NBS TECHNICAL NOTE 673

U.S. DEPARTMENT OF COMMERCE / National Bureau of Standards

Using Six-Port and Eight-Port Junctions To Measure Active and Passive Circuit Parameters

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Using Six-Port and Eight-Port Junctions To Measure Active and Passive Circuit Parameters

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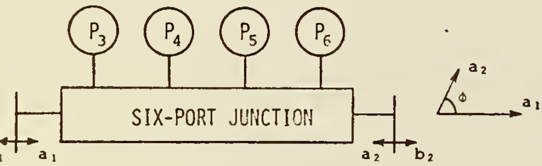
eneralized Six-Port Vector Voltmeter



$$|a_1|^2 = \sum \rho_i P_i$$

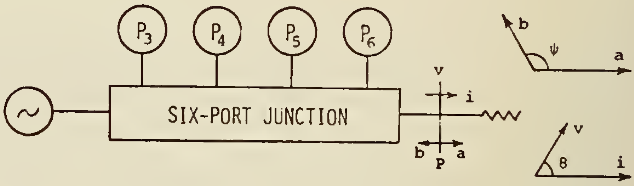
$$|a_2|^2 = \sum \sigma_i P_i$$

$$a_1 a_2| \cos\phi = \sum x_i P_i \qquad i = 3 \ldots 6$$

$$a_1 a_2| \sin\phi = \sum y_i P_i$$

$$= \frac{a_2}{a_1} = \frac{\sum (x_i + j\, y_i) P_i}{\sum \rho_i P_i}$$

Generalized Six-Port Reflectometer



$$|a|^2 = \sum \alpha_i P_i \qquad |v|^2 = \sum \nu_i P_i$$

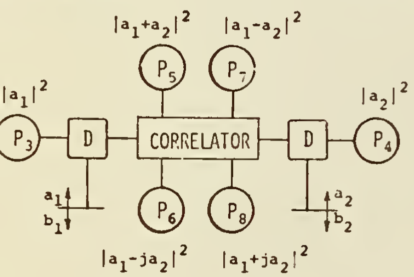
$$|b|^2 = \sum \beta_i P_i \qquad |i|^2 = \sum \mu_i P_i$$

$$|ab| \cos\psi = \sum c_i P_i \qquad P = |vi| \cos\theta = \sum q_i P_i$$

$$|ab| \sin\psi = \sum s_i P_i \qquad P'' = |vi| \sin\theta = \sum r_i P_i$$

$$\Gamma = \frac{b}{a} = \frac{\sum (c_i + j\, s_i) P_i}{\sum \alpha_i P_i} \qquad Z = \frac{v}{i} = \frac{\sum (q_i + j\, r_i) P_i}{\sum \mu_i P_i}$$

deal Eight-Port Vector Voltmeter



$$|a_1|^2 = k\, P_3$$

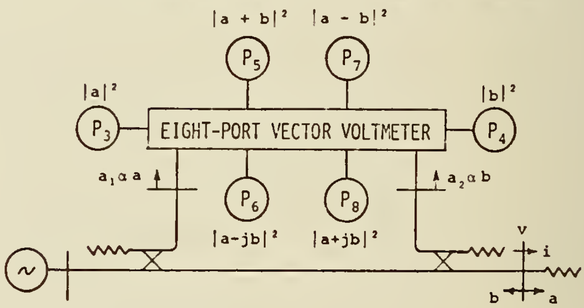
$$|a_2|^2 = k\, P_4$$

$$a_1 a_2| \cos\phi = k\,(P_5 - P_7)$$

$$a_1 a_2| \sin\phi = k\,(P_6 - P_8)$$

$$) = \frac{(P_5 - P_7) + j\,(P_6 - P_8)}{P_3}$$

Ideal Eight-Port Reflectometer



$$|a|^2 = K\, P_3 \qquad |v|^2 = 4K\, P_5$$

$$|b|^2 = K\, P_4 \qquad |iZ_0|^2 = 4K\, P_7$$

$$|ab| \cos\psi = K\,(P_5 - P_7) \qquad |viZ_0| \cos\theta = K\,(P_3 - P_4)$$

$$|ab| \sin\psi = K\,(P_6 - P_8) \qquad |viZ_0| \sin\theta = 2K\,(P_6 - P_8)$$

$$\Gamma = \frac{(P_5 - P_7) + j\,(P_6 - P_8)}{P_3} \qquad \frac{Z}{Z_0} = \frac{(P_3 - P_4) + 2j(P_6 - P_8)}{4\, P_7}$$

mmary of equations for generalized six-port junctions and also two corresponding ideal eight-port nctions. The coefficients of the power readings $P_3 \ldots P_8$ are constants which are determined by calibrating the junction for the intended application.

USING SIX-PORT AND EIGHT-PORT JUNCTIONS TO MEASURE ACTIVE AND PASSIVE CIRCUIT PARAMETERS

Cletus A. Hoer

This review paper brings together a number of old and new methods for measuring voltage, current, power, impedance, and phase angle using only amplitude type detectors. Vector voltmeters and reflectometers can be constructed to measure all of these quantities in terms of four amplitude measurements made on four arms of a six-port junction. Whereas previous uses of this type of instrument depended on precision components for accuracy, new equations switch this dependence primarily to detector linearity and only secondarily on the properties of the measuring device itself.

KEY WORDS: Admittance; automated precision measurements; correlator; current; impedance; microwave circuit parameters; microwave measurements; phase angle; power; ratio; reflection coefficient; reflectometer; self-calibration; six-port; vector voltmeter; voltage.

I. INTRODUCTION

Perhaps the greatest impact in the field of UHF and microwave measurement in recent years has been the introduction of the Automatic Network Analyzer. In contrast with the prior art where the key to improved accuracy was usually an improved hardware item, the procedure now is to measure the hardware imperfections and adjust the measurement results in such a way as to account for them. The key to this correction process is in the measurement of the phase as well as amplitude of the complex parameters involved. The measurement of this phase has generally involved conversion to a lower frequency which complicates the detection process. Although phase detection circuits which operate directly at the UHF and microwave frequencies are well known, these have generally assumed ideal components, and are not particularly suitable for automation.

Recent theoretical studies of an arbitrary six-port junction have provided an alternative method of obtaining the phase information without requiring either frequency conversion or ideal components. The theory shows that any six-port junction which has four linearly independent ports terminated with power detectors can be used to measure active and passive circuit parameters such as voltage, current, power, impedance, phase angle, and complex voltage ratios. Phase angle as well as amplitude of the desired circuit parameters are determined by the four power readings. Accuracy of the measurements is primarily a function of the linearity of the power detectors, and only slightly dependent on the properties of the six-port junction itself. Precision components are not required to make precision measurements. Components are available to build six-port junctions up to 200 GHz.

One of the unexpected results of this study is that most of the earlier designs for getting phase information from simple amplitude measurements provide a set of data which is ill conditioned from the viewpoint of the more general theory. Fortunately, the theory also suggests how these designs can be modified to eliminate this condition. Some of these earlier phase detection circuits will be reviewed to show how one can obtain phase information directly at the measurement frequency using only amplitude (e.g. power) detectors. This review of ideal circuits will lay the groundwork for understanding how a properly designed linear six-port junction can be used to make the same measurements more accurately without requiring the use of precision components.

Section II discusses six-port junctions which have been used in the past to make voltage ratio measurements and measurements of impedance. Some new ideas are added to make these old circuits more useful, leading to the design of a vector voltmeter and a reflectometer each having eight ports. Section III goes into the generalized six-port theory, showing how an arbitrary six-port junction can be used to measure complex voltage ratios. Section IV shows how an arbitrary six-port junction can be used to measure complex impedance or reflection coefficients. Equations for the generalized six-port are summarized in Table 1 which also gives examples of corresponding ideal eight-port junctions and their equations.

Partial support of this work by the Naval Sea Systems Command, USAFSAM, and the Army Metrology & Calibration Center is gratefully acknowledged.

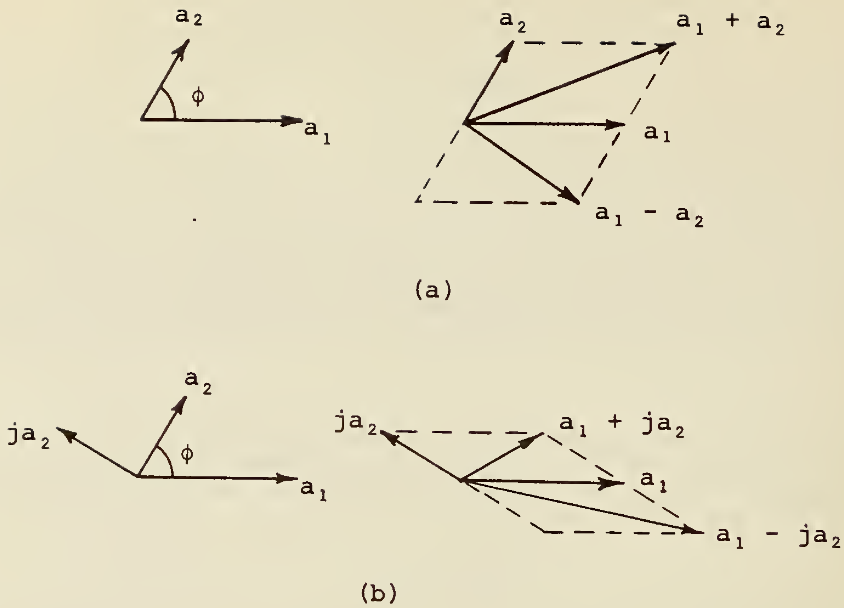


Figure 1. Getting the phase angle ϕ from the law of cosines.

