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Considerations for the Precise Measurement of Amplifier Noise

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Considerations for the Precise Measurement of Amplifier Noise

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CONSIDERATIONS FOR THE PRECISE MEASUREMENT OF AMPLIFIER NOISE

For the best accuracy in measuring noise figure, attention needs to be given to the choice of the hot and cold noise standards and to mismatch problems. Tables and graphs are presented to aid in choosing the proper measurement conditions, and an example is given to demonstrate their use. This paper essentially supplements a previous paper (included in an appendix), treating in more detail topics that become important when state-of-the-art measurements are required.

Key words: Amplifier noise; effective input noise temperature; mismatch error; mismatch uncertainty; noise figure.

1. Introduction

The problem of measuring amplifier noise is extensively documented in the literature [1,2]. For the reader's convenience, a general overview of this problem is reprinted in Appendix E (along with other related papers). This technical note provides additional detailed information concerning the accuracy of measuring amplifier noise with the various noise standards presently available. This additional information should be helpful in evaluating or designing measurements of amplifier noise as accurate as ± 0.3 dB in noise figure or ± 7% in effective input noise temperature.

With the present state-of-the-art, the least inaccuracy to which amplifier noise figures can be measured is approximately 0.1 dB in noise figure or 2% in effective input noise temperature. This paper is especially addressed to those who intend to make such a state-of-the-art noise measurements.

1.1 The Basic Measurement

The most accurate measurements of amplifier noise use some version of the so-called Y-factor method. In this method, two noise standards with noise temperatures of T_{hot} and T_{cold} are sequentially connected to the input of the unknown amplifier*, and the ratio of the noise powers, Y, out of the unknown amplifier is measured. This parameter Y may be written in terms of the Effective Input Noise Temperature** of the unknown amplifier, T_e, as follows:

$$Y = (T_{hot} + T_e)/(T_{cold} + T_e). \tag{1}$$

Because T_{hot} and T_{cold} are known, and Y is measured, then T_e is known:

$$T_e = (T_{hot} - Y\, T_{cold})/(Y - 1). \tag{2}$$

Also in common use as a measure of amplifier noise is noise figure, F_{dB}. To avoid the ambiguities and difficulties of the IEEE definition of F_{dB} discussed in [1], the definition of F_{dB} used in [1] is used here, namely

$$F_{dB} \equiv 10 \log[1 + T_e/290]. \tag{3}$$

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*In this paper, the term "unknown amplifier" is used to refer to the amplifier whose Effective Input Noise Temperature, or Noise Figure we want to know.

**The definition of T_e and F_{dB} are discussed in [1] which is reprinted in Appendix E.

1.2 Accuracy Considerations

In crude terms, measurements of T_e within 2% or F_{dB} within 0.1 dB require $T_{hot} >> T_e >> T_{cold}$, and require that the reflection coefficients of the noise standards closely match the reflection coefficient of the "antenna"* in both magnitude and phase angle.

There are at least seven important sources of error in a typical measurement of T_e: (1) uncertainty in the value of T_{hot}, (2) uncertainty in the value of T_{cold}, (3) uncertainty in the measurement of Y, (4) amplifier-gain instability during the time required to measure the ratio Y, (5) inequality of the reflection coefficients of the "antenna" and the two standard noise sources, (6) uncertainties in input connector losses with the "antenna" and each of the two standard noise sources connected to the unknown amplifier, and (7) errors in correcting for the noise originating in the measuring system (cascade error). As we shall see, the magnitude of these various errors depend on the choice of T_{hot} and T_{cold}.

To minimize the measurement error requires two steps. First the hot and cold standards are selected, then the other measurement conditions are investigated. Some major factors that influence measurement accuracy are discussed first, then the state-of-the-art is discussed, and last a measurement example is given to clarify the use of this technical note.

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*The term "antenna" in this paper refers to all of the components that will be attached to the input of the unknown amplifier when it is being used in its intended application. (If the end use is not known, then it is usual to assume that the "antenna" is reflectionless. However, the concept of an unknown "antenna" is not in the spirit of a precision amplifier noise measurement.)

To avoid breaking up the text, the long series of tables or graphs are collected in the appendices.

2. Hot and Cold Standards

Some of the choices for the hot and cold standard noise temperatures that are available are shown in Table 1.

Table 1. Thermal Noise Sources

Source*	Typical Effective Temperature and state-of-the-art uncertainty
1. Neon gas-discharge	18,000 ± 270 K (1.50%)
2. Argon gas-discharge	11,000 ± 165 (1.50%)
3. WR15, WR62, WR90--NBS Standards	1250 ± 3 (0.24%)
4. WR284--NBS Standard	692 ± 0.9 (0.13%)
5. Commercial coaxial hot standard	373 ± 0.5 (0.13%)
6. 14 mm coaxial--NBS Standard	373 ± 0.15 (0.04%)
7. Unregulated ambient temperature load	300 ± 1 (0.33%)
8. Regulated ambient temperature load	300 ± 0.1 (0.03%)
9. Commercial LN_2 load	80 ± 1 (1.25%)
10. 14 mm coaxial, WR90--NBS LN_2 Standards	80 ± 0.2 (0.25%)
11. Commercial LH_e standard	4 ± 0.5 (12.5%)
12. WR90--NBS LH_e standard	4 ± 0.1 (2.50%)

Four of the error contributions to T_e or F_{dB}, namely the uncertainty in the values of T_{hot}, T_{cold}, Y, and gain instability, depend on the choice of T_{hot} and T_{cold}. Appendix A

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*Frequency coverage for the NBS 14 mm coaxial source is d.c. to 1.2 GHz, for WR284 is 2.60 to 3.95 GHz, for WR90 is 8.2 to 12.4 GHz, for WR62 is 11.9 to 18.0 GHz, and for WR15 is 50 to 75 GHz.

contains tables that list these dependent measurement error contributions for various values of T_e or F_{dB} for combinations of T_{hot} and T_{cold} selected from Table 1. Table 2 summarizes some of the results given in Appendix A. The error listed under F_{dB} in Table 2 is the magnitude of the uncertainty of F_{dB}. Thus by $F_{dB} = 8 \pm 0.1$ dB, we mean the true value of F_{dB} is between 7.9 and 8.1 dB. The uncertainty in Y, and variations in amplifier gain used in Table 2 and Appendix A approximate the present state-of-the-art. For different uncertainties in the parameters used, one needs to refer back to Appendix A and make the modifications explained there.

3. Mismatch Ambiguity

Mismatch ambiguity (or uncertainty as it is called in [1]) is the ambiguity in effective input noise temperature because of an ambiguity in specifying the reflection coefficient of the "antenna," Γ_{ant}. It is best to measure the mismatch ambiguity (see [1] in Appendix E), but to give an idea of the magnitude of the mismatch ambiguity, it is examined below.

The simplest assumption for amplifier noise is that the amplifier is linear (that is, its output voltage y is related to its input voltage x in the form $y = ax + b$, where a and b are constants). If in addition to being linear, we assume isolation such that an impedance change at the amplifier's

Table 2. Errors for Various T_{hot}, T_{cold} Combinations

The resultant error in T_e and F_{db} listed is caused by the uncertainties (as indicated) in T_{hot} and T_{cold}, a 0.01 dB uncertainty in the measurement of Y, and 0.1% variation in amplifier gain during the measurement time.

		T_e in K		F_{dB} in dB		Reference Table
T_{hot} (K)	T_{cold} (K)	(Range)	(Error)	(Range)	(Error)	in Appendix A
18,000 ± 270	300 ± 1	1500-20,000	(± 2.3%)	8-19	(± .1)	A-I
18,000 ± 270	80 ± 1	700-20,000	(± 2.2%)	4-19	(± .1)	A-II
18,000 ± 270	4 ± 0.5	150-20,000	(± 2.2%)	1.5-19	(± .1)	A-III
11,000 ± 165	300 ± 1	2000-10,000	(± 2.3%)	1.8-14	(± .1)	A-IV
11,000 ± 165	80 ± 1	700-15,000	(± 2.3%)	5-17	(± .1)	A-V
11,000 ± 165	4 ± 0.5	150-15,000	(± 2.3%)	1.5-17	(± .1)	A-VI
1,250 ± 3	300 ± 0.1	200-3000	(± 2%)	2.3-10	(± .08)	A-VII
1,250 ± 3	80 ± 0.2	150-2000	(± 1.2%)	1.8-9	(± .05)	A-VIII
1,250 ± 3	4 ± 0.1	30-1500	(± 1%)	0.4-8	(± .04)	A-IX
692 ± 0.9	300 ± 0.1	300-1000	(± 2.2%)	2-6.5	(± .08)	A-X
692 ± 0.9	80 ± 0.2	150-1000	(± 1.2%)	1-6	(± .04)	A-XI
692 ± 0.9	4 ± 0.1	30-1000	(± 1%)	.4-7	(± .04)	A-XII
373 ± 0.5	80 ± 1	200-1000	(± 2.3%)	2-6.5	(± .08)	A-XIII
373 ± 0.5	4 ± 0.5	15-5000	(± 5%)	.2-12	(± .2)	A-XIV
373 ± 0.15	300 ± 0.1	100-1500	(± 10%)	1-8	(± .4)	A-XV
373 ± 0.15	80 ± 0.2	50-1000	(± 2%)	.5-6.5	(± .06)	A-XVI
373 ± 0.15	4 ± 0.1	20-700	(± 1%)	.3-5	(± .03)	A-XVII
300 ± 1	80 ± 1	70-2000	(± 5%)	.9-9	(± .2)	A-XVIII
300 ± 0.1	80 ± .2	50-1000	(± 2.3%)	.7-6.5	(± .08)	A-XIX
300 ± 1	4 ± .5	50-1000	(2.0%)	.7-6.5	(± .07)	A-XX
300 ± 0.1	4 ± 0.1	20-500	(1%)	.3-4.4	(± .03)	A-XXI

output termination does not alter T_e, then the most general dependence of T_e with input reflection coefficient is [3]

$$T_e(\mathrm{ant}) = \frac{T_a(1+b|\Gamma'_{\mathrm{ant}}-\beta|^2)}{1 - |\Gamma'_{\mathrm{ant}}|^2} \qquad (4)$$

where the parameters T_a, b, $|\Gamma'_{\mathrm{ant}}-\beta|$ and $|\Gamma'_{\mathrm{ant}}|$ are chosen to be terminal invariant (i.e., their value does not depend on the location of the input or output terminals provided the choice is limited to lossless regions). The parameter T_a is the amplifier's characteristic noise temperature, bT_a is the magnitude of reverse radiation (i.e., the noise temperature of the radiation from the amplifier as seen by the "antenna"), β can be thought of as a measure of the correlation of the reverse radiation with the internal noise or alternately it may be thought of as a measure of the difference in conditions for maximum power transfer and minimum noise figure, and

$$\Gamma'_{\mathrm{ant}} \equiv \frac{\Gamma_{\mathrm{ant}} - \Gamma^*_{\mathrm{amp}}}{1 - \Gamma_{\mathrm{ant}}\Gamma_{\mathrm{amp}}}, \qquad (5)$$

where the asterisk implies the complex conjugate of the "amplifier's" reflection coefficient. Note that $\Gamma'_{\mathrm{ant}} = 0$ for maximum power transfer, and has magnitude unity when $|\Gamma_{\mathrm{ant}}| = 1$. For the case that Γ_{ant} and Γ_{amp} are small, then it is convenient to use the approximate form of equation (5),

$$\Gamma'_{\mathrm{ant}} \simeq \Gamma_{\mathrm{ant}} - \Gamma^*_{\mathrm{amp}}. \qquad (6)$$

In Appendix B are graphs which can be used to estimate mismatch ambiguity.

Frankly, it is a bit of a problem to decide what value of b, and β to use for any given amplifier because so little is known about what are typical values of b and β. Engen [3] has measured $|\beta| = 0.13$, $b = 0.65$, $T_a = 496$ K for an X-band crystal mixer amplifier. For an X-band tunnel-diode amplifier $|\beta| = 0.03$, $b = 0.35$, $T_a = 825$ K was measured. For a 30 MHz vacuum tube amplifier, $|\beta| = 0.22$, $b = 0.59$, $T_a = 161$ K was measured.

For any case, and a typical value of $|\Gamma_{ant} - \Gamma^*_{amp}| \simeq 0.1$, the mismatch ambiguity is less than about 2% of T_e (0.1 dB of F_{dB}) if the magnitude of the uncertainty of Γ_{ant} is less than 0.02. This means that precise measurements of amplifier noise require fairly accurate knowledge of Γ_{ant}.

4. Mismatch Error

Mismatch error is the error resulting when the "antenna," the hot standard, and the cold standard do not have identical reflection coefficients. The maximum mismatch error is listed in tables B-III to B-XXII for ideal linear amplifiers.

As pointed out in [1] (see Appendix E), estimates of mismatch errors using the tables in Appendix B are hazardous. When it is practical, the measurements suggested in [1] are preferable.

5. The Bandwidth Problem

One problem that complicates the measurement of amplifier noise figure is caused by the dependence of the amplifier's noise on the variations in the "antenna" reflection coefficient versus frequency. For example, if local oscillator power in the superheterodyne amplifier leaks out the signal input port, then the mixer diode bias depends in part on the amplitude and phase of the reflection coefficient of the "antenna." This causes the noise figure and other amplifier parameters to vary depending on the phase of the "antenna" reflection coefficient at the local oscillator frequency. In addition, the noise figure depends on the phase of the "antenna" reflection coefficient at the signal frequency. The resulting combination effect modulates the amplifier noise figure and other parameters with a period characteristic of the IF amplifier frequency. In addition, the noise figure in theory can depend on the reflection coefficients of any pair of frequencies within the passband of the amplifier so that to check out reflection coefficient dependent effects, one needs to consider an unwieldly range of possible frequency dependences. These difficulties can be eliminated or reduced if the reflection coefficient of the standard noise sources can be adjusted to equal that of the "antenna" at all frequencies. There is evidence in the literature that significant measurement errors can occur by not using the correct frequency dependent reflection coefficient [9].

6. The State-of-the-Art

Several factors which affect the accuracy of measuring amplifier noise have been mentioned, and details of their effects examined in the appendicies. In figures 1-4, the resulting error caused by the uncertainties of T_{hot}, T_{cold}, Y, gain, connector loss, and mismatch are shown for four combinations of hot and cold noise standards. For these figures it is assumed that Y is measured within 0.01 dB, gain is stable within 0.1% over the measurement time, and the worst case uncertainty in loss in the connectors joining the hot or cold standards to the amplifier during the measurement is 0.01 dB (see Appendix C for the effect of connector loss on measurement accuracy). The mismatch error selected corresponds to $|\Gamma_{ant}-\Gamma_{std}| < 0.01$ and $|\Gamma'_{ant}| < 0.05$. For the amplifier properties, $b = \beta = 0$, were selected not because these are typical, but because the error of this assumption is within a factor of 2 of the probable situation for the various amplifiers represented in the range of amplifier noise indicated. In figures 1-2, the measurement error is expressed in decibels. As for Table 2, a 0.1 dB error say at 8 dB means the amplifier noise is between 7.9 and 8.1 dB. For values of F_{dB} where the error is a sizable fraction of F_{dB}, the figures no longer have a simple interpretation other than "this is no way to be measuring amplifiers with such small noise figures."

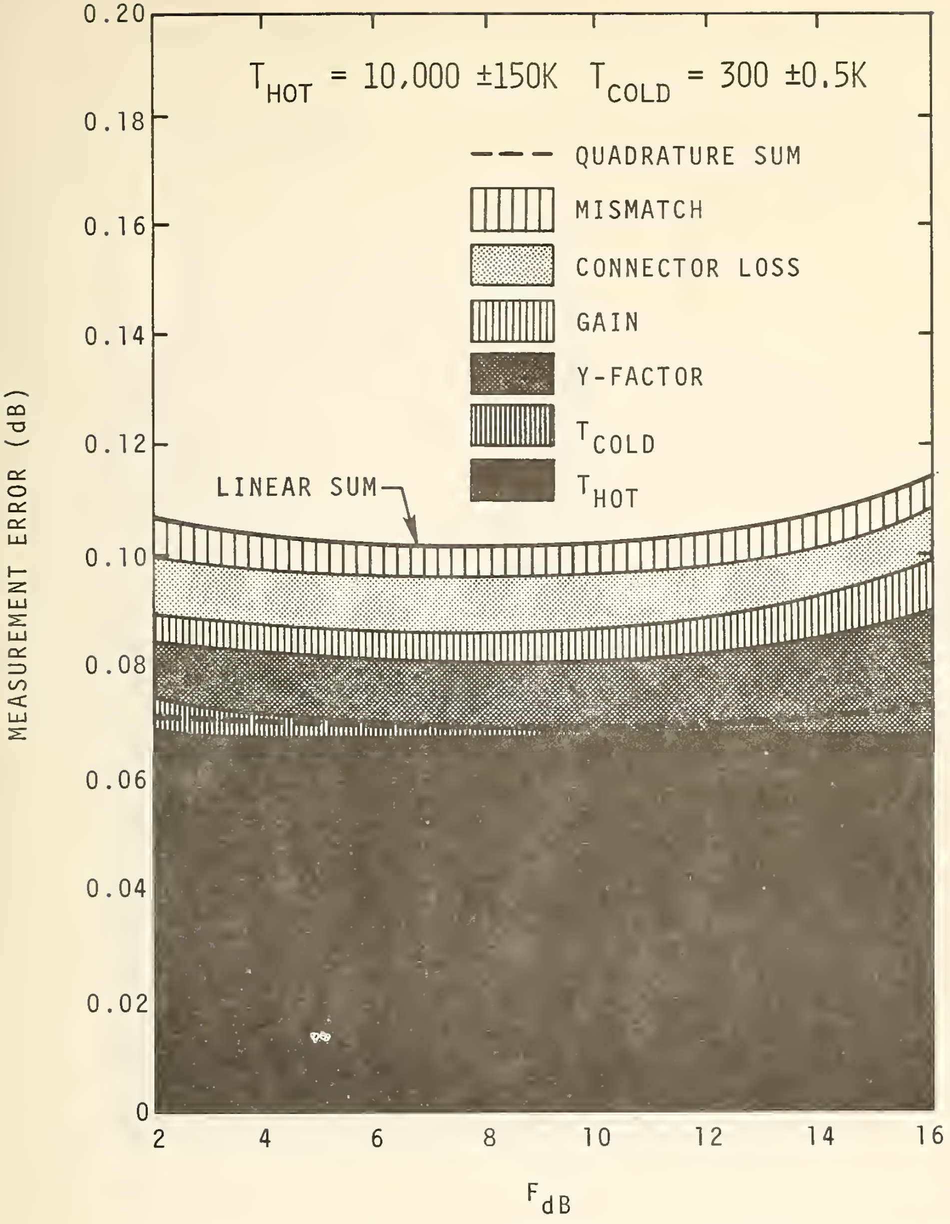

Figure 1. The state-of-the-art errors in measuring noise figure, F_{dB}, using an argon gas discharge noise source (about 10,000 K) as the hot standard, T_{hot}, and a room temperature resistive termination (about 300 K) for the cold standard, T_{cold}. The uncertainty assumed for Y-factor, gain, connector loss, and mismatch are ± 0.01 dB, ± 0.1%, 0.01 dB, and (for mismatch) $|\Gamma_{ant} - \Gamma_{std}| < 0.1$ and $|\Gamma'_{ant}| < 0.05$ as discussed in the text.

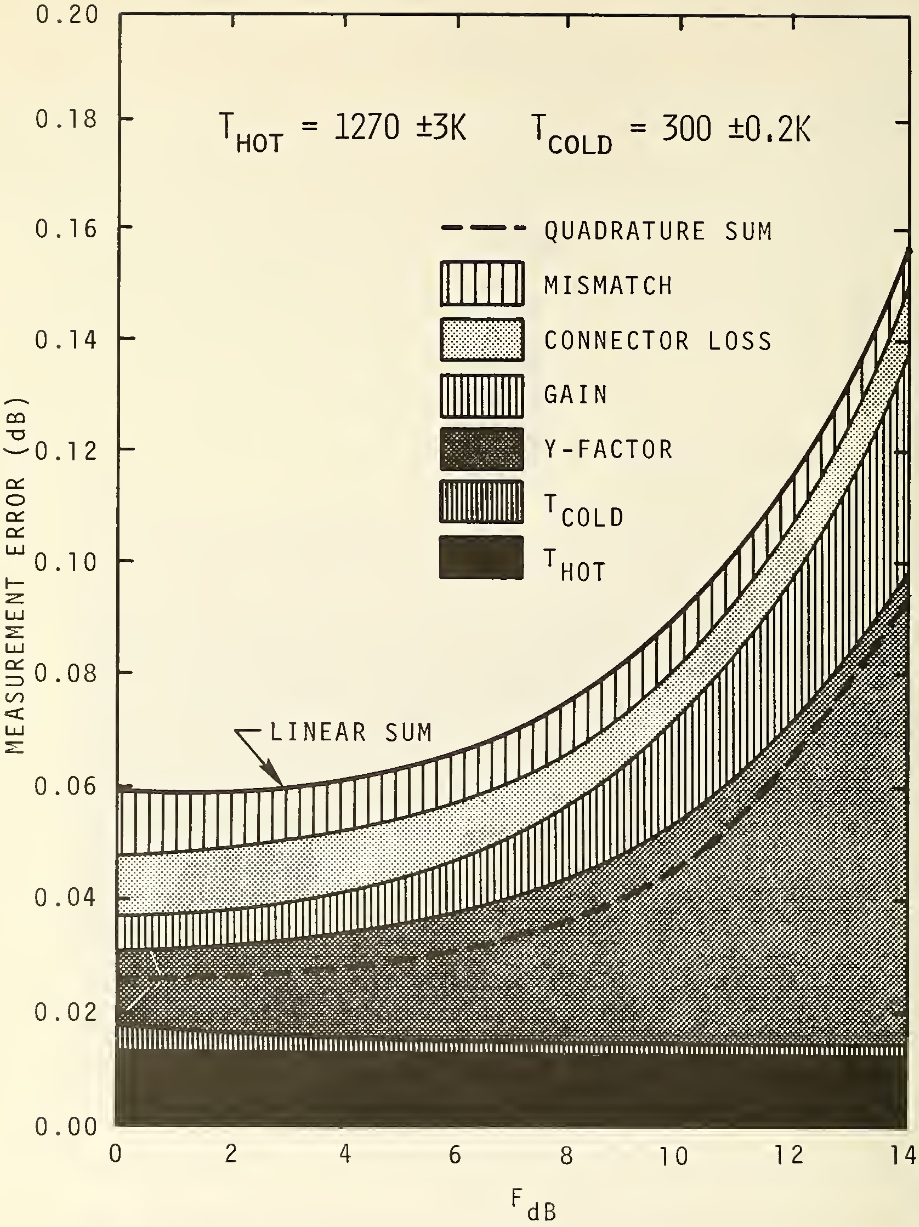

Figure 2. The state-of-the-art errors in measuring noise figure using the National Bureau of Standards' WR15, WR62 or WR90 "black-body" standard as the hot standard, and a room temperature resistive termination as the cold standard. Other parameters as noted in figure 1.

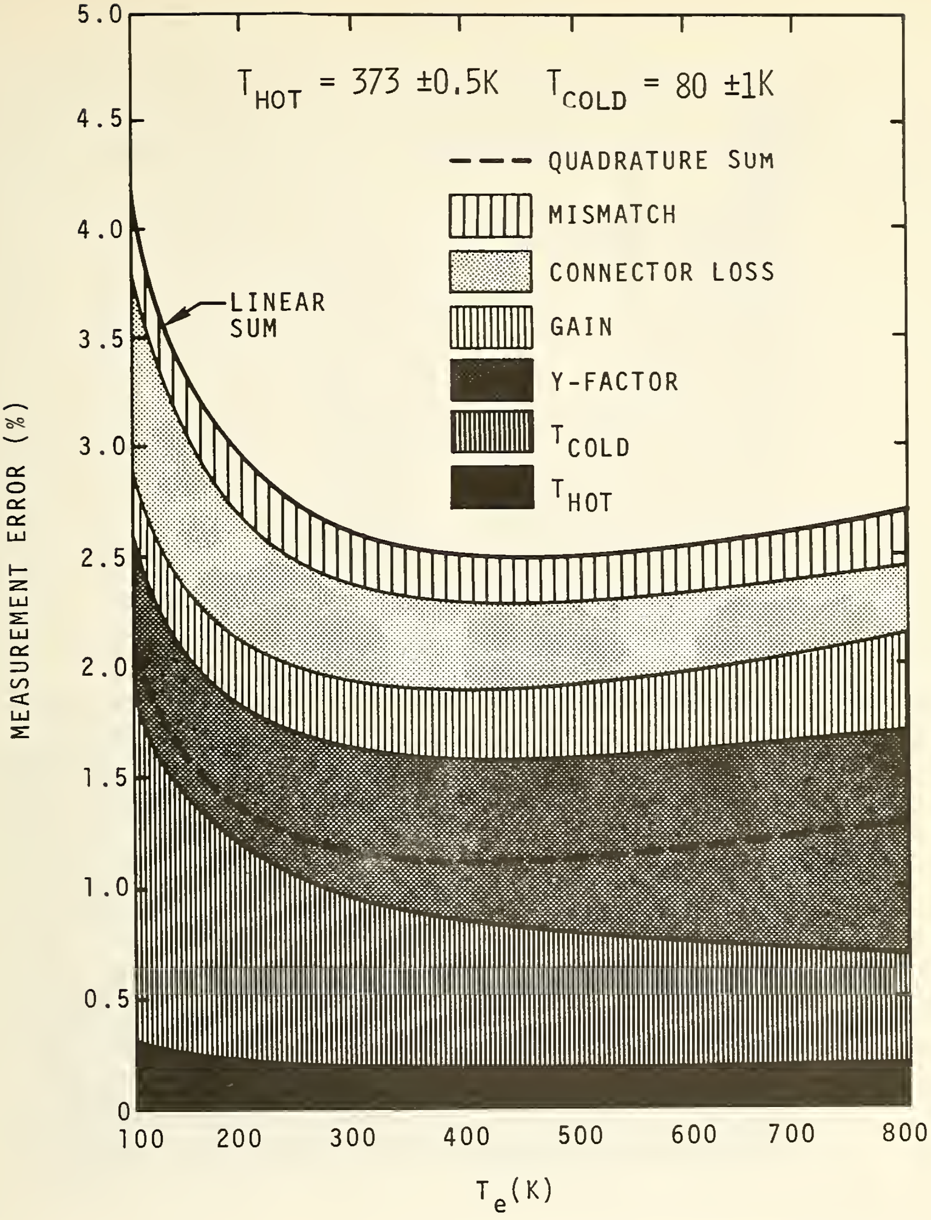

Figure 3. The state-of-the-art errors in measuring the effective input noise temperature, T_e, using a commercial hot standard near boiling water temperature and a cold standard near liquid nitrogen boiling temperature. Other parameters as noted in figure 1.

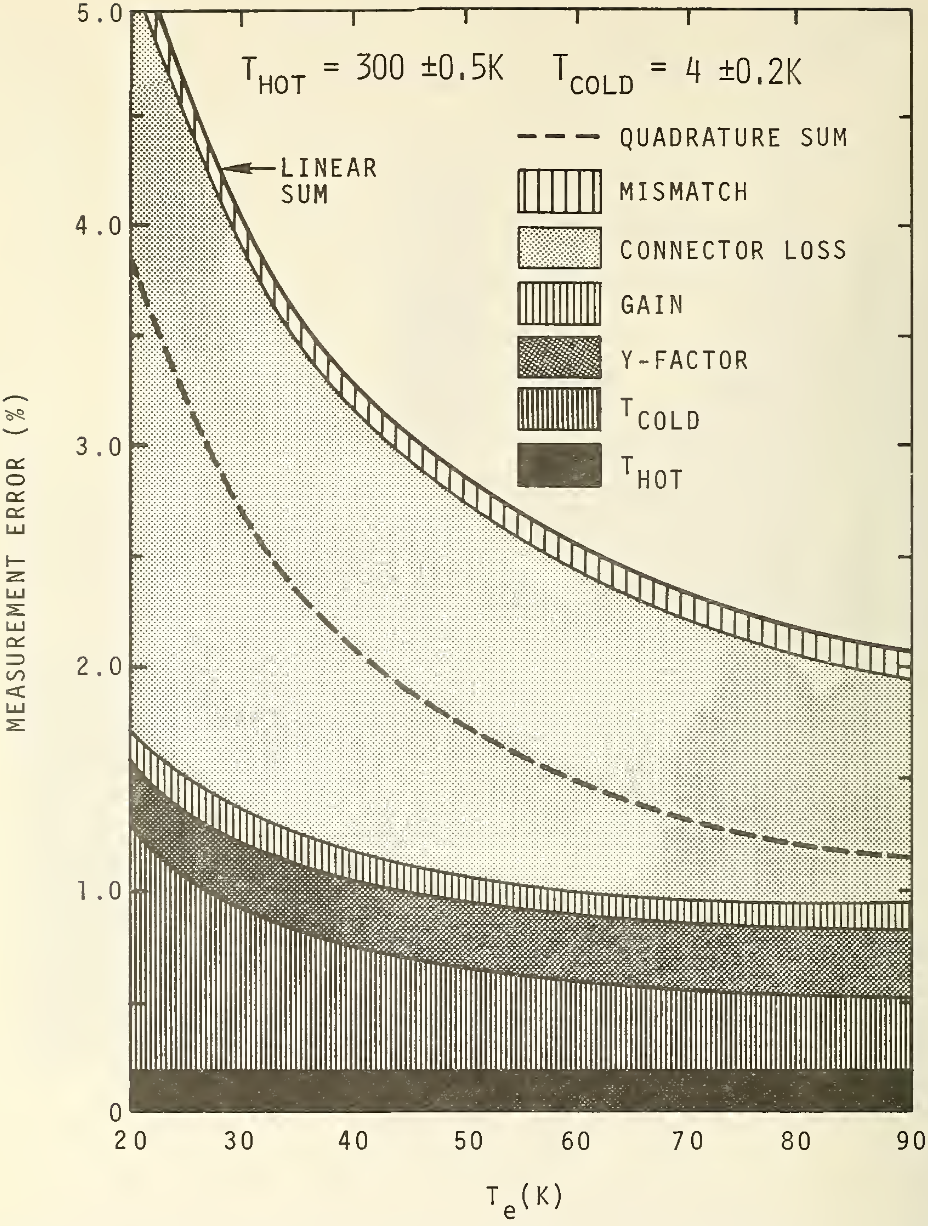

Figure 4. The state-of-the-art errors in measuring T_e using a room temperature resistive termination as the hot standard, and a liquid helium cooled resistive termination as the cold standard. The other parameters as noted in figure 1.

The inaccuracy of hot and cold standards used is indicated in each figure. In general, the measurement situation depicted is typical of the best measurement conditions, and at some frequencies (e.g., where no calibration service for noise sources exists) somewhat better than the best conditions.

The four combinations of hot and cold noise standards selected are 10,000 K and 300 K, 1270 K and 300 K, 373 K and 80 K, and 300 K and 4 K. The combination 1270 K and 300 K differs from the others in that it assumes the use of the NBS primary standards, or if you will -- the best NBS can measure noise figure. The results from these figures can be converted to either noise figure or effective input noise temperature using table 3. In figures 1-4, the dashed line is the quadrature sum (root mean square addition) of the errors. As we see, the quadrature sum is typically half of the linear addition of errors. The quadrature sum is the appropriate sum if the errors meet three criteria: (1) the error sources are independent, (2) the errors are equally likely to be positive or negative, and (3) the errors are more likely to be small rather than large. In other words, quadrature addition is appropriate if the error distributions are something like a gaussian distribution about the value used in the measurement calculation.

Unfortunately, three corrections, namely clipping, connector loss, and mismatch correction, seldom satisfy the

Table 3. Translation between effective input noise temperature, T_e(K), and noise figure, F_{dB}. The asymmetry in the error statements, because of the logarithmic non-linearity of F_{dB}, is avoided by using the slope of F_{dB} at the corresponding T_e.

T_e(K)	=	F_{dB}	F_{dB}	=	T_e(K)
10 ± 1%	=	0.15 ± .0014 dB	1 ± .1 dB	=	75 ± 11.22%
15 ± 1%	=	0.22 ± .0021 dB	2 ± .1 dB	=	169 ± 6.24%
20 ± 1%	=	0.29 ± .0028 dB	3 ± .1 dB	=	238 ± 4.62%
30 ± 1%	=	0.43 ± .0041 dB	4 ± .1 dB	=	433 ± 3.83%
50 ± 1%	=	0.69 ± .0064 dB	5 ± .1 dB	=	627 ± 3.37%
70 ± 1%	=	0.94 ± .0084 dB	6 ± .1 dB	=	864 ± 3.07%
100 ± 1%	=	1.29 ± .0111 dB	7 ± .1 dB	=	1163 ± 2.83%
150 ± 1%	=	1.81 ± .0143 dB	8 ± .1 dB	=	1539 ± 2.74%
200 ± 1%	=	2.28 ± .0177 dB	9 ± .1 dB	=	2013 ± 2.63%
300 ± 1%	=	3.08 ± .0221 dB	10 ± .1 dB	=	2609 ± 2.56%
500 ± 1%	=	4.35 ± .0275 dB	11 ± .1 dB	=	3360 ± 2.50%
700 ± 1%	=	5.33 ± .0307 dB	12 ± .1 dB	=	4306 ± 2.46%
1000 ± 1%	=	6.48 ± .0337 dB	13 ± .1 dB	=	5496 ± 2.42%
1500 ± 1%	=	7.90 ± .0364 dB	14 ± .1 dB	=	6994 ± 2.40%
2000 ± 1%	=	8.97 ± .0379 dB	15 ± .1 dB	=	8880 ± 2.33%
3000 ± 1%	=	10.55 ± .0396 dB	16 ± .1 dB	=	11255 ± 2.36%
5000 ± 1%	=	12.61 ± .0410 dB	17 ± .1 dB	=	14244 ± 2.35%
7000 ± 1%	=	14.00 ± .0417 dB	18 ± .1 dB	=	18007 ± 2.34%
10000 ± 1%	=	15.50 ± .0422 dB	19 ± .1 dB	=	22745 ± 2.33%
15000 ± 1%	=	17.22 ± .0426 dB	20 ± .1 dB	=	28709 ± 2.33%
20000 ± 1%	=	18.45 ± .0428 dB	21 ± .1 dB	=	36218 ± 2.32%
30000 ± 1%	=	20.19 ± .0430 dB	22 ± .1 dB	=	45671 ± 2.32%
50000 ± 1%	=	22.39 ± .0432 dB	23 ± .1 dB	=	57572 ± 2.31%
70000 ± 1%	=	23.84 ± .0433 dB	24 ± .1 dB	=	72554 ± 2.31%

quadrature conditions. In principle this can be arranged but it is more practical to keep the level of these errors low, and then add their contributions linearly to the quadratic sum of the remaining errors.

Frequently the errors due to the uncertainty in the hot noise standard, the cold noise standard, the measurement of Y, and the amplifier gain instability add in quadrature. In this case, the contributions from these four sources will be about half of their linear accumulation of error.

7. An Example

To illustrate a measurement problem we begin with the following measurement specification provided by a "buyer".

