Superconducting Quantum Interference Devices: an Operational Guide for rf-Biased Systems
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Superconducting Quantum Interference Devices: an Operational Guide for rf-Biased Systems

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Superconducting Quantum Interference Devices: An Operational Guide for rf-Biased Systems

D. B. Sullivan

The report discusses a number of practical considerations concerning the operation and application of rf-biased Superconducting QUantum Interference Devices (SQUID's). In the course of routine operation of these devices one amasses a set of operational rules, many of which never reach the open literature. This report is aimed at filling that void. Topics of discussion include: the readout circuitry, operational limits of the SQUID, rf-coupling to the SQUID, flux transformers, and shielding.

Key Words: Electrical measurements; quantum interference devices; superconductivity.

I. INTRODUCTION

This treatise is intended to serve as a practical operational guide for the use of rf-biased Superconducting QUantum Interference Devices (SQUID's). Part of this material may be found in a scattering of publications and part of it is an outgrowth of the experience of people who use the devices routinely. It is the author's intention to dwell on a number of the practical aspects from this latter category which will, hopefully, help people to avoid some of the pitfalls through which a number of us have already stumbled. The theory of operation of the device has been adequately covered elsewhere [1,2]. The SQUID also has a mechanical analog which can be helpful in developing some insight into its operating characteristics [3].

1Figures in brackets indicate the literature references at the end of this paper.
Before starting into the discussion, it should be pointed out that at this date two companies manufacture complete rf SQUID systems. These devices are adaptable to the measurement of a number of electrical quantities and may suit the needs of many experimentalists.

The SQUID systems may be divided into two parts; the room temperature readout circuitry and the SQUID coupled to a resonant circuit. The readout circuitry which is described in this treatise is homemade, but is quite similar in some respects to its commercial counterparts. The workings of this circuitry are discussed, but a preference is not necessarily suggested over the available commercial circuits. The emphasis in this discussion will be on the SQUID, the resonant circuit coupled to it, and how one uses the SQUID to make measurements. If commercial systems are suited to the application, the purchase of one would be the quickest and least painful solution to the measurement problem. There may be occasion to design the SQUID and input circuitry for a particular application, and it is hoped that this discussion will be of value for such cases. The commercial readout circuitry would most probably be suitable in any case. Considerable effort has already been expended on the design of these circuits and if economy of effort is a consideration, then one should consider these commercial units carefully.
II. FLUX-STATE READOUT SYSTEM

In essence, the SQUID is a small superconducting inductance connected in series with one of a few types of weak Josephson [4] junctions. The junction is a parametric inductance with an impedance which is a periodic function of the flux linking the series inductor. The period of the response is \( \Phi_0 = \hbar / 2e = 2.07 \times 10^{-15} \) webers.

The flux-state readout is accomplished by measuring the radio frequency impedance of the ring containing the junction. The basic circuit for this readout is shown in figure 1. The SQUID is coupled inductively to the coil of a resonant circuit and the impedance changes of the junction are reflected into that circuit to be picked up as rf level changes by the amplifier system.

The usual frequency range for this measurement is 15 to 30 MHz which will be seen to be a convenient choice. The resonant circuit consists of a capacitor, the coil coupled to the SQUID, a coaxial transmission line, and the input of the rf preamplifier. This entire 'front end' circuit is tuned to resonate at the frequency which matches the rf oscillator and the center of the rf amplifier passband. The circuit will behave in a simple fashion as a lumped element resonant circuit as long as the coaxial line is shorter than a quarter wavelength at the oscillator frequency. At 30 MHz, \( \lambda / 4 \approx 2.5 \) meters and one can see that this requirement may become difficult to meet at higher frequencies. In any case the coaxial line in this type of system should be kept as short as is practical.

With the oscillator and tuned circuit set to match the center of the rf amplifier passband, the system behaves as follows. For certain levels of rf oscillation in the tuned circuit, the rf voltage across the tuned circuit is a periodic function of the flux applied to the SQUID. The rf oscillator is very weakly coupled to the tuned circuit by the variable
Figure 1. Basic rf readout circuit for a SQUID. The SQUID is coupled inductively to a resonant circuit which is driven by an rf current source. The changes of impedance of the SQUID, induced by altering the flux linking the SQUID, are reflected into the resonant circuit and produce a change of the rf voltage at the rf amplifier input. The amplifier and detector produce a signal at the output which is a periodic function of the flux linking the SQUID.
coupling capacitor shown in figure 1. Thus, the rf drive source is constant current in nature and the detected rf level is essentially proportional to the rf voltage appearing across the tank circuit. Figure 2 shows the response of the system to applied flux and variations of rf level. The detected output rises from zero as the oscillator level is increased (by varying the coupling capacitor). At some critical rf current value, the voltage across the resonant circuit becomes a periodic triangular function of the applied flux. On further increase of the rf current, this periodic response disappears and then reappears at a yet higher level with a reversal in phase (or an offset of $\varphi_o/2$). Further increase of the rf level yields subsequent patterns which are similar to the first one. The device is thus seen to respond periodically to radio frequency and dc fields, in fact, the rf field periodicity is also $\varphi_o$.

