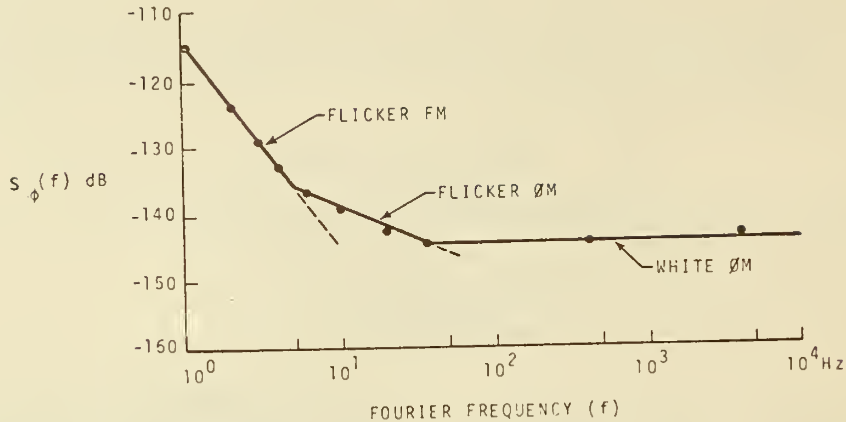


FIGURE 15

We can make the following general remarks about power-law noise processes:

1. Random walk FM ($1/f^4$) noise is difficult to measure since it is usually very close to the carrier. Random walk FM usually relates to the *oscillator's physical environment.* If random walk FM is a predominant feature of the spectral density plot then *mechanical shock, vibration, temperature,* or other environmental effects may be causing "random" shifts in the carrier frequency.

2. Flicker FM ($1/f^3$) is a noise whose physical cause is usually not fully understood but may typically be related to the *physical resonance mechanism of an active oscillator* or the *design or choice of parts used for the electronics, or even environmental properties.* Flicker FM is common in high-quality oscillators, but may be masked by white FM ($1/f^2$) or flicker ΦM ($1/f$) in lower-quality oscillators.

3. White FM ($1/f^2$) noise is a common type found in *passive-resonator frequency standards.* These contain a slave oscillator, often quartz, which is locked to a resonance feature of another device which behaves much like a high-Q filter. Cesium and rubidium standards have white FM noise characteristics.

4. Flicker ΦM ($1/f$) noise may relate to a physical resonance mechanism in an oscillator, but it usually is added by *noisy electronics.* This type of noise is common, even in the highest quality oscillators, because in order to bring the signal amplitude up to a usable level, amplifiers are used after the signal source. Flicker ΦM noise may be introduced in these stages. It may also be introduced in a frequency multiplier. Flicker ϕM can be reduced with good low-noise amplifier design (e.g., using rf negative feedback) and hand-selecting transistors and other electronic components.

5. White ΦM (f^0) noise is broadband phase noise and has little to do with the resonance mechanism. It is probably produced by similar phenomena as flicker ΦM ($1/f$) noise. *Stages of amplification* are usually responsible for white ΦM noise. This noise can be kept at a very low value with good amplifier design, hand-selected components, the addition of narrowband filtering at the output, or increasing, if feasible, the power of the primary frequency source.

VII. Translation from Frequency Domain Stability Measurement to Time Domain Stability Measurement

This section assumes the reader has some knowledge of the measurement of time domain stability. Our attention will be on how to translate a plot of $S_\phi(f)$ to $\sigma_y^2(\tau)$, the pair variance*. Elaboration on time domain stability measurements is beyond the intent of this Technical Note. A suggested reference is NBS Technical Note #669, "The Measurement of Frequency and Frequency Stability of Precision Oscillators," by D.W. Allan. Other references are listed in the Bibliography.

From the plot of phase spectral density, it is possible to make general comments about the time domain stability of an oscillator. Remarks are presented in the previous section. Recall that inferences of time domain stability are made based on specific kinds of power-law noise processes over a range of Fourier frequencies. More specifically, when dealing with noise processes, a power-law in the frequency domain corresponds to a particular power-law in the time domain. A convenient measure of frequency stability in the time domain is the pair variance, $\sigma_y^2(\tau)$. We have

$$\sigma_y^2(\tau) = \frac{1}{2(M-1)} \sum_{k=1}^{M-1} (\bar{y}_{k+1} - \bar{y}_k)^2 \tag{8}$$

where M is the number of data values and $\bar{y}_k$ is the average over time τ of the kth data point (similarly, $\bar{y}_{k+1}$ corresponds to kth + 1 point.)