WR15 mixer-preamplifier specifications:

Local Oscillator:	60 GHz
I.F. Bandpass:	10-110 MHz
Maximum Noise Figure:	< 8 dB (1500 K) for $\|\Gamma_{ant}\| < 0.1$
Input $\|\Gamma\|$:	$< .1$
Output $\|\Gamma\|$:	$< .1$
Gain:	> 10 dB
Gain Instability:	< 0.005 dB/min
Measurement Accuracy Goal:	0.1 dB

First a comment on the specified accuracy goal. From reference [1] (see Appendix E) we note that a calibration from

NBS exists so figures 1-4 can reasonably represent the state-of-the-art. Using figure 1, we see that 0.1 dB error is near the state-of-the-art using an argon gas discharge noise standard and a room temperature standard providing some of the error contributions can be placed in quadrature. Further, from figure 2, it appears NBS should be able to provide a measurement verification if needed. However, it is clear that the specified accuracy goal is going to be difficult to obtain.

To compare the specified accuracy goal with the estimated mismatch ambiguity, I would use figure B4 (it is my guess that $bT_a \simeq 300$ K so that $b = 0.2$, and $\beta = 0.2$ to be consistent with Engen's measurement [1]) with $|\Gamma_{ant} - \Gamma^*_{amp}| \simeq 0.2$, and $|\Gamma_{std} - \Gamma_{ant}| \simeq 0.1$. The mismatch ambiguity indicated is about 7.7%. This is several times the accuracy goal specified. At this point one should stop and reevaluate whether it is economically more effective to lower the amplifier noise figure to allow for the mismatch ambiguity, or to specify the antenna impedance more precisely. If the specified accuracy goal is maintained, a difficult measurement is being undertaken with a real risk that the "buyer" cannot make use of it. In a situation like this there must be a very clear mutual understanding with the "manufacturer" of the conditions under which the amplifier will be utilized. Otherwise the measurement can unintentionally become invalid.

As a final consideration of the accuracy goal, we note from [10] or from table 2 in [1] (see Appendix E) that connector losses as low as 0.01 dB have been measured in WR15. But from experience at NBS, a 0.1 dB loss can occur if connectors from different manufacturers are mated without careful inspection to see that the flanges close properly. Thus, flanges must be examined and handled with "laboratory" care, right through the time the amplifier is finally installed.

The gain of the WR15 mixer-preamplifier in this example is not great enough so that the cascade noise contribution due to a post amplifier can be ignored. This contribution is approximately equal to T_e(post) divided by the power gain of the WR15 mixer-preamplifier. For a F_{dB}(post) of 5 dB (627 K), the post amplifier contribution of 63 K represents a 0.1 dB increase of F_{dB}(spec + post) over F_{dB}(spec). We need to know T_e(post) and the gain of the specified amplifier within about 10%.

At this point, we can list a set of conditions which will make it probable that a measurement of F_{dB} is within .1 dB.

1. A <u>quadrature error</u> situation using an argon-gas discharge noise standard and room temperature standard. For example
    a. An argon-standard adequately calibrated by a standards laboratory.
    b. A Y-factor measurement system accurate to within 0.01 dB (including resolution limitations).

c. A thermometer accurate to within about 0.1 K to measure the room temperature standard.

2. Mismatch ambiguity measured for $|\Gamma_{std}| = 0.1$ (with the most unfavorable phase) preferably to better than 1%. (Note: to meet the specification, the measured F_{dB} must be lower than that specified by this mismatch ambiguity.)

3. Connector losses should be less than about 0.02 dB.

4. Cascade correction

   a. 60 GHz amplifier gain measured within 10%.

   b. Post amplifier noise temperature measured within 10%.

7.1 Measurement of the Post Amplifier Noise

The example of measuring the WR15 mixer-preamplifier is completed. But the problem of measuring the post amplifier noise temperature within 10% (± 0.3 dB at 5 dB noise figure) is a sufficiently different problem that it merits further discussion. To be specific, assume the manufacturers specifications of the amplifier chosen for this task are:

Bandpass 10-110 MHz
Noise Figure < 5 dB (627 K)
Input $|\Gamma| < 0.05$
Output $|\Gamma| < .1$
Gain > 70 dB
Gain instability < 0.005 dB/min
1 dB gain compression at 100 mw or greater.

The gain selected permits one to operate a bolometer power bridge with 0.1 milliwatt output when a room temperature noise source is connected to the WR15 mixer-preamplifier. The gain compression level selected is such that less than 0.1% of the noise power will be clipped when the hot noise source is connected to the WR15 mixer-preamplifier.*

To estimate the mismatch uncertainty for the post amplifier (due to the low frequency noise standard's reflection coefficient being different than the output reflection coefficient of the mixer-preamplifier), I would use figure B7 (no hard information for this choice) with $|\Gamma_{ant} - \Gamma^*_{amp}| \approx |\Gamma_{ant} - \Gamma_{std}| \approx 0.1$. The result is off the graph but extrapolates to be near 13% (or .4 dB via table 3). A 10% overall accuracy is our goal so we would like to tune the standards (which decreases their accuracy) to equal the output impedance of the mixer-preamplifier. But to match the output impedance from 10-110 MHz is probably impractical so like it or not, we may have to accept the 13%.

For the post amplifier noise measurement, a solid-state noise source with T_{hot} near 10,000 K, and a room temperature cold standard are reasonable choices. Scaling the errors from

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*From Cohn [4] average amplitude/saturation amplitude is approximately equal to 0.07 for 0.1% clipping in a square law detector. Bolometer-power bridges work accurately up to 10 mw of power so this suggests > 147 mw saturation level. This hard limiting model is not easy to interpret for real amplifiers. From our experience, 1 dB compression at 100 mw is adequate to keep clipping correction to less than 0.1%. For other discussions of the effects of clipping see Deutsch and Hance [5], Bell [6], Van Vleck and Middleton [7].

table A-IV for T_e = 700 K, T_{hot} ± 3% contributes 4.40%, T_{cold} ± 10 K contributes 1.6%, Y ± 0.1 dB contributes 3.6%, and gain stability of 1% over the measurement time contributes 1.6%, connector loss of 0.05 dB contributes .05 dB to F_{dB} (see appendix C) or 1.69% via table 3. If the noise standards both have $|\Gamma_{std}| < 0.02$, then from table B-IX, $\beta = .3$, $bT_a/T_e = 1$ (this is just a guess but in line with the measurements in [1]) and using F(dB) = 6 (because 5 not listed), for $|\Gamma'_{ant}| > .1$ a mismatch error of 0.068 dB (2.1% using table 3) needs to be added to the 13% mismatch uncertainty for a total mismatch error of about 15%. The grand total measurement error expected is 27.9%. If the errors due to the hot source (4.42%), the cold source (1.6%), the Y-factor (3.6%) and the gain instability (1.6%) are in quadrature, then the grand total error reduces to 22.8%, or still uncomfortably large compared with the 10% goal.

8. Conclusions

Under favorable measuring situations, noise figure measurements within 0.1 dB or effective input noise temperature within 2.5% is about the best that can be done. One of the greatest problems to accurate measurements is mismatch error. This problem is compounded by the general ignorance of the magnitude and variation of the pertinent amplifier parameters.

The accuracy achieved in a particular measurement of amplifier noise depends not only on whether a commercial automatic noise figure meter is being used, or whether a refined Y-factor measurement scheme is utilized, but also on whether a national reference noise source exists at the frequency of interest. It also depends on the stability of the amplifier properties during the measurement. But in addition, the accuracy depends on the experience and skill of the metrologist. This experience and skill needs to be learned, preserved, and shared if accurate amplifier noise measurements are to become a reality. This paper provides information on the accuracy of the noise figure measurement in terms of the uncertainties of various measurement parameters without stating how to estimate the uncertainties of these measurement parameters. For these important estimates we anticipate that the average metrologist will utilize the information provided by the manufacturers of the test equipment he chooses to use. If the metrologist hopes to have individual error contributions combined in quadrature, then an even greater skill and understanding is required. We hope that for some of the amplifiers whose noise figure he has measured, he will have the National Bureau of Standards verify the measurement. This provides an opportunity to assure the metrologist that his methods and his equipment are adequate, and to give him the confidence to share his ability with others.

9. References

[1]* D. F. Wait, "The Measurement of Amplifier Noise," Microwave Journal, pp. 25-29, Jan. 1973.

[2] M.G. Arthur, "A Guide to the Measurement of Noise Performance Factors," NBS Monograph __, 1973.

[3]* a. Glenn F. Engen, "A New Method of Characterizing Amplifier Noise Performance," IEEE Trans. Instr. & Meas., Vol. IM-19, No. 4, pp. 344-349, Nov. 1970.

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[4] Charles Erwin Cohn, "Errors in Noise Measurements Due to the Finite Amplitude Range of the Measuring Instrument," Rev. Sci. Instr., Vol. 35, pp. 701-703, June 1964.

[5] Ralph Deutsch & Harold V. Hance, "A Note on Receivers for Use in Studies of Signal Statistics," IRE Convention Record, Part I, Information Theory, Vol. 1, pp. 7-13, 1953.

[6] R. L. Bell, "Linearity Range of Noise-Measuring Amplifiers," Wireless Engineer, pp. 119-122, April 1947.

[7] J. H. Van Vleck & David Middleton, "The Spectrum of Clipped Noise," Proc. IEEE, Vol. 54, No. 1, pp. 2-19, Jan. 1966.

[8]* D.F. Wait, "Thermal Noise from a Passive Linear Multiport," IEEE Trans. Microwave Theory & Techniques, Vol. MTT-16, No. 9, pp. 687-691, Sept. 1968.

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*Copies of these papers are in Appendix E.

[9] C.H. Mayer, "Improved Noise Measurements Using Ferrites," IEEE Trans. Microwave Theory & Techniques, Vol. MTT-4, pp. 24-28, Jan. 1956.

[10] B.C. Yates and C.J. Counas, "Summary of WR15 Flange Evaluation at 60 GHz," NBS Technical Note 642, Oct. 1973.

Appendix A. Measurement Errors that Depend on T_{hot} and T_{cold}

The tables in this appendix are summarized in part in Table 2 on page 6. These tables are computer print outs using the program noted in Appendix D. The symbols have the following meanings (see equations (1), (2), and (3) in text):

Symbol	Meaning	
TH	T_{hot}	(A-1)
TC	T_{cold}	(A-2)
DY = .01 dB	Y-factor measurement inaccuracy of ± 0.01 dB	(A-3)
DG = .10%	Amplifier gain instability of 0.1%	(A-4)
TE(K)	T_e expressed in degrees Kelvin	(A-5)
F(DB)	F_{dB}	(A-6)
Y(DB)	Y expressed in decibels, i.e. $Y_{dB} = 10 \log Y$	(A-7)
ETH	Error to T_e due to uncertainty in T_{hot}	(A-8)
ETC	Error to T_e due to uncertainty in T_{cold}	(A-9)
EY	Error to T_e due to uncertainty in Y	(A-10)
EG	Error to T_e due to uncertainty in amplifier gain	(A-11)
+	±	(A-12)

The error listed next to value of T_e under TE(K) is the sum of the errors listed under ETH, ETC, EY and EG. Then this error is converted into the corresponding error in F_{dB} and listed next to the value of the corresponding F_{dB}.

As an example of using Table A-1, note that an amplifier with an effective input noise temperature near 7000 K (noise figure of 14.0 dB) can be measured to an inaccuracy of

± 2.1% (± 0.087 dB in noise figure) if the hot standard is 18000 ± 270 K, the cold standard is 300 ± 1 K, the Y-factor (near 5.35 dB) is measured with an inaccuracy of 0.01 dB, and the gain is unstable within ± 0.1%. Of the 2.1% inaccuracy, 1.59% is caused by the ± 270 K uncertainty in the hot noise standard, 0.02% is caused by the ± 1 K uncertainty in the cold noise standard, 0.34% is caused by the 0.01 dB in measuring the Y-factor, and the remaining 0.15% is caused by the gain instability of 0.1%. Mismatch error, connector loss error, and cascade error are neglected in these tables. If instead of the ± 270 K, ± 1 K, ± 0.01 dB, and ± 0.1% inaccuracies assumed in Table A-1 for T_{hot}, T_{cold}, Y(dB), and gain respectively we had ± 270K, ± 2 K, ± 0.005 dB, and ± 0.5%, then the corresponding error contributions to T_e in our example would expand or contract in proportion to 1.59%, 0.04%, 0.17%, and 0.75% for a total error of ± 2.55%. Similarly for any other measurement uncertainty situation the tables can be modified to obtain the appropriate error contributions to T_e.

Table A-I

```
                          TH=18000 + 270.00 K (1.50%)
                          TC=  300 +   1.00 K (0.33%)

                          DY=.01 DB        DG=.10%


                                                ERROR CONTRIBUTIONS TO TE
  TE(K)           F(DB)            Y(DB)       ETH      ETC       EY       EG


    10 +67.9%    0.15 +.098       17.64      47.29%   10.18%    7.26%    3.15%
    15 +45.9%    0.22 +.098       17.57      32.03%    6.79%    4.92%    2.14%
    20 +34.9%    0.29 +.098       17.51      24.41%    5.09%    3.75%    1.63%

    30 +23.9%    0.43 +.097       17.37      16.78%    3.40%    2.58%    1.12%
    50 +15.1%    0.69 +.096       17.12      10.68%    2.04%    1.64%    0.71%
    70 +11.3%    0.94 +.095       16.89       8.06%    1.46%    1.24%    0.54%

   100 + 8.5%    1.29 +.094       16.56       6.10%    1.02%    0.94%    0.41%
   150 + 6.3%    1.81 +.093       16.06       4.58%    0.68%    0.71%    0.31%
   200 + 5.2%    2.28 +.092       15.61       3.81%    0.51%    0.59%    0.26%

   300 + 4.1%    3.08 +.090       14.84       3.05%    0.34%    0.48%    0.21%
   500 + 3.2%    4.35 +.088       13.64       2.44%    0.21%    0.39%    0.17%
   700 + 2.8%    5.33 +.087       12.72       2.18%    0.15%    0.35%    0.15%

  1000 + 2.6%    6.48 +.086       11.65       1.98%    0.11%    0.32%    0.14%
  1500 + 2.3%    7.90 +.085       10.35       1.83%    0.07%    0.30%    0.13%
  2000 + 2.2%    8.97 +.085        9.39       1.75%    0.06%    0.30%    0.13%

  3000 + 2.1%   10.55 +.085        8.04       1.68%    0.04%    0.30%    0.13%
  5000 + 2.1%   12.61 +.086        6.37       1.62%    0.03%    0.32%    0.14%
  7000 + 2.1%   14.00 +.087        5.35       1.59%    0.02%    0.34%    0.15%

 10000 + 2.1%   15.50 +.090        4.34       1.57%    0.02%    0.38%    0.16%
 15000 + 2.2%   17.22 +.094        3.34       1.56%    0.01%    0.44%    0.19%
 20000 + 2.3%   18.45 +.098        2.72       1.55%    0.01%    0.50%    0.22%

 30000 + 2.5%   20.19 +.106        2.00       1.54%    0.01%    0.63%    0.27%
 50000 + 2.8%   22.39 +.122        1.31       1.53%    0.01%    0.89%    0.39%
 70000 + 3.2%   23.84 +.138        0.93       1.53%    0.01%    1.15%    0.50%
```

Table A-II

```
                           TH=18000 + 270.00 K (1.50%)
                           TC=   80 +   1.00 K (1.25%)

                           DY=.01 DB       DG=.10%


                                                  ERROR CONTRIBUTIONS TO TE
 TE(K)            F(DB)           Y(DB)           ETH      ETC       EY        EG


   10 +26.6%    0.15 +.039       23.01          13.56%  10.05%    2.08%     0.90%
   15 +18.3%    0.22 +.039       22.78           9.54%   6.70%    1.47%     0.64%
   20 +14.2%    0.29 +.040       22.56           7.53%   5.03%    1.16%     0.50%

   30 +10.1%    0.43 +.041       22.15           5.52%   3.35%    0.85%     0.37%
   50 + 6.8%    0.69 +.043       21.43           3.92%   2.01%    0.60%     0.26%
   70 + 5.4%    0.94 +.045       20.81           3.23%   1.44%    0.50%     0.22%

  100 + 4.3%    1.29 +.048       20.02           2.71%   1.01%    0.42%     0.18%
  150 + 3.5%    1.81 +.052       18.97           2.31%   0.68%    0.36%     0.16%
  200 + 3.1%    2.28 +.055       18.13           2.11%   0.51%    0.33%     0.14%

  300 + 2.7%    3.08 +.059       16.83           1.91%   0.34%    0.30%     0.13%
  500 + 2.3%    4.35 +.065       15.04           1.75%   0.21%    0.28%     0.12%
  700 + 2.2%    5.33 +.068       13.80           1.68%   0.15%    0.27%     0.12%

 1000 + 2.1%    6.48 +.071       12.45           1.63%   0.11%    0.26%     0.11%
 1500 + 2.0%    7.90 +.074       10.91           1.59%   0.07%    0.26%     0.11%
 2000 + 2.0%    8.97 +.076        9.83           1.57%   0.06%    0.27%     0.12%

 3000 + 2.0%   10.55 +.079        8.34           1.55%   0.04%    0.28%     0.12%
 5000 + 2.0%   12.61 +.082        6.56           1.53%   0.03%    0.30%     0.13%
 7000 + 2.0%   14.00 +.084        5.48           1.52%   0.02%    0.32%     0.14%

10000 + 2.1%   15.50 +.087        4.44           1.52%   0.02%    0.36%     0.16%
15000 + 2.1%   17.22 +.091        3.40           1.51%   0.01%    0.43%     0.19%
20000 + 2.2%   18.45 +.095        2.77           1.51%   0.01%    0.49%     0.21%

30000 + 2.4%   20.19 +.104        2.03           1.51%   0.01%    0.62%     0.27%
50000 + 2.8%   22.39 +.120        1.33           1.51%   0.01%    0.88%     0.38%
70000 + 3.1%   23.84 +.136        0.99           1.51%   0.01%    1.13%     0.49%
```

Table A-III

TH=18000 + 270.00 K (1.50%)
TC= 4 + 0.50 K (12.50%)

DY=.01 DB DG=.10%

			ERROR CONTRIBUTIONS TO TE			
TE(K)	F(DB)	Y(DB)	ETH	ETC	EY	EG
10 + 7.6%	0.15 +.011	31.09	2.10%	5.00%	0.32%	0.14%
15 + 5.7%	0.22 +.012	29.77	1.90%	3.34%	0.29%	0.13%
20 + 4.7%	0.29 +.013	28.76	1.80%	2.50%	0.28%	0.12%
30 + 3.7%	0.43 +.015	27.25	1.70%	1.67%	0.26%	0.11%
50 + 3.0%	0.69 +.019	25.24	1.62%	1.00%	0.25%	0.11%
70 + 2.7%	0.94 +.022	23.88	1.59%	0.72%	0.24%	0.11%
100 + 2.4%	1.29 +.027	22.41	1.56%	0.50%	0.24%	0.10%
150 + 2.2%	1.81 +.033	20.71	1.54%	0.34%	0.24%	0.10%
200 + 2.1%	2.28 +.038	19.50	1.53%	0.25%	0.24%	0.10%
300 + 2.0%	3.08 +.045	17.80	1.52%	0.17%	0.24%	0.10%
500 + 2.0%	4.35 +.054	15.65	1.51%	0.10%	0.24%	0.10%
700 + 1.9%	5.33 +.059	14.24	1.51%	0.07%	0.24%	0.10%
1000 + 1.9%	6.48 +.064	12.77	1.51%	0.05%	0.24%	0.11%
1500 + 1.9%	7.90 +.069	11.13	1.50%	0.04%	0.25%	0.11%
2000 + 1.9%	8.97 +.072	9.99	1.50%	0.03%	0.26%	0.11%
3000 + 1.9%	10.55 +.076	8.45	1.50%	0.02%	0.27%	0.12%
5000 + 1.9%	12.61 +.080	6.62	1.50%	0.01%	0.29%	0.13%
7000 + 2.0%	14.00 +.082	5.53	1.50%	0.01%	0.32%	0.14%
10000 + 2.0%	15.53 +.085	4.47	1.50%	0.01%	0.36%	0.16%
15000 + 2.1%	17.22 +.090	3.42	1.50%	0.01%	0.42%	0.18%
20000 + 2.2%	18.45 +.094	2.79	1.50%	0.01%	0.49%	0.21%
30000 + 2.4%	20.19 +.103	2.04	1.50%	0.00%	0.61%	0.27%
50000 + 2.8%	22.39 +.119	1.34	1.50%	0.00%	0.87%	0.38%
70000 + 3.1%	23.84 +.135	0.99	1.50%	0.00%	1.13%	0.49%

Table A-IV

```
                      TH=11000 + 165.00 K (1.50%)
                      TC=  300 +   1.00 K (0.33%)

                      DY=.01 DB       DG=.10%


                                                ERROR CONTRIBUTIONS TO TE
  TE(K)           F(DB)           Y(DB)      ETH       ETC       EY        EG


     10 +68.6%    0.15 +.099      15.50     47.80%   10.29%    7.34%     3.19%
     15 +46.4%    0.22 +.099      15.44     32.38%    6.86%    4.93%     2.16%
     20 +35.3%    0.29 +.099      15.37     24.67%    5.15%    3.79%     1.65%

     30 +24.1%    0.43 +.098      15.24     16.96%    3.44%    2.61%     1.13%
     50 +15.2%    0.69 +.097      14.99     10.79%    2.07%    1.66%     0.72%
     70 +11.4%    0.94 +.097      14.76      8.15%    1.43%    1.26%     0.55%

    100 + 8.6%    1.29 +.096      14.43      6.17%    1.04%    0.96%     0.41%
    150 + 6.4%    1.81 +.094      13.94      4.63%    0.69%    0.72%     0.31%
    200 + 5.2%    2.28 +.093      13.50      3.86%    0.52%    0.60%     0.26%

    300 + 4.1%    3.03 +.091      12.75      3.03%    0.35%    0.49%     0.21%
    500 + 3.3%    4.35 +.089      11.58      2.47%    0.21%    0.40%     0.17%
    700 + 2.9%    5.33 +.088      10.63      2.20%    0.16%    0.36%     0.16%

   1000 + 2.6%    6.48 +.087       9.65      2.00%    0.11%    0.34%     0.15%
   1500 + 2.4%    7.90 +.087       8.42      1.85%    0.08%    0.32%     0.14%
   2000 + 2.3%    8.97 +.087       7.52      1.77%    0.06%    0.32%     0.14%

   3000 + 2.2%   10.55 +.088       6.28      1.70%    0.04%    0.33%     0.14%
   5000 + 2.2%   12.61 +.090       4.80      1.63%    0.03%    0.36%     0.16%
   7000 + 2.2%   14.00 +.092       3.92      1.61%    0.02%    0.40%     0.13%

  10000 + 2.3%   15.50 +.096       3.09      1.59%    0.02%    0.47%     0.20%
  15000 + 2.4%   17.22 +.103       2.30      1.57%    0.02%    0.57%     0.25%
  20000 + 2.6%   18.45 +.109       1.84      1.57%    0.01%    0.68%     0.29%

  30000 + 2.8%   20.19 +.123       1.31      1.56%    0.01%    0.89%     0.39%
  50000 + 3.5%   22.39 +.149       0.84      1.55%    0.01%    1.32%     0.57%
  70000 + 4.1%   23.84 +.176       0.62      1.55%    0.01%    1.75%     0.76%
```

Table A-V

TH=11000 + 165.00 K (1.50%)
TC= 80 + 1.00 K (1.25%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE ETH	ETC	EY	EG
10 +26.7%	0.15 +.039	20.88	13.60%	10.08%	2.09%	0.91%
15 +18.4%	0.22 +.039	20.64	9.57%	6.72%	1.47%	0.64%
20 +14.3%	0.29 +.040	20.42	7.55%	5.05%	1.16%	0.50%
30 +10.1%	0.43 +.041	20.01	5.54%	3.37%	0.85%	0.37%
50 + 6.8%	0.69 +.044	19.29	3.93%	2.02%	0.61%	0.26%
70 + 5.4%	0.94 +.046	18.68	3.24%	1.45%	0.50%	0.22%
100 + 4.3%	1.29 +.043	17.90	2.72%	1.02%	0.42%	0.18%
150 + 3.5%	1.81 +.052	16.86	2.32%	0.68%	0.36%	0.15%
200 + 3.1%	2.28 +.055	16.02	2.12%	0.51%	0.33%	0.14%
300 + 2.7%	3.08 +.059	14.73	1.91%	0.34%	0.30%	0.13%
500 + 2.4%	4.35 +.065	12.97	1.75%	0.21%	0.28%	0.12%
700 + 2.2%	5.33 +.069	11.76	1.68%	0.15%	0.27%	0.12%
1000 + 2.1%	6.48 +.072	10.46	1.63%	0.11%	0.27%	0.12%
1500 + 2.1%	7.90 +.075	8.98	1.59%	0.08%	0.28%	0.12%
2000 + 2.0%	8.97 +.077	7.96	1.57%	0.06%	0.29%	0.12%
3000 + 2.0%	10.55 +.080	6.58	1.55%	0.04%	0.30%	0.13%
5000 + 2.1%	12.61 +.084	4.98	1.54%	0.03%	0.34%	0.15%
7000 + 2.1%	14.00 +.088	4.05	1.53%	0.02%	0.38%	0.17%
10000 + 2.2%	15.50 +.092	3.19	1.52%	0.02%	0.45%	0.19%
15000 + 2.3%	17.22 +.099	2.37	1.52%	0.02%	0.55%	0.24%
20000 + 2.5%	18.45 +.106	1.89	1.52%	0.01%	0.66%	0.29%
30000 + 2.8%	20.19 +.119	1.35	1.52%	0.01%	0.87%	0.33%
50000 + 3.4%	22.39 +.145	0.36	1.51%	0.01%	1.29%	0.56%
70000 + 4.0%	23.84 +.172	0.63	1.51%	0.01%	1.71%	0.74%

Table A-VI

TH=11000 + 165.00 K (1.50%)
TC= 4 + 0.50 K (12.50%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE			
			ETH	ETC	EY	EG
10 + 7.6%	0.15 +.011	28.96	2.10%	5.01%	0.32%	0.14%
15 + 5.7%	0.22 +.012	27.63	1.90%	3.34%	0.29%	0.13%
20 + 4.7%	0.29 +.013	26.62	1.80%	2.51%	0.28%	0.12%
30 + 3.7%	0.43 +.015	25.11	1.70%	1.67%	0.26%	0.11%
50 + 3.0%	0.69 +.019	23.11	1.62%	1.00%	0.25%	0.11%
70 + 2.7%	0.94 +.022	21.75	1.59%	0.72%	0.25%	0.11%
100 + 2.4%	1.29 +.027	20.28	1.56%	0.50%	0.24%	0.10%
150 + 2.2%	1.81 +.033	18.60	1.54%	0.34%	0.24%	0.10%
200 + 2.1%	2.28 +.038	17.40	1.53%	0.25%	0.24%	0.10%
300 + 2.0%	3.03 +.045	15.70	1.52%	0.17%	0.24%	0.10%
500 + 2.0%	4.35 +.054	13.58	1.51%	0.10%	0.24%	0.11%
700 + 1.9%	5.33 +.060	12.21	1.51%	0.08%	0.25%	0.11%
1000 + 1.9%	6.48 +.065	10.77	1.51%	0.05%	0.25%	0.11%
1500 + 1.9%	7.90 +.070	9.20	1.50%	0.04%	0.26%	0.11%
2000 + 1.9%	8.97 +.073	8.12	1.50%	0.03%	0.27%	0.12%
3000 + 1.9%	10.55 +.077	6.68	1.50%	0.02%	0.29%	0.13%
5000 + 2.0%	12.61 +.082	5.05	1.50%	0.01%	0.34%	0.15%
7000 + 2.1%	14.00 +.086	4.10	1.50%	0.01%	0.38%	0.16%
10000 + 2.1%	15.50 +.090	3.22	1.50%	0.01%	0.44%	0.19%
15000 + 2.3%	17.22 +.098	2.39	1.50%	0.01%	0.54%	0.24%
20000 + 2.4%	18.45 +.104	1.90	1.50%	0.01%	0.65%	0.28%
30000 + 2.7%	20.19 +.118	1.36	1.50%	0.01%	0.86%	0.37%
50000 + 3.3%	22.39 +.144	0.86	1.50%	0.01%	1.28%	0.55%
70000 + 3.9%	23.84 +.170	0.63	1.50%	0.01%	1.70%	0.74%

Table A-VII

TH= 1250 + 3.00 K (0.24%)
TC= 300 + 0.10 K (0.03%)

DY=.01 DB   DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE ETH	ETC	EY	EG
10 +24.7%	0.15 +.036	6.09	9.79%	1.33%	9.47%	4.11%
15 +15.8%	0.22 +.036	6.04	6.63%	0.89%	6.44%	2.90%
20 +12.8%	0.29 +.036	5.99	5.05%	0.67%	4.93%	2.14%
30 + 8.3%	0.43 +.036	5.89	3.47%	0.45%	3.41%	1.48%
50 + 5.6%	0.69 +.036	5.70	2.21%	0.27%	2.21%	0.96%
70 + 4.3%	0.94 +.036	5.52	1.67%	0.20%	1.69%	0.73%
100 + 3.3%	1.29 +.037	5.23	1.26%	0.14%	1.31%	0.57%
150 + 2.5%	1.81 +.037	4.93	0.95%	0.10%	1.02%	0.44%
200 + 2.1%	2.28 +.038	4.62	0.79%	0.08%	0.88%	0.38%
300 + 1.8%	3.03 +.039	4.12	0.63%	0.05%	0.75%	0.33%
500 + 1.5%	4.35 +.042	3.40	0.51%	0.04%	0.68%	0.29%
700 + 1.4%	5.33 +.044	2.90	0.45%	0.03%	0.68%	0.29%
1000 + 1.5%	6.48 +.049	2.38	0.41%	0.02%	0.71%	0.31%
1500 + 1.5%	7.90 +.056	1.84	0.38%	0.02%	0.80%	0.35%
2000 + 1.7%	8.97 +.064	1.50	0.36%	0.02%	0.91%	0.39%
3000 + 2.0%	10.55 +.079	1.10	0.35%	0.01%	1.13%	0.49%
5000 + 2.7%	12.61 +.109	0.72	0.33%	0.01%	1.61%	0.70%
7000 + 3.3%	14.00 +.139	0.53	0.33%	0.01%	2.09%	0.91%
10000 + 4.4%	15.50 +.184	0.38	0.33%	0.01%	2.81%	1.22%
15000 + 6.1%	17.22 +.260	0.26	0.32%	0.01%	4.02%	1.75%
20000 + 7.8%	18.45 +.336	0.20	0.32%	0.01%	5.24%	2.27%
30000 +11.3%	20.19 +.488	0.13	0.32%	0.01%	7.69%	3.33%
50000 +18.5%	22.39 +.797	0.08	0.32%	0.01%	12.69%	5.44%
70000 +25.8%	23.84 +1.115	0.06	0.32%	0.01%	17.88%	7.57%

Table A-VIII

TH= 1250 + 3.00 K (0.24%)
TC= 80 + 0.20 K (0.25%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE			
			ETH	ETC	EY	EG
10 + 7.7%	0.15 +.011	11.46	2.31%	2.15%	2.23%	0.97%
15 + 5.3%	0.22 +.011	11.24	1.62%	1.44%	1.58%	0.68%
20 + 4.2%	0.29 +.012	11.04	1.28%	1.09%	1.25%	0.54%
30 + 3.0%	0.43 +.012	10.66	0.94%	0.73%	0.92%	0.40%
50 + 2.1%	0.69 +.013	10.00	0.67%	0.44%	0.67%	0.29%
70 + 1.7%	0.94 +.014	9.44	0.55%	0.32%	0.56%	0.24%
100 + 1.4%	1.29 +.015	8.75	0.46%	0.23%	0.48%	0.21%
150 + 1.2%	1.81 +.017	7.84	0.39%	0.16%	0.42%	0.18%
200 + 1.1%	2.28 +.019	7.14	0.36%	0.12%	0.40%	0.17%
300 + 1.0%	3.08 +.021	6.11	0.32%	0.09%	0.39%	0.17%
500 + 0.9%	4.35 +.026	4.80	0.30%	0.06%	0.40%	0.17%
700 + 0.9%	5.33 +.029	3.98	0.29%	0.05%	0.43%	0.19%
1000 + 1.0%	6.48 +.034	3.19	0.28%	0.04%	0.48%	0.21%
1500 + 1.1%	7.90 +.041	2.41	0.27%	0.03%	0.57%	0.25%
2000 + 1.2%	8.97 +.047	1.94	0.27%	0.03%	0.67%	0.29%
3000 + 1.5%	10.55 +.060	1.40	0.26%	0.02%	0.86%	0.37%
5000 + 2.1%	12.61 +.085	0.90	0.26%	0.02%	1.25%	0.54%
7000 + 2.6%	14.00 +.110	0.66	0.26%	0.02%	1.64%	0.71%
10000 + 3.5%	15.50 +.147	0.48	0.26%	0.02%	2.23%	0.97%
15000 + 4.9%	17.22 +.208	0.32	0.26%	0.02%	3.22%	1.40%
20000 + 6.3%	18.45 +.270	0.25	0.26%	0.02%	4.21%	1.82%
30000 + 9.1%	20.19 +.393	0.17	0.26%	0.02%	6.19%	2.68%
50000 +14.9%	22.39 +.642	0.10	0.26%	0.02%	10.21%	4.40%
70000 +20.7%	23.84 +.896	0.07	0.26%	0.02%	14.32%	6.12%

Table A-IX

TH= 1250 + 3.00 K (0.24%)
TC= 4 + 0.10 K (2.50%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE ETH	ETC	EY	EG
10 + 1.8%	0.15 +.003	19.54	0.34%	1.01%	0.33%	0.14%
15 + 1.4%	0.22 +.003	18.23	0.30%	0.68%	0.30%	0.13%
20 + 1.2%	0.29 +.003	17.24	0.29%	0.51%	0.28%	0.12%
30 + 1.0%	0.43 +.004	15.76	0.27%	0.34%	0.27%	0.12%
50 + 0.8%	0.69 +.005	13.82	0.26%	0.21%	0.26%	0.11%
70 + 0.8%	0.94 +.007	12.51	0.25%	0.15%	0.26%	0.11%
100 + 0.7%	1.29 +.008	11.13	0.25%	0.11%	0.26%	0.11%
150 + 0.7%	1.81 +.010	9.59	0.25%	0.07%	0.27%	0.12%
200 + 0.7%	2.28 +.012	8.52	0.25%	0.06%	0.27%	0.12%
300 + 0.7%	3.08 +.015	7.07	0.24%	0.04%	0.29%	0.13%
500 + 0.7%	4.35 +.020	5.41	0.24%	0.03%	0.33%	0.14%
700 + 0.8%	5.33 +.024	4.42	0.24%	0.02%	0.36%	0.16%
1000 + 0.9%	6.48 +.029	3.50	0.24%	0.02%	0.42%	0.18%
1500 + 1.0%	7.90 +.036	2.62	0.24%	0.01%	0.51%	0.22%
2000 + 1.1%	8.97 +.042	2.10	0.24%	0.01%	0.60%	0.26%
3000 + 1.4%	10.55 +.055	1.51	0.24%	0.01%	0.79%	0.34%
5000 + 1.9%	12.61 +.078	0.97	0.24%	0.01%	1.16%	0.50%
7000 + 2.4%	14.00 +.102	0.71	0.24%	0.01%	1.53%	0.66%
10000 + 3.2%	15.50 +.136	0.51	0.24%	0.01%	2.08%	0.90%
15000 + 4.6%	17.22 +.194	0.35	0.24%	0.01%	3.01%	1.30%
20000 + 5.9%	18.45 +.252	0.26	0.24%	0.01%	3.93%	1.71%
30000 + 8.6%	20.19 +.368	0.18	0.24%	0.01%	5.80%	2.51%
50000 +13.9%	22.39 +.601	0.11	0.24%	0.01%	9.56%	4.12%
70000 +19.4%	23.84 +.838	0.08	0.24%	0.01%	13.40%	5.74%