For the moment we continue to ignore the conditions necessary for this behavior (the response outlined above has been optimized by adjustment of input circuit parameters). The basic periodic readout can be used directly or we can add some simple circuitry to obtain a readout which is linear in the dc flux applied to the SQUID. Our complete readout package is shown schematically in figure 3. The low frequency oscillator can be used to sweep the field applied to the SQUID (the low frequency current is passed through the rf coil to generate this low frequency field at the SQUID). This operation allows one to generate oscilloscope pictures such as shown in figure 2. The same low frequency oscillator can be used to weakly modulate the SQUID flux for a phase sensitive detector (PSD). With the circuit arranged as shown in figure 3, and the modulation level adjusted to $1\varphi_o$ peak-to-peak, the phase sensitive detector (sensing the fundamental oscillation) will generate a positive or negative output as the dc flux level shifts the modulating signal to the right or left of the symmetry point shown in the figure. If this PSD output is fed back to generate a dc flux through the SQUID (with
Figure 2. Periodic Response of a SQUID. The output of the circuit of figure 1 (which is proportional to the rf voltage level across the tuned circuit) is plotted vertically. The horizontal axis represents the field applied to the SQUID. The different periodic patterns are obtained at different rf current levels (i.e., by varying the coupling capacitor shown in fig. 1). The parameter $\beta$ is discussed in section III.
Figure 3. Analog readout circuit. The low frequency oscillator is set at a modulation level of $\phi_0/2$. The phase sensitive detector (PSD) then produces a periodic response to the average or dc flux linking the SQUID. If the switch $S$ is closed the circuit acts as a servosystem which maintains a constant dc flux through the SQUID. Thus, an external flux applied to the SQUID will result in a dc feedback current (through the rf coil coupled to the SQUID) which generates a field opposing the applied field. The feedback current is therefore a linear function of the applied field.
Figure 4. Circuit for generation of the rf voltage-current display. An external rf oscillator is 100% modulated at an audio frequency and the detected rf voltage is displayed as a function of the rf current level. The essential information in this display is the same as that of figure 2 (generated by the circuit of fig. 1). However, this circuit provides continuous display of the response at different rf current levels and is convenient for SQUID adjustment (see sec. V). The PSD (phase sensitive detector) and low frequency oscillator are not used but are shown for completeness.
an appropriate feedback resistor as shown in the figure) then the system will lock onto a peak (or valley) of the periodic response function. Now, as an external field is applied to the SQUID, a feedback current will flow to oppose the applied field and the flux linking the SQUID will not change. This feedback current can then be monitored and is a linear function of the applied field. The switch, shown in the figure, interrupts the feedback current and allows the system to be locked onto the flux quantum which is nearest the one corresponding to zero feedback current. A detailed discussion of the servo loop is contained in a paper by Giffard, Webb, and Wheatley [5].

Before proceeding to the next section, an alternate mode of operation will be mentioned. The output information shown in figure 2 involves two independent variables, the applied flux and the rf current level. Suppose we plot the rf voltage output at a fixed dc flux as a function of the rf current. Figure 4 shows the electronics arrangement for such a readout. In this case we turn off the internal oscillator and use a modulated external oscillator to provide the rf current. The modulating signal can be used to control the horizontal sweep on an oscilloscope and one thus obtains a picture such as shown in figure 5. The two interwoven curves represent states which are separated by one-half quantum of flux \( \varphi_0/2 \) linking the SQUID. The pattern displayed in this form is somewhat more convenient for preliminary adjustment of the SQUID since the rf current is continually sweeping through all of the states shown in figure 2. This means that one does not have to continually readjust the rf current during tune-up procedures. This mode of display is generally not convenient for actual measurements with the SQUID. It is possible to include within the electronics package, a modulator (for the internal rf oscillator) which is controlled by the internal low frequency oscillator. Thus this mode of operation could be obtained with a simple flick of a switch.
Figure 5. RF Current-Voltage characteristic of a SQUID. The two interwoven characteristics are separated by one-half quantum of flux linking the SQUID. These patterns are generated by the circuit of figure 4.
III. THE SQUID AND ITS OPERATIONAL LIMITS

Figure 6 shows four rf-biased SQUID's, three of which are in routine use at the present time. The first and simplest (which is not in general use now) is the single-hole, point-contact SQUID. This is one of the earliest types of SQUID's developed by Silver and Zimmerman [6] and because of its simplicity we will discuss its operational limits.