Example: Find the pair variance, $\sigma_y^2(\tau)$, of the following sequence of fractional frequency fluctuation values $\bar{y}_k$, each value averaged over one second.

$\bar{y}_1 = 4.36 \times 10^{-5}$	$\bar{y}_5 = 4.47 \times 10^{-5}$
$\bar{y}_2 = 4.61 \times 10^{-5}$	$\bar{y}_6 = 3.96 \times 10^{-5}$
$\bar{y}_3 = 3.19 \times 10^{-5}$	$\bar{y}_7 = 4.10 \times 10^{-5}$
$\bar{y}_4 = 4.21 \times 10^{-5}$	$\bar{y}_8 = 3.08 \times 10^{-5}$

(Assume no dead-time in measurement of averages)

Since each average of the fractional frequency fluctuation values is for one second, then the first pair variance calculation will be at τ = 1s. We are given M = 8 (eight values);

---

*Often called the Allan variance.

therefore, the number of pairs in sequence is M-1 = 7. We have:

Data values $\bar{y}_k$ ($\times 10^{-5}$)	First differences $(\bar{y}_{k+1} - \bar{y}_k)$ ($\times 10^{-5}$)	First differences squared $(\bar{y}_{k+1} - \bar{y}_k)^2$ ($\times 10^{-10}$)
4.36	--	--
4.61	0.25	0.06
3.19	-1.42	2.02
4.21	1.02	1.04
4.47	0.26	0.07
3.96	-0.51	0.26
4.10	0.14	0.02
3.08	-1.02	1.04

$$\sum_{k=1}^{M-1} (\bar{y}_{k+1} - \bar{y}_k)^2 = 4.51 \times 10^{-10}$$

Therefore,

$$\sigma_y^2(1s) = \frac{4.51 \times 10^{-10}}{2(7)} = 3.2 \times 10^{-11}$$

and

$$[\sigma_y^2(1s)]^{1/2} = \sqrt{3.2 \times 10^{-11}} = 5.6 \times 10^{-6}$$

Using the same data, one can calculate the pair variance for $\tau = 2s$ by averaging pairs of adjacent values and using these new averages as data values for the same procedure as above. For three second averages ($\tau = 3s$) take adjacent threesomes and find their averages and proceed in a similar manner. More data must be acquired for longer averaging times.

One sees that with large numbers of data values, it is helpful to use a computer or programmable calculator. The confidence of the estimate of the pair variance improves nominally as the square root of the number of data values used. In this example, M=8 and the confidence can be expressed as being no better than $1/\sqrt{8} \times 100\% = 35\%$. This then is the allowable error in our estimate for the $\tau = 1s$ average.

Knowing how to measure $S_\phi(f)$ for a pair of oscillators, let us see how to translate the power-law noise process to a plot of $\sigma_y^2(\tau)$. First, consider $S_y(f)$, the spectral density of frequency fluctuations. There are two quantities which completely specify $S_y(f)$ for a particular power-law noise process: (1) the slope on a log-log plot for a given range of f and (2) the amplitude. The slope we shall denote by "α"; therefore f^α is the straight line (on log-log scale) which relates $S_y(f)$ to f. The amplitude will be denoted "h_α"; it is simply the coefficient of f for a range of f. When we examine a plot of spectral density of frequency fluctuations, we are looking at a representation of the addition of all the power-law processes. We have

$$S_y(f) = \sum_{\alpha = -\infty}^{\infty} h_\alpha f^\alpha \tag{9}$$

In the preceeding section, five power-law noise processes were outlined with respect to $S_\phi(f)$. These five are the common ones encountered with precision oscillators. Equation (7) relates these noise processes to $S_y(f)$. One obtains

1.	Random Walk FM	(f^{-2})	. . .	$\alpha = -2$
2.	Flicker FM	(f^{-1})	. . .	$\alpha = -1$
3.	White FM	(1)	. . .	$\alpha = 0$
4.	Flicker ϕM	(f)	. . .	$\alpha = 1$
5.	White ϕM	(f^2)	. . .	$\alpha = 2$

with respect to $S_y(f)$ ↑ (middle column); ↑ slope (α)