Table A-X

TH= 692 + 0.90 K (0.13%)
TC= 300 + 0.10 K (0.03%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE			
			ETH	ETC	EY	EG
10 +27.2%	0.15 +.039	3.55	7.12%	1.79%	12.78%	5.55%
15 +18.5%	0.22 +.040	3.51	4.82%	1.20%	8.72%	3.79%
20 +14.2%	0.29 +.040	3.47	3.67%	0.91%	6.69%	2.91%
30 + 9.8%	0.43 +.040	3.40	2.53%	0.61%	4.67%	2.03%
50 + 6.4%	0.69 +.041	3.26	1.61%	0.38%	3.05%	1.33%
70 + 4.9%	0.94 +.041	3.14	1.21%	0.28%	2.37%	1.03%
100 + 3.8%	1.29 +.042	2.97	0.92%	0.20%	1.86%	0.81%
150 + 3.0%	1.81 +.044	2.72	0.69%	0.14%	1.48%	0.64%
200 + 2.6%	2.28 +.045	2.51	0.57%	0.11%	1.31%	0.57%
300 + 2.2%	3.08 +.049	2.18	0.46%	0.08%	1.17%	0.51%
500 + 2.0%	4.35 +.056	1.73	0.37%	0.06%	1.12%	0.49%
700 + 2.1%	5.33 +.063	1.44	0.33%	0.05%	1.17%	0.51%
1000 + 2.2%	6.48 +.074	1.14	0.30%	0.04%	1.29%	0.56%
1500 + 2.5%	7.93 +.092	0.86	0.23%	0.04%	1.55%	0.67%
2000 + 2.9%	8.97 +.110	0.63	0.26%	0.03%	1.82%	0.79%
3000 + 3.7%	10.55 +.147	0.49	0.25%	0.03%	2.39%	1.04%
5000 + 5.4%	12.61 +.220	0.31	0.24%	0.03%	3.55%	1.54%
7000 + 7.0%	14.00 +.293	0.23	0.24%	0.03%	4.72%	2.05%
10000 + 9.6%	15.50 +.404	0.16	0.24%	0.03%	6.49%	2.81%
15000 +13.8%	17.22 +.539	0.11	0.23%	0.03%	9.43%	4.09%
20000 +18.2%	18.45 +.777	0.08	0.23%	0.03%	12.52%	5.37%
30000 +27.0%	20.19 +1.163	0.05	0.23%	0.03%	18.82%	7.96%
50000 +46.4%	22.39 +2.001	0.03	0.23%	0.03%	32.87%	13.23%
70000 +69.4%	23.84 +3.000	0.02	0.23%	0.03%	50.39%	18.71%

Table A-XI

TH= 692 + 0.90 K (0.13%)
TC= 80 + 0.20 K (0.25%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE ETH	ETC	EY	EG
10 + 7.0%	0.15 +.010	8.92	1.32%	2.29%	2.38%	1.03%
15 + 4.9%	0.22 +.010	8.72	0.93%	1.54%	1.68%	0.73%
20 + 3.8%	0.29 +.011	8.52	0.74%	1.16%	1.34%	0.53%
30 + 2.8%	0.43 +.011	8.17	0.54%	0.79%	1.00%	0.43%
50 + 1.9%	0.69 +.012	7.56	0.38%	0.48%	0.73%	0.32%
70 + 1.6%	0.94 +.013	7.06	0.32%	0.36%	0.61%	0.27%
100 + 1.3%	1.29 +.014	6.43	0.26%	0.25%	0.54%	0.23%
150 + 1.1%	1.81 +.016	5.64	0.23%	0.18%	0.49%	0.21%
200 + 1.0%	2.28 +.018	5.03	0.21%	0.15%	0.47%	0.20%
300 + 1.0%	3.08 +.021	4.17	0.19%	0.11%	0.47%	0.21%
500 + 1.0%	4.35 +.027	3.13	0.17%	0.08%	0.52%	0.23%
700 + 1.1%	5.33 +.033	2.52	0.16%	0.06%	0.58%	0.25%
1000 + 1.2%	6.48 +.040	1.95	0.16%	0.06%	0.69%	0.30%
1500 + 1.4%	7.90 +.053	1.42	0.15%	0.05%	0.87%	0.38%
2000 + 1.7%	8.97 +.065	1.12	0.15%	0.04%	1.05%	0.46%
3000 + 2.2%	10.55 +.08[illegible]	0.79	0.15%	0.04%	1.43%	0.62%
5000 + 3.3%	12.61 +.136	0.49	0.15%	0.04%	2.13%	0.95%
7000 + 4.4%	14.00 +.183	0.36	0.15%	0.04%	2.93%	1.27%
10000 + 6.0%	15.50 +.253	0.26	0.15%	0.03%	4.06%	1.76%
15000 + 8.7%	17.22 +.371	0.17	0.15%	0.03%	5.96%	2.58%
20000 +11.4%	18.45 +.490	0.13	0.15%	0.03%	7.86%	3.40%
30000 +17.0%	20.19 +.729	0.09	0.15%	0.03%	11.73%	5.04%
50000 +28.4%	22.39 +1.224	0.05	0.15%	0.03%	19.82%	8.35%
70000 +40.6%	23.84 +1.754	0.04	0.15%	0.03%	28.65%	11.72%

Table A-XII

TH= 692 + 0.90 K (0.13%)
TC= 4 + 0.10 K (2.50%)

DY=.01 DB DG=.13%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE ETH	ETC	EY	EG
10 + 1.7%	0.15 +.002	17.00	0.18%	1.02%	0.33%	0.14%
15 + 1.3%	0.22 +.003	15.71	0.17%	0.69%	0.30%	0.13%
20 + 1.1%	0.29 +.003	14.72	0.16%	0.52%	0.29%	0.12%
30 + 0.9%	0.43 +.004	13.27	0.15%	0.35%	0.27%	0.12%
50 + 0.7%	0.69 +.005	11.38	0.14%	0.22%	0.27%	0.12%
70 + 0.7%	0.94 +.006	10.13	0.14%	0.16%	0.27%	0.12%
100 + 0.6%	1.29 +.007	8.82	0.14%	0.12%	0.28%	0.12%
150 + 0.6%	1.81 +.009	7.33	0.13%	0.08%	0.29%	0.13%
200 + 0.6%	2.28 +.011	6.41	0.13%	0.06%	0.30%	0.13%
300 + 0.7%	3.08 +.015	5.14	0.13%	0.05%	0.34%	0.15%
500 + 0.7%	4.35 +.020	3.74	0.13%	0.03%	0.40%	0.17%
700 + 0.8%	5.33 +.026	2.96	0.13%	0.03%	0.47%	0.20%
1000 + 1.0%	6.48 +.033	2.27	0.13%	0.02%	0.57%	0.25%
1500 + 1.2%	7.90 +.044	1.64	0.13%	0.02%	0.74%	0.32%
2000 + 1.4%	8.97 +.055	1.28	0.13%	0.02%	0.90%	0.39%
3000 + 1.9%	10.55 +.076	0.90	0.13%	0.02%	1.24%	0.54%
5000 + 2.9%	12.61 +.113	0.56	0.13%	0.02%	1.91%	0.83%
7000 + 3.8%	14.00 +.160	0.41	0.13%	0.02%	2.58%	1.12%
10000 + 5.3%	15.50 +.223	0.29	0.13%	0.02%	3.58%	1.56%
15000 + 7.7%	17.22 +.328	0.19	0.13%	0.02%	5.27%	2.23%
20000 +10.1%	18.45 +.433	0.15	0.13%	0.02%	6.96%	3.01%
30000 +15.4%	20.19 +.645	0.10	0.13%	0.01%	10.33%	4.47%
50000 +25.0%	22.39 +1.030	0.06	0.13%	0.01%	17.47%	7.41%
70000 +35.6%	23.34 +1.539	0.04	0.13%	0.01%	25.36%	10.33%

Table A-XIII

```
TH=  373 +   0.50 K (0.13%)
TC=   80 +   1.00 K (1.25%)

DY=.01 DB        DG=.10%


                                            ERROR CONTRIBUTIONS TO TE
TE(K)            F(DB)          Y(DB)       ETH      ETC      EY       FG


   10 +13.5%     0.15 +.027     6.29        1.54%   13.07%    2.71%    1.18%
   15 +12.7%     0.22 +.027     6.11        1.03%    8.83%    1.93%    0.84%
   20 + 9.8%     0.29 +.027     5.94        0.85%    6.71%    1.54%    0.67%

   30 + 6.9%     0.43 +.028     5.64        0.63%    4.58%    1.16%    0.50%
   50 + 4.6%     0.69 +.029     5.12        0.44%    2.39%    0.86%    0.38%
   70 + 3.6%     0.94 +.030     4.70        0.37%    2.16%    0.75%    0.32%

  100 + 2.9%     1.29 +.032     4.20        0.31%    1.61%    0.67%    0.29%
  150 + 2.4%     1.81 +.035     3.57        0.26%    1.19%    0.63%    0.27%
  200 + 2.1%     2.28 +.038     3.11        0.24%    0.98%    0.63%    0.27%

  300 + 1.9%     3.08 +.043     2.48        0.22%    0.77%    0.67%    0.29%
  500 + 1.9%     4.35 +.053     1.78        0.20%    0.60%    0.80%    0.35%
  700 + 2.1%     5.33 +.063     1.39        0.19%    0.52%    0.94%    0.41%

 1000 + 2.3%     6.43 +.078     1.04        0.18%    0.47%    1.17%    0.51%
 1500 + 2.8%     7.90 +.103     0.74        0.18%    0.43%    1.55%    0.67%
 2000 + 3.4%     8.97 +.128     0.57        0.18%    0.40%    1.94%    0.84%

 3000 + 4.5%    10.55 +.177     0.39        0.18%    0.38%    2.72%    1.18%
 5000 + 6.7%    12.61 +.275     0.24        0.17%    0.37%    4.30%    1.86%
 7000 + 9.0%    14.00 +.374     0.18        0.17%    0.36%    5.83%    2.55%

10000 +12.4%    15.50 +.522     0.12        0.17%    0.35%    8.27%    3.57%
15000 +18.1%    17.22 +.773     0.08        0.17%    0.35%   12.33%    5.29%
20000 +24.0%    18.45 +1.029     0.06        0.17%    0.35%  16.50%    7.31%

30000 +36.4%    20.19 +1.566     0.04        0.17%    0.35%  25.38%   10.50%
50000 +65.3%    22.39 +2.313     0.03        0.17%    0.34%  47.02%   17.74%
70000 +105.3%    23.34 +4.575     0.02        0.17%    0.34%  79.76%   25.50%
```

Table A-XIV

TH= 373 + 0.50 K (0.13%)
TC= 4 + 0.50 K (12.50%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE			
			ETH	ETC	EY	EG
10 + 5.9%	0.15 +.008	14.37	0.19%	5.19%	0.33%	0.15%
15 + 4.1%	0.22 +.009	13.10	0.17%	3.50%	0.31%	0.13%
20 + 3.2%	0.29 +.009	12.14	0.16%	2.66%	0.29%	0.13%
30 + 2.4%	0.43 +.010	10.74	0.15%	1.82%	0.29%	0.12%
50 + 1.7%	0.69 +.011	8.94	0.15%	1.15%	0.29%	0.12%
70 + 1.4%	0.94 +.012	7.77	0.14%	0.86%	0.29%	0.13%
100 + 1.2%	1.29 +.014	6.58	0.14%	0.64%	0.31%	0.13%
150 + 1.1%	1.81 +.016	5.31	0.14%	0.47%	0.34%	0.15%
200 + 1.0%	2.28 +.019	4.49	0.14%	0.39%	0.36%	0.16%
300 + 1.1%	3.08 +.023	3.45	0.14%	0.30%	0.43%	0.18%
500 + 1.2%	4.35 +.032	2.39	0.14%	0.24%	0.55%	0.24%
700 + 1.3%	5.33 +.040	1.83	0.14%	0.21%	0.67%	0.29%
1000 + 1.6%	6.48 +.052	1.36	0.14%	0.19%	0.86%	0.37%
1500 + 2.0%	7.90 +.072	0.95	0.14%	0.17%	1.17%	0.51%
2000 + 2.4%	8.97 +.092	0.73	0.14%	0.16%	1.48%	0.64%
3000 + 3.3%	10.55 +.131	0.50	0.14%	0.15%	2.11%	0.92%
5000 + 5.1%	12.61 +.209	0.31	0.14%	0.15%	3.36%	1.46%
7000 + 6.9%	14.00 +.287	0.22	0.14%	0.14%	4.61%	2.00%
10000 + 9.6%	15.50 +.405	0.16	0.14%	0.14%	6.50%	2.81%
15000 +14.1%	17.22 +.602	0.11	0.14%	0.14%	9.68%	4.17%
20000 +18.7%	18.45 +.802	0.08	0.14%	0.14%	12.92%	5.54%
30000 +28.2%	20.19 +1.214	0.05	0.14%	0.14%	19.66%	8.29%
50000 +49.1%	22.39 +2.113	0.03	0.14%	0.14%	34.88%	13.91%
70000 +74.5%	23.84 +3.221	0.02	0.14%	0.14%	54.41%	19.78%

Table A-XV

TH= 373 + 0.15 K (0.04%)
TC= 300 + 0.10 K (0.03%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE ETH	 ETC	 EY	 EG
10 +65.3%	0.15 +.095	0.92	6.37%	5.25%	37.46%	16.26%
15 +44.7%	0.22 +.096	0.91	4.32%	3.54%	25.70%	11.16%
20 +34.4%	0.29 +.096	0.89	3.29%	2.69%	19.84%	8.61%
30 +24.2%	0.43 +.098	0.87	2.26%	1.84%	13.98%	6.07%
50 +16.0%	0.69 +.102	0.82	1.44%	1.16%	9.34%	4.06%
70 +12.5%	0.94 +.106	0.78	1.09%	0.87%	7.39%	3.21%
100 +10.0%	1.29 +.112	0.73	0.82%	0.65%	5.97%	2.59%
150 + 8.2%	1.81 +.121	0.65	0.62%	0.48%	4.95%	2.15%
200 + 7.4%	2.28 +.131	0.59	0.51%	0.39%	4.52%	1.96%
300 + 6.8%	3.08 +.150	0.50	0.41%	0.31%	4.25%	1.84%
500 + 6.9%	4.35 +.189	0.38	0.33%	0.24%	4.41%	1.91%
700 + 7.4%	5.33 +.229	0.31	0.29%	0.21%	4.84%	2.10%
1000 + 8.5%	6.48 +.288	0.24	0.27%	0.19%	5.64%	2.45%
1500 +10.6%	7.90 +.386	0.17	0.25%	0.17%	7.11%	3.08%
2000 +12.8%	8.97 +.485	0.14	0.24%	0.16%	8.66%	3.74%
3000 +17.3%	10.55 +.686	0.10	0.23%	0.15%	11.34%	5.39%
5000 +26.7%	12.61 +1.096	0.06	0.22%	0.15%	18.50%	7.84%
7000 +36.6%	14.00 +1.523	0.04	0.21%	0.14%	25.64%	10.64%
10000 +53.0%	15.50 +2.238	0.03	0.21%	0.14%	37.74%	14.93%
15000 +87.5%	17.22 +3.727	0.02	0.21%	0.14%	64.66%	22.47%
20000 +142.2%	18.45 +6.035	0.02	0.21%	0.14%	111.11%	30.70

Table A-XVI

TH= 373 + 0.15 K (0.04%)
TC= 80 + 0.20 K (0.25%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE			
			ETH	ETC	EY	EG
10 + 7.0%	0.15 +.010	6.29	0.46%	2.61%	2.71%	1.13%
15 + 4.9%	0.22 +.010	6.11	0.32%	1.77%	1.93%	0.34%
20 + 3.3%	0.29 +.011	5.94	0.26%	1.34%	1.54%	0.67%
30 + 2.8%	0.43 +.011	5.64	0.19%	0.92%	1.16%	0.50%
50 + 2.0%	0.59 +.012	5.12	0.13%	0.58%	0.36%	0.38%
70 + 1.6%	0.94 +.014	4.70	0.11%	0.43%	0.75%	0.32%
100 + 1.4%	1.29 +.015	4.20	0.09%	0.32%	0.67%	0.29%
150 + 1.2%	1.81 +.018	3.57	0.08%	0.24%	0.63%	0.27%
200 + 1.2%	2.23 +.021	3.11	0.07%	0.20%	0.63%	0.27%
300 + 1.2%	3.08 +.026	2.48	0.06%	0.15%	0.67%	0.29%
500 + 1.3%	4.35 +.036	1.78	0.06%	0.12%	0.30%	0.35%
700 + 1.5%	5.33 +.046	1.39	0.06%	0.10%	0.94%	0.41%
1000 + 1.8%	6.48 +.061	1.04	0.06%	0.09%	1.17%	0.51%
1500 + 2.4%	7.90 +.086	0.74	0.05%	0.09%	1.55%	0.57%
2000 + 2.9%	8.97 +.111	0.57	0.05%	0.08%	1.94%	0.34%
3000 + 4.0%	10.55 +.160	0.39	0.05%	0.08%	2.72%	1.13%
5000 + 6.3%	12.61 +.253	0.24	0.05%	0.07%	4.30%	1.36%
7000 + 8.6%	14.00 +.357	0.18	0.05%	0.07%	5.88%	2.55%
10000 +12.0%	15.50 +.505	0.12	0.05%	0.07%	8.27%	3.57%
15000 +17.7%	17.22 +.756	0.08	0.05%	0.07%	12.33%	5.29%
20000 +23.6%	18.45 +1.012	0.06	0.05%	0.07%	16.50%	7.01%
30000 +35.0%	20.19 +1.549	0.04	0.05%	0.07%	25.38%	10.50%
50000 +64.9%	22.39 +2.301	0.03	0.05%	0.07%	47.02%	17.74%
70000 +105.4%	23.84 +4.558	0.02	0.05%	0.07%	79.76%	25.50%

Table A-XVII

TH= 373 + 0.15 K (0.04%)
TC= 4 + 0.10 K (2.50%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE ETH	ETC	EY	EG
10 + 1.6%	0.15 +.002	14.37	0.06%	1.04%	0.33%	0.15%
15 + 1.2%	0.22 +.003	13.10	0.05%	0.70%	0.31%	0.13%
20 + 1.0%	0.29 +.003	12.14	0.05%	0.53%	0.29%	0.13%
30 + 0.8%	0.43 +.003	10.74	0.05%	0.36%	0.29%	0.12%
50 + 0.7%	0.69 +.004	8.94	0.04%	0.23%	0.29%	0.12%
70 + 0.6%	0.94 +.005	7.77	0.04%	0.17%	0.29%	0.13%
100 + 0.6%	1.29 +.007	6.53	0.04%	0.13%	0.31%	0.13%
150 + 0.6%	1.81 +.009	5.31	0.04%	0.09%	0.34%	0.15%
200 + 0.6%	2.28 +.011	4.49	0.04%	0.08%	0.36%	0.16%
300 + 0.7%	3.08 +.016	3.45	0.04%	0.06%	0.43%	0.18%
500 + 0.9%	4.35 +.024	2.39	0.04%	0.05%	0.55%	0.24%
700 + 1.0%	5.33 +.032	1.83	0.04%	0.04%	0.67%	0.29%
1000 + 1.3%	6.43 +.044	1.36	0.04%	0.04%	0.86%	0.37%
1500 + 1.8%	7.93 +.064	0.95	0.04%	0.03%	1.17%	0.51%
2000 + 2.2%	8.97 +.083	0.73	0.04%	0.03%	1.48%	0.64%
3000 + 3.1%	10.55 +.123	0.50	0.04%	0.03%	2.11%	0.92%
5000 + 4.9%	12.61 +.201	0.31	0.04%	0.03%	3.36%	1.46%
7000 + 6.7%	14.00 +.279	0.22	0.04%	0.03%	4.61%	2.00%
10000 + 9.4%	15.50 +.396	0.16	0.04%	0.03%	6.50%	2.81%
15000 +13.9%	17.22 +.593	0.11	0.04%	0.03%	9.68%	4.17%
20000 +18.5%	18.45 +.793	0.08	0.04%	0.03%	12.92%	5.54%
30000 +28.0%	20.19 +1.205	0.05	0.04%	0.03%	19.66%	8.29%
50000 +48.9%	22.39 +2.110	0.03	0.04%	0.03%	34.89%	13.91%
70000 +74.3%	23.84 +3.212	0.02	0.04%	0.03%	54.41%	19.73%

Table A-XVIII

TH= 300 + 1.00 K (0.33%)
TC= 80 + 1.00 K (1.25%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE ETH	ETC	EY	EG
10 +22.4%	0.15 +.032	5.37	4.09%	14.09%	2.92%	1.27%
15 +15.4%	0.22 +.033	5.21	2.88%	9.55%	2.09%	0.91%
20 +11.9%	0.29 +.033	5.05	2.27%	7.27%	1.67%	0.73%
30 + 8.5%	0.43 +.035	4.77	1.67%	5.00%	1.27%	0.55%
50 + 5.7%	0.69 +.037	4.30	1.18%	3.18%	0.95%	0.41%
70 + 4.6%	0.94 +.039	3.92	0.97%	2.40%	0.83%	0.36%
100 + 3.7%	1.29 +.041	3.47	0.82%	1.82%	0.75%	0.33%
150 + 3.1%	1.81 +.046	2.91	0.70%	1.36%	0.72%	0.31%
200 + 2.8%	2.28 +.050	2.52	0.64%	1.14%	0.73%	0.32%
300 + 2.6%	3.08 +.058	1.93	0.58%	0.91%	0.80%	0.35%
500 + 2.6%	4.35 +.073	1.40	0.53%	0.73%	0.97%	0.42%
700 + 2.8%	5.33 +.087	1.08	0.51%	0.65%	1.17%	0.51%
1000 + 3.2%	6.48 +.107	0.81	0.49%	0.59%	1.47%	0.64%
1500 + 3.9%	7.90 +.141	0.57	0.48%	0.55%	1.99%	0.86%
2000 + 4.6%	8.97 +.174	0.44	0.47%	0.52%	2.50%	1.09%
3000 + 6.1%	10.55 +.240	0.30	0.47%	0.50%	3.55%	1.54%
5000 + 9.0%	12.61 +.371	0.18	0.46%	0.48%	5.65%	2.45%
7000 +12.1%	14.00 +.503	0.13	0.46%	0.47%	7.77%	3.36%
10000 +16.6%	15.50 +.703	0.09	0.46%	0.47%	10.99%	4.73%
15000 +24.5%	17.22 +1.042	0.06	0.46%	0.46%	16.52%	7.02%
20000 +32.6%	18.45 +1.396	0.05	0.46%	0.46%	22.34%	9.34%
30000 +50.3%	20.19 +2.165	0.03	0.46%	0.46%	35.35%	14.07%
50000 +93.0%	22.39 +4.232	0.02	0.46%	0.46%	72.95%	24.15%
70000 +197.1%	23.84 +8.526	0.01	0.46%	0.46%	160.61%	35.6

Table A-XIX

TH= 300 + 0.10 K (0.03%)
TC= 80 + 0.20 K (0.25%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE			
			ETH	ETC	EY	EG
10 + 7.4%	0.15 +.011	5.37	0.41%	2.32%	2.92%	1.27%
15 + 5.2%	0.22 +.011	5.21	0.29%	1.91%	2.09%	0.91%
20 + 4.1%	0.29 +.011	5.05	0.23%	1.45%	1.67%	0.73%
30 + 3.0%	0.43 +.012	4.77	0.17%	1.00%	1.27%	0.55%
50 + 2.1%	0.69 +.014	4.30	0.12%	0.64%	0.95%	0.41%
70 + 1.8%	0.94 +.015	3.92	0.10%	0.48%	0.83%	0.36%
100 + 1.5%	1.29 +.017	3.47	0.08%	0.36%	0.75%	0.33%
150 + 1.4%	1.81 +.020	2.91	0.07%	0.27%	0.72%	0.31%
200 + 1.3%	2.28 +.024	2.52	0.06%	0.23%	0.73%	0.32%
300 + 1.4%	3.08 +.030	1.98	0.06%	0.13%	0.80%	0.35%
500 + 1.6%	4.35 +.044	1.40	0.05%	0.15%	0.97%	0.42%
700 + 1.9%	5.33 +.057	1.03	0.05%	0.13%	1.17%	0.51%
1000 + 2.3%	6.48 +.077	0.81	0.05%	0.12%	1.47%	0.64%
1500 + 3.0%	7.90 +.109	0.57	0.05%	0.11%	1.99%	0.86%
2000 + 3.7%	8.97 +.142	0.44	0.05%	0.10%	2.50%	1.09%
3000 + 5.2%	10.55 +.207	0.30	0.05%	0.10%	3.55%	1.54%
5000 + 8.2%	12.61 +.333	0.18	0.05%	0.10%	5.65%	2.45%
7000 +11.3%	14.00 +.470	0.13	0.05%	0.09%	7.77%	3.36%
10000 +15.9%	15.50 +.670	0.09	0.05%	0.09%	10.99%	4.73%
15000 +23.7%	17.22 +1.009	0.06	0.05%	0.09%	16.52%	7.02%
20000 +31.3%	18.45 +1.362	0.05	0.05%	0.09%	22.34%	9.34%
30000 +49.6%	20.19 +2.132	0.03	0.05%	0.09%	35.35%	14.07%
50000 +97.2%	22.39 +4.199	0.02	0.05%	0.09%	72.95%	24.15%
70000 +196.4%	23.84 +8.492	0.01	0.05%	0.09%	160.61%	35.60

Table A-XX

TH= 300 + 1.00 K (0.33%)
TC= 4 + 0.50 K (12.50%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE ETH	ETC	EY	EG
10 + 6.2%	0.15 +.009	13.45	0.47%	5.24%	0.34%	0.15%
15 + 4.4%	0.22 +.009	12.20	0.43%	3.55%	0.31%	0.13%
20 + 3.5%	0.29 +.010	11.25	0.41%	2.70%	0.30%	0.13%
30 + 2.7%	0.43 +.011	9.87	0.38%	1.86%	0.29%	0.13%
50 + 2.0%	0.69 +.013	8.12	0.36%	1.18%	0.29%	0.13%
70 + 1.7%	0.94 +.014	6.99	0.36%	0.89%	0.30%	0.13%
100 + 1.5%	1.29 +.017	5.85	0.35%	0.68%	0.32%	0.14%
150 + 1.4%	1.81 +.020	4.66	0.35%	0.51%	0.36%	0.16%
200 + 1.3%	2.28 +.024	3.89	0.34%	0.42%	0.40%	0.17%
300 + 1.4%	3.08 +.030	2.95	0.34%	0.34%	0.47%	0.21%
500 + 1.5%	4.35 +.042	2.01	0.34%	0.27%	0.63%	0.27%
700 + 1.7%	5.33 +.052	1.52	0.34%	0.24%	0.73%	0.34%
1000 + 2.0%	6.43 +.068	1.12	0.34%	0.22%	1.02%	0.44%
1500 + 2.6%	7.90 +.093	0.78	0.34%	0.20%	1.40%	0.61%
2000 + 3.1%	8.97 +.113	0.60	0.34%	0.19%	1.79%	0.73%
3000 + 4.2%	10.55 +.167	0.41	0.34%	0.19%	2.57%	1.12%
5000 + 6.4%	12.61 +.264	0.25	0.34%	0.18%	4.13%	1.79%
7000 + 8.7%	14.00 +.362	0.18	0.34%	0.18%	5.70%	2.47%
10000 +12.1%	15.50 +.509	0.13	0.34%	0.17%	8.07%	3.49%
15000 +17.8%	17.22 +.757	0.08	0.34%	0.17%	12.08%	5.18%
20000 +23.6%	18.45 +1.010	0.06	0.34%	0.17%	16.70%	6.39%
30000 +35.8%	20.19 +1.540	0.04	0.34%	0.17%	24.96%	10.34%
50000 +64.2%	22.39 +2.772	0.03	0.34%	0.17%	46.21%	17.49%
70000 +103.7%	23.84 +4.485	0.02	0.34%	0.17%	78.02%	25.16%

Table A-XXI

TH= 300 + 0.10 K (0.03%)
TC= 4 + 0.10 K (2.50%)

DY=.01 DB DG=.10%

TE(K)	F(DB)	Y(DB)	ERROR CONTRIBUTIONS TO TE ETH	ETC	EY	EG
10 + 1.6%	0.15 +.002	13.45	0.05%	1.05%	0.34%	0.15%
15 + 1.2%	0.22 +.003	12.20	0.04%	0.71%	0.31%	0.13%
20 + 1.0%	0.29 +.003	11.25	0.04%	0.54%	0.30%	0.13%
30 + 0.8%	0.43 +.003	9.87	0.04%	0.37%	0.29%	0.13%
50 + 0.7%	0.69 +.004	8.12	0.04%	0.24%	0.29%	0.13%
70 + 0.7%	0.94 +.005	6.99	0.04%	0.18%	0.30%	0.13%
100 + 0.6%	1.29 +.007	5.35	0.04%	0.14%	0.32%	0.14%
150 + 0.7%	1.81 +.010	4.66	0.03%	0.10%	0.36%	0.16%
200 + 0.7%	2.28 +.012	3.89	0.03%	0.08%	0.40%	0.17%
300 + 0.3%	3.08 +.017	2.95	0.03%	0.07%	0.47%	0.21%
500 + 1.0%	4.35 +.027	2.01	0.03%	0.05%	0.63%	0.27%
700 + 1.2%	5.33 +.037	1.52	0.03%	0.05%	0.78%	0.34%
1000 + 1.5%	6.48 +.052	1.12	0.03%	0.04%	1.02%	0.44%
1500 + 2.1%	7.90 +.076	0.78	0.03%	0.04%	1.40%	0.61%
2000 + 2.6%	8.97 +.100	0.60	0.03%	0.04%	1.79%	0.78%
3000 + 3.3%	10.55 +.149	0.41	0.03%	0.04%	2.57%	1.12%
5000 + 6.0%	12.61 +.246	0.25	0.03%	0.04%	4.13%	1.79%
7000 + 8.2%	14.00 +.344	0.18	0.03%	0.04%	5.70%	2.47%
10000 +11.6%	15.50 +.490	0.13	0.03%	0.03%	8.07%	3.49%
15000 +17.3%	17.22 +.738	0.08	0.03%	0.03%	12.08%	5.18%
20000 +23.2%	13.45 +.991	0.06	0.03%	0.03%	16.20%	6.39%
30000 +35.4%	20.19 +1.521	0.04	0.03%	0.03%	24.96%	10.34%
50000 +63.8%	22.39 +2.753	0.03	0.03%	0.03%	46.21%	17.49%
70000 +103.2%	23.84 +4.466	0.02	0.03%	0.03%	73.02%	25.16%

Appendix B. Mismatch

In this appendix, different mismatch errors are expressed in a set of figures and tables. The derivations of the equations used in the computer program are presented last. The computer program actually used is contained in Appendix D.

B.1 Mismatch Uncertainty

The following figures, referred to as the "Mismatch Uncertainty" figures, are based on Eq. (4) in the text. This set can be used to estimate the mismatch ambiguity discussed in Section 3 or they can be used to estimate the error caused by using noise standards that have a different reflection coefficient than the "antenna". The maximum uncertainty in T_e versus $|\Gamma_{ant} - \Gamma^*_{amp}|$ is graphed for various values of β, b, and $|\Gamma_{std} - \Gamma_{ant}|$. For estimating mismatch ambiguity $|\Gamma_{std} - \Gamma_{ant}|$ is the magnitude of the uncertainty of Γ_{ant}. An upper bound for the mismatch ambiguity is obtained using the maximum value for $|\Gamma_{ant} - \Gamma^*_{amp}|$ consistent with uncertainty of Γ_{ant}.

The values of β and b for the various mismatch uncertainty figures is listed in Table BI.

Table B-I. Key to mismatch uncertainty figures

Figure	β	b
B1	0	0
B2	.1	.2
B3	.1	1
B4	.2	.2
B5	.2	1
B6	.3	.2
B7	.3	1

As an example of how to use these figures, consider an amplifier where $\beta = 0.2$ and $b = 0.2$. From Table B-I we see that figure B4 is appropriate. For this amplifier if $|\Gamma_{std}| \leq 0.01$, $|\Gamma_{amp}| \leq 0.05$, and $|\Gamma_{ant}| \leq 0.1$, then $|\Gamma_{ant} - \Gamma_{amp}^*| \leq 0.15$ and $|\Gamma_{std} - \Gamma_{ant}| \leq 0.11$. Thus using figure B4 the maximum mismatch error caused by using a standard with a reflection coefficient different from the "antenna" is about ± 7%. For this same example, to estimate the mismatch ambiguity (because Γ_{amp} can have any amplitude a phase restricted only by $|\Gamma_{ant}| \leq 0.1$) we use $|\Gamma_{ant} - \Gamma_{amp}^*| \leq 0.15$ and $|\Gamma_{std} - \Gamma_{ant}| \leq 0.1$. The mismatch ambiguity is about ± 6%.

B.2. Mismatch Error

Mismatch error is the error in measuring either T_e or F_{dB} caused when the reflection coefficients of the "antenna", the hot standard, and the cold standard differ from each other. Part of the mismatch error, namely the error caused because the "antenna" reflection coefficient differs from the reflection coefficient of the two standards, is already given in the mismatch uncertainty figures, and this contribution is NOT included in the mismatch error tables. In other words, the tables only include the error because the impedance of the hot and cold standards differ from each other. The magnitude of the error depends on the magnitudes of T_{hot}, T_{cold}, β, and bT_a/T_e. A key to the 19 mismatch error tables is provided in Table B-II. The errors listed in the tables are decibel errors to F_{dB}. The meaning of the symbols used in the mismatch error tables compared with the symbols used in Eqs. (1), (3), and (4) are as follows:

Symbols	Meaning
T(HOT)	T_{hot}
T(COLD)	T_{cold}
BETA	β
BTA/TE	$bT_a/T_e \simeq b$
ERR	$\lvert\Gamma_{hot} - \Gamma_{cold}\rvert$
ANT	$\lvert\Gamma'_{ant}\rvert \simeq \lvert\Gamma_{ant} - \Gamma^*_{amp}\rvert$
F(DB)	F_{dB}

As an example of how to use these tables, consider an amplifier where $\beta = 0.2$ and $b = 0.2$, and where a hot standard of 10,000 K and a cold standard of 300 K will be used for the measurement of noise figure. Then from Table B-II (page 62) we note that Table B-VI is appropriate for use with these given conditions. If the amplifier has a noise figure near 6 dB, and Γ_{hot}, Γ_{cold}

and $\Gamma_{amp} \leq 0.05$ and $\Gamma_{ant} \leq 0.15$ so that "ERR" $\leq |\Gamma_{hot}-\Gamma_{cold}| = 0.1$, "ANT" $= |\Gamma_{ant}-\Gamma^*_{amp}| = 0.2$, the maximum mismatch error is ± 0.281 dB.