As mentioned earlier the junction is a parametric inductor. The form for this inductance is given by

\[ L(i) = \frac{\Phi_0}{2\pi i_c} \left[ 1 - \left( \frac{i}{i_c} \right)^2 \right]^{-1/2} \]

where \( i_c \) is the critical current of the junction. Now one would expect that the signal derived at the SQUID readout would be large when this parametric inductance is of the order of the inductance in series with the junction. The ratio formed between the magnitude of this parametric inductance and the series inductance can be written

\[ \beta = \frac{2\pi L_i}{\Phi_0} \frac{1}{i_c} \]

This parameter is convenient because it can be estimated directly from the readout display. Figure 3 shows the method for making this estimate.

It turns out that the point contact type SQUID operates with an optimum signal output for values of \( \beta \) in the range from approximately 1 to 10. For \( \beta < 1 \) the response is smaller and is often obtained only if the resonant circuit is slightly detuned. In this situation, the impedance reflected into the tank circuit is primarily reactive and causes a shift in its resonant frequency (this accounts for the failure to see much
Figure 6. Different types of rf-SQUID's. The thin film or microbridge SQUID is evaporated on a small quartz rod. The other three SQUID's use point contact junctions. The one-hole SQUID was one of the earliest types. The two-hole SQUID offers the advantage of mechanical rigidity and permanent adjustment, while the multihole SQUID offers increased signal-to-noise ratio.
effect when the oscillator is set at the tuned circuit's resonant frequency). For $\beta > 1$ the signal reflected into the tank circuit is primarily resistive and produces variations in the amplitude of the resonance.

The character of a point contact limits $\beta$ to be less than approximately 10. For $\beta < 10$, the damping of the point contact junction allows only a single quantum of flux to move in and out of the ring during each cycle (assuming the rf bias current is set at the lowest periodic pattern). For $\beta > 10$, this situation does not hold and multiple quantum transitions are observed. The onset of these transitions limits the useful range of $\beta$ for the point contact SQUID.

The large $\beta$ limitation occurs at a much higher level for the microbridge [7] type of SQUID [8] shown in figure 6. This is due to the fact that its effective shunt conductance is large and it is therefore heavily damped. There is, however, a practical limit to $\beta$ in this case. For large $\beta$, the signal amplitude relative to the bias level becomes small and the system is subject to output changes caused by amplitude variations of the rf bias oscillator. In addition, rf amplifiers eventually saturate when the bias level becomes too large. Thus, the range of operation of this type of SQUID is greater than for point contact devices, but is still bounded.

A third type of SQUID which was developed by Zimmerman, et al. [2] is the symmetric 2-hole SQUID shown in figure 6. The main attribute of this SQUID is its inherent mechanical rigidity. Once adjusted (and this is done at room temperature) it can be cycled between low temperature and room temperature without loss of adjustment. By virtue of its topology, it does not respond to a uniform field, but rather responds to a difference in the flux linking the two holes. The rf tank circuit coil (10 - 15 turns) is coupled into one of the two holes just as is done with the single hole SQUID and a dc coil or flux transformer coil is
plugged into the second hole (see section VI) to produce the necessary flux difference in the two holes.

The 8-hole SQUID [9] shown in figure 6 is designed to enhance the signal. It turns out that the readout signal amplitude is inversely proportional to the SQUID inductance and one thus gains by reducing the inductance. In this case the volume of flux, to which the device couples, remains large. The 8 holes form 8 loops which are connected in parallel to reduce the inductance. This device exhibits a significantly enhanced signal-to-noise ratio and offers an added advantage in the symmetry of the junction location. The junction bridges the plane separating the two halves of the SQUID and is thus not subjected to fields which are applied along the SQUID axis (for instance, by a concentric coil). This is an advantage if one wishes to operate the device over a very large flux range. If the junction is subjected to the applied field it may produce a second dc modulation on the SQUID response. This modulation results from quantum interference effects within the area of the junction and causes the normal periodic response to die out at intervals which are many times the usual periodicity.

The field periodicity of all these devices depends upon the area of their series inductance. The typical SQUID hole is 2 - 3 mm in diameter which results in periodicities of the order of $10^{-10}$ tesla. At first glance it would appear to be of advantage to just increase this hole size to obtain a finer field periodicity and indeed this does result. But the signal amplitude will be reduced because this will increase the SQUID inductance and we then lose resolution in subdividing the flux quantum. The hole size can be increased if the overall inductance isn't. This is done by connecting holes in parallel as described above. A further consideration is the thermal noise generated in the loop. The SQUID has one 'degree of freedom and thus the energy in the inductance $\Phi^2/2L$
must be equated to $\frac{1}{2}kT$. One immediately sees that the flux noise increases as $L$ is increased. The present devices described herein are not limited by this noise, but rather are restricted by the preamplifier noise. However, improvement of the amplifier may eventually bring the limitation to the thermal noise in the SQUID.