Table 1 is a list of coefficients for translation from $\sigma_y^2(\tau)$ to $S_y(f)$ and from $S_\phi(f)$ to $\sigma_y^2(\tau)$. The left column is the designator for the power-law process. Using the middle column, we can solve for the value of $S_y(f)$ by computing the coefficient "a" and using the measured time domain data $\sigma_y^2(\tau)$. The rightmost column yields a solution for $\sigma_y^2(\tau)$ given frequency domain data $S_\phi(f)$ and a calculation of the appropriate "b" coefficient.

$S_y(f) = h_\alpha f^\alpha$ $\alpha =$	$S_y(f) = a\,\sigma_y^2(\tau)$ $a =$	$\sigma_y^2(\tau) = b\,S_\phi(f)$ $b =$
2 (white phase)	$\frac{(2\pi)^2 \tau^2 f^2}{3 f_h}$	$\frac{3 f_h}{(2\pi)^2 \tau^2 \nu_o^2}$
1 (flicker phase)	$\frac{(2\pi)^2 \tau^2 f}{3.81 + 3 \ln(\omega_h \tau)}$	$\frac{[3.81 + 3 \ln(\omega_h \tau)] f}{(2\pi)^2 \tau^2 \nu_o^2}$
0 (white frequency)	2τ	$\frac{f^2}{2 \tau \nu_o^2}$
-1 (flicker frequency)	$\frac{1}{2 \ln(2) \cdot f}$	$\frac{2 \ln(2) \cdot f^3}{\nu_o^2}$
-2 (random walk frequency)	$\frac{6}{(2\pi)^2 \tau f^2}$	$\frac{(2\pi)^2 \tau f^4}{6 \nu_o^2}$

TABLE 1. Conversion table from time-domain to frequency-domain and from frequency-domain to time-domain for common kinds of integer power law spectral densities; f_h $(= \omega_h/2\pi)$ is the measurement system bandwidth. Measurement response should be within 3 dB from D.C. to f_h (3dB down high-frequency cutoff is at f_h).

$$S_\phi(f) = \frac{\nu_o^2}{f^2} \quad S_y(f)$$

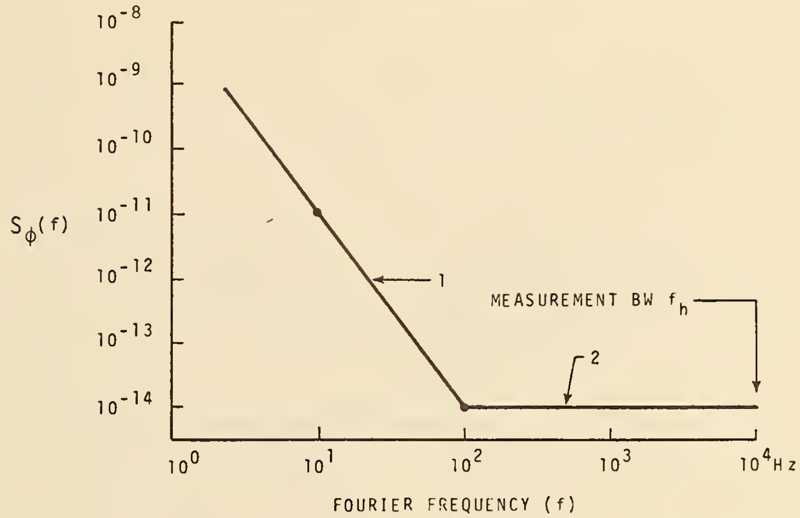


FIGURE 16

In the phase spectral density plot of figure 16, there are two power-law noise processes. For region 1, we see than when f increases by one decade (that is, from 10 Hz to 100 Hz), $S_\phi(f)$ goes down by three decades (that is, from 10^{-11} to 10^{-14}). Thus, $S_\phi(f)$ goes as $1/f^3 = f^{-3}$. For region 1, we identify this noise process as flicker FM. The rightmost column of Table 1 relates $\sigma_y^2(\tau)$ to $S_\phi(f)$. The row designating flicker frequency noise yields:

$$\sigma_y^2(\tau) = \frac{2\ell n(2) \cdot f^3}{\nu_0^2} S_\phi(f)$$

One can now pick (arbitrarily) a convenient Fourier frequency f and determine the corresponding values of $S_\phi(f)$ given by the plot of figure 18. Say, f = 10, thus $S_\phi(10) = 10^{-11}$. Solving for $\sigma_y^2(\tau)$, given ν_0 = 1 MHz, we obtain:

$$\sigma_y^2(\tau) = 1.39 \times 10^{-20}$$

therefore, $\sigma_y(\tau) = 1.18 \times 10^{-10}$. For region 2, we have white ϕM. The relationship between $\sigma_y^2(\tau)$ and $S_\phi(f)$ for white ϕM is:

$$\sigma_y^2(\tau) = \frac{3f_h}{(2\pi)^2 \tau^2 \nu_0{}^2} S_\phi(f)$$

Again, we choose a Fourier frequency, say f = 100, and see that $S_\phi(100) = 10^{-14}$. Assuming $f_h = 10^4$ Hz, we thus obtain:

$$\sigma_y^2(\tau) = 7.59 \times 10^{-24} \cdot \frac{1}{\tau^2}$$

therefore,

$$\sigma_y(\tau) = 2.76 \times 10^{-12} \frac{1}{\tau}.$$

The resultant time domain characterization is shown in figure 17.

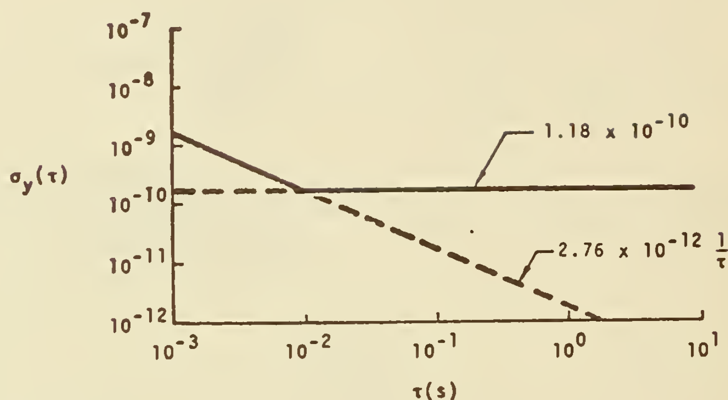


FIGURE 17. The translation of $S_\phi(f)$ of figure 16 yields this $\sigma_y(\tau)$ plot.

Figures 18 and 19 show a plot of time-domain stability and a translation to frequency domain. The procedure is left as an exercise for the reader.

VIII. Conclusion

Starting with the basic concepts associated with the sine-wave signal generator, this writing attempts to present fundamental aspects of spectral analysis with only light mathematical treatment. The RF spectrum is not a useful means of inferring the frequency stability of an oscillator, so a discussion of $S_\phi(f)$ and $S_y(f)$ is put forth as well as the technique for measurement. It was felt that there was a need for a tutorial writing that explains frequency-domain measurements without elaborating on existing time-domain techniques. A table for translating data from one domain to the other has been included, and the use of this table requires knowledge of the Allan Variance. The author has tried to be general in the treatment of topics, and a bibliography is attached for readers who would like details about specific items.

