Critical Two-Phase Flow
for Cryogenic Fluids

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CRITICAL TWO-PHASE FLOW FOR CRYOGENIC FLUIDS

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This work presents a state-of-the-art survey intended to be useful
to a designer of equipment involving two-phase flow of cryogenic fluids. It
is desirable to assess the probability of critical, or choking, flow in such
a system and, if possible, estimate the critical flow rate.

The literature is surveyed, primarily since Smith (1963), and the
predictive results for several analytical models are evaluated and com-
pared with experimental data. These results are discussed; however,
no firm conclusions are reached because, often, the spread of experimental
data is equivalent to that of the predictive results from the models.

Finally, computer evaluations are presented for oxygen, hydrogen
and helium along with some design recommendations.

Key Words: Analyses; analytical models; choking flow; critical flow;
cryogenic fluids; helium; hydrogen; oxygen; reviews; two-phase flow.

1. INTRODUCTION

1.1 Definition and Comparisons with Single-Phase Critical Flow

Critical, or choking, flow may occur for any compressible fluid whether it is flowing
by itself or in a multi-phase mixture. It will occur when there are sufficient pressure differ-
ences in a conduit which is either of constant cross-sectional flow area or one in which the
cross-sectional area is converging in the direction of the flow. Thus, for example, in a
constant area or a converging section with a given upstream pressure, the rate of flow for
subcritical conditions will depend on the downstream pressure. For this case, the velocity
of the fluid at the exit will be increased as the downstream pressure is reduced. This velo-
city will continue to increase until the situation is reached where further reductions in the
downstream pressure will no longer increase the exit velocity or the mass rate of flow. At
this point, and for all lower downstream pressures, critical flow exists at the exit of the
section.

Modern technological developments demand compact units with higher efficiencies.
This in turn often results in designs which must consider higher and higher fluid velocities.
Thus, critical flow conditions are more probable in modern designs and critical two-phase
flow may occur in a wide variety of situations including expanders in refrigeration systems,
pumps for cryogenic equipment, venting systems in normal operation or in failure conditions
and many other applications discussed in somewhat more detail later. Further, critical
two-phase flow is a bench mark in the flow processes involving more than one phase similar
to the sonic velocity in the single-phase gas flow. The latter, of course, becomes the de-
nominator in the expression for the Mach Number.

The purpose of this study is to provide information on critical two-phase flow required by designers of systems involving cryogenic fluids. In many cases, the designer will need to know whether or not critical flow is at all likely in a specific component. This requires more than a cursory examination because, intuitively, he may feel that the critical two-phase flow velocity would be somewhere between the critical velocity for the gas and the critical velocity for the liquid. This concept is entirely misleading because the actual effective critical two-phase flow velocities are much lower than either of these two velocities and may be, at low qualities with a single component fluid, very low indeed if equilibrium conditions are approximately maintained during the pressure drop. For an investigation to determine whether or not critical flow is likely, detailed charts, graphs, and other computational aids for the specific cryogenic fluids involved have been prepared. A second case for design use may be where critical flow is deliberately intended or cannot be avoided. In this case, the best possible predictive models must be employed to indicate the condition of the fluid in the region of the critical or choking point. These needs will be served by presenting analytical models and analytical expressions which appear to be the most reliable in terms of the data which have been reported and, again, to present these data in the most usable form.

Critical two-phase flow has significant similarities to critical single-phase flow. A discussion of the similar and dissimilar analytical expressions and experimental observations follow:

1. For both single-phase and two-phase flow, there is no increase in mass flow with a reduction in the downstream pressure, that is,

$$\left(\frac{\partial G}{\partial p}\right) = 0 \ . \tag{1}$$

2. In both single- and two-phase critical flow, the condition may be thought of as the point where the potential energy supply rate is exceeded by the kinetic energy requirements as the pressure is further reduced, or mathematically, for the very simple case for gas flow with no heat or work energy flow, frictional forces or elevation change,

$$- \int v dp = \int d(u^2/2). \tag{2}$$

At critical flow the potential energy supply ratio, left side of (2), is just equal to the kinetic energy demands, right side of (2). For further pressure differences, the "demand" exceeds the "supply".

At the point of critical flow,

$$u^2 = (\partial p / \partial \rho)_{process} \ . \tag{3}$$

For reference, the sonic velocity expression is

$$a^2 = (\partial p / \partial \rho)_s \ . \tag{4}$$

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3.   For both single-phase and critical two-phase flow, the expression

$$(dp/dA)$$

undergoes a change in sign. An example of this is in a converging-
diverging venturi, where the flow in the diverging section is con-
tinuously accelerated into the supercritical or essentially supersonic
range.

4.   In general, for critical flow, there will be evidence of shocks in the
system for both the single-phase and the two-phase case. An excep-
tion would be an ideally designed venturi.

5.   For single-phase gas flow, pressure signals cannot be transmitted
upstream from a position downstream of the critical flow. For the
two-phase case, it is quite feasible that a pressure signal can be
transmitted upstream through an essentially continuous phase;
however, experimental evidence indicates that, in the critical
condition, signals are not effectively transmitted upstream — that
is, downstream pressure changes do not produce a pressure change
upstream.

6.   The critical pressure ratio for gas is quite predictable and for an
ideal gas follows

$$p_c/p_0 = \left(\frac{2}{\gamma+1} \right)^{\gamma/\gamma-1} \qquad \text{for isentropic flow,} \tag{5}$$

and

$$p_c/p_0 = e^{-1/2} = 0.607 \qquad \text{for isothermal flow.} \tag{6}$$

For the two-phase case, no such simple expression exists and the
critical pressure ratio is known to be a function of properties of the
mixture such as quality.

For reference, for an ideal gas, the processes of (5) and (6) produce
simple expressions in (3) and (4). That is:

$$(\partial p/\partial \rho)_s = \gamma RT \qquad \text{for isentropic flow,} \tag{7}$$

and

$$(\partial p/\partial \rho)_T = RT \qquad \text{for isothermal flow.} \tag{8}$$

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7. For single-phase gas flow, simple expressions exist for the sonic velocity and there is essentially an equivalence between the critical fluid velocity and the sonic velocity in the fluid.

For the two-phase case, some slower interface processes, such as mass transfer (vaporization or condensation), will not occur at a rate sufficient for the two-phase mixture to maintain equilibrium during the very rapid pressure changes near the point of critical flow. Therefore, calculations of critical flow rates and of sonic velocity must consider these process rates rather than equilibrium conditions, in turn influenced by the rate of pressure change which drives these interface processes. The rate of pressure change may not be the same for the critical flow and for the sonic velocity case.

In addition to the similarities between critical two-phase flow and critical gas flow, all critical flow phenomena have something in common with other critical phenomena such as critical fields, currents, thermodynamic properties, etc. This similarity is, for example, found through the mathematics in that the approach to the critical point is characterized by very rapid changes in the behavior, and there is essentially a discontinuity in the mathematics at the critical point.

For example, with single-phase, ideal flow, as in (2), the momentum expression is

$$- dp/\rho = u_g \ du_g \ .$$

Also, the conservation of mass is

$$dp/\rho + du/u + dA/A = 0.$$

Combining,

$$du_g \ = \ \frac{-dA \ u}{A[1 - u^2/(dp/d\rho)_{process}]} \ . \tag{9}$$

This simple example shows that at critical flow, the sonic velocity from (3) is

$$u^2 = (\partial p/\partial \rho)_{process}$$

and the denominator in (9) becomes zero.

1.2 Purpose of This Study

The specific purpose of this study is to review the proposed predictive analytical models and the reported experimental data by detailed examination and comparisons, and from these results provide design recommendations and computational aids. These recommendations and aids will be specifically concerned with the cryogenic fluids: helium, hydrogen and oxygen.

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1.3 Applications

This section will deal with industrial applications in which critical two-phase flow may be involved. The treatment will not be complete but should serve to indicate some examples in the broad range of conditions in which critical two-phase flow may occur. For the most general case, one would look at components or systems which use two-phase fluids in their operations. These would include steam power plants, refineries, and refrigeration and cryogenic systems. It is possible that in many existing systems, critical two-phase flow has occurred and gone undetected, often recorded as an unusually high pressure drop. This latter concept, however, will lead to the wrong treatment if the real case is critical two-phase flow. In this general category of systems, one should check for critical two-phase flow in any restriction in the system, at the exit of a constant flow area system through which there is a pressure drop approximately one-half the initial pressure, or where a substantially lower pressure drop occurs when there is a rather low quality of fluid flowing. As a general guide, figures 1.3-1 and 1.3-2 show computed critical pressures for H_2 and H_2O computed from the models of Moody (1965, 1969), Henry (1968, 1970) and Henry and Fauske (1971). Additionally, very significant two-phase flow problems would occur in the failure of two-phase or single-phase high pressure systems, for example, where a rupture occurs involving the discharge of a radioactive fluid or a fluid with a high potential for chemical reaction such as oxygen or hydrogen. For these systems, the venting or containment situation must be well designed to prevent further catastrophy as a result of such accidental ruptures. A very substantial amount of the critical two-phase flow work has been supported by the Atomic Energy Commission under the general heading of Reactor Safety — for example, the extensive works authored by Fauske, Henry, Moulton, Zaloudek, Levy and Moody. Ryley (1961, 1964) has indicated steam, geothermal wells may be in critical flow at some points. Critical two-phase flow has also been reported in food dehydration plants where very rapid evaporation is necessary to preserve the maximum nutritional benefits of the food; such flow conditions have occurred in conduits carrying the liquid and vapor away from the food undergoing dehydration.

As mentioned previously, critical two-phase flow may be the limiting condition for many cryogenic pumps in that they routinely pump a certain quantity of vapor; however, if the two-phase inlet mixture does undergo critical flow, that section of the pump will operate quite differently and probably much less efficiently than expected as indicated by Bissel et al. (1970). Interestingly enough, there have been suggestions for the design of cryogenic pumps where a critical two-phase flow shock is deliberately introduced in the inlet section to re-condense all of the vapor in the two-phase mixture so that the pump handles only liquid thereafter. Critical two-phase flow has also occurred in exploratory systems with steam turbines reported by Ryley and Parker (1968). They introduced slots to remove the liquid

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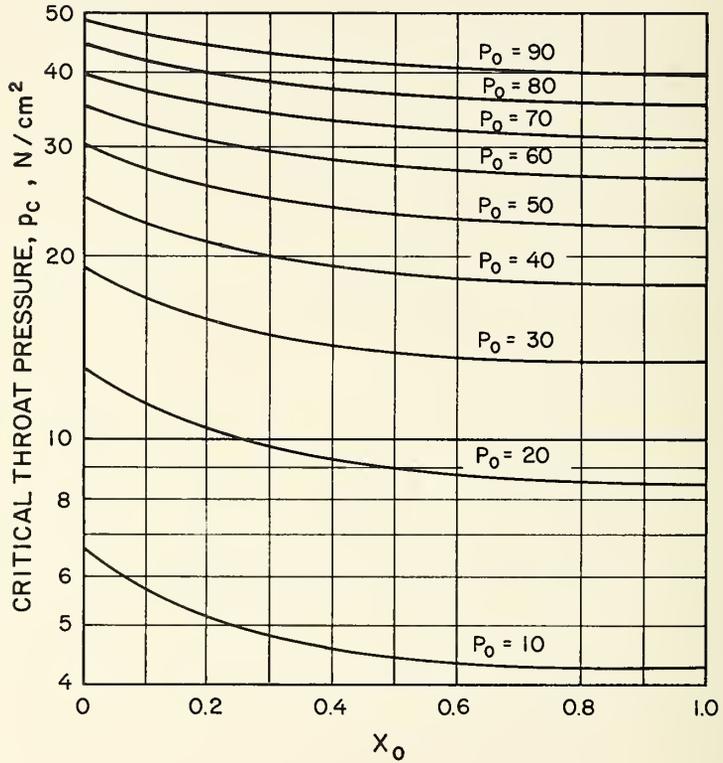
Figure 1.3-1   Critical throat pressure versus stagnation quality as predicted by Moody (1965, 1969) from Ward (1966). [Fluid: hydrogen].
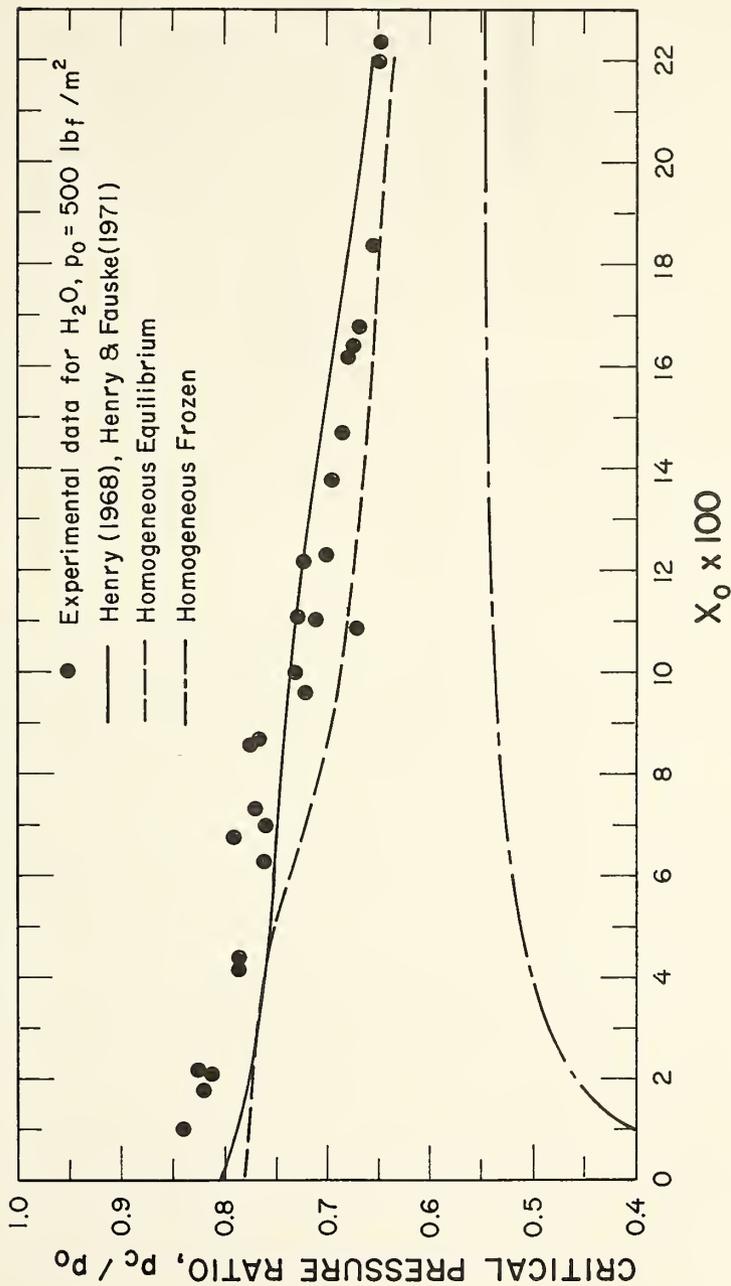
Figure 1.3-2 Predicted and experimental critical pressure ratios for water from Henry and Fauske (1971).

from the stationary turbine blades, thus reducing the erosion of metal which occurs when too much liquid is in the fluid flowing through the blades.

Another interest in critical two-phase flow involves the design of units where high velocity gas is used to accelerate a slower moving liquid. For example, many magneto-hydrodynamic schemes employ this concept and some jet pumps have used this principle for many years. Again, for compact, high velocity systems, critical flow is much more pro-bable than in the past. Warner and Netzer (1963) reported such a study. Critical two-phase flow may also occur in fuel systems. A simple example of this would be the carburetor of an internal combustion engine or in the fuel line of an aircraft. Another, and perhaps the most sophisticated example, would be in the case of a start-up of a spacecraft engine which utilizes a liquefied gas as a fuel or as a working fluid. If the fuel or liquefied gas is intro-duced into a relatively warm system, substantial vaporization will occur until the system has been cooled to approximately the liquefaction temperature. During such a start-up, critical two-phase flow is very likely to take place in one or more of the flow passages. Further, critical two-phase flow might easily occur in one of the heat exchangers as the liquefied gas undergoes the transition from liquid to vapor. This could happen in flow through the coolant passages for the rocket engine.

1.4 Review of Reported Experimental and Analytical Work

Since critical flow of the single, gas phase is reasonably well understood and pre-dictable, it seems reasonable to approach the problem from that vantage point. In such an approach, one would attempt to sort the differences or additional complications introduced by the presence of the second phase and incorporate these into the analytical model and the experimental procedure to be used. This would also hold true for the experimental part of the two-phase flow investigation. This indeed has been the general process followed by two-phase critical flow investigators. The first analytical model proposed was a very simple and natural one, that being to assume that the two-phase mixture could be described by effective or average properties and that the mixture was homogeneous and in thermal equilibrium. It was discovered rather early in the work that this homogeneous, thermal equilibrium model, in general, underpredicted the experimental critical mass flow rate. The predictions were not so far in error for high qualities and were very substantially in error for low quality flow situations, as shown by figure 2.4-1. These were previously documented by Faletti and Moulton (1963), Zaloudek (1961) and Smith (1963).

Experimental difficulties were encountered also. Perhaps the primary problem was at the critical condition where the quality is unknown and is usually calculated as though the fluid were in thermal equilibrium. Pressure measurements in the direction of flow for the two-phase mixture were also very difficult near the point of critical flow, and extrapolations from points just upstream were shown to be often subject to considerable uncertainty.

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Most of the subsequent analytical investigations have been associated with further examination of the two primary assumptions of the homogeneous, thermal equilibrium model. These assumptions were that of thermal equilibrium (which seems unlikely because of the time available) and that of homogeneity (since it is generally accepted that the gas phase is often traveling at a considerably higher velocity on the average than the liquid phase).

A second, very simple analytical model was introduced early in the studies, for example, Ryley (1952), Zaloudek (1961) and Smith (1963). This usually has the title of the vapor choking model. This model does not treat two-phase fluid as a mixture with effective properties but instead examines the behavior of each phase separately. Also, rather than assuming that critical flow occurs when the mixture is in critical flow as described by the mixture equations with effective properties, the vapor choking model assumes critical flow when the gas is in critical flow. This appears to be a reasonable model as long as the phases are essentially flowing separately. It was found that this model produced reasonably good results at high qualities and, by early treatments, very substantially overpredicted the mass critical flow rate at lower qualities (Smith, 1963). These treatments used slip (phase velocity) ratios from an extrapolation of the Martinelli and Nelson (1948) correlations for void fractions. The point of departure of reliable predictions from this model was at somewhat higher qualities than one would expect to find as the lower limit for separated flow.

Since the first two simple models failed to predict the experimental data for critical two-phase flow, even allowing some uncertainty in the experimental data because of the quality and pressure determinations mentioned earlier, historically and in retrospect it seemed proper to reexamine the entire program. In doing so, one can make a series of observations which follow:

1. For both approaches previously mentioned, whether one used single properties for the mixture or studied the gas and liquid phase separately, a major deviation from the single phase flow is introduced because there are additional degrees of freedom of the interface processes which involve momentum, energy, and mass transport. Further, estimates of these processes are fairly difficult with the current knowledge of two-phase flow and particularly difficult at or near the point of critical flow.

2. It seems difficult to know which basic kind of model to choose — that of the effective mixture properties or of the separated phase flow. Perhaps more realistically it should be assumed that one is proper in one case and the other is proper in another, but then the limitations of each usage are unknown.

3. Another very serious difficulty associated with (2) above is that the description of the flow pattern (liquid-gas distribution and flow velocities) in all two-phase flow cases is difficult, and in critical two-phase flow the information is essentially non-existent. It would appear little improvement has been made on the now-classical plots prepared by Baker (1954). If flow pattern information were known, analysis and modeling would be considerably easier and also experimental programs could be planned more intelligently and proper measurements made more accurately.

4. Extending the reasoning in (1), which had to do with the interface processes, the alternative path to assuming equilibrium would be to try to determine the rate of the interface processes or, put another way, to determine the relaxation times for these processes. If these relaxation times were known, comparison could be made with the time available during flow and an estimate could be given for the degree of thermal equilibrium in the fluid.

1.4.1 Experimental Data.

Almost all of the early experimental data were reported from steam-water and air-water systems (Smith, 1963). Except for comparisons of performance with analytical models, it is still difficult to assess the effect of employing different fluids. This study was planned specifically to investigate the behaviour and predictability of cryogenic fluid flow where properties are substantially different. These differences are shown in table 1. In the steam-water and air-water systems, it was soon established that critical two-phase flow did indeed occur in the systems by qualitative observations of shock and flow rate measurements which showed the mass-limiting condition. Shock observations were elaborately reported by Muir and Eichorn (1963) and Eddington (1967). All investigators found critical or mass limiting conditions except Marriot (1970) and Murdock (1962) for flow through orifices. For discussion of two-phase orifice flow, see Chisholm and Watson (1965). It is not surprising that there are not mass-limiting conditions for two-phase orifice flow, as single-phase gas orifice flow does not exhibit mass-limiting flow.

Following these investigations, a number of studies were made, primarily with straight tubes and nozzles, and better experimental data were obtained. There were some difficulties in determining the critical quality and pressures previously mentioned. At that time, an empirical correlation was suggested by Faletti and Moulton (1963) which plotted quality versus the ratio of the mass rate of flow per unit area measured at the critical point to that predicted by the homogeneous thermal equilibrium model, see figure 2.4-1. From

this plot, a reasonably good correlation was achieved. Further refinements in the experimental data have primarily been to improve the pressure measurements by a more exact measurement at or near the critical point where the pressure gradients are very, very steep. There have been some preliminary attempts to experimentally investigate the interface processes and flow patterns (Smith, 1968), (Henry, 1968), (Carofano and McManus, 1969).

The experimental data at that point did give some indications of the primary and the secondary variables which may be involved in critical two-phase flow. The success of the correlation of Faletti and Moulton (1963) indicated that quality certainly is a primary variable in the two-phase flow case. Secondly, it indicated that, while fluid properties were significant variables as the pressure and temperature were changed, these changes were reasonably well accounted for by use of the homogeneous thermal equilibrium model.

To investigate the effect of geometry, the tests were run on straight sections (Fauske, 1961, 1965), (Zaloudek, 1961, 1963), on nozzles (Silver and Mitchell, 1945), (Edmonds and Smith, 1965), on deLaval nozzles (Tangren et al., 1949), on venturies (Carofano and McManus, 1969), (Smith, 1968), and on very short sections (Silver and Mitchell, 1945), (Chisholm and Watson, 1965; Min et al., 1965). Except for the very short sections which experienced orifice-like flow and should be considered as a separate case, it appeared that the geometry was a secondary variable for the critical two-phase flow. In summary, properties, aside from quality, apparently are accounted for reasonably well by the homogeneous thermal equilibrium model. Quality (or perhaps void fraction) is a primary variable and geometry is probably a secondary variable.

Some experimental work has been reported regarding slip velocity, with investigations using radiation attenuation or momentum balances (Fauske, 1965), (Klingbiel and Moulton, 1971).

Although the very early experimental data and that subsequent to it did indicate the probable behaviour for some cases and, at least, the primary and secondary variables, experimental data have been markedly scarce and must be rated as largely unsuccessful in describing the more sophisticated aspects of critical two-phase flow, for example, in studies of the interface processes and their relaxation times. Similarly, very little information has been obtained from experimental systems regarding the flow pattern of critical two-phase flow. Even the division between effectively separated flow and effectively mixture flow has not been successfully studied experimentally.

1.4.2 Analytical Models

1.4.2.1 Mixture Models - The effective property primarily involved in any of the expressions is the density. Assessment of effective density to describe the two-phase mixture inevitably leads one to describe, in some way, the two major influences on that mixture

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density. These are the flow pattern (including the phase velocities) and state of equilibrium for the fluid. This again assumes that other property variations can be handled reasonably well by the simple approach of using averaged or effective property values for the mixture. One may be able to divide the various approaches involving these systems into the following:

1. Systems which assume that the density, pressure and temperature relationship for the two-phase mixture may be treated entirely as a single phase — that is, for example, establishing an isentropic density relationship similar to that for an ideal gas.

    For example, using mixture specific volumes,

    $$p_1/p_2 = (v_2/v_1)^\gamma$$

    for an isentropic process with thermal equilibrium. Similar treatments could be made assuming that the mixture was undergoing an isothermal process with thermal equilibrium or a polytropic process assuming thermal equilibrium. In the latter, the exponent would then be largely an empirical factor. This method was used by Hesson and Peck (1958). A difficulty of this treatment is the handling of the quality. One simple method is to assume that the fluid is in frozen flow which simply means that there will be no mass transport at the interfaces, or, in another way, no condensation or vaporization, see for example Starkman et al., (1963).

