UNITED STATES
DEPARTMENT OF
COMMERCE
PUBLICATION

Computation of
Spectral Data for a
Josephson Junction Circuit

U.S.
DEPARTMENT
OF
COMMERCE

National
Bureau
of
Standards

NATIONAL BUREAU OF STANDARDS

The National Bureau of Standards[1] was established by an act of Congress March 3, 1901. The Bureau's overall goal is to strengthen and advance the Nation's science and technology and facilitate their effective application for public benefit. To this end, the Bureau conducts research and provides: (1) a basis for the Nation's physical measurement system, (2) scientific and technological services for industry and government, (3) a technical basis for equity in trade, and (4) technical services to promote public safety. The Bureau consists of the Institute for Basic Standards, the Institute for Materials Research, the Institute for Applied Technology, the Center for Computer Sciences and Technology, and the Office for Information Programs.

THE INSTITUTE FOR BASIC STANDARDS provides the central basis within the United States of a complete and consistent system of physical measurement; coordinates that system with measurement systems of other nations; and furnishes essential services leading to accurate and uniform physical measurements throughout the Nation's scientific community, industry, and commerce. The Institute consists of a Center for Radiation Research, an Office of Measurement Services and the following divisions:

Applied Mathematics—Electricity—Heat—Mechanics—Optical Physics—Linac Radiation[2]—Nuclear Radiation[2]—Applied Radiation[2]—Quantum Electronics[3]—Electromagnetics[3]—Time and Frequency[3]—Laboratory Astrophysics[3]—Cryogenics[3].

THE INSTITUTE FOR MATERIALS RESEARCH conducts materials research leading to improved methods of measurement, standards, and data on the properties of well-characterized materials needed by industry, commerce, educational institutions, and Government; provides advisory and research services to other Government agencies; and develops, produces, and distributes standard reference materials. The Institute consists of the Office of Standard Reference Materials and the following divisions:

Analytical Chemistry—Polymers—Metallurgy—Inorganic Materials—Reactor Radiation—Physical Chemistry.

THE INSTITUTE FOR APPLIED TECHNOLOGY provides technical services to promote the use of available technology and to facilitate technological innovation in industry and Government; cooperates with public and private organizations leading to the development of technological standards (including mandatory safety standards), codes and methods of test; and provides technical advice and services to Government agencies upon request. The Institute also monitors NBS engineering standards activities and provides liaison between NBS and national and international engineering standards bodies. The Institute consists of a Center for Building Technology and the following divisions and offices:

Engineering Standards Services—Weights and Measures—Invention and Innovation—Product Evaluation Technology—Electronic Technology—Technical Analysis—Measurement Engineering—Fire Technology—Housing Technology[4]—Federal Building Technology[4]—Building Standards and Codes Services[4]—Building Environment[4]—Structures, Materials and Life Safety[4]—Technical Evaluation and Application[4].

THE CENTER FOR COMPUTER SCIENCES AND TECHNOLOGY conducts research and provides technical services designed to aid Government agencies in improving cost effectiveness in the conduct of their programs through the selection, acquisition, and effective utilization of automatic data processing equipment; and serves as the principal focus within the executive branch for the development of Federal standards for automatic data processing equipment, techniques, and computer languages. The Center consists of the following offices and divisions:

Information Processing Standards—Computer Information—Computer Services—Systems Development—Information Processing Technology.

THE OFFICE FOR INFORMATION PROGRAMS promotes optimum dissemination and accessibility of scientific information generated within NBS and other agencies of the Federal Government; promotes the development of the National Standard Reference Data System and a system of information analysis centers dealing with the broader aspects of the National Measurement System; provides appropriate services to ensure that the NBS staff has optimum accessibility to the scientific information of the world, and directs the public information activities of the Bureau. The Office consists of the following organizational units:

Office of Standard Reference Data—Office of Technical Information and Publications—Library—Office of International Relations.

Computation of Spectral Data
for a Josephson Junction Circuit

E.G. Johnson, Jr.
Quantum Electronics Division

D.G. McDonald
Cryogenics Division

Institute for Basic Standards
U.S. National Bureau of Standards
Boulder, Colorado 80302

t Technical note no 627

NBS Technical Notes are designed to supplement the Bureau's regular publications program. They provide a means for making available scientific data that are of transient or limited interest. Technical Notes may be listed or referred to in the open literature.

National Bureau of Standards Technical Note 627

Nat. Bur. Stand. (U.S.), Tech. Note 627, 63 pages (November 1972)
CODEN: NBTNAE

Table of Contents

Computation of Spectral Data for a Josephson Junction Circuit

E. G. Johnson, Jr. and D. G. McDonald

A computer program has been developed to study power flow between different frequency channels in a Josephson junction circuit. This paper discusses the mathematical assumptions used to get such results. They are the trapezoidal approximation from spline theory and the use of a finite range of frequencies to characterize the frequency spectrum. This paper describes the program and provides the FORTRAN listing, flow charts, and discusses how to use the program. A discussion of possible sources of errors is also included.

Key Words: Differential equation; fast Fourier transform; Josephson junction; nonlinear integral-differential equation; spline theory.

1. INTRODUCTION

The central problem of interest is studying the power flow between various frequency channels in a Josephson junction circuit that is appropriate for harmonic mixing. To be realistic, the circuit must include shunt capacitance and shunt resistance as well as source and load impedances all of which may be frequency dependent. The general circuit we have chosen to investigate is shown in figure 1.

If the impedance Z_i, of figure 1, are not frequency dependent, it is relatively easy to solve the differential equation for the circuit and this solution is included as an appendix. The result is what we term the transient program, i.e., it gives the behavior of the circuit as a function of time starting from specified initial conditions. After a few oscillations, the transient behavior reaches a good approximation to a steady state oscillation and thus provides an estimate of a consistent set of currents, of currents, voltages, and phases which we use in the main program, the steady state program.

The steady state program imposes strict periodicity on the solution as required for Fourier harmonic analysis. Steady state solutions are generated in two steps, first with the Z_i frequency independent and second, using the full integro-differential equation for general Z_i. In the notation of figure 1, we want the steady state power for each frequency present in the detector, Z_3, and in the Josephson junction when we are given the applied signal V_s. The V_s is specialized to be

$$V_s = V_o \left[b_o + b_L \cos (N_L s + \theta_L) + b_U \cos (N_U s + \theta_U) \right] . \tag{1}$$

Here V_o is the voltage scale factor, b_o is the dimensionless d.c. voltage, b_L is the lower applied r.f. amplitude, θ_L is its phase, and b_U, θ_U are the same quantities for the upper applied r.f.. The integers, "frequencies" N_L and N_U generate multiples of the phase s, $o \le s = \omega_o t \le 2\pi$ where $\frac{2\pi}{\omega_o}$ is the base period of the system. Since our strategy is to perform a discrete Fourier analysis, obviously we must adjust the discrete spectrum to be consistent with the applied frequencies N_L and N_U, the rotation frequency N_1 of the self oscillation $\omega = \frac{2eV}{\hbar} = N_1 \omega_o$ and all possible linear combinations of these. The b_o is adjusted to allow only periodic solutions with each frequency related to this base frequency, ω_o, by an integer. Note that b_o is a function of b_U, b_L, θ_L, θ_U, and N_1, as well as the impedances Z_i.
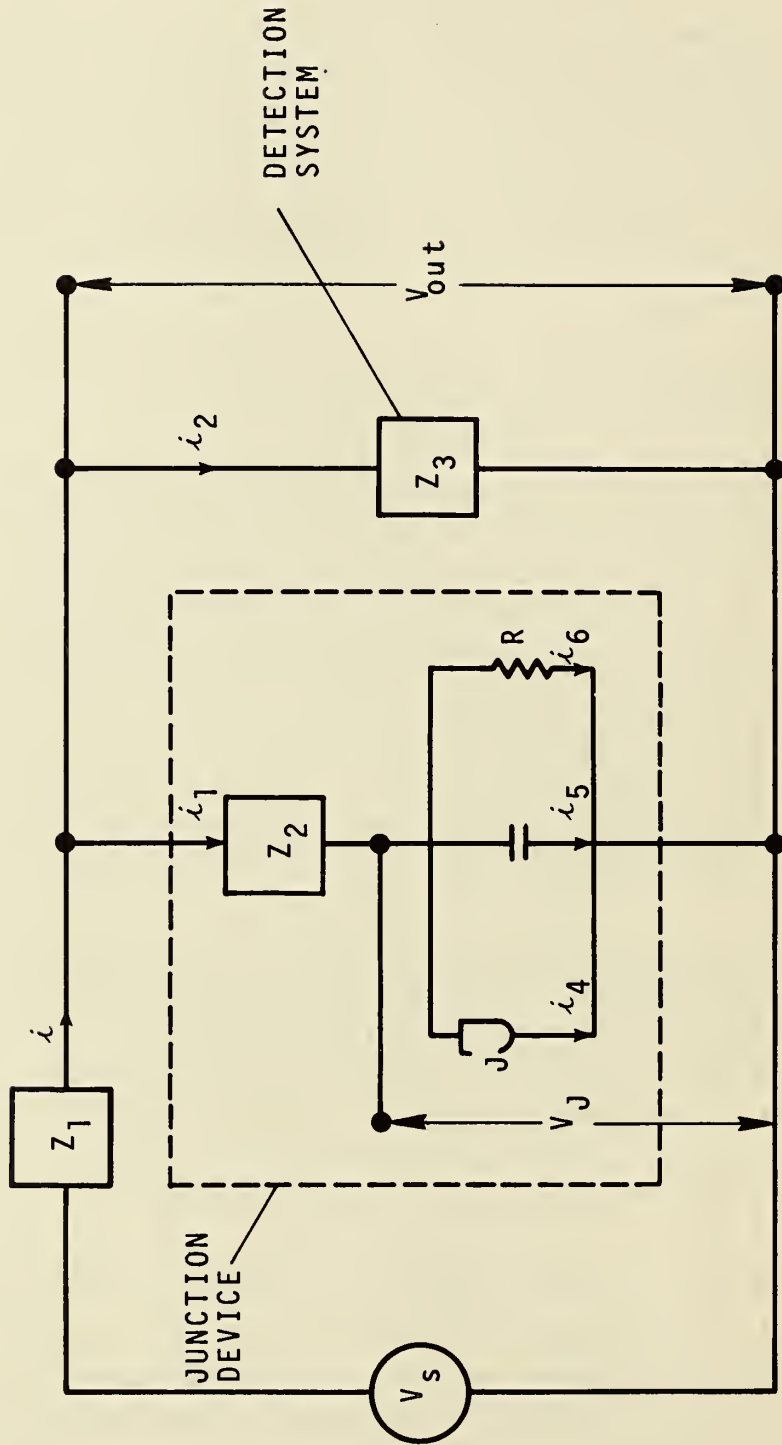
Figure 1. Josephson Circuit.

This report is subdivided into six additional sections. (2) The scale adjustment is defined and the basic equation system is determined. (3) The approximations necessary to solve the basic equation system are given. (4) The FORTRAN Program and flow chart used to solve the equation system are described. (5) We present a sample collection of output from the computer showing the following for each case: (a) the time plot of i_4, (b) the magnitude of the frequency spectra for i_2, (c) the magnitude of the frequency spectra for V_{out}, (d) the relative phase of $(i_2 - V_{out})$ at each frequency, (e) the amplitude spectrum of $\frac{d\varphi}{ds}$, (f) the steady state power into the detector Z_3, (g) the steady state power across the junction device, and finally (h) a summary chart showing such things as the scales of amperes, volts, resistance, frequency, the b_o, b_L, b_U, θ_U, θ_L and the power and other items at the following selected frequencies, namely d.c., N_L, N_U, the direct beat frequency N_B, and the rotation frequency N_1. (6) The summary gives general conclusions and recommendations. (7) Finally, we have the appendix describing the transient program.

2. THE BASIC EQUATION SYSTEM

2.1 Definitions

The following definitions are noted:

Z_i are the frequency dependent impedances for the circuit.

R is the shunt resistance of the Josephson Junction.

Z_2 represents any residual impedance of the Junction device.

C is the capacitance of the Josephson Junction.

φ is the Josephson phase function.

J is the Josephson critical current.

V_o is the voltage scale.

W$\emptyset$,$(\omega_o)^*$ is the radian frequency scale.

R_o is the resistance scale.

A_o is the current scale.

P_o is the power scale.

K is the impedance operator.

K_o is the d.c. part of the impedance operator.

D is the integral equation operator in the circuit system.

U is the net d.c. current needed to drive the system at the steady state rotation frequency N_1.

a is the characteristic radian frequency squared for critical current in dimensionless form.

ANU (ν) is the characteristic frequency for loss due to RC type effects.

ANUP (ν_p) is the characteristic frequency for loss due to the rest of the circuit.

ANUT (ν_T) is the total loss characteristic frequency.

V, VA (v, v_a) are the various voltages in dimensionless form.

---

*(Note that the first symbol is the FORTRAN one and the second is the algebraic form. This is only shown for the less obvious cases.)

3

PHA (φ_a) is the phase with the rotation and directly induced r.f. signal removed.

PHB (φ_b) is the directly induced r.f. signal phase plus the rotation and the initial phase at s = 0.

PHC (φ_c) is the initial phase at s = 0.

THL, THU (θ_L, θ_U) is the phase of the lower, upper r.f. frequency.

2.2. Equation System

We define

$$S = \omega_o t, \ V_o \equiv \frac{\omega_o}{2e}, \ A_o = V_o / R_o, \ Z_i = z_i R_o, \ J = A_o a,$$

$$C = c_1 / R_o \omega_o, \ R = R_o r_1, \ i_\ell = A_o I_\ell \ (\ell = 1 \text{ to } 6), \text{ and } \omega_o = 2\pi / T .$$

(2)

Note that a, R_o, and ω_o are fixed by requiring c_1 to be unity in the dimensionless system, by requiring $ar_1 = \frac{10}{a_1}$, and by using a_1 near 1. This allows a good initial guess for starting slope and d.c. current in the program. Near the hysteresis region[1] a_1 would have to be carefully adjusted to allow study of this region.

Further definitions are:

$$V_J = \frac{\hbar}{2e} \frac{d\varphi}{dt} = V_o \frac{d\varphi}{ds} = V_o V, \ i_4 = A_o a \sin \varphi ,$$

(3)

$$i_5 = C \frac{dV_2}{dt} = V_o C \frac{dV_2}{ds} \omega_o = A_o \frac{dV}{ds}, \text{ and } i_6 = \frac{V_L}{R} = \frac{A_o}{r_1} \frac{d\varphi}{ds} .$$

Therefore
$$i_1 = i_4 + i_5 + i_6 = A_o \left[a \sin \varphi + \frac{V}{r_1} + \frac{dV}{ds} \right] .$$
(4)

Now
$$V_s \equiv V_o V_1, \ i = i_1 + i_2, \ i_2 = A_o I_2, \ i_1 = A_o I_1 ,$$

(5)

$$i = A_o I_o, \ V_{out} = V_o V_2 .$$

Therefore we have the dimensionless operator equations:

$$I_2 = z_3^{-1} \left[z_2 I_1 + V \right] ,$$

$$I_1 = a \sin \varphi + V / r_1 + \frac{dV}{ds} ,$$

(6)

$$I_0 = (1 + z_3^{-1} z_2) I_1 + z_3^{-1} V,$$

$$V_1 = z_1 I_0 + V_2 , \text{ and}$$

$$V_2 = z_3 I_2 = z_2 I_1 + V .$$

4

These are reduced to a single operator form.

$$V_1 = z_1 (1 + z_3^{-1} z_2) I_1 + z_1 z_3^{-1} V + z_2 I_1 + V, \text{ or}$$

$$V_1 = \left[z_1 + z_2 + z_1 z_3^{-1} z_2 \right] \left[a \sin \varphi + \frac{dV}{ds} \right] \tag{7}$$

$$+ \left\{ \left[z_1 + z_2 + z_1 z_3^{-1} z_2 \right] / r_1 + z_1 z_3^{-1} + 1 \right\} V .$$

We finally define

$$K \equiv z_1 + z_2 + z_1 z_3^{-1} z_2, \quad \nu \equiv 1/r_1, \quad D \equiv K^{-1}(z_1 z_3^{-1} + 1) - \frac{1}{K_o} \left(z_1^{o}(z_3^{o})^{-1} + 1 \right), \text{ and } \nu_P \equiv \frac{1}{K_o} \left(z_1^{o}(z_3^{o})^{-1} + 1 \right) \tag{8}$$

as the d. c. part of operator, where z_j^{o} is the D. C. value of z_j, and

$$\nu_T = \nu + \nu_P$$

to get the following pair of operator equations

$$K^{-1}(V_1) = a \sin \varphi + \frac{dV}{ds} + \nu_T V + DV, \text{ and}$$

$$V = \frac{d\varphi}{ds} . \tag{9}$$

As the reader can see, if the z_i are frequency independent then the D operator is zero and we have a pure differential equation. If, as would usually be the case, there is frequency dependence, then D is not zero. However, if we put in estimates for the values of z_i, namely $z_1 \sim 10^4$, $z_2 \lesssim 1$, and $z_3 \sim 10^8$, we have $D \sim 10^{-4}$. If $a \gtrsim 1$, then our problem is dominated by the differential equation and the integral equation part can be used in an iteration procedure. The acid test is to run the program.