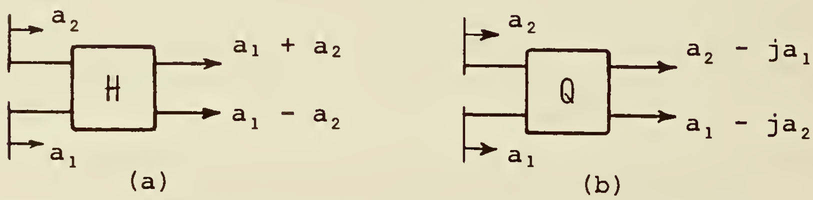


Figure 2. Using a 180° hybrid (H) and a quadrature (90°) hybrid (Q) to get the sum and difference vectors shown in Figure 1.

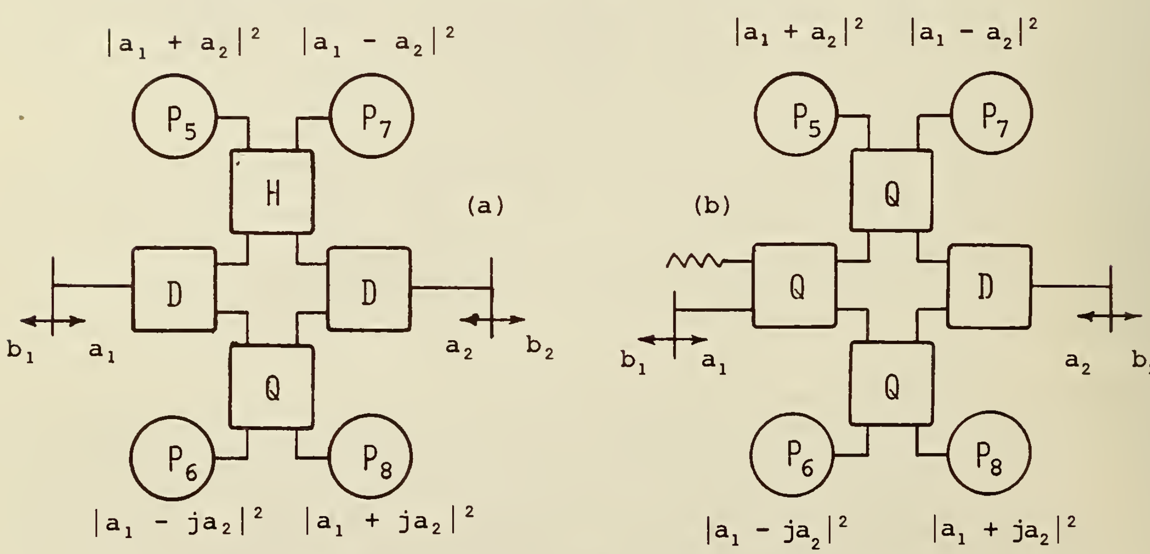


Figure 3. Correlators constructed from hybrid junctions (H), quadrature hybrids (Q), and in-phase power dividers (D).

II. PHASE FROM AMPLITUDE MEASUREMENTS - IDEAL CIRCUITS

Basic Idea

Consider two sinusoidal voltages a_1 and a_2 having the same frequency and related in phase as shown in figure 1a. The phase angle ϕ between a_2 and a_1 can be determined by measuring the amplitude of a_1, a_2, and either $a_1 - a_2$ or $a_1 + a_2$ and calculating the angle from the law of cosines;

$$|a_1 - a_2|^2 = |a_1|^2 + |a_2|^2 - 2|a_1a_2| \cos \phi \tag{1}$$

$$|a_1 + a_2|^2 = |a_1|^2 + |a_2|^2 + 2|a_1a_2| \cos \phi \tag{2}$$

If the amplitude of both the sum and difference vectors are measured, the phase angle can be obtained from

$$4|a_1a_2| \cos \phi = |a_1 + a_2|^2 - |a_1 - a_2|^2 \tag{3}$$

which is (2) minus (1). One disadvantage of calculating ϕ from any one of these three equations is that $\cos \phi$ is sensitive to angles near ±90° but insensitive to angles near 0° and 180°. Also, the cosine does not give the sign of the phase angle, only its magnitude. What is needed in addition to $\cos \phi$ is the $\sin \phi$ which is sensitive to angles near 0° and 180° and which also gives the sign of the phase angle.

One way to get $\sin \phi$ is to shift either a_1 or a_2 90° and then combine to get the sum and/or difference. Figure 1b shows the result of shifting a_2 90°. The sum and difference amplitudes are

$$|a_1 + ja_2|^2 = |a_1{}^2| + |a_2|^2 - 2|a_1a_2| \sin \phi \tag{4}$$

$$|a_1 - ja_2|^2 = |a_1|^2 + |a_2|^2 + 2|a_1a_2| \sin \phi \tag{5}$$

which combine to give

$$4|a_1a_2| \sin \phi = |a_1 - ja_2|^2 - |a_1 + ja_2|^2 \tag{6}$$

To get the sum and difference of two signals, a 180° hybrid junction or magic tee can be used as shown in figure 2a. To shift one of the signals 90° relative to the other and then combine to get the sum and difference, a quadrature (90°) hybrid can be used as shown in figure 2b.

Correlator

Quadrature hybrids and 180° hybrids can be combined in different ways to make a device called a correlator or phase-discriminator [1] whose outputs give both $\cos \phi$ and $\sin \phi$ from amplitude measurements. Two correlators are shown in figure 3. Both have the same output amplitudes, but the one in figure 3b is more practical at frequencies above about 1 GHz where quadrature hybrids are less expensive than 180° hybrids. The boxes labeled "D" are power dividers which split an input signal into two output signals each having the same amplitude and phase angle relative to the input signal. The amplitudes of the four output signals are measured with power detectors $P_5 \ldots P_8$. The phase angle ϕ between the two input signals a_1 and a_2 is calculated from (3) and (6) which can be written

$$|a_1a_2| \cos \phi = R\ (P_5 - P_7) \rightarrow X_\phi \tag{7}$$

$$|a_1a_2| \sin \phi = R\ (P_6 - P_8) \rightarrow Y_\phi \tag{8}$$

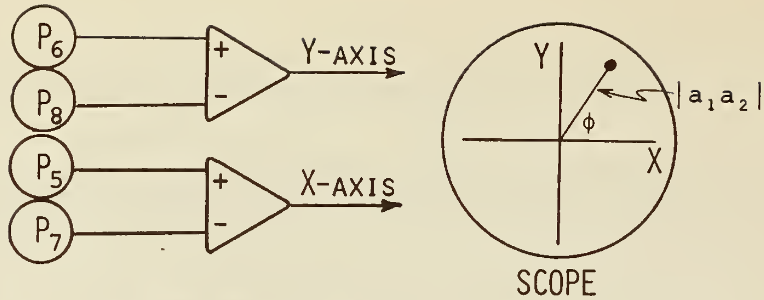


Figure 4. Using the four outputs of a correlator to display a dot indicating phase angle ϕ and the amplitude $|a_1a_2|$.

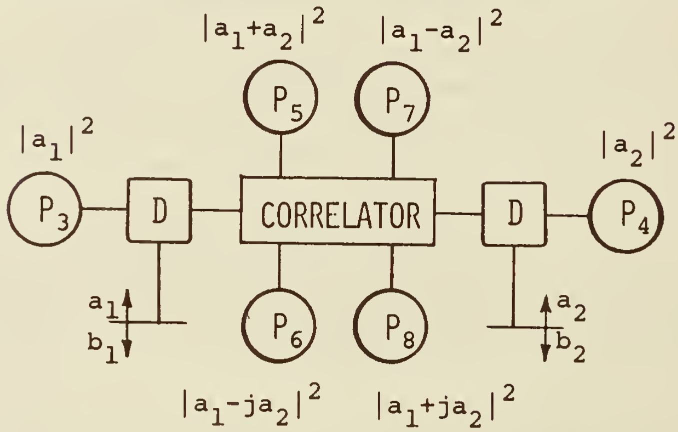


Figure 5. Adding two power dividers (D) and detectors P_3 and P_4 to the correlator to make an eight-port vector voltmeter.

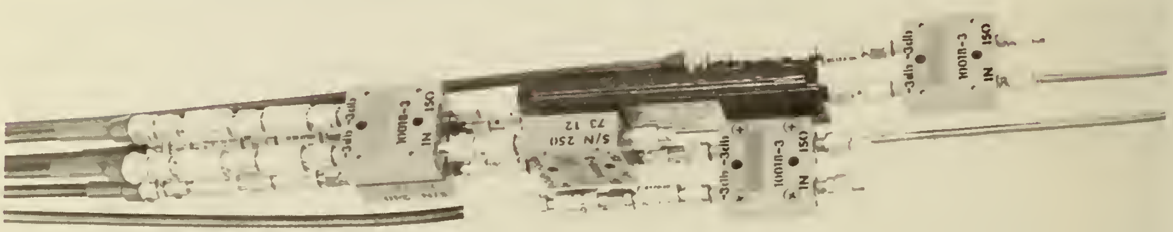


Figure 6. Experimental eight-port vector voltmeter for the 8 - 12 GHz range.