B.3. Derivations

The output power, P_{out}, out of a linear amplifier when a standard with temperature T_{std} and reflection coefficient Γ_{std} is attached to it is [3]

$$P_{out} = kGBM_{std}(T_{std}+T_e) \tag{B.1}$$

where k is Boltzmann's constant, B is the appropriate bandwidth, G is the appropriate gain, and M_{std} is the mismatch factor ($M_{std} = 1 - |\Gamma'_{std}|^2$), where Γ'_{std} defined in eq. (5) except with the subscript "ant" replaced by "std"). Using eq. (4) except with the subscript "ant" replaced by "std":

$$T_e(\mathrm{std}) = \frac{T_a(1+b|\Gamma'_{std}-\beta|^2)}{1 - |\Gamma'_{std}|^2}, \tag{B.2}$$

then

$$\begin{aligned} P_{out}/(GBk) = {} & T_{std} + T_a - |\Gamma'_{std}|^2(T_{std}-bT_a) \\ & + bT_a[|\beta|^2-2\mathrm{Re}(\beta^*\Gamma'_{std})] \end{aligned} \tag{B.3}$$

where Re( ) implies the real part of the vector in parenthesis, and the asterisk indicates the complex conjugate. If we let

$$\Gamma'_{std} = \Gamma'_{ant} + \varepsilon'_{std}, \tag{B.4}$$

then using eq. (B.3) and eq. (B.4)

$$P_{out}/(GBk) = M_{ant}(T_{std}+T_e-\Delta T_{std}) \tag{B.5}$$

where M_{ant} and $T_e(ant)$ are the mismatch factor and effective input noise temperature when the "antenna" is connected to the amplifier,

$$M_{ant}\Delta T_{std} \equiv T_{std}(1-bT_a/T_{std})[|\varepsilon'_{std}|^2+2\mathrm{Re}(\Gamma'_{ant}\varepsilon'^{*}_{std})]$$
$$+ 2bT_a[\mathrm{Re}(\beta^{*}\varepsilon'_{std})]. \tag{B.6}$$

If G and B do not change as Γ_{ant} changes, then

$$Y = \frac{T_{hot} + T_e - \Delta T_{hot}}{T_{cold} + T_e - \Delta T_{cold}}, \tag{B.7}$$

where the subscript "std" is changed to "hot" or "cold" as appropriate. Solving for T_e,

$$T_e = \frac{(T_{hot}-\Delta T_{hot}) - Y(T_{cold}-\Delta T_{cold})}{Y - 1}. \tag{B.8}$$

For the computer calculations, eq. (B.6) is modified so that

$$\mathrm{Re}(\Gamma'_{ant}\varepsilon'^{*}_{std}) \rightarrow \pm \Gamma'_{ant}\varepsilon_{std},$$

and

$$\mathrm{Re}(\beta^{*}\varepsilon'_{std}) \rightarrow \pm \beta\varepsilon'_{std}.$$

For <u>mismatch error</u> we assume $|\varepsilon'_{hot}| = |\varepsilon'_{cold}|$, and T_e in Eq. (B.8) is computed for the eight sign combinations of the parameters Γ'_{ant}, $\pm\ \varepsilon'_{hot}$, $\pm\ \varepsilon'_{cold}$, and β, and the greatest difference from the value of T_e with $\varepsilon'_{hot} = \varepsilon'_{cold} = 0$ is used as the mismatch error.

For <u>mismatch uncertainty</u> $\varepsilon'_{hot} \equiv \varepsilon'_{cold} = \varepsilon'$ so that using eqs. (B.2) and (B.4)

$$T_e(\mathrm{std}) = \frac{T_a[1+b|\Gamma'_{ant}-\beta|^2+b|\varepsilon'|^2+2b\mathrm{Re}(\varepsilon'^*(\Gamma'_{ant}-\beta))]}{1 - |\Gamma'_{ant}|^2 - |\varepsilon'|^2 - 2\mathrm{Re}(\Gamma'^*_{ant}\varepsilon')} \qquad (B.9)$$

Using eq. (B.2) and eq. (B.9),

$$\frac{T_e(\mathrm{ant})-T_e(\mathrm{std})}{T_a} = -\frac{L+Lb|\Gamma'_{ant}-\beta|^2+b|\varepsilon'|^2+2b\mathrm{Re}[\varepsilon'^*(\Gamma'_{ant}-\beta)]}{(1-|\Gamma'_{ant}|^2)(1-L)} \qquad (B.10)$$

where

$$L = \frac{|\varepsilon'|^2 + 2\mathrm{Re}(\Gamma'^*_{ant}\varepsilon')}{1 - |\Gamma'_{ant}|^2}.$$

Equation (B.10) is the mismatch uncertainty and unlike the mismatch error calculated from eq. (B.8), the mismatch uncertainty is independent of the values for T_{hot} and T_{cold}. For the computer calculation, eq. (B.10) is modified so that

$$\mathrm{Re}(\Gamma'^*_{ant}\varepsilon') \rightarrow \Gamma'_{ant}\varepsilon'$$

and

$$\mathrm{Re}[\varepsilon'^*(\Gamma'_{ant}-\beta)] \rightarrow \varepsilon'(\Gamma'_{ant}-\beta).$$

Then eq. (B.10) is computed for the four different sign combinations of the parameters Γ'_{ant}, $\pm\varepsilon'$, and $\pm\beta'$, and the greatest value is used as the mismatch uncertainty.

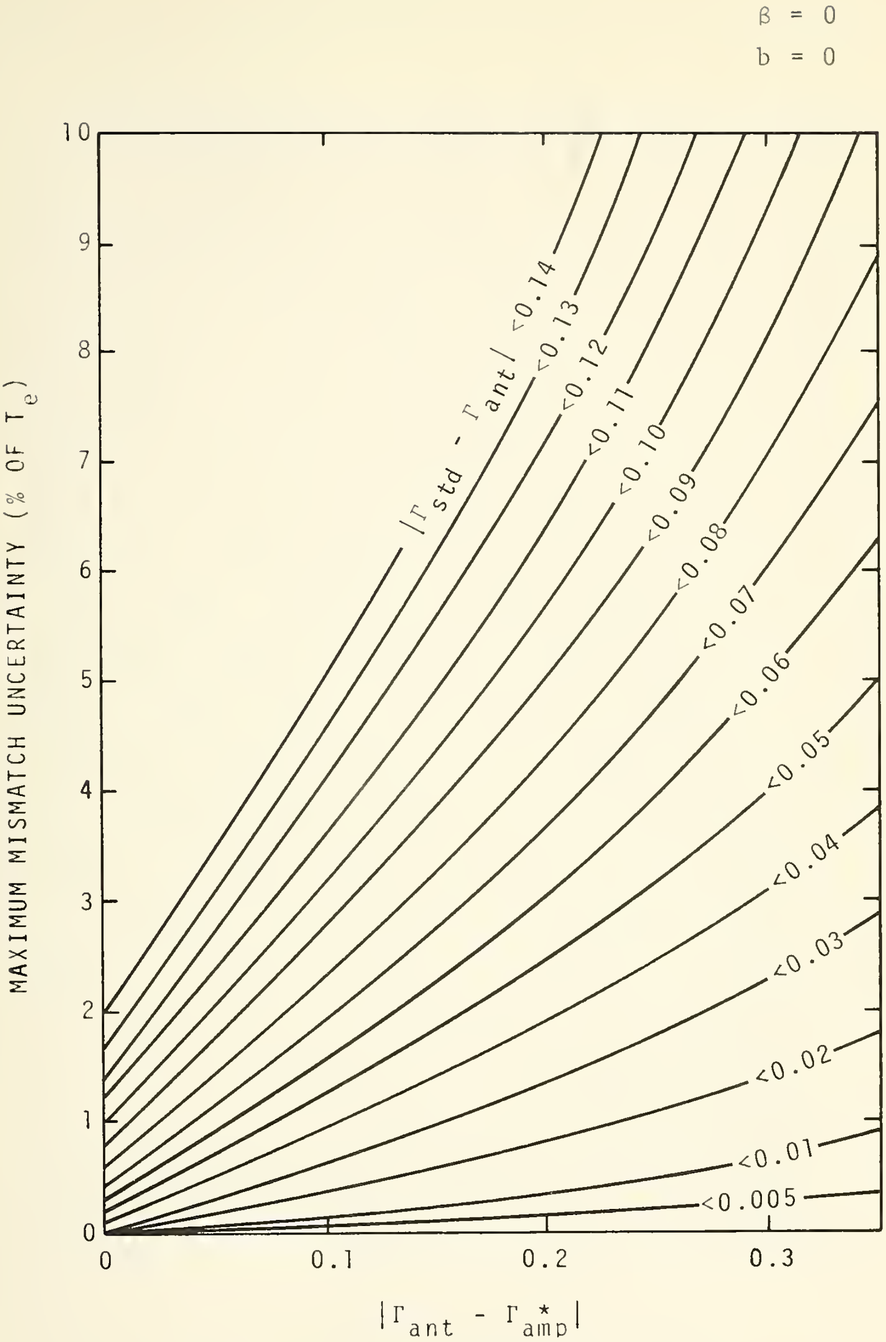

Fig. B1. Mismatch Uncertainty.
Explanation on p. 49.

$\beta = 0.1$

$b = 0.2$

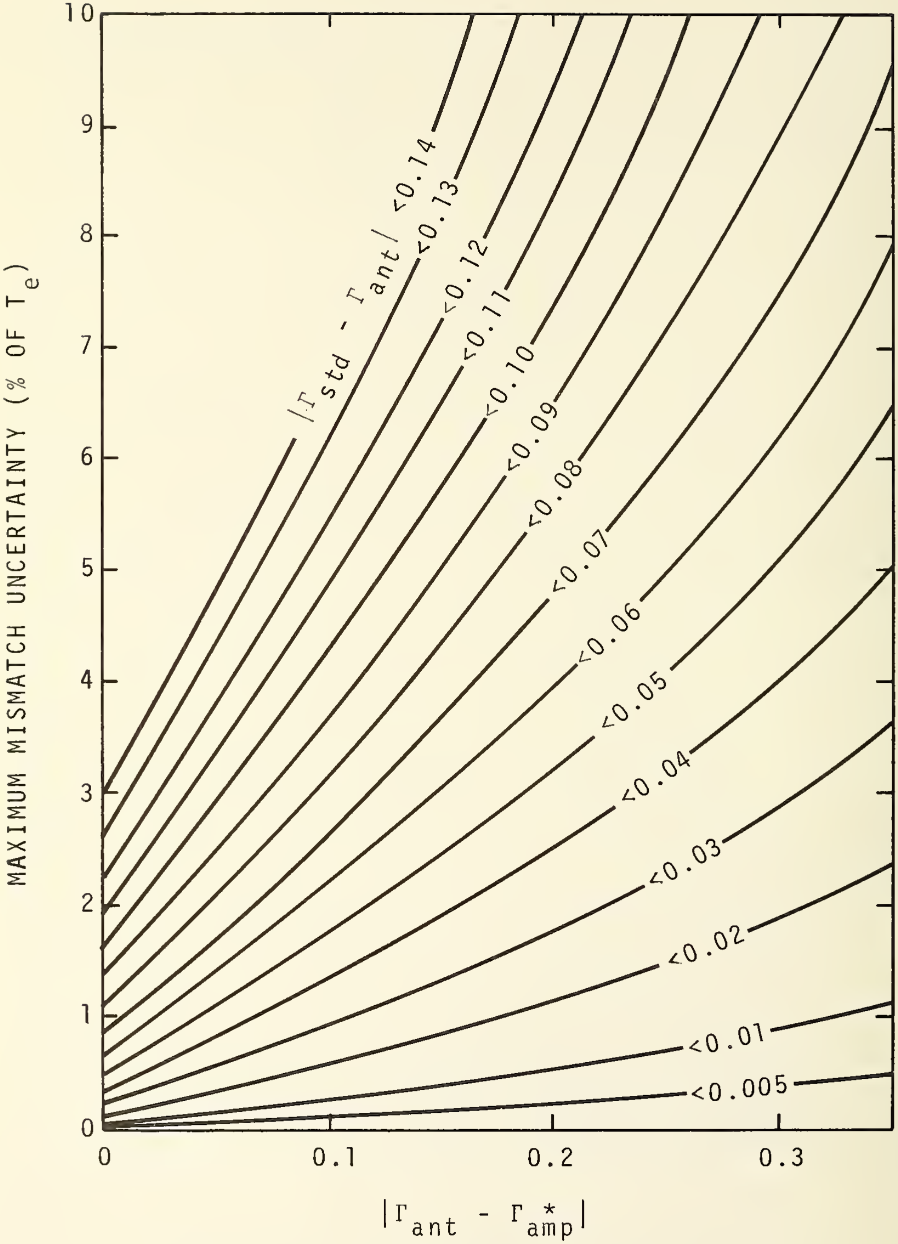

Fig. B2. Mismatch Uncertainty.
Explanation on p. 49.

$\beta = 0.1$

$b = 1$

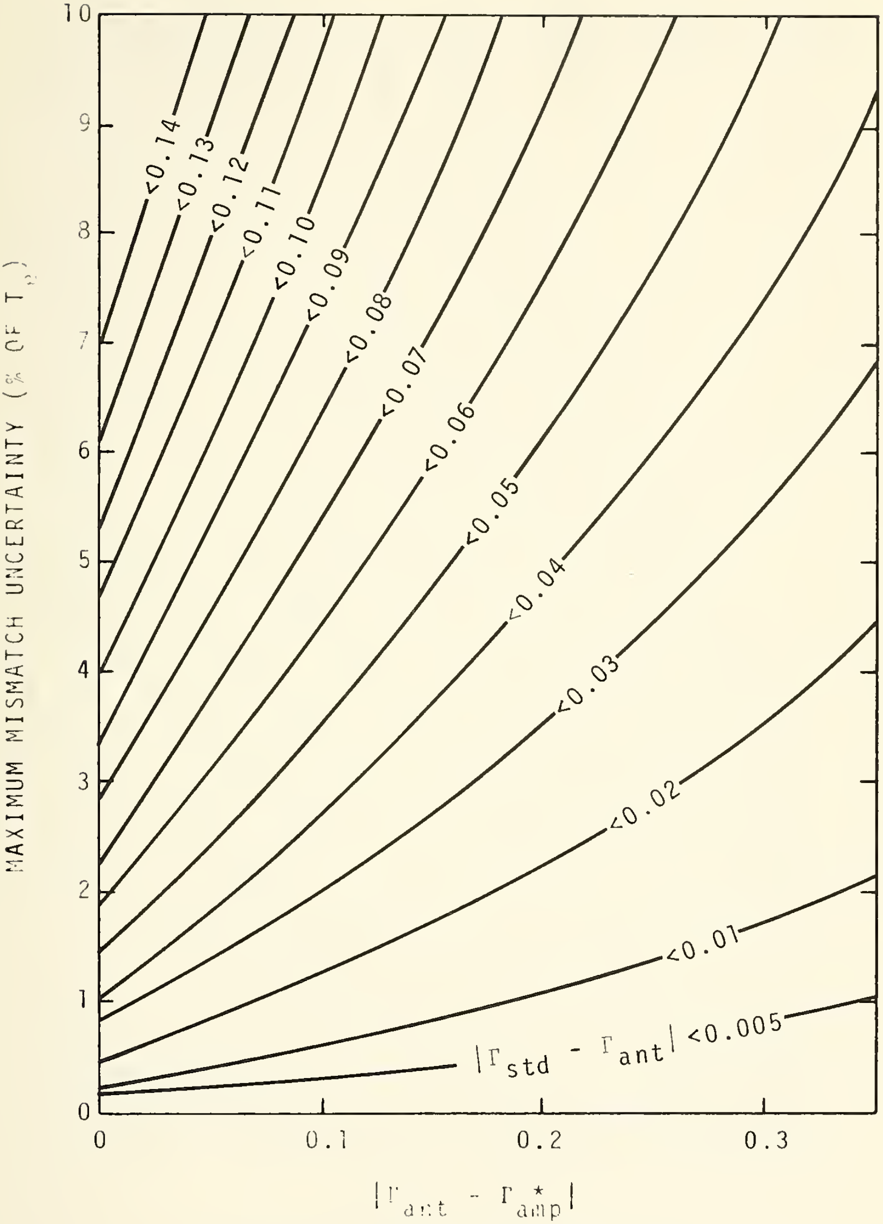

Fig. B3. Mismatch Uncertainty.
Explanation on p. 49.

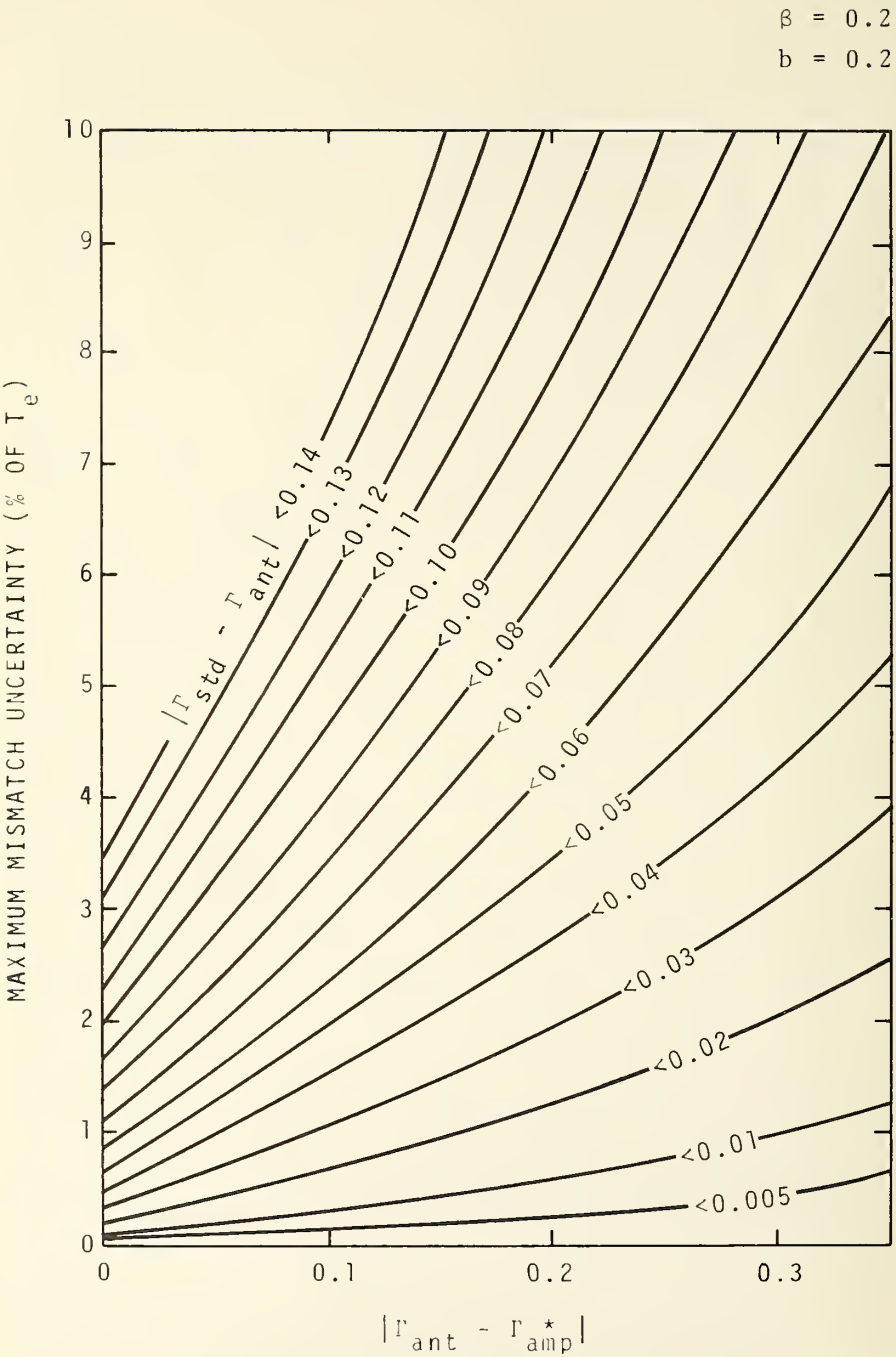

Fig. B4. Mismatch Uncertainty.
Explanation on p. 49.

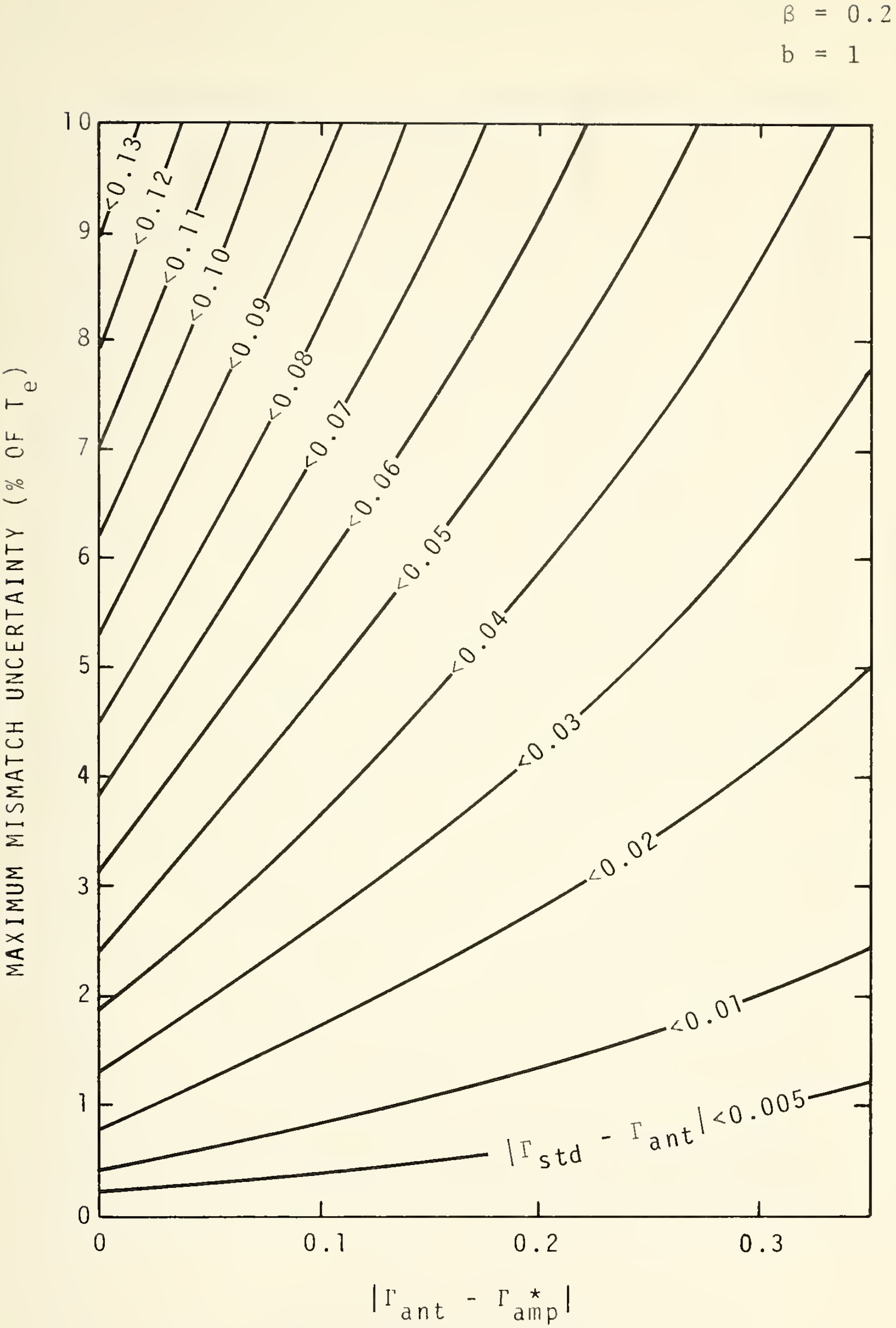

Fig. B5. Mismatch Uncertainty.
Explanation on p. 49.

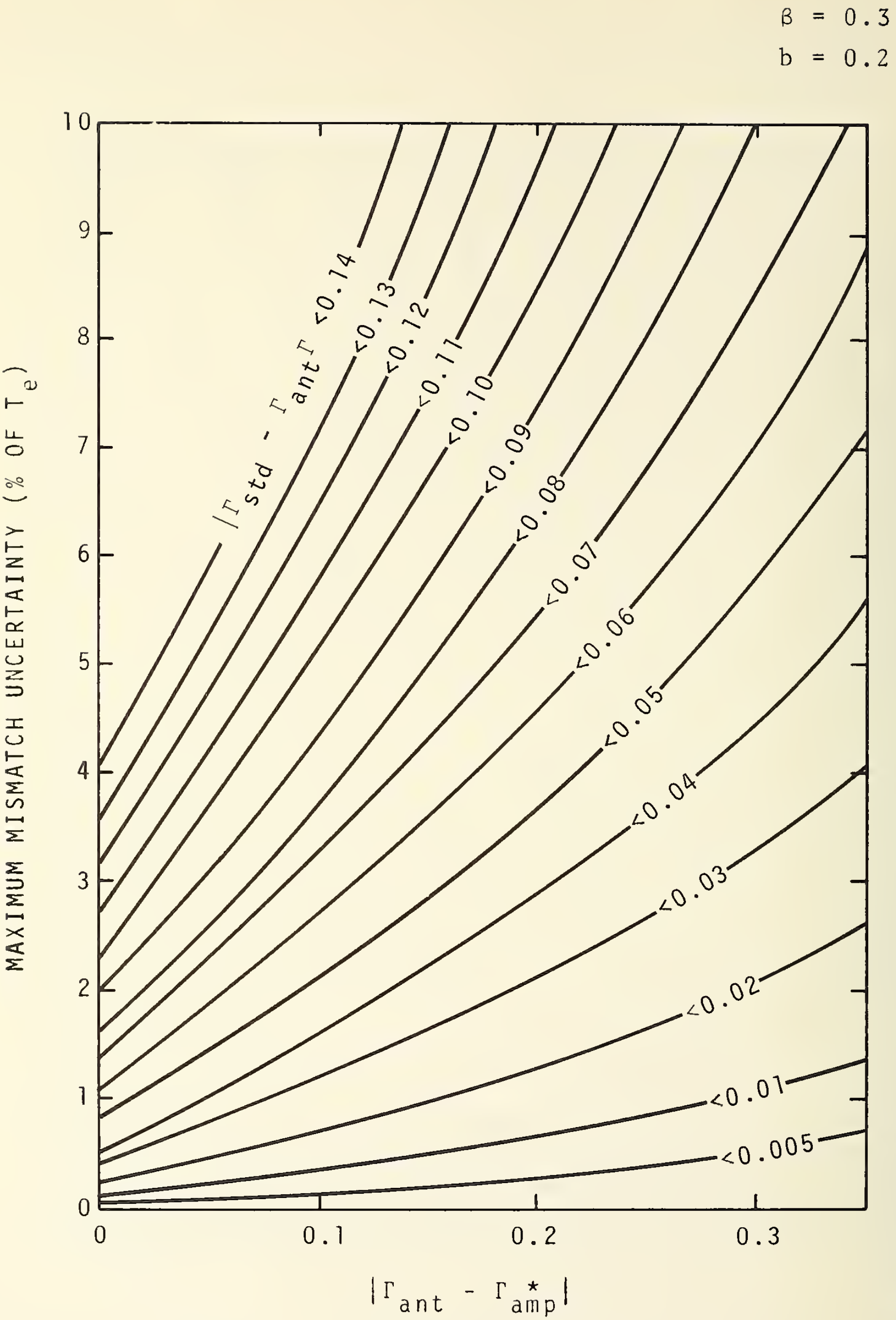

Fig. B6. Mismatch Uncertainty.
Explanation on p. 49.

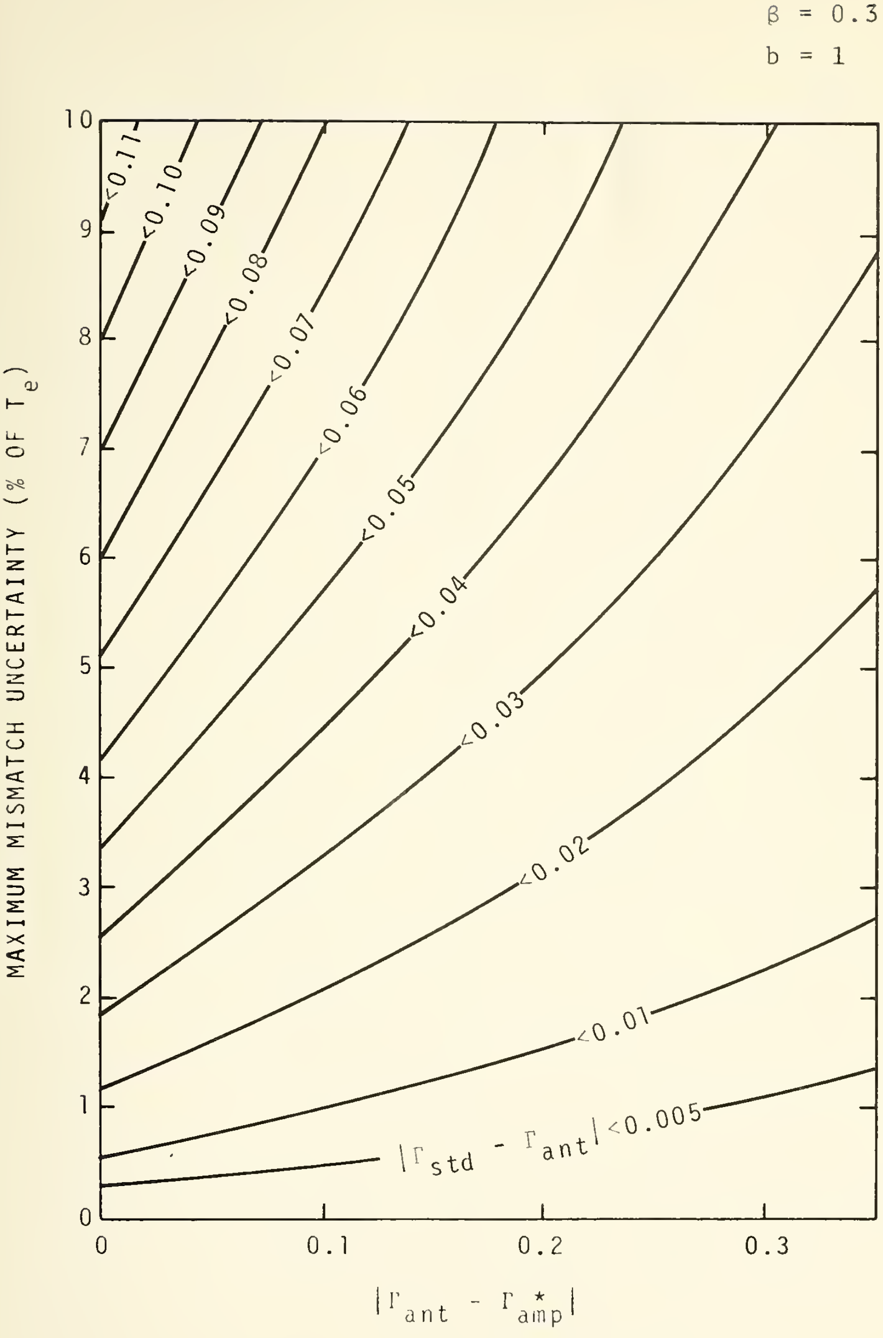

Fig. B7. Mismatch Uncertainty.
Explanation on p. 49.