As we are interested only in strictly periodic solutions, we assume the following forms for the φ and V. First we split $\varphi = \varphi_a + \varphi_b$; $V = V_a + V_b$ and note that $\varphi_b = N_1 s + \varphi_c + \varphi_d$ and $V_b = N_1 + \frac{d\varphi}{ds} d \equiv \frac{d\varphi_b}{ds}$. We require φ_d to be defined by

$$K^{-1}(V_{1 \text{ A. C.}}) = \frac{d^2 \varphi_d}{d^2 s} + \nu_T \frac{d\varphi_d}{ds} + D \frac{d\varphi_d}{ds} , \tag{10}$$

and φ_a to be defined by

$$u = \frac{dV_a}{ds} + a \sin (\varphi_a + \varphi_b) + \nu_T V_a + DV_a . \tag{11}$$

Here

$$u \equiv K_o^{-1} b_o - \nu_T N_1 .$$

The purpose of this split is to remove a ramp term and to get the first order induced phase function, so that the remaining part, $\varphi_a V_a$ are periodic and have a chance for rapid convergence to the correct solution. Since V_a, φ_a, φ_d are all periodic function of s, the exact solution would be periodic. We use the Fourier space form to get an idea of the structure of D. Thus with $-\infty \leq \ell \leq \infty$,

$$\varphi_a(s) = \sum_\ell e^{-i\ell s}\, \tilde{\varphi}_a(\ell), \quad V_a(s) = \sum_\ell e^{-i\ell s}\, \tilde{V}_a(\ell), \quad \text{(note that } \tilde{V}_a(0) = 0. \text{)}$$

$$\varphi_d(s) = e^{-iN_L s}\, \tilde{\varphi}_d(N_L) + e^{iN_L s}\, \tilde{\varphi}_d^*(N_L) + e^{-iN_U s}\, \tilde{\varphi}_d(N_U) + e^{iN_U s}\, \tilde{\varphi}_U^*(N_U) , \tag{12}$$

and $D(s) = \sum e^{i\ell s}\, \tilde{D}(\ell)$. (We have made $DV = \displaystyle\int_0^{2\pi} \frac{ds'}{2\pi}\, D(s - s')\, V(s').$)

The equation (10) defines $\tilde{\varphi}_d$ as

$$K^{-1}(N_L)\, \frac{b_L}{2}\, e^{-i\theta_L} = \left[(-iN_L) + \nu_T + \tilde{D}(N_L)\right] (-iN_L)\, \tilde{\varphi}_d(N_L). \tag{13}$$

The $\tilde{\varphi}_d(N_U)$ is gotten by U replacing L in the above expression. $\tilde{\varphi}^*$ is the complex conjugate. We note that N_U and N_L cannot be zero.

The equation (11) can be written as

$$u\, \delta_o^\ell = \left[(-i\ell)^2 + \nu_T(-i\ell) + \tilde{D}(\ell)\, (-i\ell)\right] \varphi_a(\ell) + a\int_0^{2\pi} \frac{ds}{2\pi}\, e^{i\ell s}\, \sin(\varphi_a + \varphi_b) . \tag{14}$$

where δ_o^ℓ is the Kronecker delta. We further note that $\tilde{D}(\ell)$ is given as

$$\tilde{D}(\ell) = \left(z_1(\ell)z_3^{-1}(\ell) + 1\right)/K(\ell) - \left(z_1(o)z_3^{-1}(o) + 1\right)/K(o),$$

where

$$K(\ell) = z_1(\ell) + z_2(\ell) + z_1(\ell)\, z_2(\ell)\, z_3(\ell)^{-1} \text{ and}$$

$$K(o) = K_o .$$

It appears to be best to do the calculation in the time domain and then do the spectral transform. Thus we have

$$\varphi_d(s) = 2\, \text{Real}\left[\tilde{\varphi}_d(N_L)\, e^{-iN_L s} + \tilde{\varphi}_d(N_U)\, e^{-iN_U s}\right],$$

$$\varphi_b(s) = \varphi_d(s) + N_1 s + \varphi_c,$$

$$u = \frac{d}{ds}\, V_a + a\, \sin(\varphi_a + \varphi_b(s)) + \nu_T V_a + \int_0^{2\pi} \frac{ds'}{2\pi}\, D(s-s')\, V_a(s'), \tag{15}$$

and

$$V_a = \frac{d}{ds}\, \varphi_a$$

as our basic equation system. To progress further, we must make numerical approximations.

6

3. THE APPROXIMATIONS NEEDED FOR ACCURATE NUMERICAL RESULTS

We have to choose a discrete set of s-points, N_2, because we must take a finite amount of computation time and must keep the entire data set in the machine to allow fast fourier transformation. The set of points has to be equidistant and equal to 2^{N_O}, where N_O is some integer. Practical considerations restrict the range of N_O from 8 to 11.

3.1 Constraints

A discrete set of points implies frequency folding, hence we require the following conditions to minimize this effect.

(a) The highest frequency of interest must be $\leq \dfrac{N_2}{8}$. All frequencies above this value are to be treated as unreliable, due to the method of analysis. Computational noise must also be small.

(b) Our demand accuracy for analysis is near 5%. We make checks on the results by increasing N_2 as the occasion demands. If the spectrum does not change to the demand accuracy, our analysis is treated as adequate and accurate to that order. Incidently, if the value of U is correct to 5% or less, then the lower frequency spectra is also correct to that accuracy.

(c) This 5% computational accuracy level suggests using $N_2 = 256$ or 512 with an occasional extension to 2048 points for special studies.

(d) We allow $z_i(\ell)$ to have arbitrary values for $0 \leq \ell \leq \dfrac{N_2}{2}$ and require the remaining ℓ values to be fixed by $z_i^*(\ell) = z_i(N_2 - \ell)$.

(e) In $\tilde{D}(\ell)$ and $\tilde{\varphi}_d(\ell)$, we use in place of $(-i\ell)$ the periodic expression $(e^{-ih\ell}-1)/h$, where $h \equiv 2\pi/N_2$ is the interval between points in the time space. This allows folding and preservation of reality conditions. The sensitivity to this change is checked by the doubling of points.

(f) Because $D(s - s')$ is a small correction to the differential equation system, we replace, with no qualms, the integral by a sum using the trapezoidal rule. Thus

$$\int_0^{2\pi} \frac{ds'}{2\pi} D(s - s') V_a(s') = \sum_{j'=0}^{N_2-1} \frac{1}{N_2} D(j - j') V_a(j') . \tag{16}$$

(g) To complete the approximations, we must decide how to approximate $\dfrac{dV}{ds}$ type terms. Finite differencing does not allow high enough precision in the fit. Since the spectral information for power uses $\dfrac{dV}{ds}$ in its current, we need to have smoothness in the results to at least order h^2. Additionally, we want to avoid absolute instabilities and permit the application of quasilinear theory[3] to get the b_o value needed to allow strict periodicity. These requirements make it necessary to use the spline method[2] using at least the trapezoidal rule. We assume the trapezoidal rule everywhere in the calculation. The results of this analysis should be smooth functions (accuracy order, h^2), hence a frequency separation of true results from noise due to the discrete analysis of the system and an inherent overall accuracy of h^2. Periodicity should tend to force uniform error throughout the time interval. Sharp spikes in the time dependence of φ_a will negate the uniformity of the error and make higher frequencies spectral information less accurate.

(h) Now the problem is split into a part (α) which neglects D and a part (β) that takes D into account.

7

3.2 Part (α)

Here we have the simplified system of equations using the trapezoidal rule.

$$\varphi_a(j+1) = \varphi_a(j) + \frac{h}{2}\left[V_a(j+1) + V_a(j)\right] \tag{17}$$

$$V_a(j+1) = V_a(j) + \frac{h}{2}\left[q_1(j+1) + q_1(j)\right] \tag{18}$$

$$q_1(j) = -a\sin\left[\varphi_a(j) + \varphi_b(j)\right] - \nu_T V_a(j) + u. \tag{19}$$

The boundary conditions are

$$V_a(o) = V_a(N_2), \text{ and } \varphi_a(o) = \varphi_a(N_2).$$

The u is adjusted to allow this result. The eq (19) can be rewritten with definitions

$$h_1 = (2 - h\nu_T)/(2 + h\nu_T), \; h_2 = -ah/(2 + h\nu_T), \; h_3 = 2h/(2 + h\nu_T),$$

as

$$V_a(j+1) = h_1 V_a(j) + h_3 u + h_2\left[\sin\varphi(j+1) + \sin\varphi(j)\right].$$

We generate two solutions starting with $\varphi_a(o)$ fixed and unchanged for a given calculation run. To avoid numerical noise problems, experience shows that it is absolutely necessary to fix the $\varphi_a(o)$, when u is adjustable.

3.3 Calculation Sequence

(1) We iterate for a given j, j+1 pair until $\left|\varphi_a(j+1) - \varphi_a(j)\right|$ of the n+1 iteration changes by less than EPI = 10^{-8}.

(2) We compute two quasilinear solutions using the following starting conditions

$$\delta\varphi_a^{(1)}(o) = 0, \; \delta V_a^{(1)}(o) = 1, \text{ and } \delta\varphi_a^{(2)}(o) = \delta V_a^{(2)}(o) = 0.$$

In case one $\delta u = 0$ and in case two $\delta u = 1$. The two equation sets are, with $q_2(j) \equiv \cos\varphi(j)$,

$$\delta\varphi_a^{(i)}(j+1) = \delta\varphi_a^{(i)}(j) + \frac{h}{2}\left[\delta V_a^{(i)}(j+1) + \delta V_a^{(i)}(j)\right], \tag{20}$$

and

$$\delta V_a^{(i)}(j+1) = h_1 \delta V_a^{(i)}(j) + h_3 \delta_2^i + h_2\left[q_2(j+1)\delta\varphi^{(i)}(j+1) + q_2(j)\delta\varphi^{(i)}(j)\right].$$

These can be rewritten as

$$\delta\varphi_a^{(i)}(j+1) = \delta\varphi_a^{(i)}(j) + \frac{h}{2}\left[\delta V_a^{(i)}(j+1) + \delta V_a^{(i)}(j)\right],$$

8

$$\delta V_a^{(i)}(j+1) = T_1^{(i)} / T_2^{(i)} ,$$

with
$$T_1^{(i)} \equiv \left[h_1 + h_2 \frac{h}{2} q_2(j+1) \right] \delta V_a^{(i)}(j) + h_3 \delta_2^i + h_2 \delta \varphi^{(i)}(j) \left[q_2(j+1) + q_2(j) \right], \qquad (21)$$

and
$$T_2^{(i)} \equiv 1 - h_2 \frac{h}{2} q_2(j+1) .$$

(3) We use the values computed at $\varphi_a(N_2)$, $\delta \varphi^{(i)}(N_2)$, etc. to satisfy the following requirements and hence fix β and δu:

$$\varphi_a(N_2) - \varphi_a(o) + \beta \, \delta \varphi_a^{(1)}(N_2) + \delta u \, \delta \varphi_a^{(2)}(N_2) = 0 ,$$

and
$$V_a(N_2) - V_a(o) + \beta (\delta V_a^{(1)}(N_2) - 1) + \delta u \, \delta V_a^{(2)}(N_2) = 0 . \qquad (22)$$

The new $\varphi_a(j)$, $V_a(j)$, and u are now defined by

$$\varphi_a(j) = \varphi_a(j) \, (\text{old}) + \beta \, \delta \varphi_a^{(1)}(j) + \delta u \, \delta \varphi_a^{(2)}(j) ,$$

$$V_a(j) = V_a(j) \, (\text{old}) + \beta \, \delta V_a^{(1)}(j) + \delta u \, \delta V_a^{(2)}(j) , \qquad (23)$$

and
$$u = u \, (\text{old}) + \delta u .$$

In the computer program the corrections to the initial guesses of u and $V_a(o)$ are bounded because initially the first guess may not be in the circle of convergence except when the computer program in the appendix is used. Thus, it is necessary to have built into the calculational procedure a means to search for the region where the quasilinear technique will work. The computer program has allowed jumps in δu and in $\delta V_a(o)$ that shrink automatically as each computation is made with a new u and $V_a(o)$. If after ten iterations the convergence has not taken place, then the $V_a(o)$ is set equal to $V_a(N_2)$ using the idea that it is more likely to be compatible with a periodic solution because the transient should be less. Then we repeat the above quasilinear process and the window shrinking process for up to eight times. If the convergence is particularly difficult, the shift in $V_a(o)$ can take place up to ten times. Finally, a relaxed test for acceptance of the results as converged is made so that a spectral analysis can be made.

3.4 Problems

There are two main sources for convergence problems: (1) The number of computed points is too small when there are sharp spikes in the V_a and $\frac{dV_a}{ds}$. This causes extensive computational noise. (2) The initial $V_a(o)$ and u are badly guessed. Both problems are particularly bad when there are strong B_V, B_L terms and/or when a, ν_T are very large numbers.

Here we use the full quasilinear theory.[3] First, we define

$$q_1(j) = -a \sin\varphi(j) - \sum_{j'=0}^{N_2-1} D(j-j') \frac{1}{N_2} V_a(j') + u - \nu_T V_a(j)$$

$$q_2(j) = \cos\varphi(j)$$

$$q_3(j) = h_3 \left\{ \frac{1}{h} \left(V_a(j+1) - V_a(j) \right) - 0.5 \left(q_1(j+1) + q_1(j) \right) \right\}$$

$$q_4(j) = h \left\{ \frac{1}{h} \left(\varphi_a(j+1) - \varphi_a(j) \right) - 0.5 \left(V_a(j+1) + V_a(j) \right) \right\}$$ (24)

We note that q_3, q_4 are the errors due to not fitting the equation system to a quadratic spline. We use these terms to define the error signal in the quasilinear theory. In addition to the terms needed to correct the error signal, we must add effects due to needed changes in $V_a(o)$, and u to allow periodic solutions. Since the quasilinear theory integrates over the entire domain, the u is sensitive to the global solution, and periodicity tends to force global accuracy; we should retain a global accuracy of order h^2, and a smooth spectrum to that same order. This should be accurate enough to separate the true solution from the effects of the finite analysis. Again, spiking will make the higher frequencies less accurate.

The quasilinear differential equations are replaced by the trapezoidal rule. The basic equations become

$$\delta\varphi_a^{(i)}(j+1) = \delta\varphi_a^{(i)}(j) + \frac{h}{2} \left[\delta V_a^{(i)}(j) + \delta V_a^{(i)}(j+1) \right] - q_4(j+1) \delta_1^i$$

and

$$\delta V_a^{(i)}(j+1) = h_1 \delta V_a^{(i)}(j) + h_2 \left[q_2(j) \delta\varphi_a^{(i)}(j) + q_2(j+1) \delta\varphi_a^{(i)}(j+1) \right]$$ (25)

$$+ h_3 \delta_2^i - q_3(j+1) \delta_1^i , \text{ for } i = 1, 2, 3.$$

The initial conditions are $\delta\varphi_a^{(i)}(o) = 0$, for all i; the $\delta V_a^{(i)} = 0$ for i = 1, 2, and the $\delta V_a^{(3)}(o) = 1$. The periodic conditions at $j = N_2$ requires

$$\delta\varphi^{(1)}(N_2) + \delta u \, \delta\varphi^{(2)}(N_2) + \beta \, \delta\varphi^{(3)}(N_2) = 0,$$

$$\delta V^{(1)}(N_2) + \delta u \, \delta V^{(2)}(N_2) + \beta \, (\delta V^{(3)}(N_2) - 1) = 0.$$ (26)

We use these equations to fix δu and β. We compute for the different α, a new $\varphi_a \equiv$ old $\varphi_a + \alpha(\delta^{(1)}\varphi_a + \delta u \delta^{(2)}\varphi_a + \beta\delta^{(3)}\varphi_a)$, and a similarily constructed new V_a which are then inserted to compute a new q_3 and q_4 set for each α. The sum of these; namely, $P(\alpha) = \sum_{j=0}^{N_2-1} (q_3^2 + q_4^2)$, gives us an error signal. Our choice for the correct φ_a, V_a is fixed by the requirement the correct α has; $P(\alpha) < P(o)$. The smallest $P(\varphi)$ improves the calculation the most and guarantees convergence.

The α chosen, is then used to construct the new φ_a and V_a and the entire part (β) quasilinear analysis is repeated. This process is continued until 10 iterations have been performed or until $P(o) < 2h^2$. The results of this process constitutes the best accuracy that can be obtained for a global solution subject to the inherent errors of the trapezoidal method.

Next we compute by fast Fourier transform the desired spectra. This completes the discussion of the assumptions made to get numerical results. The next section has the FORTRAN program listing and a flow chart of the basic program flow. In particular, we show the input data requests.

4. THE FORTRAN PROGRAM AND FLOW CHART

Here we have listed the Fortran programs that give the main details of the computation. The microfilm and fast Fourier transform programs are not listed. The former depends on the computer installation and the latter should be available and optimized at each computer installation. After the Fortran list, figure 2, a flow chart, figure 3, shows the bulk data and computational flows between programs.

5. THE SAMPLE COLLECTION OF OUTPUT

This section discussed the printed and the microfilm output for a single computed case. The purpose is to give general information to allow use of the Fortran program by an interested party. No attempt is made to report all the possible data that exists as a consequence of the computations. Selected information generated as a consequence of this program will be described in separate reports. To use the program correctly in detail, one must read the Fortran listing.

5.1 The Printed Output

Figure 4 shows the printed output from a single computed case. The first row of data gives the number of data points used, the number (11) corresponds to 2048 points in time and 1024 different frequencies. The J (1.-4) is the supercurrent in amperes. The Al (1.25-0) gives the scale adjustment defined on page 4. C (8.2277-15) is the capacitance in farads. R (1. ± 1) is the number of ohms. EB1 (1.-8) is the demand accuracy for each step interation and scales the overall demand accuracies in the various iterations of part (α) and part (β). The EP6 (1.-0) can be set to increase demand accuracy of part (β). EP6 equal 1. uses the values stated on page 10 for the converged exit. EP6 less than one increases the accuracy demand and EP6 greater than one decreases accuracy demand.

The first Z1, etc., row gives, with I9 equal 1, the complex values of Z1, Z2, and Z3 for all impedances as if they were frequency independent. Subsequent rows of Z1 type data show specific frequency exceptions to the frequency independence assumption. These are overlaid on the previous values. Please note that I9 is one unit larger than the corresponding frequency number, thus 1 is the d. c. case. In our sample output we have changed the first harmonic of the Z3 impedance from (1.+8, 0.) to (1.+5, 0.). The others have remained unchanged.