2. Another approach is similar to that of (1), but the interface processes are examined in somewhat greater detail and so are considerations for the flow pattern. Some of the models which followed the homogeneous thermal equilibrium model concentrated essentially on the flow pattern and specifically on the slip velocity between the vapor and the liquid; these essentially all assume thermal equilibrium for the mixture and attempt in various ways to estimate the slip velocity which would occur at critical two-phase flow. Analytically, using various assumptions, several investigators, Cruver and Moulton (1967), Moody (1965, 1969) and Zivi (1964), determined that the maximum slip velocity would be indicated by the following expression:

    $$(u_g/u_f) = (\rho_f/\rho_g)^{1/3} .$$

    One other investigator, Fauske (1961), proposed this relationship to be

    $$(u_g/u_f) = (\rho_f/\rho_g)^{1/2} .$$

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Both of the above treatments do reasonably well in predicting most of the experimental data for the flow at the critical point. Compensating errors are quite possible because thermal equilibrium (although unlikely) is assumed in the reduction of the experimental data, and in the analytical process.

Subsequent to this, several people began to investigate the validity of the assumption of thermal equilibrium (Smith, 1963, Cruver and Moulton, 1966, Fauske, 1965). Simple calculations, Smith (1963), for example, indicated insufficient time to achieve equilibrium, particularly that required for mass transfer for condensation or vaporization to occur. Thus it was quite unlikely that thermal equilibrium existed at the point of critical flow. Cruver and Moulton (1966) showed that if one assumed frozen flow as previously defined and used the experimental measurements for the slip velocity (Klingbiel and Moulton, 1971), (Fauske, 1965), rather than the higher slip velocity as indicated by the analytical procedures shown previously as a function of the density ratios, they, too, could match experimental data reasonably well.

The current state-of-the-art is primarily involved with approaches which try to describe these interface processes in specific detail and study their behaviour from some upstream point to the point of critical flow, for example, Henry, (1968), Henry and Fauske (1971) and Moody (1965, 1969). This has several advantages. For example, one can compare the entire experimental and analytically determined pressure profiles which gives considerably more confidence in the results than simply being able to match the flow at the critical point. The latter would be subject to more compensating errors. Progress in this direction is difficult, however, because so little information is available regarding the interface processes, and estimates of the process rates are arbitrary. Therefore, with so many things floating in this sense, it is difficult to know whether one set of assumptions is really superior to another.

1.4.2.2 Separated Phase Models - These, of course, are simply extensions of the previously mentioned vapor choking model but, more recently, with an attempt to describe the interface processes rather simply and with generalized assumptions. Early investigators, (Zaloudek, 1961), (Smith, 1963), used the Martinelli and Nelson (1948) slip ratio in the vapor choking expression. Results using these models have been good only at high qualities,

with rather radical overpredictions at lower quality flow. Thermal equilibrium was generally assumed in these very early models.

More recent investigators, Carofano and McManus (1969), Smith, Smith et al., (1968, 1972), have attempted to follow the flow from an upstream point and describe the interface processes in detail. Of course, analytical expressions for the processes must be largely based on estimates and, with so many variables, it again is difficult to determine which of several models really best describes the process involved. With reasonably high quality flow and for limited data with relatively low qualities (Smith, 1968, 1972), it certainly is true that the pressure profile to the critical point is well described by use of these models and by using estimates for the interface processes. This was not the case for some of the effective property mixture models where the pressure profile was better described by one model, for example, the homogeneous one, and the critical condition best described by the frozen flow model.

1.5 Summary

In summary of the section on analytical models and their comparison with experimental data, we conclude:

1. The descriptions of the interface processes and the flow pattern are the difficult points in any current approach.

2. Very little experimental data have been reported to help in the analytical expressions of these interface processes.

3. Some success has been achieved for both the effective mixture properties model and the separated model, both of which make estimates for these interface processes. This would indicate their accurate description may be a variable of secondary importance for many flow cases.

4. Both homogeneity and equilibrium are unlikely to be valid assumptions, yet their use has produced reasonably good results for predicting flow rates at the point of critical flow.

5. Although some success has been achieved by using assumptions discussed in (4) above, that approach has been extended about as far as is likely to be productive. It would seem that any future progress would have to be made specifically considering the interface processes from an upstream point to the point of critical flow.

6. Better experimental data are needed to determine information on flow pattern and the rate of interface processes near the point of critical flow.

7. It would seem reasonable to believe that there will be regions where the effective mixture properties model will be more successful than the separated flow model and vice versa. It would be highly desirable to determine the regions of effectiveness for both models.

8. Most of this discussion has been concerned with fluids other than cryogenic fluids because so little data exist for cryogenic fluids. It would appear that the cryogenic fluids, although having somewhat different property values, may be generally described by systems used for other fluids. The use of the quality in correlations for cryogenic fluids, where the ratio of the liquid to gas density is quite different from conventional fluids, may be a dangerous procedure, but there are no reported experimental data to show this danger. Table 1 indicates the differences in the liquid and gas density ratios between cryogenic and more conventional two-phase fluids.

Table 1 – Density Ratio and Other Properties at the Normal Boiling Point

Fluid	Normal Boil. Point Sat. Temperature	Heat of Vaporization	Density Liquid	Density Gas	Density Ratio	Molecular Weight
He	4.224 K	$82.3 \frac{J}{g\text{-mol}}$	$31.406 \frac{g\text{-mol}}{\text{liter}}$	$4.2245 \frac{g\text{-mol}}{\text{liter}}$	7.434	4.0026
	7.603°R	$8.842 \frac{Btu}{lb_s}$	$7.8478 \frac{lb_s}{ft^3}$	$1.0556 \frac{lb_s}{ft^3}$		
H_2	20.268 K	$898.3 \frac{J}{g\text{-mol}}$	$0.0351 \frac{g\text{-mol}}{cm^3}$	$6.636 \times 10^{-4} \frac{g\text{-mol}}{cm^3}$	52.91	2.01571
	36.482°R	$191.63 \frac{Btu}{lb_s}$	$4.418 \frac{lb_s}{ft^3}$	$0.0835 \frac{lb_s}{ft^3}$		
N_2	77.36 K	$5581.13 \frac{J}{g\text{-mol}}$	$0.02882 \frac{g\text{-mol}}{cm^3}$	$1.6434 \times 10^{-4} \frac{g\text{-mol}}{cm^3}$	175.5	28.016
	139.25°R	$85.668 \frac{Btu}{lb_s}$	$50.40 \frac{lb_s}{ft^3}$	$0.2874 \frac{lb_s}{ft^3}$		
O_2	90.180 K	$6812.1 \frac{J}{g\text{-mol}}$	$0.03566 \frac{g\text{-mol}}{cm^3}$	$1.399 \times 10^{-4} \frac{g\text{-mol}}{cm^3}$	254.94	31.9988
	162.324°R	$91.588 \frac{Btu}{lb_s}$	$71.23 \frac{lb_s}{ft^3}$	$0.2794 \frac{lb_s}{ft^3}$		
H_2O	373.15 K	$40649.2 \frac{1}{g\text{-mol}}$	$0.0532 \frac{g\text{-mol}}{cm^3}$	$3.317 \times 10^{-5} \frac{g\text{-mol}}{cm^3}$	1602.87	18.0156
	671.69°R	$970.3 \frac{Btu}{lb_s}$	$59.88 \frac{lb_s}{ft^3}$	$0.0373 \frac{lb_s}{ft^3}$		

2. EXPERIMENTAL DATA, ANALYTICAL DATA AND COMPARISONS

2.1 Experimental Systems

Table 2 shows experimental systems reported since Smith (1963). The table is restricted to those papers reporting well-defined critical flow data. Further references may be found in the section of the Bibliography dealing with experimental papers.

Table 2 - Table of Reported Experimental Data [Since Smith (1963)]

Author	Geometry	Range	Fluid
Bonnet (1967)	Orifice and Orifice with Baffle	$75 < p_0 < 170 \ lb_f/in^2$ $0.0 < x_0 < 0.20$	N_2
Brennan et al., (1968)	Tube L/D = 20	$14 < p_c < 80 \ lb_f/in^2$ $0.025 < x_c < 0.30$	N_2 H_2
Campbell and Overcamp (1966)	Square Edged Orifice Assume Orifice Coef = 0.6	$3.5 < p_0 < 17 \ lb_f/in^2$ $0.0 < x_0 < 1.0$	N_2
Carofano and McManus (1969)	Venturi	$20 < p_c < 40 \ lb_f/in^2$ $0.64 < x_c < 1.0$	Air-Water H_2O
Edmonds and Smith (1965)	Tube L/D = 20 Long and short Nozzle	$30 < p_c < 45 \ lb_f/in^2$ $0.042 < x_c < 0.16$	R-11
Fauske (1965)	Channel and Tubes L/D > 10	$p_c \sim 17 \ lb_f/in^2$ $0.0 < x_c < 1.00$	Air-Water
Hendricks et al., (1972)	Venturi	$70 < p_c < 276 \ lb_f/in^2$ $0.01 < x_c < 0.10$	N_2
Henry (1968) ANL-7430	Tube L/D > 40	$40 < p_c < 150 \ lb_f/in^2$ $0.009 < x_c < 0.22$	H_2O
Henry (1970) ANL-7740	Tube L/D > 100	$150 < p_c < 300 \ lb_f/in^2$ $0.003 < x_c < 0.1467$	H_2O
Klingbiel and Moulton (1971)	Tube L/D = 40	$28 < p_c < 75 \ lb_f/in^2$ $0.01 < x_c < 0.99$	H_2O
Ryley (1964)	Long Tube (Geothermal Well)	$55 < p_c < 80 \ lb_f/in^2$ $0.11 < x_c < 0.14$	H_2O
Ryley and Parker (1968)	Slots in turbine blade	$1 < p_0 < 3 \ lb_f/in^2$ $0.003 < x_0 < 0.3$	H_2O
Smith, Cousins and Hewitt (1968) Smith (1968, 1972)	Annular Venturi	$15 < p_c < 30 \ lb_f/in^2$ $0.018 < x_c < 1.00$	Air-Water
Smith (1971)	Venturi	$p_c \approx 15 \ lb_f/in^2$ $0.35 < x_c < 0.64$	H_2O
Starkman et al., (1963)	Venturi	$100 < p_0 < 1000 \ lb_f/in^2$ $0.0 < x_c < 0.2$	H_2O

2.2 Analytical Models (Mixture)

The analytical models chosen for this study were those which appeared to have been most often used and referenced at the time of this report. Except for the preliminary studies using a separated phase model, all of the analytical approaches investigated used the mixture concept, that is, a final single set of equations describing the flow of both phases. These analytical models are the homogeneous, thermal equilibrium model; the homogeneous, non-equilibrium model; and those reported by Fauske (1961); Moody (1965, 1969); Cruver and Moulton (1967); Henry (1968); and Henry and Fauske (1971). Table 3 briefly describes these models. A description of the formulation assumptions and final expression of the models follow:

Homogeneous, Thermal Equilibrium Model

The homogeneous thermal equilibrium model is the earliest and most used of all models in critical two-phase flow studies. It is the best defined and simplest in its assumptions. It can also be used to study the flow from upstream conditions. There is generally a consistent pattern that the model shows the generally correct form for the upstream pressure profile but substantially underpredicts the critical two-phase flow rate, especially at low quality. This is shown in figure 2.4-1.

For this study, two general expressions were used for the homogeneous, thermal equilibrium critical flow. They were

$$G_c^2 = \left(\frac{\partial p}{\partial v} \right)_s \qquad (10)$$

$$G_c^2 = \frac{-1}{\left\{ x \left(\frac{\partial v_g}{\partial p} \right)_s + (v_g - v_f) \left(\frac{\partial x}{\partial p} \right)_s + (1 - x) \left(\frac{\partial v_f}{\partial p} \right)_s \right\}_{P_c}} \cdot \qquad (11)$$

The results of these expressions were tested using the properties of several fluids, particularly the cryogenic fluids, and it was found that the differences in the results for the computed critical flow rate were within approximately two or three percent. For cryogenic fluids, the compressibility of the liquid (last term in the denominator of eq. 11) is a significant term. Predicted values for critical flow rates are shown in figures 2.2-1 through 2.2-3A.

Fauske (1961)

In his treatment Fauske used a final mixture expression but attempted to improve on the homogeneous, thermal-equilibrium model by introducing a slip ratio between the liquid and the gas which would alter the critical flow density and the rate of change of the critical flow density in the expression

$$G_c = - (\partial p / \partial v)_s$$

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Table 3 - Table of Analytical Models Recommended for Use in Design Studies

Model Name/Author	Physical Model	Key Relationships/Assumptions	Equation	Figures
Homogeneous, Thermal Equilibrium	Mixture. Phases at same velocity, and in equilibrium.	$u_g = u_f$ Mass, energy & momentum transport rates sufficient to maintain equilibrium at all times.	11	2.2-1 through 2.2-3A
Fauske (1961)	Mixture. Mixture properties (essentially ρ) from consideration of a separated-phase model.	$u_g/u_f = (\rho_f/\rho_g)^{\frac{1}{2}}$ Mass, energy & momentum transport rates sufficient to maintain equilibrium at all times.	12	2.2-4 through 2.2-6
Cruver & Moulton (1967) Moody (1965, 1969)	Mixture. Mixture properties (essentially ρ) from consideration of a separated-phase model.	$u_g/u_f = (\rho_f/\rho_g)^{\frac{1}{3}}$ Mass, energy & momentum transport rates sufficient to maintain equilibrium at all times. [Cruver & Moulton offer several approaches to effective ρ value.]	13 14	2.2-7 through 2.2-9
Moody (1965, 1969) Stagnation	Mixture. Mixture properties (essentially ρ) from consideration of a separated-phase model.	$u_g/u_f = (\rho_f/\rho_g)^{\frac{1}{3}}$ Mass, energy & momentum transport rates sufficient to maintain equilibrium at all times.	15	2.2-10 through 2.2-12
Henry (1968) Henry & Fauske (1971)	Mixture. Mixture properties (essentially ρ) from consideration of a separated-phase model. Evaluation possible from stagnation conditions.	$u_g = u_f$ Mass & energy transport rates estimated from available experimental data.	16	2.2-13 through 2.2-15
Homogeneous, Non-Equilibrium (Metastable, Frozen Flow)	Mixture. Phases at same velocity. Equilibrium not possible at or near point of critical flow.	$u_g = u_f$ Mass transport rates negligible at critical flow. Momentum & energy transport rates sufficient to maintain equilibrium.	17	None; Equation simple & easily evaluated.
Separated Phase Carofano & McManus (1969) Sullivan & Wallis (1972) Smith (1968, 1972)	Separated phase. Critical flow occurs when gas in critical flow. Limited to separated phase cases. Numerical evaluation possible from upstream conditions.	Mass, energy & momentum transport rates estimated from available experimental data.	18	None; Equation simple & easily evaluated.

18

The slip ratio which he proposed was

$$u_g/u_f = (\rho_f/\rho_g)^{1/2}.$$

Henry and Fauske (1971) have later proposed critical flow-rate expressions which do not assume thermal equilibrium and which they generally believe to be a more accurate description of the case. The expression is included here because of its wide use and because, despite its vulnerability because of its assumptions, it still retains relatively good predictive qualities for critical two-phase flow. The final expression of Fauske (1961) programmed for the figures and for the comparative studies is as follows:

$$G_{CF}^2 = \cfrac{-g \cdot dp}{\cfrac{x^2 v_g}{1 + \cfrac{1-x}{x} \cfrac{v_f}{v_g}^{1/2}} + \cfrac{(1-x)^2 v_f}{1 - \cfrac{1}{1 + \cfrac{1-x}{x} \cfrac{v_f}{v_g}^{1/2}}}} \tag{12}$$

Predicted values for critical flow rates are shown in figures 2.2-4 through 2.2-6.

Moody (1965, 1969) and Cruver and Moulton (1967)

Both of these models follow the same general pattern as that of Fauske (1961). That is, a single expression is used for the critical two-phase flow, thermal equilibrium is assumed and a slip ratio is introduced which changes the density and the rate of change of density from that which one would use in the thermal homogeneous thermal equilibrium expression. Essentially, the difference between these treatments and that of Fauske (1961) is that in this case the slip ratio is shown as

$$k = u_g /u_f = (\rho_f /\rho_g)^{1/3}.$$

The same maximum flow slip ratio has been reported by these and other investigators using considerably different approaches; for example, that of Zivi (1964) who approached the problem from a point of view of maximum energy production. It should be pointed out that the slip ratios from the above expressions and from that used by Fauske (1961) all show very substantially higher values for the slip ratio than those which have been reported as measured by Klingbiel (1964, 1971) and by Fauske (1965) and are considerably different than that used by Fauske and Henry in their later works (Henry and Fauske, 1971) and for Cruver and Moulton (1967). The equation programmed for Cruver and Moulton is:

$$G_{CM}^2 = \cfrac{-g \cdot [xv_g + (1-x)v_f]}{v_f[1+x(k^2-1)]^2 [x/k (\frac{\partial v_g}{\partial p})_s + \frac{3}{2} v_f (k^2-1) (\frac{\partial x}{\partial p})_s + (1-x) (\frac{\partial v_f}{\partial p})_s]} \tag{13}$$

That for Moody (1965, 1969) at critical flow was

$$G_{CM}^2 = \frac{-2g_c \ (778) \ (v_f + x \ v_{fg})}{778/144 \ [kv_f + x(v_g - kv_f)] \left([kv_f + x(v_g - kv_f)] \left\{ \left[\dfrac{s_g'}{k^2 s_{fg}} - \dfrac{s_f'}{s_{fg}} - \dfrac{(s_{fg}k^2)'}{k^4 s_{fg}} \right] + x \left[\dfrac{(s_{fg}k^2)'}{s_{fg}k^2} - \dfrac{s_{fg}'}{s_{fg}} \right] \right\} \right.}$$

$$\frac{-2g_c \ (778) \ (v_f + x \ v_{fg})}{\left. +2 \left[\dfrac{1}{k^2} + x(1 - \dfrac{1}{k^2}) \right] \left\{ \left[s_{fg} \left(\dfrac{kv_f}{s_{fg}} \right)' + \dfrac{kv_f}{s_{fg}} s_g' - \left(\dfrac{v_g}{s_{fg}} \right) s_f' \right] + x \left[s_{fg} \left(\dfrac{v_g}{s_{fg}} \right) - s_{fg} \left(\dfrac{kv_g}{s_{fg}} \right)' \right] \right\} \right)} \tag{14}$$

where the primes represent incremental differences with respect to incremental pressure changes. Predicted values for critical flow rates from (13) and (14) are shown in figures 2.2-7 through 2.2-9. These expressions used essentially the same assumptions and produced similar predictive critical flow rates. A detailed comparison of these models is made by Cruver and Moulton (1967).

A further stagnation model of Moody (1965, 1969) which allows prediction of critical flow from stagnation conditions rather than at the critical condition was programmed and reported which used the following expression

$$G_{CM_o}^2 = \frac{2g_c \ \left[h_o - h_f - \dfrac{h_{fg}}{s_{fg}} \ (s_o - s_f) \right] (778)}{\left[\dfrac{k(s_g - s_o)v_f}{s_{fg}} + \dfrac{(s_o - s_f)v_g}{s_{fg}} \right]^2 \left[\dfrac{s_o - s_f}{s_{fg}} + \dfrac{s_g - s_o}{k^2 s_{fg}} \right]} \ . \tag{15}$$

Predicted values for critical flow rates are shown in figures 2.2-10 through 2.2-12.

Henry (1968), Henry and Fauske (1971)

This is another model which uses stagnation properties and thus allows prediction of a critical two-phase flow from stagnation conditions rather than at the critical point. In this model both the variations from the homogeneous, thermal equilibrium model in equilibrium and slip ratio were considered. The rate of mass transfer was expressed as

$$dx/dp \big|_c = N \frac{dx_E}{dp} \big|_c$$

where

$$N = x_E / 0.14 \ (\text{for } x_E > 0.14, \ N = 1)$$

and

$$dx_E / dp \big|_c \ \text{is the equilibrium rate of mass transfer.}$$

The slip ratio was generally based on the data of Vogrin (1963) which showed a critical slip ratio to approach unity and a rate of change of slip ratio at the critical point to be zero. The final expression used for programming and comparative studies is shown in the following equation:

20

$$G_{cH_o}^2 = -\left\{ k\left[[1+x(k-1)]\, x\, \frac{dv_g}{dp} + \left[v_g\{1+2x(k-1) + kv_f\ 2(x-1) + k(1-2x)\}\right]\frac{dx}{dp}\right.\right.$$

$$\left.\left.+ k[1+x(k-2) - x^2(k-1)]\frac{dv_f}{dp} + x(1-x)\ (kv_f - \frac{v_g}{k})\frac{dk}{dp}\right]^{-1}\right\}_c$$ (16)

$$v_{gc} = v_{go}\ (p_o/p_e)^\gamma$$

$$\frac{dv_{gc}}{dp} = \frac{v_{gc}}{np_c}$$ where $n = \dfrac{(1-x)\dfrac{c_f}{c_{pg}} + 1}{(1-x)\dfrac{c_f}{c_{pg}} + \dfrac{1}{\gamma}}$

$$\frac{ds_f}{dp} = const.$$ $$\frac{d\rho_f}{dp} = 0$$

$$\frac{ds}{dp} = \frac{c_{pg}}{p_c}\ [\frac{1}{n} - \frac{1}{\gamma}]$$ $$\frac{dk}{dp} = 0.$$

Predicted critical flow rates are shown in figures 2.2-12 through 2.2-15.

Homogeneous, Non-Equilibrium Model (Metastable, Frozen Flow)

This is a mixture model which generally follows the development of Tangren et al. (1949). It was first developed for an air-water system. Thus, it assumes the mass transfer rate (condensation or evaporation) is zero at the point of critical flow. When applied to a single component mixture, in this report (as in Smith, 1963), the critical quality x_2 is calculated assuming thermal equilibrium and then used in the expression

$$G_{cHx} = \dfrac{\dfrac{x_c}{1-x_c}\ C_{pg} + C_{yf}}{\dfrac{x_c}{1-x_c}\ C_{vg} + C_{yf}}\quad p/x_c\ v_g$$ (17)

which assumes the mass transfer rate to be zero at the point of critical flow.

No computer evaluations are shown, as for the previous models, because the equation is quite simple. Some evaluation charts are shown in Smith (1963).

Appendix A provides an example problem using the figures showing predicted critical flow rates for cryogenic fluids (hydrogen in this case).

Figures 2.2-16 through 2.2-21 show the ratio of predicted critical flow rates from the Fauske (1961) and Cruver and Moulton (1967) and Moody (1965, 1969) models over those from the homogeneous, thermal equilibrium model. These figures show that the model results predict the ratio approaches one as the pressure goes higher and the liquid to gas density ratio approaches one. They also show the predictions are in the same general range as the experimental data.
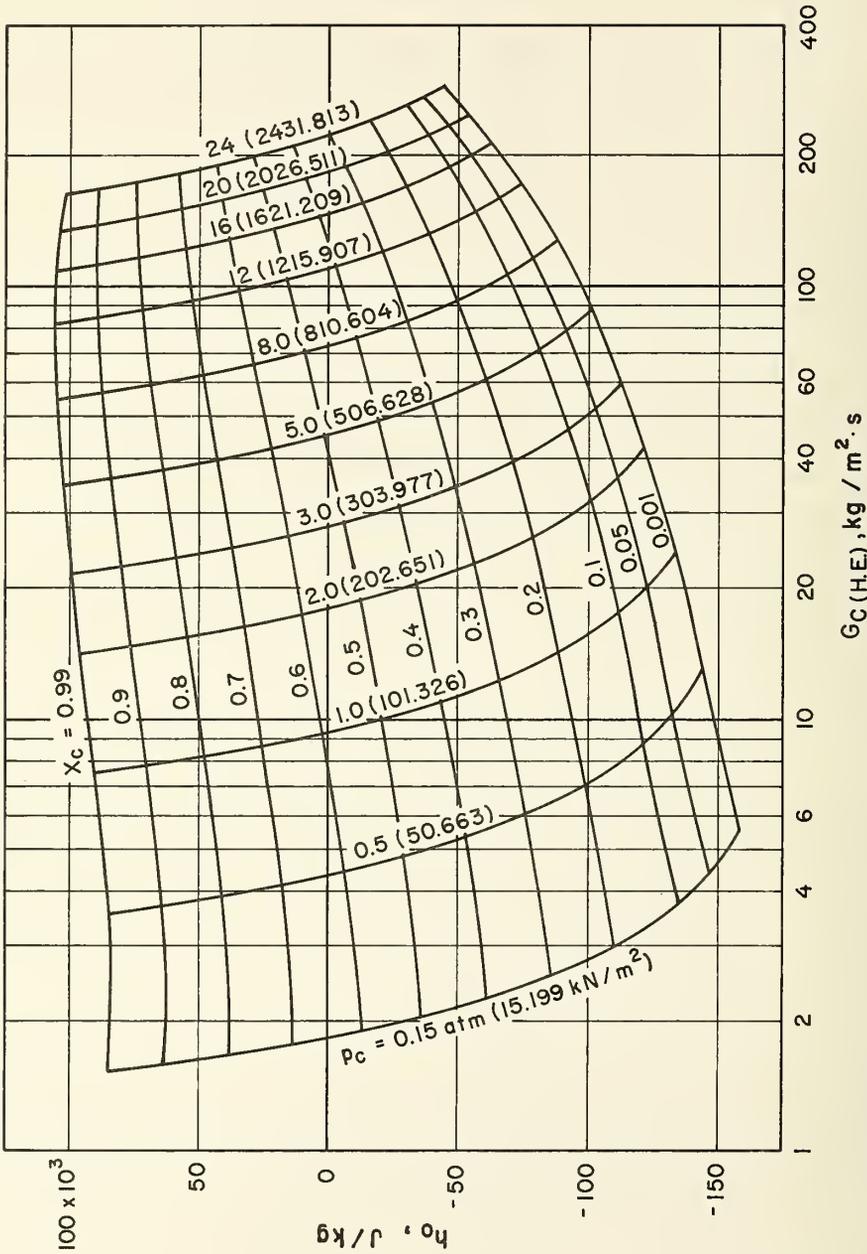
Figure 2. 2-1   Predicted critical two-phase flow rates using homogeneous, thermal equilibrium model for oxygen [SI units].