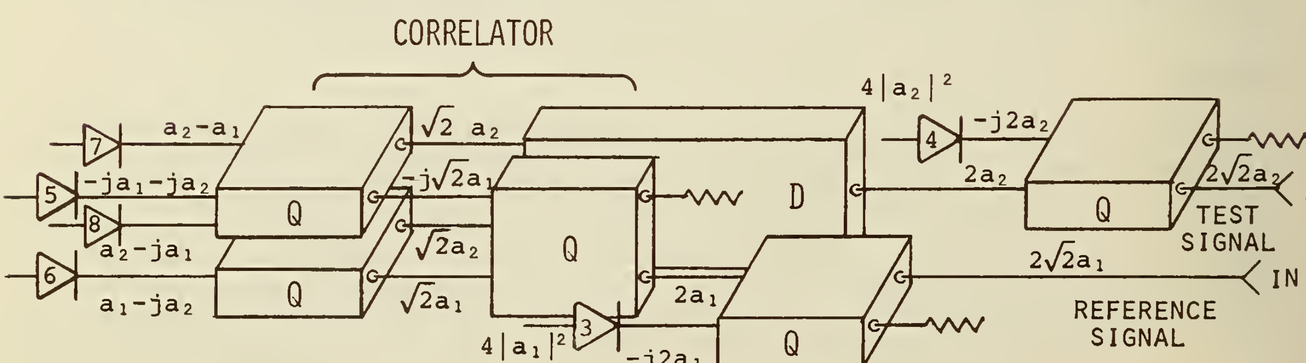


Figure 7. Combining quadrature hybrids (Q) and a power divider (D) to make an eight-port vector voltmeter having two input ports and six output ports which are terminated with diode detectors. The signals labeled at different parts of the junction are those one would obtain from ideal components if the input signals are $2 \cdot 2a_1$ and $2 \cdot 2a_2$.

where each detector is assumed to have the same input resistance R. If each power detector has an output voltage proportional to the rf input power, these voltages can be combined as indicated in (7) and (8) to get two voltages X_ϕ and Y_ϕ. When X_ϕ and Y_ϕ are connected to the horizontal and vertical inputs of a scope, it will display a dot which indicates both the phase angle ϕ (0 to 360°) and the amplitude $|a_1 a_2|$ as shown in figure 4.

A Vector Voltmeter

The correlator is unable to measure $|a_1|$ or $|a_2|$, only the product $|a_1 a|$. To separate $|a_1|$ and $|a_2|$, one can add two more power dividers and detectors to the correlator as shown in figure 5. This circuit acts like a vector voltmeter in that it measures the amplitude of the two input signals as well as the phase angle between them. If P_3 is used to level the generator so that $|a_1|$ is held constant, the scope display in figure 4 indicates the amplitude and phase of the complex voltage ratio a_2/a_1. Amplitude and phase information are calculated from the following equations which are for an ideal eight-port vector voltmeter.

$$|a_1|^2 = 2RP_3 \tag{9a}$$

$$|a_2|^2 = 2RP_4 \tag{9b}$$

$$|a_1 a_2| \cos\phi = 2R(P_5 - P_7) \tag{9c}$$

$$|a_1 a_2| \sin\phi = 2R(P_6 - P_8) \tag{9d}$$

The complex voltage ratio a_2/a_1 is given by

$$\frac{a_2}{a_1} = \frac{|a_1 a_2|(\cos\phi + j\sin\phi)}{|a_1|^2} = \frac{(P_5 - P_7) + j(P_6 - P_8)}{P_3} \tag{10}$$

Figure 6 is a photograph of an eight-port vector voltmeter constructed from commercially available miniature coaxial X-band quadrature hybrids and a power divider. Back diodes are used for power detectors. The components in the photograph are identified in figure 7 which also shows how the signals are combined to get the six desired outputs.

Six-Port Reflectometer

The four-probe reflectometer [2] shown in figure 8 is a six-port junction used to display complex reflection coefficients in the same way that a correlator is used to display complex voltage ratios. The reflection coefficient, Γ, is a complex voltage ratio, the ratio of the voltage wave, b, reflected from a device to the voltage wave, a, incident on the device,

$$\Gamma = \frac{b}{a} = \left|\frac{b}{a}\right| e^{j\Psi} \tag{11}$$

where the angle Ψ between a and b is the phase angle of the reflection coefficient. Equations for getting Ψ in terms of amplitude measurements are identical to (1) through (6) if a_1 is replaced by a, a_2 by b, and ϕ, by Ψ. Equations (3) and (6) then become

$$4|ab|\cos\Psi = |a + b|^2 - |a - b|^2 \tag{12}$$

$$4|ab|\sin\Psi = |a - jb|^2 - |a + jb|^2 \tag{13}$$

The four amplitudes on the right of (12) and (13) can be obtained with four identical voltage probes spaced an eighth of a wavelength apart in a uniform lossless transmission line as shown in figure 8. If the outputs from these probes are measured with power detectors $P_5 \ldots P_8$, equations (12) and (13) become

$$|ab|\cos\Psi = K(P_5 - P_7) \rightarrow X_\Psi \tag{14}$$

$$|ab|\sin\Psi = K(P_6 - P_8) \rightarrow Y_\Psi \tag{15}$$

where K is determined by the coupling between the probes on the transmission line. If the voltage X_Ψ and Y_Ψ are connected to a scope, it will display a dot indicating the phase angle Ψ, and the amplitude $|ab|$ which is proportional to $|\Gamma|$.

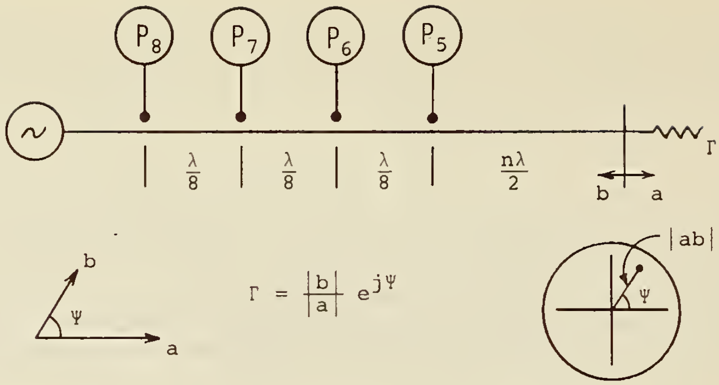


Figure 8. A four-probe reflectometer consisting of four identical voltage probes spaced an eighth of a wavelength apart beginning at a distance $n\lambda/2$ from the reference plane.

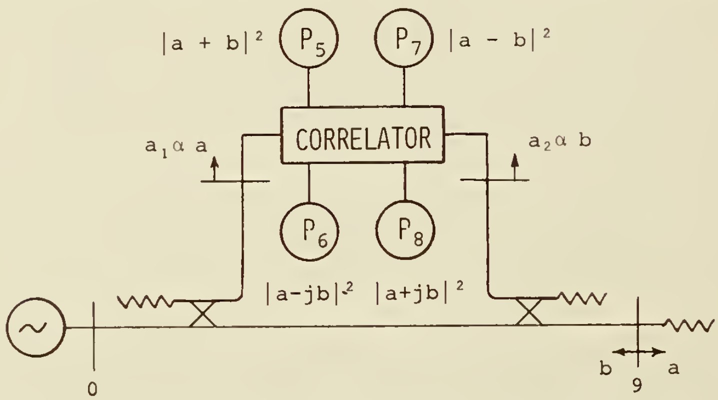


Figure 9. Using two directional couplers with a correlator to get the same outputs as the four-probe reflectometer.

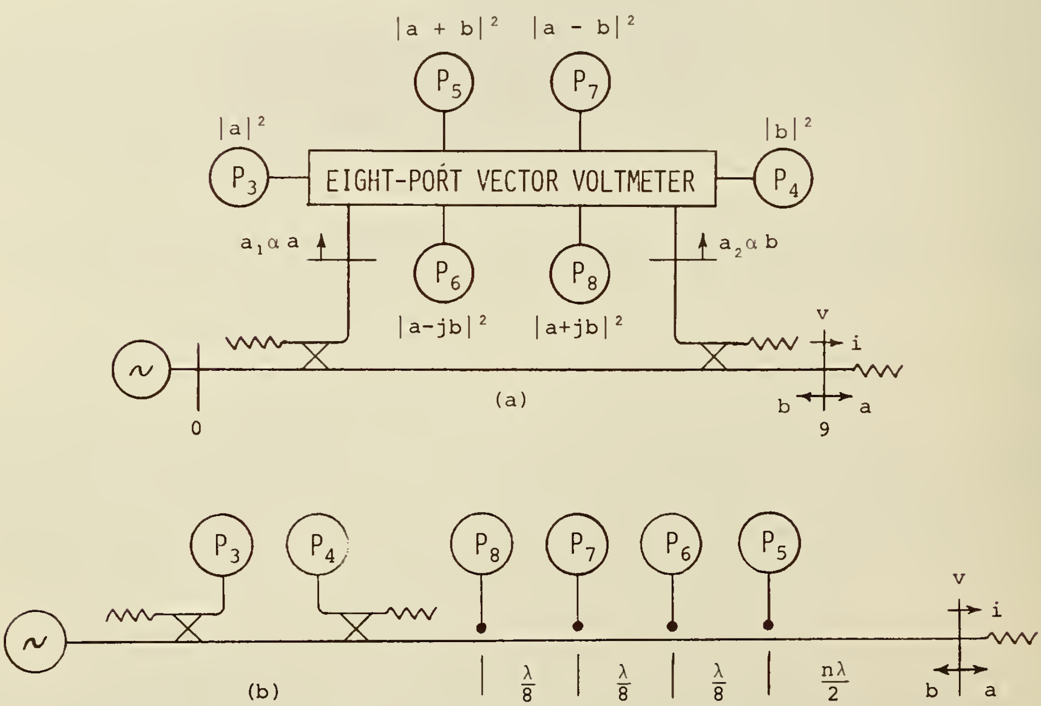


Figure 10. Eight-port reflectometers capable of displaying Γ, Z or Y directly on a scope.