We continue the row by row analysis of the printed output. After the impedance spectrum is defined, we next specify the lower frequency, NL(4), the upper frequency, NU(5), at which the two r. f.

```
           PROGRAM JOSPHC
C          THIS PROGRAM COMPUTES FREQUENCY SPECTRA FOR A CIRCUIT MODELING
C          THE POINT CONTACT JOSEPHSON JUNCTION. THE ANALYSIS USES A TIME
C          PERIOD WITH N2 POINTS. NO IS THE ORDER 2**NO=N2  NO.LE. 11 .
           DATA(PI=3.14159265359),(IM=(0.,1.)),(PHS=2.06778E-15)
           DIMENSION Z1(1025),Z2(1025),Z3(1025)
           COMMON /5/ BI,S1,S2,S3
          1,A1
           COMMON /16/ V16,P16,H8
           TYPE COMPLEX Z1,Z2,Z3,K,DT,ZF,ZH,ZG,KL,AN2,ATMP,KU,ATN,AN6,AN7,ZI
          1,CBU,CBL ,ZD
           TYPE REAL K0
           COMMON /DAT/ PI,IM,PHS,U
           DIMENSION K(1025),DT(1025),KL(3),KU(3)
          1  ,I2T(1025),VAT(1025)
           EQUIVALENCE (K(1),DPH(1)),(DT(1),DVA(1))
           COMMON /1/                        PHC,VA(2050),DPH(4,2050),
          1DVA(4,2050),H,H1,H2,H3,H4,EP1,EP2,EP3,LBX(5),LBTR(5),EP4(6)
           EQUIVALENCE (DPH(6151),Z1(1)),(DVA(6151),Z2(1)),(Q4(1),Z3(1))
           COMMON /2/D(2050),PHA(2050),PHB(2050),Q1(2050),Q2(2050),Q3(2050),
          1Q4(2050),KEY(6),NO,N1,N2,NH,NP,P0,V0,A0,V,   THL,THU,B0,BL,BU,NL,NU
          2,NB,KL,KU, AN5,H5,   ANU,ANUP,ANUT,A,R1,R0,C,W0,T,   R,PI2,AN2,AN4,
          3AN3,MM,N5,N6,N7,ISKPD,SC,                OUT(6,11),NR,   ICONV,AJ,ICASE,
          4POW1,POW2,POWJ,POWIN,K0,EP5,H7
          5,CBU,CBL
           EQUIVALENCE( X(1),DPH(1)),(Y(1),DPH(2051)),(VAT(1),DVA(1)),
          1(I1T(1),DVA(2051)),(VOUT(1),DVA(4101)),(X1(1),DPH(1025))
           TYPE COMPLEX VAT,I1T,VOUT,I2T ,IM
           DIMENSION  X(1025),Y(1025),I1T(1025),VOUT(1025),X1(1025),Y1(1025)
           EQUIVALENCE(Y1(1),DPH(4101)),(I2T(1),Q3(1))
1          READ 2,NO,AJ,A1,C,R,EP1,EP6
2          FORMAT(I5,6E12.4)
           PRINT 200,NO,AJ,A1,C,R,EP1,EP6
200        FORMAT(* NO,J,A1,C,R,EP1,EP6 */I5,6E12.4)
           N2= 2**NO $NH=N2/2   $AN3=N2  $PI2= 2.*PI $AN4=PI2/AN3 $N5=N2+1
           N6=N2+2   $N7=N2-1   $AN2=-IM*AN4 $MM=NO-1 $SC=.5/AN3 $NP=NH+1
           XT=PI2*C  $ R7=PHS/(XT*AJ)   $ A=R7/(R*A1*.1)**2
           R0=SQRTF(A*R7) $T=R0*XT  $W0=PI2/T      $NR=NH/2
           R1= R/R0  $ ANU=1./R1  $V0=PHS/T  $A0=V0/R0  $P0=V0*A0  $AN8=NH
           EP2=EP1*A  $EP5=.1*A
           S1=R1*SQRTF(A) $S2=A1/10.  $ S3=EXPF(.2-S1)
C          SET I8=I9=1 FIRST TIME TO GENERATE ALL THE SAME Z1,Z2,Z3.
C          TO ALLOW SELECTED FREQUENCY DIFFERENCE USE I8=0 I9 THE PARTICULAR
C          HARMONIC. TO EXIT USE I8=-1 AND I9 THE LAST PARTICULAR FREQUENCY
C          HERE    I9=NP AND Z1,Z2,Z3 ARE REAL AS MUST BE Z1(1) ETC.
C          I9=1 IS D.C. CASE ADD 1 TO HARMONIC ORDER TO GET CORRECT
C          LOCATION IN DATA STORE.
5          READ 3,I8,I9,Z1(I9),Z2(I9),Z3(I9)$PRINT199,I9,Z1(I9),Z2(I9),Z3(I9)
3          FORMAT(I3,I5,3C(E12.3,E12.3))
           IF(I8)4,5,6
6          DO 7 I=2,NP $Z1(I)=Z1(1) $Z2(I)=Z2(1) $Z3(I)=Z3(1)
7          CONTINUE     $GO TO 5
4          DO 8 I=1,NP $Z1(I)=Z1(I)/R0 $Z2(I)=Z2(I)/R0 $Z3(I)=Z3(I)/R0
           ATMP=1.+Z1(I)/Z3(I) $K(I)=Z1(I)+Z2(I)*ATMP $DT(I)=ATMP/K(I)
8          CONTINUE     $ANUP=DT(1) $ANUT=ANUP+ANU $DO 9 I=2,NP
9          DT(I)=DT(I)-DT(1) $K0=K(1)    $ DT(1)=0.
           READ 10,NL,NU,ISKPD,ICASE
C          NL IS LOWER APPLIED FREQUENCY   NU IS UPPER
10         FORMAT(4I5)
C          ISKPD=+1 DO THE D STRUCTURES. ISKPD=0 SKIP D STRUCTURES HERE D=0.
           PRINT 201,NL,NU,ISKPD,ICASE
201        FORMAT(* NL,NU,ISKPD,ICASE *,4I5)
C          ICASE IS INCREMENTED TO LABEL EACH   COMPUTED CASE LOAD LAST VALUE
           IF(NL-NU)11,12,12
12         PRINT 202,NL,NU      $CALL EXIT
202        FORMAT( * NL TOO LARGE * ,2I5)
```

Figure 2. The steady state Fortran program listing.

```
11        IF(NU-NR)13,14,14
14        PRINT 203,NU,NR
203       FORMAT( * NU TOO LARGE *  2I5)
13        NB=NU-(NU/NL)*NL
          KEY(1)=0  $KEY(2)=NB $KEY(3)=NL $KEY(4)=NU   $KEY(5)=NH
          IF(ISKPD)15,19
15        DT(NP)=.5*DT(NP) $ZH=CEXP(AN2) $ZG=CONJG(ZH)$DO 16 J=1,N2 $D(J)=0.
          ZG=ZG*ZH $ZF=1. $DO 17 I=2,NP $ZF=ZF*ZG $ZD=DT(I)*ZF
17        D(J)=D(J)+REAL(ZD)
16        D(J)=D(J)/AN8       $ DT(NP)=2.*DT(NP)
          READ 18,IDRED $GO TO (19,20),IDRED
C         DECIDE PRINT OF D IDRED=2 YES  =1 NO
20        PRINT 204,(D(J),J=1,N2)
204       FORMAT( * D SET * /(10E12.3))
19        CONTINUE         $H=AN4   $EP3=2.*H*H*EP6
          H4=H*.5  $ HT=H*ANUT $ HU=1./(2.+HT) $H3=2.*H*HU $H2=-A*HU*H
          H1=(2.-HT)*HU    $H5=H4*H2 $ H7=HU
          AN7= AN2*NU    $ATMP=(CEXP(AN7)-1.)/H $ATN=(ATMP+ANUT+DT(NU+1))
          KU= 0.5/(ATMP*K(NU+1)*ATN) $KU(2)=ATMP*KU $KU(3)=ATMP*KU(2)
          AN6=AN2*NL    $ATMP=(CEXP(AN6)-1.)/H $ATN=(ATMP+ANUT+DT(NL+1))
          KL=.5/(ATMP*K(NL+1)*ATN)   $KL(2)=KL*ATMP   $KL(3)=ATMP*KL(2)
C         N1 IS RAMP FREQUENCY RELATIVE TO BASE   USE PHC=-PI IS N1 NOT ZERO
C         PH0=PHC AT S=0.  BL,THL IS LOWER APPLIED RELATIVE VOLT AND PHASE
C         BU ,THU ARE THE UPPER
          CALL CHARTA
C         NOTE BL=K0*BL USED IN TRANSIENT PROGRAM. SAME FOR BU. HERE K0 IS
C         Z1/R0.  HERE R0=R*A1*A/10.
2022      READ 21,N1,PHC,BL,THL,BU,THU
          BL= K0*BL  $ BU=K0*BU
C         ADD EACH TIME ICOUNT =1 IGUS=-1 FOR SMALL POWER INCREMENTS.
C         ADD ICOUNT =1 IGUS=1 AND USE RESULTS OF TRANSIENT PROGRAM.
C         ADD ICOUNT =1  IGUS=0   TO START NON RF GUESS SEQUENCE.
C         ADD ICOUNT =0,-1 TO GET NEW POWER VALUES
2020      READ 2041,ICOUNT,IGUS,BI,V16,P16
2041      FORMAT(2I5,3E20.9)
          IF(EOF,60)22,23
22        CALL EXIT
23        IF(ICOUNT)2022,2022,2023
2023      AN5=H*N1 $ KEY(6)=N1
          S4=N1*S2
          PRINT 2111,N1,PHC,BL,THL,BU,THU
2111      FORMAT(* N1,PHC,BL,THL,BU,THU * I5,5E12.4)
21        FORMAT(I5,E20.9 /2E20.9/2E20.9 )
          CBU=BU*CEXP(-IM*THU)  $ CBL=BL*CEXP(-IM*THL)
          ZF=CEXP(AN6) $ZG=CEXP(AN7) $ZH=CONJG(ZF)*KL*CBL
          ZI=CONJG(ZG)*CBU*KU          $DO 24 I=1,N5 $AI=I-1
          ZH=ZH*ZF $ZI=ZI*ZG $ E=2.*(ZH+ZI)
24        PHB(I)=E+PHC+AI*AN5    $ICONV=30       $ICASE=ICASE+1
          IF(IGUS)302,300,303
302       BI=U $V16=VA(1) $P16=PHA(1) $ GO TO 303
300       BI=A*S3*(SQRTF(1.+S4*S4)-S4)
301       V16=P16=0.
303       CONTINUE
          PRINT 2113 ,ICOUNT,IGUS,BI,V16,P16
2113      FORMAT(* ICOUNT,IGUS,BI,V16,P16 */2I5,3E20.9)
          CALL PARTA    $IF (ICONV) 25,26,26
25        PRINT 205, N1,ICASE
205       FORMAT( * NO CONVERGENCE FOR N1= * I5, * AND CASE NUMBER= * I5 )
          GO TO 2020
26        ICONV=10  $CALL PARTB  $IF(ICONV)27,31,31
27        PRINT 205,N1,ICASE
31        CALL CHARTB
          GO TO 2020
18        FORMAT(I5)
199       FORMAT(* Z1,Z2,Z3(I9)= * I5,3C(E12.3,E12.3))
          END
```

Figure 2. (Continued)

13

```
         SUBROUTINE SCALE(I9,I99)
C        PURPOSE IS TO CONVERT THE DC DOWN BY 10-7 AND FORM LOG10(Y1)
C          THE AMPLITUDE OR POWER SPECTRA 5 DECADES ONLY.
         TYPE REAL  MIN,MAX
         COMMON /4/ ICLE(8,2,6),IDC(8)
         COMMON /1/ E(6151),Y1(1025),F(11299)
         GO TO (1,2,3,4,5,6)I99
3        IDC(3)=ICLE(I9+1,1,3)=ICLE(I9+1,2,3)=0   $ RETURN
1        CONTINUE
2        CONTINUE
4        CONTINUE
5        CONTINUE
6        CONTINUE
C        POWER CASE AND VOLT CASE
7        MIN=0. $ MAX=0.
         IF(I9)8,9
9        IDC(I99)=-7   $ Y1(3)=1.E-7*Y1(3)
8        DO 11 I=1,1024
         IF(Y1(I))12,11,13
12       IF(MIN-Y1(I))11,11,14
14       MIN=Y1(I) $ GO TO 11
13       IF(MAX-Y1(I))15,11,11
15       MAX=Y1(I)
11       CONTINUE
         IF(MIN)31,32,32
31       IMIN=ALOG10(-MIN) $ MIN=-1.*(10.)**IMIN $ICLE(I9+1,1,I99)=IMIN-5
32       IF(MAX)33,33,34
34       IMAX=ALOG10(MAX)$MAX=(10.)**IMAX $ICLE(I9+1,2,I99)=IMAX -5
33       DO 21 I=1,1024   $ IF(Y1(I))22,21,23
22       Y1(I)=-(ALOG10(Y1(I)/MIN)+5.)    $ IF(Y1(I))21,21,25
25       Y1(I)=0. $ GO TO 21
23       Y1(I)=ALOG10(Y1(I)/MAX)+5.   $ IF(Y1(I))24,21,21
24       Y1(I)=0.  $ GO TO 21
21       CONTINUE  $ RETURN $END



         SUBROUTINE PART
         DATA( ALPHA=0.,1.,.2,.1,.01,.005)
         DIMENSION      ALPHA(6)
         DIMENSION Z1(1025),Z2(1025),Z3(1025)
         COMMON /5/ BI,S1,S2,S3
        1,A1
         COMMON /16/ V16,P16  ,H8
         TYPE COMPLEX Z1,Z2,Z3,K,DT,ZF,ZH,ZG,KL,AN2,ATMP,KU,ATN,AN6,AN7,ZI
        1,CBU,CBL
         TYPE REAL K0
         COMMON /DAT/ PI,IM,PHS,U
         DIMENSION K(1025),DT(1025),KL(3),KU(3)
        1   ,I2T(1025),VAT(1025)
         EQUIVALENCE (K(1),DPH(1)),(DT(1),DVA(1))
         COMMON /1/                          PHC,VA(2050),DPH(4,2050),
        1DVA(4,2050),H,H1,H2,H3,H4,EP1,EP2,EP3,LBX(5),LBTR(5),EP4(6)
         EQUIVALENCE (DPH(6151),Z1(1)),(DVA(6151),Z2(1)),(Q4(1),Z3(1))
         COMMON /2/D(2050),PHA(2050),PHB(2050),Q1(2050),Q2(2050),Q3(2050),
        1Q4(2050),KEY(6),N0,N1,N2,NH,NP,P0,V0,A0,V,  THL,THU,B0,BL,BU,NL,NU
        2,NB,KL,KU,AN5,H5,  ANU,ANUP,ANUT,A,R1,R0,C,W0,T,  R,PI2,AN2,AN4,
        3AN3,MM,N5,N6,N7,ISKPD,SC,           OUT(6,11),NR,   ICONV,AJ,ICASE,
        4POW1,POW2,POWJ,POWIN,K0,EP5,H7
        5,CBU,CBL
         EQUIVALENCE( X(1),DPH(1)),(Y(1),DPH(2051)),(VAT(1),DVA(1)),
        1(I1T(1),DVA(2051)),(VOUT(1),DVA(4101)),(X1(1),DPH(1025))
         TYPE COMPLEX VAT,I1T,VOUT,I2T ,IM
         DIMENSION  X(1025),Y(1025),I1T(1025),VOUT(1025),X1(1025),Y1(1025)
         EQUIVALENCE(Y1(1),DPH(4101)),(I2T(1),Q3(1))
         DIMENSION T1(4),T2(4),T3(4)
```

```
        ENTRY PARTA
C       THIS PART USES PURE DIFFERENTIAL EQUATION AND TRAPAZOIDAL SPLINE
C       APPROACH TO GET PHA,VA SHAPE IN TIME
        DO 1 I=1,N6    $VA(I)=0.
1       PHA(I)=0.
        EP77=EP5
        EP78=.01*A $ZP=EP78*.3
        EP74=EP78
        IF(N1)2,3
3       U= A*SINF(PHB(1)) $GOTO 483
2       U=BI
        VA(N5)=V16
483     ICONV1=12
        DO 460 I19=1,10
        BET=GAM=XI=0.
        PHA(1)=P16                            $ VA(1)=VA(N5)   $ VA(N5)=0.
        ICONV1=ICONV1-2 $ IF(ICONV1)61,61,620
61      ICONV=2 $ GO TO 4
620     ICONV=ICONV1
4       ICONV=ICONV-1 $IF(ICONV)460,461,461
460     CONTINUE $ IF(ABSF(PHA(N5)-PHA(1))-100.*EP2)411,413,413
411     IF(ABSF(VA(N5)-VA(1))-100.*EP2)412,413,413
412     ICONV=1 $ GO TO 405
413     ICONV=-1$ GO TO 405
461     U=U+GAM
        VA(1)=VA(1)+XI
        TA=H3*U $TB=PHA(1)+PHB(1)  $Q3(1)=SINF(TB)
401     Q2(1)=COSF(TB)                $DPH(1,1)=DPH(2,1)=DVA(2,1)=0.
        DVA(1,1)=1.  $ T3(2)=H3 $T3(1)=0.
        DO 6 I=1,N2   $I3=I+1   $TD=H1*VA(I)+TA+H2*Q3(I)   $ IV=5
8       TB=PHA(I3)+PHB(I3) $Q2(I3)=COSF(TB) $ TC=1.-H5*Q2(I3)
        IV=IV-1
        IF( IV)10,11,11
10      PRINT 13    $CALL EXIT
13      FORMAT(* CONVERGENCE PROBLEMS IN PART A* )
11      Q3(I3)=SINF(TB)
        VA(I3)=TD+H2*Q3(I3)
        TB=PHA(I)-PHA(I3)+H4*(VA(I3)+VA(I))
        DEL=TB/TC
        PHA(I3)=PHA(I3)+DEL           $            DEL=ABSF(DEL)
        IF(DEL-EP1)7,8,8
7       TB=PHA(I3)+PHB(I3)
        Q3(I3)=SINF(TB)$TC=   Q2(I)+Q2(I3)$VA(I3)=TD+H2*Q3(I3)
        Q2(I3)=COSF(TB)
        TB=H5*Q2(I3)              $DO 5 I1=1,2
        T1(I1)=DVA(I1,I)*(H1+TB)+T3(I1)+H2*DPH(I1,I)*TC
        T2(I1)=1.-TB
        DVA(I1,I3)=T1(I1)/T2(I1)
5       DPH(I1,I3)=DPH(I1,I)+H4*(DVA(I1,I3)+DVA(I1,I))
6       CONTINUE      $GO TO 12
12      B1=PHA(1)-PHA(N5) $B2=VA(1)-VA(N5)
1450    A11=DPH(1,N5)                        $A12=DPH(2,N5)
        A21=DVA(1,N5)-1.                     $A22=DVA(2,N5)
        DET=A11*A22-A12*A21 $B1=B1/DET $ B2=B2/DET
        XI =A22*B1-A12*B2  $GAM=A11*B2-A21*B1
        IF(ABSF(GAM)-EP5)17,17,19
17      IF(ABSF(XI)-EP74)14,14,19
19      GAM=SIGNF(EP77,PHA(1)-PHA(N5))
        IF(ABSF(PHA(1)-PHA(N5))-.1)199,199,198
198     XI=SIGNF(ZP,VA(4)*(VA(1)+VA(4)-VA(3)-VA(2)))
        GO TO 200
199     XI=SIGNF(EP78,(VA(1)+VA(4)-VA(3)-VA(2))*VA(4))
        EP78=.9*EP78
200     EP77=.90*EP77
14      PRINT 1000,U,VA(1),PHA(1),GAM,BET,XI
       1 ,VA(N5),PHA(N5)
1000    FORMAT(* U,VA,PHA * /(6E13.4))
        IF(ABSF(GAM)-EP2)400,4,4
400     IF(ABSF(XI)-EP1)40,4,4
```