22
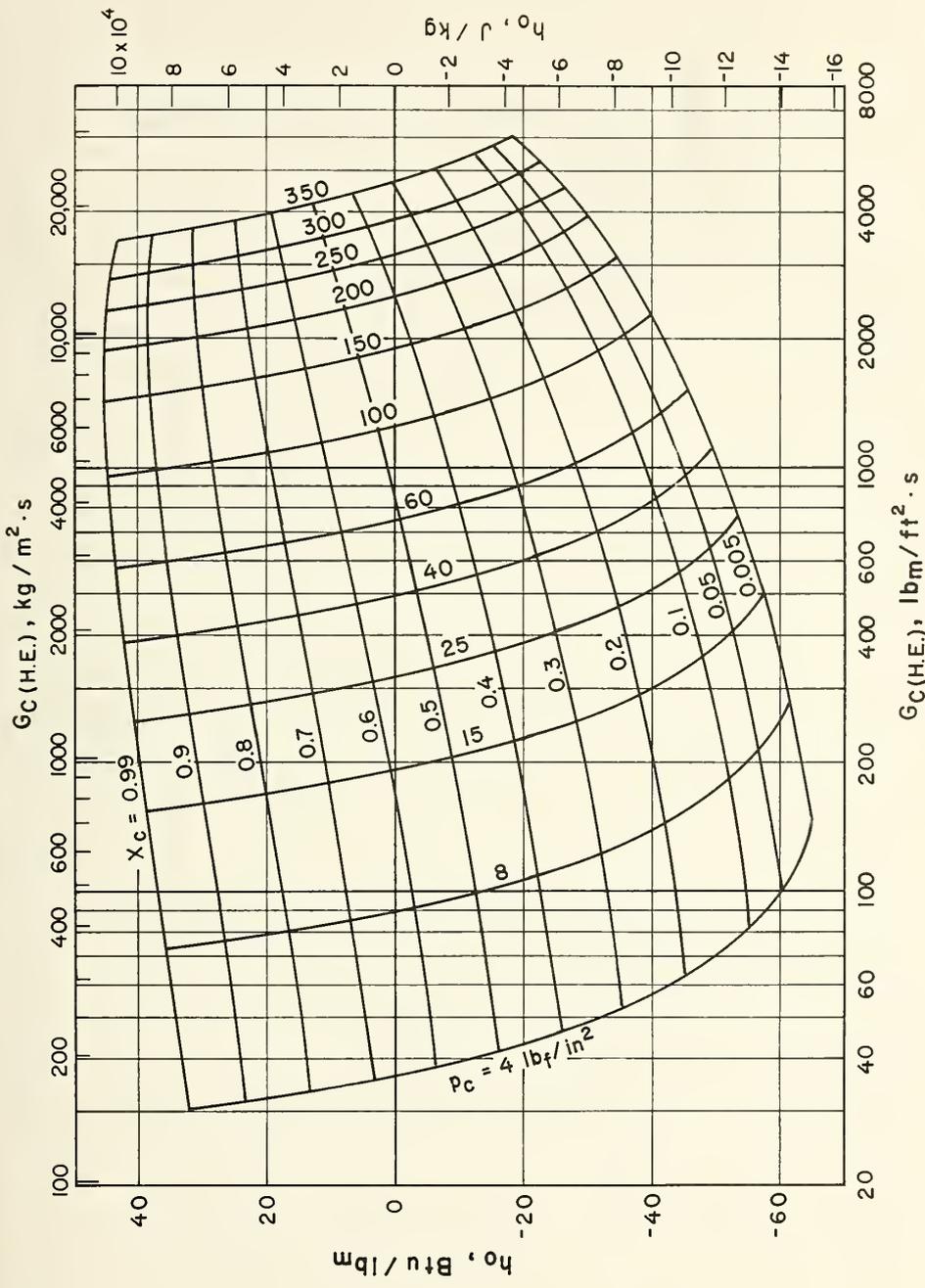
Figure 2.2-1A Predicted critical two-phase flow rates using homogeneous, thermal equilibrium model for oxygen [British units].

23
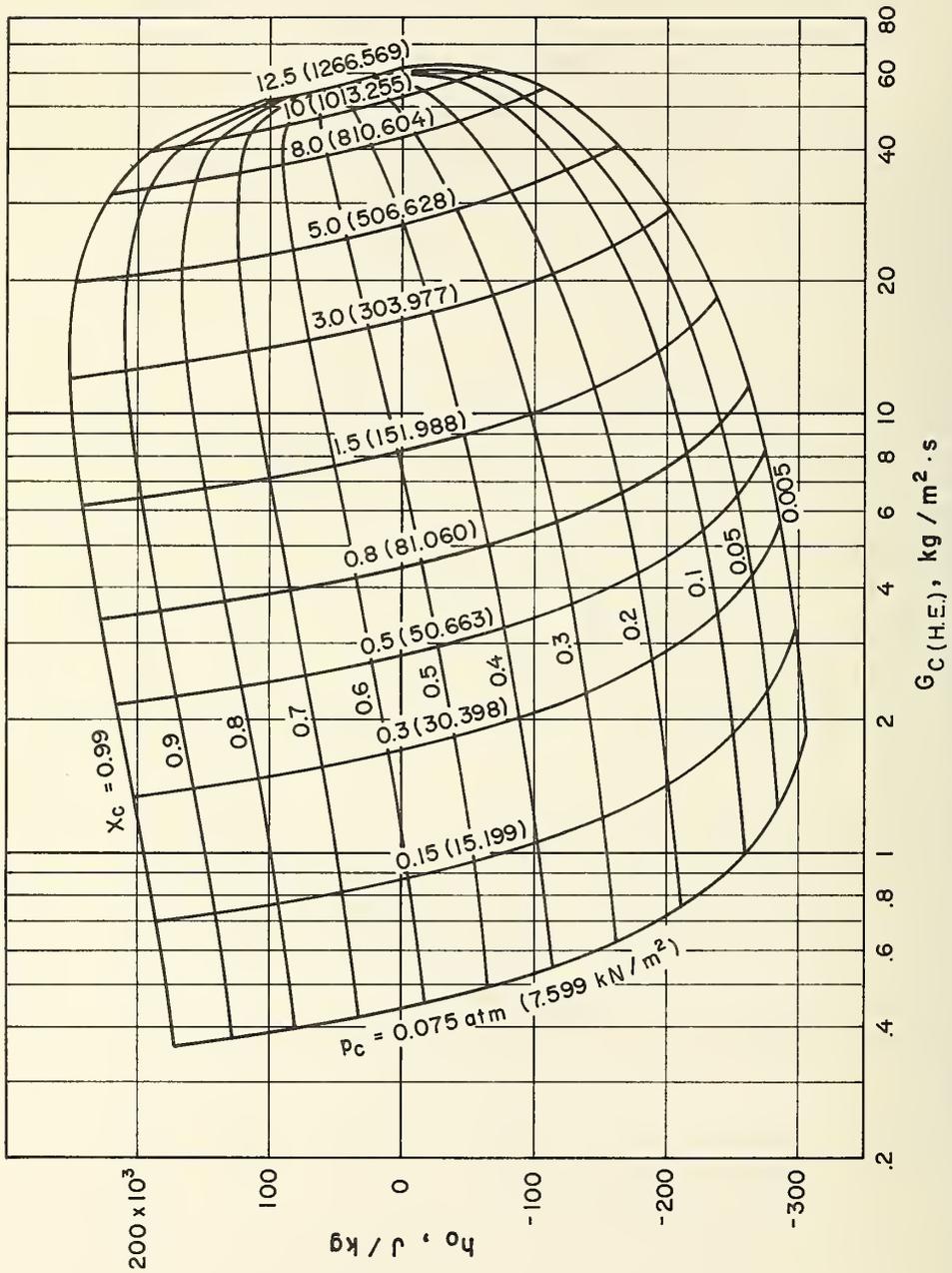
Figure 2.2-2   Predicted critical two-phase flow rates using homogeneous, thermal equilibrium model for hydrogen [SI units].

24
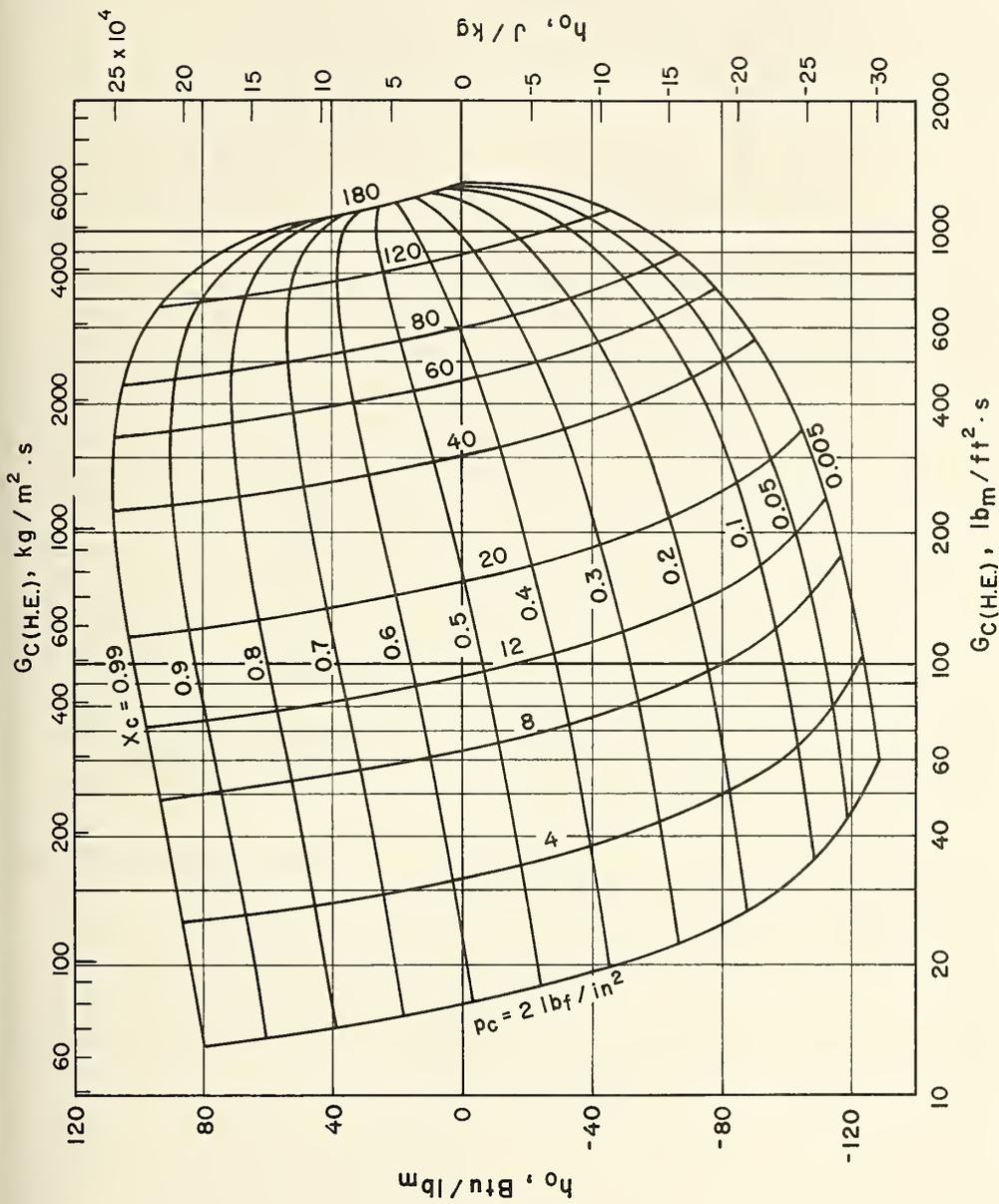
Figure 2.2-2A   Predicted critical two-phase flow rates using homogeneous, thermal equilibrium model for hydrogen [British units].

25
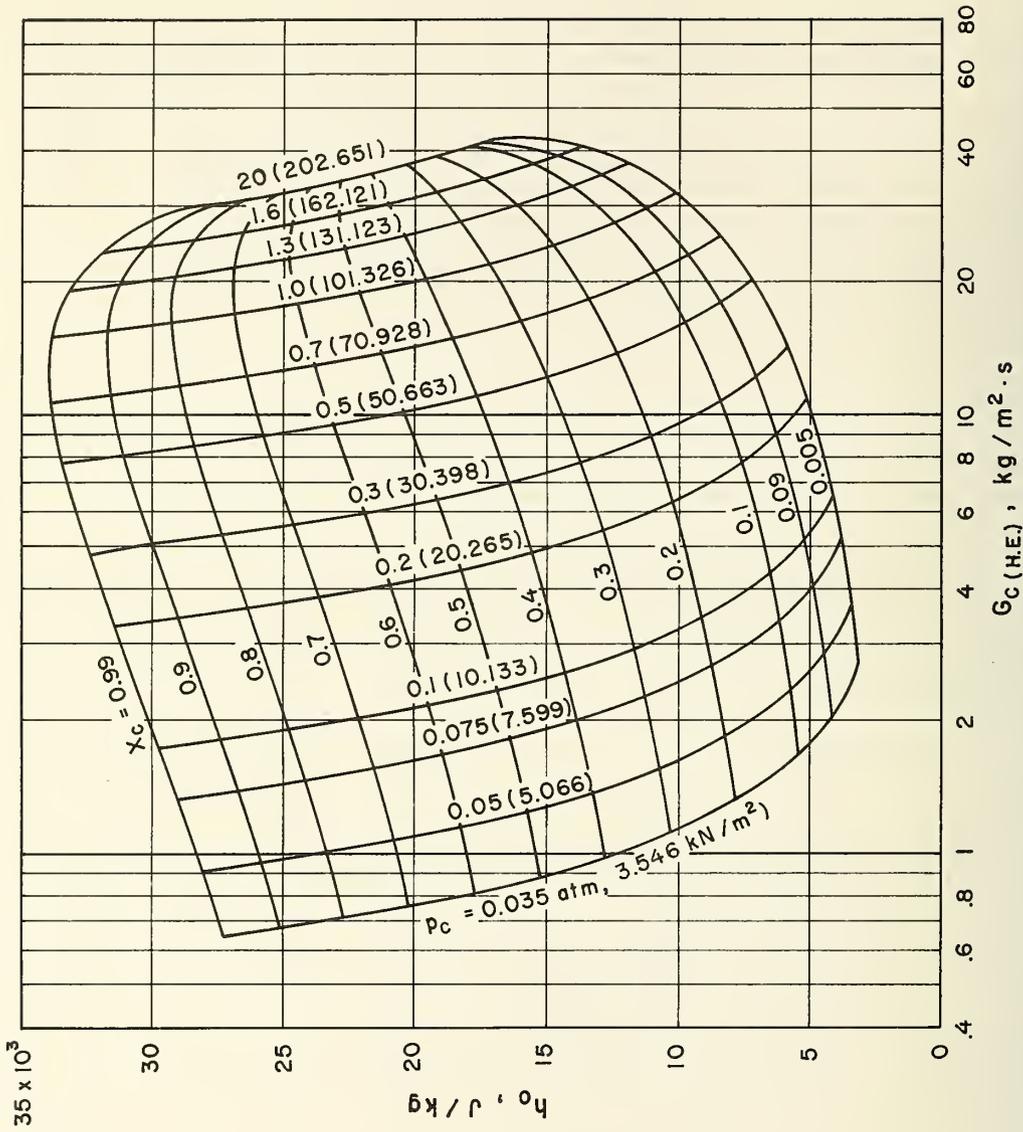
Figure 2.2-3   Predicted critical two-phase flow rates using homogeneous, thermal equilibrium model for helium [SI units].
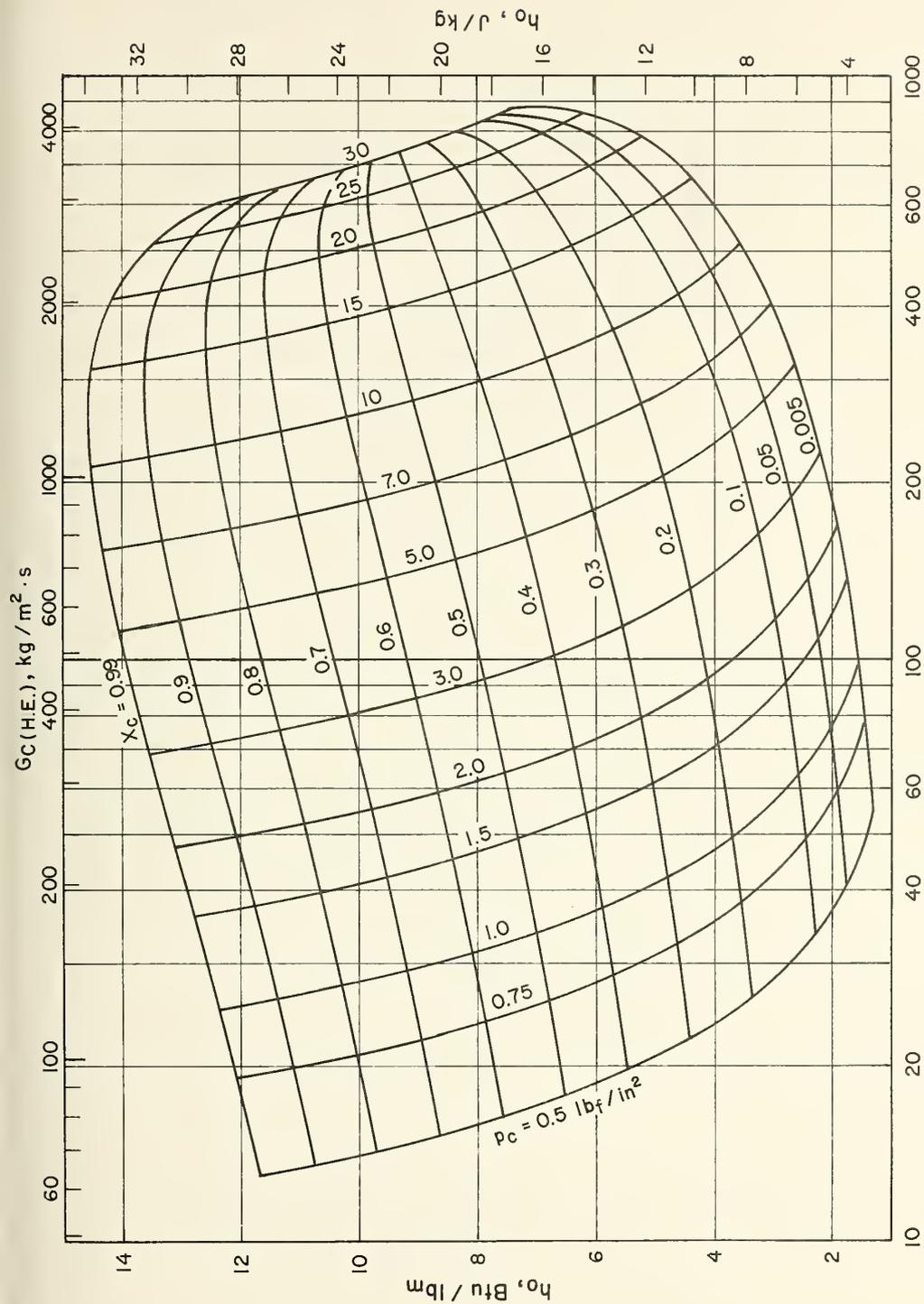
Figure 2.2-3A   Predicted critical two-phase flow rates using homogeneous, thermal equilibrium model for helium [British units].

27
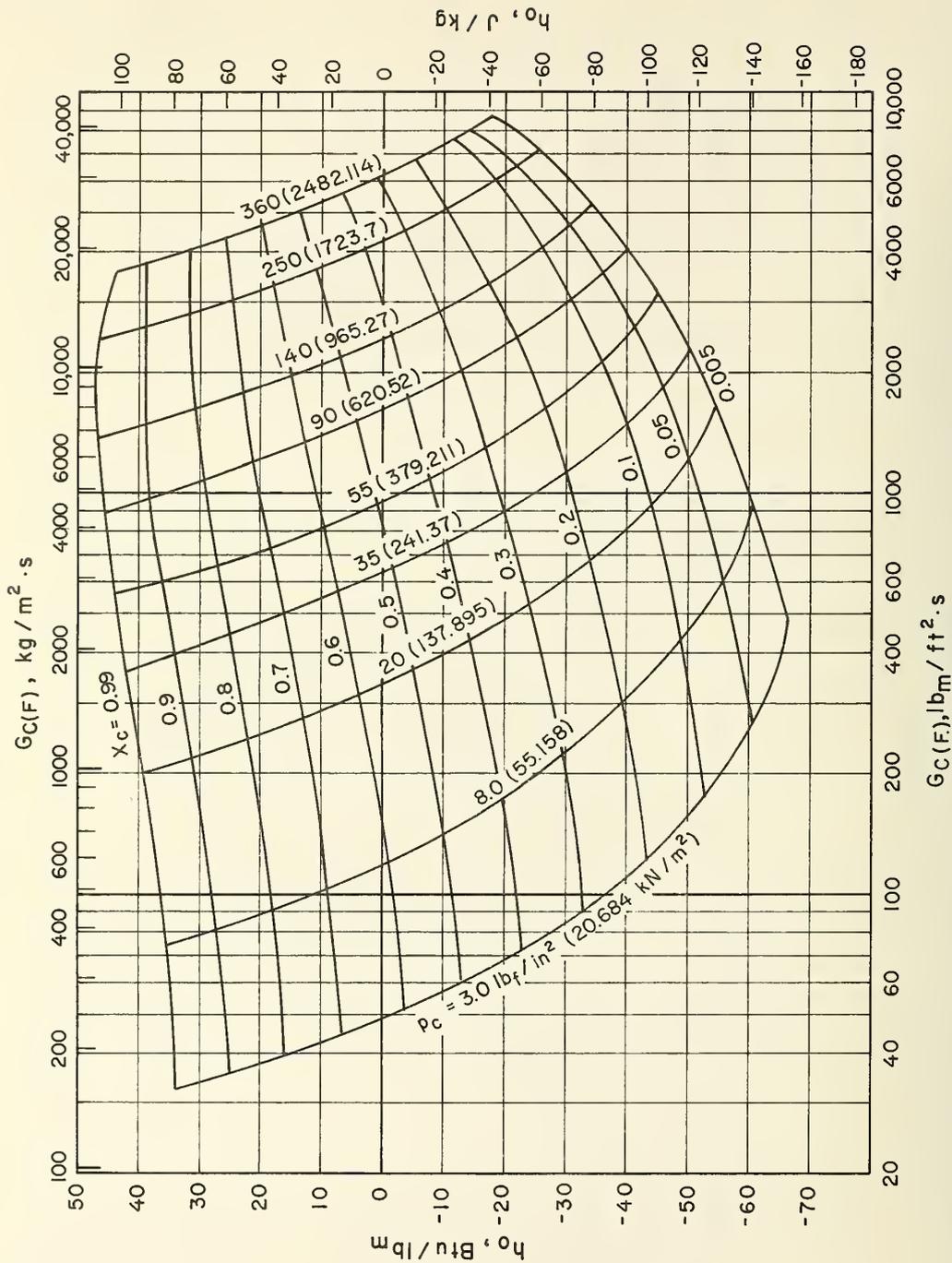
Figure 2.2-4   Predicted critical two-phase flow rates using Fauske (1961) for oxygen.