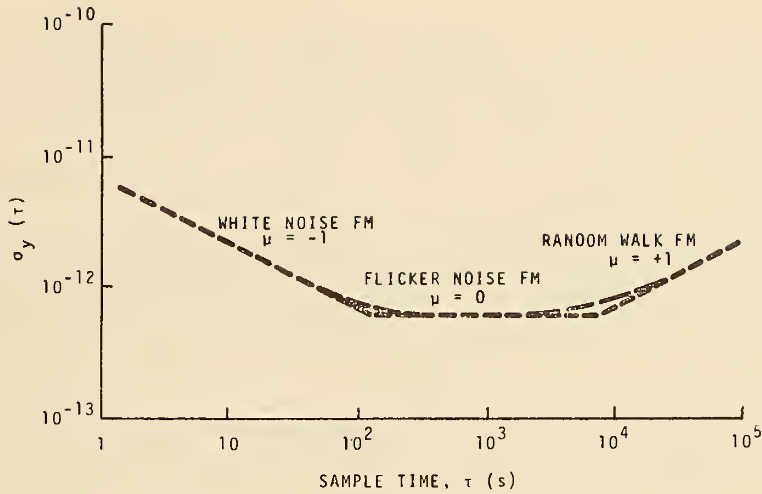


FIGURE 18. A $\sigma_y(\tau)$ versus τ plot of some actual oscillator data. Notice that if $\sigma_y^2(\tau) \approx \tau^{\mu}$, then $\sigma_y(\tau) \simeq \tau^{\mu/2}$ hence the $\tau^{-1/2}$ slope for $\mu = -1$, etc.

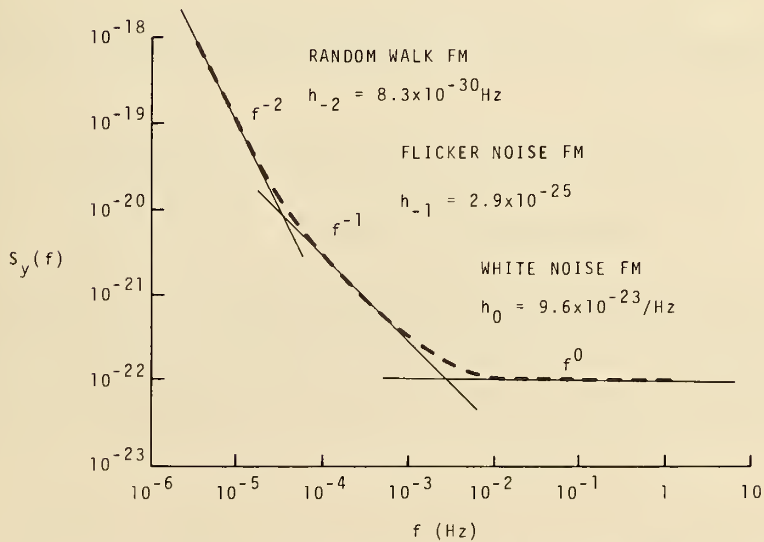


FIGURE 19. A plot of $S_y(f)$ as transformed from the time-domain data plotted in Figure 18.

BIBLIOGRAPHY

General References

A. November or December of even-numbered years, IEEE Transactions on Instrumentation and Measurement (Conference on Precision Electromagnetic Measurements, held every 2 years).

B. Proceedings of the IEEE. Special issue on frequency stability, vol. 54, February 1966.

C. Proceedings of the IEEE. Special issue on time and frequency, vol. 60, no. 5, May 1972.

D. Proceedings of the IEEE-NASA Symposium on the Definition and Measurement of Short-Term Frequency Stability at Goddard Space Flight Center, Greenbelt, Maryland, November 23-24, 1964. Prepared by Goddard Space Flight Center (Scientific & Technical Information Division, National Aeronautics and Space Administration, Washington, D.C., 1965). Copies available for $1.75 from the U.S. Government Printing Office, Washington, D.C., 20402.

E. Proceedings of the Annual Symposium on Frequency Control. Prepared by the U.S. Army Electronics Command, Fort Monmouth, New Jersey. Copies available from Electronics Industry Association, 2001 Eye Street, N.W., Washington, D.C., 20006, starting with 23rd Annual. Price per copy, $8.00.

F. J.A. Barnes, A.R. Chi, L.S. Cutler, et. al., "Characterization of Frequency Stability," IEEE Trans. on Instr. and Meas. IM-20, No. 2, pp. 105-120 (May 1971). Prepared by the Subcommittee on Frequency Stability of the Institute of Electrical and Electronics Engineers.

G. Time and Frequency: Theory and Fundamentals, Byron E. Blair, Editor, NBS Monograph 140, May 1974.

Additional Specific References (selected)

1. D.W. Allan, "Statistics of Atomic Frequency Standards," Proc. IEEE, vol. 54, pp. 221-230, February 1966.

2. D.W. Allan, "The Measurement of Frequency and Frequency Stability of Precision Oscillators," NBS Tech. Note 669, July 1975.

3. J.R. Ashley, C.B. Searles, and F.M. Palka, "The Measurement of Oscillator Noise at Microwave Frequencies," IEEE Trans. on Microwave Theory and Techniques MTT-16, pp. 753-760, September 1968.