Figure 2. (Continued)

15

```
40      RETURN
405     PRINT 999,(PHA(I7),VA(I7),I7=1,N5)
999     FORMAT(8E12.4)
        ENTRY PARTB
C       THIS PART GETS THE INTEGRAL EQUATION PART USING QUASILINEAR
C       ANALYSIS AND THE TRAPAZOIDAL METHOD FOR THE ERROR ESTIMATE
        DPH(1,1)=DPH(2,1)=DPH(3,1)=DVA(2,1)=DVA(1,1)=0. $DVA(3,1)=1.
60      J6=1            $GO TO 44
30      GO TO (21,28,28,28,28,24,24,60,35)IWAY
21      A12=-H4$A11=A22=DET1= 1. $ T2(2)=T2(3)=0. $T3(2)=H3  $T3(3)=0.
        DVA(2,1)=DPH(2,1)=0.
        DO 22 I=1,N2    $I2=I+1 $A21=-H2*Q2(I2) $DET=DET1-A12*A21
        T3(1)=-Q3(I2)
        T2(1)=-Q4(I2) $DO  23 I6=1,3
        B1=(DPH(I6,I)+H4*DVA(I6,I)+T2(I6))/DET
        B2=(H1*DVA(I6,I)+T3(I6)+H2*Q2(I)*DPH(I6,I))/DET
        DPH(I6,I2)=  B1*A22 -B2*A12
        DVA(I6,I2)=  B2*A11 -B1*A21
23      CONTINUE
22      CONTINUE
        B1=                  -DPH(1,N5)  $B2=                  -DVA(1,N5)
        A11=DPH(3,N5)                    $A12=DPH(2,N5)
        A21=DVA(3,N5)-1.                 $A22=DVA(2,N5)
        DET=A11*A22-A12*A21 $B1=B1/DET $ B2=B2/DET
        XI =A22*B1-A12*B2 $GAM=A11*B2-A21*B1
        I99=2  $ GO TO 42
24      TEMP=EP4(1)$I99=1$ICONV=ICONV-1 $J6=1$IWAY=8     $IF(ICONV)39,26,26
26      DO 250 L6=2,6  $ IF(EP4(L6)-TEMP)25,250,250
25      TEMP=EP4(L6) $J6=L6
250     CONTINUE    $ IF (J6-1) 62,62,28
28      DO 32 I=1,N2
        DPH(2,I)=   ALPHA(J6)* DPH(1,I)+PHA(I)
32      DVA(2,I)=   ALPHA(J6)* DVA(1,I)+VA(I)       $TC=U+ALPHA(J6)*GAM
        GO TO (46,49)I99
29      EP4(J6)=0. $DO 34 I=1,N2
        SMALL=Q4(I)**2+Q3(I)**2  $ IF(SMALL-EP1 )341,341,344
344     PRINT 345,SMALL,U,I,J6
345     FORMAT(* ERROR AT POINT * ,2E12.3,2I5)
341     EP4(J6)=EP4(J6)+SMALL
34      CONTINUE
        TEST=EP4(J6)  $ IWAY=J6  $ J6=J6+1
        IF( TEST    -EP3)51,30,30
51      IWAY=9 $GO TO 46
62      PRINT 63,EP4,EP3  $RETURN
39      PRINT 41,EP4,EP3
41      FORMAT( *   10 LOOPS IN QUASI NO CONVERGENCE */(6E13.4))
35      RETURN
42      DO 43 I=1,N2
        DPH(1,I)=DPH(1,I)+XI *DPH(3,I)+GAM*DPH(2,I)
43      DVA(1,I)=DVA(1,I)+XI *DVA(3,I)+GAM*DVA(2,I)
        IWAY=J6=2 $ GO TO 30
44      DO 45 I=1,N2
        DPH(2,I)=PHA(I) $DVA(2,I)=VA(I)
45      CONTINUE    $ TC=U $GO TO 49
46      DO 47 I=1,N2  $PHA(I)=DPH(2,I) $VA(I)=DVA(2,I)
47      CONTINUE  $U=TC                        $GO TO 30
49      DO  52 I=1,N2  $TB=DPH(2,I)+PHB(I) $Q2(I)=COSF(TB)
        Q1(I)=-A*SINF(TB)+TC   -ANUT*DVA(2,I)
        IF(ISKPD)54,52
54      DO  53 I3=1,N2
        L=I-I3
        IF(L)55,56,56
55      L=N2+L
56      CONTINUE
53      Q1(I)=Q1(I)-D(L+1)*DVA(2,I3)
52      CONTINUE
        DVA(2,N5)=DVA(2,1)
```

Figure 2.   (Continued)

```
                 DVA(2,N6)=DVA(2,2)
                 DPH(2,N5)=DPH(2,1)
                 DPH(2,N6)=DPH(2,2)
      63    FORMAT(* NO CONVERGENCE POSSIBLE  * /(6E13.4))
                 Q2(N5)=Q2(1)   $Q1(N5)=Q1(1) $Q1(N6)=Q1(2)
                 DO 50 I=1,N2   $ I2=I+1 $I3=I2+1
                 Q4(I2)=DPH(2,I2)-DPH(2,I)-H4*(DVA(2,I2)+DVA(2,I))
      50    Q3(I2)=H7*(DVA(2,I2)-DVA(2,I)-H4*(Q1(I2)+Q1(I)) )
                 Q3(1) =Q3(N5)$Q4(1) =Q4(N5) $GO TO 29    $END


                 SUBROUTINE CHART
                 DIMENSION Z1(1025),Z2(1025),Z3(1025)
                 COMMON /5/ BI,S1,S2,S3
               1,A1
                 COMMON /4/ ICLE(8,2,6),IDC(8)
                 COMMON /DDC/LU,LUC,IFL
                 COMMON /DD/ IN,IOR,IT,IS,IC,ICC,IX,IY
                 COMMON /TAB/ V1(60),V2(121),V3(19)
                 TYPE COMPLEX Z1,Z2,Z3,K,DT,ZF,ZH,ZG,KL,AN2,ATMP,KU,ATN,AN6,AN7,ZI
               1,CBU,CBL
                 TYPE REAL K0
                 COMMON /DAT/ PI,IM,PHS,U
                 DIMENSION K(1025),DT(1025),KL(3),KU(3)
               1  ,I2T(1025),VAT(1025)
                 EQUIVALENCE (K(1),DPH(1)),(DT(1),DVA(1))
                 COMMON /1/                              PHC,VA(2050),DPH(4,2050),
               1DVA(4,2050),H,H1,H2,H3,H4,EP1,EP2,EP3,LBX(5),LBTR(5),EP4(6)
                 EQUIVALENCE (DPH(6151),Z1(1)),(DVA(6151),Z2(1)),(Q4(1),Z3(1))
                 COMMON /2/D(2050),PHA(2050),PHB(2050),Q1(2050),Q2(2050),Q3(2050),
               1Q4(2050),KEY(6),N0,N1,N2,NH,NP,P0,V0,A0,V,   THL,THU,BO,BL,BU,NL,NU
               2,NB,KL,KU, AN5,H5,   ANU,ANUP,ANUT,A,R1,R0,C,W0,T,  R,PI2,AN2,AN4,
               3AN3,MM,N5,N6,N7,ISKPD,SC,            OUT(6,11),NR,    ICONV,AJ,ICASE,
               4POW1,POW2,POWJ,POWIN,K0,EP5,H7
               5,CBU,CBL
                 EQUIVALENCE( X(1),DPH(1)),(Y(1),DPH(2051)),(VAT(1),DVA(1)),
               1(I1T(1),DVA(2051)),(VOUT(1),DVA(4101)),(X1(1),DPH(1025))
                 TYPE COMPLEX VAT,I1T,VOUT,I2T ,IM
                 DIMENSION  X(1025),Y(1025),I1T(1025),VOUT(1025),X1(1025),Y1(1025)
                 EQUIVALENCE(Y1(1),DPH(4101)),(I2T(1),Q3(1))
                 ENTRY CHARTA
                 REWIND 1
                 WRITE TAPE 1,Z1,Z2,Z3
                 REWIND 1
                 IN=IOR=IT=IC=0 $IS=1 $LU=LUC=0         $IFL=30
                 CALL DDINIT(3,20H EGJOHNSON,EXT 3706      )
                 S1=S1/(R1*ANUT)
                 PRINT 100,A1,S1  $ PRINT 110,A,ANUT
      110   FORMAT(* A,ANUT *,2E12.4)
                 ENCODE(40,100,V1)A1,S1
      100   FORMAT(16H A1,SQR(A)/NUT'/ ,2E12.4)
                 IX=89 $ IY=500 $ CALL DDTAB $ CALL DDTEXT(5,V1)
                 ENCODE(480,1,V1)N0,N2,NB,NL,NU,ANU,ANUT,A,V0,A0,P0,W0,R0,C,R,H,AJ,
               1T
      1     FORMAT(89H THE JOSEPHSON SPECTRUM CONTROL PARAMETERS THAT REMAIN
               1CONSTANT '/FOR THE ENTIRE RUN.'/ ,40H NO,DATA POINTS,BEAT,LOWER,U
               2PPER FREQ.'/ ,5I5, 48H'/ NU,NU TOTAL,SCALED A,VOLTS,AMPERES, WATTS
               3'/  , 6E12.4, 62H '/RADIAN FREQ,OHMS, CAPACITANCE, RESISTANCE, ST
               4EP INTERVAL'/ , 5E12.4,36H'/SUPER CURRENT AMP.,PERIOD SEC.'/
               5   2E12.4,24H '/CASES TO FOLLOW.'.                          )
                 ENCODE(32,99,LBX(1))
      99    FORMAT(32H JOSEPHSON CURRENT IN TIME CASE           )
                 IX=89$IY=950 $CALL DDTAB $CALL DDTEXT(60,V1) $CALL DDFR $RETURN
                 ENTRY CHARTB
                 READ TAPE 1,Z1,Z2,Z3
                 REWIND 1
      C     Q2(I) IS  A*SINF(PH)
```

Figure 2. (Continued)

17

```
          DO 2 I=1,N2
2         Q2(I)=A*SINF(PHA(I)+PHB(I))
          X1(1)=0.  $DO 3 I=2,256
3         X1(I)=X1(I-1)+4.                   $IFL=60
          ENCODE(8,98,LBX(5))ICASE
98        FORMAT(I5)
          NS=N2/257+1 $DO 4 I8=1,NS $I9=256*(I8-1) $DO 5 I=1,256 $I1=I+I9
5         Y1(I)=Q2(I1)   $CALL FILMGRAF(X1,Y1,256,LBX,1,0)
4         CONTINUE
          DO 6 I=1,N2
6         Q2(I)=Q2(I)+Q1(I)+ANU*VA(I)
          DO 7 I=1,NH  $X(I)=Q2(I)
7         CONTINUE $DO 8 I=NP,N2
8         Y(I-NH)=Q2(I)   $CALL REORDER(X,Y,MM) $CALL CFFTRC(X,Y,MM,SC,1)
          CALL REALTRAN(X,Y,MM,1,1) $DO 90I=1,NP $ I1T(I)=CMPLX(X(I),Y(I))
90        CONTINUE
C            CALL REORDER(A,B,M)
C            REVERSIBLE PERMUTATION OF REAL SEQUENCE
C            FROM FIRST-LAST-NORMAL SEQUENCE
C            TO ODD-EVEN-REVERSE BINARY SEQUENCE.
C            SEQUENCE LENGTH IS N = 2**M
C            CALL REALTRAN(A,B,M,NE,INV)
C            IF(INV.GT.0) UNSCRAMBLE THE TRANSFORM OF A REAL SEQU_NCE.
C            IF(INV.LT.0) SCRAMBLE THE TRANSFORM OF A REAL SEQUENCE.
C            INPUT AND OUTPUT ARE IN NORMAL SEQUENCE,
C            SEQUENCE LENGTH IS N = 2**M
C            NE MUST AGREE WITH SIGN OF EXPONENT IN TRANSFORM DEFINITION.
C            INNER LOOP SINES AND COSINES COMPUTED
C              RECURSIVELY BY SINGLETONS 2ND DIFFERENCE ALGORITHM,
C               INITIALIZED FROM A DATA TABLE.
C            DISCRETE COMPLEX FAST FOURIER TRANSFORM.
C            CALL CFFTRC(A,B,M,SC,NX)
C            INPUT A(J) + I*B(J) IN REVERSE BINARY SEQUENCE.
C            OUTPUT A(K) + I*B(K) IN NORMAL SEQUENCE.
C            SEQUENCE LENGTH IS N = 2**M
C            SC IS REAL SCALING MULTIPLIER.
C            NX IS THE SIGN OF THE EXPONENT IN THE TRANSFORM DEFINITION.
C            INNER LOOP SINES AND COSINES COMPUTED
C              RECURSIVELY BY SINGLETONS 2ND DIFFERENCE ALGORITHM,
C               INITIALIZED FROM A DATA TABLE.
          I1T(1)= I1T(1)+N1*ANU  $I1T(NU+1)=I1T(NU+1' (KU(2)*ANU+KU(3))*CBU
          I1T(NL+1)=I1T(NL+1)+(KL(2)*ANU+KL(3))*CBL
          DO 10 I=1,NH $X(I)=VA(I)
10        CONTINUE $DO 11 I=NP,N2
11        Y(I-NH)=VA(I) $CALL REORDER(X,Y,MM) $CALL CFFTRC(X,Y,MM,SC,1)
          CALL REALTRAN(X,Y,MM,1,1) $DO 12 I=1,NP $VAT(I)=CMPLX(X(I),Y(I))
12        CONTINUE
          VAT(1)=VAT(1)+N1  $VAT(NU+1)=VAT(NU+1)+KU(2) *CBU
          VAT(NL+1)=VAT(NL+1)+KL(2)  *CBL
          VJUN=V0*VAT(1)   $ AIJUN=A0*I1T(1) $ ENCODE(64,9111,V1)VJUN,AIJUN
9111      FORMAT(30H VJUN(VOLTS), IJUN(AMPERES)*/     , 2E17.5)
          DO 13 I=1,NP
          VOUT(I)= Z2(I)*I1T(I)+VAT(I)     $I2T(I)=VOUT(I)/Z3(I)
13        CONTINUE  $DO 14 I=1,1024     $X1(I)=I-1
14        Y1(I)=0.
C         DROP NP POINT IN THESE PLOTS. SEE PRINT OUT FOR IT.
          POW1=POW2=POWJ=0.
          NS=NH/129+1 $DO 15 I8=1,NS  $I9=128*(I8-1)  $K7=I8-1
          DO 21 I=1,128  $ I2=(I-1)*7+3  $I1=I +I9
21        Y1(I2)=CABS(I2T(I1))  $ENCODE(40,97,LBTR(1))ICASE,I8
97        FORMAT(26H AMPLITUDE I2TRAN, ICASE= 2I5, 4HPART  )
          CALL SCALE(K7,1)
          CALL FILMGRAF(X1,Y1,1024,LBTR,1,0)
C         PLOT OF CURRENT,I2, AMPLITUDE THROUGH DETECTOR Z3, AS A FUNCTION
C         FREQUENCY
          DO 22 I=1,128 $I2=(I-1)*7+3 $I1=I+I9
22        Y1(I2)=CABS(VOUT(I1)) $ENCODE(40,96,LBTR(1))ICASE,I8
96        FORMAT(26H AMPLITUDE VOUTTR, ICASE=   2I5, 4HPART  )
```

Figure 2. (Continued)