28
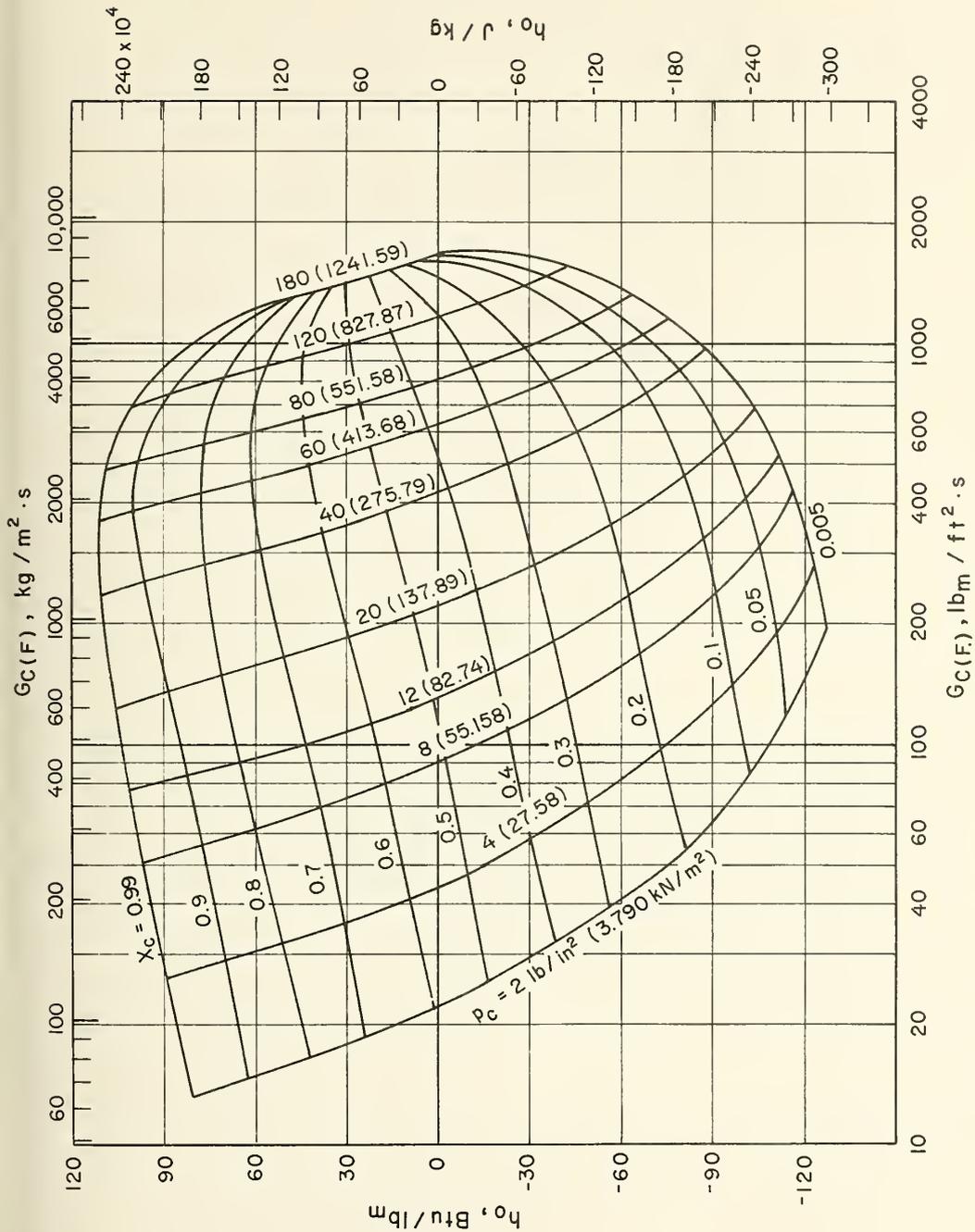
Figure 2.2-5   Predicted critical two-phase flow rates using Fauske (1961) for hydrogen.

29
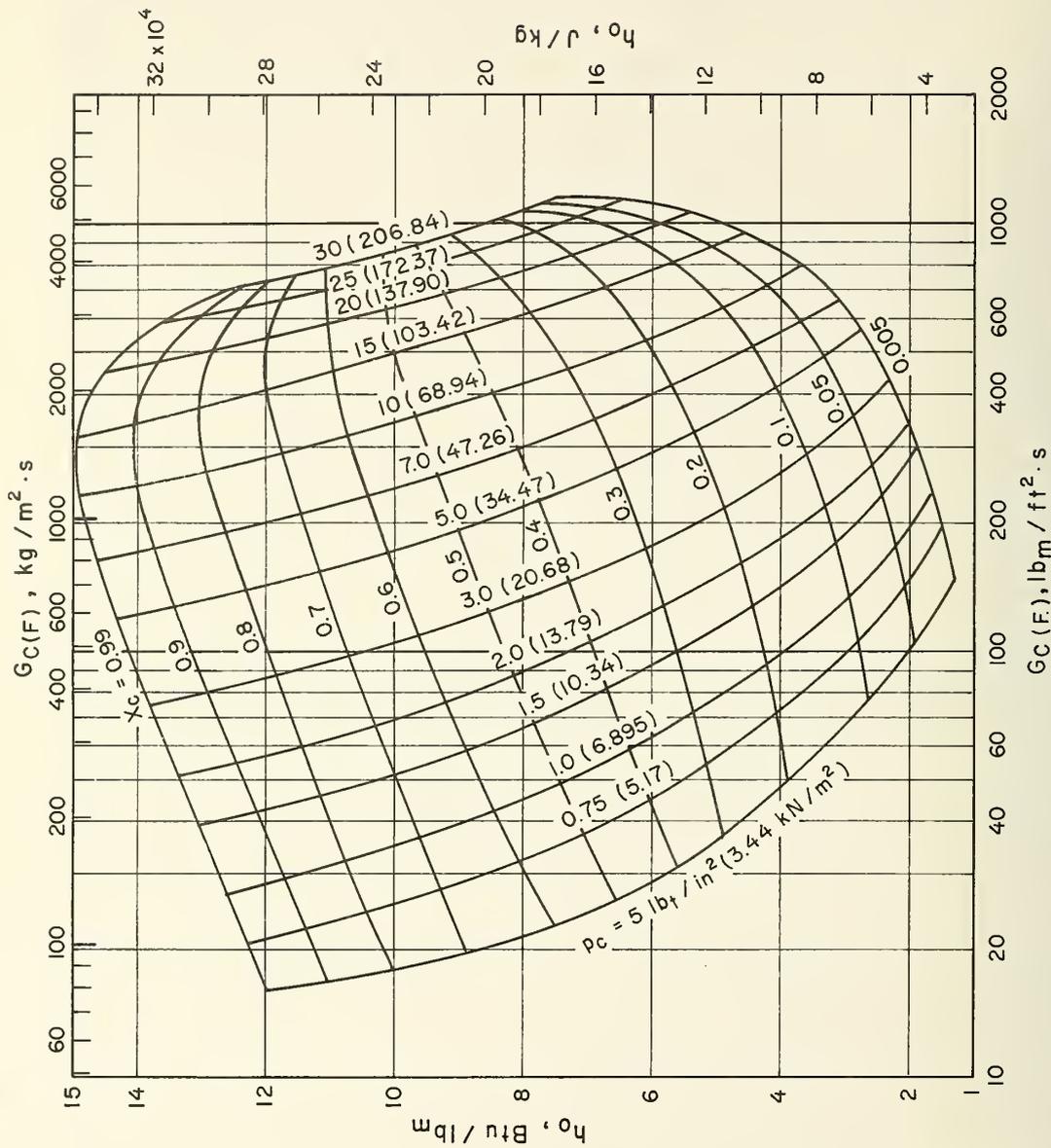
Figure 2.2-6   Predicted critical two-phase flow rates using Fauske (1961) for helium.

30
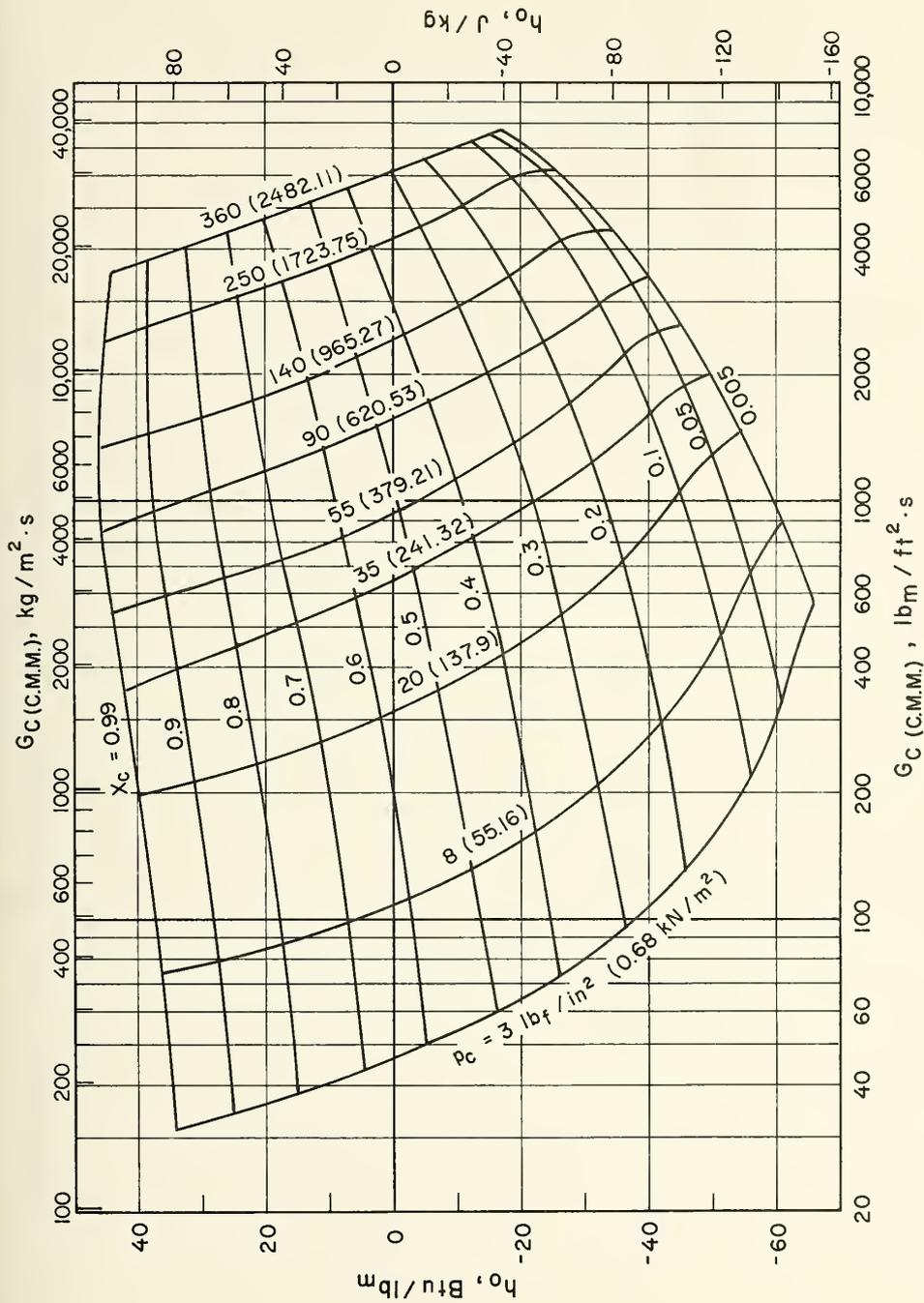
Figure 2.2-7   Predicted critical two-phase flow rates using Cruver and Moulton (1967) and Moody (1965, 1969) for oxygen.

31
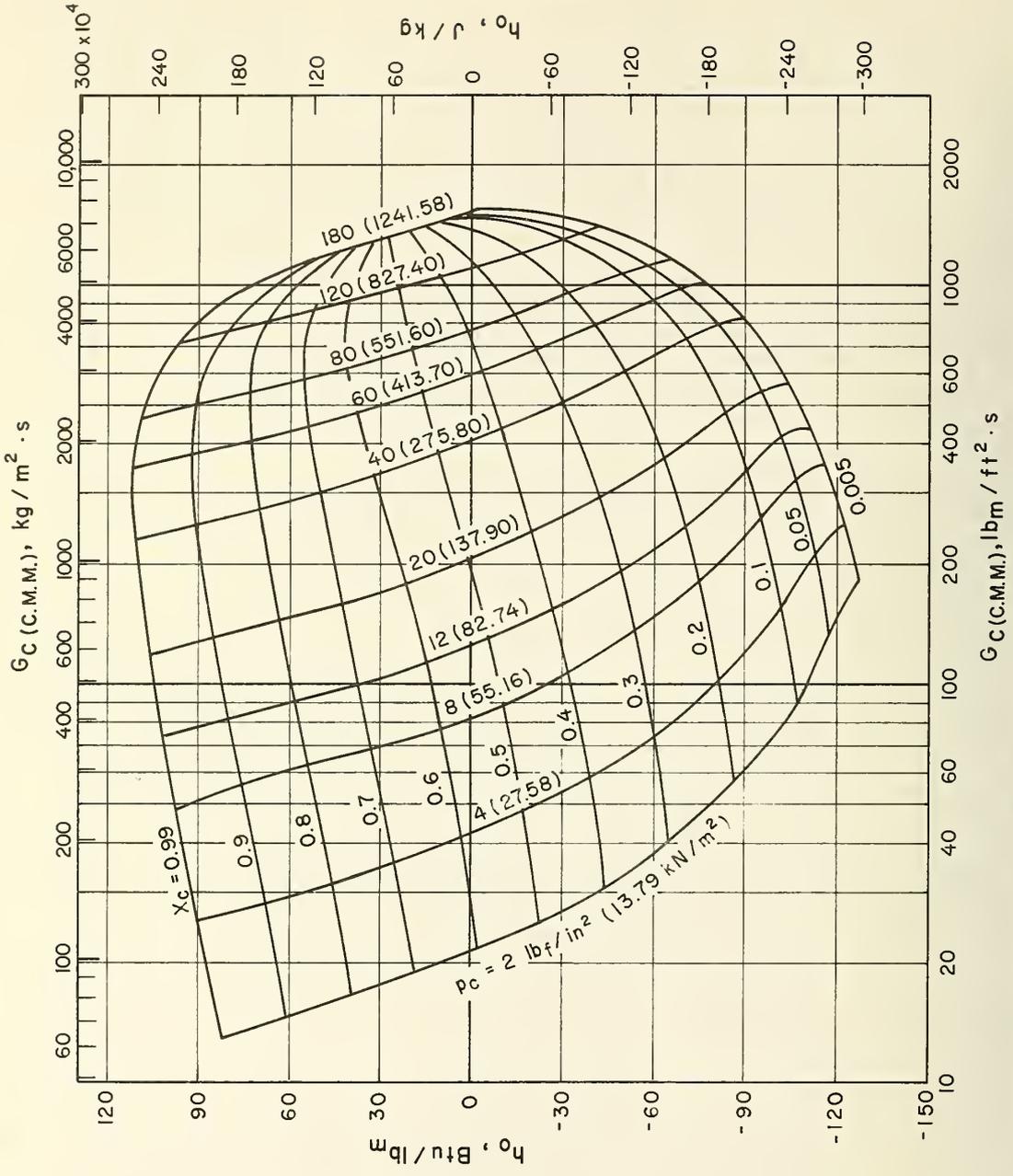
Figure 2.2-8   Predicted critical two-phase flow rates using Cruver and Moulton (1967) and Moody (1965, 1969) for hydrogen.

32
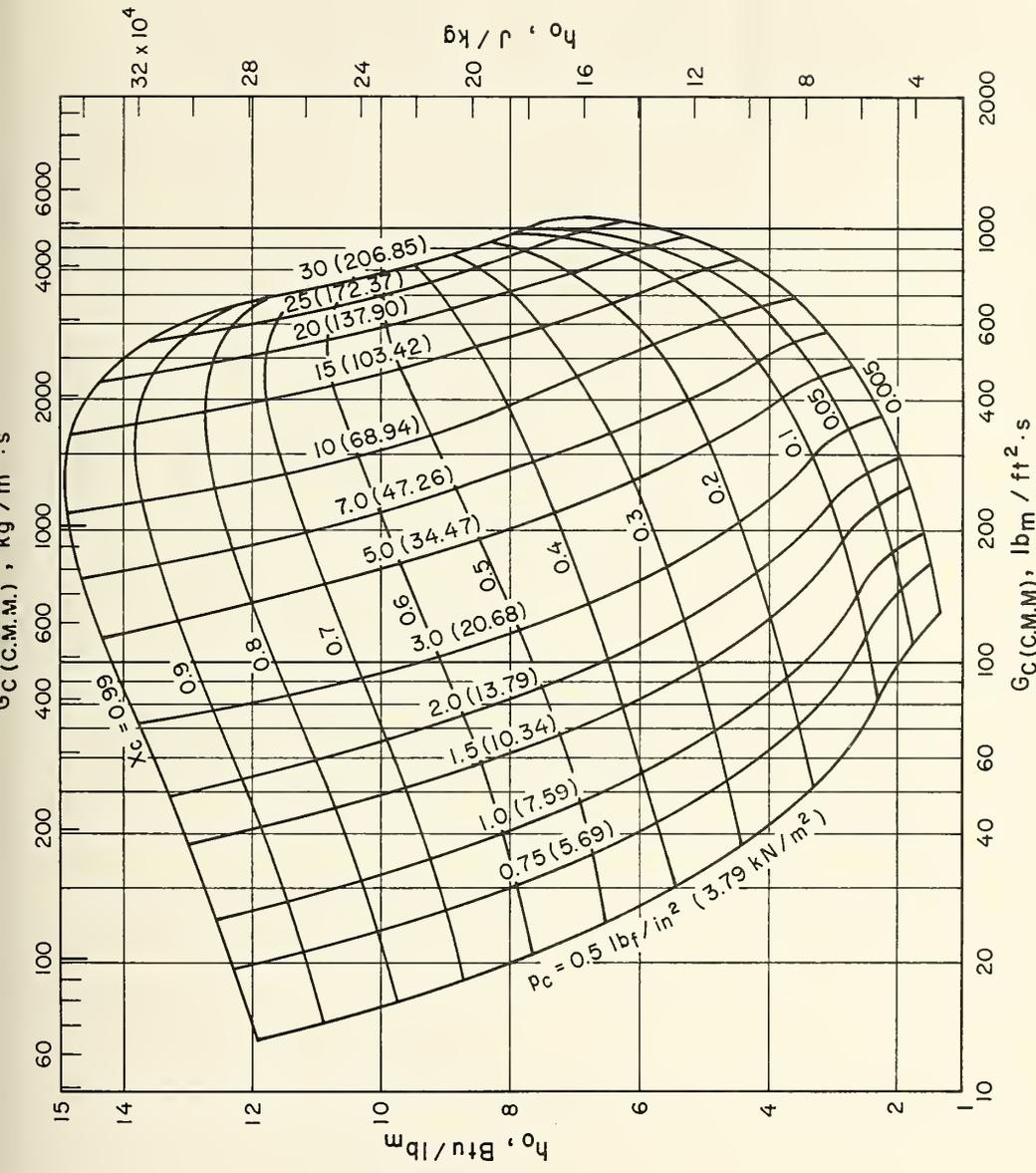
Figure 2.2-9   Predicted critical two-phase flow rates using Cruver and Moulton (1967) and Moody (1965, 1969) for helium.

33
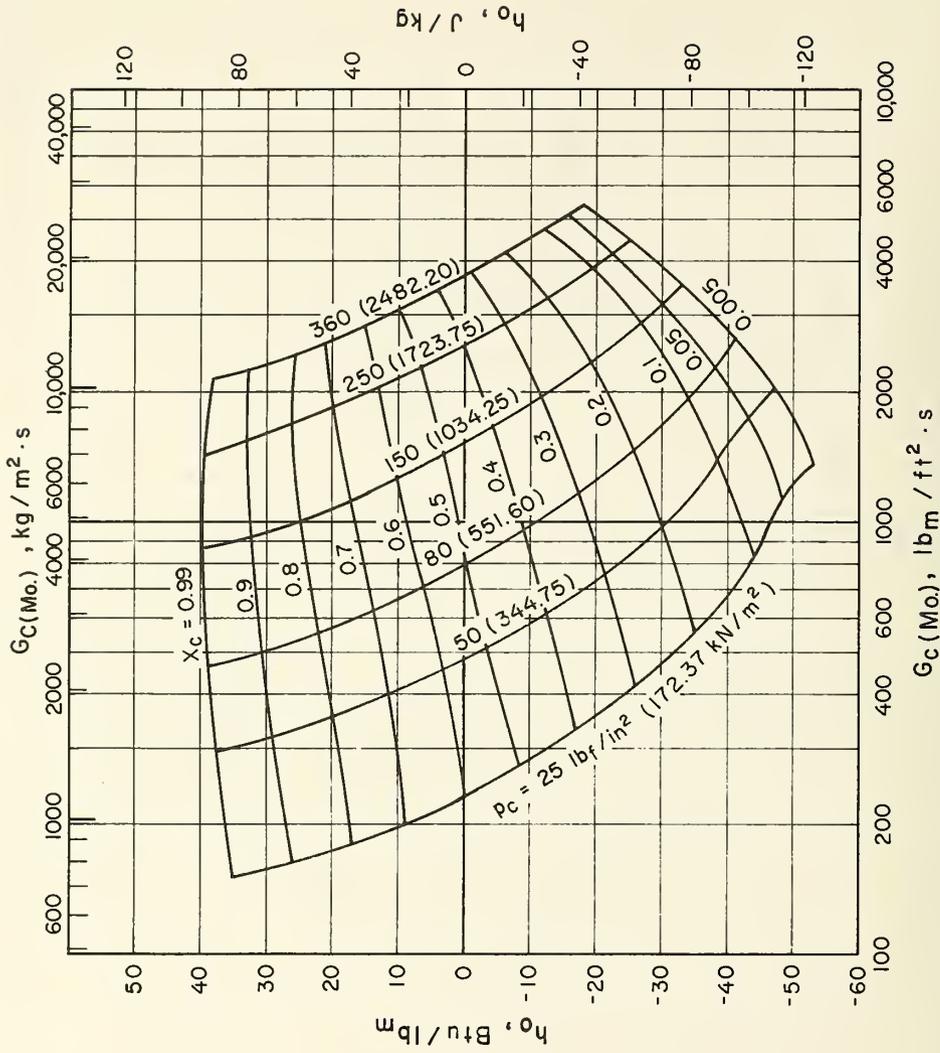
Figure 2.2-10   Predicted critical two-phase flow rates using Moody (1965, 1969)[stagnation properties] for oxygen.

34
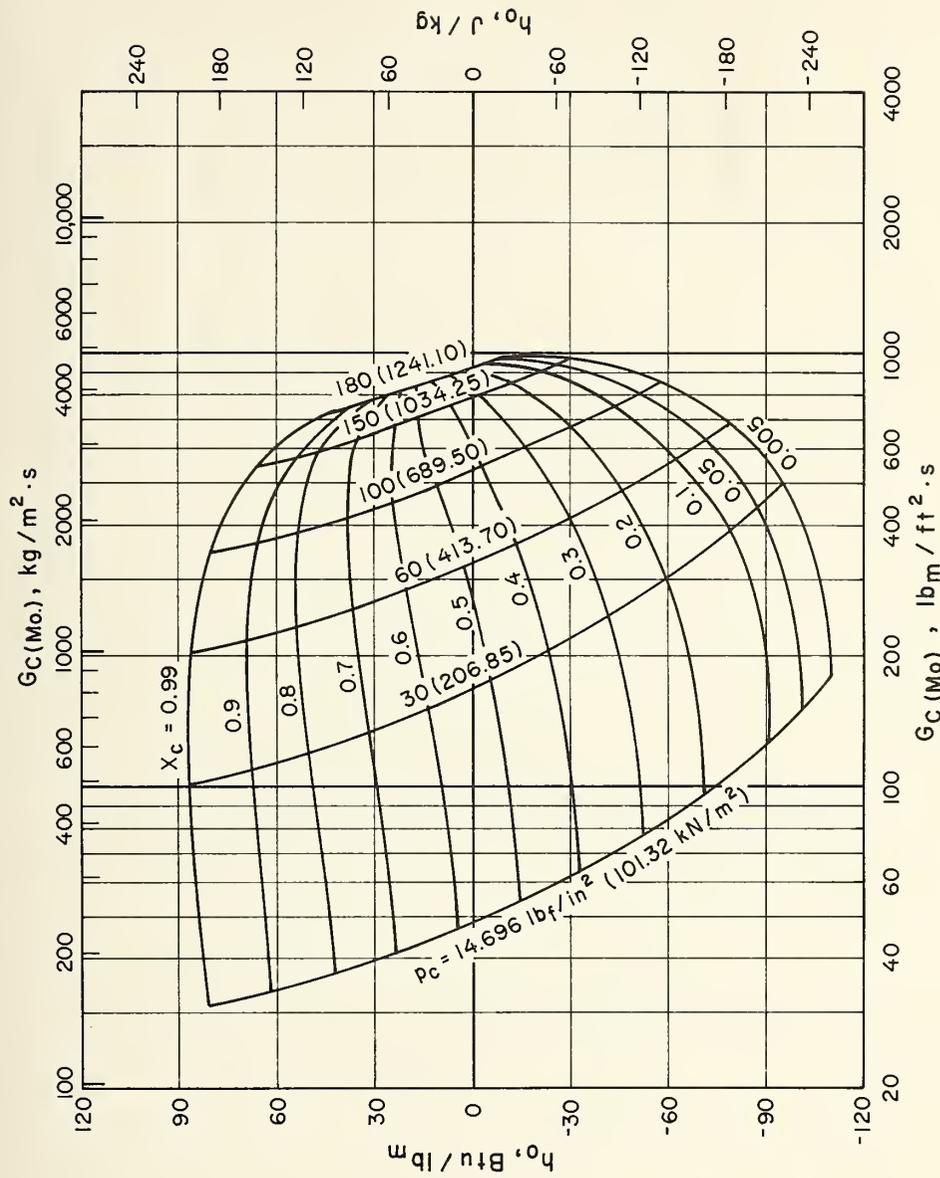
Figure 2.2-11   Predicted critical two-phase flow rates using Moody (1965, 1969)[stagnation properties] for hydrogen.