Another way to make a six-port reflectometer for measuring complex reflection coefficients in terms of amplitude measurements is to use a correlator with two directional couplers as shown in figure 9. The outputs of the two (ideal) couplers, which become the inputs to the correlator, are proportional to a and b;

$$a_1 = \frac{S_{10}}{S_{90}}\, a \quad , \quad a_2 = S_{29}\, b \tag{16}$$

where the S's are scattering coefficients between ports indicated by the subscripts. If the scattering coefficients satisfy

$$\frac{S_{10}}{S_{90}} = S_{29} \quad , \tag{17}$$

then the correlator measures the ratio $a_2/a_1 = b/a$ which is Γ. Equation (17)is satisfied by adding attenuation and phase at the output of one of the couplers. Equations for calculating or displaying Ψ and $|ab|$ are identical to (14) and (15) with $K = R/|S_{29}|^2$. One advantage of the correlator-coupler design is that broadband components can be used in its construction, whereas the four-probe design has the correct probe spacing only at one frequency.

Eight-Port Reflectometer

An eight-port reflectometer can be constructed by connecting the eight-port vector voltmeter of figure 5 to the output of two directional couplers as shown in figure 10. This arrangement has several advantages over the six-port reflectometers in figure 8 and 9. The additional two detectors P_3 and P_4 make it possible to separate $|a|$ and $|b|$ and also to display impedance, Z, and admittance, Y, as well as Γ directly on a scope.

If (17) is satisfied, the six outputs of the ideal eight-port reflectometer give a set of equations similar to those in (9) and (10) for the ideal eight-port vector voltmeter;

$$|a|^2 = K\,P_3 \tag{18a}$$

$$|b|^2 = K\,P_4 \tag{18b}$$

$$|ab|\cos\Psi = K(P_5 - P_7) \tag{18c}$$

$$|ab|\sin\Psi = K(P_6 - P_8) \tag{18d}$$

where

$$K = \frac{2R}{|S_{29}|^2} \tag{19}$$

The reflection coefficient can be written

$$\Gamma = \frac{|ab|(\cos\Psi + j\sin\Psi)}{|a|^2} \tag{20a}$$

$$= \frac{(P_5 - P_7) + j(P_6 - P_8)}{P_3} \; . \tag{20b}$$

Leveling the generator with P_3 so that $|a|$ is held constant at the reference plane will give a scope display (using X_Ψ and Y_Ψ as before) whose radius is proportional to $|\Gamma|$.

Recall that the voltage, v, and the current, i, at the reference plane are related to the incident and reflected waves by

$$v = a + b \tag{21}$$

$$iZ_0 = a - b \quad . \tag{22}$$

where Z_0 is a constant usually chosen equal to the characteristic impedance at the reference plane. With the help of (21) and (22), one obtains a set of equations similar to (18) giving v, i, and the phase angle θ between v and i in terms of the six amplitude measurements*:

$$|v|^2 = 4KP_5 \tag{23a}$$

$$|iZ_0|^2 = 4KP_7 \tag{23b}$$

$$|viZ_0| \cos\theta = K(P_3 - P_4) \tag{23c}$$

$$|viZ_0| \sin\theta = 2K(P_6 - P_8) \tag{23d}$$

Equations for Z and Y are similar to (20) for Γ.

$$\frac{Z}{Z_0} = \frac{(P_3 - P_4) + 2j(P_6 - P_8)}{4\ P_7} \tag{24}$$

$$YZ_0 = \frac{(P_3 - P_4) - 2j(P_6 - P_8)}{4\ P_5} \tag{25}$$

If X_θ is a voltage corresponding to $(P_3 - P_4)/2$, and Y_θ ($=Y_\psi$) is a voltage corresponding to $P_6 - P_8$, these two voltages can be applied to the X and Y axis of a scope to display a dot indicating the phase angle θ and the amplitude $|viZ_0|$. Leveling the generator with P_7 will hold the current constant at the reference plane, and give a scope display whose radius is proportional to $|Z|$. Leveling the generator with P_5 will hold the voltage constant at the reference plane, and give a scope display whose radius is proportional to $|Y|$. Since the phase angle of Y is $-\theta$, the voltage Y_θ on the Y axis of the scope must be reversed to get the correct sign of the admittance phase angle.

Another way of making an eight-port reflectometer is by adding two directional couplers to the four-probe reflectometer as shown in figure 10b. If (17) is satisfied for the couplers, and if each probe coupling is 6 dB less than the coupler coupling, S_{29}, all of equations (18) - (25) apply to figure 10b also. The design in figure 10b is easier to construct than that in figure 10a, but the above equations will hold only at the frequency where the probe spacing is as indicated.

Limitations of Ideal Circuits

One problem with all of the previous circuits is that it is difficult to get components good enough for precision measurements. Tuners and other hardware can be added to force one or more of the outputs to read exactly as desired [3], but that requires time and patience, and is usually correct at only one fixed frequency.

In attempting to correct for non-ideal components, a detailed circuit analysis of some of the preceding networks was performed at NBS several years ago. This analysis led to the surprising result that, with few exceptions, any linear six-port junction with four ports terminated with amplitude measuring detectors (e.g. power detectors) can be described by

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* Since (17) must be satisfied for (23a) and (23b) to hold true, these equations provide an easy way of determining when (17) is satisfied. With a short at the reference plane so that v=0, attenuation and phase are added to the output of one of the couplers until P_5=0. Likewise, P_7 must equal zero with an open at the reference plane.

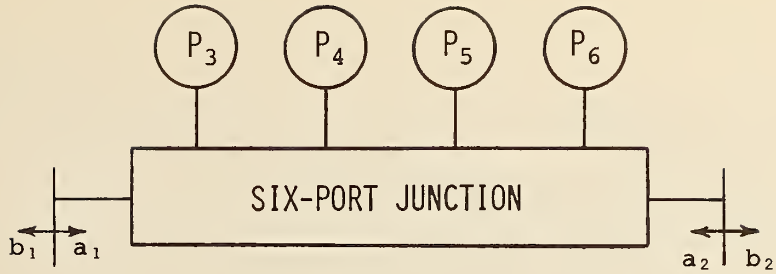

Figure 11. An arbitrary six-port junction with power meters on four of the six ports, being used as a vector voltmeter.

the same set of rather simple but exact equations. The contents of the junction do not matter. It is not necessary to have precision components to make precision measurements. The remainder of this paper will derive the general six-port equations and describe techniques for calibrating an arbitrary six-port for several different applications. An arbitrary six-port will first be analyzed as a vector voltmeter, and then as a reflectometer.

III. ARBITRARY SIX-PORT AS A VECTOR VOLTMETER

In this section we will show that it is possible to use an arbitrary six-port junction as a vector voltmeter, measuring the amplitude of two input signals as well as the phase angle between them in terms of power measurements made on four of the six ports [4].

General Theory

Consider the arbitrary six-port junction shown in figure 11, where four ports are terminated with power meters. If the junction is linear and only one mode is present at each port, the scattering equations for the junction can be written

$$b_i = \sum_{j=1}^{6} S_{ij} a_j \, , \qquad i = 1 \ldots 6, \tag{26}$$

where a_j and b_i are the complex incident and emergent wave amplitudes and the S_{ij} are the scattering parameters of the junction. Assuming that the power meters on arms 3 ... 6 are permanently connected,

$$a_j = b_j \Gamma_j \, , \qquad j = 3 \ldots 6 \tag{27}$$

where Γ_i is the reflection coefficient of the power meter on port j. Equations (26) and (27) represent a collection of ten linear equations in terms of the twelve variables a_i, b_i, $i = 1 \ldots 6$. This system of equations may be solved for any ten of these variables as functions of the remaining two. In particular, it is possible to write

$$b_i = A_i a_1 + B_i a_2 \, , \ i = 3 \ldots 6, \tag{28}$$

where A_i and B_i are functions of the scattering parameters of the junction and the reflection coefficients of the power meters. Multiplying (28) by its complex conjugate yields

$$|b_i|^2 = |A_i|^2 |a_1|^2 + A_i B_i^* a_1 a_2^* + A_i^* B_i a_1^* a_2 + |B_i|^2 |a_2|^2 \qquad 1 = 3 \ldots 6 \tag{29}$$

where (*) indicates complex conjugate.