4. E.J. Baghdady, R.D. Lincoln, and B.D. Nelin, "Short-Term Frequency Stability: Theory, Measurement, and Status," Proc. IEEE, vol. 53, pp. 704-722, 2110-2111, 1965.

5. J.A. Barnes, "Atomic Timekeeping and the Statistics of Precision Signal Generators," Proc. IEEE, vol. 54, pp. 207-220, February 1966.

6. J.A. Barnes, "Tables of Bias Functions, B_1 and B_2, for Variances Based on Finite Samples of Processes with Power Law Spectral Densities," NBS Tech. Note 375, January 1969.

7. J.A. Barnes and D.W. Allan, "A Statistical Model of Flicker Noise," Proc. IEEE, vol. 43, pp. 176-178, February 1966.

8. J. A. Barnes and R. C. Mockler, "The Power Spectrum and its Importance in Precise Frequency Measurements," IRE Trans. on Instrumentation I-9, pp. 149-155, September 1960.

9. W. R. Bennett, Electrical Noise. New York: McGraw-Hill, 1960.

10. C. Bingham, M. D. Godfrey, and J. W. Tukey, "Modern Techniques of Power Spectrum Estimation," IEEE Trans. AU, vol. 15, pp. 56-66, June 1967.

11. R. B. Blackman and J. W. Tukey, The Measurement of Power Spectra. New York: Dover, 1958.

12. E. O. Brigham and R. E. Morrow, "The Fast Fourier Transform," IEEE Spectrum, vol. 4, pp.63-70, December 1967.

13. R. A. Campbell, "Stability Measurement Techniques in the Frequency Domain," Proc. IEEE-NASA Symposium on the Definition and Measurement of Short-Term Frequency Stability, NASA SP-80, pp. 231-235, 1965.

14. L. S. Cutler and C. L. Searle, "Some Aspects of the Theory and Measurement of Frequency Fluctuations in Frequency Standards," Proc. IEEE, vol. 54, pp. 136-154, February 1966.

15. W. A. Edson, "Noise in Oscillators," Proc. IRE, vol. 48, pp. 1454-1466, August 1960.

16. C. H. Grauling, Jr. and D. J. Healey, III, "Instrumentation for Measurement of the Short-Term Frequency Stability of Microwave Sources," Proc. IEEE, vol. 54, pp. 249-257, February 1966.

17. H. Hellwig, "Atomic Frequency Standards: A Survey," Proc. IEEE, vol. 63, pp. 212-229, February 1975.

18. H. Hellwig, "A Review of Precision Oscillators," NBS Tech Note 662, February 1975.

19. H. Hellwig, "Frequency Standards and Clocks: A Tutorial Introduction," NBS Tech Note 616-R, March 1974.

20. Hewlett-Packard, "Precision Frequency Measurements," Application Note 116, July 1969.

21. G. M. Jenkins and D. G. Watts, Spectral Analysis and its Applications. San Francisco: Holden-Day, 1968.

22. S. L. Johnson, B. H. Smith, and D. A. Calder, "Noise Spectrum Characteristics of Low-Noise Microwave Tubes and Solid-State Devices," Proc. IEEE, vol. 54, pp. 258-265, February 1966.

23. P. Kartaschoff and J. Barnes, "Standard Time and Frequency Generation," Proc. IEEE, vol. 60, pp. 493-501, May 1972.

24. D. B. Leeson, "A Simple Model of Feedback Oscillator Noise Spectrum," Proc. IEEE L., vol. 54, pp. 329-330, February 1966.

25. P. Lesage and C. Audoin, "Characterization of Frequency Stability: Uncertainty due to the Finite Number of Measurements," IEEE Trans. on IM, vol. IM-22, pp. 157-161, June 1973.

26. D. G. Meyer, "A Test Set for Measurement of Phase Noise on High Quality Signal Sources," IEEE Trans. on Instrumentation and Measurement IM-19, No. 4, pp. 215-227, November 1970.