35
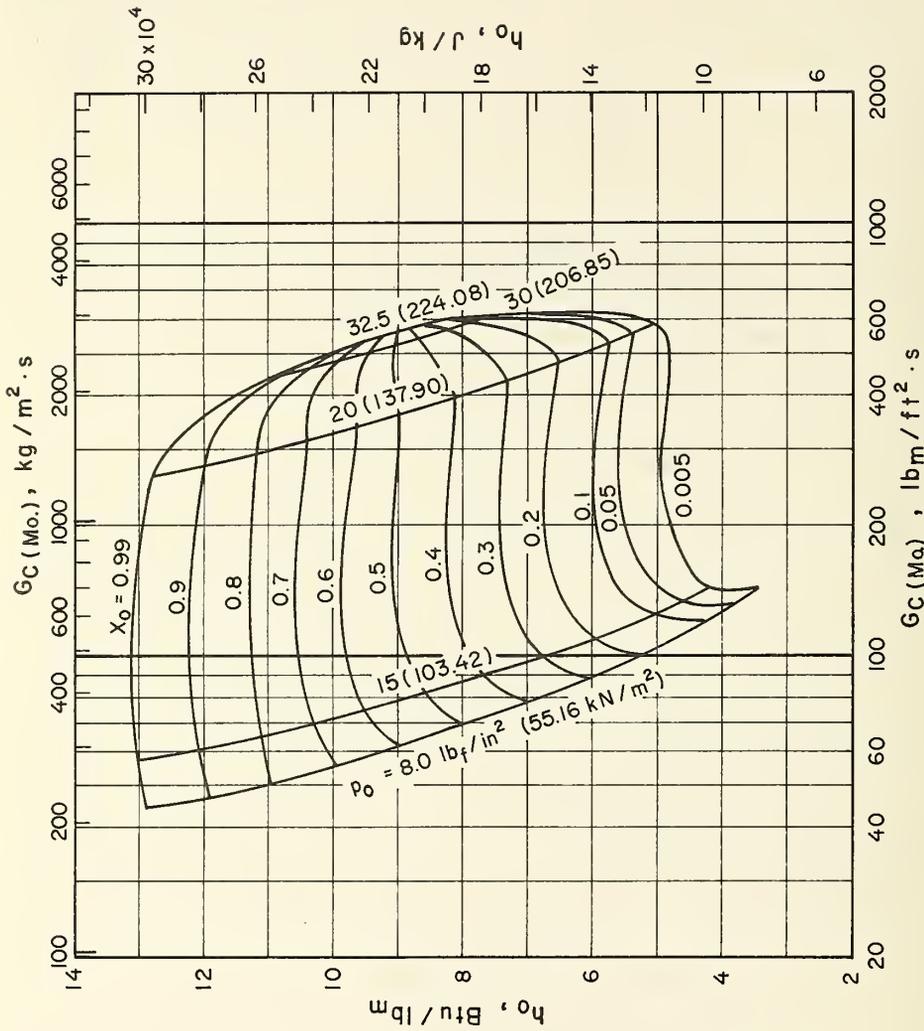
Figure 2.2-12   Predicted critical two-phase flow rates using Moody (1965, 1969)[stagnation properties] for helium.
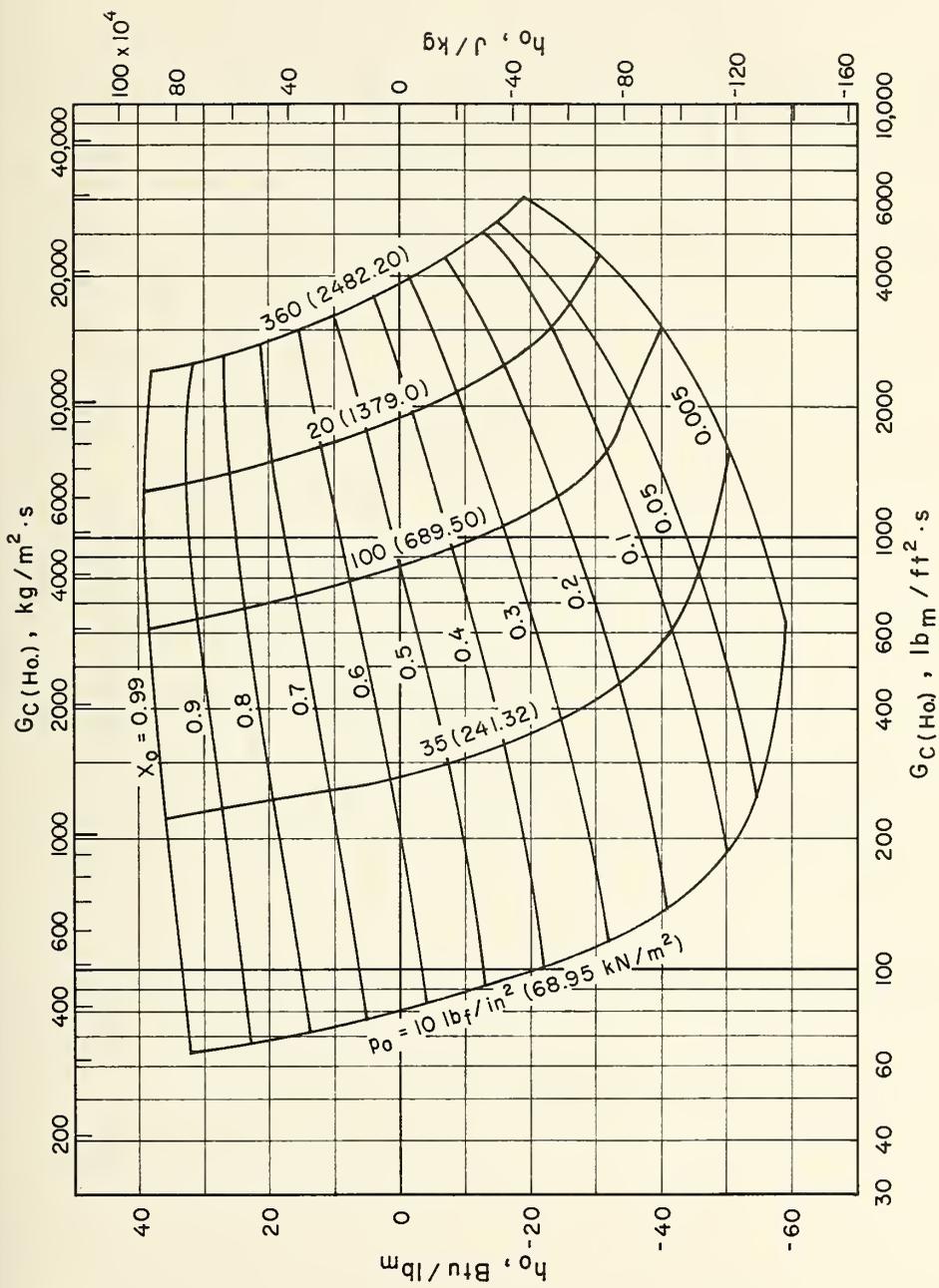
Figure 2.2-13   Predicted critical two-phase flow rates using Henry (1968), Henry and Fauske (1971) for oxygen.

37
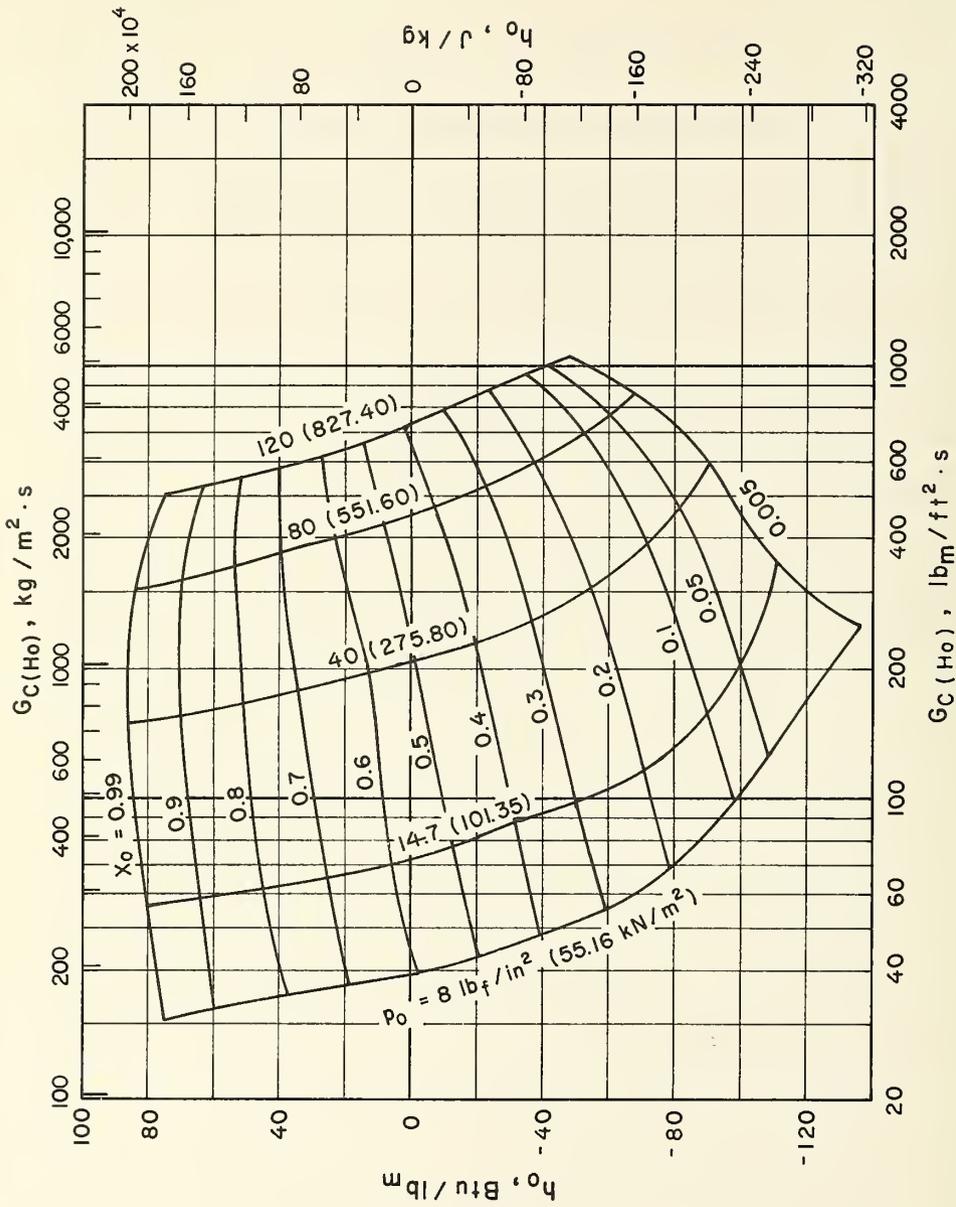
Figure 2.2-14   Predicted critical two-phase flow rates using Henry (1968), Henry and Fauske (1971) for hydrogen.
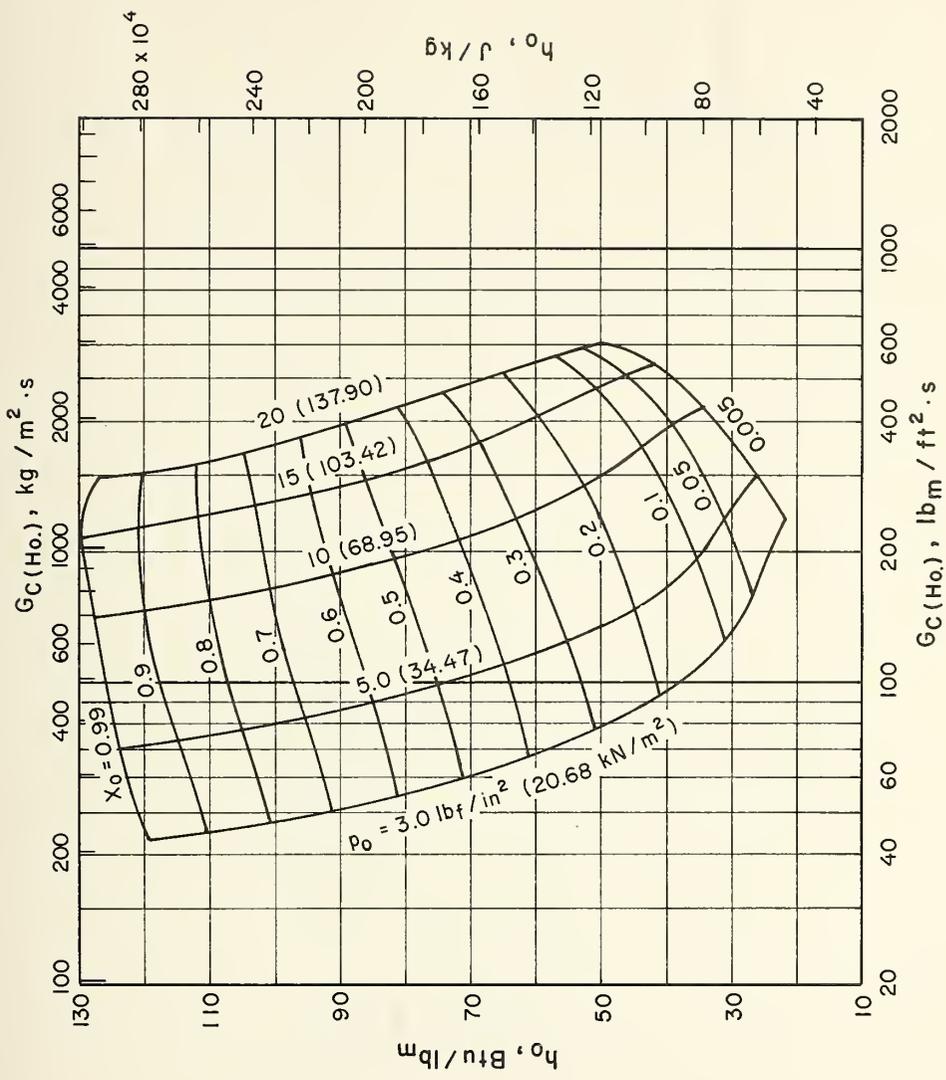
Figure 2.2-15   Predicted critical two-phase flow rates using Henry (1968), Henry and Fauske (1971) for helium.

39
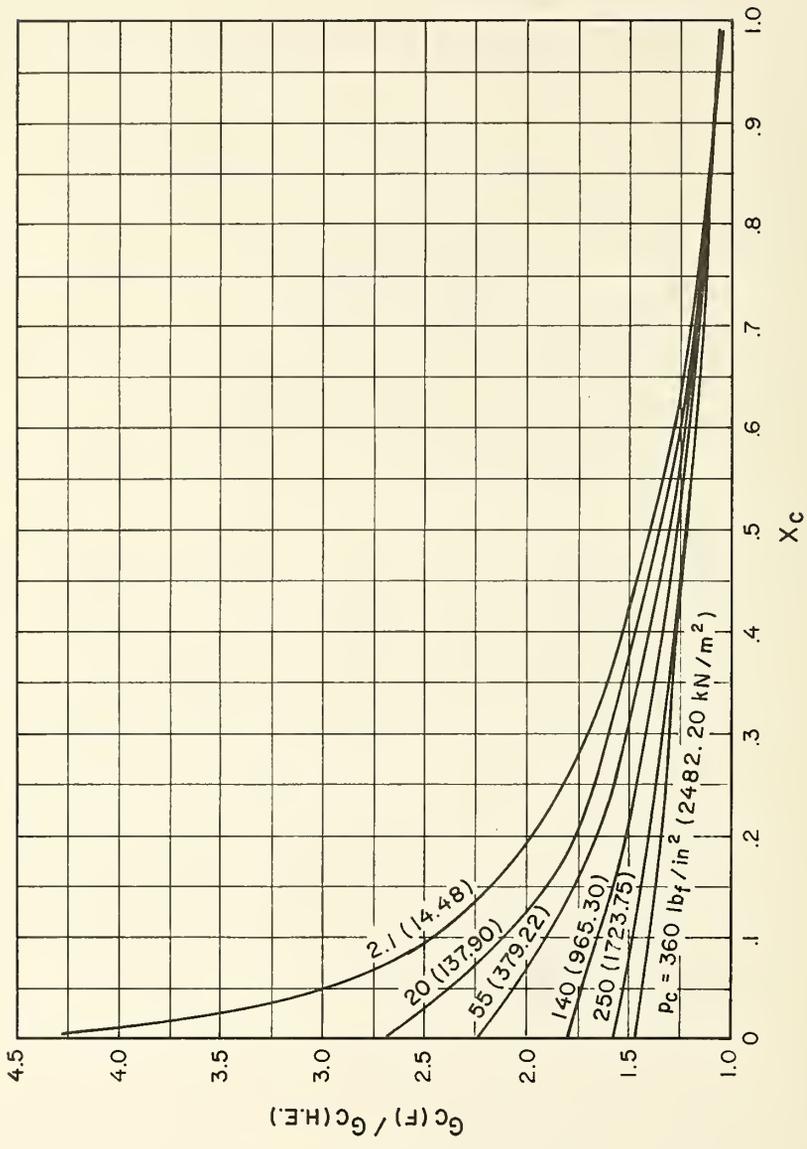
Figure 2.2-16   Predicted ratios of Fauske (1961) to homogeneous, thermal equilibrium model results for oxygen.

40
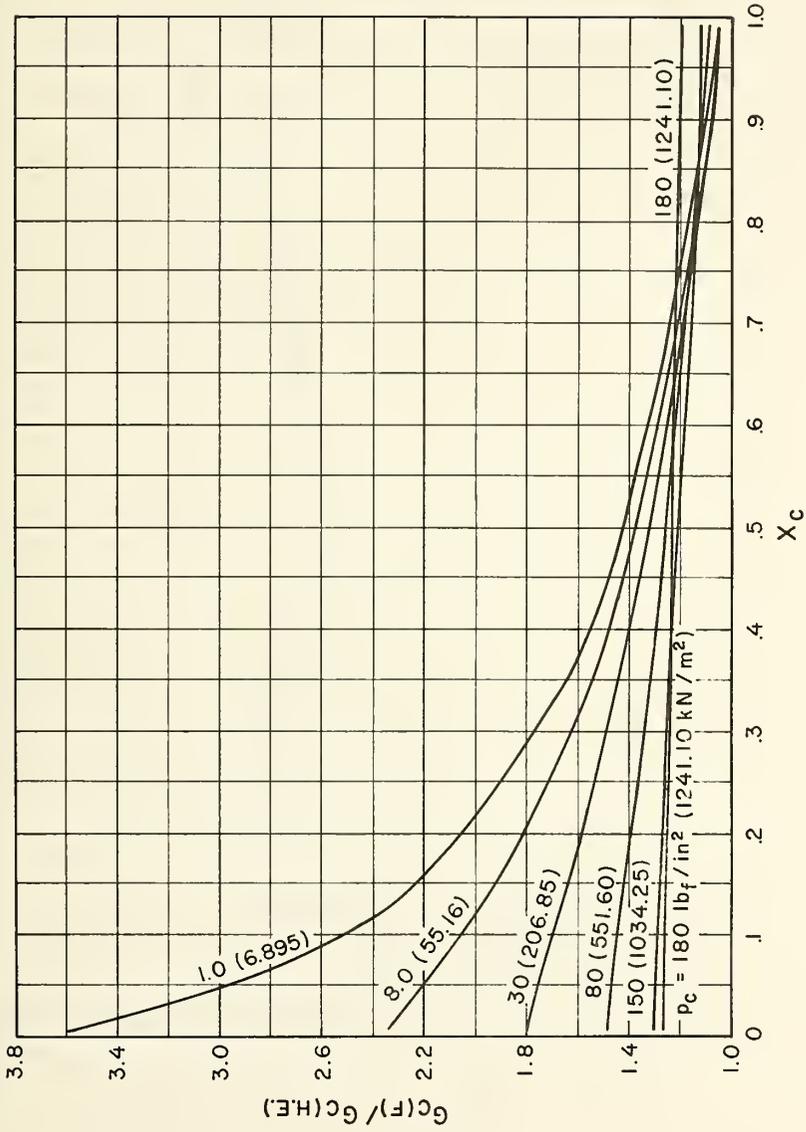
Figure 2.2-17   Predicted ratios of Fauske (1961) to homogeneous, thermal equilibrium model results for hydrogen.

41
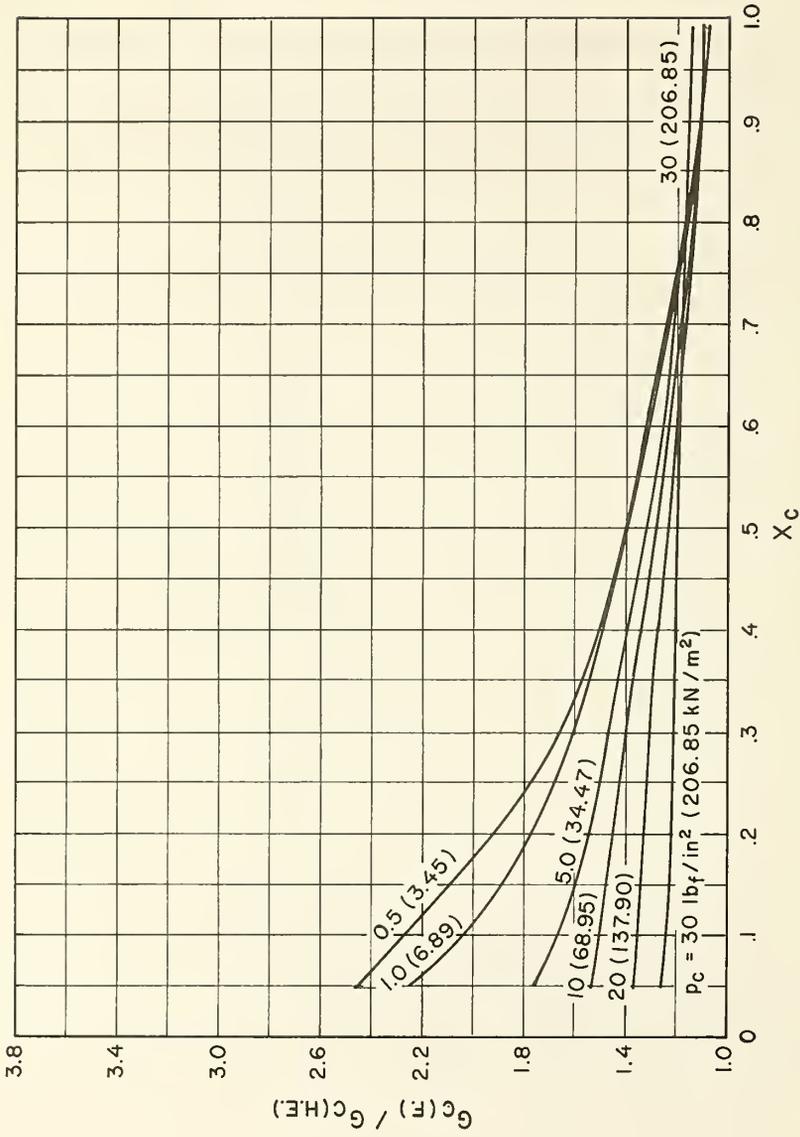
Figure 2.2-18   Predicted ratios of Fauske (1961) to homogeneous, thermal equilibrium model results for helium.
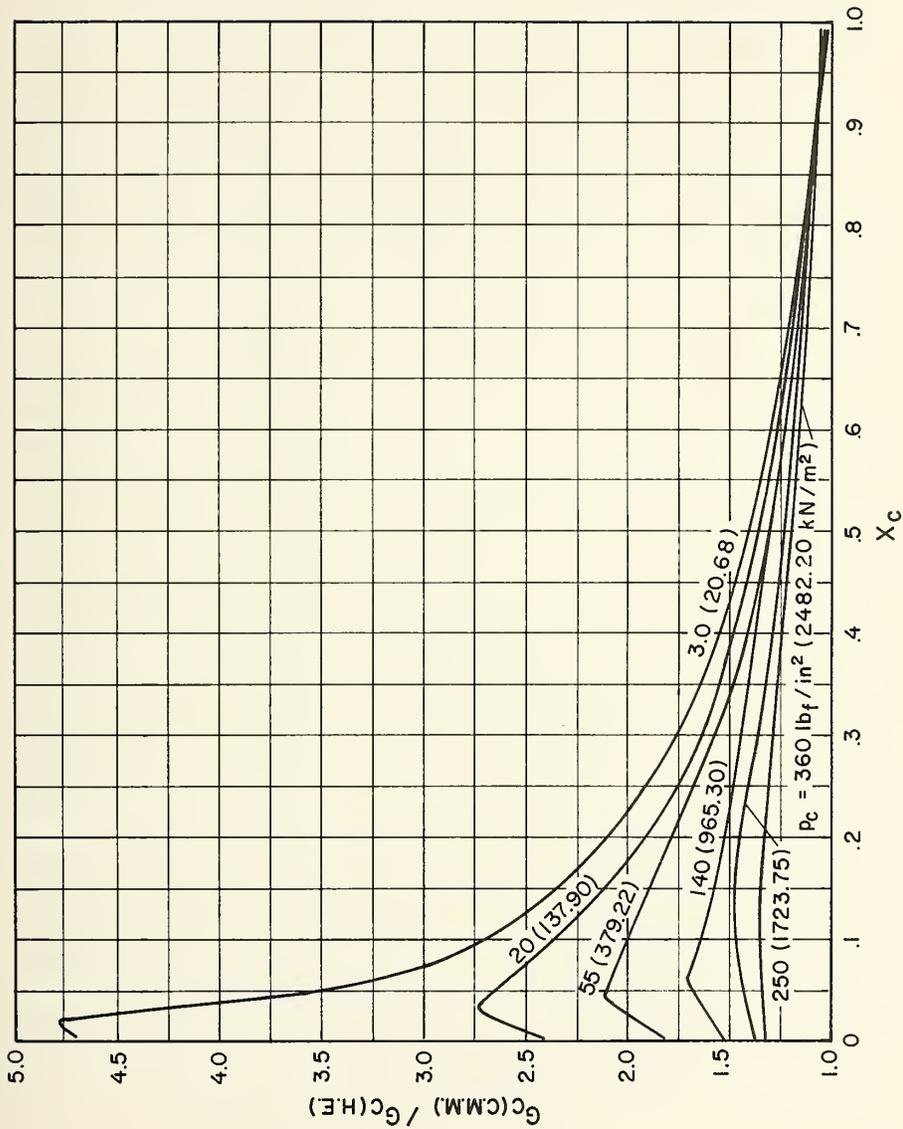
Figure 2.2-19   Predicted ratios of Cruver and Moulton (1967) and Moody (1965, 1969) to homogeneous, thermal equilibrium model results for oxygen.