Table B-II. Key to mismatch error tables

Table	T_{hot}	T_{cold}	β	bT_a/T_e
B-III	10,000	300	0	0
IV	10,000	300	.1	.2
V	10,000	300	.1	1
VI	10,000	300	.2	.2
VII	10,000	300	.2	1
VIII	10,000	300	.3	.2
IX	10,000	300	.3	1
X	373	80	0	0
XI	373	80	.1	.2
XII	373	80	.1	1
XIII	373	80	.2	.2
XIV	373	80	.2	1
XV	373	80	.3	.2
XVI	373	80	.3	1
XVII	1,270	300	0	0
XVIII	1,270	300	.2	1
XIX	1,270	80	0	0
XX	1,270	80	.2	1
XXI	300	4	0	0
XXII	300	4	.2	1

Table B-III

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 10000 K
T(COLD)= 300 K

BETA= 0
BTA/TE= 0

--GAMMA'--		--F(DB)--					
ERR	ANT	1	2	4	6	8	10
.005	0	0	0	0	0	0	0
	.02	.002	.002	.001	.001	.001	.001
	.05	.004	.004	.003	.003	.003	.003
	.1	.008	.008	.007	.006	.005	.005
	.2	.017	.016	.014	.012	.011	.011
	.35	.033	.03	.026	.023	.021	.02
.01	0	0	0	0	0	0	0
	.02	.003	.003	.003	.003	.003	.002
	.05	.008	.008	.007	.006	.006	.005
	.1	.017	.015	.013	.012	.011	.011
	.2	.035	.032	.027	.024	.023	.022
	.35	.066	.06	.052	.046	.043	.041
.02	0	0	.001	.001	.001	.001	.002
	.02	.007	.007	.006	.006	.006	.006
	.05	.017	.016	.014	.013	.012	.012
	.1	.034	.031	.027	.025	.023	.022
	.2	.07	.064	.055	.05	.046	.044
	.35	.133	.121	.104	.094	.087	.083
.05	0	.002	.004	.007	.008	.009	.01
	.02	.019	.019	.019	.02	.02	.02
	.05	.044	.042	.039	.037	.036	.035
	.1	.086	.08	.072	.067	.063	.061
	.2	.175	.161	.141	.129	.121	.116
	.35	.334	.306	.265	.24	.224	.214
.1	0	.009	.016	.026	.033	.037	.039
	.02	.042	.046	.052	.056	.058	.059
	.05	.092	.092	.091	.091	.09	.09
	.1	.177	.169	.157	.15	.145	.142
	.2	.355	.331	.297	.275	.261	.252
	.35	.673	.62	.545	.498	.468	.45

Table B-IV

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 10000 K
T(COLD)= 300 K

BETA= .1
BTA/TE= .2

--GAMMA'--		F(DB)					
ERR	ANT	1	2	4	6	8	10
.005	0	0	0	.001	.001	.001	.001
	.02	.002	.002	.002	.002	.002	.002
	.05	.004	.004	.004	.003	.003	.003
	.1	.008	.008	.007	.006	.006	.006
	.2	.017	.015	.013	.011	.01	.011
	.35	.033	.029	.024	.021	.019	.02
.01	0	0	.001	.002	.002	.002	.003
	.02	.004	.004	.004	.004	.004	.005
	.05	.009	.008	.007	.007	.007	.007
	.1	.017	.015	.013	.012	.011	.012
	.2	.034	.031	.026	.023	.021	.022
	.35	.065	.058	.048	.042	.037	.04
.02	0	.001	.002	.004	005	.006	.007
	.02	.008	.008	.008	009	.009	.01
	.05	.017	.017	.015	.015	.015	.015
	.1	.034	.031	.027	.025	.023	.024
	.2	.069	.062	.053	.046	.042	.044
	.35	.131	.117	.098	.085	.076	.081
.05	0	.005	.008	.014	.018	.021	.024
	.02	.021	.023	.026	.028	.03	.032
	.05	.045	.044	.043	.043	.043	.043
	.1	.087	.081	.073	.068	.065	.067
	.2	.174	.159	.137	.122	.112	.118
	.35	.329	.296	.25	.219	.197	.209
.1	0	.015	.026	.043	.055	.064	.072
	.02	.047	.055	.067	.075	.081	.087
	.05	.096	.098	.102	.105	.108	.11
	.1	.179	.172	.162	.156	.152	.158
	.2	.353	.327	.289	.265	.248	.261
	.35	.664	.604	.517	.46	.419	.445

Table B-V

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 10000 K
T(COLD)= 300 K

BETA= .1
BTA/TE= 1

--GAMMA'--		F(DB)					
ERR	ANT	1	2	4	6	8	10
.005	0	.001	.002	.003	.004	.005	.006
	.02	.002	.003	.004	.005	.005	.007
	.05	.005	.005	.005	.007	.008	.008
	.1	.008	.008	.008	.01	.011	.012
	.2	.016	.014	.014	.017	.019	.021
	.35	.03	.025	.024	.03	.034	.036
.01	0	.002	.004	.007	.009	.011	.013
	.02	.005	.006	.008	.01	.011	.014
	.05	.009	.01	.011	.014	.015	.016
	.1	.017	.016	.016	.02	.023	.024
	.2	.033	.028	.028	.035	.039	.042
	.35	.061	.051	.048	.06	.067	.072
.02	0	.005	.008	.014	.019	.023	.028
	.02	.01	.013	.017	.021	.023	.029
	.05	.019	.02	.023	.029	.032	.034
	.1	.034	.032	.034	.042	.047	.051
	.2	.066	.057	.057	.07	.079	.085
	.35	.122	.102	.098	.122	.137	.146
.05	0	.014	.026	.043	.057	.069	.082
	.02	.029	.037	.05	.062	.069	.084
	.05	.051	.055	.066	.082	.092	.098
	.1	.088	.084	.092	.115	.129	.138
	.2	.168	.147	.15	.186	.209	.224
	.35	.309	.26	.253	.315	.354	.379
.1	0	.037	.067	.113	.146	.174	.203
	.02	.067	.09	.126	.156	.175	.208
	.05	.111	.125	.157	.196	.22	.235
	.1	.186	.185	.211	.263	.295	.316
	.2	.345	.311	.327	.407	.457	.489
	.35	.628	.538	.536	.667	.75	.802

Table B-VI

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 10000 K
T(COLD)= 300 K

BETA= .2
BTA/TE= .2

--GAMMA'--		----------------------F(DB)----------------------					
ERR	ANT	1	2	4	6	8	10
.005	0	0	.001	.001	.002	.002	.003
	.02	.002	.002	.002	.003	.003	.003
	.05	.004	.004	.004	.004	.004	.005
	.1	.009	.008	.007	.007	.007	.007
	.2	.017	.016	.014	.012	.011	.012
	.35	.033	.03	.025	.022	.02	.021
.01	0	.001	.002	.003	.004	.004	.005
	.02	.004	.004	.005	.006	.006	.007
	.05	.009	.009	.009	.009	.009	.009
	.1	.017	.016	.015	.014	.013	.013
	.2	.035	.032	.027	.024	.023	.023
	.35	.066	.059	.05	.044	.04	.042
.02	0	.002	.004	.006	.008	.01	.012
	.02	.008	.009	.011	.012	.013	.015
	.05	.018	.018	.018	.018	.019	.02
	.1	.035	.033	.03	.028	.027	.027
	.2	.07	.064	.055	.05	.046	.043
	.35	.132	.119	.1	.088	.08	.084
.05	0	.007	.012	.02	.026	.031	.037
	.02	.023	.026	.032	.036	.04	.044
	.05	.047	.048	.049	.051	.053	.056
	.1	.089	.085	.079	.076	.075	.076
	.2	.176	.162	.143	.131	.123	.126
	.35	.331	.3	.256	.228	.209	.213
.1	0	.018	.033	.055	.071	.084	.097
	.02	.051	.062	.079	.091	.102	.112
	.05	.1	.106	.114	.121	.128	.136
	.1	.183	.179	.174	.172	.173	.175
	.2	.357	.334	.302	.281	.269	.277
	.35	.668	.612	.531	.478	.442	.463

Table B-VII

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 10000 K
T(COLD)= 300 K

BETA= .2
BTA/TE= 1

--GAMMA'--		F(DB)					
ERR	ANT	1	2	4	6	8	10
.005	0	.002	.004	.006	.008	.01	.013
	.02	.003	.005	.007	.009	.01	.013
	.05	.006	.007	.008	.01	.011	.013
	.1	.009	.009	.011	.013	.015	.016
	.2	.017	.016	.016	.02	.023	.025
	.35	.032	.027	.027	.034	.038	.04
.01	0	.004	.007	.013	.017	.021	.026
	.02	.007	.01	.014	.017	.021	.026
	.05	.011	.013	.016	.02	.023	.027
	.1	.019	.019	.022	.027	.03	.032
	.2	.035	.032	.033	.041	.046	.05
	.35	.063	.055	.054	.067	.076	.081
.02	0	.008	.015	.026	.035	.043	.053
	.02	.014	.02	.029	.036	.044	.054
	.05	.023	.027	.034	.042	.047	.056
	.1	.038	.039	.044	.055	.062	.066
	.2	.07	.064	.068	.084	.094	.101
	.35	.127	.11	.11	.136	.153	.164
.05	0	.024	.043	.073	.097	.119	.145
	.02	.038	.055	.08	.1	.12	.147
	.05	.061	.072	.092	.114	.128	.151
	.1	.098	.102	.119	.148	.165	.178
	.2	.178	.166	.177	.22	.247	.265
	.35	.32	.28	.283	.352	.396	.423
.1	0	.056	.102	.173	.226	.275	.328
	.02	.086	.126	.187	.233	.276	.333
	.05	.13	.161	.21	.261	.293	.341
	.1	.205	.22	.264	.328	.369	.395
	.2	.365	.348	.331	.474	.533	.57
	.35	.65	.578	.596	.741	.833	.891

Table B-VIII

```
                              MAXIMUM MISMATCH ERROR (DB)

                                   T(HOT)= 10000 K
                                   T(COLD)= 300 K

                                   BETA= .3
                                   BTA/TE= .2


--GAMMA'--     -----------------------F(DB)------------------------
ERR    ANT       1        2        4        6        8        10


.005   0       .001     .001     .002     .003     .003     .004
       .02     .002     .003     .003     .004     .004     .005
       .05     .005     .005     .005     .005     .005     .006

       .1      .009     .008     .008     .008     .008     .008
       .2      .018     .016     .014     .013     .012     .012
       .35     .033     .03      .026     .023     .021     .022


.01    0       .001     .002     .004     .005     .006     .008
       .02     .005     .005     .006     .007     .008     .01
       .05     .009     .01      .01      .01      .011     .012

       .1      .018     .017     .016     .015     .015     .016
       .2      .035     .032     .029     .026     .025     .025
       .35     .066     .06      .051     .046     .042     .043


.02    0       .003     .005     .008     .011     .014     .017
       .02     .009     .011     .013     .015     .017     .02
       .05     .019     .019     .02      .021     .023     .025

       .1      .036     .034     .032     .031     .032     .033
       .2      .07      .065     .058     .053     .051     .051
       .35     .133     .12      .103     .092     .085     .088


.05    0       .008     .015     .026     .034     .041     .049
       .02     .025     .03      .038     .044     .05      .057
       .05     .049     .051     .055     .059     .063     .069

       .1      .09      .088     .085     .085     .085     .088
       .2      .178     .166     .149     .139     .133     .134
       .35     .333     .304     .263     .237     .22      .227


.1     0       .022     .04      .066     .087     .104     .122
       .02     .055     .069     .091     .107     .122     .137
       .05     .104     .113     .126     .137     .148     .161

       .1      .186     .186     .187     .189     .193     .2
       .2      .361     .342     .315     .298     .29      .293
       .35     .673     .62      .544     .496     .465     .481
```

Table B-IX

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 10000 K
T(COLD)= 300 K

BETA= .3
BTA/TE= 1

--GAMMA'--		--------F(DB)--------					
ERR	ANT	1	2	4	6	8	10
.005	0	.003	.005	.009	.012	.015	.019
	.02	.004	.007	.01	.013	.015	.019
	.05	.007	.008	.011	.013	.015	.02
	.1	.01	.011	.013	.017	.019	.02
	.2	.018	.018	.019	.024	.027	.029
	.35	.033	.029	.03	.037	.042	.045
.01	0	.006	.011	.019	.025	.031	.038
	.02	.009	.013	.02	.026	.031	.039
	.05	.013	.017	.022	.027	.031	.04
	.1	.021	.023	.027	.034	.038	.041
	.2	.037	.035	.039	.048	.054	.058
	.35	.065	.059	.06	.075	.084	.09
.02	0	.012	.022	.038	.051	.063	.078
	.02	.018	.027	.041	.052	.064	.079
	.05	.027	.034	.045	.055	064	.081
	.1	.042	.046	.055	.068	.077	.084
	.2	.074	.072	.078	.098	.11	.117
	.35	.131	.118	.122	.151	.17	.182
.05	0	.033	.061	.103	.137	.17	.207
	.02	.043	.072	.11	.141	.17	.21
	.05	.07	.09	.121	.147	.171	.214
	.1	.108	.12	.145	.181	.203	.221
	.2	.188	.134	.204	.254	.256	.335
	.35	.331	.3	.313	.389	.437	.468
.1	0	.076	.138	.233	.307	.376	.453
	.02	.105	.161	.247	.314	.377	.458
	.05	.149	.196	.268	.326	.39	.466
	.1	.225	.256	.317	.394	.443	.482
	.2	.385	.334	.436	.542	.639	.651
	.35	.672	.619	.655	.815	.916	.93

Table B-X

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 373 K
T(COLD)= 80 K

BETA= .2
BTA/TE= .2

--GAMMA'-- ERR	ANT	F(DB) 1	2	4	6	8	10
.005	0	.001	.002	.005	.01	.018	.03
	.02	.001	.002	.005	.01	.018	.032
	.05	.002	.004	.006	.01	.019	.035
	.1	.004	.005	.008	.011	.021	.04
	.2	.008	.009	.011	.012	.024	.05
	.35	.015	.016	.017	.017	.031	.07
.01	0	.002	.004	.01	.02	.036	.061
	.02	.003	.005	.011	.02	.037	.065
	.05	.005	.007	.013	.021	.038	.07
	.1	.009	.011	.016	.022	.041	.079
	.2	.016	.019	.022	.025	.048	.1
	.35	.029	.032	.034	.035	.062	.14
.02	0	.003	.008	.02	.04	.072	.123
	.02	.006	.011	.023	.041	.074	.13
	.05	.01	.015	.026	.042	.078	.141
	.1	.017	.022	.032	.045	.084	.16
	.2	.032	.037	.045	.05	.098	.201
	.35	.059	.065	.069	.07	.125	.28
.05	0	.01	.022	.055	.107	.187	.314
	.02	.017	.029	.061	.109	.193	.332
	.05	.027	.04	.07	.112	.201	.359
	.1	.045	.058	.085	.117	.216	.436
	.2	.082	.097	.113	.131	.251	.511
	.35	.148	.166	.178	.132	.32	.739
.1	0	.025	.054	.126	.233	.396	.652
	.02	.039	.068	.133	.237	.407	.633
	.05	.06	.089	.156	.243	.424	.742
	.1	.097	.126	.136	.254	.454	.836
	.2	.17	.203	.252	.232	.524	1.046
	.35	.303	.343	.374	.385	.665	1.444

Table B-XI

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 373 K
T(COLD)= 80 K

BETA= .1
BTA/TE= .2

--GAMMA'--		--------F(DB)--------					
ERR	ANT	1	2	4	6	8	10
.005	0	0	.001	.002	.005	.009	.015
	.02	.001	.002	.003	.005	.009	.017
	.05	.002	.003	.004	.005	.01	.02
	.1	.004	.004	.005	.006	.012	.024
	.2	.008	.008	.009	.009	.015	.034
	.35	.014	.015	.014	.016	.021	.052
.01	0	.001	.002	.005	.01	.018	.031
	.02	.002	.003	.006	.01	.019	.034
	.05	.004	.005	.008	.011	.021	.04
	.1	.008	.009	.011	.012	.024	.049
	.2	.015	.017	.017	.018	.03	.069
	.35	.028	.03	.029	.033	.042	.105
.02	0	.002	.004	.011	.021	.037	.062
	.02	.005	.007	.013	.022	.039	.069
	.05	.009	.011	.017	.023	.042	.08
	.1	.016	.019	.023	.025	.048	.099
	.2	.031	.034	.035	.037	.061	.138
	.35	.057	.061	.058	.067	.085	.211
.05	0	.006	.013	.032	.058	.099	.163
	.02	.013	.021	.037	.06	.104	.181
	.05	.024	.031	.046	.063	.113	.208
	.1	.041	.049	.061	.068	.127	.253
	.2	.078	.088	.093	.098	.159	.353
	.35	.144	.156	.151	.174	.22	.536
.1	0	.018	.037	.079	.136	.22	.35
	.02	.032	.051	.09	.14	.231	.385
	.05	.053	.072	.108	.146	.248	.439
	.1	.088	.108	.138	.156	.276	.531
	.2	.162	.185	.202	.217	.341	.73
	.35	.295	.324	.32	.371	.465	1.099

Table B-XII

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 373 K
T(COLD)= 80 K

BETA= .1
BTA/TE= 1

--GAMMA'--		--F(DB)--					
ERR	ANT	1	2	4	6	8	10
.005	0	.002	.004	.012	.024	.044	.076
	.02	.002	.004	.013	.028	.052	.09
	.05	.003	.004	.015	.034	.063	.111
	.1	.004	.005	.019	.043	.083	.147
	.2	.006	.008	.026	.064	.125	.223
	.35	.01	.015	.04	.101	.202	.363
.01	0	.004	.009	.024	.049	.089	.152
	.02	.005	.009	.027	.056	.104	.18
	.05	.006	.009	.031	.068	.127	.222
	.1	.008	.01	.038	.087	.166	.294
	.2	.012	.017	.053	.123	.251	.447
	.35	.02	.03	.08	.203	.404	.726
.02	0	.003	.019	.05	.099	.179	.306
	.02	.01	.019	.055	.114	.21	.361
	.05	.012	.019	.063	.137	.256	.446
	.1	.016	.021	.077	.175	.334	.589
	.2	.025	.035	.107	.257	.503	.896
	.35	.041	.06	.162	.407	.809	1.455
.05	0	.023	.051	.132	.258	.459	.776
	.02	.027	.052	.145	.295	.535	.915
	.05	.033	.052	.165	.351	.65	1.126
	.1	.043	.057	.2	.447	.847	1.484
	.2	.065	.092	.275	.653	1.268	2.253
	.35	.104	.155	.414	1.029	2.026	3.65
.1	0	.055	.119	.29	.549	.954	1.592
	.02	.063	.119	.316	.623	1.106	1.87
	.05	.074	.12	.357	.735	1.337	2.291
	.1	.095	.13	.426	.927	1.73	3.008
	.2	.139	.2	.577	1.34	2.574	4.547
	.35	.219	.329	.857	2.096	4.113	7.345

Table B-XIII

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 373 K
T(COLD)= 80 K

BETA= .2
BTA/TE= .2

--GAMMA'--		F(DB)					
ERR	ANT	1	2	4	6	8	10
.005	0	.001	.002	.005	.01	.018	.03
	.02	.001	.002	.005	.01	.018	.032
	.05	.002	.004	.006	.01	.019	.035
	.1	.004	.005	.008	.011	.021	.04
	.2	.008	.009	.011	.012	.024	.05
	.35	.015	.016	.017	.017	.031	.07
.01	0	.002	.004	.01	.02	.036	.061
	.02	.003	.005	.011	.02	.037	.065
	.05	.005	.007	.013	.021	.038	.07
	.1	.009	.011	.016	.022	.041	.079
	.2	.016	.019	.022	.025	.048	.1
	.35	.029	.032	.034	.035	.062	.14
.02	0	.003	.008	.02	.04	.072	.123
	.02	.006	.011	.023	.041	.074	.13
	.05	.01	.015	.026	.042	.078	.141
	.1	.017	.022	.032	.045	.084	.16
	.2	.032	.037	.045	.05	.098	.201
	.35	.059	.065	.059	.07	.125	.28
.05	0	.01	.022	.055	.107	.187	.314
	.02	.017	.029	.061	.109	.193	.332
	.05	.027	.04	.07	.112	.201	.359
	.1	.045	.058	.085	.117	.216	.406
	.2	.082	.097	.118	.131	.251	.511
	.35	.148	.166	.178	.132	.32	.709
.1	0	.025	.054	.126	.233	.396	.652
	.02	.039	.068	.133	.237	.407	.688
	.05	.06	.089	.156	.243	.424	.742
	.1	.095	.126	.136	.254	.454	.836
	.2	.17	.203	.252	.232	.524	1.046
	.35	.303	.343	.374	.385	.665	1.444

Table B-XIV

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 373 K
T(COLD)= 80 K

BETA= .2
BTA/TE= 1

--GAMMA'--		--------F(DB)--------					
ERR	ANT	1	2	4	6	8	10
.005	0	.004	.009	.024	.049	.088	.152
	.02	.004	.009	.025	.052	.096	.165
	.05	.005	.009	.027	.058	.108	.187
	.1	.006	.009	.031	.068	.128	.223
	.2	.008	.01	.039	.089	.171	.302
	.35	.012	.017	.054	.129	.252	.449
.01	0	.008	.018	.048	.098	.177	.303
	.02	.008	.018	.051	.105	.192	.331
	.05	.01	.018	.055	.116	.215	.374
	.1	.012	.018	.062	.136	.255	.447
	.2	.016	.02	.078	.178	.342	.605
	.35	.024	.033	.107	.258	.504	.899
.02	0	.015	.036	.097	.196	.355	.609
	.02	.017	.036	.102	.211	.386	.664
	.05	.019	.036	.111	.234	.432	.749
	.1	.024	.037	.125	.273	.512	.895
	.2	.032	.041	.156	.358	.686	1.212
	.35	.049	.067	.216	.518	1.01	1.8
.05	0	.041	.095	.25	.501	.9	1.533
	.02	.045	.095	.264	.538	.976	1.672
	.05	.051	.095	.284	.594	1.092	1.884
	.1	.062	.096	.32	.692	1.292	2.249
	.2	.084	.109	.398	.905	1.727	3.341
	.35	.125	.174	.549	1.305	2.538	4.512
.1	0	.091	.205	.527	1.034	1.836	3.135
	.02	.099	.236	.554	1.108	1.988	3.334
	.05	.111	.287	.594	1.221	2.22	3.508
	.1	.132	.209	.666	1.416	2.62	4.537
	.2	.177	.234	.824	1.845	3.492	6.123
	.35	.261	.365	1.128	2.646	5.117	9.069

Table B-XV

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 373 K
T(COLD)= 80 K

BETA= .3
BTA/TE= .2

--GAMMA'--		--------F(DB)--------					
ERR	ANT	1	2	4	6	8	10
.005	0	.001	.003	.007	.015	.027	.046
	.02	.002	.003	.008	.015	.027	.047
	.05	.003	.004	.009	.015	.028	.05
	.1	.005	.006	.01	.016	.03	.055
	.2	.008	.01	.014	.017	.033	.066
	.35	.015	.017	.02	.02	.041	.087
.01	0	.002	.005	.015	.029	.053	.091
	.02	.004	.007	.016	.03	.054	.095
	.05	.006	.009	.017	.03	.056	.1
	.1	.009	.013	.021	.032	.059	.11
	.2	.017	.02	.027	.035	.067	.132
	.35	.03	.034	.04	.041	.082	.174
.02	0	.005	.011	.03	.06	.108	.184
	.02	.008	.014	.032	.06	.11	.191
	.05	.012	.018	.036	.062	.113	.202
	.1	.019	.026	.042	.064	.119	.221
	.2	.034	.041	.055	.07	.134	.264
	.35	.06	.069	.08	.083	.165	.349
.05	0	.014	.031	.079	.155	.275	.466
	.02	.021	.038	.085	.157	.281	.484
	.05	.031	.049	.094	.16	.289	.511
	.1	.049	.067	.109	.166	.305	.559
	.2	.086	.106	.142	.181	.343	.668
	.35	.153	.176	.205	.215	.421	.831
.1	0	.033	.071	.174	.33	.573	.955
	.02	.047	.085	.185	.334	.584	.991
	.05	.067	.107	.203	.34	.601	1.046
	.1	.103	.143	.234	.352	.632	1.142
	.2	.177	.222	.301	.383	.703	1.361
	.35	.311	.363	.425	.451	.866	1.769

Table B-XVI

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 373 K
T(COLD)= 80 K

BETA= .3
BTA/TE= 1

--GAMMA'--		---F(DB)---					
ERR	ANT	1	2	4	6	8	10
.005	0	.006	.013	.036	.073	.132	.227
	.02	.006	.013	.037	.077	.14	.241
	.05	.007	.013	.039	.082	.152	.263
	.1	.008	.013	.043	.092	.172	.3
	.2	.01	.014	.051	.114	.217	.331
	.35	.014	.018	.067	.156	.302	.535
.01	0	.011	.026	.072	.146	.265	.455
	.02	.012	.026	.074	.153	.28	.483
	.05	.013	.026	.079	.165	.304	.525
	.1	.015	.027	.086	.185	.344	.6
	.2	.02	.028	.102	.229	.434	.763
	.35	.023	.037	.134	.313	.605	1.071
.02	0	.023	.053	.144	.293	.532	.911
	.02	.024	.053	.15	.308	.562	.967
	.05	.027	.054	.158	.331	.609	1.053
	.1	.031	.054	.173	.371	.69	1.201
	.2	.04	.056	.206	.459	.87	1.527
	.35	.057	.075	.27	.628	1.211	2.145
.05	0	.06	.138	.369	.743	1.34	2.29
	.02	.064	.138	.382	.78	1.417	2.43
	.05	.07	.139	.403	.837	1.534	2.643
	.1	.08	.14	.439	.936	1.737	3.013
	.2	.103	.145	.522	1.158	2.186	3.83
	.35	.146	.192	.694	1.531	3.04	5.375
.1	0	.123	.292	.764	1.518	2.717	4.619
	.02	.136	.292	.791	1.592	2.87	4.893
	.05	.148	.293	.832	1.707	3.104	5.325
	.1	.169	.296	.905	1.906	3.51	6.066
	.2	.215	.307	1.071	2.349	4.41	7.7
	.35	.303	.402	1.398	3.2	6.122	10.794

Table B-XVII

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 1273 K
T(COLD)= 300 K

BETA= 0
BTA/TE= 0

--GAMMA'--		-----------	-----------	F(DB)	-----------	-----------	-----------
ERR	ANT	1	2	4	6	8	10
.005	0	0	0	0	0	0	0
	.02	.002	.002	.002	.002	.002	.002
	.05	.005	.005	.005	.004	.004	.004
	.1	.011	.01	.009	.008	.008	.008
	.2	.023	.021	.019	.017	.016	.016
	.35	.043	.04	.036	.033	.031	.03
.01	0	0	0	0	0	0	0
	.02	.004	.004	.004	.004	.003	.003
	.05	.011	.01	.009	.009	.008	.008
	.1	.022	.02	.018	.017	.016	.016
	.2	.045	.042	.037	.035	.033	.032
	.35	.086	.08	.071	.066	.062	.06
.02	0	0	.001	.001	.001	.001	.002
	.02	.009	.009	.008	.008	.008	.008
	.05	.022	.021	.019	.018	.017	.017
	.1	.044	.041	.037	.035	.033	.032
	.2	.09	.084	.075	.07	.066	.064
	.35	.173	.161	.143	.133	.126	.121
.05	0	.002	.004	.007	.008	.009	.01
	.02	.024	.024	.024	.025	.025	.025
	.05	.056	.054	.051	.049	.048	.047
	.1	.111	.105	.097	.091	.088	.086
	.2	.227	.213	.193	.18	.172	.166
	.35	.433	.404	.363	.337	.321	.31
.1	0	.009	.016	.026	.033	.037	.039
	.02	.052	.056	.062	.065	.068	.069
	.05	.117	.116	.116	.115	.115	.114
	.1	.227	.219	.206	.199	.194	.191
	.2	.459	.434	.399	.376	.362	.353
	.35	.871	.817	.741	.693	.662	.643

Table B-XVIII

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 1270 K
T(COLD)= 300 K

BETA= .2
BTA/TE= 1

--GAMMA'-- ERR	ANT	F(DB) 1	2	4	6	8	10
.005	0	.003	.006	.013	.022	.035	.055
	.02	.005	.008	.014	.023	.037	.059
	.05	.003	.01	.015	.024	.04	.065
	.1	.013	.013	.016	.025	.045	.076
	.2	.023	.021	.019	.029	.057	.099
	.35	.04	.035	.027	.037	.079	.142
.01	0	.007	.013	.027	.045	.071	.11
	.02	.01	.016	.028	.046	.075	.118
	.05	.016	.02	.029	.048	.081	.13
	.1	.025	.027	.032	.051	.091	.151
	.2	.045	.042	.038	.059	.114	.198
	.35	.081	.069	.054	.074	.158	.234
.02	0	.013	.027	.055	.091	.143	.222
	.02	.021	.032	.057	.093	.151	.238
	.05	.032	.04	.06	.097	.163	.262
	.1	.051	.055	.065	.103	.184	.304
	.2	.091	.085	.076	.119	.23	.397
	.35	.162	.14	.11	.149	.318	.57
.05	0	.036	.071	.145	.237	.369	.567
	.02	.055	.085	.15	.243	.388	.607
	.05	.082	.106	.157	.252	.418	.668
	.1	.13	.141	.17	.269	.471	.773
	.2	.229	.217	.199	.307	.587	1.034
	.35	.403	.355	.253	.384	.809	1.433
.1	0	.081	.153	.316	.507	.774	1.173
	.02	.118	.186	.325	.519	.813	1.253
	.05	.174	.227	.34	.537	.873	1.375
	.1	.268	.298	.365	.57	.973	1.586
	.2	.468	.451	.426	.647	1.212	2.049
	.35	.927	.723	.596	.805	1.659	2.92

Table B-XIX

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 1270 K
T(COLD)= 80 K

BETA= 0
BTA/TE= 0

--GAMMA'--		F(DB)					
ERR	ANT	1	2	4	6	8	10
.005	0	0	0	0	0	0	0
	.02	.001	.001	.001	.001	.001	.001
	.05	.002	.002	.002	.002	.002	.002
	.1	.003	.004	.004	.004	.005	.005
	.2	.006	.007	.008	.009	.01	.01
	.35	.012	.014	.016	.017	.018	.019
.01	0	0	0	0	0	0	0
	.02	.001	.002	.002	.002	.002	.002
	.05	.003	.004	.004	.005	.005	.005
	.1	.006	.007	.008	.009	.01	.01
	.2	.013	.014	.017	.018	.019	.02
	.35	.024	.028	.032	.035	.037	.038
.02	0	0	.001	.001	.001	.001	.002
	.02	.003	.003	.004	.005	.005	.005
	.05	.006	.008	.009	.01	.011	.011
	.1	.013	.015	.017	.019	.02	.021
	.2	.026	.029	.034	.037	.039	.041
	.35	.049	.055	.065	.071	.074	.077
.05	0	.002	.004	.007	.008	.009	.01
	.02	.008	.011	.015	.017	.018	.019
	.05	.017	.021	.027	.03	.032	.033
	.1	.033	.039	.047	.052	.055	.057
	.2	.066	.076	.09	.099	.104	.108
	.35	.124	.141	.166	.182	.192	.198
.1	0	.009	.016	.026	.033	.037	.039
	.02	.021	.03	.042	.05	.055	.058
	.05	.039	.05	.066	.076	.082	.086
	.1	.071	.086	.107	.12	.129	.134
	.2	.136	.15	.193	.214	.228	.236
	.35	.253	.292	.348	.383	.405	.419

Table B-XX

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 1270 K
T(COLD)= 80 K

BETA= .2
BTA/TE= 1

--GAMMA'-- ERR	ANT	F(DB) 1	2	4	6	8	10
.005	0	.002	.005	.01	.017	.027	.043
	.02	.003	.005	.01	.018	.029	.047
	.05	.003	.005	.01	.019	.032	.052
	.1	.004	.007	.011	.021	.037	.061
	.2	.006	.01	.016	.026	.047	.08
	.35	.01	.017	.027	.036	.067	.117
.01	0	.005	.009	.02	.034	.055	.087
	.02	.005	.01	.02	.036	.059	.094
	.05	.007	.011	.021	.038	.064	.104
	.1	.009	.013	.022	.043	.074	.122
	.2	.013	.02	.033	.053	.095	.161
	.35	.021	.033	.054	.072	.135	.234
.02	0	.01	.019	.041	.07	.112	.176
	.02	.011	.02	.042	.073	.119	.189
	.05	.014	.022	.043	.078	.13	.21
	.1	.018	.027	.045	.087	.149	.246
	.2	.026	.041	.063	.107	.192	.324
	.35	.042	.067	.11	.145	.272	.47
.05	0	.027	.054	.111	.185	.29	.451
	.02	.031	.056	.113	.193	.309	.485
	.05	.037	.059	.116	.205	.336	.537
	.1	.047	.073	.121	.228	.385	.626
	.2	.063	.109	.177	.277	.491	.822
	.35	.107	.174	.283	.373	.692	1.108
.1	0	.063	.123	.243	.402	.617	.941
	.02	.071	.128	.252	.413	.654	1.009
	.05	.082	.135	.258	.444	.71	1.113
	.1	.102	.162	.269	.488	.806	1.292
	.2	.145	.234	.381	.583	1.02	1.684
	.35	.225	.365	.596	.783	1.426	2.42

Table B-XXI

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 300 K
T(COLD)= 4 K

BETA= 0
BTA/TE= 0

--GAMMA'--		---F(DB)---					
ERR	ANT	1	2	4	6	8	10
.005	0	0	0	0	0	0	0
	.02	0	0	.001	.001	.001	.001
	.05	.001	.001	.001	.002	.002	.002
	.1	.001	.002	.003	.003	.004	.004
	.2	.002	.004	.006	.007	.008	.008
	.35	.004	.007	.011	.014	.015	.016
.01	0	0	0	0	0	0	0
	.02	0	.001	.001	.002	.002	.002
	.05	.001	.002	.003	.004	.004	.004
	.1	.002	.004	.006	.007	.008	.009
	.2	.004	.007	.012	.014	.016	.017
	.35	.008	.014	.022	.027	.031	.033
.02	0	0	.001	.001	.001	.001	.002
	.02	.001	.002	.003	.004	.004	.005
	.05	.002	.004	.007	.008	.009	.01
	.1	.004	.008	.012	.015	.017	.018
	.2	.009	.015	.024	.029	.033	.035
	.35	.017	.028	.045	.055	.062	.066
.05	0	.002	.004	.007	.008	.009	.01
	.02	.004	.007	.012	.015	.017	.018
	.05	.007	.013	.02	.025	.028	.03
	.1	.012	.021	.034	.042	.047	.051
	.2	.023	.04	.064	.079	.088	.094
	.35	.043	.073	.116	.144	.161	.172
.1	0	.009	.016	.026	.033	.037	.039
	.02	.013	.023	.037	.046	.052	.055
	.05	.019	.033	.054	.066	.074	.08
	.1	.03	.051	.082	.101	.113	.121
	.2	.052	.088	.141	.174	.195	.208
	.35	.091	.156	.243	.306	.343	.366

Table B-XXII

MAXIMUM MISMATCH ERROR (DB)

T(HOT)= 300 K
T(COLD)= 4 K

BETA= .2
BTA/TE= 1

--GAMMA'--		--------F(DB)--------					
ERR	ANT	1	2	4	6	8	10
.005	0	.003	.007	.021	.045	.084	.146
	.02	.003	.007	.023	.049	.091	.16
	.05	.003	.008	.025	.054	.103	.181
	.1	.004	.009	.029	.064	.123	.217
	.2	.006	.011	.038	.086	.165	.296
	.35	.009	.017	.055	.127	.247	.441
.01	0	.006	.014	.042	.09	.168	.293
	.02	.006	.015	.045	.098	.183	.32
	.05	.006	.016	.05	.109	.206	.362
	.1	.007	.018	.053	.129	.246	.435
	.2	.011	.022	.076	.173	.333	.592
	.35	.019	.033	.11	.254	.494	.883
.02	0	.012	.029	.086	.131	.337	.587
	.02	.012	.031	.092	.197	.368	.642
	.05	.012	.033	.101	.22	.414	.726
	.1	.015	.037	.117	.26	.494	.871
	.2	.023	.045	.153	.347	.668	1.185
	.35	.037	.067	.22	.539	.991	1.768
.05	0	.032	.078	.222	.463	.854	1.478
	.02	.033	.081	.237	.501	.93	1.617
	.05	.034	.087	.261	.559	1.046	1.828
	.1	.041	.096	.332	.659	1.246	2.189
	.2	.06	.118	.391	.877	1.681	2.976
	.35	.097	.174	.56	1.284	2.439	4.434
.1	0	.073	.171	.469	.958	1.744	2.996
	.02	.074	.173	.5	1.035	1.897	3.273
	.05	.076	.189	.548	1.151	2.129	3.694
	.1	.09	.239	.63	1.351	2.529	4.415
	.2	.13	.252	.81	1.789	3.4	5.992
	.35	.234	.365	1.149	2.605	5.019	8.912

Appendix C. Effect of Front End Loss on Noise Figure and Effective Input Noise Temperature

If the reference plane used to define the input of the amplifier is changed, then the noise figure, F_{dB}, and the effective input noise temperature, T_e changes if there is any loss between these reference planes. These changes in amplifier noise are calculated in this appendix. One application of the results of this appendix is to estimate the uncertainty in amplifier noise because of the uncertainties in the connector loss.