These SQUID's must all be operated in the superconducting state (below $T_c$, usually about 9K). The critical current is temperature dependent for both types of junction but only weakly so for point contacts and thus temperature regulation is not needed for their operation. The critical current of a microbridge is, however, highly temperature dependent and the junction is usually operated near the critical temperature of the material. This does not necessarily mean that the entire environment must be controlled at that temperature, but only that the local temperature of the junction must be regulated. This can be done by isolating the junction from the bath and using a very simple servo-system (involving a small heater) to control the local temperature.

The thin-film SQUID and the 2-hole SQUID were chosen for the commercial systems because of their "permanent" nature. The multi-hole SQUID must be adjusted from run to run, a proposition which is a disadvantage but is much simpler than one would expect. Details concerning adjusting mechanisms can be found in a paper by Zimmerman [10].
Figure 7. Simplified schematic of the rf input circuit. Optimum coupling of the resonant circuit and SQUID is achieved when the energy dissipated in the load resistor is equal to the energy dissipated in the SQUID junction. M is the mutual inductance between the two inductors.
IV. RADIO FREQUENCY COUPLING TO THE SQUID

The optimum signal from the SQUID is derived for the coupling condition \( K^2 Q \approx 1 \) where the \( Q \) is that of the input resonant circuit and \( K \) is the coupling coefficient between the SQUID and rf coil. This condition was arrived at empirically by Simmonds and Parker [11] and later derived in a simple fashion by Kamper [12]. Since Kamper's argument is unpublished it is displayed below. The input circuit is shown in figure 7. In terms of the individual and mutual inductances, the coupling coefficient is given by \( K^2 = M^2 / L_1 L_2 \). The SQUID operation for optimum signal is approximated as a shorted junction which allows the inductive energy stored in the ring \( \frac{1}{2} L_1 i_1^2 \) to be dumped during each rf cycle. The voltage, \( V_1 \), across this short is given by

\[
V_1 = j \omega L_1 i_1 + j \omega M i_2 = 0
\]

or

\[
i_2^2 = i_1^2 \frac{L_2^2}{M^2} = i_1^2 \frac{L_1}{K^2 L_2}
\]

It has been assumed that \( K \) is small so that energies in the individual inductors can be expressed as independent of the mutual inductance. The energy stored in the tank circuit is \( \frac{1}{2} L_2 i_2^2 \) and optimum energy transfer is derived with a dissipation in the load, \( R \), of \( \frac{1}{2} L_1 i_1^2 \) for each cycle of rf. Since the \( Q \) is defined as the ratio of stored to dissipated energy we have

\[
Q = \frac{\frac{1}{2} L_2 i_2^2}{\frac{1}{2} L_1 i_1^2} = \frac{L_2}{L_1} \frac{i_1^2 L_1}{K^2 L_2} = \frac{1}{K^2}
\]
Figure 8. Variation of the SQUID readout with $K^2Q$. Optimum coupling is achieved at $K^2Q \approx 1$. The form of the display allows one to discern whether the coupling is too large or too small.
\[ K^2 Q = 1 \]

Since the tuned circuit \( Q \) is usually of order 100, then \( K \approx 0.1 \) for optimum coupling, and our assumption of weak coupling is justified.

Fortunately, we need not worry about the values of \( K \) and \( Q \) individually. Again the readout patterns allow us to decide whether \( K^2 Q \) is greater than or less than 1. The arguments which follow are from the analog computer studies of the input circuit by Simmonds and Parker [11]. The situation is summarized by the output pictures sketched in figure 8. Optimum coupling (\( K^2 Q \approx 1 \)) results in a triangular periodic response pattern as shown in figure 8b. This optimum coupling is further characterized by the very close spacing between the periodic responses at difference rf current levels. If the coupling is too loose (\( K^2 Q \ll 1 \)), then the output signal becomes clipped (trapezoidal) and the patterns become widely separated as in figure 8a. For \( K^2 Q \gg 1 \), the triangular pattern is washed out by noise coupled from the room temperature circuitry and the sinusoidal display of figure 8c results. The rf current-voltage curves for each case (same information) are shown adjacent to the periodic field displays. The optimum (or nearly optimum) signal is obtained over a modest range of the parameter \( K^2 Q \) and thus does not require critical tuning. For a given type of SQUID, the proper size rf coil and capacitance can be arrived at with perhaps one or two changes from an initial guess.

One cannot readily lay out a specific set of details for the coupling circuit since the resonant circuit is widely distributed from the SQUID to the amplifier input. Thus the \( Q \) will certainly vary from
system to system and since $L_2$ (see fig. 7) includes the inductance in the coax and amplifier input, then the coupling constant, $K$, can vary with each system (without even changing that part of $L_2$ which is wound as the rf coil). In general, though, the one and two hole SQUID's use coils of 10 - 15 turns which slip inside the hole and the microbridge-type SQUID uses a similar coil wound about the outside. The coupling coil for the multihole SQUID uses only 3 or 4 turns wound about its outside. One system of adjusting $K^2Q$ is to slightly overcouple to the SQUID at first and then reduce the coupling by withdrawing the coil from the SQUID to obtain the condition $K^2Q \approx 1$. One minor complication (to be discussed in section VI) is that subsequent windings on the SQUID (for instance; a flux transformer) may tend to change the rf coupling coefficient since the induced rf currents will be distributed into the two systems (SQUID and transformer coil).
V. TYPICAL PROCEDURE FOR SETTING UP
THE rf INPUT CIRCUIT

In the account which follows a number of seemingly trivial points will be belabored with the aim of avoiding some of the more common problems. When the system doesn't work as was advertised earlier, the problem will usually be due to some small oversight.