43
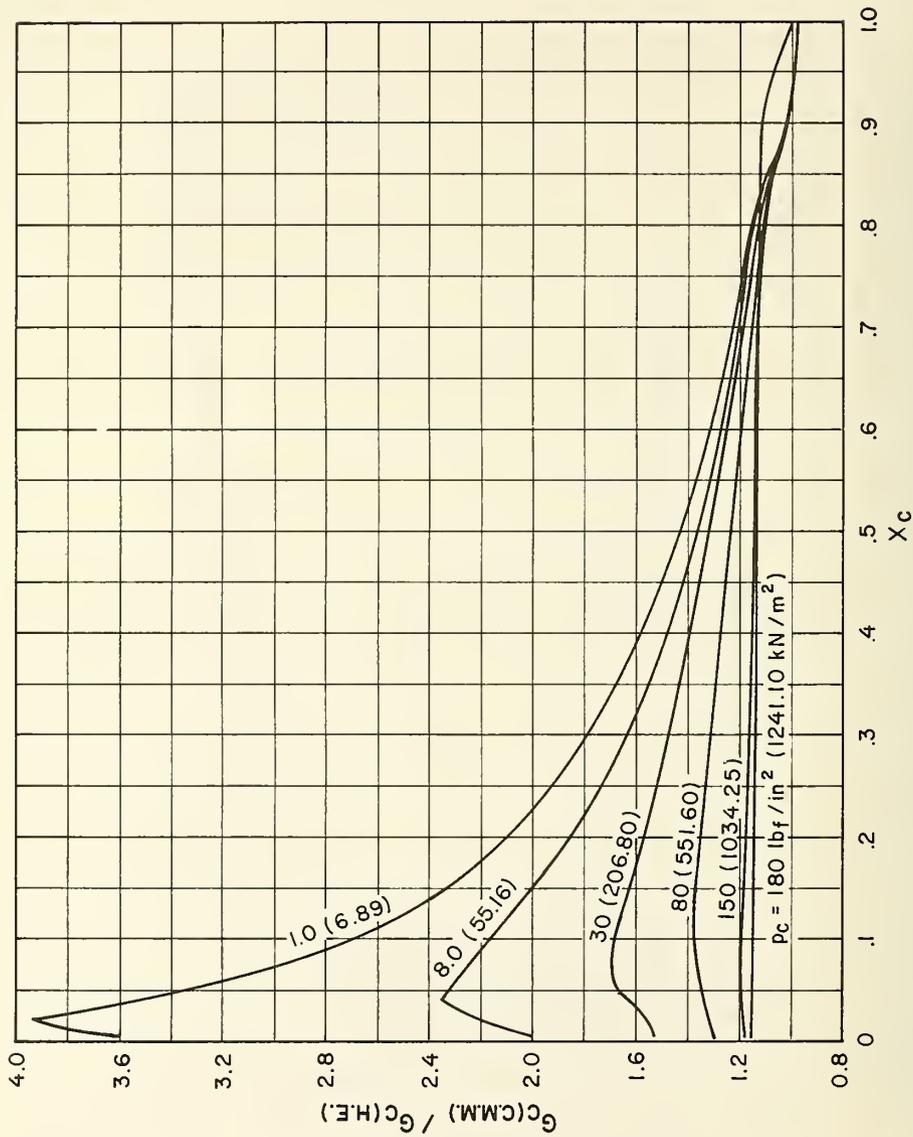
Figure 2. 2-20   Predicted ratios of Cruver and Moulton (1967) and Moody (1965, 1969) to homogeneous, thermal equilibrium model results for hydrogen.

44
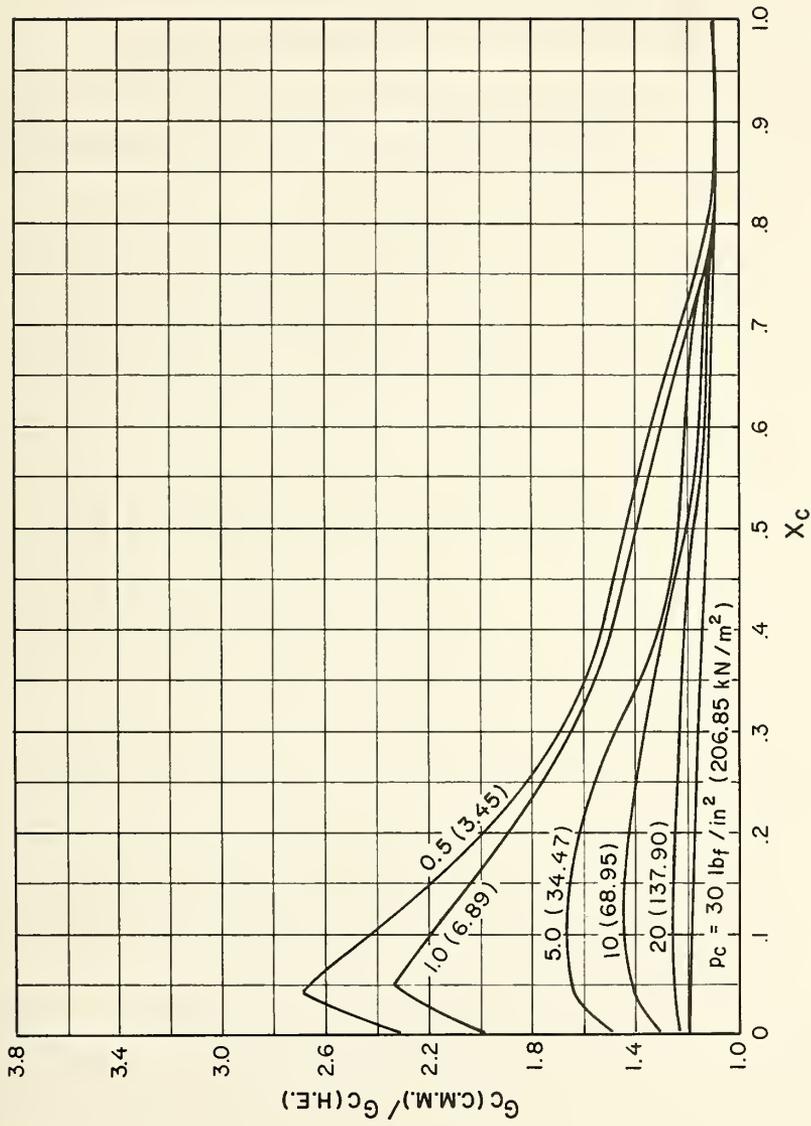
Figure 2.2-21   Predicted ratios of Cruver and Moulton (1967) and Moody (1965, 1969) to homogeneous, thermal equilibrium model results for helium.

For all analytical models the following properties data were used:

Helium - McCarty (1970)

Hydrogen - Roder, Weber and Goodwin (1965)

Oxygen - McCarty and Weber (1971).

2.3 Gas Flow Correction Factor for Separated Flow Model

This section reports the results of a preliminary study to investigate the feasibility of employing a general correction factor (CF) to account for liquid blockage of gas flow and, thus, to predict critical two-phase flow ratio assuming the gas, with its adjusted flow area, is at that point in critical flow.

Those investigators using analytical models which assume separated phases, first, introduce data which would have tended to substantiate the hypothesis that the gas is in critical flow when the two-phase system is in critical flow and that the critical and near critical flow could be adequately described by sets of equations for each phase together with expressions for the interface transport processes. In works reported by Smith (1968, 1972), Carofano and McManus (1969) and Sullivan and Wallis (1972) it was found that, with simple treatment, the equations tended to overpredict the critical flow rate. It was suggested both by Smith (1968, 1971) and Sullivan and Wallis (1972) that this discrepancy was essentially the result of the analytically-unaccounted-for blockage of the gas flow by the liquid. The equations of Smith (1968, 1972), for example, initially (before introducing a gas-flow correction) considered all the liquid to be flowing in a stream with a smooth interface. The liquid blockage of the gas flow would obviously be substantially greater, flowing, as it was experimentally shown to be, partially in droplets and partially at the walls with wavy surfaces.

All three investigators further reported that the effect of the interface processes appeared to be significant, but secondary, with respect to the critical and near critical two-phase flow condition. For this report, it was therefore decided to investigate the possible use of a very simple critical two-phase flow expression similar to that proposed earlier for the vapor choking model, for example, by Collingham and Fiery (1963), Zaloudek (1961), and Smith (1963), where the Martinelli and Nelson (1948) correlation was used to express the gas flow factor. The chief differences for the present case were that considerable work now has been reported indicating that the proposed critical flow mechanism appears to be valid for separated phase flow, and secondly, a correlating factor to be used in this case to account for gas flow blockage is derived from actual critical flow data rather than extrapolations of the general two-phase flow data as proposed by Martinelli and Nelson.

The simple mathematics for the case may be expressed as follows:

$$G_{(tp)(t)} = \frac{\dot{m}_{g(tA)}(CF)}{x_c A_t} \tag{18}$$

where $\dot{m}_{g(tA)}$ is the critical gas flow rate based on the total flow area

$$\dot{m}_{g(tA)} = \rho_g \, u_{gc} \, A_t$$

and

$$u_{gc} = a_g .$$

The term (CF) accounts for the liquid blockage of gas flow.

For the sonic gas velocity calculation, the ideal, isentropic gas relationship

$$a_g = \sqrt{\gamma R T}$$

was used. Additionally, for hydrogen, more accurate gas, sonic velocity calculations based on "real" gas properties were taken from Roder et al. (1965). The latter values are shown to improve the predictions at higher pressures, see figure 2.3-3.

For these sonic velocity determinations, the critical flow gas temperature was determined by calculating the temperature for an isentropic process to the critical pressure.

This selection was somewhat arbitrary — a more accurate appraoch would be to use the isentropic temperature (and isentropic sonic velocity expression) for higher qualities and isothermal flow process and isothermal sonic velocity for lower qualities. The isothermal sonic velocity for an ideal gas is

$$a_g = \sqrt{R T} .$$

With lower qualities, more interface energy transport is possible from liquid to gas, producing near-isothermal conditions for the gas. This pattern of behavior (isentropic, - high quality; isothermal, - low quality) is indicated by the critical pressure ratios reported by Smith (1968, 1972). Such an approach would improve the predictions, lowering the critical mass flow rates at lower qualities for the near-isothermal case.

This portion of Smith's report was considered preliminary, however, and such process refinements premature. Thus, the isentropic relationships were used both for the near-critical flow process and for the sonic velocity determination. The correction factor represents the ratio of the effective gas flow area to the total flow area available for the two-phase mixture. The correction factor (CF) was developed using the critical flow data reported by Smith (1968, 1972) using air-water flow. It is shown in figure 2.3-1.

This gas flow correction factor was then tested for possible use as a general correction factor for critical two-phase flow. The results for hydrogen and nitrogen are shown in figures 2.3-2 through 2.3-5.
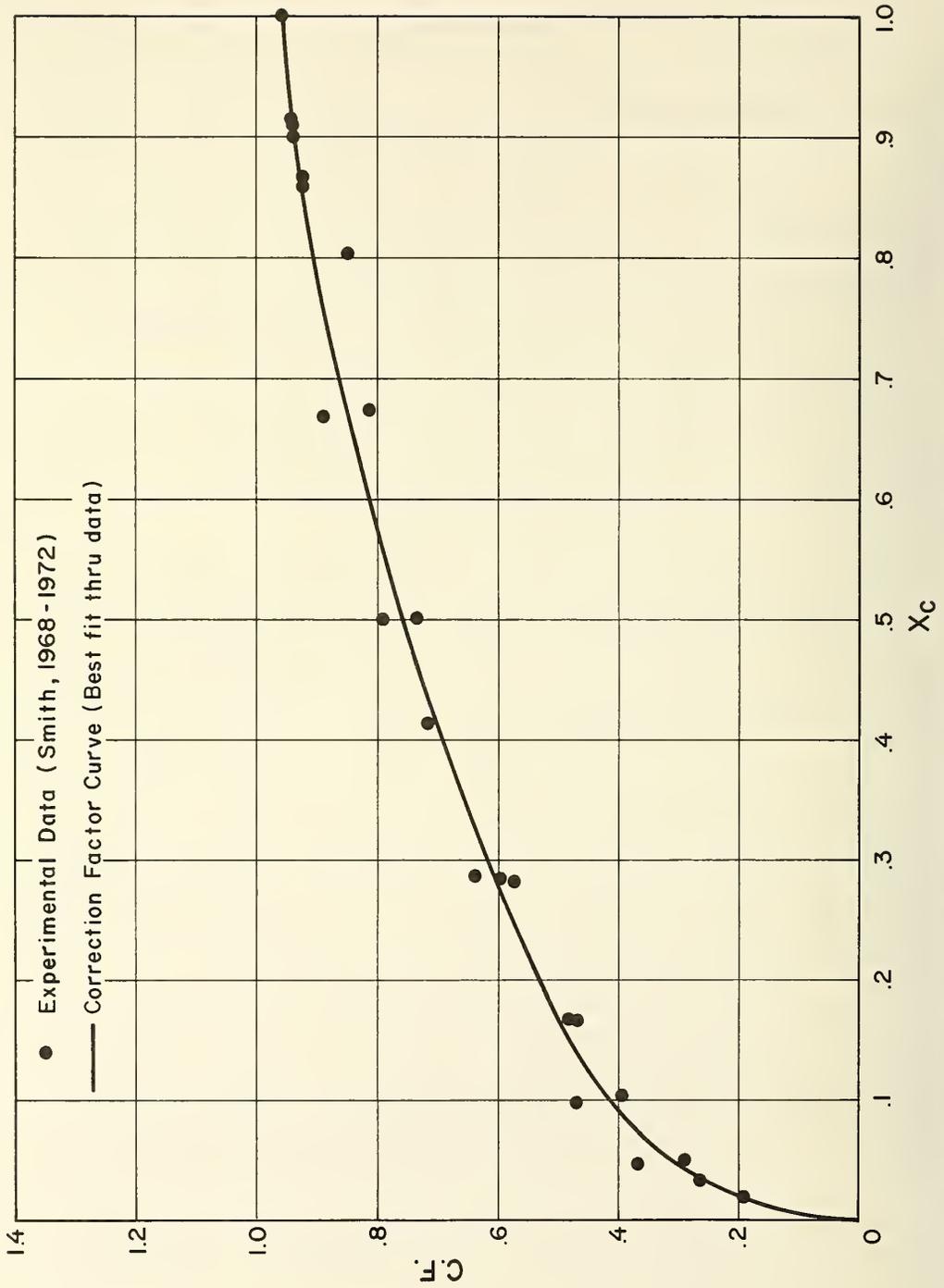
Figure 2.3-1   Empirical correction factor (CF) curve compared with experimental air-water data from Smith (1968, 1972).
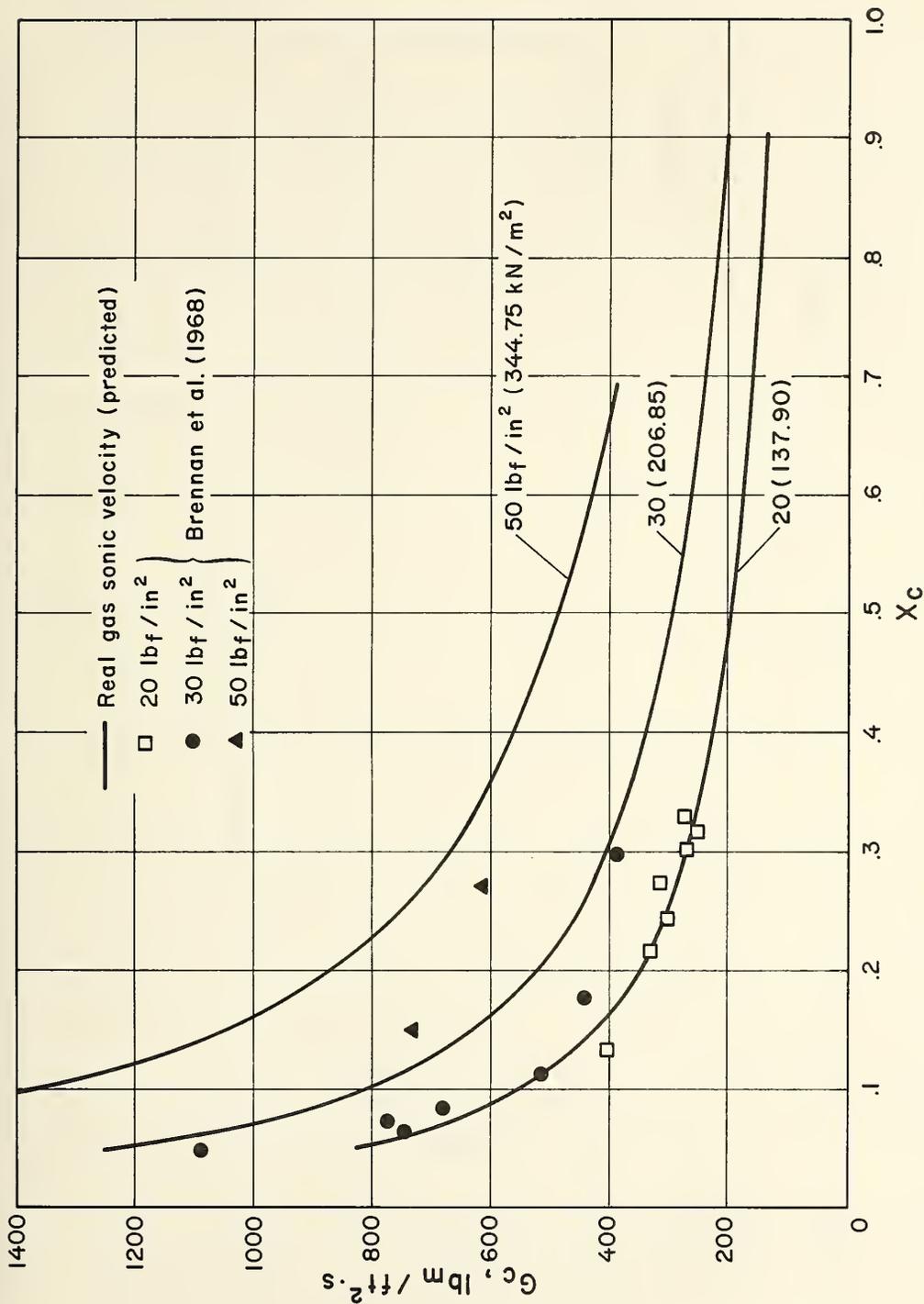
Figure 2.3-2   Predicted and experimental values of critical two-phase hydrogen flow rates using separated-phase model with correction factor from figure 2.3-1. Critical pressures are shown.
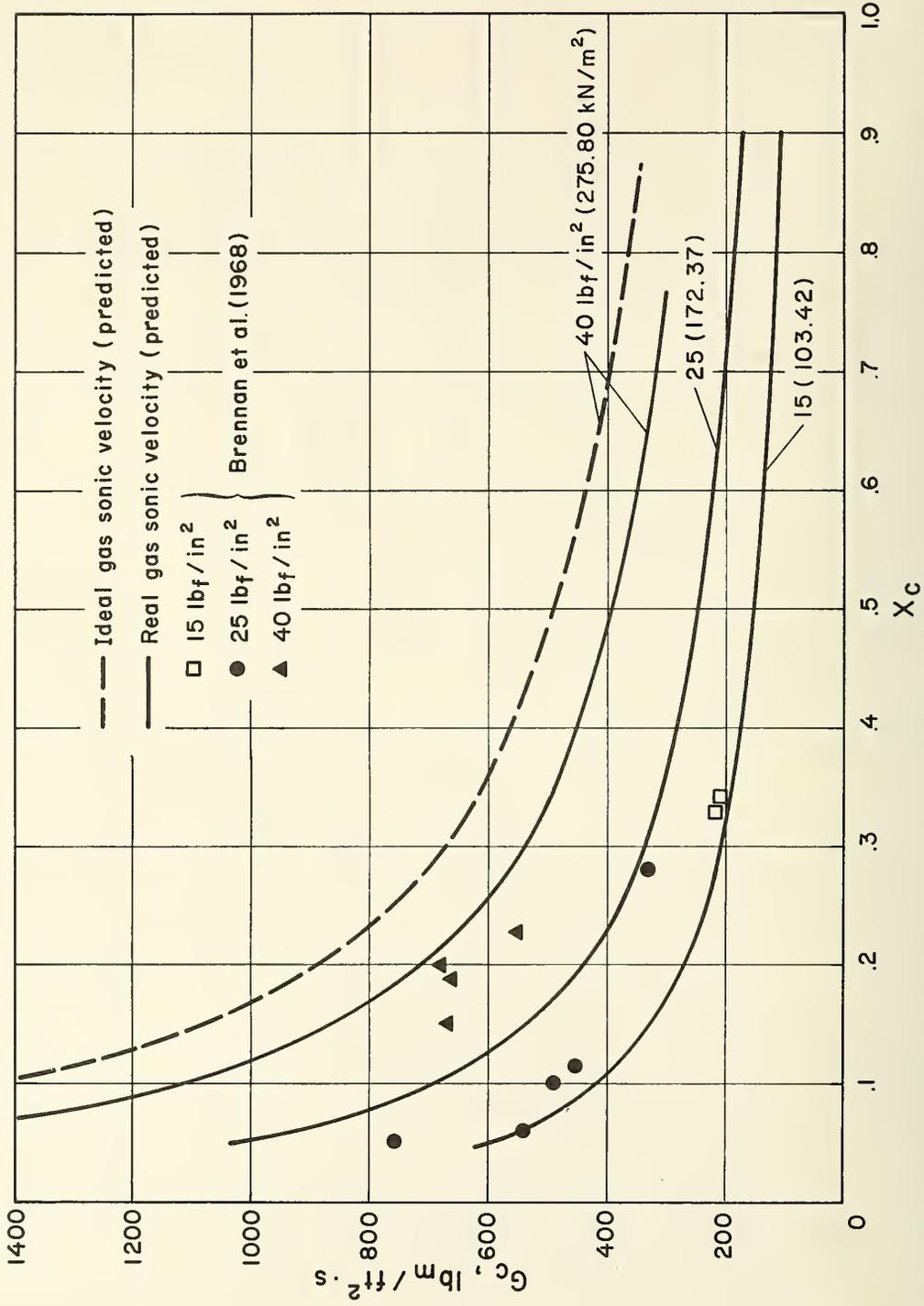
Figure 2.3-3 Predicted and experimental values of critical two-phase hydrogen flow rates using separated-phase model with correction factor from figure 2.3-1. Upper curves illustrate differences between real and ideal gas property values in sonic velocity determination. Critical pressures are shown.
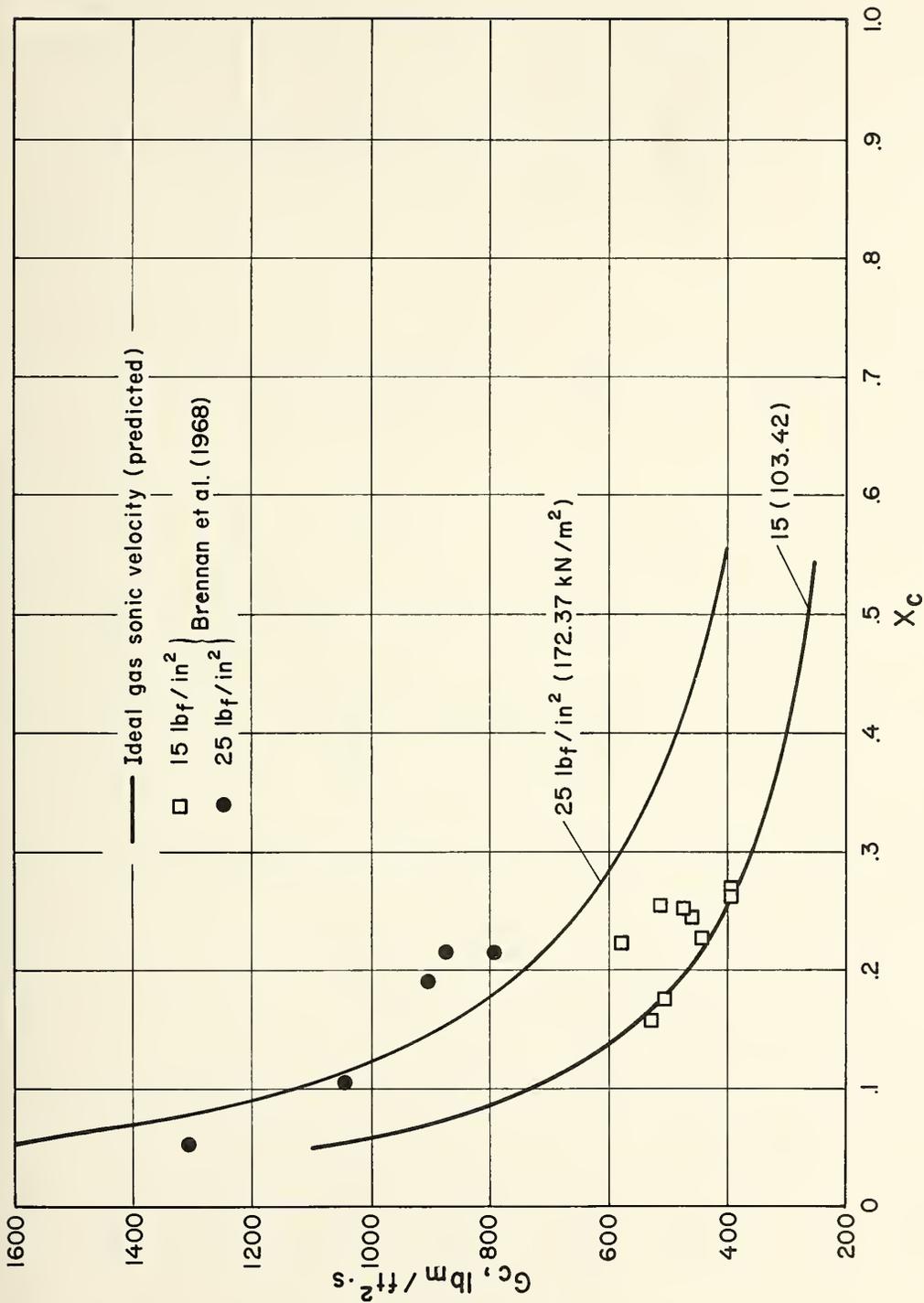
Figure 2.3-4   Predicted and experimental values of critical two-phase nitrogen flow rates using separated-phase model with correction factor from figure 2.3-1.   Critical pressures are shown.
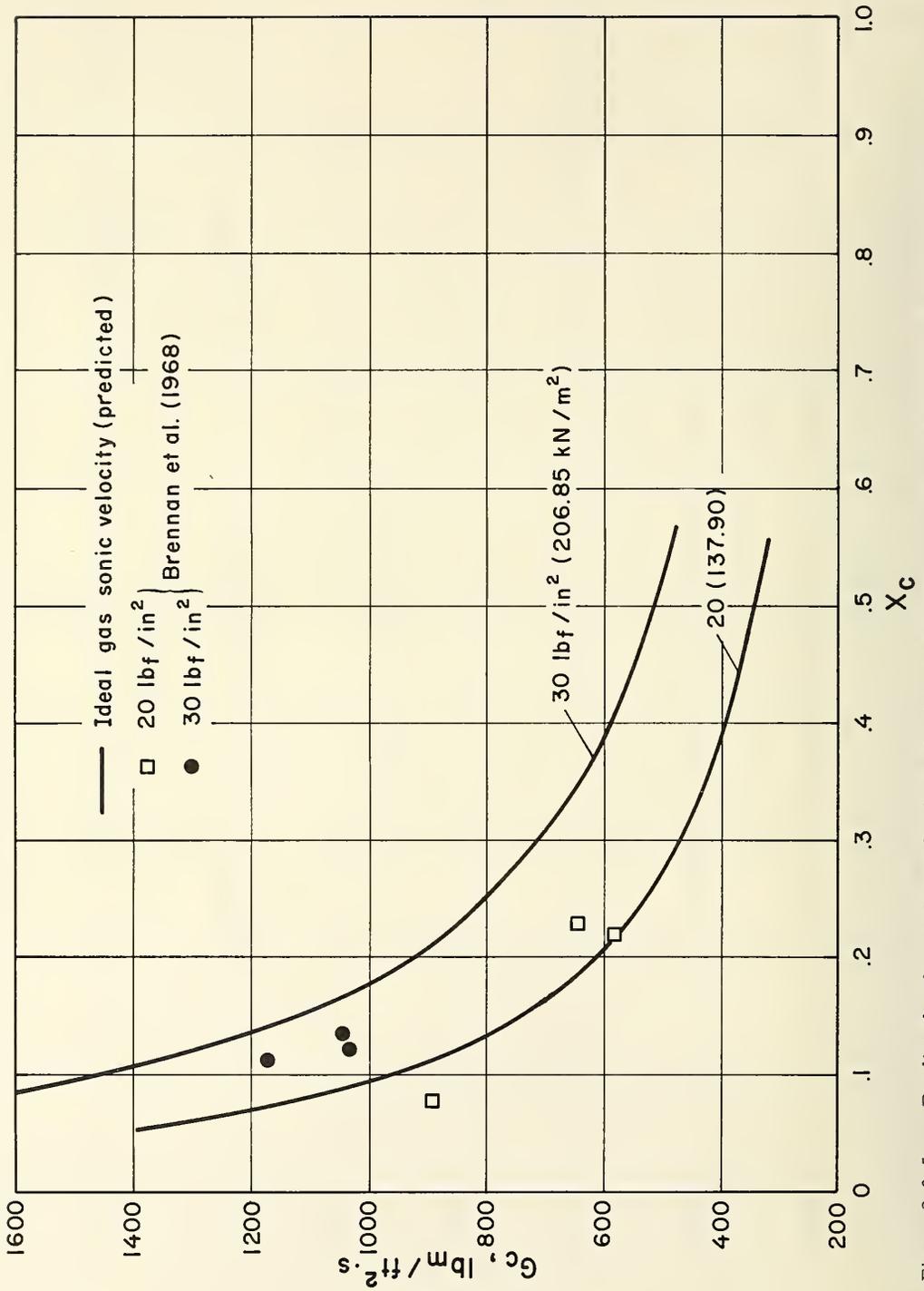
Figure 2.3-5   Predicted and experimental values of critical two-phase nitrogen flow rates using separated-phase model with correction factor from figure 2.3-1. Critical pressures are shown.