27. C. D. Motchenbacher and F. C. Fitchen, Low-Noise Electronic Design. New York: John Wiley & Sons, 1973.

28. J. Shoaf, "Specification and Measurement of Frequency Stability," NBS Internal Report 74-396, November 1974.

29. J. L. Stewart, "Frequency Modulation Noise in Oscillators," Proc. IRE, vol. 44, pp. 372-376, March 1956.

30. R. F. C. Vessot, L. Mueller, and J. Vanier, "The Specification of Oscillator Characteristics from Measurements in the Frequency Domain," Proc. IEEE, pp. 199-206, February 1966.

31. F. Walls and A. Wainwright, "Measurement of the Short-Term Stability of Quartz Crystal Resonators and the Implications for Quartz Oscillator Design and Applications," IEEE Trans. on IM, pp. 15-20, March 1975.

NBS-114A (REV. 7-73)

U.S. DEPT. OF COMM. BIBLIOGRAPHIC DATA SHEET	1. PUBLICATION OR REPORT NO. NBS TN-679	2. Gov't Accession No.	3. Recipient's Accession No.

4. TITLE AND SUBTITLE	5. Publication Date
"Frequency Domain Stability Measurements: A Tutorial Introduction"	March 1976
	6. Performing Organization Code: 277.04
7. AUTHOR(S) David A. Howe	8. Performing Organ. Report No.
9. PERFORMING ORGANIZATION NAME AND ADDRESS NATIONAL BUREAU OF STANDARDS DEPARTMENT OF COMMERCE WASHINGTON, D.C. 20234	10. Project/Task/Work Unit No. 2774148 11. Contract/Grant No.
12. Sponsoring Organization Name and Complete Address (*Street, City, State, ZIP*) Same as Item 9.	13. Type of Report & Period Covered 14. Sponsoring Agency Code

15. SUPPLEMENTARY NOTES

16. ABSTRACT (*A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.*)

This report introduces the concept of stability measurements of oscillators by spectral analysis. Development of topics does not rely heavily on mathematics. The equipment and set-up for stability measurements in the frequency domain are outlined. Examples and typical results are presented. Physical interpretations of common noise processes are discussed. The last section provides a table by which typical frequency domain stability characteristics may be translated to time domain stability characteristics.

17. KEY WORDS (*six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons*)

Fractional frequency fluctuations; frequency stability; phase fluctuations; power law noise processes; spectral density; spectrum analysis.

18. AVAILABILITY [X] Unlimited	19. SECURITY CLASS (THIS REPORT)	21. NO. OF PAGES
[ ] For Official Distribution. Do Not Release to NTIS	UNCLASSIFIED	27
[X] Order From Sup. of Doc., U.S. Government Printing Office Washington, D.C. 20402, SD Cat. No. C13.46:679	20. SECURITY CLASS (THIS PAGE)	22. Price
[ ] Order From National Technical Information Service (NTIS) Springfield, Virginia 22151	UNCLASSIFIED	$0.65

NBS TECHNICAL PUBLICATIONS

PERIODICALS

JOURNAL OF RESEARCH reports National Bureau of Standards research and development in physics, mathematics, and chemistry. It is published in two sections, available separately:

• **Physics and Chemistry (Section A)**

Papers of interest primarily to scientists working in these fields. This section covers a broad range of physical and chemical research, with major emphasis on standards of physical measurement, fundamental constants, and properties of matter. Issued six times a year. Annual subscription: Domestic, $17.00; Foreign, $21.25.

• **Mathematical Sciences (Section B)**

Studies and compilations designed mainly for the mathematician and theoretical physicist. Topics in mathematical statistics, theory of experiment design, numerical analysis, theoretical physics and chemistry, logical design and programming of computers and computer systems. Short numerical tables. Issued quarterly. Annual subscription: Domestic, $9.00; Foreign, $11.25.

DIMENSIONS/NBS (formerly Technical News Bulletin)—This monthly magazine is published to inform scientists, engineers, businessmen, industry, teachers, students, and consumers of the latest advances in science and technology, with primary emphasis on the work at NBS. The magazine highlights and reviews such issues as energy research, fire protection, building technology, metric conversion, pollution abatement, health and safety, and consumer product performance. In addition, it reports the results of Bureau programs in measurement standards and techniques, properties of matter and materials, engineering standards and services, instrumentation, and automatic data processing.

Annual subscription: Domestic, $9.45; Foreign, $11.85.

NONPERIODICALS

Monographs—Major contributions to the technical literature on various subjects related to the Bureau's scientific and technical activities.