Let us assume we have wound the rf coupling coil on the SQUID. The circuit would then be tuned (at room temperature) to oscillate near (preferably above) the rf frequency (let's say 30 Mhz). This is done by putting a fixed capacitor across the rf coil. The dipped-mica capacitors seem to be quite suitable at low temperature, and typical values range from 50 to 500 pf. It is best to start with a lower value of capacitance than necessary for exact tuning of the coupling circuit since the resonant frequency can be reduced by adding parallel capacitance at the pre-amplifier input. The SQUID readout system can be used to find the resonant frequency of the coupling circuit. With the internal oscillator turned off, use an external variable frequency oscillator to drive the system through the external rf input. The amplifier system is tuned but the bandwidth is large and an output will exist well off the center frequency of the amplifier if the rf drive amplitude is large. Thus one can sweep the frequency of the external rf oscillator and look for the resonance of the input circuit. Its resonance will show up as a considerably narrower response than the frequency response of the amplifier.

Figure 9 shows a typical response for a 30 MHz amplifier with the resonant circuit tuned to 35 MHz (for such a system one can find the input resonance over a range of ~15 MHz to ~45 MHz). The Q of this resonance will of course increase significantly when the SQUID goes superconducting (the frequency will also be shifted).
Figure 9. Input circuit resonance superimposed on the amplifier bandpass. Through such a measurement one can locate the input resonance and then adjust it to coincide with the center frequency of the amplifier passband.
With the resonant frequency set, as discussed above, to 5 to 10 MHz above the oscillator frequency, the system is then cooled in liquid helium. Final tuning is accomplished by adding further fixed capacitance across the preamplifier input to bring the resonance down to the oscillator frequency. A small variable capacitor ~ 5 pf to ~ 30 pf is used for the final trim of the input resonance. The SQUID should now be ready to operate if it is of the permanent type or ready for contact adjustment if it is not.

A common mistake is to fail to operate the system on the coupling circuit resonance. The amplifier response although quite broad (~ 1 MHz) looks like a resonant response but should not be confused with the resonance of the coupling circuit. Furthermore, if an external oscillator is not used, it is easy to misinterpret the response of the system to the fine tuning capacitor. For example, with the internal oscillator running at some fixed level a dc output is observed from the rf detector; now, the fine trim capacitor (for the input resonant frequency), when adjusted, will cause a change in the output level. This capacitor is usually of the type involving two semicircular plates which overlap and one of them rotates continuously through 360°. Even when off resonance, the system response will peak up once for every revolution of this capacitor. The response will include a true resonance if the system exhibits two peaks for each 360° rotation of the tuning capacitor since these capacitors have two identical values of capacitance for each complete rotation. This simple test will insure that the system is properly tuned. Other common difficulties at the input are shorted or open circuits in the coax or coupling circuits.

For the adjustable SQUID, the contact can be closed once the resonant circuit is tuned. The adjustment of the contact is made with the circuit connected in either of two configurations, as shown in figure 3.
or figure 4 (the phase sensitive detector is not used for this operation). With the circuit of figure 3, the output of the rf detector is displayed vertically on the oscilloscope and the low frequency oscillator is used to control the horizontal axis. With this arrangement the display patterns of figure 2 will result when the SQUID is properly adjusted. Prior to adjustment the response will be a straight horizontal line which moves up and down the screen as the rf current level is increased and decreased. Figure 4 shows the appropriate oscilloscope connections for the other display mode. For this arrangement the display patterns of figure 5 will be obtained when the SQUID is in proper adjustment. With the contact open, the response will not contain the step-like structure but will simply reflect the detector characteristics (straight line at higher rf levels with some curvature as rf voltage goes to zero). For either display, the vertical sensitivity should be adjusted so that noise is discernible (as shown in figs. 2 and 4).