Unfortunately, most of the reported data fall in a lower quality region where use of the separated phase model may be uncertain or marginal. It would be expected that the correlation would improve at higher qualities. Results of Wallis (1972) and Sullivan and Wallis (1972) indicate that in spite of the uncertainty of separated flow, they show good air-water results for the lower quality as did Smith (1968, 1972). Both reported good results for air-water critical flow rates with x_c down to 0.02. Since this is only preliminary, the results are not conclusive but do indicate considerable promise for this very simplified correlation.

From figures 2.3-2 through 2.3-5, the agreement between predicted and analytical results appears reasonably good. The predictive reliability improves at lower pressures and higher qualities. This improvement may be the result of a more distinctly separated flow for these cases of relatively higher values of gas flow. One might expect, roughly, the same kind of predictive accuracy that would be achieved from the other correlations considered in this report.

In examining the curves in figures 2.3-2 through 2.3-5, one notes the agreement is better at low pressures than at high pressures. The high pressure predictions are improved by use of "real" gas properties in determining the gas sonic velocity, which is encouraging, but the analytical-experimental agreement remains better for the lower pressure data. Improvement in the lower pressure case may indicate that the correction factor would be better if made a function of the density ratio. Another way of achieving that objective would be to develop the original correction factor as a function of a defined void fraction (for example, a slip ratio of one) rather than using quality, as was the case for this preliminary study.

It should be pointed out, however, that if this correlation is valid, it is valid only for the case where the phases are essentially flowing separated. That is, for the generally high void fraction and the high quality flow cases. Visual observations by Smith (1968) indicated that perhaps the lower limit of applicability of this expression may be in the range of quality from 0.1 to 0.2. Below this quality, the flow could not be said to be clearly separated based on visual observations. Other work, Wallis (1972), mentioned earlier, indicates the limit may be considerably lower than that.

This gas correction factor could also be used to express the behavior of near critical as well as critical two-phase flow. For example, the expression could be used for sets of very simple expressions for the separated phases which treat the interface processes as negligable, as indicated by Smith (1968, 1972) and Carofano and McManus (1969) to be secondary. By this means, one could write a simple expression for the gas flow in which the area of the gas flow would be adjusted by the correction factor. This would allow one

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to study critical flow from upstream or stagnation conditions and predict both the pressure profile in the near critical flow region and the critical flow rate and the point of critical flow. Some preliminary work indicates that this method would be successful for the data of Smith (1968, 1972) in describing the near-critical pressure profile as well as the critical flow condition.

In summary, this extremely simple approach which assumes separated-phase flow, gas choking and the general use of an empirical correction factor looks promising and may be as accurate as the other, more complex, predictive methods.

2.4 Comparison of Analytical and Experimental Data

The ultimate objective of this section is to compare the analytical model results with the reported experimental data for cryogenic fluids. The analytical models compared will be limited to "mixture" models. All of the comparisons for the separated models are shown in the previous section. Since the data for cryogenic fluids were so limited and uncertain in quality, it was decided to begin these cryogenic studies with a general background of all critical flow data.

In figure 2.4-1 the experimental data to be compared in this report are shown over a background of all reported two-phase experimental critical flow data. The ordinate is the mass flow rate per unit area over that rate predicted by the homogeneous, thermal equilibrium model for that condition. This dimensionless plot tends to normalize the effects of fluid property variations. All data are compared at the point of critical flow using quality data computed on the basis of equilibrium (discussed later). It may be seen that the cryogenic fluid and air-water data, to be compared, fall in the general range of that reported for other fluids, except for some points from the Hendricks et al. (1972) data and that reported by Bonnet (1967). The Hendricks et al. (1972) data are generally for high pressure inlet conditions with a good deal of subcooling at the inlet. A substantial part of the pressure drop is not associated with fluid compressibility and it seems unlikely that such data should be compared with the other data reported here, which are almost entirely from fluids saturated or two-phase at the inlet. The determination of the quality based on equilibrium assumptions, for example, would be expected to be in much greater error for the Hendricks, et al. (1972) data. A better comparison would be with models using fluid stagnation conditions. This is done in the Hendricks, et al. (1972) paper. In this, they conclude that the homogeneous equilibrium model is fairly representative for the case of fluids above the thermodynamic critical region and the Henry (1968) model reasonably satisfactory for pressure and temperature below the critical point. In spite of the possible inappropriateness of showing these data on this plot, the data do not radically exceed the range of the other reported data.

The uncertainties of the reported critical pressure measurements of Bonnet (1967) may account for some of the deviation of that data. Also, the Bonnet (1967) data are from orifice or orifice-like flow which may not be true critical flow as discussed previously.

The nitrogen data of Campbell and Overcamp (1966) are not shown. These data are for orifice flow, and only stagnation (not critical) conditions were reported. These data are compared with the results of models using stagnation properties in figure 2.4-6. The experimental hydrogen data reported by Evans (1969) did not include either stagnation or critical conditions and could not be used in this report.

Figures 2.4-1 through 2.4-5 all compare critical flow data at the point of critical flow. As mentioned previously, a major problem is in the determination of the quality. In order to minimize this problem, the data to be compared are from air-water systems with negligible mass transfer and from straight sections where more time would be available to achieve the equilibrium upon which the quality calculations are based. Thus these comparisons, except for the air-water case, are made assuming thermal equilibrium in reducing the experimental data for quality. In spite of this approximation it is felt that reasonably good comparisons can be made. This approach has become fairly conventional and by this means all data are compared on the same basis. Comparison of data at stagnation conditions avoids this problem but introduces uncertainties regarding the flow process from stagnation to critical conditions. For further background information, figure 2.4-2 shows non-cryogenic fluid, single component data reported since Smith (1963) with a background of data reported previous to 1963. It should be noted again that the range of reported data has not changed substantially. There is a trend for the steam-water data to approach the values of the homogeneous, equilibrium model as the pressure goes higher. One explanation for this is that the homogeneous predictions are approached as the liquid-gas density ratio approaches unity. If this is the case, cryogenic fluid data, with density ratios much nearer to one than essentially all steam-water data, could be expected to lie closer to the homogeneous equilibrium model predictions. Further, hydrogen and helium data could be expected to be nearest to the homogeneous, thermal equilibrium model predictions of all fluids at the same pressure and quality. Not enough data are reported to either refute or substantiate this hypothesis. Continuing the general background study, in figure 2.4-3 the air-water data are compared with results from the Fauske (1961) and the Cruver and Moulton (1967) and Moody (1965, 1969) model. Air-water was chosen because it represented a system with one less degree of freedom (mass transfer — condensation or vaporization). Unfortunately, for this case and for subsequent figures in this section, the range of experimental data is roughly the same as the range of the results from the analytical models. This does not allow specific conclusions with respect to the models. In general the agreement is not bad and the trends of the experimental and analytical data are similar. There are major

discrepancies at low qualities indicating that the suggestions of Cruver and Moulton (1966) and Fauske (1965) may be correct — that models compared have slip ratio errors which compensate for the error in assuming equilibrium. For the air-water case

$$dx/dp = 0.$$

The value of dx/dp has the greatest influence on the predicted single component critical flow rate at low quality. Therefore if dx/dp is not evaluated properly in the expression and that error is compensated for by an incorrect evaluation of another term (for example a high value for slip velocity), then the compensating error would appear in the air-water case where dx/dp is zero. This may be the reason the models using equilibrium assumptions for evaluating dx/dp show relatively poor prediction reliability at low quality for the air-water data. In addition to comparison with the "mixture" analytical models discussed, the separated flow models of Smith (1968, 1972) and Carofano and McManus (1969) have been shown to match their own data well and, noting the overlap of their experimental data, would be expected to fit the total air-water data of both papers. Both experimental and analytical data follow the same trend. The homogeneous frozen flow model and the Henry model at critical conditions show close predictive values, as they should, because the Henry model assumes a slip ratio of unity (essentially homogeneous) and a low value for dx/dp (approaching frozen flow). These both underpredict Brennan et al. (1968) data which probably reported more accurate critical flow pressures than that of Bonnet (1967).

Figure 2.4-4 shows the nitrogen data compared with results from the analytical models of Fauske (1961), Cruver and Moulton (1967) with Moody (1965, 1969), Henry (1968) and Henry and Fauske (1971) and the homogeneous, non-equilibrium (frozen flow) model. Again, because of the experimental data spread and uncertainty, no firm conclusions can be made regarding the reliability of the models. The other two models, Fauske and Cruver and Moulton with Moody show somewhat better predictions and overpredict the Brennan et al. (1968) data. Other plots show this pattern of comparative plots generally holds for data at other pressures. It seems any conclusions, other than one might expect roughly a ± 30% reliability from the models, would not be warranted. Figure 2.4-5 shows the hydrogen data of Brennan et al. (1968) compared to the same models as previously mentioned. Better general agreement is achieved than with nitrogen, with the homogeneous or near homogeneous (Henry) models showing slightly better predictive results. Conclusions again are very dangerous but it may be that predictions improve as the density ratio gets smaller as in the case of hydrogen compared with nitrogen at the same pressure. The reliability here appears to be on the order of ± 20%. Figure 2.4-6 displays experimental data on the basis of stagnation conditions rather than critical conditions shown in the previous plots. The experimental data are from Bonnet (1967) and Campbell and Overcamp (1966), both, as previously mentioned, from orifice or orifice-like flow. Uncertainties again prevent
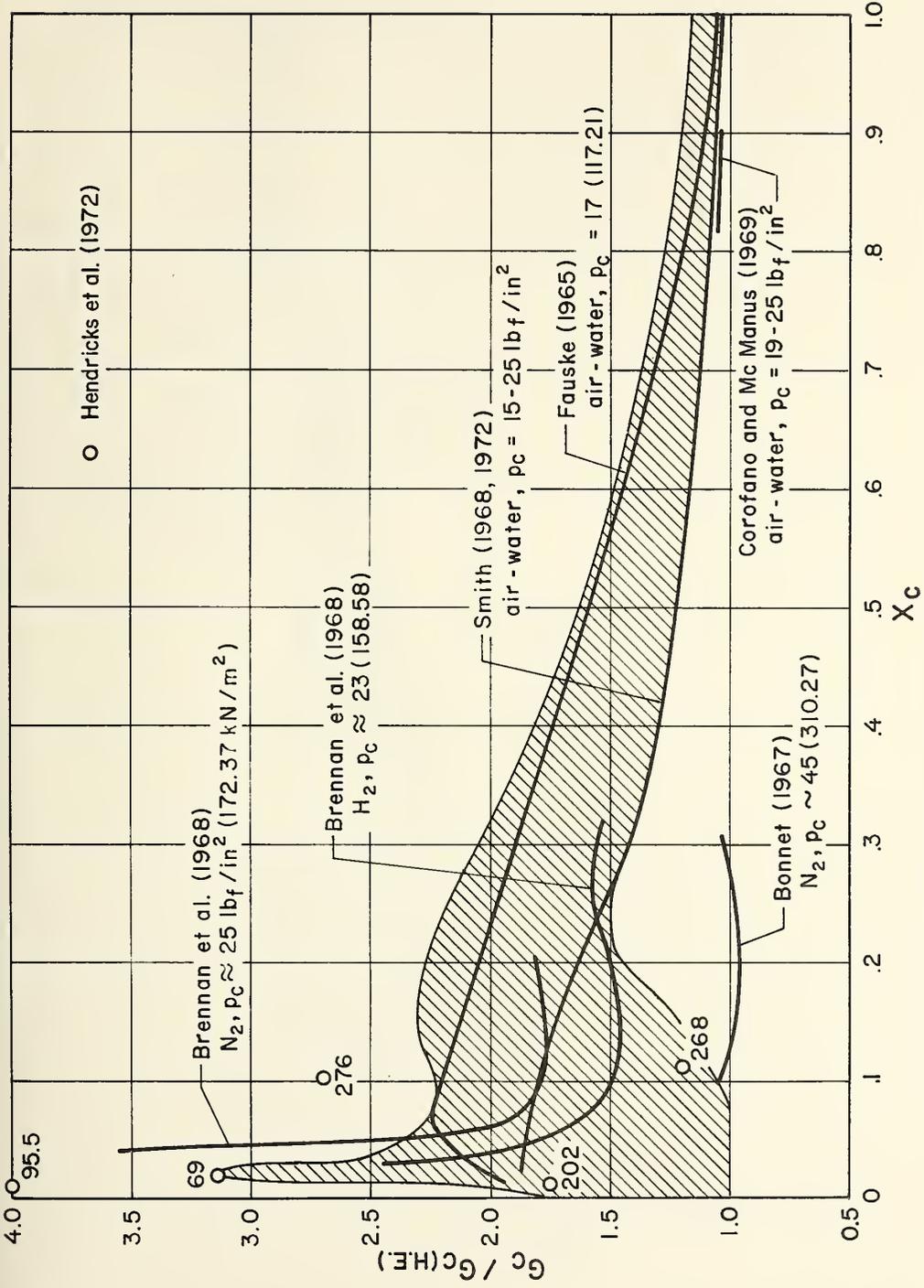
Figure 2.4-1   Experimental data used in comparative studies in this report shown on a background of all reported critical flow data. Numbers near Hendricks et al.(1972) data points indicate critical pressures in lb$_f$/in^2.
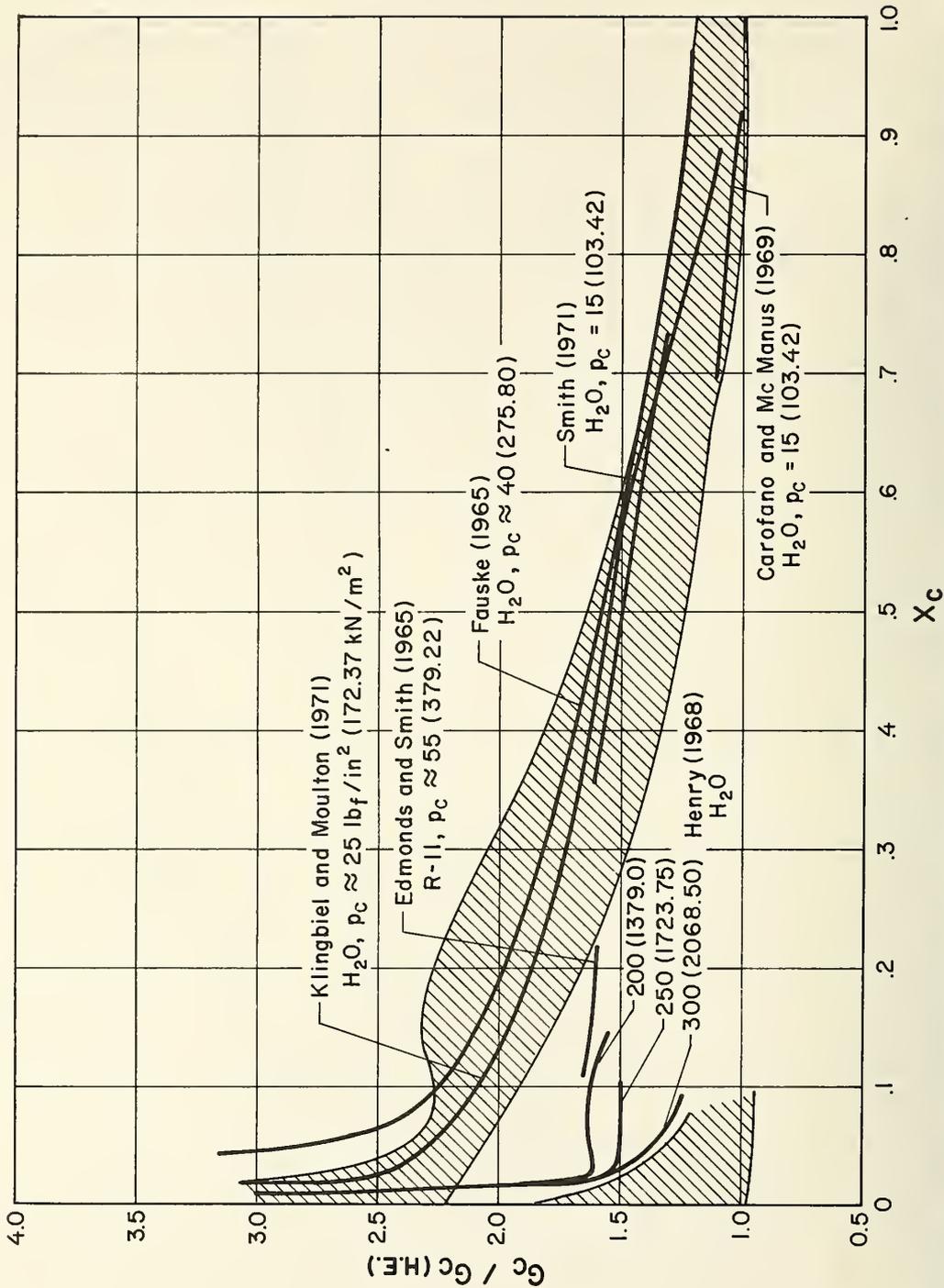
Figure 2.4-2   Experimental critical flow data reported since Smith (1963) on a background of data reported prior to 1963. Data shown in figure 2.4-1 used in comparative studies in this report are not included.