Using figure C-1, the effective input noise temperature at reference plane 1 is T_e, and at reference plane 2 is T_e'. The two-port between 1 and 2 has the physical temperature T_{conn} and an absorption loss of A so that the thermal radiation from the two-port expressed relative to port 1 is A T_{conn} [8]. The effective input noise temperature at reference plane 2, T_e', is the amplifier noise and the two-port noise expressed relative to port 2, namely

$$T_e' = (A\ T_{conn}+T_e)/(1-A) \tag{C.1}$$

or

$$T_e' - T_e = \frac{A}{1 - A}(T_{conn}+T_e). \tag{C.2}$$

Expressing the absorption loss in decibels,

$$A_{dB} = -10\ \log_{10}(1-A), \tag{C.3}$$

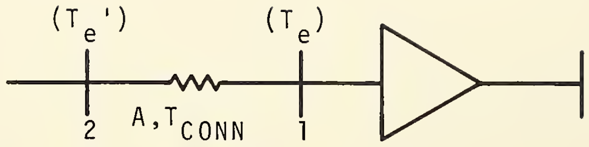

Figure C1

then

$$T_e' - T_e = (10^{A_{dB}/10} - 1)(T_{conn}+T_e). \tag{C.4}$$

For small losses (i.e., $A_{dB} << 10$)

$$T_e' - T_e \simeq 0.2303\ (T_{conn}+T_e)\ A_{dB}. \tag{C.5}$$

To express the difference in noise figure at reference plane 2 from plane 1, recall that $F_{dB} = 10 \log_{10} F$, where $F = 1 + (T_e/T_o)$, and where $T_o = 290$ K. Because $dF_{dB} = (10 \log_{10} e)dF/F$, then for small losses

$$F_{dB}' - F_{dB} \simeq 4.343\ (T_e' - T_e)/(T_o+T_e) \tag{C.6}$$

or using eq. (C.5)

$$F_{dB}' - F_{dB} \simeq \left(\frac{T_{conn} + T_e}{T_o + T_e}\right) A_{dB}. \tag{C.7}$$

If the two-port's physical temperature, T_{conn}, is nearly equal $T_o = 290$ K, then

$$F_{dB}' - F_{dB} \simeq A_{dB}. \tag{C.8}$$

Thus an amplifier which includes a two-port with 0.1 dB absorption loss at 290 K (16.8 C or 62.3 F) has a noise figure 0.1 dB greater than the same amplitude excluding this two-port.

Appendix D. Computer Programs

This appendix contains the computer programs used for the figures and tables in this paper. The programs are in BASIC language.

D.1. Figures 1 Through 4

For figures 1-4, the program DW11 in table D-I was used. A typical output of this program is shown in table D-II.

The basic relationships used for this table are as follows: using eq. (1) of reference 1 and changing the notation to correspond with the remark statements in Table D-I, and adding the gain function G [cf step 4400 in Table D-I]

$$Y = G*(TH+TE)/(TC+TE), \qquad (D.1)$$

or rearranging [cf steps 4300, 4440, 4450, 4480, 4490, 4520, 4530, 4560, 4570]

$$TE = (G*TH-Y*TC)/(Y-G). \qquad (D.2)$$

This eq. (D.2) is used to calculate the error contributions from TH, TC, Y and G. Equation (2) of reference 1 (cf steps 4340 and 4330) is used to convert from F(DB) to TE,

$$F(DB) = 10*CLG(1+TE/290), \qquad (D.3)$$

and the derivative of eq. (D.3) is used to convert errors in F(DB) to errors in TE (cf step 4427 which is also divided by 100 to convert to percent)

$$DF(DB) = 10[(F-1)/(F*Log(10))]\ DTE/TE, \qquad (D.4)$$

where the symbol D denotes the derivative, and F is defined via (cf step 4425)

$$F(DB) = 10\ CLG(F). \qquad (D.5)$$

The error caused by loss in the connector uses the following relationships. First converting loss in decibels to a ratio (step 4435) uses

$$CONNECTOR\ LOSS\ (DB) = -10\ CLG(A). \qquad (D.6)$$

The changed noise temperature TH (or similar for TC) caused by a loss at 300 K is given by [8] (cf step 4640)

$$TH + DTH = A*TH + (1-A)*300. \qquad (D.7)$$

For mismatch error, eq. (A.6) is used as explained in appendix A (of steps 1000 through 1150).

Table D-I. Computer program for figures 1-4.

```
DW11        11:00  CSS     FRI.11/24/72

5 REM MODIFIED STEPS 74,95,96,4315,4597-4630,1000+,FROM ↑NBS 5891
10 REM  USES NBS 5853 FOR MISMATCH
20 N=3
30 M=3
40 C=62
42 V=0
45 REM I=1 FOR TE,I=2 FOR F(DB),I=3 FOR TE RECYCLE,I=4 FOR F(DB) RECYCLE
50 PRINT
60 PRINT
65 REM      B2=BETA
70 PRINT
71 REM      T=TE
72 REM      T1=TH
73 REM      T2=TC
74 REM      T3=BTA
75 REM      T4=TA
76 REM
77 REM      D1=DTH
78 REM      D2=DTC
79 REM      D3=DY(DB)
80 REM      D4=DG(%)
81 REM      D5=CONNECTOR LOSS (DB)
82 REM      D6=CLIPPING (%)
83 REM
84 REM      E1=ETH
85 REM      E2=ETC
86 REM      E3=EY
87 REM      E4=EG
88 REM      E5=E(MISMATCH)
89 REM      E6=E(CONNECTOR)
90 REM      E7=E(CLIPPING)
91 REM
92 REM      F=F
93 REM      F1=F(DB)
94 REM
95 REM      G1=GAMMA'(ANT)
96 REM      G2=GAMMA'(HOT)-GAMMA'(ANT)
97 REM      G5=GAMMA'(COLD)-GAMMA'(ANT)
98 REM
100 REM      Y=Y
102 REM      Y2=DY
104 REM
105 REM
106 REM      T5=PERTURBED TH
108 REM      T6=PERTURBED TC
150 GOSUB 4300
190 READ G1,G2,T3,D3,D4,D5,D6
195 B2=0
200 READ T1,D1,T2,D2
205 READ I,H1,H2,H3
245 GOSUB 5000
247 ON I GOTO 250,810,250,810
250 PRINT USING 5550
260 PRINT
```

Table D-I (Continued)

```
300 FOR T=H1 TO H2 STEP H3
330 GOSUB 4340
340 PRINT USING 5560,T,E1,E1+E2,E1+E2+E3,E8,E8+E6,E8+E6+E7,E9,P
350 GOSUB 3000
770 NEXT T
775 IF I=3 GOTO 205
800 GOSUB 3080
802 GOTO 190
810 PRINT USING 5570
815 PRINT
820 M=M+2
830 FOR F1=H1 TO H2 STEP H3
840 GOSUB 4330
890 PRINT USING 5530,F1,R1,R1+R2,R1+R2+R3,R8,R8+R6,R8+R6+R7,R9,P*B
895 GOSUB 3000
900 NEXT F1
901 IF I=4 GOTO 205
905 GOSUB 3030
910 GOTO 190
1000 K=0
1005 G5=G2
1010 FOR J=1 TO 8
1020 B2=-B2
1030 K=K+1
1040 IF K=2 GOTO 1060
1045 K=1
1050 G5=-G5
1060 IF J<>5 GOTO 1080
1070 G2=-G2
1080 T5=T1+FNM(T1,G2,B2)
1090 T6=T2+FNM(T2,G5,B2)
1100 W=ABS(FNT(T5,T6,Y,1)-T)
1110 IF J<>1 GOTO 1130
1120 X=W
1130 IF X>W GOTO 1150
1140 X=W
1150 NEXT J
1160 RETURN
3000 M=M+1
3010 V=V+1
3020 IF V<>3 THEN 3070
3030 V=0
3040 PRINT
3050 PRINT
3060 M=M+2
3070 IF M<C-3 THEN 3120
3080 FOR Q=M TO C+6
3090 PRINT
3100 NEXT Q
3110 M=3
3120 RETURN
4300 DEF FNT(H,C,Y,G)=(G*H-Y*C)/(Y-G)
4310 DEF FNE(W,X,Z)=50*ABS((W-X)/Z)
4315 DEF FNM(X,Y,Z)=((X-T*T3)*(Y*Y+2*G1*Y)+2*T*T3*Z*Y)/(1-G1*G1)
4320 RETURN
```

Table D-I (Continued)

```
4330 T=290*(10↑(F1/10)-1)
4340 F1=10*CLG(1+T/290)
4400 Y=(T1+T)/(T2+T)
4410 Y1 =10*CLG(Y)
4415 Y2 = Y* LOG(10)* D3/10
4425 F=10↑(F1/10)
4427 B=0.1*(F-1)/(F* LOG(10))
4430 T4=T-G1↑2*(T+T3)
4435 A=10↑(-D5/10)
4440      W=FNT(T1+D1,T2,Y,1)
4450      X=FNT(T1-D1,T2,Y,1)
4460      Z=T
4470 E1=FNE(W,X,Z)
4475 R1=E1*B
4480      W=FNT(T1,T2+D2,Y,1)
4490      X=FNT(T1,T2-D2,Y,1)
4510 E2=FNE(W,X,Z)
4515 R2=E2*B
4520      W=FNT(T1,T2,Y+Y2,1)
4530      X=FNT(T1,T2,Y-Y2,1)
4550 E3 =FNE(W,X,Z)
4555 R3=E3*B
4560      W=FNT(T1,T2,Y,1+D4/100)
4570      X=FNT(T1,T2,Y,1-D4/100)
4590 E4=FNE(W,X,Z)
4595 R4=E4*B
4600 GOSUB 1000
4610 E5=100*X/T
4635 R5=E5*B
4640      W=FNT(A*T1+(1-A)*300,A*T2+(1-A)*300,Y,1)
4650      X=T
4660      Z=T
4670 E6=2*FNE(W,X,Z)
4675 R6=E6*B
4680      W=((1-D6/100)*T1-Y*T2)/(Y-1)
4710 E7=2*FNE(W,X,Z)
4715 R7=E7*B
4717 E9=E1+E2+E3+E4+E5+E6+E7
4718 R9=E9*B
4720 E8=E1+E2+E3+E4
4725 R8=B*E3
4727 P=SQR(E1↑2+E2↑2+E3↑2+E4↑2+E5↑2+E6↑2+E7↑2)
4730 RETURN
5000 PRINT USING 5500,T1,D1,100*D1/T1
5005 PRINT USING 5510,T2,D2,100*D2/T2
5010 PRINT
5020 PRINT USING 5520,T3,G2,G1,B2
5050 PRINT USING 5530,D4,D3,D6,D5
5060 PRINT
5065 M=M+8
5070 RETURN
5080 PRINT USING 5550
5090 PRINT
5100 PRINT
5110 RETURN
5500:                             TH=##### + ###.# K (#.##%)
5510:                              TC=##### + #.# K (#.##%)
5520:     BTA/TE=##.#  G(STD-ANT)=#.###   G(ANT)=#.###       BETA=.##
5530:      DG=##.#%      DY=.## DB        CLIPPING=##.#%     LOSS=.### DB
5550:TE(K)    ETH    ETC     EY     EG   LOSS    CLIP   MATCH QUAD-E
5560:##### ###.## ###.## ###.## ###.## ###.## ###.## ###.## ###.##
5570:F(DB)   ETH     ETC    EY     EG   LOSS    CLIP   MATCH  QUAD-E
5580:##.#   .####  .####  .####  .####  .####  .####  .####   .####
```

Table D-I (Continued)

```
8000 DATA .05,.01,0,.01,.1,.01,0
8010 DATA 10000,150,300,.5
8015 DATA 2,2,16.5,.5
8020 DATA .05,.01,.0,.01,.1,.01,.0
8030 DATA 373,.5,80,1
8040 DATA 3,100,300,20
8050 DATA 1,300,800,50
8060 DATA .05,.01,0,.01,.1,.01,0
8070 DATA 1270,3,300,.2
8072 DATA 4,.1,1,.1
8074 DATA 2,1,14,.5
8075 END

READY
```

Table D-II. Print out of D-I program.

```
READY
RUN

DWI1       10:44   CSS     FRI.11/24/72



                              TH=10000 + 150.0 K (1.50%)
                               TC=  300 + 0.5 K (0.17%)

   BTA/TE= 0.0  G(STD-ANT)=0.010    G(ANT)=0.050        BETA=.00
    DG= 0.1%        DY=.01 DB       CLIPPING= 0.0%      LOSS=.010 DB

F(DB)  ETH      ETC     EY      EG     LOSS     CLIP    MATCH   QUAD-E

 2.0  .0686   .0736   .0843   .0889   .0991   .0991   .1069    .0709
 2.5  .0685   .0729   .0836   .0883   .0985   .0985   .1059    .0707
 3.0  .0683   .0723   .0831   .0878   .0979   .0979   .1051    .0705


 3.5  .0682   .0718   .0826   .0873   .0975   .0975   .1044    .0704
 4.0  .0681   .0713   .0822   .0869   .0971   .0971   .1038    .0702
 4.5  .0680   .0709   .0818   .0866   .0967   .0967   .1033    .0701


 5.0  .0679   .0705   .0816   .0864   .0965   .0965   .1029    .0700
 5.5  .0678   .0701   .0813   .0862   .0963   .0963   .1025    .0700
 6.0  .0677   .0698   .0811   .0861   .0961   .0961   .1022    .0699


 6.5  .0677   .0696   .0810   .0860   .0960   .0960   .1020    .0698
 7.0  .0676   .0693   .0809   .0860   .0960   .0960   .1019    .0698
 7.5  .0676   .0691   .0809   .0860   .0960   .0960   .1019    .0698


 8.0  .0675   .0689   .0809   .0861   .0961   .0961   .1019    .0698
 8.5  .0675   .0688   .0810   .0862   .0963   .0963   .1019    .0698
 9.0  .0675   .0686   .0811   .0865   .0965   .0965   .1021    .0698


 9.5  .0674   .0685   .0812   .0867   .0968   .0968   .1023    .0698
10.0  .0674   .0684   .0814   .0871   .0971   .0971   .1026    .0698
10.5  .0674   .0683   .0817   .0875   .0975   .0975   .1029    .0699


11.0  .0673   .0682   .0820   .0880   .0980   .0980   .1034    .0699
11.5  .0673   .0681   .0823   .0885   .0986   .0986   .1039    .0700
12.0  .0673   .0680   .0828   .0892   .0992   .0992   .1045    .0701


12.5  .0673   .0679   .0833   .0900   .1000   .1000   .1053    .0703
13.0  .0673   .0679   .0839   .0908   .1008   .1008   .1061    .0704
13.5  .0673   .0678   .0846   .0918   .1018   .1018   .1071    .0706


14.0  .0673   .0678   .0853   .0929   .1029   .1029   .1082    .0708
14.5  .0672   .0677   .0862   .0942   .1042   .1042   .1094    .0711
15.0  .0672   .0677   .0872   .0956   .1056   .1056   .1108    .0714


15.5  .0672   .0677   .0883   .0973   .1073   .1073   .1124    .0718
```

Table D-II (Continued)

```
                         TH=  373 +   0.5 K (0.13%)
                          TC=   80 + 1.0 K (1.25%)

    BTA/TE= 0.0  G(STD-ANT)=0.010   G(ANT)=0.050       BETA=.00
     DG= 0.1%      DY=.01 DB        CLIPPING= 0.0%     LOSS=.010 DB

TE(K)    ETH     ETC      EY      EG    LOSS     CLIP    MATCH  QUAD-E

  100    0.31    1.92    2.59    2.88    3.80    3.80    4.17    2.05


  120    0.28    1.69    2.33    2.61    3.42    3.42    3.75    1.82
  140    0.27    1.52    2.15    2.43    3.15    3.15    3.46    1.65
  160    0.26    1.39    2.02    2.29    2.96    2.96    3.25    1.53


  180    0.25    1.30    1.92    2.20    2.81    2.81    3.09    1.44
  200    0.24    1.22    1.85    2.12    2.70    2.70    2.96    1.37
  220    0.23    1.15    1.79    2.06    2.61    2.61    2.87    1.32


  240    0.23    1.10    1.74    2.02    2.54    2.54    2.79    1.28
  260    0.22    1.05    1.70    1.99    2.48    2.48    2.73    1.24
  280    0.22    1.02    1.68    1.96    2.44    2.44    2.68    1.22


  300    0.22    0.98    1.65    1.94    2.40    2.40    2.64    1.20
                         TH=  373 +   0.5 K (0.13%)
                          TC=   80 + 1.0 K (1.25%)

    BTA/TE= 0.0  G(STD-ANT)=-.010   G(ANT)=0.050       BETA=.00
     DG= 0.1%      DY=.01 DB        CLIPPING= 0.0%     LOSS=.010 DB

TE(K)    ETH     ETC      EY      EG    LOSS     CLIP    MATCH  QUAD-E

  300    0.22    0.98    1.65    1.94    2.40    2.40    2.64    1.20
  350    0.21    0.91    1.61    1.92    2.34    2.34    2.57    1.16


  400    0.20    0.86    1.59    1.91    2.31    2.31    2.53    1.15
  450    0.20    0.83    1.59    1.92    2.30    2.30    2.51    1.14
  500    0.20    0.79    1.59    1.94    2.30    2.30    2.51    1.15


  550    0.20    0.77    1.60    1.96    2.32    2.32    2.52    1.16
  600    0.19    0.75    1.61    1.99    2.33    2.33    2.53    1.18
  650    0.19    0.73    1.63    2.02    2.36    2.36    2.56    1.20


  700    0.19    0.71    1.65    2.06    2.39    2.39    2.58    1.23
  750    0.19    0.70    1.68    2.10    2.42    2.42    2.62    1.25
  800    0.19    0.69    1.70    2.14    2.46    2.46    2.65    1.28
```

D.2. Figures B1 through B7

For figures B1 through B7, the program DW21.3 listed in table D-III was used. A minicomputer was used for this calculation and call 11 in step 1052 reads a switch position on the front panel for a diagnostic routine. A typical output from this program is listed in table D-IV. For the calculation, eq. (B.9) is used as explained in appendix B.

Table D-III. Computer program for figures B1-B7.

```
1   PRINT "DW21.3  (29 NOV 72)"
2   REM   MISMATCH UNCERTAINTY; SWITCH 0 UP FOR DIAGNOSTICS
10   REM   FROM NBS PAGE 5927, PROGRAM MODIFICATION FO 5836
12   REM        B1=BETA
14   REM        G1=GAMMA'(ANT)-GAMMA'(STD)
16   REM        G2=GAMMA'(ANT)
17   REM        B=REVERSE RADIATION PARAMETER
90   DEF  FNA(C)= INT (10000*C+.5)/100
100   DIM D[25]
110   FOR I=1 TO 23
120     READ D[I]
130   NEXT I
140   READ B1
200   PRINT
210   PRINT
220   PRINT
230   PRINT  TAB (18);
240   PRINT "MAXIMUM MISMATCH UNCERTAINTY (% OF TE)"
250   PRINT
260   PRINT
270   PRINT ,,"FOR BETA=";B1
280   PRINT
290   PRINT
300   PRINT
310   PRINT
320   PRINT " --GAMMA'--";
325   PRINT "    ";
330   PRINT "------------REVERSE RADIATION PARAMETER,B------------"
340   PRINT " ERR    ANT";
350   FOR K=1 TO 6
360     PRINT  TAB (D[17+K]+1);D[11+K];
370   NEXT K
380   PRINT
385   PRINT
390   PRINT
400   FOR I=1 TO 5
405     FOR J=1 TO 6
410       LET G1=D[I]
420       LET G2=D[5+J]
430       IF J<>1 GOTO  450
440       PRINT G1;
450       PRINT  TAB (7),G2;
460       FOR K=1 TO 6
470         LET B=D[11+K]
480         GOSUB  1000
485         LET X= FNA(E)+.0001
490         IF X> 0 GOTO  510
500         LET X=.001
510         PRINT  TAB (D[17+K]- LOG (X)/ LOG (10)), FNA(E);
520         PRINT "%";
```

Table D-III (Continued)

```
530       NEXT K
540       PRINT
550       LET V=V+1
560       IF V=3 GOTO  610
570       IF V=6 GOTO  590
580       GOTO  620
590       LET V= 0
600       PRINT
610       PRINT
620     NEXT J
630   NEXT I
640   FOR M=1 TO 17
650     PRINT
660   NEXT M
670   GOTO  140
1000   LET E= 0
1005   FOR Q=1 TO 4
1010     LET G1=-G1
1020     IF Q<>3 GOTO  1040
1030     LET B1=-B1
1040     LET L=(G1*G1+2*G2*G1)/(1-G2*G2)
1041     LET E2=L*B*(G2-B1)*(G2-B1)
1042     LET E3=B*G1*G1
1043     LET E4=2*B*G1*(G2-B1)
1044     LET E5=(1-G2*G2)*(1-L)
1050     LET E1=(L+E2+E3+E4)/E5
1052     CALL 11, 0,P
1053     IF P= 0 GOTO  1060
1054     PRINT
1055     PRINT "E="E;"E"Q;"="E1;"G1="G1;"G2="G2;"B1="B1;"B="B;"L="L;
1056     PRINT
1057     PRINT "E2="E2;"E3="E3;"E4="E4;"E5="E5
1060     IF  ABS (E1)<E GOTO  1080
1070     LET E= ABS (E1)
1080   NEXT Q
1090   RETURN
5000   REM     G1=GAMMA'(ANT)-GAMMA'(STD) FOLLOWS
5010   DATA  .005, .01, .02, .05, .1
5020   REM    G2=GAMMA"(ANT) FOLLOWS
5030   DATA  0, .02, .05, .1, .2, .35
5040   REM     REVERSE RADIATION PARAMETER B FOLLOWS
5050   DATA  0, .2, .5, 1, 2, 5
5070   REM    TAB POSITIONS FOLLOW
5075   DATA  17, 27, 36, 45, 55, 64
5080   REM    B1=BETA FOLLOWS
5090   DATA  0, .1, .2, .3, .4, .5
```

Table D-IV. Print out of D-III program.

5964

```
RUN
DV21.3   (30 NOV 72)
←NBS PAGE 5940, RECHECK 5928


                          MAXIMUM MISMATCH UNCERTAINTY (% OF TE)


                                    FOR BETA= 0



--GAMMA'--     ------------REVERSE RADIATION PARAMETER,B-----------
ERR     ANT         0          .2         .5          1          2           5


.03     0          .09 %      .11 %      .14 %      .18 %      .27 %      .54 %
        .02       .21 %      .25 %      .32 %      .42 %      .63 %      1.26 %
        .05       .39 %      .47 %      .59 %      .79 %      1.18 %     2.36 %

        .1        .71 %      .85 %      1.06 %     1.42 %     2.13 %     4.25 %
        .2        1.42 %     1.7 %      2.13 %     2.84 %     4.26 %     8.51 %
        .35       2.92 %     3.5 %      4.38 %     5.83 %     8.75 %     17.5 %


.04     0         .16 %      .19 %      .24 %      .32 %      .48 %      .96 %
        .02       .32 %      .39 %      .48 %      .64 %      .96 %      1.93 %
        .05       .57 %      .68 %      .85 %      1.13 %     1.7 %      3.4 %

        .1        .99 %      1.19 %     1.48 %     1.98 %     2.97 %     5.93 %
        .2        1.95 %     2.33 %     2.92 %     3.89 %     5.84 %     11.67 %
        .35       3.98 %     4.77 %     5.97 %     7.96 %     11.93 %    23.87 %


.12     0         1.46 %     1.75 %     2.19 %     2.92 %     4.38 %     8.77 %
        .02       1.96 %     2.35 %     2.94 %     3.92 %     5.88 %     11.76 %
        .05       2.73 %     3.27 %     4.09 %     5.45 %     8.18 %     16.35 %

        .1        4.08 %     4.89 %     6.11 %     8.15 %     12.23 %    24.46 %
        .2        7.24 %     8.69 %     10.86 %    14.48 %    21.72 %    43.45 %
        .35       14.39 %    17.27 %    21.59 %    28.79 %    43.18 %    86.36 %


.13     0         1.72 %     2.06 %     2.58 %     3.44 %     5.16 %     10.31 %
        .02       2.26 %     2.71 %     3.39 %     4.52 %     6.79 %     13.57 %
        .05       3.1 %      3.72 %     4.65 %     6.2 %      9.29 %     18.59 %

        .1        4.58 %     5.49 %     6.86 %     9.15 %     13.73 %    27.45 %
        .2        8.05 %     9.67 %     12.08 %    16.11 %    24.16 %    48.33 %
        .35       15.98 %    19.17 %    23.97 %    31.96 %    47.93 %    95.87 %


.14     0         2 %        2.4 %      3 %        4 %        6 %        12 %
        .02       2.59 %     3.1 %      3.88 %     5.17 %     7.76 %     15.52 %
        .05       3.49 %     4.19 %     5.24 %     6.99 %     10.48 %    20.97 %

        .1        5.1 %      6.12 %     7.65 %     10.2 %     15.31 %    30.61 %
        .2        8.9 %      10.69 %    13.36 %    17.81 %    26.71 %    53.43 %
        .35       17.64 %    21.16 %    26.45 %    35.27 %    52.91 %
105.82 %
```

D.3. Table 3

For table 3, the program DW13 listed in table D-V was used. Essentially eq. (D.4) is used in step 320.

Table D-V. Computer program for table 3.

```
                                                LIST

DW13        15:00   CSS    THU.05/04/72

50 N=1
60 V=0
100 PRINT
110 PRINT
120 PRINT
130 PRINT USING 140
140:    TE(K)      =       F(DB)                   F(DB)       =      TE(K)
150 PRINT
160 FOR J=1 TO 4
170 FOR K=1 TO 6
180 ON K GOTO 200,210,220,230,240,250
200 T=10↑J
205 GOTO 300
210 T=1.5*10↑J
215 GOTO 300
220 T=2*10↑J
225 GOTO 300
230 T=3*10↑J
235 GOTO 300
240 T=5*10↑J
245 GOTO 300
250 T=7*10↑J
255 GOTO 300
300 F1=10*CLG(T/290+1)
310 F=1+T/290
320 B=.1*(F-1)/(F* LOG(10))
330 T5=290*(10↑(.1*N)-1)
340 F5=1+T5/290
350 T6=F5*LOG(10)/(F5-1)
360 PRINT USING 370,T,F1,B,N,T5,T6
370:##### + 1%      ##.## + .#### DB          ## + .1 DB    ##### + ##.##%
380 N=N+1
390 V=V+1
400 IF V<>3 THEN 430
410 V=0
420 PRINT
425 PRINT
430 NEXT K
440 NEXT J
500 FOR I=1 TO 10
510 PRINT
520 NEXT I
999 END

READY
500 FOR I=1 TO 20
```

D.4. Tables AI--AXXI

For table A-I through A-XXI the program DW10 listed in table D-VI was used. This program is very similar to parts of DW11 listed in table D-I.

Table D-VI. Computer program for tables A-I through A-XXI.

```
LIST

DW10      08:50  CSS    THU.04/20/72

20 N=3
30 M=3
40 C=62
42 V=0
45 I=1
50 PRINT
60 PRINT
70 PRINT
71 REM     T=TE
72 REM     T1=TH
73 REM     T2=TC
74 REM     T3=BTA
75 REM     T4=TA
76 REM
77 REM     D1=DTH
78 REM     D2=DTC
79 REM     D3=DY(DB)
80 REM     D4=DG(%)
81 REM     D5=CONNECTOR LOSS (DB)
82 REM     D6=CLIPPING (%)
83 REM
84 REM     E1=ETH
85 REM     E2=ETC
86 REM     E3=EY
87 REM     E4=EG
88 REM     E5=E(MISMATCH)
89 REM     E6=E(CONNECTOR)
90 REM     E7=E(CLIPPING)
91 REM
92 REM     F=F
93 REM     F1=F(DB)
94 REM
95 REM     G1=GAMMA  (AMP)
96 REM     G2=GAMMA (STDS)
97 REM     Y=Y
98 REM     Y1=Y(DB)
99 REM     Y2=DY
100 DEF FNT(H,C,Y,G)=(G*H-Y*C)/(Y-G)
110 DEF FNE(W,X,Z)=50*ABS((W-X)/Z)
120 DEF FNF(T)=10*CLG(1+T/290)
190 READ G1,G2,T3,D3,D4,D5,D6
200 READ T1,D1,T2,D2
245 GOSUB 5000
300 FOR J=1 TO 4
310 FOR K=1 TO 6
320 ON K GOTO 1000,1020,1040,1060,1080,1100
330 GOSUB 4400
340 PRINT USING 5560,T,E8,F1,R8,Y1,E1,E2,E3,E4
350 M=M+1
355 V=V+1
360 IF V<>3 THEN 760
370 V=0
380 PRINT
390 PRINT
```

Table D-VI (Continued)

```
400 M=M+2
760 IF M<= C-3 THEN 770
763 IF M> C-3 THEN 765
765 GOSUB 3000
770 NEXT K
780 NEXT J
800 GOSUB 3000
802 GOTO 200
803 GOTO 200
804 GOSUB 5000
1000 T=10↑J
1010 GOTO 330
1020 T=1.5*10↑J
1030 GO TO 330
1040 T=2*10↑J
1050 GOTO 330
1060 T=3*10↑J
1070 GOTO 330
1080 T=5*10↑J
1090 GOTO 330
1100 T=7*10↑J
1110 GOTO 330
3000 FOR Q=M TO C+6
3010 PRINT
3020 NEXT Q
3025 M=3
3040 RETURN
4400 Y=(T1+T)/(T2+T)
4410 Y1 =10*CLG(Y)
4415 Y2 = Y*LOG(10)*D3/10
4420 F1=FNF(T)
4425 F=10↑(F1/10)
4427 B=0.1*(F-1)/(F*LOG(10))
4430 T4=(1-G1↑2)*T-T3*G1
4435 A=10↑(-D5/10)
4440     W=FNT(T1+D1,T2,Y,1)
4450     X=FNT(T1-D1,T2,Y,1)
4460     Z=T
4470 E1=FNE(W,X,Z)
4480     W=FNT(T1,T2+D2,Y,1)
4490     X=FNT(T1,T2-D2,Y,1)
4510 E2=FNE(W,X,Z)
4520     W=FNT(T1,T2,Y+Y2,1)
4530     X=FNT(T1,T2,Y-Y2,1)
4550 E3 =FNE(W,X,Z)
4560     W=FNT(T1,T2,Y,1+D4/100)
4570     X=FNT(T1,T2,Y,1-D4/100)
4590 E4=FNE(W,X,Z)
4600     W=(T4+T3*((G1+G2)/(1-G1*G2)))/(1-((G1+G2)/(1-G1*G2))↑2)
4610     X=(T4+T3*ABS(G1-G2))/(1-(ABS(G1-G2))↑2)
4630 E5=FNE(W,X,Z)
4640     W=FNT(A*T1+(1-A)*300,A*T2+(1-A)*300,Y,1)
4650     X=T
4660     Z=T
4670 E6=2*FNE(W,X,Z)
4680     W=((1-D6/100)*T1-Y* T2)/(Y-1)
4710 E7=2*FNE(W,X,Z)
4720 E8=E1+E2+E3+E4
4725 R8=B*E8
4730 RETURN
```

Table D-VI (Continued)

```
5000 PRINT USING 5500,T1,D1,100*D1/T1
5005 PRINT USING 5510,T2,D2,100*D2/T2
5010 PRINT
5020 PRINT USING 5520,D3,D4
5050 PRINT
5060 PRINT
5065 M=M+8
5070 PRINT USING 5540
5080 PRINT USING 5550
5090 PRINT
5100 PRINT
5110 RETURN
5500:                           TH=##### + ### K (#.##%)
5510:                           TC=##### + #.# K (#.##%)
5520:                           DY=.## DB       DG=.##%
5540:                                          ERROR CONTRIBUTIONS TO TE
5550:    TE(K)         F(DB)         Y(DB)     ETH     ETC      EY     6
5560:##### +##.#%     ##.# + .##     ##.#     ##.##%  ##.##%  ##.##%  #6
8010 DATA .1,.1,300,.01,.1,.02,.1
8020 DATA 18000,270,300,1
8030 DATA 18000,270,80,.5
8040 DATA 18000,270,4,.5
8050 DATA 10000,150,300,1
8060 DATA 10000,150,80,.5
8070 DATA 10000,150,4,.5
8080 DATA 1270,3,300,1
8090 DATA 1270,3,80,.5
8100 DATA 1270,3,4,.5
8110 DATA 373,.1,290,.1
8120 DATA 373,.1,80,.5
8130 DATA 373,.1,4,.5
8140 DATA 300,1,80,.5
8150 DATA 300,1,4,.5
9999 END

READY
RUN
```

D.5. Tables BIII--BXXII

For tables BIII -- BXXII, the program DW22 listed in table D-VII was used. This uses the same mismatch error calculation from appendix B that was used in DW11 listed in table D-I.