18

```
         CALL SCALE(K7,2)
         CALL FILMGRAF(X1,Y1,1024,LBTR,1,0)
C        PLOT OF OUTPUT VOLTAGE ACROSS DETECTOR
         DO 23 I=1,128 $I2=(I-1)*7+3  $I1=I+I9
23       Y1(I2)= CANG(I2T(I1))-CANG(VOUT(I1))
         ENCODE(40,85,LBTR(1))ICASE,I8
85       FORMAT(26H PHASE I2-VOUT TRAN ICASE= 2I5, 4HPART   )
         CALL SCALE(K7,3)
         CALL FILMGRAF(X1,Y1,1024,LBTR,1,0)
C        PHASE OF I2-VOUT
         DO 24 I=1,128 $I2=(I-1)*7+3 $I1=I+I9
24       Y1(I2)= CABS(VAT(I1)) $ENCODE(40,95,LBTR(1))ICASE,I8
95       FORMAT(26H AMPLITUDE VPHTR, ICASE=    2I5, 4HPART   )
         CALL SCALE(K7,4)
         CALL FILMGRAF(X1,Y1,1024,LBTR,1,0)
C        PLOT OF AMPLITUDE OF VOLTAGE ACROSS JUNCTION,PH(DOT)
         DO 25 I=1,128 $I2=(I-1)*7+3 $I1=I+I9
         TEP= 2.*REAL( I2T(I1)*CONJG(VOUT(I1))) $POW2=POW2+TEP
25       Y1(I2)=TEP  $ IF(I9)26,27
27       TEP= Y1(3)=.5*Y1(3)  $POW2=POW2-TEP
26       ENCODE(40,94,LBTR(1))ICASE,I8
94       FORMAT(26H POWER I2VOUT SSTR, ICASE= 2I5, 4HPART     )
         CALL SCALE(K7,5)
         CALL FILMGRAF(X1,Y1,1024,LBTR,1,0)
C        DETECTOR POWER FOR EACH FREQUENCY. POW2 IS TOTAL POWER.
         DO 28 I=1,128 $I2=(I-1)*7+3 $I1=I+I9
         TEP=2.*REAL(I1T(I1)*CONJG(VOUT(I1))) $POW1=POW1+TEP
28       Y1(I2)=TEP $IF(I9)29,30
30       TEP=Y1(3)=.5*Y1(3) $POW1=POW1-TEP
29       ENCODE(40,93,LBTR(1))ICASE,I8
93       FORMAT(26H POWER I1VOUT SSTR, ICASE=    2I5, 4HPART   )
         CALL SCALE(K7,6)
         CALL FILMGRAF(X1,Y1,1024,LBTR,1,0)
C        POWER IN Z2+JUNCTION   POW1 IS TOTAL POWER IN POWJ IS TOTAL IN JUN
C        KO IS THE EFFECTIVE IMPEDENCE.  NOTE ALL PLOTS ARE  LOG10 EXCEPT
C        PHASE CASE.  THE POSITIVE ,UP CASE IS SCALED INDEPENDENT OF
C        THE NEGATIVE CASE,DOWN CASE.  THIS IS DONE IS SCALE SUBROUTINE.
C        FILMGRAF   THEN DOES A SCALE CHANGE TO MAKE THE DATA FULL SCALE
15       CONTINUE
         OUT(1,1)=(U+N1*ANUT)*KO $OUT(2,1)=OUT(5,1)=OUT(6,1)=0.
         OUT(3,1)=BL $OUT(4,1)=BU $OUT(3,2)=THL $OUT(1,2)=PHC+PHA(1)
         OUT(4,2)=THU $OUT(2,2)=OUT(5,2)=OUT(6,2)=0.
         OUT(1,3)=OUT(1,1)*(I1T(1)+I2T(1))   $OUT(2,3)=OUT(5,3)=OUT(6,3)=0.
         OUT(3,3)=BL*REAL((I1T(NL+1)+I2T(NL+1))*CEXP(IM*THL))
         OUT(4,3)=BU*REAL((I1T(NU+1)+I2T(NU+1))*CEXP(IM*THU))
         POWIN=OUT(1,3)+OUT(3,3)+OUT(4,3)
         POW1=POW1+REAL( I1T(NP)*CONJG(VOUT(NP)))
         POW2=POW2+REAL( I2T(NP)*CONJG(VOUT(NP)))
         POWJ=REAL( I1T(1)*CONJG(VAT(1)  )+I1T(NP)*CONJG(VAT(NP)))
         DO 32 I=2,NH
32       POWJ=POWJ+2.*REAL(I1T(I)*CONJG(VAT(I)))
         DO 34 I4=1,6  $ I5=KEY(I4)+1
         OUT(I4,4)= 2.*REAL(I1T(I5)*CONJG(VOUT(I5)))
         OUT(I4,5)= 2.*REAL(I2T(I5)*CONJG(VOUT(I5)))
         OUT(I4,6)= REAL(Z1(I5))   $OUT(I4,7)=AIMAG(Z1(I5))
         OUT(I4,8)= REAL(Z2(I5))   $OUT(I4,9)=AIMAG(Z2(I5))
         OUT(I4,10)=REAL(Z3(I5))   $OUT(I4,11)=AIMAG(Z3(I5))
34       CONTINUE
         DO 35 I4=1,6,4 $ DO 36 I7=4,5
36       OUT(I4,I7)=.5*OUT(I4,I7)
35       CONTINUE
         ENCODE(968 ,9,V2)POWJ,POW1,POW2,POWIN,KO,(KEY(I),(OUT(I,J),J=1,11)
        1,I=1,6)
9        FORMAT(28H POWJ,POW1,POW2,POWIN,KO,'/ 5E12.3,2H'/    ,
        138HKEY,B,PHC,POWIN,POW1,POW2,'/Z1,Z2,Z3'/          ,
        2 6(I4,5E12.3,2H'/,6E12.3,2H'/   ))
         ENCODE(152,91,V3)EP1,EP2,EP3,EP4
```

Figure 2. (Continued)

19

```
91      FORMAT(40H ERROR LIMITS EP1,EP2,EP3 '/ EP4(6) '/    ,3E12.3,2H'/
        16E12.3, 2H'. )
        IX=89   $ IY=950  $ CALL DDTAB $ CALL DDTEXT(121,V2)
        IX=89 $ IY=600 $ CALL DDTAB $ CALL DDTEXT(8,V1)
        IX=89   $IY=500     $CALL DDTAB $CALL DDTEXT(19,V3)
        IX=89 $IY=400 $ ENCODE(480,911,V2)(IDC(I),I=1,6),(((ICLE(I,J,L),I=
        1 1,8),J=1,2),L=1,6)
        CALL DDTAB $ CALL DDTEXT(60,V2) $ CALL DDFR
911     FORMAT(58H SCALE EXP ONLY DC SET THEN MIN,MAX SET FOR EACH GRAPH
        1'/  , 6I4,2H'/ ,6(16I4,2H'/))
        RETURN
        END
```

Figure 2.   (Continued)

THE MAIN PROGRAM JOSPHC

1 (READ) NO,

5 (READ) Z1,

(IF(I8)) 4,5,6

(READ) NL,

(IF(ISKPD)) 15,19

(MAY BE SKIPPED)   15 (READ) IRED

19 (CONTINUE)

(CALL) CHARTA

(REPEAT AS   2022 (READ) ,N1
  NECESSARY)
(ICOUNT CONTROLS   2020 (READ) ICOUNT,IGUS
  PATHS)

(IGUS CONTROLS
  INITIAL DATA
  GUESSES)   CALL PARTA
                              PARTB
                              CHARTB

(GO TO) 2020

THE MICROFILM PLOT
ROUTINES

DDINIT
FILMGRAF
DDTAB
DDTEXT
DDFR
SCALE

THE FAST FOURIER
TRANSFORM ROUTINES

REORDER
CFFTRC
REALTRAN

THE CHART SUB-
ROUTINE CALLS
THE MICROFILM
AND THE FAST
FOURIER TRANSFORM
TO GENERATE THE
SPECTRAL DATA
ON MICROFILM.

ENTRY CHARTA
ENTRY CHARTB

THE PART SUB-
ROUTINE GENERATES
THE INITIAL
GUESS FOR PH
AND THE QUASI-
LINEAR PROCEDURE
FOR THE SELF-
CONSISTANT PH.

ENTRY PARTA
ENTRY PARTB

Figure 3.   The flow chart for Josephson steady state program.

21

```
NO,J,A1,C,R,EP1,EP6
  11  1.0000-004  1.2500+000   8.2277-015  1.0000+001  1.0000+000
Z1,Z2,Z3(I9)=   1   1.000+005  -0.000+000   0.000+000  -0.000+000  1.000+008  -0.000+000
Z1,Z2,Z3(I9)=   2   1.000+005  -0.000+000   0.000+000  -0.000+000  1.000+005  -0.000+000
NL,NU,ISKPD,ICASE   4   5   -0   5
N1,PHC,BL,THL,BU,THU   4  -3.1416+000   2.4001+005  -0.0000+000  -0.0000+000  -0.0000+000
ICOUNT,IGUS,BI,V16,P16
   1   1  -2.600000000+001   4.950000000+000  -3.620000000+000
U,VA,PHA
 -2.6000+001   4.9500+000  -3.6200+000   1.6177+000   0.0000+000  -2.2528-001
  4.6127+000  -3.6492+000
U,VA,PHA
 -2.4382+001   4.7247+000  -3.6200+000   2.1460-003   0.0000+000  -1.8820-003
  4.7227+000  -3.6200+000
U,VA,PHA
 -2.4380+001   4.7228+000  -3.6200+000   9.7014-009   0.0000+000  -3.3108-009
  4.7228+000  -3.6200+000
```

Figure 4. Sample printed output from a single computational case - steady state.

signals are to be applied, whether we want to compute the integral equation part, (ISKP is zero means that the frequency spectra of impedance is to be ignored. ISKP equal 1 means that it is considered and the integral equation is now used.); and finally what the initial case number label for this computation run is to be, ICASD(5). The remaining cases are incremented by one for each new case.

The N1, PHC, etc., row is repeated for each new case. Here N1(4) is the chosen rotation frequency for the Josephson oscillator, PHC (-3. 1416-0) is the chosen initial phase assuming PHA is zero initial value. BL is the corrected value for the lower frequency r. f. applied voltage as defined on page 1. This BL (2. 4+5) is changed from the $\overline{BL}$ (7. 68+2) put into the program and used in the transient program in the appendix. The lower r. f. phase is THL(0.) in this example. The BU(0.) is likewise scaled from $\overline{BU}$(0.) and finally we have the upper initial phase THU(0.).

The next data row has two control variables ICOUNT and IGUS as well as input data that is used as initial guesses for U, $V_a(1)$, and $\varphi_a(1)$. The ICOUNT(1) has two options--if it is zero, it causes the program to request a new set of N1, PHC, $\overline{BL}$, THL, $\overline{BU}$, THU; and if it is equal to one it causes the program to proceed to compute the case just read in. The IGUS has three options--if it is -1, it causes the program to use the computed previous values of U, VA(1), PHA(1) as the initial guesses for the new set of data; (this option is useful if you are simply making small changes in previous data cases), if IGUS is zero, it uses some computed guesses based on the approximation that the r. f. power is not importantly affecting the initial data, hence the d. c. case can be a good basis for initial data guesses; and finally, if IGUS is one, it uses the currently read in values BI for U, V16 for VA(1), and P16 for PHA(1). These values are gotten from the program in the appendix or from other calculations.

The final rows of information in the printed output depends on how the calculation proceeds. If the calculation is normal, then you get what is shown in the sample case. If the calculation does not work normally, then you will get error messages and some detailed printouts. Please look at the Fortran listings to understand these messages and printouts. The U, VA, PHA sequence is repeated until there is convergence in the PARTA subroutine. The data printed out is the current U(-2. 6+1), VA(1) (4. 96), PHA(1) (-3. 62, δu (1. 6177), change in $\varphi_a(1)$, the change in VA(1), the VA(N5) (4. 6127), and finally PHA(N5) (-3. 6492). Please note that convergence occurs when the VA(1) equals VA(N5) and PHA(1) equals PHA(N5). Also note that the phase PHA(1) is unchanged for the entire calculation sequence. Here (1, N5) is equivalent to $(0, N_2)$ in the analysis.

5. 2 The Microfilm Output

We look at selected plots of the microfilm output of a single computer run. This data corresponds to the same run as the printed output. Figure 5 has information that remains constant throughout the different runs. The only information that may not be self evident is the SQR(A)/ NUT term (. 5) which is the characteristic frequency ratio of the circuit. It is $\sqrt{a}/\nu_T$ and it measures the capacitance level; the smaller it is the less important is the capacitance term in the circuit.

Figure 6 has 8 plots of 256 points each of a single period of computation. Please note that the FILMGRAF routine will independently scale the individual plots so that the variations are full scale in the y axis. If the range of y variables are both negative and positive then the scale is plus and minus one. If the y variables are only one sign then the range is zero to minus one or one to zero

THE JOSEPHSON SPECTRUM CONTROL PARAMETERS THAT REMAIN CONSTANT
FOR THE ENTIRE RUN
  NO.DATA POINTS.BEAT.LOWER.UPPER FREQ
   11 2000   1   4   5
  NU.NU TOTAL.SCALED A.VOLTS.AMPERES. WATTS
     9 1999-001 9.2002-001 2 9999-002 1 2900-004 9 9964-30* 4 9092-011
RADIAN FREQ.OHMS. CAPACITANCE. RESISTANCE. STEP INTERVAL
     9.7909-011 9.1999-002 0 2277-015 1 0000-001 9 0600-009
SUPER CURRENT AMP .PERIOD SEC
   1.0000-004 1.6542-011
CASES TO FOLLOW

A1.SORT(A).GT
  1.2900-000 9 0001-001

Figure 5. Sample microfilm output - the control parameters.

24

depending on the sign. We emphasize that it is possible for the different plots to have very different scales for their respective data. In this sample case the figure 6 set has exactly the same scale for all 8 plots.

Figures 7 and 13 are the first two plots of a series of 8 for the amplitude at each frequency of the current through the detector. The first figure gives the first 128 harmonics, the latter figure gives the next 128 harmonics. The remaining 768 harmonics are not shown. The d. c. line amplitude is reduced by 10^{-7} in the first plot. Then, in this set of 128 harmonics as well as each succeeding set of 128 harmonics, the largest positive value is found, the $\log_{10}$ is taken and the integer part is extracted. Then all positive nonzero y values are scaled relative to this integer. Only the top five decades of $\log_{10}$ amplitude values are computed and put into the y axis. The remaining y values that are less than this allowed 5 dacades are set equal to zero and is thus plotted that way. For those plots which have negative nonzero y values the largest negative value is extracted, the integer part is removed, these y values are scaled down by that number, and the $\log_{10}$ is taken and those numbers that are 5 decades less than this negative maximum are set to zero. The numbers are then made negative. Both the negative and positive values that have been processed as above are now subject to further scaling in the subroutine FILMGRAF. That scaling is the same as already discussed for the time series plots.

The above scaling takes place for all the figures 7 through 18 as well as those plots that have not been shown. Figure 19 has at its bottom a set of scaling information which in conjunction with the information given at the bottom of each figure permits the relative determination of the amplitudes for each spectral line that has been plotted. The row of -7's with a zero mean that only the phase difference series of plots does not scale the d. c. frequency line; the rest are scaled by this 10^{-7}. This was done to allow detailed plot information of the higher frequencies without the d. c. values causing scaling problems. After the -7 row are six rows of numbers. The first eight numbers of each row defines the scaling used on the negative numbers in the spectrum. Note that only the last row which corresponds to the power spectra across the junction has negative values. The last eight values correspond to the scaling of positive numbers. The rows are related to the plots as follows.

Row	Plot
1	Amplitude of the current across detector (AMPLITUDE I2TRAN)
2	Amplitude of the Voltage across detector (AMPLITUDE VOUTTR)
3	Phase difference between detector current and voltage (PHASE I2-VOUT TRAN)
4	Voltage across R (AMPLITUDE VPHTR)
5	Power across detector (POWER I2VOUT SSTR)
6	Power across Josephson junction (POWER I1VOUR SSTR)

We illustrate the reading of the plots to get an absolute number for the power across the junction at the 4, 8 harmonics and at the 132, 133 harmonics respectively. The first two are gotten from figure 12 and the latter from figure 18. The conversion to watts is given in figure 5 as (4. 8830 -11). The fourth harmonic has value full scale (5. 918). The eighth harmonic has value (4. 7 $\times$ (4. 4410)/6. 2 = 3. 34).
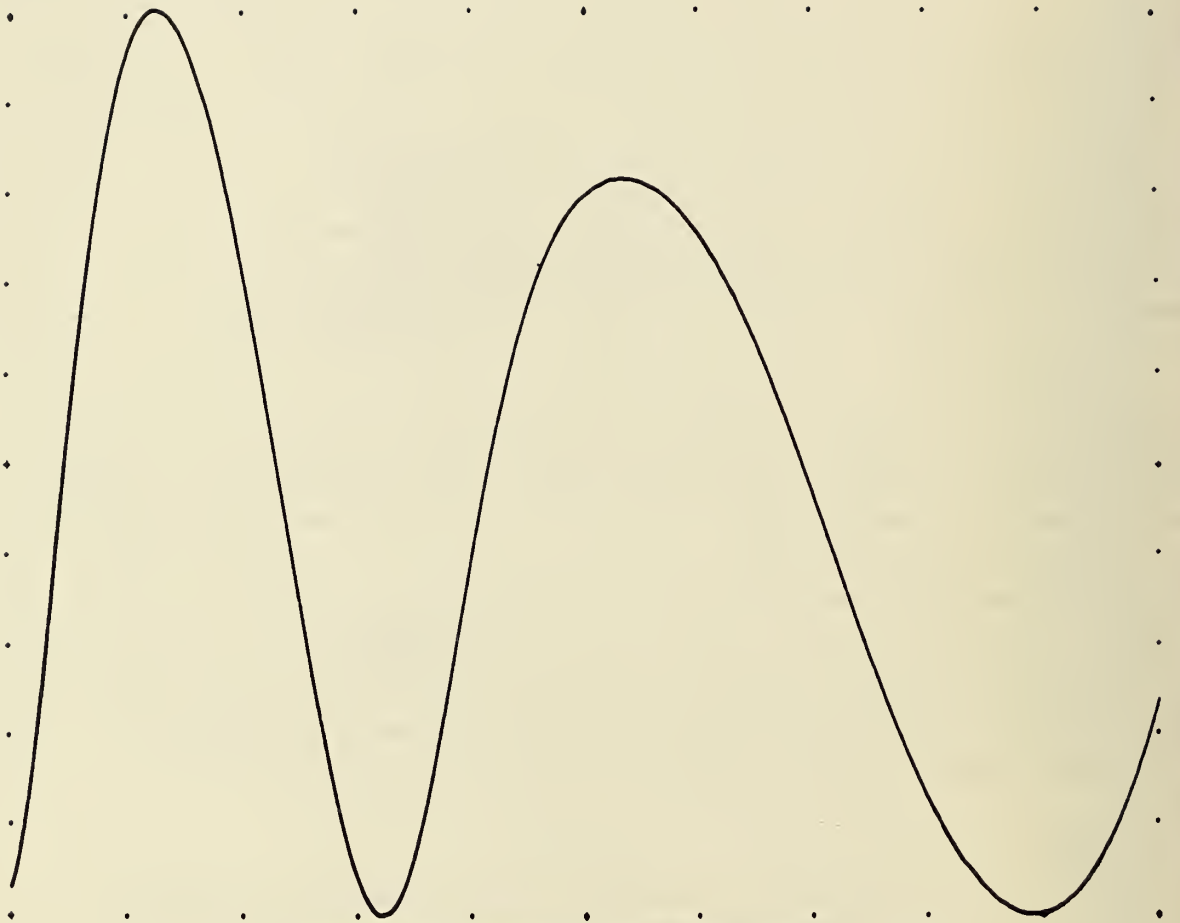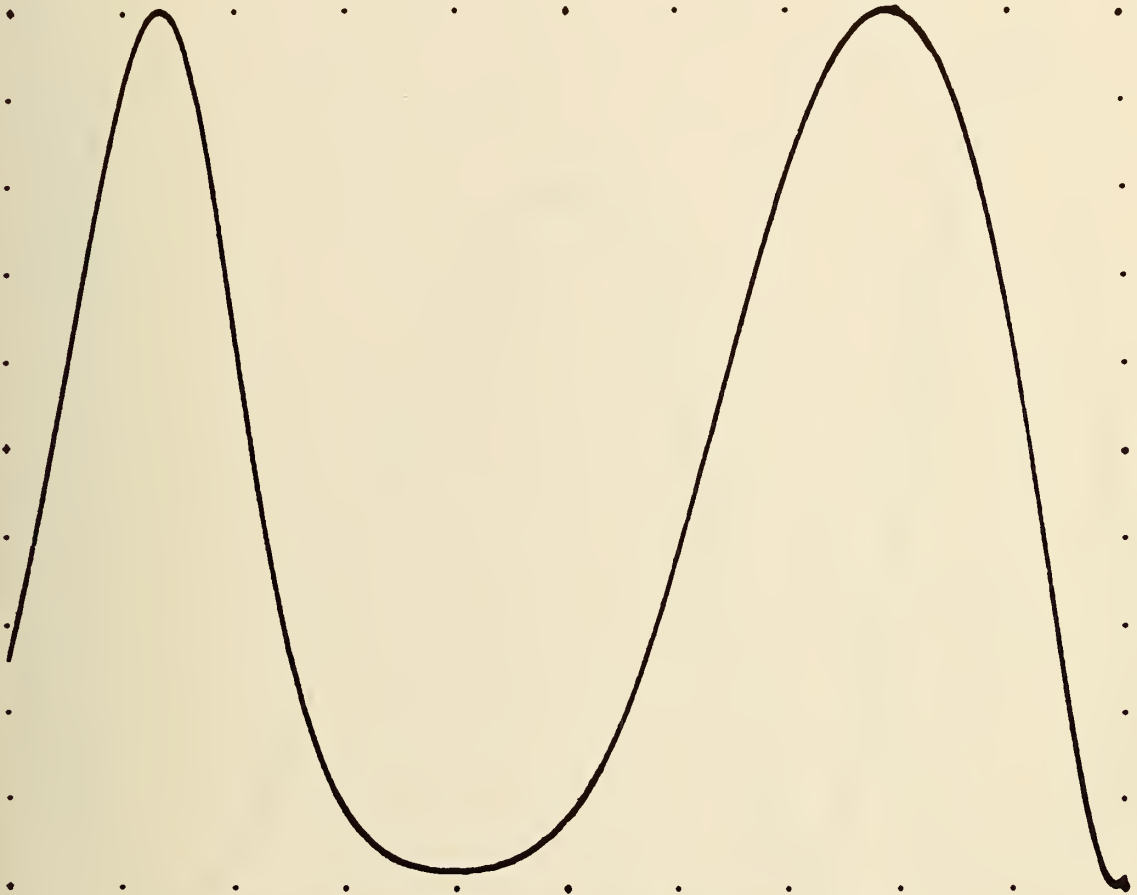
JOSEPHSON CURRENT IN TIME CASE   6