If the phase angles ϕ_1, ϕ_2 and ϕ are defined such that

$$a_1 = |a_1| e^{j\phi_1} \quad , \quad a_2 = |a_2| e^{j\phi_2} \tag{30}$$

and $\phi = \phi_2 - \phi_1$, then (29) becomes

$$\frac{P_i}{K_i} = |A_i|^2|a_1|^2 + (A_iB_i^* + A_i^*B_i)\ |a_1a_2|\ \cos\phi$$

$$+ |B_i|^2|a_2|^2 + j(A_i^*B_i - A_iB_i^*)\ |a_1a_2|\ \sin\phi\ , \qquad i = 3 \ldots 6 \tag{31}$$

where $P_i \equiv K_i|b_i|^2$ is the power indicated by the meter on the i^{th} port, and K_i is a constant. This expression represents a linear system of four equations in the four unknowns $|a_1|^2$, $|a_2|^2$, $|a_1a_2|\cos\phi$ and $|a_1a_2|\sin\phi$. If these equations are independent†, they may be inverted to obtain each of the four unknowns as a linear function of the four P_i ($i = 3 \ldots 6$). The result is

$$|a_1|^2 = \sum \rho_iP_i \tag{32a}$$

$$|a_2|^2 = \sum \sigma_iP_i \qquad i = 3 \ldots 6 \tag{32b}$$

$$|a_1a_2|\ \cos\phi = \sum x_iP_i \tag{32c}$$

$$|a_1a_2|\ \sin\phi = \sum y_iP_i \tag{32d}$$

where each sum in (32) and throughout this paper is over the four sidearm power readings. The coefficients of P_i are real numbers which are functions of the parameters S_{ij} and Γ_j. Equations (32) constitute the desired result and are valid for any linear six-port junction subject to the conditions mentioned.

Note that it is not necessary to measure absolute power at the sidearms. As long as P_i indicated by the sidearm power detector is proportional to the absolute power, the proportionality constant is absorbed in the coefficient of each P_i in (32).

Using (32)in (10a) gives

$$\frac{a_2}{a_1} = \frac{\sum (x_i + jy_i)P_i}{\sum \rho_iP_i}\ , \tag{33}$$

Equations (32) and (33) should be compared with (9) and (10) for the ideal eight-port vector voltmeter.

A more useful form of equation (33) for calibration purposes is obtained by factoring $x_m + jy_m$ out of the top sum and factoring ρ_n out of the bottom sum to get

$$\frac{a_2}{a_1} = k\,\frac{\sum z_iP_i}{\sum w_iP_i} \tag{34}$$

where k, z_i, and w_i are new constants such that $z_i = 1$ when $i = m$, and $w_i = 1$ when $i = n$, so that only three complex z_i and three real w_i need to be determined by some calibration process. For many applications the complex constant k does not need to be known.

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† This condition is not satisfied by the ideal correlation discussed in section II. For any correlator, the matrix to be inverted in (31) is either singular or nearly so, leading to the ill condition mentioned in the introduction. The "Six-Port Design Criteria" below shows how to modify the correlator to eliminate this ill condition.

Complex Voltage Ratios

Using (21), the voltage at the two input ports can be written

$$v_i = a_i + b_i \qquad\qquad i = 1,\ 2 \tag{35}$$

$$= a_i(1 + \Gamma_i) \tag{36}$$

where Γ_i is the complex ratio b_i/a_i at port i;

$$\Gamma_1 = S_{11} + S_{12}\,\frac{a_2}{a_1} \tag{37}$$

$$\Gamma_2 = S_{22} + S_{21}\,\frac{a_1}{a_2} \tag{38}$$

The scattering parameters in (37) and (38) are those of the equivalent two-port obtained by terminating the four sidearms of the six-port with power meters. The ratio of the two input voltages is

$$\frac{v_2}{v_1} = \frac{a_2(1+\Gamma_2)}{a_1(1+\Gamma_1)}\ . \tag{39}$$

The voltage ratio will be proportional to a_2/a_1 provided that there is sufficient isolation between input ports 1 and 2 so that any change in Γ_1 or Γ_2 due to a change in a_1 or a_2 is negligible.

Self-Calibration Procedure

All of the constants in (34) except k can be determined by a self-calibration technique which does not require any standards. A calibration circuit such as shown in figure 12 is used. A signal is divided into two channels which are connected to the inputs of the six-port. The signal a_1 in one channel is held constant by internally leveling the generator and isolating it from the signal a_2 in the other channel which contains a level set attenuator, α_0, phase shifter, ϕ_0, and a two-position insertion device. Data for calibrating the six-port is obtained by noting the value of all P_i for the two positions of the insertion device at different settings of α_0 and ϕ_0. The value of the insertion device does not need to be known, but it must be highly reproducible and independent of signal level.

The initial value of a_2 relative to a_1 is determined by the setting of α_0 and ϕ_0, which also need not be known. When the insertion device is switched to its second position, a_2 changes to a_2' and the power readings change from P_i to P_i'. Assuming that a_1 is constant during the time it takes to read P_i and P_i', (32a) gives

$$\frac{|a_1|^2}{\rho_n} = \sum w_i P_i = \sum w_i P_i'\ . \tag{40}$$

The ratio of a_2'/a_2 obtained from (34) is

$$L \equiv \frac{a_2'}{a_2} = \frac{\sum z_i P_i'}{\sum z_i P_i}\ . \tag{41}$$

This L is the change in insertion ratio of the two-position insertion device.

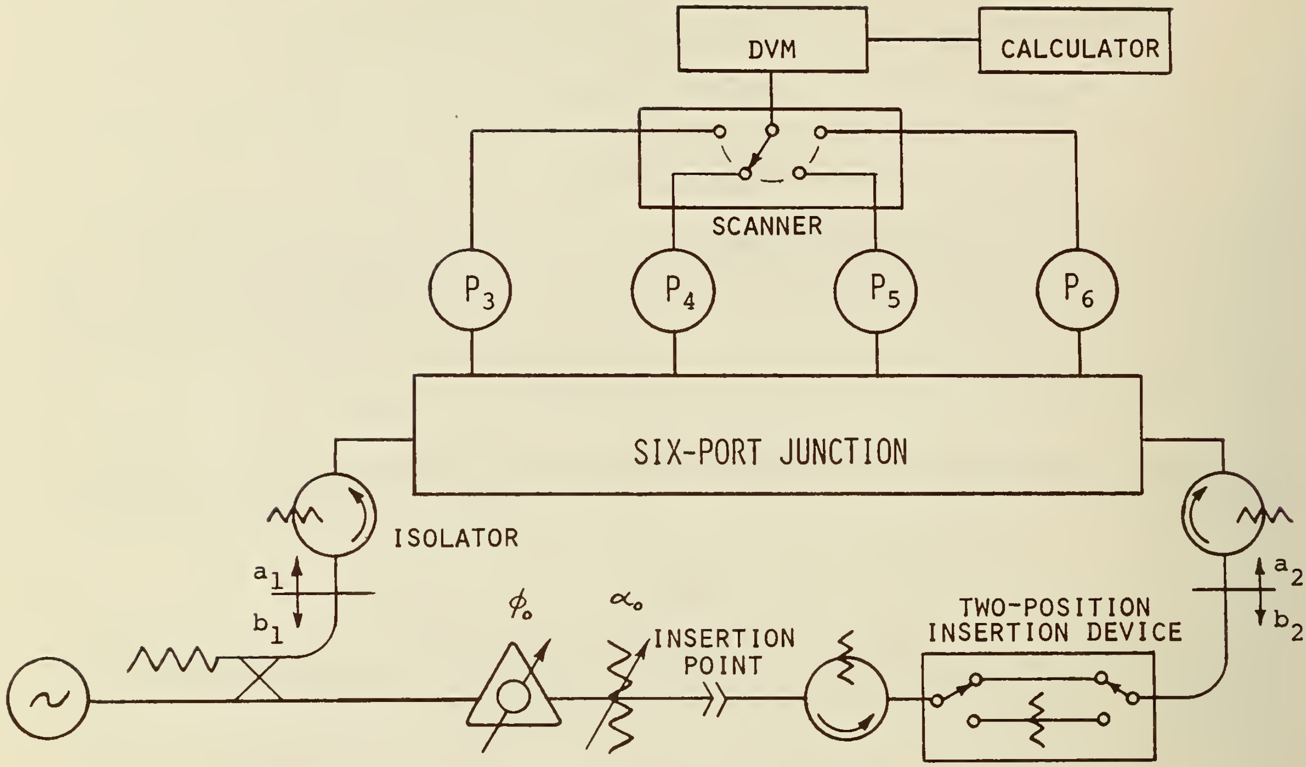

Figure 12. Setup for calibrating and using a six-port junction to measure complex voltage ratios.

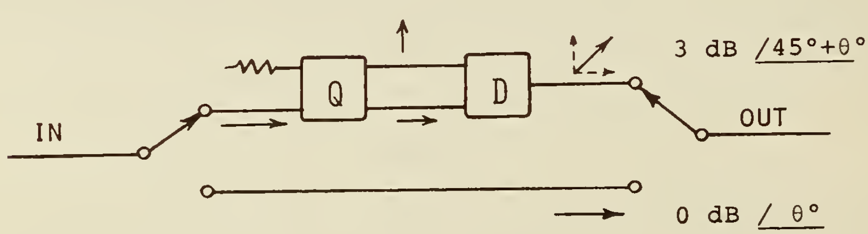

Figure 13. Using a quadrature hybrid (Q) and a power divider (D) to make an insertion ratio of 3 dB with 45° phase shift.