Table D-VII. Computer program for tables B-III through B-XXII.

```
LIST
1  PRINT "DW22(3 DEC 72)"
2  PRINT "←PAGE 5856,CK 5891"
5  REM :MISMATCH ERROR; SWITCH 0 UP FOR DIAGNOSTICS
10  REM : MOD DW11 (PAGE 5985), DW21.3 (PAGE 5973)
12  REM        B1=BETA
14  REM      G1=GAMMA'(HOT) - GAMMA'(ANT)
16  REM        G2=GAMMA'(ANT)
17  REM      G3=GAMMA'(COLD) - GAMMA'(ANT)
18  REM        B=REVERSE RADIATION PARAMETER
20  REM
21  REM      T=T3
22  REM      T1=T(HOT)
24  REM      T2=T(COLD)
26  REM      T5=PERTURBED T(HOT)
28  REM      T6=PERTURBED T(COLD)
30  REM      T7=PERTURBED TE
38  REM      F=F
40  REM      F1=F(DB)
50  REM      Y=Y
90  DEF  FNA(X)= INT (1000*X+.5)/1000
100  DIM D[25]
110  FOR I=1 TO 23
120    READ D[I]
130  NEXT I
140  READ T1,T2,B1,B
200  PRINT
210  PRINT
220  PRINT
230  PRINT  TAB (13);
240  PRINT "MAXIMUM MISMATCH ERROR (DB)"
250  PRINT
260  PRINT
270  PRINT ,"          T(HOT)="T1;"K"
272  PRINT ,"          T(COLD)="T2;"K"
274  PRINT
276  PRINT ,"          BETA="B1
278  PRINT ,"          BTA/TE="B
280  PRINT
290  PRINT
300  PRINT
310  PRINT
320  PRINT " --GAMMA'--";
325  PRINT "   ";
330  PRINT "-----------------------F(DB)-----------------------"
340  PRINT " ERR    ANT";
350  FOR K=1 TO 6
360    PRINT  TAB (D[17+K]+1);D[11+K];
370  NEXT K
380  PRINT
385  PRINT
390  PRINT
```

Table D-VII (Continued)

```
400  FOR I=1 TO 5
405    FOR J=1 TO 6
410      LET G1=D[I]
420      LET G2=D[5+J]
430      IF J<>1 GOTO  450
440      PRINT G1;
450      PRINT  TAB (7),G2;
460      FOR K=1 TO 6
470        LET F1=D[11+K]
472        LET T=290*(10↑(F1/10)-1)
473        LET F=10↑(F1/10)
474        LET Y=(T1+T)/(T2+T)
476        LET E4=10*(F-1)/(F* LOG (10))
480        GOSUB  1000
485        LET X= FNA(E)+.0001
490        IF X> 0 GOTO  510
500        LET X=.001
510        PRINT  TAB (D[17+K]), FNA(E);
530      NEXT K
540      PRINT
550      LET V=V+1
560      IF V=3 GOTO  610
570      IF V=6 GOTO  590
580      GOTO  620
590      LET V= 0
600      PRINT
610      PRINT
620    NEXT J
630  NEXT I
640  FOR M=1 TO 17
650    PRINT
660  NEXT M
670  GOTO  140
1000  LET Q= 0
1002  LET E3= 0
1005  LET G3=G1
1006  DEF  FNE(X)=(X*X+2*G2*X)
1007  LET E1=1-G2*G2
1010  FOR R=1 TO 8
1020    LET B1=-B1
1030    LET S=S+1
1040    IF S=2 GOTO  1060
1045    LET S=1
1050    LET G3=-G3
1053    IF P= 0 GOTO  1060
1054    PRINT
1056    PRINT
1060    IF R<>5 GOTO  1100
1070    LET G1=-G1
1100    DEF  FNF(X)=2*B*T*X*G1
1120    LET T5=((T1-B*T)* FNE(G1)+ FNF(G1))/E1
1125    LET T5=T1-T5
1130    LET T6=((T2-B*T)* FNE(G3)+ FNF(G3))/E1
1135    LET T6=T2-T6
1150    LET T7=(T5-Y*T6)/(Y-1)
1160    LET E2= ABS ((T7-T)/T)
1170    CALL 11, 0,P
1200    IF P= 0 GOTO  1230
1205    PRINT
1210    PRINT "F="F;"F1="F1;"T="T;"T1="T1;"T2="T2
1215    PRINT "T5="T5;"T6="T6;"T7="T7;"Y="Y
1220    PRINT "G1="G1;"G3="G3;"G2="G2;"B1="B1;"B="B
1225    PRINT "E("R;")="E;"E1="E1;"E2="E2;"E3="E3;"E4="E4
1230    IF E2<E3 GOTO  1250
1240    LET E3=E2
1250    LET E=E3*E4
```

Table D-VII (Continued)

```
1260  NEXT R
1270  RETURN
5000  REM      G1=GAMMA'(ANT)-GAMMA'(STD) FOLLOWS
5010  DATA  .005, .01, .02, .05, .1
5020  REM     G2=GAMMA"(ANT) FOLLOWS
5030  DATA  0, .02, .05, .1, .2, .35
5040  REM :F1=F(DB) FOLLOWS
5050  DATA  1, 2, 4, 6, 8, 10
5070  REM     TAB POSITIONS FOLLOW
5075  DATA  17, 27, 36, 45, 55, 64
5080  REM : SETS OF T1,T2,BETA,BTA/TE FOLLOW
5090  DATA  10000, 300, 0, 0
5100  DATA  10000, 300, .1, .2
5110  DATA  10000, 300, .1, 1
5120  DATA  10000, 300, .2, .2
5130  DATA  10000, 300, .2, 1
5140  DATA  10000, 300, .3, .2
5150  DATA  10000, 300, .3, 1
5160  DATA  373, 80, 0, 0
5170  DATA  373, 80, .1, .2
5180  DATA  373, 80, .1, 1
5190  DATA  373, 80, .2, .2
5200  DATA  373, 80, .2, 1
5210  DATA  373, 80, .3, .2
5220  DATA  373, 80, .3, 1
5230  DATA  1270, 300, 0, 0
5240  DATA  1270, 300, .2, 1
5300  DATA  1270, 80, 0, 0
5310  DATA  1270, 80, .2, 1
5400  DATA  300, 4, 0, 0
5410  DATA  300, 4, .2, 1
```

Appendix E. Reprints of Useful Papers

References [1, 3, and 8] are reprinted here.

Reference [1] Reprinted with Permission of the Publisher

Measurement of Amplifier Noise

mismatch errors and amplifier noise measurements are interrelated -- other errors are exposed

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Measurement of amplifier noise, in particular a state-of-the-art measurement using hot-cold noise standards of 10,000 K - 300 K or 373 K - 80 K, is discussed. An error that is especially difficult to estimate is the mismatch error. Measurements are suggested to help with this problem. Also, the traditional NBS thermal noise services are briefly sketched.

In its RF and microwave thermal-noise services, the National Bureau of Standards has in the past concentrated on developing means for transferring the fundamental standards of noise power to the scientific and engineering community of the Nation. With substantial capability for doing this now on hand, NBS has turned its attention to the application of these standards, and of related techniques, to measurements of more direct concern to the "ultimate user." One of the key objectives of these more recent efforts is to transfer to the user the ability to make his own measurements and/or to verify that these are indeed valid. In special instances NBS may simply make measurements for the user; in others it may repeat the customer's measurement, if necessary at the customer's own laboratory, to validate his technique and results.

This paper outlines and discusses the amplifier noise figure that has been used by NBS in the validation process and that, most important, can be used by others. It is typical of the new directions in NBS noise services.

The emphasis in this paper is on *how* to make amplifier noise measurements and on sources of measurement error and uncertainty. But because these and so many other similar measurements critically depend on the availability of noise standards, the standards that can be made available through the more traditional NBS calibration services are first briefly sketched.

TRADITIONAL CAPABILITIES IN NOISE

The National Bureau of Standards maintains reference noise standards and calibration radiometers, and has been active in developing the theory to support these areas. The most utilized noise service is that of noise-source calibration, although more and more frequently NBS is asked to analyze a radiometer rather than to calibrate a noise source.

Reference Standards

All NBS noise standards are resistive elements maintained at a uniform and known temperature T[1-5]. The theoretical power P_a available from these blackbody radiators is $P_a = kTB$ where k is Boltzmann's constant, and B is the bandwidth being viewed. The cold standards maintained by NBS are liquid-nitrogen cooled and have noise temperature near 80 K. The temperatures of the hot standards vary: the NBS 14-mm coaxial hot standard operates at 373 K; the WR284 standard (2.60 - 3.95 GHz) at 692 K; and the WR90 (8.2 - 12.5 GHz), WR62 (11.9 - 18.0 GHz), and WR15 (50 - 75 GHz) standards near 1250 K. Figure 1 indicates frequencies at which NBS maintains the hot and cold primary standards.

Calibration Services

The comparison radiometer, used to calibrate an unknown noise source relative to the primary standard, is

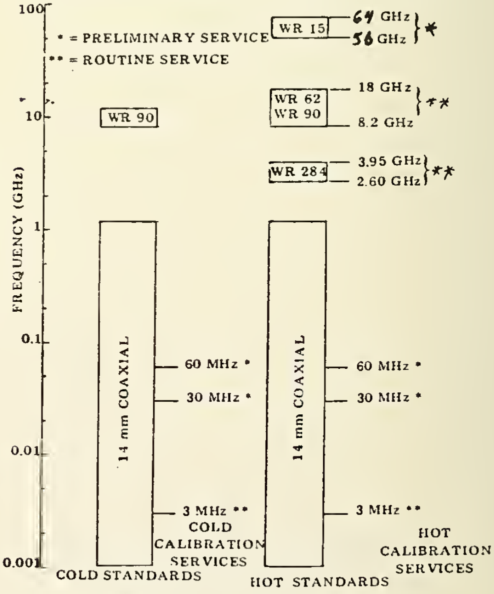

Fig. 1 NBS calibration services and standards.

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the most elaborate part of any calibration service, and the need to understand its operation spawns most of the NBS experimental and theoretical noise studies[4-16]. The frequencies at which both reference standards and comparison radiometers exist are the frequencies of the available noise calibration services shown in Figure 1. Typically noise temperatures between 1000 to 300,000 K are calibrated to within 2% total uncertainty (excess noise ratios between 4 and 30 dB to within 0.1 dB)[17]. The calibration uncertainty is degraded somewhat if the connector of the source to be calibrated differs from that of the corresponding NBS standard. For frequencies at which cold-source calibrations are indicated in Figure 1, noise sources are typically calibrated to within 1% at temperature above 75 K.

MEASUREMENT OF AMPLIFIER NOISE

At present, the National Bureau of Standards can measure effective input noise temperature only in WR15 waveguide but, in principle, the noise figure service can be readily implemented at any frequency where an NBS primary noise source exists (see Figure 1). Thus, the potential frequency coverage of NBS noise-figure-measurement capability is greater than the coverage of the noise-source calibration services, which are limited by the frequency ranges of the comparison radiometers.

An Overview

The most crisply defined measure of amplifier noise is T_e, the effective input noise temperature.* If two noise standards with corresponding noise temperatures T_{hot} and T_{cold} are sequentially connected to the input of an amplifier, then the change in output power of the amplifier, measured as a ratio Y, is

$$Y = (T_{hot} + T_e)/(T_{cold} + T_e) \qquad (1)$$

We see that T_e is essentially a measure of the amplifier's internal noise expressed in terms of the *available*** power density of a noise generator at the *input* of the amplifier.

Another popular measure of amplifier noise is noise figure[18], but the ambiguities and difficulties[19-26] with the IEEE definition of noise figure limits the usefulness of noise figure. The most serious ambiguity is related to superheterodyne amplifiers for which two different input channels contribute to the same output channel. The noise figure for the same amplifier differs depending on whether the user expects useful information through only the signal channel, or through both channels. Nevertheless, noise figure is still used as a measure of amplifier noise. To bypass the past difficulties an unorthodox definition is used here, namely

$$F_{dB} \equiv 10 \log [1 + T_e/290]. \qquad (2)$$

This definition corresponds approximately to *double-channel noise figure* as discussed by Mumford and Sheibe[24], and is equivalent to the IEEE definition of noise figure for a single-response amplifier. Defined this way, F_{dB} can be viewed as a change in scale of T_e. The conversions between T_e and F_{dB} are given in Table 1.

TABLE 1

T_e (K) =	F_{dB}	F_{dB} =	T_e (K)
10 ±1% =	0.15 ±.0014 dB	1 ±.1 dB =	75 ±11.20%
15 ±1% =	0.22 ±.0021 dB	2 ±.1 dB =	169 ± 6.24%
20 ±1% =	0.29 ±.0028 dB	3 ±.1 dB =	283 ± 4.62%
30 ±1% =	0.43 ±.0041 dB	4 ±.1 dB =	438 ± 3.83%
50 ±1% =	0.69 ±.0064 dB	5 ±.1 dB =	627 ± 3.37%
70 ±1% =	0.94 ±.0084 dB	6 ±.1 dB =	864 ± 3.07%
100 ±1% =	1.29 ±.0111 dB	7 ±.1 dB =	1163 ± 2.88%
150 ±1% =	1.81 ±.0148 dB	8 ±.1 dB =	1539 ± 2.74%
200 ±1% =	2.28 ±.0177 dB	9 ±.1 dB =	2013 ± 2.63%
300 ±1% =	3.08 ±.0221 dB	10 ±.1 dB =	2609 ± 2.56%
500 ±1% =	4.35 ±.0275 dB	11 ±.1 dB =	3360 ± 2.50%
700 ±1% =	5.33 ±.0307 dB	12 ±.1 dB =	4306 ± 2.46%
1000 ±1% =	6.48 ±.0337 dB	13 ±.1 dB =	5496 ± 2.42%
1500 ±1% =	7.90 ±.0364 dB	14 ±.1 dB =	6994 ± 2.40%
2000 ±1% =	8.97 ±.0379 dB	15 ±.1 dB =	8880 ± 2.38%
3000 ±1% =	10.55 ±.0396 dB	16 ±.1 dB =	11255 ± 2.36%
5000 ±1% =	12.61 ±.0410 dB	17 ±.1 dB =	14244 ± 2.35%
7000 ±1% =	14.00 ±.0417 dB	18 ±.1 dB =	18007 ± 2.34%
10000 ±1% =	15.50 ±.0422 dB	19 ±.1 dB =	22745 ± 2.33%
15000 ±1% =	17.22 ±.0426 dB	20 ±.1 dB =	28709 ± 2.33%
20000 ±1% =	18.45 ±.0428 dB	21 ±.1 dB =	36218 ± 2.32%
30000 ±1% =	20.19 ±.0430 dB	22 ±.1 dB =	45671 ± 2.32%
50000 ±1% =	22.39 ±.0432 dB	23 ±.1 dB =	57572 ± 2.31%
70000 ±1% =	23.84 ±.0433 dB	24 ±.1 dB =	72554 ± 2.31%

Conversion between *effective input noise temperature*, T_e (K), and noise figure, F_{dB}. The asymmetry in the error statements resulting from the logarithmic non-linearity of F_{dB} is avoided by using the slope of F_{dB} at the corresponding T_e.

The Accuracy Goal

The most accurate and easiest measurements of T_e are based on Equation 1. The effective input noise temperature T_e is known from a measurement of Y because

$$T_e = (T_{hot} - Y\,T_{cold})/(Y-1) \qquad (3)$$

There are seven important sources of error in a measurement of T_e: uncertainty in the value of T_{hot}; uncertainty in the value of T_{cold}, uncertainty in the measurement of Y; the amplifier-gain instability during the time required to measure the ratio Y; the inequality of the three reflec-

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* "Effective Input Noise Temperature, Average T_e (of a multiport transducer with one port designated as the output port). The *noise temperature* in degrees Kelvin which, assigned simultaneously to the specified impedance terminations at all frequencies at all accessible ports except the designated output port of a noisefree equivalent of the transducer, would yield the same total noise power in a specified output band delivered to the output termination as that of the actual transducer connected to noisefree equivalents of the terminations at all ports except the output port."[13]

** The noise temperature of a noise source is defined to be the physical temperature of a blackbody radiator which has the same *available* power density as the noise source (at the frequency in question).

tion coefficients, vis., the hot and cold noise standards, and the reflection coefficient "seen" by the amplifier input port when it is in actual use (mismatch error); difference in losses associated with the connector of the hot and cold standards and the amplifier under operating conditions are either not reproducible or are unknown; uncertainty in the noise generated in the measuring system is not accurately known (cascade error).

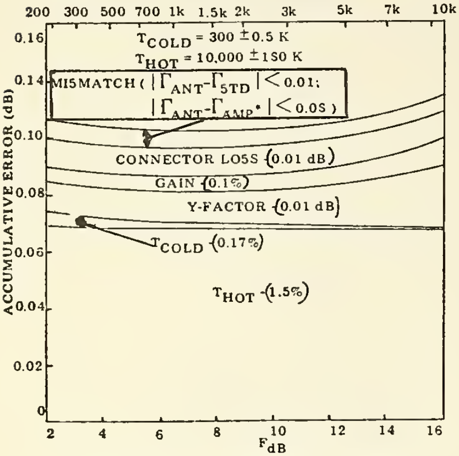

Fig. 2 Physical parameter contributions to noise-figure measurement limit of error.

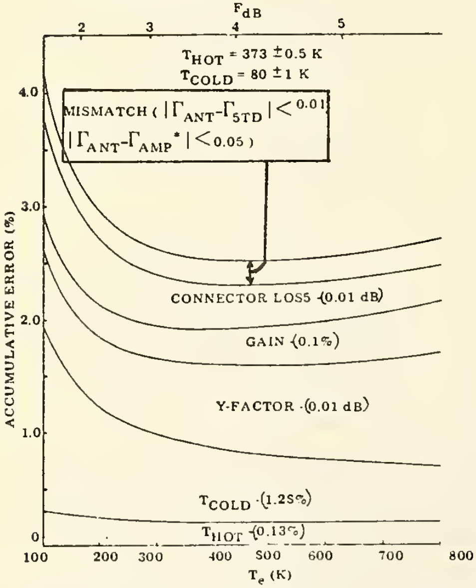

Fig. 3 Physical parameter contributions to effective-input noise-temperature measurement limit of error.

In setting an accuracy goal for the measurement of amplifier noise, it is useful to keep in mind the state of the art, the magnitude of mismatch uncertainty that can be expected, and the quality of available connectors.

The State of the Art In establishing an accuracy goal, one of the best guides is an awareness of how closely one wishes to approach the state-of-the-art.

Figures 2 and 3 show the accumulated errors for state-of-the-art measurements of F_{dB} and T_e, using two frequently employed hot-cold standard combinations; namely, 10,000 K and 300 K, or 373 K and 80 K. With one of these combinations, measurement uncertainties of about 0.1 dB in noise figure or effective temperature uncertainties of about 2.5% are state of the art.

Mismatch Uncertainty The effective noise temperature T_e is a function of the reflection coefficient (i.e., impedance) of the "antenna" (components attached to the input of the amplifier). If the reflection coefficient of this "antenna" is not known exactly, then the T_e that the amplifier will have when in use cannot be known exactly. This uncertainty in effective input noise temperature is referred to here as the *mismatch uncertainty*. One should be hesitant to set an accuracy goal better than this mismatch uncertainty.

TABLE 2

(PRELIMINARY) CONNECTOR ABSORPTION LOSS IN dB
AND
ABSORPTION COEFFICIENT A IN PERCENT

There is very little evaluated connector loss literature. The values in this table should be treated as preliminary data suggesting the general magnitude of losses to be expected.

TYPE	FREQ. (GHz)	LOSS (dB)	A (%)
1. COMMERCIAL TYPE N[a,b]	2 4	0.01, < 0.1	0.23, < 2.3
2. PRECISION TYPE N[b]	0.1	0.01	0.23
	0.2	0.02	0.46
	0.4	0.04	0.9
	1.0	0.04	0.9
	2.0	0.08	1.8
	4.0	0.11	2.6
	8.0	0.20	4.6
3. COMMERCIAL WR90[c]	8-12	< 0.01	< 0.23
COMMERCIAL WR90[d]	9.4	0.003	0.07
NBS TYPE WR90[e]			
(dirty)	9	0.003	0.07
(lapped)	9	0.001	0.023
CMR WR90[e]			
(lapped or not)	9	0.0004	0.009
4. SPECIAL CPR WR112[a]		0.0002	0.0046
5. SPECIAL WR430[a]	2.3	~ 0.00001	0.00023
6. PRECISION COAX[f]			
14 mm	0.01-8.5	$0.003\sqrt{f}$*	$0.07\sqrt{f}$
7 mm	0.01-17	$0.007\sqrt{f}$	$0.16\sqrt{f}$
7. COMMERCIAL WR15[g]	60	0.007 to 0.045	0.16 to 1.04

* in GHz

a. Charles T. Stelzreid (JPL) private communication.

b. Robert T. Adair (NBS) private communication

c. R. W. Beatty, Proc. IEEE, Vol. 53, pp. 642 643, 1965.

d. R. W. Beatty, G. F. Engen, and W. J. Anson, IRE Trans. on Instrumentation, Vol. I-9, pp. 219 226, 1960.

e. W. E. Little (NBS) private communication.

f. IEEE Standard for Precision Coaxial Connectors, IEEE Trans. on Instr. and Meas., Vol. IM-17, pp. 204 222, 1968

g. B. Yates (NBS), Private communication.

In Figure 4, the maximum mismatch uncertainty is plotted for an ideal amplifier as described by Engen[2,3]. It is clear from Figure 4 that the mismatch uncertainty becomes very important if the magnitude of the difference between the "antenna's" and standard's reflection coefficient is expected to differ by more than 0.01.

Connector Quality Table 2 is a list of connector loss measurement results. The change in F_{dB} due to the connector loss is equal to the decibel loss of the connector when it is at the temperature 290 K. Experience in waveguide has shown that a great deal of care is required to duplicate these values. A carelessly marred flange, or a poorly mating pair can easily be worse by a factor of ten. It is good practice to remeasure the noise figure after a break and a remake of some of the connectors. Bad connectors can sometimes be spotted in this way.

Measurement Technique

In principle the measurement of T_e is simple. The hot standard is connected, then the cold standard is connected and Y, the ratio of output powers, is measured. The details of changing from hot to cold (e.g., by changing current through a solid-state noise source), of measuring Y, and displaying the results depend on the equipment used. The manufacturer's literature and the prior literature[23-33] are very helpful but often the information needed to establish error limits is difficult to obtain, especially for the automatic noise-figure meters. Often the straightforward (connect a hot standard and measure the output power, then connect a cold standard and measure the output power) technique is best. One precaution however: it has been found[34] at NBS that, for 1-dB gain compression in the power measuring system, the power level needs to be about a factor of 10 greater than the mean noise power if the error from clipping the noise is to be less than 0.1%. This is also true for the commercial noise-figure measuring systems. Systems operating with too much gain can produce serious errors[35-37]

The most insidious measurement hazard is "mismatch" error,[23,38-40]. The problem of using standards having a different reflection coefficient from the "antenna" has already been discussed as mismatch uncertainty. The additional problem of the hot-standards and the cold standards having different reflection coefficients from each other is the mismatch error displayed in Figure 2 and 3. Here the two standards reflection coefficients have arbitrary amplitude and phase within the limits indicated. When the reflection coefficients are different many ugly things can happen as these standards are exchanged for each other: changes in gain, in gain stability, in bandwidth, and in linearity. In Figure 2 and 3 these ugly possibilities were ignored, and only the changes for the best possible ideal amplifier described by Engen[23] were considered. It is safer to estimate mismatch error from the measurements suggested below.

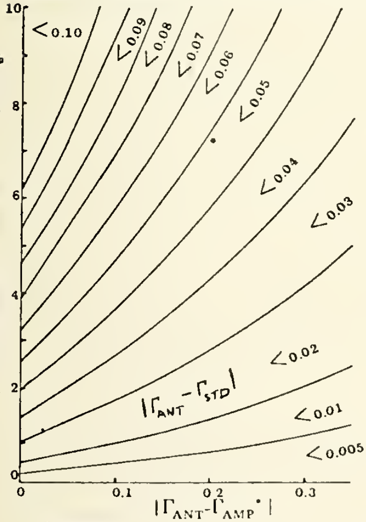

Fig. 4 Maximum mismatch uncertainty — "antenna" reflection coefficients vs amplifier input reflection complex conjugate.

The change of T_e with a change of reflection coefficient can be measured by inserting an isolator and a slide-screw tuner (or the appropriate equivalent) between the noise standards and the amplifier as shown in Figure 5. From this variation of T_e the mismatch uncertainty can be estimated. This measurement is not exact because the losses change in the tuner-isolator combination as the reflection coefficient changes, but this loss change is usually small. For example, for commercial X-band slide-screw tuners, measured changes in absorption loss are less than 0.015 dB as reflection coefficient is changed from zero to 0.5 (any phase). Because of the definition of the noise figure (Equation 2), a 0.015-dB increase in absorption loss (at 290 K) increases the noise figure by 0.015 dB.

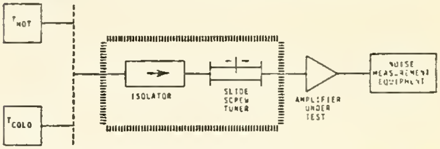

Fig. 5 Block diagram for measurement of mismatch uncertainty as related to reflection coefficient.

One estimate of mismatch error is obtained by measuring T_e with and without the isolator-tuner shown in Figure 5. If the isolator-tuner is adjusted to the "antenna" impedance, then the change in noise figure should be equal to the absorption loss of the isolator-tuner. If this absorption loss can be estimated (insertion loss for low-reflection elements is nearly equal to the absorption loss), then any serious mismatch errors will be spotted. A better method, if it can be implemented, is to attach a slide-screw tuner to the hot standard only, and to observe the change in T_e with the change of reflection coefficient. This procedure is then repeated with the slide-screw tuner attached to the cold standard only.

Estimating Measurement Error

Figures 2 and 3 are useful for estimating measurement errors because the error contributions to F_{dB} or T_e are essentially in direct proportion to the magnitudes of the respective error source parameters. Therefore, if Y is measured to within ± 0.02 dB, the Y-factor contribution error bands in Figures 2 and 3 double. However, this proportionality is not a good approximation of the mismatch-error estimate, but direct measurement of T_e versus Γ_{std} should be used for estimates of the mismatch errors. Thus, Figures 2 and 3 can be used to translate from uncertainties in the various measurement parameters into the error induced in T_e.

The cascade error is not shown in Figures 2 or 3. The desired T_e can be deduced[28,41] from the measured T_{total} when the effective input noise temperature of the measuring system is T_{e2} from

$$T_e = T_{total} - T_{e2}/G_1 ,$$

where G_1 is the available power gain of the amplifier being measured. The cascade error depends on how accurately T_{e2} and G_1 are known.

CONCLUSIONS

An accurate measurement of amplifier noise requires that particular attention be paid to the mismatch error. This error depends on several amplifier characteristics and their changes with the different terminations on the amplifier. It is therefore important to make measurements of the the change in amplifier noise with a change in terminating reflection coefficient, in order to estimate this mismatch error.

From a discussion of the state-of-the-art measurements for hot-cold standards of 10,000 K - 300 K or 373 K - 80 K, it is seen that measurements of noise figure to within about 0.1 dB, or effective input noise temperature to within about 2.5%, is the best that can be done. Such small uncertainties are possible only at those frequencies at which NBS calibration of noise sources exist.

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A New Method of Characterizing Amplifier Noise Performance

GLENN F. ENGEN

Abstract—Although the use of noise figure or noise temperature to characterize amplifier performance is a well-established practice, it is also recognized that this parameter provides only a partial description of the amplifier noise properties. In general, the noise figure (or temperature) depends upon the generator impedance and is thus a function of the signal-source and amplifier combination.

Typically, the noise figure is measured by the Y-factor method using hot and cold noise sources that are nominally matched (reflectionless). The result of this measurement is of value as a figure of merit; however, if optimum performance is to be realized, the applications engineer must know whether to adjust the signal source impedance for maximum power transfer, minimum noise figure, or some other criterion, and he must know the deterioration in performance that results if this is not done. It is the purpose of this paper to present an alternative method of characterizing amplifier noise performance in terms of parameters that provide ready answers to these questions. In addition, the measurement of these parameters via a simple extension of the Y-factor method will be described.

I. Introduction

ALTHOUGH the use of noise figure or noise temperature to describe amplifier performance is a well-established practice, it is also recognized that this parameter provides only a partial description of the amplifier noise properties. More recently, the use of *noise measure* [1] has been proposed, but again, this provides only a partial description. As usually defined [2], the noise figure depends upon the generator impedance and thus is a function of the signal-source and amplifier combination.

If optimum use is to be made of a given amplifier, its performance characteristics must be known in sufficient detail to predict its operation in arbitrary systems, and the options available in attempting to optimize the overall system performance. It is quite possible, for example, for two amplifiers to have the same noise figures and yet have markedly different sensitivities to changes in source impedance. In order to account for this and other phenomena, it is necessary to begin with a "complete" description. The remaining problem is one of putting this description in the most convenient form.

This is certainly not to suggest that this point of view is new, and indeed a number of useful descriptions have been given (for example [3], [4]). It is the purpose of this paper to put this description in an alternative form that provides additional insights for certain problem areas.

II. General Theory

Undoubtedly it would be possible to obtain the material that follows as an extension or reinterpretation of a number of earlier results. It will prove more instructive, however, to give its derivation from an elementary model; this provides the additional benefit that the presentation is "self-contained."

The model chosen for the amplifier is a two-port that is linear and active (both in that it provides gain and includes noise sources). This device, shown in Fig. 1, is conveniently described by the equations

$$b_1 = S_{11}a_1 + S_{12}a_2 + b_{1n} \tag{1}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 + b_{2n} \tag{2}$$

where b_1, b_2 are the emergent wave amplitudes at ports 1 and 2, respectively, and a_1, a_2 are the corresponding incident waves. The S_{mn} are the scattering coefficients of the two-port. Finally, the internal noise sources are represented by the terms b_{1n}, b_{2n}, such that the power spectral density (per cycle of bandwidth) delivered to passive and matched (reflectionless) terminations on ports 1 and 2 is given by $|b_{1n}|^2$ and $|b_{2n}|^2$, respectively.

It will prove convenient to express b_{2n} as the sum of two components: the first αb_{1n} is completely correlated with b_{1n}, the second b_{2o} is uncorrelated. Thus,

$$b_{2n} = \alpha b_{1n} + b_{2o}. \tag{3}$$

In keeping with the usual design objectives, it will be assumed that $|S_{12}|$ is negligibly small. In addition, it is also convenient[1] for the present discussion to assume $S_{22} = 0$.

The boundary condition imposed on b_1 and a_1 by the generator is conveniently expressed in the form:

$$a_1 = b_g + b_1\Gamma_g. \tag{4}$$

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[1] If $S_{12} = 0$, the term S_{22} is only of interest in attempting to optimize the coupling between the amplifier and the system that follows it.

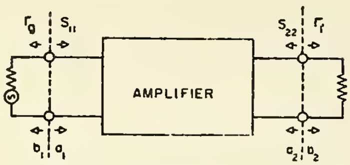

Fig. 1. Basic circuit for discussion of amplifier noise.

Here, b_g is the source term and Γ_g is the generator reflection coefficient. In order to examine the dependence of the amplifier noise upon the source impedance, it is convenient initially to assume that $b_g = 0$.

These equations and assumed conditions may now be combined to yield the following expression for the noise power delivered to a (matched) load at the amplifier output terminals:

$$|b_2|^2 = \left|\alpha + \frac{S_{21}\Gamma_g}{1 - S_{11}\Gamma_g}\right|^2 |b_{1n}|^2 + |b_{2o}|^2. \tag{5}$$

This result explicitly displays the dependence upon source impedance Γ_g and this dependence is a rather complicated one.

A substantial simplification in this expression may be effected by introducing "terminal invariant" parameters. (For an earlier discussion see [5].) The desired transformation begins with the substitution:[2]

$$\Gamma_g' = (\Gamma_g - S_{11}^*)/(1 - S_{11}\Gamma_g) \tag{6}$$

so that (5) can be written[3]

$$T_o = |b_2|^2 = T_aG(1 + b\,|\Gamma_g' - \beta|^2) \tag{7}$$

where T_o is the noise power output (to the matched load),

$$G = \frac{|S_{21}|^2}{1 - |S_{11}|^2} = \text{amplifier power gain,}$$

$$T_a = \frac{|b_{2o}|^2}{G} = \text{minimum amplifier noise temperature (as a function of source impedance) referred to amplifier input,}$$

$$b = \frac{1}{T_a}\left(\frac{|b_{1n}|^2}{1 - |S_{11}|^2}\right) = \text{ratio of the noise temperature of the input port}^4 \text{ to } T_a,$$

$$\beta = -S_{11}^* - \frac{\alpha(1 - |S_{11}|^2)}{S_{21}}.$$

The physical significance of β may be interpreted as expressing to what extent the conditions for maximum power transfer and minimum amplifier noise differ.

In (7), the parameters T_a, G, b, and $|\beta|$ are dependent on the amplifier properties only, and moreover their values are "terminal invariant" (i.e., their values do not change if a "lossless" tuner is added to the amplifier input). The entire dependence upon the generator impedance is found in the term Γ_g'.

III. Evaluation of Parameters

The terms T_a, b, and $|\beta|$ may be evaluated by an extension of the Y-factor method.

Let a temperature T_g be assigned to the generator of Fig. 1. Then (7) becomes[5]

$$T_o = T_gG(1 - |\Gamma_g'|^2) + T_aG(1 + b\,|\Gamma_g' - \beta|^2). \tag{8}$$

As in the Y-factor method, the measurement procedure calls for the use of "hot" and "cold" loads (noise sources). These will be designated T_{gh}, T_{gc}, respectively, and T_{oh}, T_{oc} the corresponding outputs when the noise sources are connected to the amplifier input. As a further condition it is assumed that the impedances of these loads is such that $\Gamma_g' = 0$ (see the Appendix). After the observation of T_{oh} and T_{oc}, a sliding short is connected to the amplifier input and the maximum and minimum values of T_o observed in response to motion of the short. These will de designated T_{oM} and T_{om}, respectively.

These four observations provide a set of simultaneous equations, which can be solved to yield

$$|\beta| = \frac{(T_{oM} - T_{om})(T_{gh} - T_{gc})}{2\{(T_{oM} + T_{om} - 2T_{oc})(T_{gh} - T_{gc}) + 2T_{gc}(T_{oh} - T_{oc})\}} \tag{9}$$

$$T_ab = \frac{(T_{oM} - T_{om})(T_{gh} - T_{gc})}{4\,|\beta|\,(T_{oh} - T_{oc})} \tag{10}$$

$$T_a = \frac{T_{gh} - (T_{oh}/T_{oc})T_{gc}}{(T_{oh}/T_{oc}) - 1} - |\beta|^2\,T_ab. \tag{11}$$

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[2] This transformation has the following properties: 1) $\Gamma_g' = 0$ when the conditions for maximum power transfer are satisfied ($\Gamma_g = S_{11}^*$), 2) $|\Gamma_g'| = 1$ when $|\Gamma_g| = 1$, 3) the value of $|\Gamma_g'|$ is "invariant" to the choice of terminal surface between generator and amplifier, provided that this choice is limited to a lossless region. For a more complete discussion, see [6].

[3] Although the functional form of this equation is equivalent to that obtained by a number of earlier authors, it is important to recognize the following distinctions. 1) The emphasis is on noise temperatures instead of noise figures. 2) In addition to "characterizing the rapidity with which T_o increases above T_aG as Γ_g' departs from β," the term b is also the ratio between the noise temperature of the input port and T_a. 3) The complete dependence upon source impedance is contained in the single term Γ_g'. 4) The behavior with either power matching or noise matching is immediately evident from inspection.

[4] This is the available noise power (per cycle of bandwidth) from the input terminals expressed as a temperature.

[5] In obtaining this result it is helpful to note that

$$(1 - |\Gamma_g'|^2) = \frac{(1 - |\Gamma_g|^2)(1 - |S_{11}|^2)}{|1 - S_{11}\Gamma_g|^2}.$$

The last two equations can also be solved explicitly for T_a and b in terms of the observed quantities; however, the suggested computation follows the indicated steps. [Note that (10) requires the value of $|\beta|$, and (11) the value of both $|\beta|$ and $T_a b$.] Finally, b is determined by taking the ratio of (10) and (11).

It is of interest to note that the first term on the right in (11) is just the usual one for determining amplifier noise temperature via the Y-factor method. In addition it may be recognized that these equations call (implicitly) only for ratios between the different output temperatures (e.g., T_{oh}/T_{oc}, etc.). If the absolute value of these terms is measured, it is also possible to obtain the amplifier gain.

The only remaining quantity to complete the description is the argument of β. This may be obtained by computing the argument of Γ_g' that provides T_{om}. This argument may, in turn, be obtained from (6) provided that S_{11} and the corresponding argument of Γ_g are known.

IV. Power Matching Versus Noise Matching

Assuming that the applications engineer wishes to obtain the best overall performance, (i.e., the maximum signal-to-noise ratio at the amplifier output) it is necessary to know whether to adjust the generator impedance for maximum power transfer, minimum amplifier noise, or according to some other criterion, and the penalty that results if this is not done.[6] These topics will now be investigated.