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.020+003	-2.560+002	2.560+002
Scale	0.000+000	1.020+003	-2.560+002	2.560+002

Run 1972 May 03 (124) at 17:50:52   L1,L2= 1, 0   256 Points, Frame   1

Figure 6.   The next 8 plots give one complete period, each plot 256
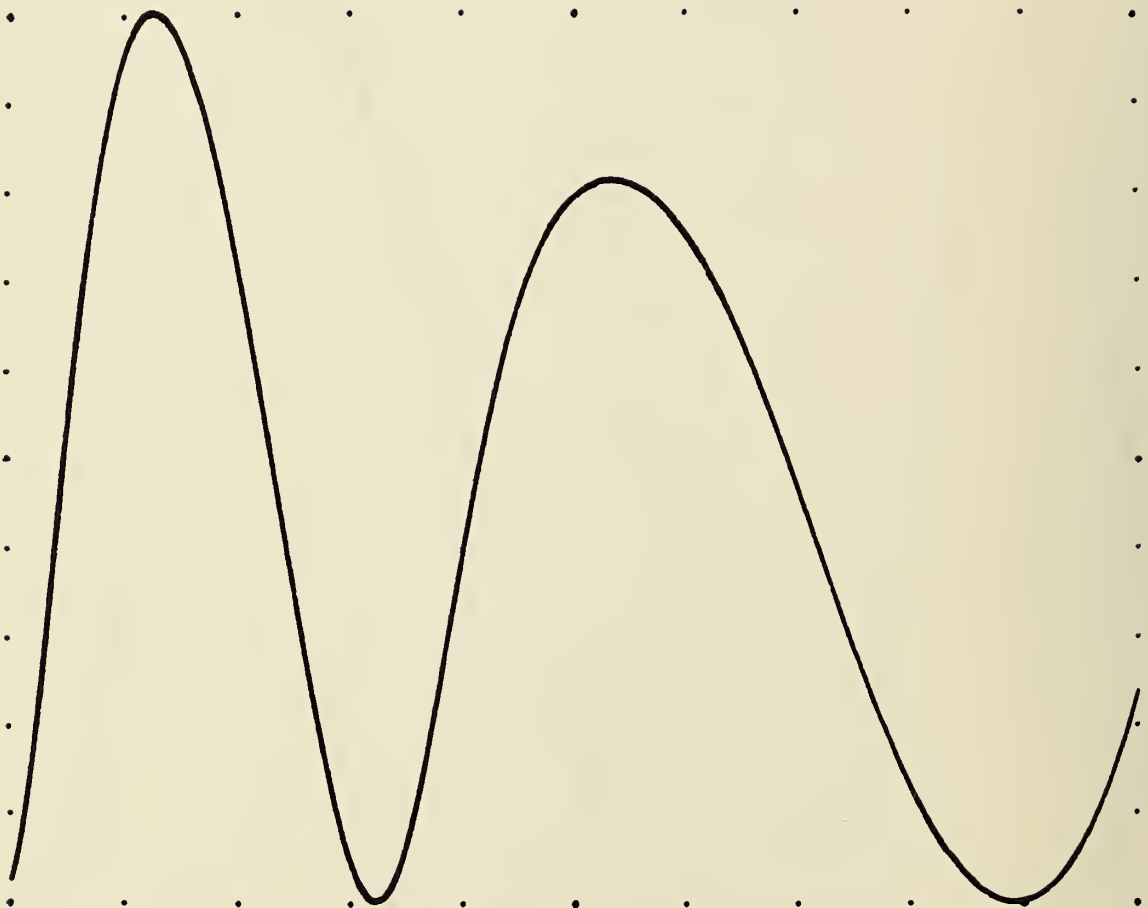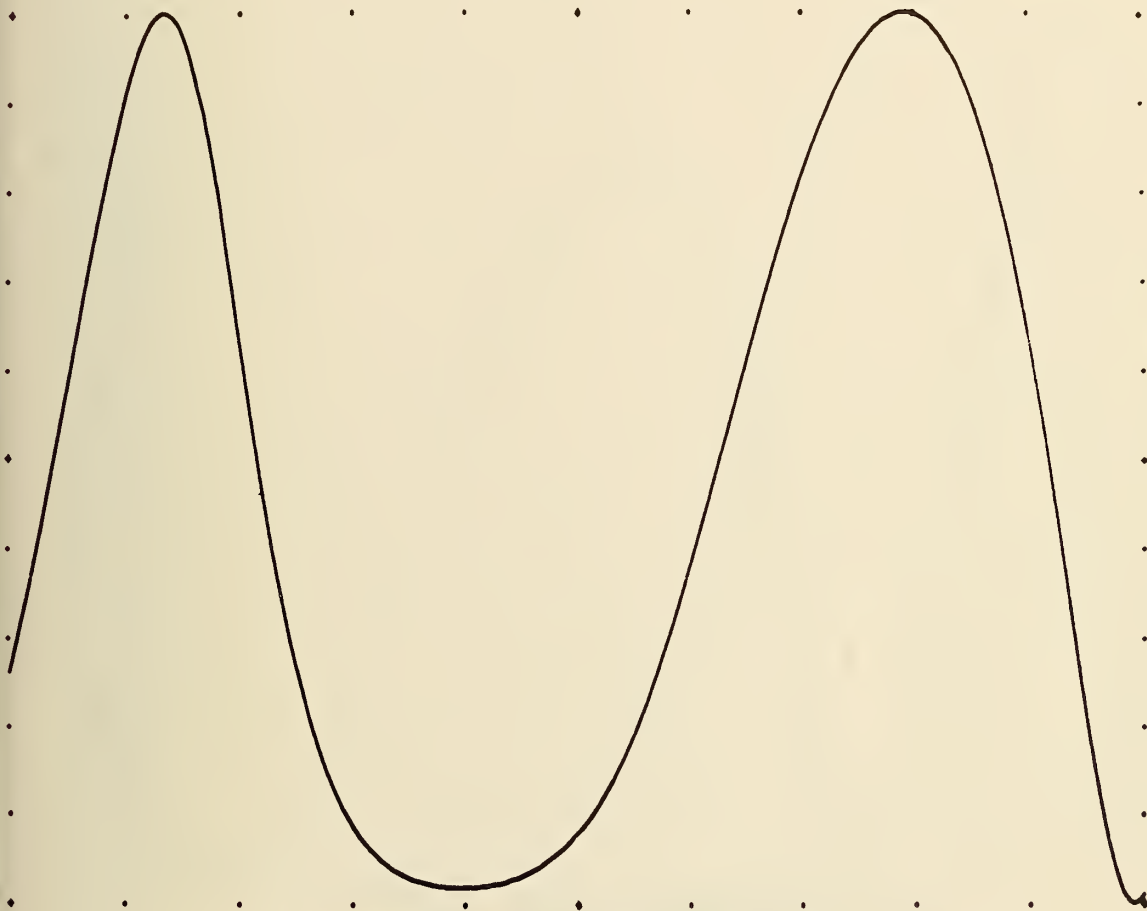   points in time.

JOSEPHSON CURRENT IN TIME CASE   6

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.020+005	-2.960+002	2.960+002
Scale	0.000+000	1.020+005	-2.960+002	2.960+002

Run 1972 May 05 :1261 at 17:50:95   L1,L2= 1, 0   206 Points, Frame   0

Figure 6.   (Continued)

27

JOSEPHSON CURRENT IN TIME CASE   6

	X - Min	Max	Y - Min	Max
Function	0 000+000	1 020+005	-2 960+002	2 960+002
Scale	0 000+000	1 020+005	-2 960+002	2 960+002

Run 1972 May 05 :1201 at 17.50.94   i1.i25 1. 0   256 Points. Frame

Figure 6. (Continued)
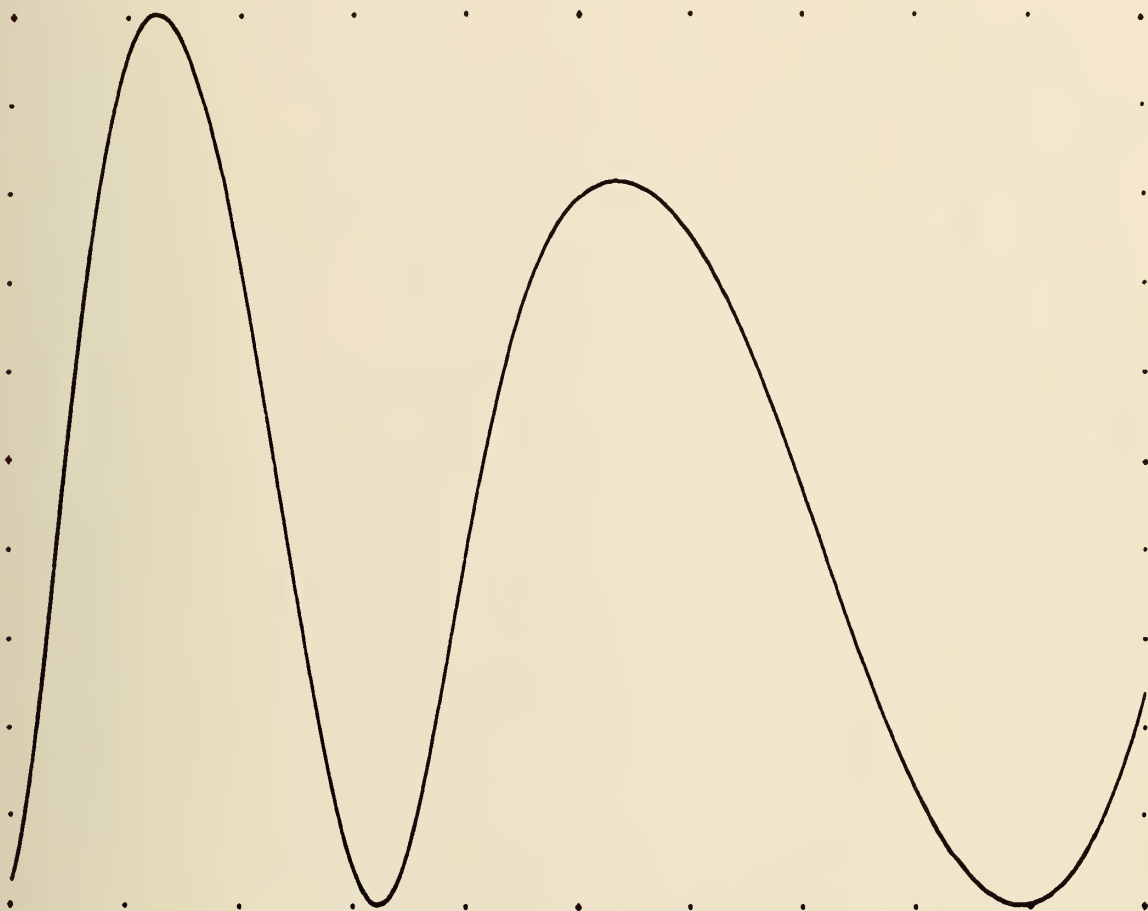
JOSEPHSON CURRENT IN TIME CASE   6

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.020+005	-2.560+002	2.560+002
Scale	0.000+000	1.020+005	-2.560+002	2.560+002

Run 1972 May 05 (124) at 17:50:55   L1,L2= 1, 0   256 Points, Frame   5
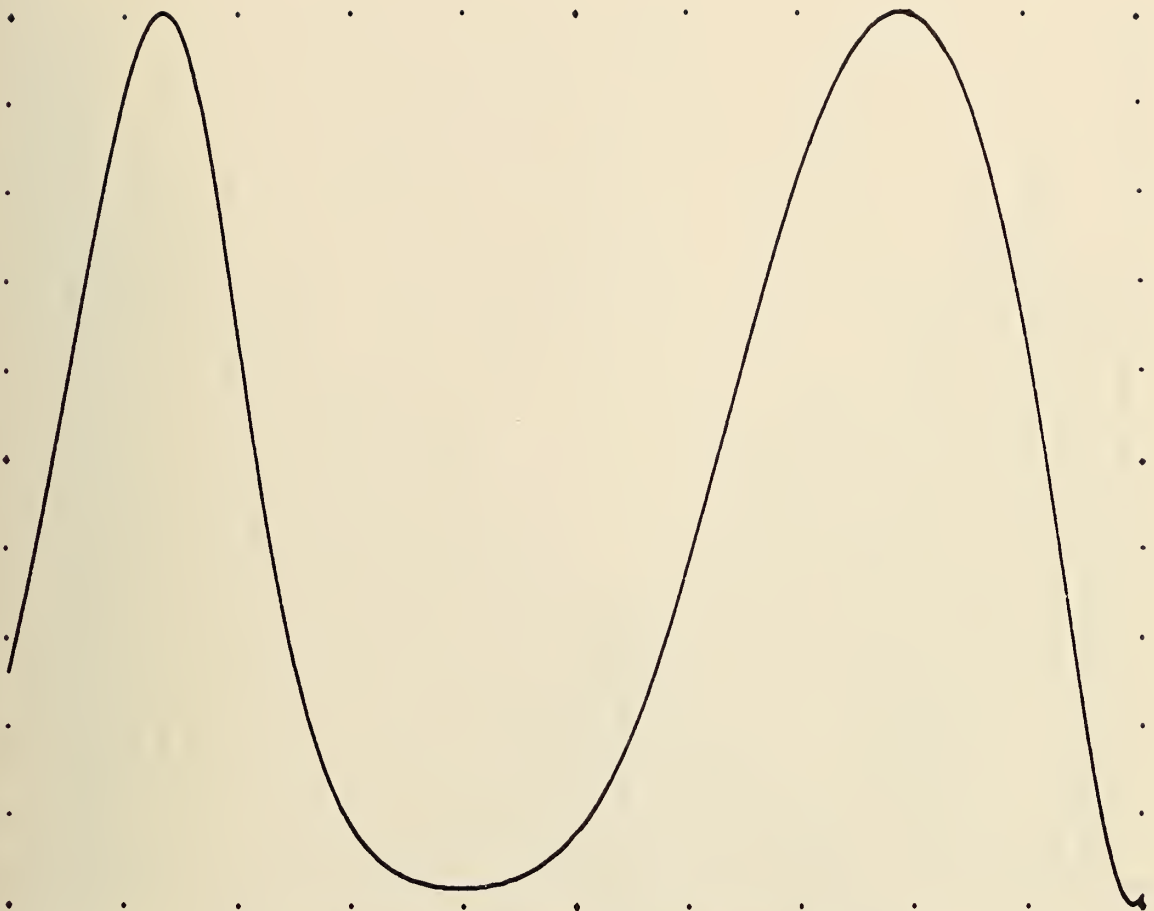
Figure 6.   (Continued)
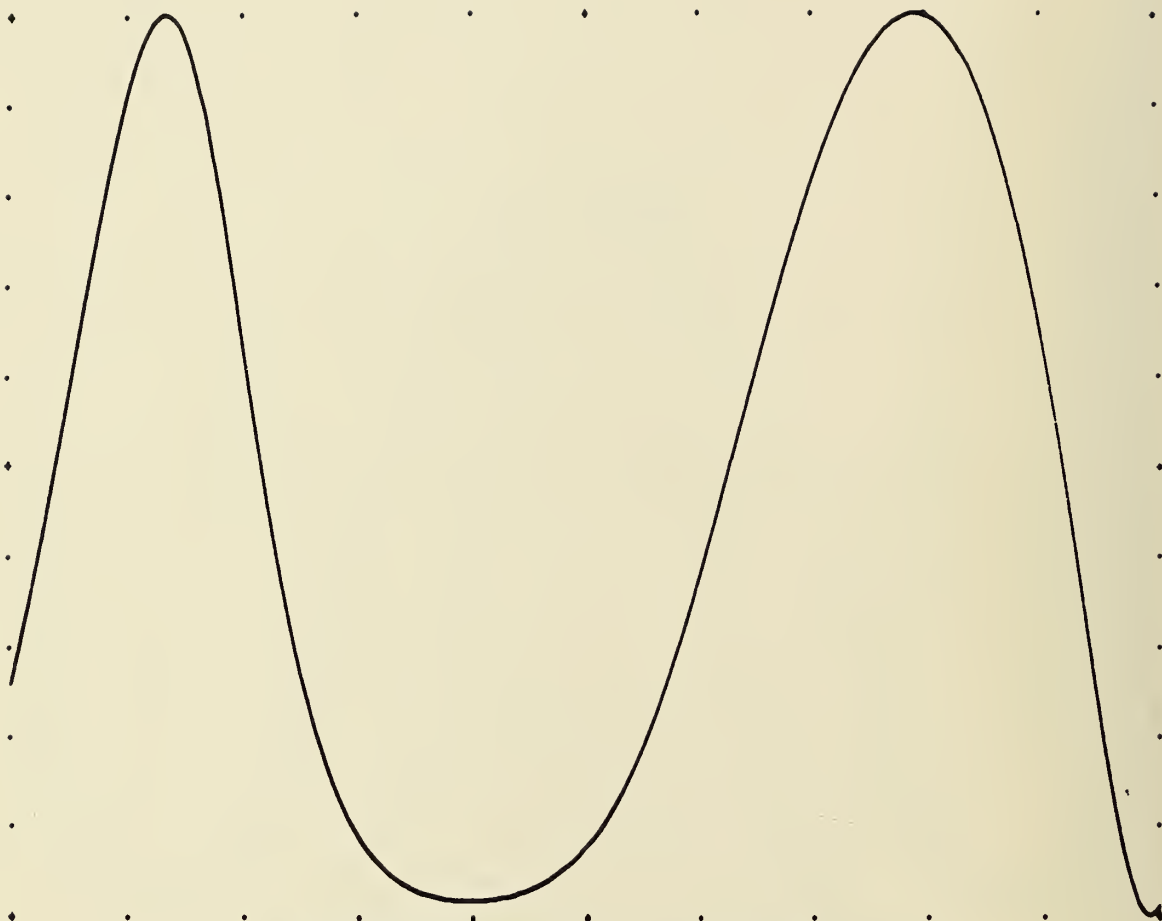
JOSEPHSON CURRENT IN TIME CASE   6