If the resonant circuit is properly tuned as described above and if the SQUID is coupled even somewhat poorly to the resonant circuit, then a clear indication of contact closure will be observable at the oscilloscope. With the contact closed, the inductance of the resonant circuit will be altered and the rf voltage level will change. This appears as a downward shift of the display for either arrangement. If the SQUID can be readily moved into or out of the helium bath, then a simple measure of the coupling of SQUID and resonant circuit can be made. The SQUID (and its circuit) are gradually lifted above the bath so that its temperature increases toward the transition temperature. As the SQUID goes through the transition temperature, the display pattern will decrease since the onset of penetration of rf field into the body of the SQUID will produce a change in inductance of the coil in the resonant circuit. If the SQUID is not properly coupled into the resonant circuit then such change will not be noted.
The contact is adjusted by very gradually twisting the screw inward so as to avoid flattening the tip of the screw. If contact indication is observed on the oscilloscope (decrease of rf voltage) without the appearance of the periodic response, then the contact should be reopened as the adjustment is probably too tight. Adjustment is continued in this manner until a periodic pattern (either fig. 2 or 4) is obtained. The value of $\beta$ (computed as shown in fig. 2 or 4) should lie in the range 1 to 10. For $\beta \geq 10$ a change in periodicity of some integral number and an increase of noise are usually noted. With the SQUID adjusted, one can observe the pattern and ascertain the state of coupling ($K^2Q$ greater than or less than one) as discussed in section V. If there appears to be a significant departure from ideal coupling, then appropriate changes should be made to the coupling coil. Once the proper coupling is achieved, the only adjustment required for subsequent runs will be that of the point contact.

It is imperative that the entire set-up procedure be performed with the SQUID in a somewhat shielded environment since external fields and high frequency radiation may drastically interfere with the desired responses. The extension of the coaxial shield to encompass the SQUID will eliminate rf interference and a simple superconducting tube could be used to reduce low frequency problems.
Figure 10. Simplified schematic of a dc (superconducting) flux transformer coupled to a SQUID. An external dc flux applied to $L_2$ produces a dc flux through $L_1$ and thus through the SQUID. Optimum flux transfer is achieved for $L_1 \approx L_2$. 

To rf readout circuit

SQUID

SUPERCONDUCTING FLUX TRANSFORMER

$L_1$ $L_2$
VI. FLUX TRANSFORMERS AND dc COILS

For many applications, the SQUID is used to sense the dc or low frequency current in a coil wound about it. The two-hole SQUID, in fact, requires a superconducting flux transformer if it is to be used as a magnetometer. Figure 10 shows schematically an arrangement of a SQUID with the rf coupling circuit and flux transformer. Zimmerman [9] has discussed the use of flux transformers and concludes that the inductances of the two coils (pick-up and SQUID coils) must be matched for optimum flux transfer. Since the application of such techniques has generated considerable misunderstanding, it would be wise to refer to Zimmerman's paper [9] before proceeding with the design of such a flux transformer.

The use of a simple dc coil to construct a current galvanometer is a simple matter; since the SQUID is at 4 K, the coil can be wound of superconducting wire to achieve any arbitrary current sensitivity with zero resistance. One may run into difficulty if distributed capacitance, inductance, and series resistance combine to yield a very short time constant. But this should cause no difficulty unless the number of turns becomes excessive ($\gg 10^4$). With a field sensitivity of $10^{-13}$ tesla (which all of these SQUID's are capable of) a current sensitivity of less than $10^{-12}$ amps should be readily achievable. The 8-hole SQUID lends itself to such application because the dc winding can be directly placed about the SQUID. Because of their smaller (and different) geometries, it is difficult to couple a large number of turns directly to the other SQUID's. In principle, this is not a severe limitation since a dc flux transformer can be used between the SQUID and the dc winding.

If the dc winding or flux transformer is coupled very tightly to the SQUID, then it will interact with the rf currents flowing in the SQUID. Since it is difficult to achieve a large coupling coefficient for one-hole,
two-hole and thin film SQUID's, because they are so small, this problem is not too severe. However, if one winds a dc coil tightly on the 8-hole SQUID, the rf coupling can be severely altered. We have found it necessary to leave a small space (~0.5 mm) between the SQUID and the dc coil (the rf coil is in between the SQUID and the dc coil) in order to reduce this effect. Even then it may be necessary to overcouple the rf coil to the SQUID to allow for the reduced coupling when the dc coil is added. Efficient flux transfer requires a tight transformer coupling \[9\] so one is forced into a situation of compromise.

In one instance of very tight flux transformer coupling to an 8-hole SQUID, it was found that the rf coil interacted with high-Q resonant modes of the superconducting transformer coil to produce resonant responses which complicated the readout picture. Such resonances were damped by winding a 0.03 mm foil of brass about the SQUID (this should be tight against the transformer winding and not directly on the SQUID or rf winding). The use of large amounts of normal metals in the vicinity of the SQUID can result in deteriorated performance due to Johnson noise (see section VII). One should not, therefore, apply such cures (the brass foil) indiscriminantly.
VII. SHIELDING THE SQUID

The SQUID is extremely sensitive to a broad range of signals from dc to X-band and this sensitivity poses one of the more difficult problems in its use. The system must be designed to respond only to those signals which are of interest and all others should be rejected.