Returning to Fig. 1, it will be assumed that the signal source is of variable impedance and that its available power includes a signal component S_g and a noise component kT_gB. The signal power S_o at the amplifier output is given by

$$S_o = S_g G(1 - |\Gamma_g'|^2) \tag{12}$$

while the noise power output N_o is given by

$$N_o = kT_gBG(1 - |\Gamma_g'|^2) + kT_aBG(1 + b\,|\Gamma_g' - \beta|^2). \tag{13}$$

The signal-to-noise ratio at the amplifier output is

$$\frac{S_o}{N_o} = \frac{S_g(1 - |\Gamma_g'|^2)}{kT_gB(1 - |\Gamma_g'|^2) + kT_aB(1 + b\,|\Gamma_g' - \beta|^2)}. \tag{14}$$

By inspection, for maximum signal-to-noise ratio, the argument of Γ_g' should equal that of β. The remaining problem is that of maximizing

$$(1 - |\Gamma_g'|^2)/[1 + b(|\Gamma_g'| - |\beta|)^2].$$

By use of the calculus, this maximum occurs for

$$|\Gamma_g'| = (1/D)[1 - \sqrt{1 - D^2}] \tag{15}$$

where

$$D = (2b\,|\beta|)/[1 + b(1 + |\beta|^2)]. \tag{16}$$

It can be shown that $D < 1$ and (15) can be expanded to yield

$$|\Gamma_g'| = \frac{b\,|\beta|}{1 + b(1 + |\beta|^2)} \cdot \left[1 + \frac{(b\,|\beta|)^2}{[1 + b(1 + |\beta|^2)]^2} + \cdots\right]. \tag{17}$$

These results can now be substituted back into (14) to obtain an expression for the optimum signal-to-noise ratio. Because of its complexity, this result will not be given. If $|\beta|$ is small, however, the following approximation is useful:

$$\left.\frac{S_o}{N_o}\right|_{\max} \simeq \frac{S_g}{kB[T_g + T_a(1 + b\,|\beta|^2/(1 + b))]}. \tag{18}$$

It is of interest to evaluate the potential improvement over matching on a maximum power transfer or minimum noise basis. The required expressions may be obtained, in turn, from (14) by letting $\Gamma_g' = 0$ and $\Gamma_g' = \beta$. If the subscripts p (for power) and n (for noise) are used to identify these two cases, a little manipulation leads to the following approximate expressions:

$$\frac{(S_o/N_o)|_p}{(S_o/N_o)|_{\max}} \simeq 1 - \left(\frac{T_a}{T_g + T_a}\right)\left(\frac{b^2\,|\beta|^2}{1 + b}\right) \tag{19}$$

and

$$\frac{(S_o/N_o)|_n}{(S_o/N_o)|_{\max}} \simeq 1 - \left(\frac{T_a}{T_g + T_a}\right)\left(\frac{|\beta|^2}{1 + b}\right). \tag{20}$$

It is worth noting that the potential improvement is small if $|\beta|$ is small, but increases rapidly as $|\beta|$ increases.

V. Experimental Results

In an effort to explore the method further, the measurement procedures described in Section III were applied to a number of different amplifiers.

The first item to be evaluated was an X-band tunnel-diode amplifier. In this case the measurement was straightforward and yielded the results: $|\beta| = 0.03$, $b = 0.35$, $T_a = 825$ K. It should be noted that if the circulator, which is required in such devices, is "ideal," this would lead to $|\beta| = 0$ and $bT_a =$ ambient temperature. The above results agree with the theoretical expectations.

The evaluation of an X-band crystal mixer gave $|\beta| = 0.13$, $b = 0.65$, $T_a = 496$ K. Again the results appear plausible enough; it may be noted that bT_a is only slightly above ambient while the value for T_a is

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[6] The point of view adopted here is that there are many situations in which the applications engineer might be willing to adjust the impedance matching but unwilling to tackle the more ambitious problem of also adding inverse feedback. For a discussion of this more general problem, see [1]; see also, J. S. Engberg, "Simultaneous input power match and noise optimization using feedback," M.S. thesis, Syracuse University, Syracuse, N. Y., July 1969.

half the usually quoted value since the measurement method utilized both the signal and image channels. In spite of this plausibility, however, these results are subject to some question in that a substantial shift in crystal bias was observed in response to the short motion. The source of this problem was due to a substantial coupling of local-oscillator power into the input arm, and when this was reflected back into the system, a shift in bias resulted. This, in turn, gives rise to some question as to whether or not the device was behaving in a linear manner.

An application of the method to a 30-MHz vacuum-tube amplifier gave $|\beta| = 0.22$, $b = 0.59$, $T_a = 161$ K. Initially these results were believed to be in error in that this implies an input-port temperature of 94 K. Some further effort was expended in attempting to refine the impedance matching, but this proved to be a difficult broad-band problem in that the bandwidth of the detection system was a significant fraction of the operating frequency. An indirect confirmation of this result was finally obtained by an evaluation of the input-port temperature using a narrow-bandwidth radiometer. This gave the value 120 K, which may be regarded as a "confirmation" even if the agreement is not very good. It is unfortunate that it has not, as yet, been possible to repeat the entire evaluation with this narrow-band radiometer.

Although these results represent only a cursory evaluation of the method, they do call attention to certain practical problems that must be considered in its application.

VI. Extension to More General Operating Conditions

In the preceding analysis, a number of simplifying approximations were introduced. These will now be examined in greater detail.

The theory can be easily generalized to account for arbitrary impedance conditions ($S_{22} \neq 0$, etc.) at the output port. In order to retain the terminal-invariant character of G, it is redefined as the ratio of the net power delivered (to the amplifier input) to the available power at the output port. (That is, the definition that follows (7) should contain the factor $(1 - |S_{22}|^2)^{-1}$). The right-hand side of (12) and (13) should be multiplied by $(1 - |\Gamma_l'|^2)$ where

$$|\Gamma_l'| = \left|\frac{\Gamma_l - S_{22}^*}{1 - S_{22}\Gamma_l}\right| \tag{21}$$

and Γ_l is the reflection coefficient of the terminating load. The same is true of (7), provided that T_o is associated with the "delivered" noise power as contrasted with "available" noise power. If the second interpretation is used, no change in (7) is required.

With regard to the $S_{12} = 0$ postulate, it can be shown that the functional dependence of output power upon Γ_g' is still in the form of (7), even when $S_{12} \neq 0$. There is, however, a dependence of the different parameters (including Γ_g') upon Γ_l.

Finally, it is possible to prescribe a more general method for evaluating T_a, b, and $|\beta|$ than that outlined in Section III. Returning to Fig. 1, it is convenient to assume for the generator a noise source of temperature T_{gs} whose impedance can be adjusted in such a way as to vary the argument of Γ_g' while keeping its magnitude constant. Inspection of (8) shows that the first term on the right will remain constant while the second will go through maxima and minima. An observation of this response can be substituted for the sliding-short operation described in Section III. The appropriate equations are

$$|\beta| = \frac{|\Gamma_g'|\,(T_{oM} - T_{om})(T_{gh} - T_{gc})}{2\{(T_{oM} + T_{om} - 2T_{oc})(T_{gh} - T_{gc}) + 2[T_{gc} - T_{gs}(1 - |\Gamma_g'|^2)](T_{oh} - T_{oc})\}} \tag{22}$$

$$T_a b = \frac{(T_{oM} - T_{om})(T_{gh} - T_{gc})}{4\,|\Gamma_g'|\,|\beta|\,(T_{oh} - T_{oc})} \tag{23}$$

$$T_a = \frac{T_{gh} - (T_{oh}/T_{oc})T_{gc}}{(T_{oh}/T_{oc}) - 1} - |\beta|^2\, T_a b. \tag{24}$$

By a proper choice of $|\Gamma_g'|$ it should be possible to avoid the nonlinear operating region noted earlier. On the other hand, the realization of the required noise source, for which the argument of Γ_g' is adjustable without changing its magnitude, is somewhat more of a problem. One possible solution is to add a tuning transformer to the amplifier input terminals and adjust for an impedance match. To the extent that it may be considered lossless, the presence of this transformer will not affect the values of T_a, b, and $|\beta|$. For the tuned amplifier, $S_{11} = 0$ and $\Gamma_g' = \Gamma_g$. An adjustable generator (at ambient temperature) is now conveniently provided by a matched attenuator followed by a sliding short.

At this point it is possible to consider the tuner either as part of the amplifier or part of the adjustable noise generator. The former interpretation is convenient when the tuner losses are negligible since the resulting impedance requirement on the "hot" and "cold" loads (required to complete the evaluation of T_a, b, $|\beta|$) is now that these are also matched. In the projected evaluation of cryogenic systems, however, these losses may not be negligible and further study is indicated.

VII. Summary

An alternative method of characterizing amplifier noise has been presented, which (assuming linearity) completely accounts for variations in source impedance and in which all the parameters have a simple physical meaning. With the exception of the argument of β, these parameters are "terminal invariant." In addition, methods for measuring these terms have been described and an analysis, leading to optimum source impedance, presented.

The preliminary experimental evaluation suggests that the methods should prove of value in system optimization, although confirmation of this awaits further work.

In particular it appears that the method may be of real value in the evaluation of parametric- and maser-type amplifiers, since here there is some reason to anticipate larger values of b and $|\beta|$ than those reported above. On the other hand, in the case of the parametric amplifier at least, this expectation is seriously qualified by recognition that these devices typically exhibit a substantial sensitivity to changes in source impedance, and the region of linear operation may be difficult to establish.

The possible use of a moving short alone for the evaluation of maser noise has been previously suggested. From (7) it appears that it is indeed possible thus to determine an upper limit to T_a, provided that G is known.

At the present, the most serious limitation to this description appears to be in the assumed amplifier linearity (in respect to changes in source impedance, not output versus input). However, if this method puts this aspect in better focus, even this may prove to be a contribution.

APPENDIX

The measurement procedure outlined in Section III calls for the adjustment of the hot and cold load impedances to equal the complex conjugate of the amplifier input reflection coefficient Γ_a. In principle, this can be effected by measuring this impedance (by use of a slotted line, for example) and then using the impedance meter to recognize when the hot and cold load impedances are the conjugate of this value. An alternative procedure follows.

Referring to Fig. 2 it is convenient to assume a system comprised of a directional coupler, tuning transformers (T_x, T_y), load isolators, generator, and detector as shown.

Scattering equations can be written for this system as follows:

$$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 \tag{25}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3. \tag{26}$$

The boundary conditions imposed by the different terminations are

$$a_1 = b_1\Gamma_d \tag{27}$$

$$a_2 = b_2\Gamma_l \tag{28}$$

$$a_3 = b_g + b_3\Gamma_g. \tag{29}$$

In the operation to follow, there is no occasion to change

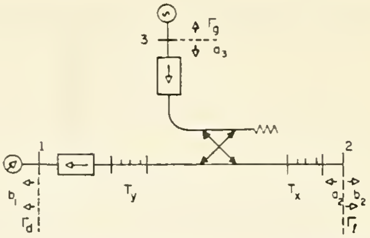

Fig. 2. Measurement system for recognizing conjugate impedance match.

either Γ_d or Γ_g; thus there is no loss in generality[7] in assuming $\Gamma_d = \Gamma_g = 0$.

Combining these results with (25) and (26) leads to

$$b_1 = b_g \frac{(S_{12}S_{23} - S_{13}S_{22})\Gamma_l + S_{13}}{1 - S_{22}\Gamma_l}. \tag{30}$$

Next it is assumed that T_x has been adjusted such that b_1 vanishes when port 2 is terminated by the load of interest ($\Gamma_l = \Gamma_a$), and T_y has been adjusted such that $|b_1|$ is constant in response to a moving short at port 2. The conditions imposed on the scattering parameters by these adjustments are, respectively,

$$S_{13} + (S_{12}S_{23} - S_{13}S_{22})\Gamma_a = 0 \tag{31}$$

and[8]

$$S_{13} + (S_{12}S_{23} - S_{1\underset{3}{2}}S_{22})S_{22}^* = 0. \tag{32}$$

By inspection,

$$S_{22} = \Gamma_a^*. \tag{33}$$

Equation (26) may now be written

$$b_2 = S_{23}b_g + \Gamma_a^* a_2 \tag{34}$$

or, stated in words, the equivalent generator that obtains at port 2 has the source reflection coefficient Γ_a^*. To obtain a termination with this reflection, it is only necessary to replace the generator on arm 3 by a passive load of the same impedance. The purpose of the isolator on arm 3 is to minimize the requirement that this impedance be the same as for the generator and to avoid possible pulling effects with the short motion. The isolator on arm 1 avoids the requirement of a constant detector impedance with changes in power level.

Finally, the adjustment of other impedances (e.g., the hot and cold loads) to equal this value may be effected by any of the procedures for recognizing the equality of two impedances.

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[7] For a discussion of this point, see [7].

[8] See, for example, [6].

ACKNOWLEDGMENT

The author expresses his appreciation to O. L. Patty and M. G. Arthur for providing the experimental results described.

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Reference [8]

Thermal Noise from a Passive Linear Multiport

DAVID F. WAIT

Abstract—**The thermal noise from passive multiports is discussed from fundamentals so that it can be understood, measured, or calculated by a microwave engineer. The multiports are assumed to have a uniform temperature, but with no restrictions on reciprocity or mismatch. The noise temperature, T_N, contributed by such a multiport is $T_N = A\,T$ where T is its physical temperature and A is its absorption coefficient. An approximate method of measuring A, and a method of measuring an A as small as 0.008 within 5 percent, are pointed out. Also, exact and approximate expressions for A in terms of scattering matrix elements and termination reflection coefficients are derived. Finally, the crosscorrelation of the noise from different ports is briefly considered.**

I. Introduction

MORE THAN 40 percent of the noise in low-noise communication systems and 95 percent of the noise in low-noise radio astronomy receivers is due to "black body" radiation from passive linear elements. Likewise, this type of noise accounts for most of the noise in a Blum radiometer [1], [2] used at the National Bureau of Standards to compare cryogenic noise sources [3].

The literature on the subject of noise is very extensive. Some of it applies to the problem of describing the noise from multiports like those used in modern low-noise communications and measuring systems [4]–[7]. But if one wants to determine how much of the noise power a three-port switchable circulator,[1] or a four-port hybrid tee, or other multiport element contributes to the system noise, the literature is sometimes inconvenient to use, especially if the elements used to terminate the multiport are mismatched. This inconvenience may be due to unfamiliar mathematical language, but more likely, it is because a conversion is required to reexpress the results in more useful terms. What is useful depends on the details of the system involved. Thus, a laboratory measurement of the scattering matrix parameters might prove helpful in estimating the noise contributed from a circulator, but relatively useless in estimating the noise contribution from a hybrid tee.

This paper will discuss the thermal noise from passive linear multiports in various terms that might be useful to a microwave engineer. The results are derived from fundamentals to make them more understandable. An exception is the discussion of the correlation of noise from different ports where the results can be obtained from the literature with little difficulty. Since a tractable extension to mismatched multiports is not known, this is not belabored.

In Section II, junctions with uniform physical temperature are considered. For them, the noise contribution is directly proportional to their physical temperature. The constant of proportionality is referred to as the absorption coefficient which may either be measured (Section III) or calculated (Section IV). The calculated form is stated in terms of scattering matrix elements and reflection coefficients of the port terminations. In Section V, the correlation of noise from different ports is discussed. Multiports with almost reflectionless terminations are treated in Section VI. In Section VII, the absorption coefficient is related to loss concepts known for two-ports. Conclusions are given in Section VIII.

II. The Noise Temperature of a Passive Multiport

Neglecting quantum effects, the power P available in each waveguide mode from a thermal source at a uniform temperature T is $P = k\,TB$, where k is Boltzman's constant and B is the bandwidth. Thus, the available power per unit bandwidth (available power spectral density) may be expressed as a temperature, and conversely, a temperature can be used to express an available power spectral density. In a somewhat similar way the available noise from a "passive" multiport is described. Specifically, if all ports of a multiport are terminated with loads at the absolute zero of temperature, while the multiport itself is at a different temperature, then the radiation emerging from any port is due only to thermal sources within the multiport. The *available* noise power spectral density[2] at, say, port i expressed as a temperature is known as the noise temperature of the multiport at port i. Thus defined, the value of the noise temperature depends on which port is being considered and also upon the reflection coefficients of the other port terminations, i.e., the reflection coefficient "looking" out of the multiport.

A multiport with independent port generators is shown in Fig. 1. Because the sources are independent (thus uncorrelated) they each make an additive contribution to the total available output of port i, so

$$T_{\text{total},\,i} = \sum_{j \neq i} \alpha_{ij} T_j + T_{N,i} \tag{1}$$

where T_j is the available power spectral density of the jth generator at its own output port and $T_{N,i}$ is the remaining contribution due to the thermal sources within the multiport itself. The proportionality factor α_{ij} is the ratio of the power spectral density available out of port i to the power spectral density available from the generator on port j when only the

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[1] The effect of the time dependence of the noise on system sensitivity is treated by [8].

[2] The available noise power spectral density is defined as the maximum noise power spectral density that could be delivered to a *load*. The circuit will deliver the maximum power when the load has the proper reflection coefficient. No manipulation of the reflection coefficient of any of the port generators is implied.

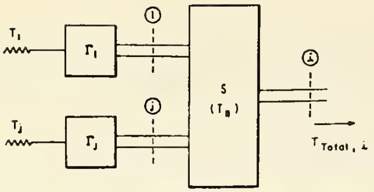

Fig. 1. A multiport with independent thermal noise generators.

generator j has nonzero amplitude. The proportionality factors α_{ij} depend on the reflection coefficients of the other port terminations. It should be intuitively obvious that α_{ij} is also equal to the ratio of available power out of port i to the available power from the generator on port j when a CW source is used on port j.[3] Thus α_{ij} will be referred to as the available power ratio. The noise temperature of the multiport $T_{N,i}$ is a characteristic of the multiport and the reflection coefficients of its terminations but independent of the amplitudes of the port generators.

To determine $T_{N,i}$, consider the multiport when it is in thermal equilibrium with all its port generators at temperature T. Then from the second law of thermodynamics, $T_{\text{total},i}$ as well as all the T_n's must equal T. Thus

$$T_{N,i} = A_i T \tag{2}$$

where

$$A_i \equiv \left\{1 - \sum_{j \neq i} \alpha_{ij}\right\}. \tag{3}$$

The constant A_i is here called the absorption coefficient for port i of the multiport. For a two-port, (1) reduces to the form given by Daywitt [10]: $T_{\text{total}} = \alpha T_S + (1-\alpha)T$, where α is the ratio of available power at the output to the available power at the input, and T_S is the temperature of the source.

III. Measurement of the Absorption Coefficient

The losses within a junction can change the power available at the output port, and the amount of the change can be used to determine the absorption loss. For example, if the junction is lossless and each port of the junction is terminated by a noise generator with the noise temperature T_g, then the output from the junction $T_{\text{total},i}$ would be T_g. But if the junction has a nonzero absorption coefficient A_i, then from Section II the difference between T_g and $T_{\text{total},i}$ is

$$T_g - T_{\text{total},i} = A_i(T_g - T) \tag{4}$$

where T is the physical temperature of the junction. Thus to measure A_i, one needs to adjust a set of noise generators[4] to have the same known noise temperature T_g, attach them to the ports of the junction to be measured, and then measure the available power spectral density [11], $T_{\text{total},i}$ from the output port. Because the absorption coefficient depends upon the reflection coefficients of the port terminations, the noise generators used in the measurements must have the appropriate reflection coefficients.

If the reflection coefficients are less than one tenth, then the small difference of $(T_g - T_{\text{total},i})$ can be measured within 2°K when T_g is 10 800°K [8] and $(T_g - T)$ can be known to 2.5 percent [12]. Thus A_i as small as 0.008 (a two-port loss of 0.03 dB) may be measured within 5 percent.

In case only an approximate measure of absorption coefficient is needed, a simpler measurement will suffice. For example, consider the problem of determining the approximate thermal contribution of a two-port placed between an antenna and a low-noise receiver as shown in Fig. 2. The two-port might be a directional coupler terminated with a gas tube which, when fired, adds a known amount of noise to the system for calibration purposes. When the tube is off, the terminated directional coupler may be treated as a two-port at uniform temperature.

For small A_i and $T_g \gg T$, (4) becomes

$$A_i \simeq 0.230 \Delta P_{\text{dB}}(1 + T/T_g) \tag{5}$$

where ΔP_{dB} is the relative change in available power expressed in decibels when the two-port is inserted into the system.

Ignoring the receiver noise, ΔP_{dB} can be determined by measuring the relative change in power when a noise source feeds the receiver directly (from reference plane 1 in Fig. 2) compared with when a source with the same noise temperature feeds the receiver through the two-port (from reference plane 2). The noise sources used must have the same reflection coefficients, amplitude, and phase as the circuit they replace[5] (Γ_1 and Γ_2, respectively) so that the same fraction of the available power is delivered to the receiver for both parts of the measurement.

The precision of measuring ΔP_{dB} depends on the precision of the readout of the receiver, on the stability and resolution of the receiver, on the noise temperature of the receiver, and on the relative accuracy of the noise sources. If a single noise source with adjustable reflection coefficient, like that shown in Fig. 3, is used for both parts of the measurement, the change in noise temperature due to the change in tuner losses is typically less than 0.15 percent at X-band for reflection coefficients in the range of zero to 0.3 [14]. The error due to neglecting the receiver noise is on the order of $(A_i\, T_{\text{rec}}/T_g)$, where T_{rec} is the receiver noise temperature.

---

[3] The rigorous mathematical proof that the value α_{ij} is the same for noise as the CW signals is beyond the scope of this paper. The foundations for such a proof are in Middleton [9], however, the route to the proof may be more obvious from the appendices of the paper by the author [8].

[4] An "internal" attenuator is the usual way to modify the noise temperature of a noise source.

[5] A simple procedure for adjusting the reflection coefficient of one circuit to be equal in amplitude and phase to another is described by Engen [13].

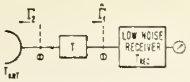

Fig. 2. A receiving system with a two-port between the antenna and the low-noise receiver.

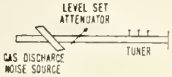

Fig. 3. An adjustable reflection coefficient noise source. If necessary, the noise temperature of the source is reduced by the level set attenuator so that the source does not saturate the receiving system.

IV. Calculation of the Absorption Coefficient

At times it is useful to know the relationship between the absorption coefficient and the parameters that describe the multiport and its terminations. To do this, the available power ratios α_{ij} are first expressed in terms of the scattering matrix elements of the multiport and the reflection coefficients of the port terminations, and then the absorption coefficient is obtained using (3).

The available power ratio will be calculated as the ratio of the CW power available at port i, $P_{b_i}{}^{\text{avail}}$, to the CW power available from the generator on port j, $P_{a_j}{}^{\text{avail}}$.

$$\alpha_{ij} = P_{b_i}{}^{\text{avail}}/P_{a_j}{}^{\text{avail}}. \tag{6}$$

The power available from port i expressed in terms of the equivalent "generator" wave[6] amplitude $\hat{b}_i$ is [16]

$$P_{b_i}{}^{\text{avail}} = |\hat{b}_i|^2/(1 - |\hat{\Gamma}_i|^2) \tag{7}$$

where $\hat{\Gamma}_i$ is the reflection coefficient "looking" into port i. Similarly, the power available from the generator in terms of its "generator" wave amplitude $\hat{a}_j$ is

$$P_{a_j}{}^{\text{avail}} = |\hat{a}_j|^2/(1 - |\Gamma_j|^2) \tag{8}$$

where Γ_j is the reflection coefficient of the generator. The relationship between $\hat{b}_i$ and $\hat{a}_j$ can be established by matrix techniques (see the Appendix). A compact and convenient expression of this relationship is

$$\hat{b}_i = \{D_{(iSj)}/D_{(ii)}\}\hat{a}_j \tag{9}$$

where $D_{(iSj)}$ is the determinant of the matrix $(I - S\Gamma)$ when its ith column is replaced by the jth column of scattering matrix S, Γ is the diagonal matrix with elements $(\Gamma)_{ii}$ equal to the reflection coefficient of the load on port i, and $D_{(ii)}$ is the determinant of $(I - S\Gamma)$ when its ith row and column are deleted. Thus for $i \neq j$

$$\alpha_{ij} = \frac{(1 - |\Gamma_j|^2)}{(1 - |\hat{\Gamma}_i|^2)} \frac{|D_{(iSj)}|^2}{|D_{(ii)}|^2} \tag{10}$$

so that

$$A_i = 1 - \sum_{k \neq i} \frac{(1 - |\Gamma_k|^2)|D_{(iSk)}|^2}{(1 - |\hat{\Gamma}_i|^2)|D_{(ii)}|^2}. \tag{11}$$

For example, for the three-port,[7]

$$D_{(iSj)} = \begin{vmatrix} S_{ij} & -S_{ik}\Gamma_k \\ S_{kj} & (1 - S_{kk}\Gamma_k) \end{vmatrix} \tag{12}$$

$$D_{(ii)} = \begin{vmatrix} (1 - S_{jj}\Gamma_j) & -S_{jk}\Gamma_k \\ -S_{kj}\Gamma_j & (1 - S_{kk}\Gamma_k) \end{vmatrix} \tag{13}$$

and

$$\hat{\Gamma}_i = [D_{(ii)}]^{-1} \begin{vmatrix} S_{ii} & -S_{ij}\Gamma_j & -S_{ik}\Gamma_k \\ S_{ji} & (1 - S_{jj}\Gamma_j) & -S_{jk}\Gamma_k \\ S_{ki} & -S_{kj}\Gamma_j & (1 - S_{kk}\Gamma_k) \end{vmatrix}. \tag{14}$$

V. Crosscorrelations for the Matched Multiport

In some applications it is interesting to know if the internal junction noise emitted from one port is crosscorrelated to the noise emitted at another port. A measure of crosscorrelation is the time average of the product of the time dependent voltages. However, the analysis of this paper is entirely in the frequency domain. In the frequency domain, the Fourier transform of the crosscorrelation is appropriate, namely, the cross-spectral density W_{ij} [17].

$$W_{ij} = \tfrac{1}{2}\{\langle n_i n_j^* \rangle_{\text{av}} + \langle n_i^* n_j \rangle_{\text{av}}\} \tag{15}$$

where $\langle\ \rangle_{\text{av}}$ denotes the ensemble average and the asterisk denotes the complex conjugate. If n_i and n_j are the voltages from port i and port j, respectively, of the generator waves due to the noise sources within a matched multiport, then [5]

$$\langle n_i n_j^* \rangle_{\text{av}} = N_{ij} T \tag{16}$$

where T is the physical temperature of the multiport and N_{ij} is an element of

$$N \equiv I - SS^\dagger \tag{17}$$

where the dagger indicates the Hermitian conjugate (transpose, complex conjugate). Bosma [5] refers to N as the noise distribution matrix. For the matched multiport, a "generator" wave is the same as the emergent wave.

The special case of $i = j$ may also be obtained from (2) and (11) by setting all Γ's equal to zero, that is

$$\langle n_i n_i^* \rangle_{\text{av}} = (1 - |\hat{\Gamma}_i|^2) T_{N,i} = \left\{1 - \sum_k S_{ik} S_{ik}^*\right\} T. \tag{18}$$

---

[6] The "generator" wave amplitude is defined as the amplitude the generator would deliver to a reflectionless load [15], [16]. In this paper, a basis has been chosen so that the characteristic impedance Z_{0i} associated with a port i will not be expressed explicitly. For those who wish an explicit statement, each voltage such as a_i should be replaced by $a_i/\sqrt{Z_{0i}}$, and each scattering matrix element S_{ij} should be replaced by $\sqrt{Z_{0i}}S_{ij}/\sqrt{Z_{0j}}$.

[7] When actually writing out $D_{(iSj)}$, one should take advantage of the property that $D_{(iSj)}$ is unchanged when the jth row and column of the determinant are deleted. This property is expressed by $D_{(iSj)} = D_{(iSj)(jj)}$.

An interesting special case is the two-port

$$\langle n_1 n_2^* \rangle_{\text{av}} = -(S_{11}S_{21}^* + S_{12}S_{22}^*)T. \tag{19}$$

Thus, if the two-port is matched to reflectionless loads so that S_{11} and S_{22} are zero, then there is no correlation[8] in the thermal noise emitted out the two ports.

VI. MULTIPORTS WITH LOW-REFLECTION PORT TERMINATIONS

In many situations the port terminations have small reflection coefficients. For this situation, the results of Section III simplify. In particular, neglecting the squares of the reflection coefficients [15],

$$D_{(iSj)}/D_{(ii)} \simeq S_{ij} + \sum_{m \neq i} S_{im}\Gamma_m S_{mj}. \tag{20}$$

If in addition $|\hat{\Gamma}_i|^2$ is negligible, then the absorption coefficient simplifies to

$$A_i \simeq A_i^0 + \epsilon_i \tag{21}$$

where A_i^0 is the absorption coefficient of the junction when all Γ's$=0$ and

$$\epsilon_i = \sum_{m \neq i} (S_{im}\Gamma_m N_{mi} + S_{im}^*\Gamma_m^* N_{mi}^*) \tag{22}$$

where N_{mi} are defined in (17). If the elements N_{mi} are not known, then it is of interest to determine an upper limit for the magnitude of the N_{ij}'s. From the definition of N_{ij} in (17) it is clear that

$$N_{ij} = N_{ji}^*. \tag{23}$$

Because $N \equiv (1 - SS^\dagger)$ is a positive semidefinite matrix for a passive multiport [5], [16], then [18]

$$|N_{ij}|^2 \leqq N_{ii}N_{jj}. \tag{24}$$

This in turn implies

$$N_{ij} + N_{ji} \leq N_{ii} + N_{jj}. \tag{25}$$

The diagonal elements of N are directly related to the absorption coefficients of the multiport, viz.,

$$N_{ii} = (1 - |S_{ii}|^2)A_i^0 \tag{26}$$

where A_i^0 is defined in (21). Thus, because $\sum_m |S_{im}|^2 \leq 1$ and all $|\Gamma_m|^2$ and A_i^0 are less than some maximums $|\Gamma_{\max}|$ and $A_{\max}^0$, then

$$|\epsilon_i| < 2(n-1)^{1/2} A_{\max}^0 |\Gamma_{\max}| \tag{27}$$

where n is the number of ports.

VII. THE ABSORPTION COEFFICIENT AS A LOSS

Many types of losses have been distinguished in the literature. Among those described by Beatty [19], the absorption coefficient is related most nearly to dissipative loss $L_D = -10 \log_{10} \eta$ (in decibels), where η is the efficiency. The absorption coefficient expressed in decibels, i.e., absorptive loss, would be $(A_{ji})_{\text{dB}} = -10 \log_{10} \alpha_{ij}$. Efficiency is a ratio of delivered powers in the same way that α_{ij} is the ratio of available powers. The efficiency and the available power ratio are related in the following way.

$$\alpha_{ij} = (M_j/M_i)\eta \tag{28}$$

where M_i and M_j are the mismatch factors of the ith and jth ports, respectively.[9] Thus, if both ports are conjugate matched [16], dissipative loss and absorptive loss are indistinguishable. Otherwise, dissipative loss is a loss that depends on the output load impedance, but independent of the input generator impedance, while absorptive loss is a loss that depends on the input generator impedance, but is independent of the output load impedance.

VIII. CONCLUSIONS

The noise from the thermal sources within a multiport element is equal to the absorption coefficient of the element, times the physical temperature of the element. The absorption coefficient may be measured or calculated. The calculated absorption coefficient is simply the sum of the available power ratios from all ports to the output port. In turn, the available power ratios are expressible in terms of scattering matrix elements and reflection coefficients. Only linearity was assumed: no restrictions on reciprocity, symmetry, loss, or condition of match are needed.

The correlation of the junction noise emitted from different ports when the junction has reflectionless terminations depends on the appropriate matrix elements of $N \equiv 1 - SS^\dagger$ and the junction temperature. When all ports of the junction have nearly reflectionless terminations, the expression for the absorption coefficient greatly simplifies, and a limit of error can be established if the port reflections are merely ignored.

APPENDIX

EQUIVALENT WAVE GENERATORS

The mathematics and the notation in this appendix have been influenced by Nemoto. Further details and interpretation of the concepts introduced here are discussed elsewhere [15].

A multiport is described by a scattering matrix that relates the emerging waves b_i to the incident waves a_i.

$$\mathbf{b} = \mathbf{S}\mathbf{a}. \tag{29}$$

The boundary conditions imposed by terminating each port with a load or a generator is described by

$$\mathbf{a} = \hat{\mathbf{a}} + \mathbf{\Gamma}\mathbf{b} \tag{30}$$

where $\hat{a}_i$ is a generator wave, that is, a wave a generator would deliver to a reflectionless load, and $\mathbf{\Gamma}$ is a diagonal matrix whose diagonal elements are the reflection coefficients

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[8] One should remember that zero correlation does not imply that the signals are unrelated. For example, for the related signals $n_1 = x + y$ and $n_2 = x - y$, where x and y are uncorrelated, $\langle n_1 n_2^* \rangle_{\text{av}} = \langle xx^* \rangle_{\text{av}} - \langle yy^* \rangle_{\text{av}}$ is zero when the power spectral density in x equals y. If the spectral densities of x and y are equal at all frequencies, then n_1 and n_2 are uncorrelated. This principle is used in the Blum radiometer [1], [2].

[9] Mismatch factor is used here in accordance with Miller *et al.* [10]. $M_i \equiv (1 - |\Gamma_i|^2)(1 - |\hat{\Gamma}_i|^2)/|1 - \Gamma_i\hat{\Gamma}_i|^2$, where Γ_i is the reflection coefficient "looking" out of the multiport and $\hat{\Gamma}_i$ is the reflection coefficient "looking" into the multiport at reference plane i.

looking out of the multiport. The generator wave $\hat{a}_i$ characterizes the generator on port i in the sense that its amplitude depends only on the output of the generator and that it is independent of the load that terminates the generator. Eliminating a between (29) and (30),

$$b = \mathcal{S}\hat{a} \tag{31}$$

where

$$\mathcal{S} \equiv (1 - S\Gamma)^{-1}S. \tag{32}$$

Because a matrix element of a product $C = A^{-1}B$, where A and B are square and have the same dimension, is $C_{ij} = \det A_{(iBj)}/\det A$, where $\det A_{(iBj)}$ is the determinant of A after the ith column of A has been replaced by the jth column of B;

$$\mathcal{S}_{ij} = D_{(iSj)}/D \tag{33}$$

where D stands for the det $(1 - S\Gamma)$.

In the same way that the inputs a_i were each divided into a generator wave plus a reflected wave, the outputs b_i can be divided into an equivalent generator wave plus a reflected wave.

$$b = \hat{b} + \hat{\Gamma}a \tag{34}$$

where $\hat{\Gamma}$ is a diagonal matrix whose elements are the reflection coefficients looking into the multiport.

Eliminating b from (31) using (30) and (34),

$$\hat{b} = G\hat{a} \tag{35}$$

where one form for G is

$$G = \{(1 - \Gamma\hat{\Gamma})\mathcal{S} - \hat{\Gamma}\}. \tag{36}$$

From the nature of the definition of $\hat{b}$ and $\hat{a}$, the diagonal elements of G must be zero. Setting the diagonal elements of (36) equal to zero and solving for $\hat{\Gamma}_i$ we obtain

$$\hat{\Gamma}_i = D_{(iSi)}/D_{(ii)} \tag{37}$$

where $D_{(ii)}$ is the determinant of $(1 - S\Gamma)$ after the ith row and ith column have been deleted.

Again looking at a diagonal element of (36) and solving for $(1 - \Gamma_i\hat{\Gamma}_i)$, replacing the remaining $\hat{\Gamma}_i$ by (37), we obtain

$$D = (1 - \Gamma_i\hat{\Gamma}_i)D_{(ii)}. \tag{38}$$

An off diagonal element of (36) is

$$G_{ij} = (1 - \Gamma_i\hat{\Gamma}_i)D_{(iSj)}/D. \tag{39}$$

Using (38) we obtain

$$G_{ij} = D_{(iSj)}/D_{(ii)}. \tag{40}$$

To obtain (9), $\hat{b}_i$ is obtained from (40) and (35) for the case when only the jth generator, and thus only the jth generator wave $\hat{a}_j$ is nonzero.

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