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.020+003	-2.559+002	2.560+002
Scale	0.000+000	1.020+003	-2.560+002	2.560+002

Run 1972 May 03 (124) at 17:50:53   L1,L2= 1, 0   256 Points, Frame   6

Figure 6.   (Continued)

30

JOSEPHSON CURRENT IN TIME CASE   6

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.020+003	-2.559+002	2.560+002
Scale	0.000+000	1.020+003	-2.560+002	2.560+002

Run 1972 May 03 (124) at 17:50:53   L1,L2= 1, 0   256 Points, Frame   4

Figure 6. (Continued)
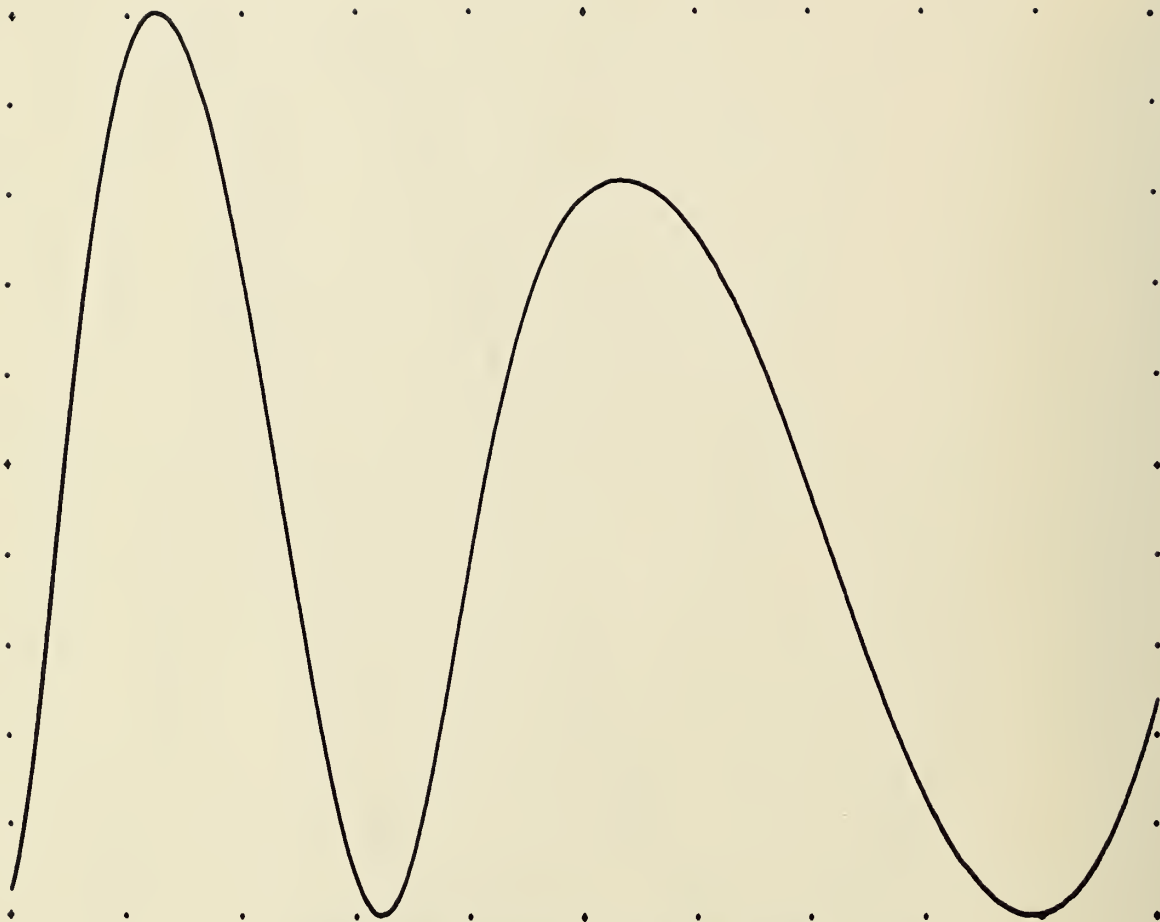
31

JOSEPHSON CURRENT IN TIME CASE   6

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.020+003	-2.560+002	2.560+002
Scale	0.000+000	1.020+003	-2.560+002	2.560+002

Run 1972 May 03 (124) at 17:50:53   L1,L2= 1, 0   256 Points, Frame   3

Figure 6.   (Continued)

JOSEPHSON CURRENT IN TIME CASE   6

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.020+005	-2.559+002	2.560+002
Scale	0.000+000	1.020+005	-2.560+002	2.560+002

Run 1972 May 05 (124) at 17:50:55   L1,L2= 1, 0   256 Points, Frame   2

Figure 6.   (Continued)
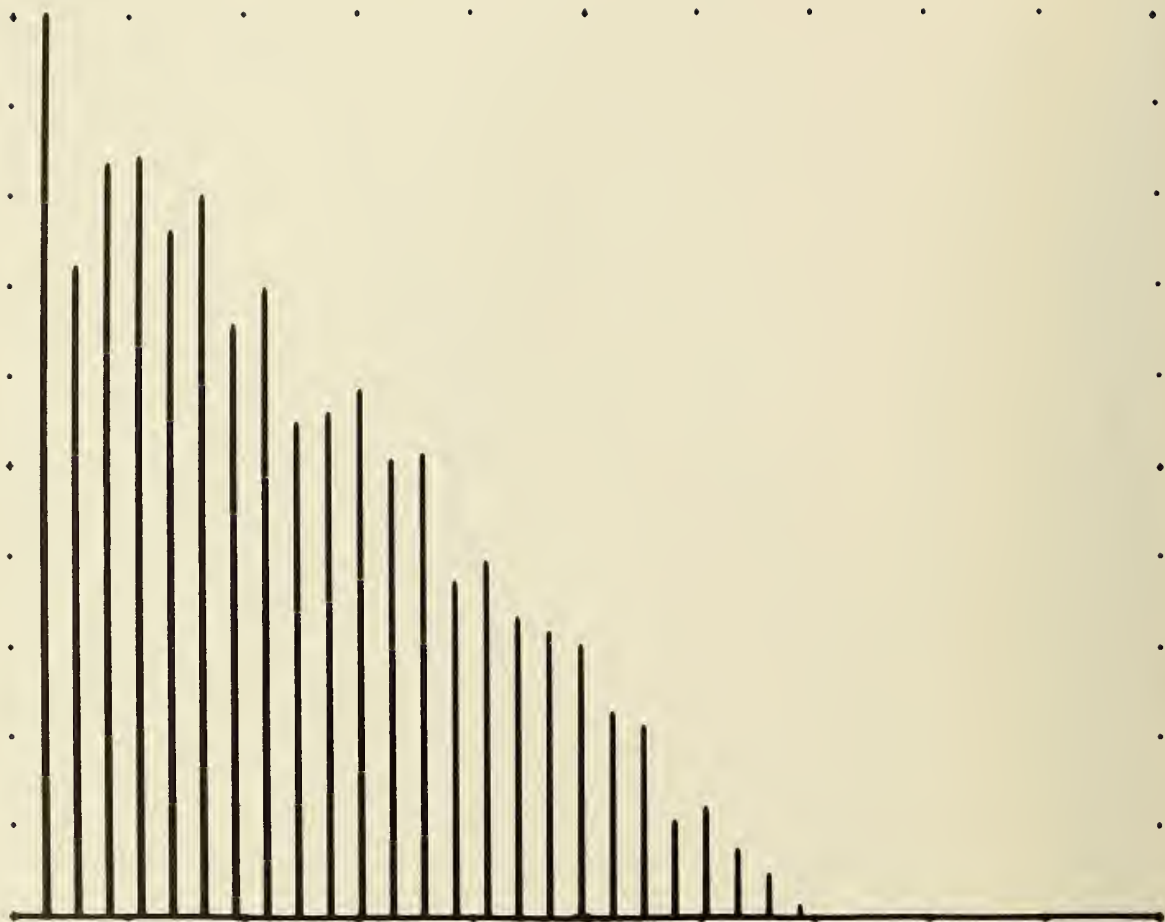
33

AMPLITUDE I2TRAN, ICASE=   6   IPART

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.023+003	0.000+000	4.540+000
Scale	0.000+000	1.023+003	0.000+000	4.540+000

Run 1972 May 05 (124) at 17:50:57   L1,L2= 1, 0   1024 Points, Frame   9

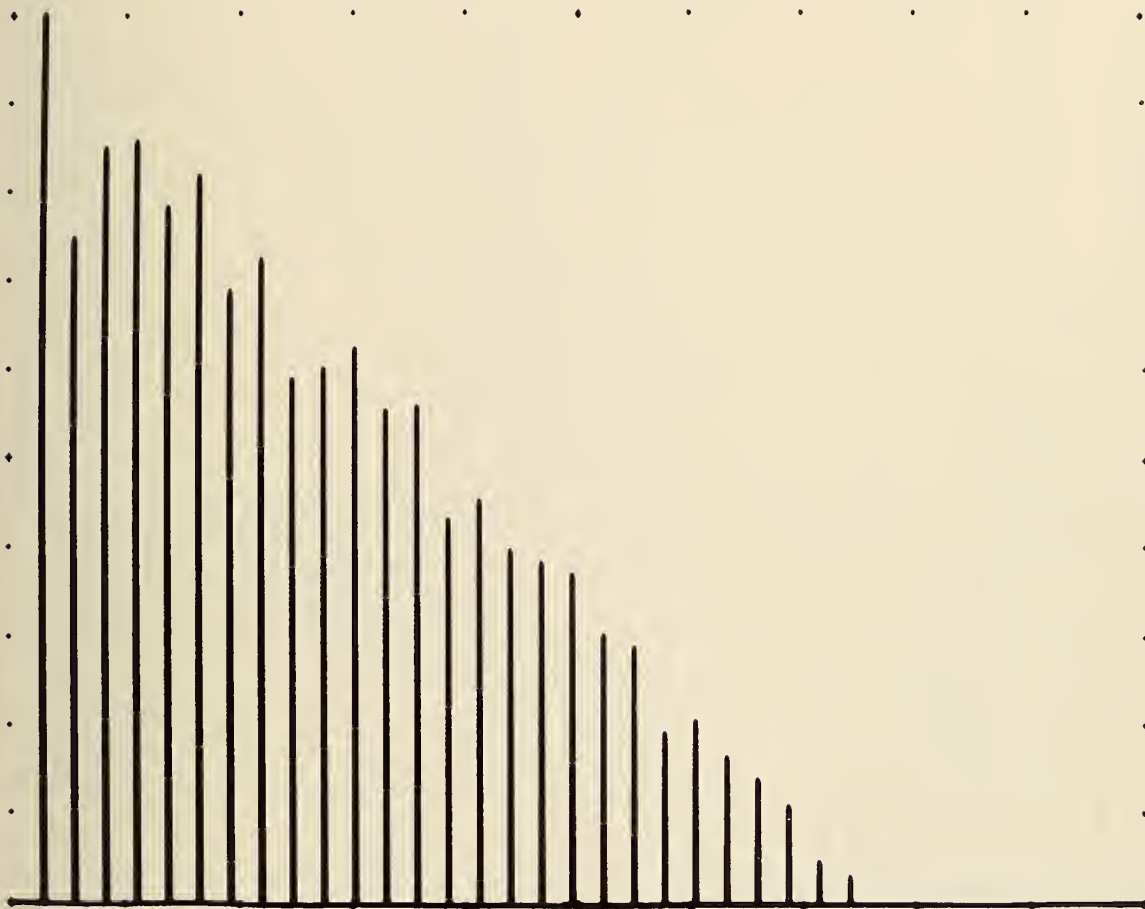Figure 7.   The first frequency block for the LOG_{10} amplitude of the current through the detector.   (Notice only $\ell = 4$, 8, etc., are non zero.) ℓ is the "frequency" relative to ω_o.
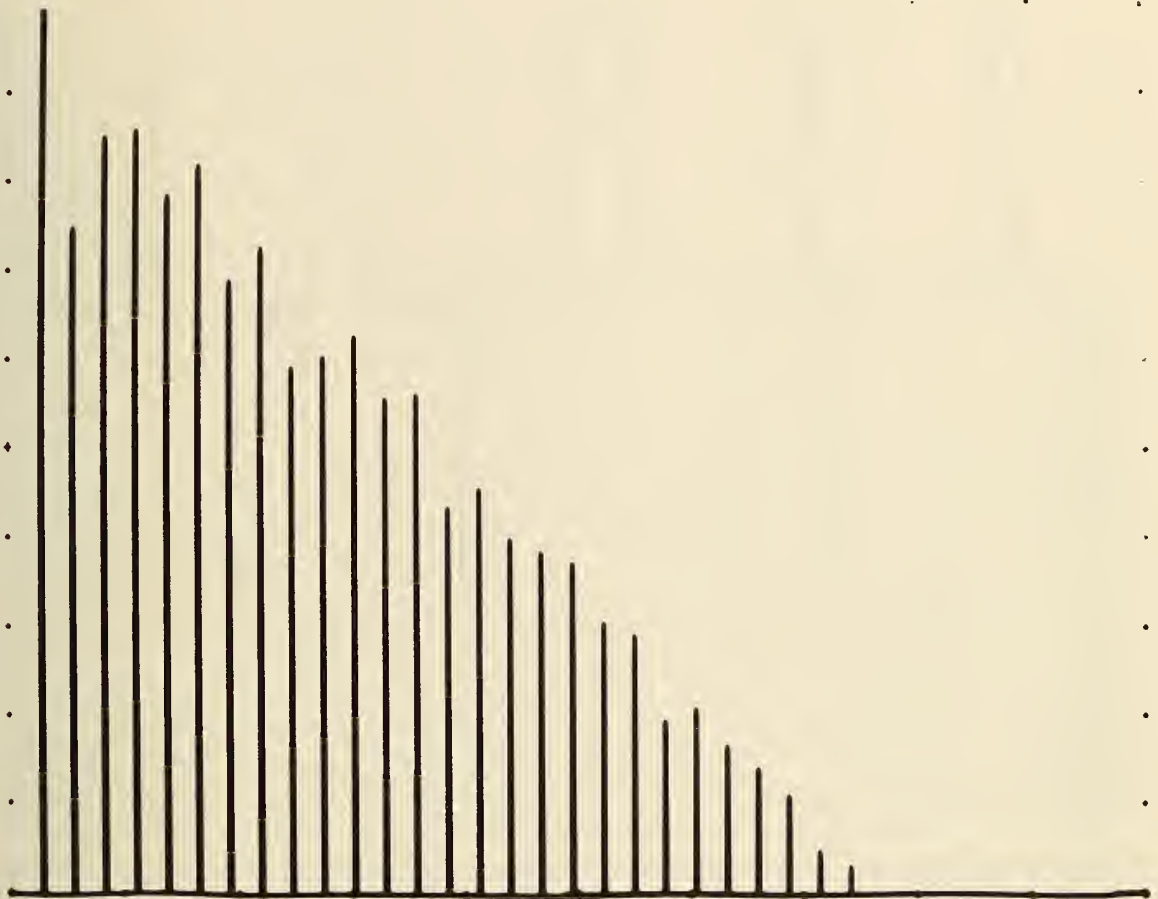
34

AMPLITUDE VOUTTR, ICASE=   6   IPART

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.023+003	0.000+000	5.035+000
Scale	0.000+000	1.023+003	0.000+000	5.035+000

Run 1972 May 03 (124) at 17:50:57   L1,L2= 1, 0   1024 Points, Frame   10

Figure 8.   The first frequency block for the LOG_{10} amplitude of the
voltage across the detector.

PHASE I2-VOUT TRAN ICASE= 6 IPART

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.023+003	-5.021-011	2.910-011
Scale	0.000+000	1.023+003	-5.021-011	2.910-011

Run 1972 May 03 (124) at 17:50:58   L1,L2= 1, 0   1024 Points, Frame   11

Figure 9.   The first frequency block for the relative phase between the
current and voltage of the detector.