The SQUID itself is readily shielded against high frequency interference by extending the outer cylinder of the coaxial line to completely enclose the SQUID. Thin-wall stainless steel or brass tubing is adequate for this purpose. As will be discussed later, there are some restrictions to observe when preparing such a shield. The more difficult problems of rf interference occur where leads are passed through the rf shield to carry other signals to the SQUID. Usually such leads constitute an antenna for coupling unwanted signals into the interior of the rf shield. If the leads go only to other apparatus which can be readily shielded against rf interference, then the coaxial rf shield can be extended to a shield which encloses that apparatus, be it in the cryostat or at room temperature. If this is not feasible, then rf filters should be used where the leads pass through the rf shield. A number of types of commercial capacitance filters will probably suffice for most applications. Lukens, Warburton and Webb [13] use a unique method of handling this problem which might be of interest. They enclose the SQUID in the rf shield as described above, but design the rf shield so that the flux transformer coil (or dc coil) can be coupled to the SQUID from outside the rf shield. For the two-hole SQUID which they use, this involves the extension of the rf shield into one of the two holes of the SQUID (in the form of a thin-wall brass tube). This necessarily results in a somewhat weaker coupling between the coil and SQUID but offers a foolproof means of avoiding rf interference.
The use of a normal cylinder between the SQUID and sensed field (for instance, that produced by a concentric coil) also offers a means of lagging the applied signal. This may be desirable if the fields to be measured undergo sudden step-like changes, since the SQUID servo loop can become unlocked by too rapid a change. The lagging shield is designed with a time constant \((L/R)\) which will restrict the rate of flux change to the limit set by the SQUID system (for example, \(10^3 \varphi_o/\text{second}\)). As discussed below, this lagging shield should be so designed that it does not contribute any noise to the system.

Suppose that a normal cylindrical tube is coupled with cylindrical symmetry to a SQUID. The Johnson noise currents flowing in the cylinder generate axial flux noise. If the tube material has resistivity, \(\rho\), the resistance around its circumference is

\[
R = \frac{\pi d \rho}{\ell t}
\]

where \(d\) is the diameter, \(\ell\) is the length and \(t\) is the wall thickness (small compared to \(d\)). The noise current is given by Nyquist's formula

\[
\langle I_N^2 \rangle = 4kT \Delta f / R
\]

where \(k\) is Boltzmann's constant and \(\Delta f\) is the bandwidth over which the noise is distributed. Now only a fraction, \(\alpha\), of the flux through the cylinder links the SQUID so that the flux noise coupled into the SQUID is

\[
\langle \varphi_N^2 \rangle = \alpha^2 L^2 \langle I_N^2 \rangle = 4kT L^2 \alpha^2 \Delta f / R
\]
Since the electronic system is capable of resolving an equivalent flux of $10^{-3}$ to $10^{-4} \phi_0$, the noise $\phi_N$ should be kept below this level. A 0.5 mm thick shield of copper when coupled loosely ($\alpha \approx 0.1$ or less) to any of the SQUID's will generate noise which many times exceeds the inherent noise of the system. To reduce this flux noise, one can reduce the coupling $\alpha$, increase $R$ by increasing $\rho$ and/or decreasing $t$, or reduce the temperature. Note that increasing the size of the shield relative to the SQUID decreases $\alpha$ while increasing $L$ and $R$. Since $\alpha$ varies at least as fast as $d^2$, this results in a net decrease of $\phi_N$.

The formulas expressed above are not exact since the current mode considered is only the basic uniform one. There are, in fact, an infinite number of cylindrical decay modes which couple with different coupling constants to the SQUID and an exact solution requires consideration of these modes. However, the formulas can be used quantitatively to make a rough estimate of the noise generated. Note that only cylindrical modes couple well to the SQUID and thus the presence of normal material affects the SQUID significantly only when such modes can be generated (e.g., a split cylinder will not couple well and windings of normal wires will not generally couple Johnson noise into the SQUID).

Shielding against dc and low frequency fields is, of course, accomplished with a superconducting shield. Niobium or lead shields are common. We prefer lead to niobium because it is difficult to make flux-tight seals between niobium pieces. In addition, lead shields can be made in any configuration by forming the shield out of thin brass and then electroplating it with lead ($\sim 2$ mm thick). A positive flux seal can be obtained by soldering joints with any of a number of superconducting solders.
Figure 11. A shielded multihole SQUID used as a galvanometer. The lead-plated shield completely encloses the SQUID and shields it from external fields. One must still prevent the coupling of unwanted high frequency fields to the SQUID. Such coupling can occur if leads (such as the dc coil leads) are left unshielded. These leads would then act as an antenna to couple high frequency fields to the interior of the shield.
One very important principle should be followed in the design of the low frequency shield. The SQUID (and coil windings) should be rigidly attached to the shield. The shield will trap some flux as it is cooled and any vibrational motion of the SQUID relative to the shield will appear as flux noise. This problem can be virtually eliminated by careful design, but should some microphonic noise persist, it can be reduced by cooling the system in a smaller field (i.e., mu metal shield). Figure 11 shows a simple superconducting shield for an 8-hole SQUID which is used as a null detector. The principles suggested here should be followed carefully in the design of flux transformer circuits, since motion of the transformers or interconnecting leads will also appear as flux noise.
Figure 12. Circuit for making precise quantum dc measurements of current or voltage. The rf circuitry in the lower right of the figure reads out the flux state of the device. The flux counter reads the change in number of flux quanta as the current from the supply is increased. At the desired current level, the supply is regulated by completing the servo-loop with the switch, S. Interpolation between adjacent quantum states is accomplished with the application of a small dc field in the rf coil. The cryogenic resistor, R, can be used to transform current ratios to voltage ratios.
VIII. COMMENTS ON ALTERNATE OPERATIONAL MODES