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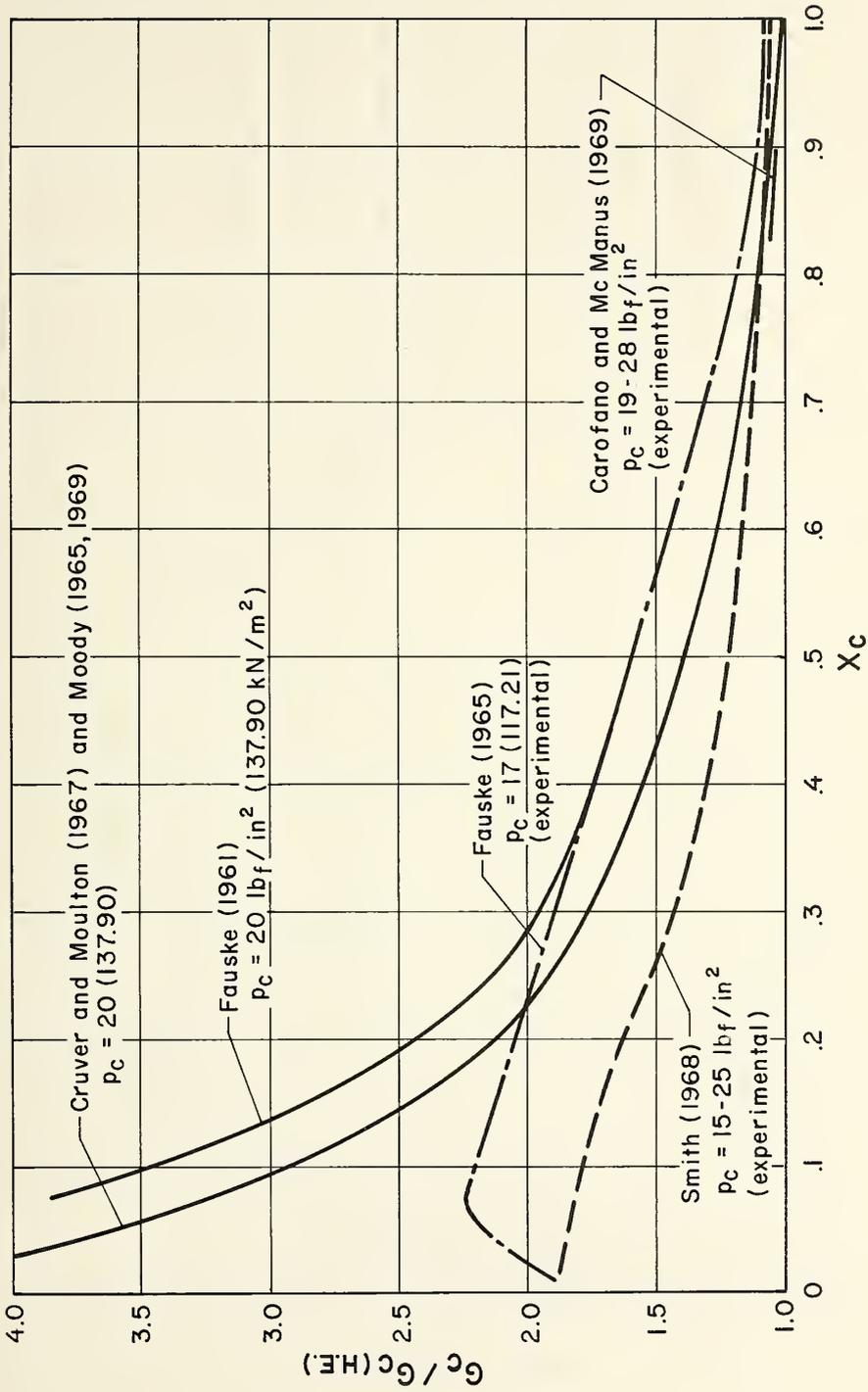
Figure 2.4-3   Comparison of the predictive curves of Fauske (1961) and Cruver and Moulton (1967) and Moody (1965, 1969) with three sets of experimental data for air-water.
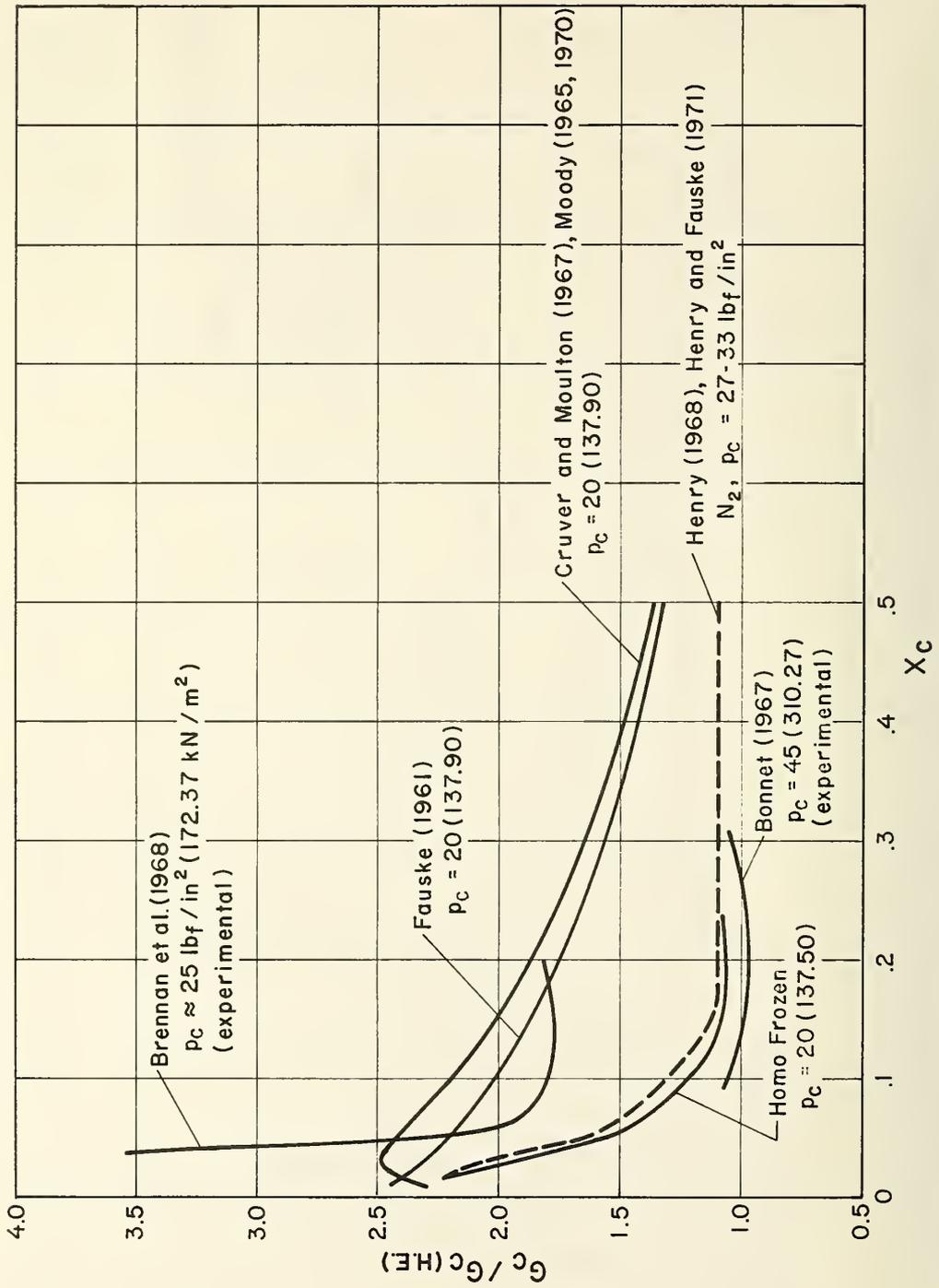
Figure 2.4-4   Experimental critical flow data for nitrogen from Brennan et al.(1968) and Bonnet (1967) shown with predicted results from four analytical models.
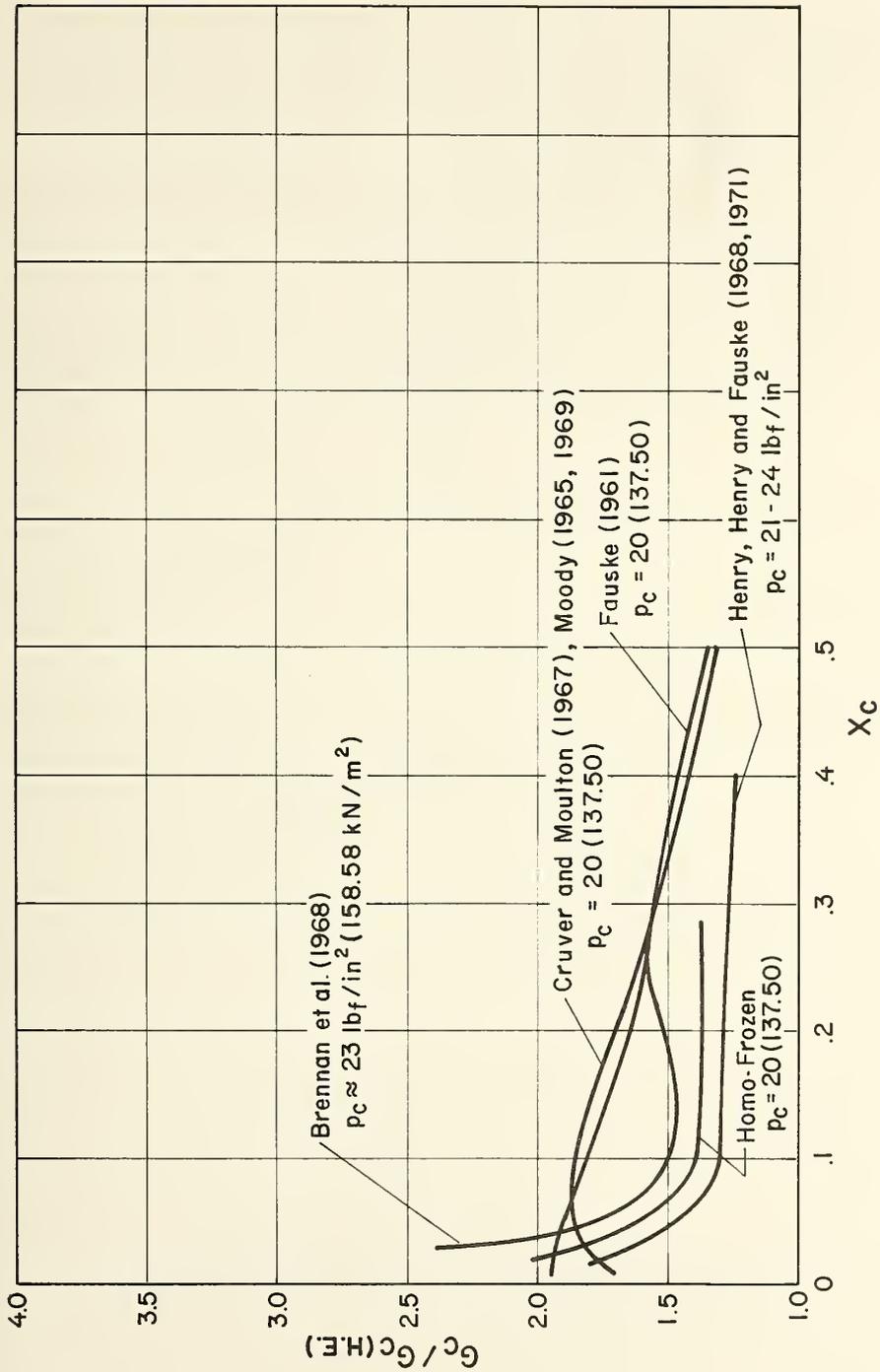
Figure 2.4-5   Experimental critical flow data for hydrogen from Brennan et al. (1968) shown with predicted results from four analytical models.
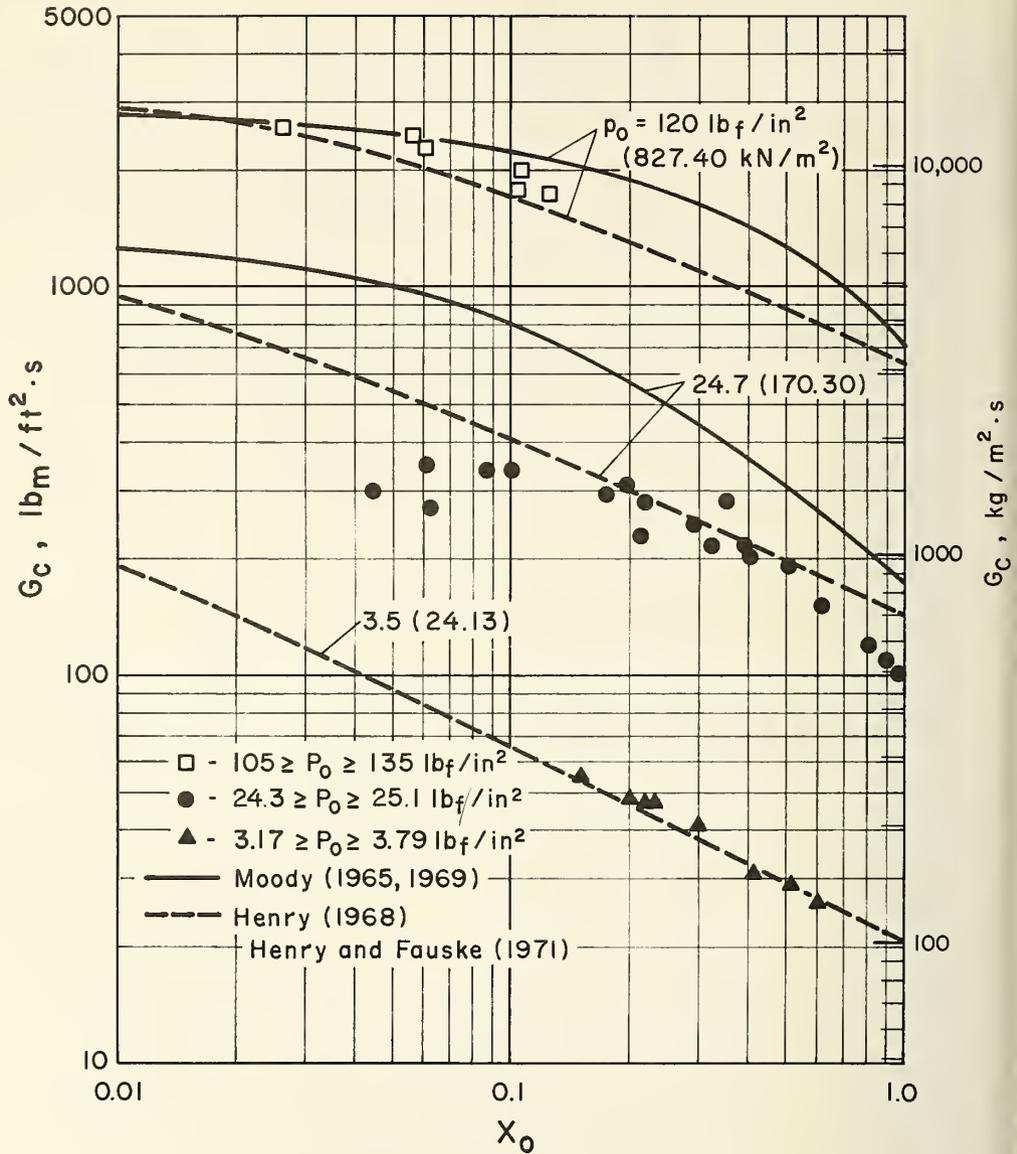
Figure 2. 4-6   Two-phase orifice flow with nitrogen.   Figure from Henry and Fauske (1971) with stagnation property curves of Moody (1965, 1969) added.

specific conclusions but the agreement appears fairly good and all data follow the same general trends. Models using stagnation properties must include a representation of the process from stagnation to critical conditions. This representation can be very useful for design purposes, but it does have one disadvantage in comparative studies such as these in that it adds another uncertainty to the list for possible compensating or additive error.

In concluding the discussion of the comparative studies for the cryogenic fluids, one must first observe that the agreement between most of the models presented and the experimental data are fairly good considering the uncertainties of both the models and the experimental data. Since the range of reported experimental data is approximately the same range of the model predictions, it would not seem reasonable to make very firm conclusions regarding the use of one model with respect to another on the basis of the critical mass flow rate.

As mentioned previously, the predictions assuming separated phases, gas choking and the use of a correction factor appear to have about the same reliability as the predictive results discussed here.

3. RECOMMENDATIONS FOR DESIGN USE AND COMPUTATIONAL AIDS

In keeping with the foregoing conclusions, (section 2.4), it seemed only reasonable for this report to present the results of a fairly wide range of models leaving the designer to choose that which best fits his particular situation after he has made his own evaluation of the adaptability and reliability of the model for his specific case. It is believed that showing all of the predictive models in a collected form, and in a form which the critical mass flow can be determined quite quickly, should be of considerable aid to designers.

On the basis of the equivalence of the model to the physical flow situation it would appear those which assume thermal equilibrium would be somewhat inferior to those which do not. It would also appear that homogeneous or near homogeneous mixture type models may be better for low quality flows and the separated models show considerable promise for the higher quality flows.

The success of the homogeneous, non-equilibrium model (frozen flow) and the Henry model, which assumes low values for mass transfer and gas-liquid velocity ratio, at lower qualities indicate they may tend to describe that flow process reasonably well. This suggests that those models which employ high slip ratios and assume equilibrium for low quality flow may be in error with their assumptions. Some experimental data by Smith (1968) indicate equilibrium may be unlikely, and some low quality studies of the slip ratio report it to be rather low, Vogrin (1963), Fauske (1965). The separated-phase models seem to describe the higher quality case reasonably well and show promise of predictive reliability. Thus, although the comparative studies do not indicate a clear superiority for any model,

if the foregoing arguments are valid, it may be safer to use the models which appear to have the most reliable assumptions when extrapolating to other fluids, geometries, and flow processes.

With respect to design utility, the models which predict the critical mass flow rate from upstream conditions must be considered superior because the data are much better known at these points and these data and analytical procedures would have more and better utility in design use.

This reasoning would show a preference for the stagnation models of Henry and Moody and for the separated-phase models.

For simplicity, the homogeneous frozen and the separated flow expressions are desirable. Their reliability is less-well established but it appears to be roughly the same as for the other methods.

4. RECOMMENDATIONS FOR FUTURE WORK

For improvements in understanding and predictive capability for critical two-phase flow, the greatest need appears to be for better experimental data. This is the only way to resolve differences between analytical approaches and to improve them.

The needed experimental data may be divided into four categories:

1) Critical flow rates in simple systems as a function of stagnation for upstream conditions and conditions at the point of critical flow. Primary variables should be flow rate as a function of pressure, temperature, quality (if possible) and fluid properties. These data are required to assess the reliability of the proposed analytical (predictive) methods. Other desired measurements of significant variables follow.

2) Flow pattern data. These data should include velocity ratios between the liquid and the gas in addition to the more conventional emphasis on liquid-gas distributions. It would be very useful to know the conditions where the phases do and do not essentially flow in separate streams. These data would be very helpful in deciding which analytical model to use.

3) Geometry effects - The simple systems in (1) may be divided into constant-area and venturi or nozzle systems. In the next division of geometry effects, one could investigate the effect of section cross sections and lengths. It would be desirable to know the limit of orifice-like flow where critical flow is not possible.

Geometry effects generally would be expected to influence the degree of equilibrium and the general flow structure (such as flow patterns and the prevalence of secondary flow). These effects so far appear to be secondary in many cases, but they are surely never negligible for accurate predictions and in some cases undoubtedly are the controlling parameter.

4) Interface processes and property effects. Significant work in this area may have to wait for further developments in the preceding (3) recommended studies, but the final key to critical two-phase flow must lie in an accurate expression for these processes. These data will finally allow the derivation of reliable relationships to predict the behavior of various fluids if the behavior of one fluid is known. Meanwhile some progress can be made with correlations based on behavior of the fluids in other flow conditions.

5. NOMENCLATURE

English Letters

a	=	Sonic velocity, L/t	$\dot{m}$	=	Mass flow rate per unit time, M/t
A	=	Area, cross-sectional, flow area, L^2	p	=	Pressure, M/Lt^2
c	=	Specific heat, $L^2/t^2\,T$	R	=	Gas constant, $ML^2/t^2\,T$
CF	=	Liquid blockage correction factor	s	=	Specific entropy, $L^2/t^2\,T$
C	=	Coefficient	t	=	Time, t
D	=	Tube diameter, L	T	=	Temperature, T
g	=	Gravitational acceleration, L/t^2	u	=	Velocity, L/t
G	=	Mass flow rate per unit area, M/L^2	v	=	Specific volume, M/L^3
h	=	Specific enthalpy, L^2/t^2	V	=	Volume, L^3
k	=	Thermal conductivity, $ML/t^3\,T$	x	=	Quality
m	=	Mass, M			

Greek Letters

γ	=	Specific heat ratio, c_p/c_v	ρ	=	Density, M/L^3

Subscripts

c	=	Critical flow	g	=	Gas
CF	=	Liquid blockage correction factor	o	=	Stagnation conditions
f	=	Liquid	p	=	Constant pressure
fg	=	Liquid-gas	t	=	Total
ft	=	Total liquid	tp	=	Two phase

<u>Subscripts for Analytical Model Nomenclature</u>

E   =   Equilibrium

H   =   Homogeneous

x   =   Constant quality (frozen, metastable)

H_o =   Henry (1968) Henry and Fauske (1971)

M_o =   Moody (Stagnation)

CMM =   Cruver and Moulton (1967), Moody (1965, 1970) (critical)

F   =   Fauske (1961) (critical)

S_{cf} =   Separate equations (gas choking) with correction factor

CM   =   Cruver and Moulton (1967) (critical)

6. BIBLIOGRAPHY

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APPENDIX A

Example Problem

A converging restriction in a hydrogen flow line creates a possible critical flow case. Explore the critical or choking flow probability for the following conditions:

$$
\begin{aligned}
\text{Upstream flow area} &= 1.44 \text{ in}^2 &= 0.01 \text{ ft}^2 \\
\text{Downstream flow area} &= 0.288 \text{ in}^2 &= 0.002 \text{ ft}^2 \\
\text{Mass flow rate} &= 0.2 \text{ lb}_m/\text{s}
\end{aligned}
$$

$$
G_{max} = \frac{\dot{m}_{tp}}{A_t} = \frac{0.2}{0.002} = 100 \text{ lb}_m/\text{ft}^2 - \text{s}
$$

Case No.	Estimated Upstream Pressure $\sim p_0$ $(\text{lb}_f/\text{in}^2)$	Estimated Upstream Quality $\sim x_0$	Estimated Exit Pressure p_e if critical $(\text{lb}_f/\text{in}^2)$	Estimated Exit Quality x_e if critical
1	14.7	0.1		
2	14.7	0.8		
3			8.0	0.1
4			8.0	0.8

TABLE OF VALUES OF G FROM ANALYTICAL MODELS

Case No.	Model	Figure	$G(\text{lb}_m /\text{ft}^2 - s)$
1	Moody Stagnation	2.2-11	130
	Henry Stagnation	2.2-14	156
2	Moody Stagnation	2.2-11	37
	Henry Stagnation	2.2-14	62
3	Homogeneous Thermal Equilibrium	2.2-2A	120
	Fauske (1961)	2.2-5	230
	Cruver-Moulton Moody	2.2-8	260
4	Homogeneous Thermal Equilibrium	2.2-2A	53
	Fauske (1961)	2.2-5	71
	Cruver-Moulton Moody	2.2-8	63
	Separated-Phase (by calculation)	computed using sonic velocity & (CF) eq (18)	61

The probability of critical flow may be determined from the table of values of G.

Case 1 appears to be safe from the critical (or choking) flow condition as both model values of G (130, 156) are substantially above the specified value (100). This assumes the estimates of p_0 and x_0 are more reliable than the margin of safety indicated by the values of G.

Case 2 values of G indicate that the flow will be critical and that the specified mass flow rate will not be achieved.

Case 4 again indicates that critical flow is almost certain and also that the desired mass flow rate cannot be achieved. Separated-phase values calculated from (18) are included in the table. They were not included in Case 1 or 2 because results from stagnation conditions, although possible, have not been computed. For Case 3, it was thought that at the lower quality (0.1), the flow may not be clearly separated and the use of the model might not be appropriate. For Case 4 it is interesting to note that both the mixture and separated models give approximately the same prediction.

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16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

This work presents a state-of-the-art survey intended to be useful to a designer of equipment involving two-phase flow of cryogenic fluids. It is desirable to assess the probability of critical, or choking, flow in such a system and, if possible, estimate the critical flow rate. The literature is surveyed, primarily since Smith (1963), and the predictive results for several analytical models are evaluated and compared with experimental data. These results are discussed; however, no firm conclusions are reached because, often, the spread of experimental data is equivalent to the predictive results from the models. Finally, computer evaluation are presented for oxygen, hydrogen and helium along with some design recommendations.

17. KEY WORDS (Alphabetical order, separated by semicolons)
Analyses; analytical models; choking flow; critical flow; cryogenic fluids; helium; hydrogen; oxygen; reviews; two-phase flow.

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