The measurements of P_i and P_i' are repeated for four or more different settings of α_0 and ϕ_0. Each different setting gives additional equations like (40) and (41) where L is the same for each measurement. These two sets of equations can be solved for the calibration constants z_i, w_i and also for L. If L is known it can be thought of as the standard in the calibration process. However the calibration process works equally well if L is not known.

To assure that L remains constant when α_0 and ϕ_0 are changed, and that a_1 remains constant at the two position of L, isolators are added as shown in figure 12.

The constants ρ_n, and $x_m + jy_m$ cannot be determined by this calibration process. However, for measuring complex insertion ratios, these constants are not needed. Complex insertion ratios of a_2'/a_2 can now be measured using the known z_i in (41). Using the z_i and w_i in (34), ratios of a_2/a_1 can be measured to within a constant k.

Broadband Two-Position Insertion Device

In calibrating the six-port, the complex insertion ratio, L, of the repeatable two-position insertion device must not have a phase angle of 0° or multiples of 90°. A phase shift of 45° works well and is easy to generate. One way of getting 45° phase shift over a broad frequency range is shown in figure 13. The two outputs of the quadrature hybrid (Q) are equal in amplitude but 90° out of phase. Adding these two signals with an in-phase power divider (D) gives a signal that is shifted 45° relative to the input signal. In addition to this 45° there will be some phase shift θ due to the lengths of line. The length of the lower path can be adjusted to give a phase shift equal to θ. The phase difference in the two switch positions will be 45° over the complete frequency range of the hybrid and divider, which can be the same as the frequency range of the six-port. The amplitude of the insertion ratio will be 3 dB.

The optimum value of $|L|$ to calibrate the six-port has not been determined. Values near 3 dB and 8 dB have been used with no noticeable difference in results.

Six-Port Design Criteria

Comparing (32) and (9) leads one to expect that the four outputs of a practical six-port vector voltmeter would be some combination of the six outputs of the ideal eight-port vector voltmeter shown in figure 5. This is indeed the case. These six outputs are listed in Table 2. It can be shown that a practical set of four outputs includes one from each group in Table 2. The fourth output can be any one of the six not already used. For example, if $|a_1| \simeq |a_2|$, a practical six-port vector voltmeter could have outputs approximately proportional to

$$|a_1|^2,\ |a_1 + a_2|^2,\ |a_1 - ja_2|^2\ ,\ \text{and}\ |a_2|^2. \tag{42}$$

Or if $|a_2| << |a_1|$ one might design the outputs to approximate

$$|a_1|^2,\ |a_1 + a_2|^2,\ |a_1 - ja_2|^2,\ \text{and} \left\{ \begin{array}{c} |a_1 - a_2|^2 \\ \text{or} \\ |a_1 + ja_2|^2 \end{array} \right\} \tag{43}$$

Here $|a_1 - a_2|^2$ or $|a_1 + ja_2|^2$ is used instead of $|a_2|^2$ because $|a_2|^2$ might be too small to measure, but $|a_1 - a_2|^2$ or $|a_1 + ja_2|^2$ would still contain useful information about a_2.

It should be emphasized that the outputs listed in Table 2 are only design goals. The actual outputs of a six-port can depart considerably from these values and still be quite useful. When the outputs depart greatly from those in Table 2, the coefficients of P_i in (32) become large so that the desired information is obtained from the difference between large terms in the sum. As the individual terms in the sum become significantly larger than the quantity on the left of the corresponding equal sign in (32), greater precision is required in measuring each P_i to obtain a given accuracy.

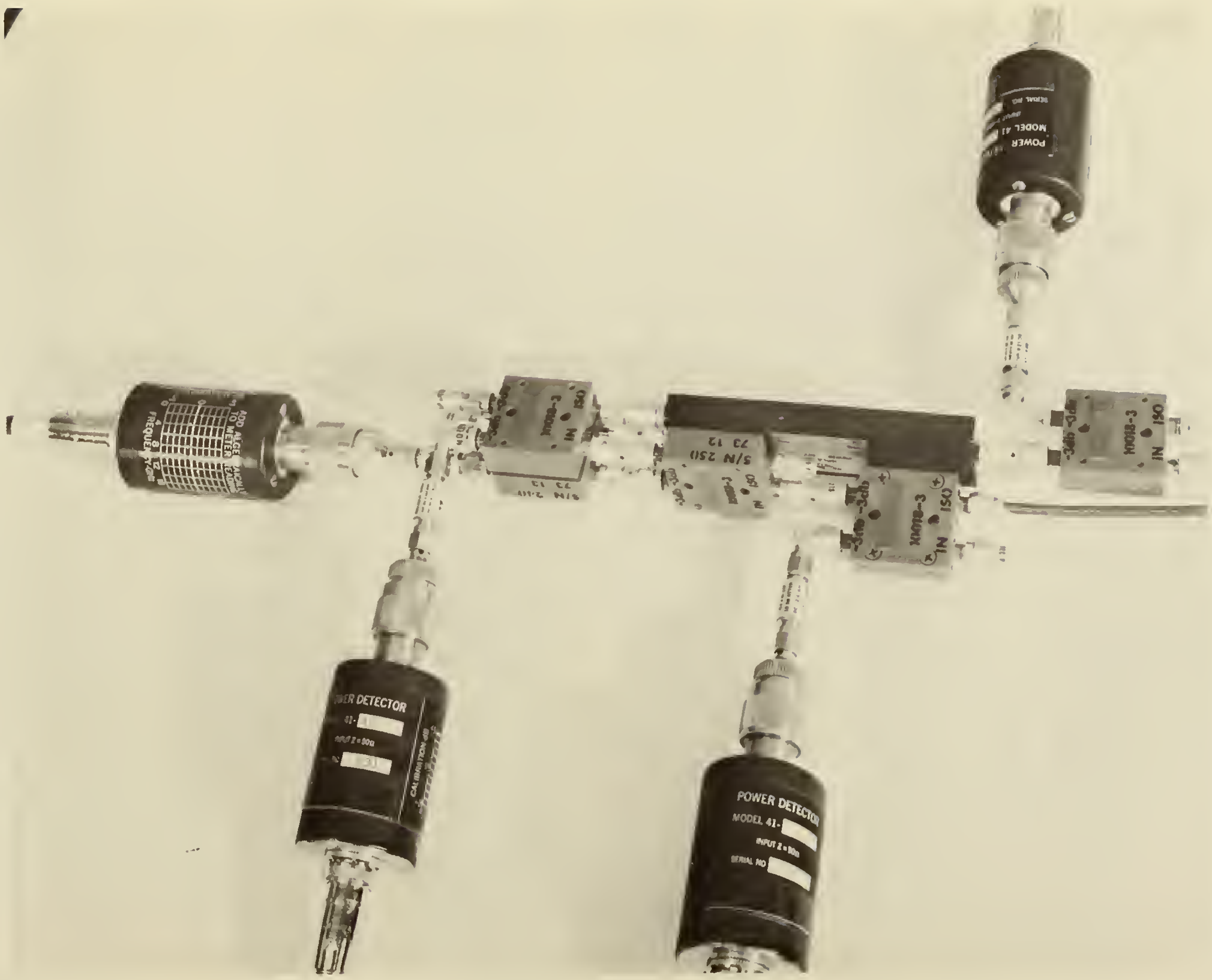

Figure 14. Experimental six-port junction and power detector mounts.