The signal processing electronics described in section II provides either a periodic or linear response to applied flux, but these modes of operation are not unique. With some minor modifications the SQUID can be adapted to control currents in external circuits, or the periodic response can be used to drive a digital flux counter. These two modes of operation will be described here.

The basic periodic response could be used to trigger an electronic counter, but the counter could not tell which direction the flux would be changing (that is, the time evolution of the response looks identical for increasing or decreasing applied flux). Forgacs and Warnick [14] have developed a scheme which provides for such sensing and thus permits an up/down counter to keep track of the flux state of the SQUID. The system uses two phase sensitive detectors to sense the fundamental and second harmonic of the modulated signal and thus generates two field-periodic responses (period of $\varphi_o$) which are in quadrature. With the appropriate logic circuitry these signals can generate a sense signal for direction of field change and thus one can couple the system to an up/down counter to record the flux state. The signal-to-noise ratio of the periodic output response limits the speed of such counting operations. The 30 MHz system with a 2-hole SQUID is limited to a flux counting rate of $\sim 10^3$ (perhaps $10^4$) $\varphi_o$/second. More sophisticated systems should be capable of rates exceeding $10^5\varphi_o$/second.

The analog mode of operation (linear output) utilizes the rf coil to feed back the correcting dc field to the SQUID. The feedback signal could just as easily be applied to some other coil about the SQUID and this simple change allows one to use the SQUID as a current regulating element. Figure 12 shows such a regulation scheme [15] employed
in a circuit for making current ratio measurements. Note that the feedback current is added to that of the current supply. If the current supply is turned on with switch $S$ open, then the current value can be set at any desired level. If we now close switch $S$, then the current through the dc coil coupled to the SQUID will be forced to remain constant, since any change in supply current will result in a feedback signal which will tend to keep the flux applied to the SQUID at a constant level. For ideal regulation, the servo loop must be type I, that is, there should be an active integrator in series with the output of the phase sensitive detector. This simply assures that for steady state operation, there will be no error in the feedback signal. At a flux level of $10^6 \varphi_0$ produced by a many turn coil, such a system has succeeded in regulating a current of $3 \times 10^{-3}$ A to one part in $10^9$. A fine trim can be added to this system by driving a small direct current through the rf coil. This results in an added flux at the SQUID which must be counteracted by an appropriate change in feedback current. The particular system shown in figure 12 is a preliminary design for a system intended for precise measurement of arbitrary current or voltage ratios.

A similar concept is discussed by Giffard, Webb and Wheatley [5]. They use the negative feedback in a dc voltmeter to generate a high input impedance, so that little current is drawn from the source of emf. Their system is similar in principle to the operation of a photovoltaic galvanometer with negative feedback.
IX. SUMMARY

The SQUID has been applied to a number of measurement problems including magnetometry and magnetocardiography [16, 17]; dc measurements [15, 18]; noise thermometry [19]; sensitive dc measurements [5, 13, 20]; and susceptibility and moment measurements [20, 21, 22]. A modified version of the SQUID (higher frequency bias) has been used to make rf measurements up to 1 GHz [23]. The references cited above, while not complete, contain details which should be of considerable value to anyone bent on using a SQUID to solve a measurement problem.

I would like to thank my colleagues, Nolan Frederick, Bob Kamper, Mike Simmonds, and Jim Zimmerman for exchange of information concerning their experience with SQUID's. I especially thank Jim Zimmerman from whom I have learned much of what I know about this marvelous sensor.
REFERENCES


ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

The report discusses a number of practical considerations concerning the operation and application of rf-biased Superconducting Quantum Interference Devices (SQUID's). The course of routine operation of these devices one amasses a set of operational tales, many of which never reach the open literature. This report is aimed at filling this void. Topics of discussion include: the readout circuitry, operational limits of the SQUID, rf-coupling to the SQUID, flux transformers, and shielding.

KEY WORDS (Alphabetical order, separated by semicolons)

electrical measurements; quantum interference devices; superconductivity.
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