Handbooks—Recommended codes of engineering and industrial practice (including safety codes) developed in cooperation with interested industries, professional organizations, and regulatory bodies.

Special Publications—Include proceedings of conferences sponsored by NBS, NBS annual reports, and other special publications appropriate to this grouping such as wall charts, pocket cards, and bibliographies.

Applied Mathematics Series—Mathematical tables, manuals, and studies of special interest to physicists, engineers, chemists, biologists, mathematicians, computer programmers, and others engaged in scientific and technical work.

National Standard Reference Data Series—Provides quantitative data on the physical and chemical properties of materials, compiled from the world's literature and critically evaluated. Developed under a world-wide program coordinated by NBS. Program under authority of National Standard Data Act (Public Law 90-396).

NOTE: At present the principal publication outlet for these data is the Journal of Physical and Chemical Reference Data (JPCRD) published quarterly for NBS by the American Chemical Society (ACS) and the American Institute of Physics (AIP). Subscriptions, reprints, and supplements available from ACS, 1155 Sixteenth St. N. W., Wash. D. C. 20056.

Building Science Series—Disseminates technical information developed at the Bureau on building materials, components, systems, and whole structures. The series presents research results, test methods, and performance criteria related to the structural and environmental functions and the durability and safety characteristics of building elements and systems.

Technical Notes—Studies or reports which are complete in themselves but restrictive in their treatment of a subject. Analogous to monographs but not so comprehensive in scope or definitive in treatment of the subject area. Often serve as a vehicle for final reports of work performed at NBS under the sponsorship of other government agencies.

Voluntary Product Standards—Developed under procedures published by the Department of Commerce in Part 10, Title 15, of the Code of Federal Regulations. The purpose of the standards is to establish nationally recognized requirements for products, and to provide all concerned interests with a basis for common understanding of the characteristics of the products. NBS administers this program as a supplement to the activities of the private sector standardizing organizations.

Federal Information Processing Standards Publications (FIPS PUBS)—Publications in this series collectively constitute the Federal Information Processing Standards Register. Register serves as the official source of information in the Federal Government regarding standards issued by NBS pursuant to the Federal Property and Administrative Services Act of 1949 as amended, Public Law 89-306 (79 Stat. 1127), and as implemented by Executive Order 11717 (38 FR 12315, dated May 11, 1973) and Part 6 of Title 15 CFR (Code of Federal Regulations).

Consumer Information Series—Practical information, based on NBS research and experience, covering areas of interest to the consumer. Easily understandable language and illustrations provide useful background knowledge for shopping in today's technological marketplace.

NBS Interagency Reports (NBSIR)—A special series of interim or final reports on work performed by NBS for outside sponsors (both government and non-government). In general, initial distribution is handled by the sponsor; public distribution is by the National Technical Information Service (Springfield, Va. 22161) in paper copy or microfiche form.

Order NBS publications (except NBSIR's and Bibliographic Subscription Services) from: Superintendent of Documents, Government Printing Office, Washington, D.C. 20402.

BIBLIOGRAPHIC SUBSCRIPTION SERVICES

The following current-awareness and literature-survey bibliographies are issued periodically by the Bureau:

Cryogenic Data Center Current Awareness Service

A literature survey issued biweekly. Annual subscription: Domestic, $20.00; foreign, $25.00.

Liquefied Natural Gas. A literature survey issued quarterly. Annual subscription: $20.00.

Superconducting Devices and Materials. A literature survey issued quarterly. Annual subscription: $20.00. Send subscription orders and remittances for the preceding bibliographic services to National Bureau of Standards, Cryogenic Data Center (275.02) Boulder, Colorado 80302.

Electromagnetic Metrology Current Awareness Service

Issued monthly. Annual subscription: $24.00. Send subscription order and remittance to Electromagnetics Division, National Bureau of Standards, Boulder, Colo. 80302.

U.S. DEPARTMENT OF COMMERCE
National Bureau of Standards
Washington, D.C. 20234

OFFICIAL BUSINESS

Penalty for Private Use, $300

POSTAGE AND FEES PAID
U.S. DEPARTMENT OF COMMERCE
COM-215

Fourth Class Mail

75 YEARS
NBS
1901-1976