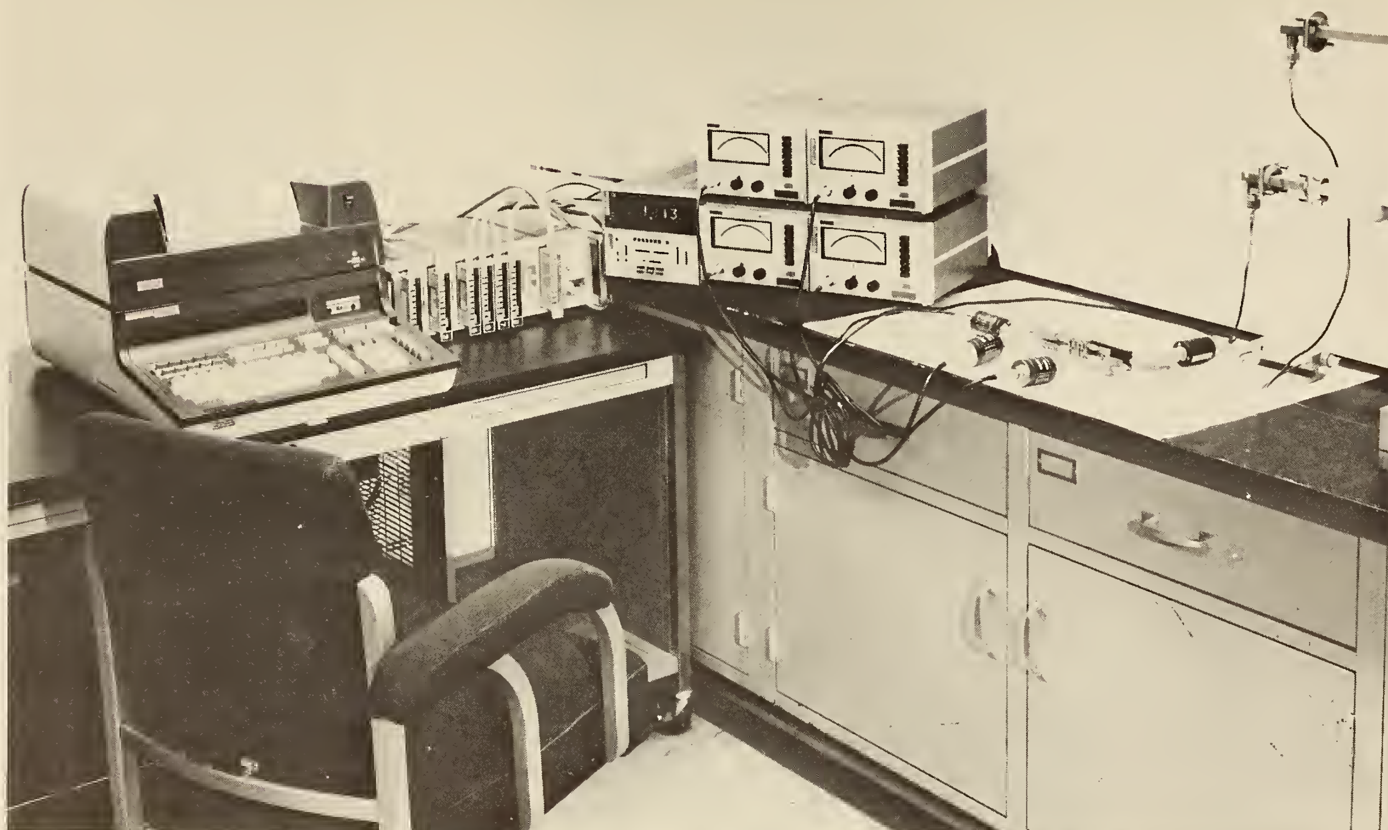

Figure 15. Experimental six-port X-band vector voltmeter setup. From left to right: Programmable calculator, Input/Output Expander, DVM and Scanner, four power meters, six-port junction (on white paper), and two input signal lines.

Group 1	Group 2	Group 3
$\lvert a_1 \rvert^2$	$\lvert a_1 + a_2 \rvert^2$	$\lvert a_1 - ja_2 \rvert^2$
$\lvert a_2 \rvert^2$	$\lvert a_1 - a_2 \rvert^2$	$\lvert a_1 + ja_2 \rvert^2$

Table 2. Outputs of an ideal eight-port vector voltmeter, used as a guide in designing a six-port vector voltmeter.

FREQUENCY, GHz,		8	9	10	11	12
ATTENUATION DB	SIX-PORT	7.82	7.58	7.52	7.91	8.53
	ANA	7.75	7.57	7.48	7.92	8.36
	DIFFERENCE	.07	.01	.04	-.01	.17
PHASE DEG.	SIX-PORT	38.15	34.13	33.19	31.49	31.00
	ANA	38.09	34.81	32.45	31.73	30.91
	DIFFERENCE	.06	-.68	.74	-.24	.09

Table 3. Amplitude and phase change in the two-position step attenuator as measured by the six-port with diode power meters and by the NBS Automatic Network Analyzer.

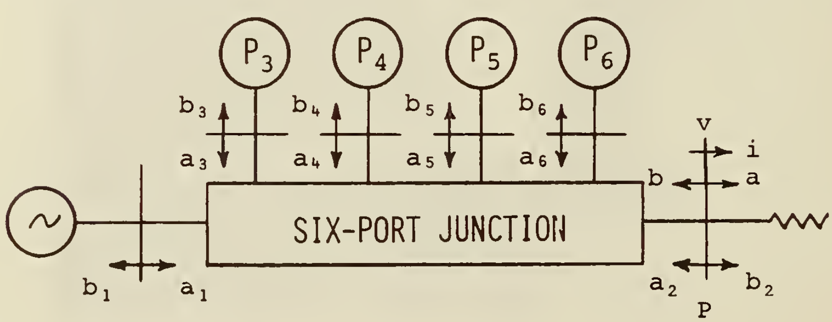

Figure 16. An arbitrary six-port junction with power meters on four of the six ports being used as a reflectometer.

Note that the correlator by itself does not satisfy the above design criteria. It has no output in group 1 of Table 2. Although the correlator is a six-port device, the equations in (32) do not apply in the limit when the correlator becomes ideal because the four outputs are not linearily independent. To see this, add (1) and (2) and also add (4) and (5) to get

$$2|a_1|^2 + 2|a_2|^2 = |a_1 + a_2|^2 + |a_1 - a_2|^2$$

$$= |a_1 + ja_2|^2 + |a_1 - ja_2|^2 \tag{44}$$

This equation shows that the ideal correlator has only three independent outputs since any one output can be obtained from the other three. In a sense, the ideal correlator is only a five-port junction. One more independent output must be added to the correlator to have (32) apply. That output should be one of those in group 1 of Table 2.

Experimental Setup

Several six-port measurement systems have been constructed at NBS, Boulder. In one experiment, the eight-port vector voltmeter shown in figure 6 was changed to a six-port by terminating ports 7 and 8 with 50 Ω loads. The remaining four outputs are proportional (roughly) to those in (42). The back diodes were replaced with diode type power meters having a linearity of ±1% from 10 nw to 10 μw. The power level into each diode was kept less than 10 μw to assure square-law operation. Figure 14 shows the six-port and detectors.

Since the six-port calibration and measurement process requires taking many sidearm power measurements, it is most desirable to have the data read directly into a computer which can then process the data. A programmable calculator is capable of taking the necessary sidearm power readings under program control, and then processing the data to give the calibration constants and measurement results.

Basic language programs have been written for calibrating the junction as a microwave vector voltmeter, and also for using it to measure complex insertion ratio. After calibrating the six-port using the setup in figure 12, the same setup is used to measure complex insertion ratios by either changing or inserting something in the test (lower) channel, and using (41) to calculate the ratio.

A picture of the setup is shown in figure 15. The same setup can be used with any type power detectors that have an output dc voltage which is a known function of the input power level.

Results

Measurements were made on this setup at 8, 9, 10, 11, and 12 GHz. The complex insertion ratio of the two-position step attenuator L, was measured with the six-port and then measured by the NBS Automatic Network Analyzer. The results are shown in Table 3. The agreement is within what one would expect using detectors accurate to 1%. Better power detectors should give greater accuracy. The comparison shows that the theory of using an arbitrary six-port junction as a vector voltmeter is correct. It also shows that the six-port junction can be calibrated without using any standards. The only precision component in the setup is the two-position step attenuator whose change in insertion ratio is repeatable to ±.001 dB.

IV. ARBITRARY SIX-PORT AS A REFLECTOMETER

General Theory

An arbitrary six-port junction can be analyzed as a reflectometer in much the same way that it was analyzed as a vector voltmeter in section III [5]. Again consider the arbitrary six-port junction shown in figure 11. Let a signal be applied to port 1, and let port 2 be a reference plane at which we wish to make measurements of a, b, and Γ (or v, i, and Z), as shown in figure 16. The derivation of the six-port reflectometer equations follows the same steps used in (26) through (32), except that in (28) b_i is written in terms of a and b instead of a_1 and a_2;

$$b_i = C_i a + D_i b, \quad i = 3 \ldots 6 \tag{45}$$

where C_i and D_i are constants which are functions of S_{ij} and Γ_j. Multiplying (45) by its complex conjugate yields four equations similar to (31) only in terms of the four unknowns $|a|^2$, $|b|^2$, $|ab| \cos \Psi$, and $|ab| \sin \Psi$. If these equations are independent,* they can be inverted to obtain each of the unknowns in terms of the four sidearm power readings. The desired result is

$$|a|^2 = \sum \alpha_i P_i \tag{46a}$$

$$|b|^2 = \sum \beta_i P_i \tag{46b}$$

$$|ab| \cos \Psi = \sum c_i P_i \tag{46c}$$

$$|ab| \sin \Psi = \sum s_i P_i \tag{46d}$$

where each sum is over the four sidearm power readings. The coefficients of P_i are again real numbers. The set of equations in (46) is valid for any linear six-port, subject to the same conditions mentioned in arriving at (32). Using (46) in (20a), the reflection coefficient at the reference plane becomes

$$\Gamma = \frac{\sum (c_i + js_i) P_i}{\sum \alpha_i P_i} \tag{47}$$

Equations (46) and (47) should be compared with (18) and (20) for the ideal eight-port reflectometer.

A set of equations similar to (46) can be obtained in terms of v, i, and θ instead of a, b, and Ψ. Using (21) and (22) in (45) gives

$$b_i = \frac{C_i + D_i}{2} v + \frac{C_i - D_i}{2} i Z_0$$

$$= E_i v + F_i i \qquad i = 3 \ldots 6 \tag{48}$$

where E_i and F_i are new constants playing the same roll as A_i and B_i in (28). Multiplying (48) by its complex conjugate and solving the resulting four equations yields

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* Like the correlator, the outputs of the four-probe reflectometer are not linearly independent and lead to a singular or nearly singular matrix in solving these equations. See "Design Criteria" below.

$$|v|^2 = \sum \nu_i P_i \tag{49a}$$

$$|i|^2 = \sum \mu_i P_i \tag{49b}$$

$$P = |vi| \cos \theta = \sum q_i P_i \tag{49c}$$

$$P'' = |vi| \sin \theta = \sum r_i P_i \tag{49d}$$

The coefficients of P_i are again real constants. Equation (49c) is recognized as the real net power, P, at the reference plane, and (49d) is the reactive power P". Equations similar to (47) are obtained from (49) for the impedance and admittance;

$$Z = \frac{\sum (q_i + jr_i) P_i}{\sum \mu_i P_i} \tag{50}$$

$$Y = \frac{\sum (q_i - jr_i) P_i}{\sum \nu_i P_i} \tag{51}$$

One can show that the constants in (49) and (46) are related by the following equations

$$\nu_i = \alpha_i + \beta_i + 2c_i \tag{52a}$$

$$Z_0{}^2\mu_i = \alpha_i + \beta_i - 2c_i \tag{52b}$$

$$Z_0 q_i = \alpha_i - \beta_i \tag{52c}$$

$$Z_0 r_i = 2\, s_i \tag{52d}$$

Calibration Techniques

The simplest calibration proceedure is that for obtaining the q_i in (49c) so that the six-port can be used to measure net power P [6]. Referring to figure 16, port 2 is first terminated with a power standard so that $P \equiv P_{std}$ is known. The corresponding values of $P_3 \ldots P_6$ are noted and used in (49c) to get

$$P_{std} = \sum q_i P_i \quad . \tag{53}$$

This proceedure is then repeated with the power standard replaced by three or more different offset shorts (sliding short or variable lossless reactances) for which P = 0 so that (53) becomes

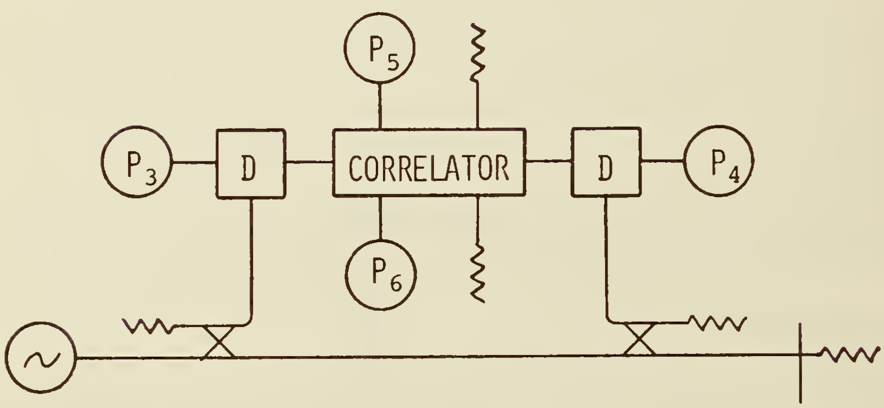
$$0 = \sum q_i P_i \tag{54}$$

equation (53) and three or more equations like (54) may be inverted to obtain the four q_i.

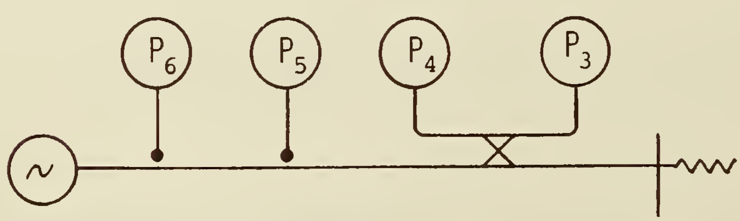
It should be noted that the standard power meter may be of arbitrary and unknown impedance. In addition, the arguments of the offset shorts are not required, although it is obvious that multiples of $\lambda/2$ must be avoided.

Group 1	Group 2	Group 3
$\lvert a\rvert^2$	$\lvert a + b\rvert^2$	$\lvert a - jb\rvert^2$
$\lvert b\rvert^2$	$\lvert a - b\rvert^2$	$\lvert a + jb\rvert^2$

Table 4. Outputs of an ideal eight-port reflectometer, used as a guide in designing a six-port reflectometer.



(a)



(b)

Figure 17. Six-port reflectometers which satisfy the design criteria in Table 4.

With the q_i known, the net power into any unknown impedance can be measured using (49c). Since the impedance of the load does not have to be known, mismatch errors [7] usually associated with power measurements are eliminated.

Calibrating the six-port to measure Γ is only slightly more difficult. In addition to observing the P_i for three or more different positions of an offset or sliding short, the P_i are also observed for two or more positions of a low reflection termination. With this set of measurements, all of the constants needed to calculate Γ may be determined. Details are given in reference [8]. The minimum of five measurements needed to calibrate a six-port reflectometer should be compared with a minimum of three measurements required to calibrate a four-port reflectometer having a complex ratio detector on its sidearms.

If the measurement with a power standard is included in the above set all 16 of the coefficients of P_i in (46) can be determined. The coefficients of P_i in (49) can be obtained from (46). The six-port reflectometer is then completely calibrated for measuring voltage, current, power, impedance, and phase angles.

Design Criteria

The goals to strive for in designing a six-port reflectometer are similar to those for designing a six-port vector voltmeter. It can be shown that a practical set of four outputs includes one from each group in Table 4 plus a fourth which can be any one of the six not already chosen. As with the six-port vector voltmeter, the actual outputs of a six-port reflectometer can depart considerably from its design goals and still be quite useful. Two examples of six-port reflectometers which satisfy the above design criteria are shown in figure 17. Both were obtained from figure 10 by terminating ports 7 and 8. The design in figure 20b is particularly interesting. Whereas the design in figure 10b is useful at only one frequency when using equations (18) - (25), the design in figure 17b is useful over at least a full waveguide band when using (46) - (52). This is because the $\lambda/8$ spacing now needs to be satisfied only approximately at the center of the band. Also the distance from the probe to the reference plan, and the relative probe couplings are all of secondary importance.

The design in figure 17a can be constructed to have a useful frequency range of four or more octaves. Precision components are not required, and components can usually be used considerably outside of their normal specified frequency range.

In trying to apply (46) and (49) to the four-probe reflectometer in figure 8, one finds that it has the same problem as the correlator in that its outputs are not all independent. The identity

$$|a + b|^2 + |a - b|^2 = |a + jb|^2 + |a - jb|^2 \tag{68}$$

shows that the four-probe reflectometer has only three independent outputs. Just like the correlator, the four-probe reflectometer is, in a sense, only a five-port junction requiring one more independent output for (46) and (49) to apply. The output needed is one from Group 1 in Table 4.

Experimental Results

The first application of the six-port concept was at NBS-Boulder using an X-band waveguide six-port reflectometer to measure net power [6]. To verify the theory, a six-port reflectometer was placed in series with a tuned four-port reflectometer, the most accurate instrument for measuring net power. Both instruments were calibrated to measure power at the same reference plane so that both instruments could measure the same power at the same time. The power into a variable impedance load was measured at 10 GHz. The agreement between the two instruments was .1% for $|\Gamma|<.1$ and .25% for $|\Gamma|<.2$ at any phase angle. These results showed that a six-port could be used to calibrate power meters (whose $|\Gamma|$ seldom exceeds 0.2) almost as accurately as a tuned four-port reflectometer without the need to tune the measuring instrument at each frequency.

At the present time work is continuing at NBS to improve and evaluate the design shown in figure 17b and to simplify the calibration software for using the six-port to measure Γ.

V. FUTURE APPLICATIONS

One obvious application of the six-port concept is in computer controlled measurement systems. If diodes or other power detectors are used which have an output dc voltage proportional to the input rf power, all desired information is obtained from four dc voltage measurements. Adding a scanner, DVM, and a programmable calculator or minicomputer to the setup such as shown in figure 12 makes an automated measurement system. All components used to calibrate the six-port vector voltmeter can be controlled by the calculator so that there is no operator involvement. In calibrating a six-port reflectomater, the operator must connect the different loads but that is required to calibrate other automated reflectometers also.

Since it is not necessary to heterodyne the signal down to a lower frequency to measure phase angles, only one signal source is required and it need not be a phase-locked source as is required in some rf measuring systems. The source could be a programmable sweep oscillator which should be accurate enough for most applications.

Another application of the six-port concept is in the millimeter wave region where it is much more difficult to get phase information by other techniques. Components are commercially available for constructing the six-port reflectometer in figure 17b up to about 200 GHz.

Finally, it should be possible to use two six-port reflectometers to measure all of the scattering parameters of a two-port device by connecting one six-port on each side of the two-port. Preliminary studies indicate that it may be even simpler to calibrate two six-ports than one.

VI. BIBLIOGRAPHY

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16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

This review paper brings together a number of old and new methods for measuring voltage, current, power, impedance, and phase angle using only amplitude type detectors. Vector voltmeters and reflectometers can be constructed to measure all of these quantities in terms of four amplitude measurements made on four arms of a six-port junction. Whereas previous uses of this type of instrument depended on precision components for accuracy, new equations switch this dependence primarily to detector linearity and only secondarily on the properties of the measuring device itself.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)* Admittance; automated precision measurements; correlator; current; impedance; microwave circuit parameters; microwave measurements; phase angle; power; ratio; reflection coefficient; reflectometer; self-calibration; six-ports; vector voltmeter; voltage.

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