36

AMPLITUDE VPHTR, ICASE=   6   IPART

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.023+003	0.000+000	5.035+000
Scale	0.000+000	1.023+003	0.000+000	5.035+000

Run 1972 May 03 (124) at 17:50:59   L1,L2= 1, 0   1024 Points, Frame   12

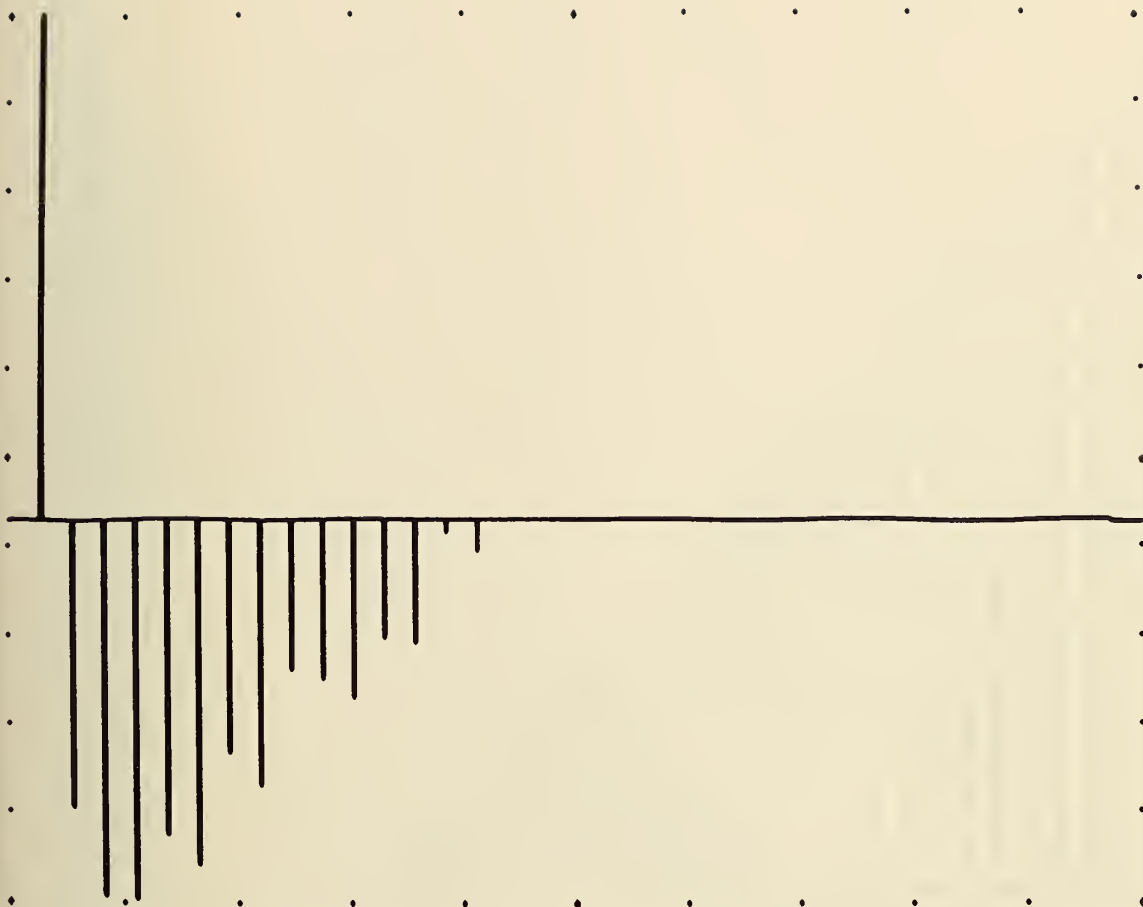Figure 10.   The first frequency block for the LOG_{10} amplitude of the voltage across the junction without z_2.

37
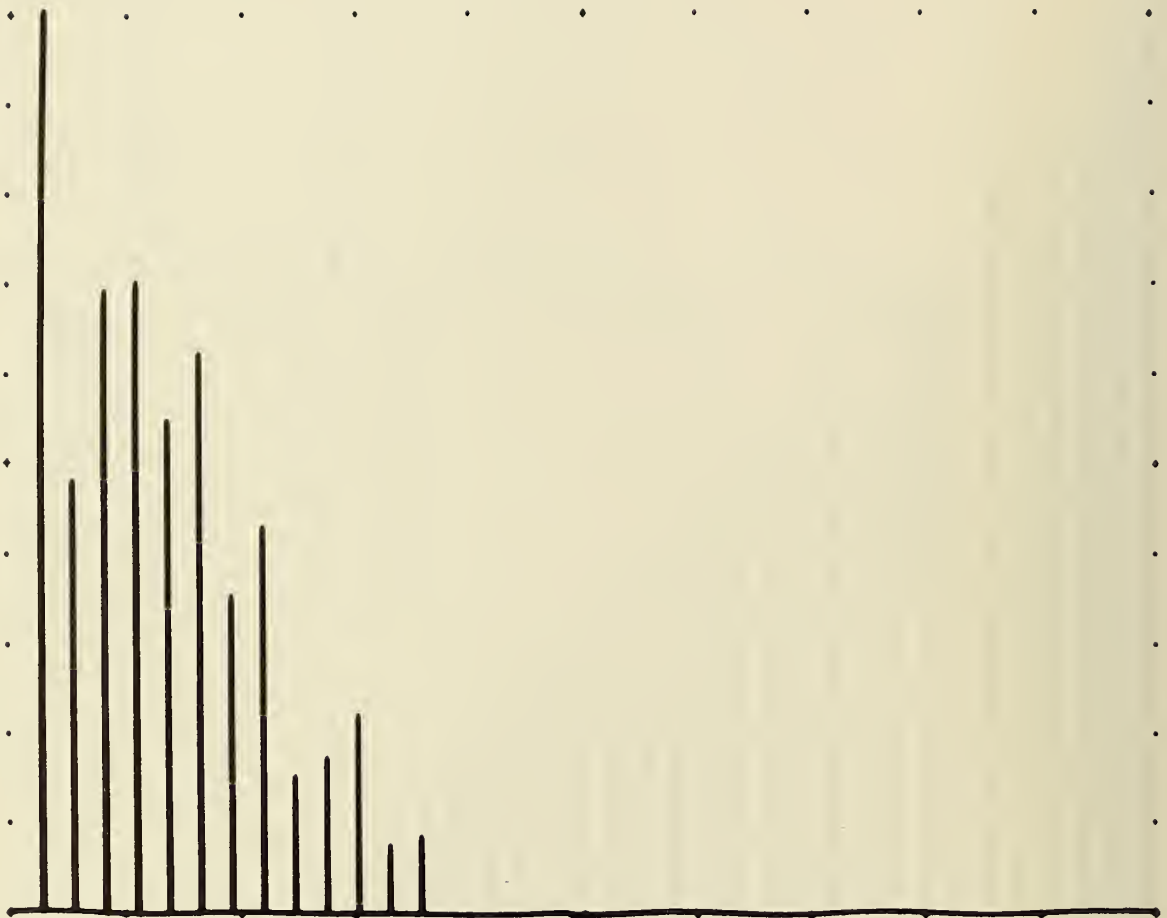
POWER I2VOUT SSTR, ICASE= 6 IPART

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.023+003	0.000+000	4.876+000
Scale	0.000+000	1.023+003	0.000+000	4.876+000

Run 1972 May 03 (124) at 17:50:59   L1,L2= 1, 0   1024 Points, Frame   13

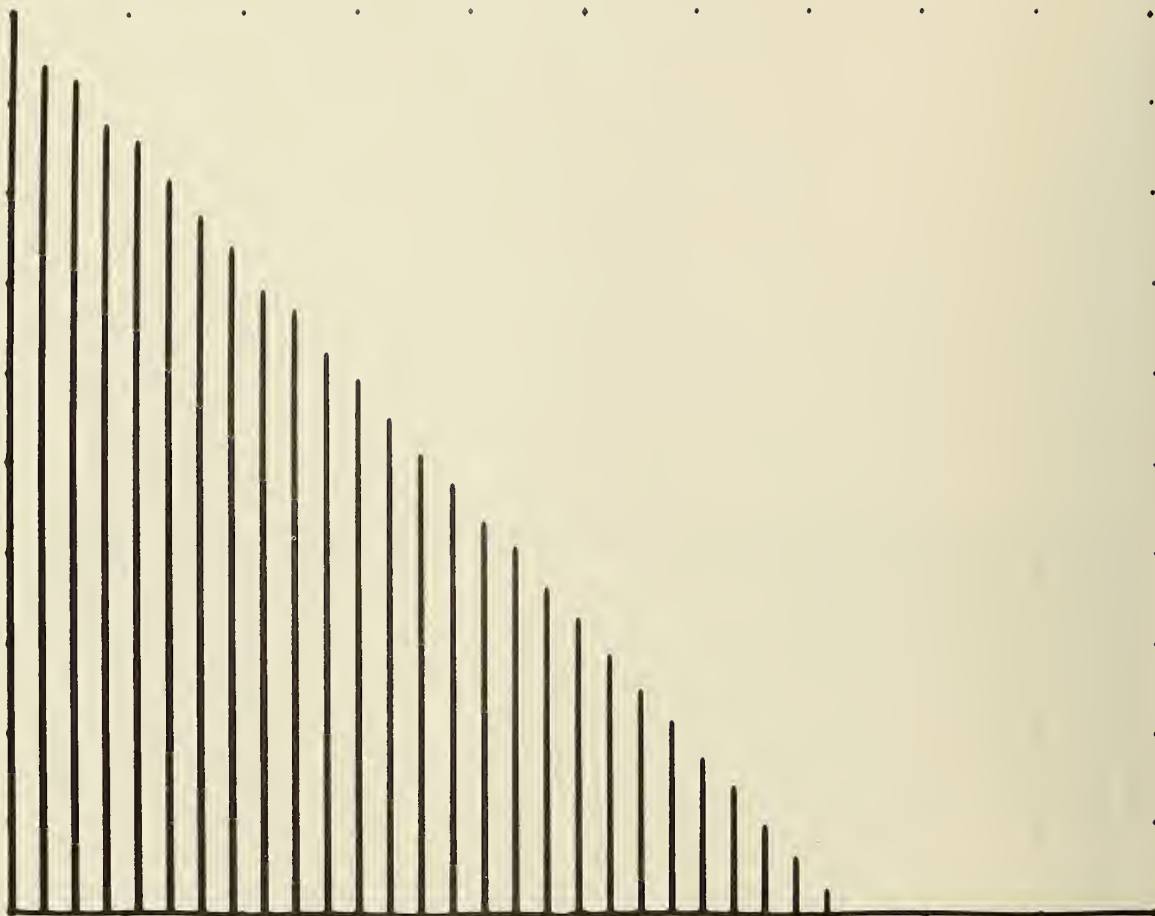Figure 11.   The first frequency block for the LOG_{10} power into the
detector.

POWER IIVOUT SSTR, ICASE= 6 IPART

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.023+003	-4.410+000	5.910+000
Scale	0.000+000	1.023+003	-4.410+000	5.910+000

Run 1972 May 05 (124) at 17:51:00   L1,L2= 1, 0   1024 Points, Frame 14

Figure 12. The first frequency block for the LOG_{10} power into the Josephson junction.
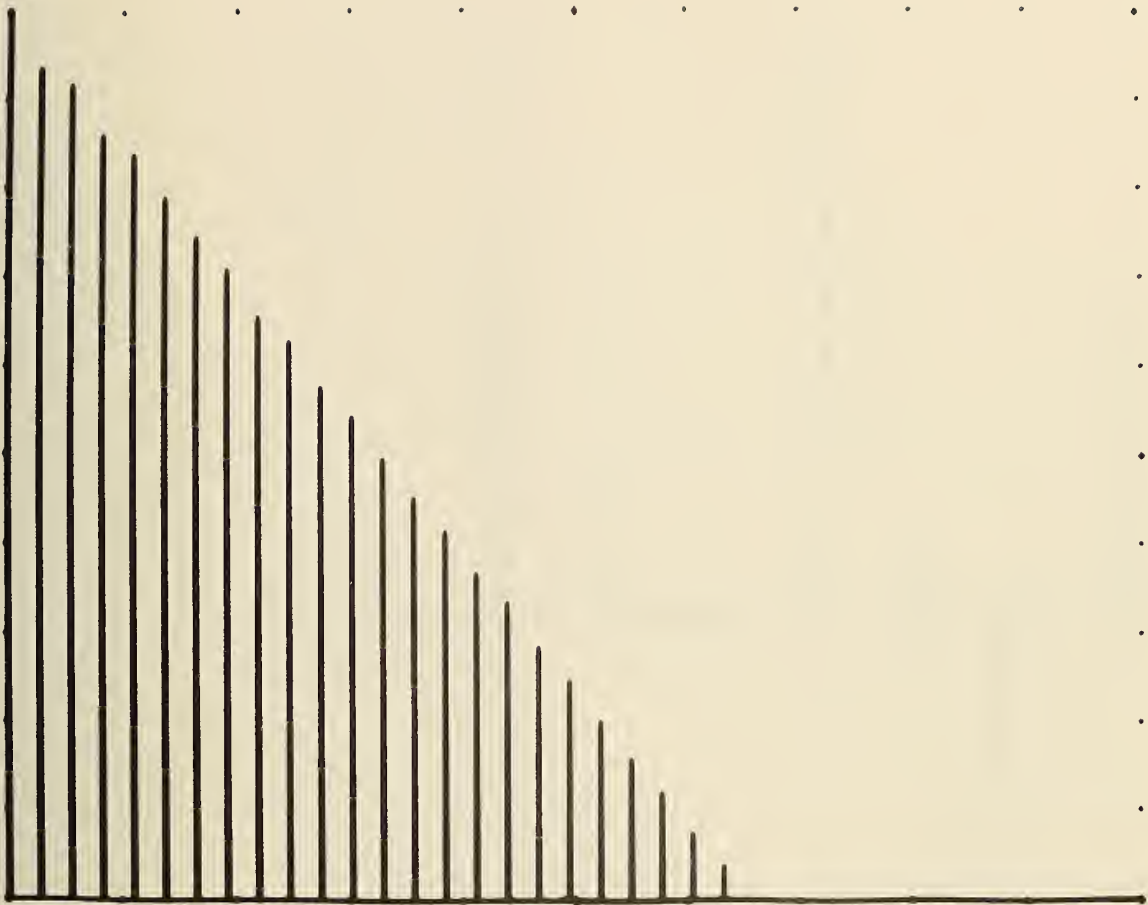
39

AMPLITUDE I2TRAN, ICASE=   6   2PART

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.023+003	0.000+000	4.777+000
Scale	0.000+000	1.023+003	0.000+000	4.777+000

Run 1972 May 03 (124) at 17:51:01   L1,L2= 1, 0   1024 Points, Frame   15

Figure 13.   The second frequency block ($128 \leq \ell < 256$) for the LOG_{10} current through the detector.
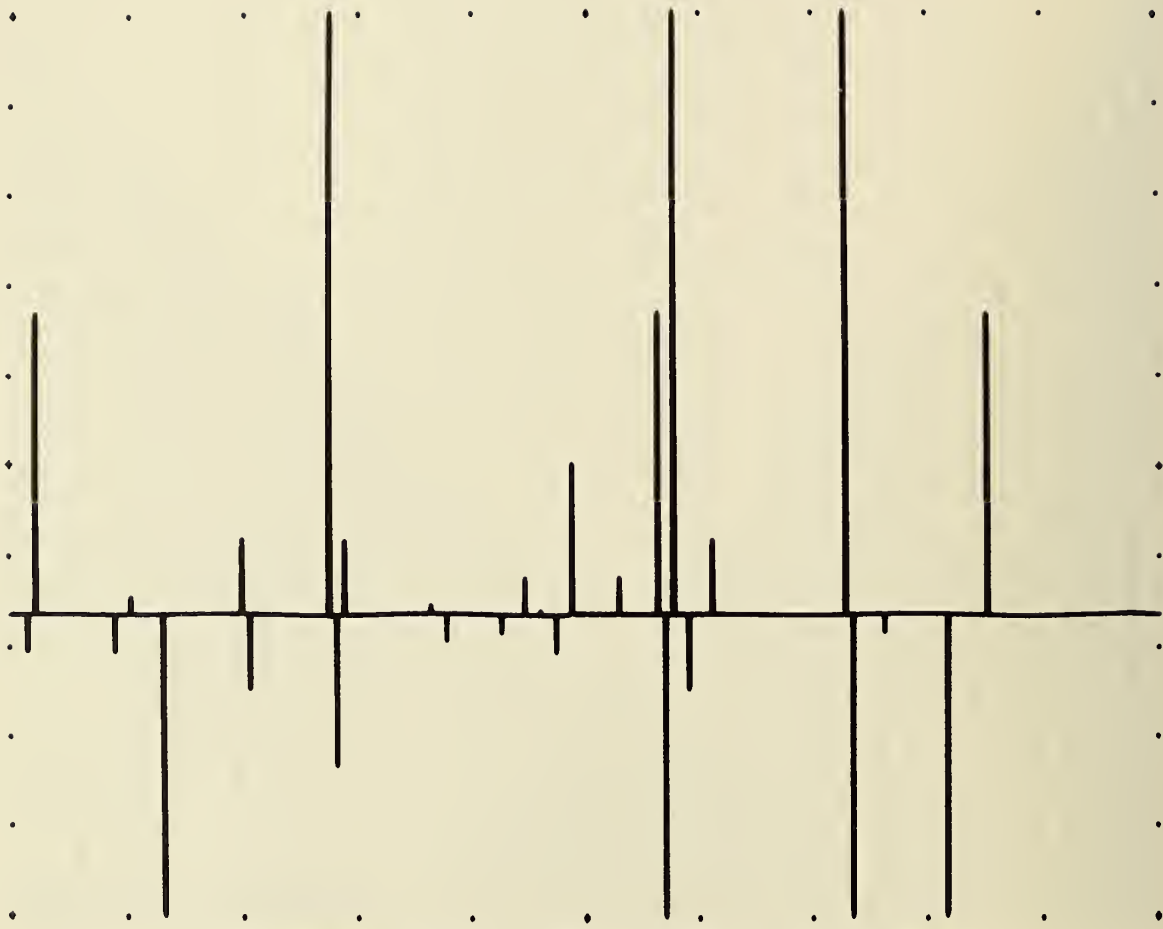
40

AMPLITUDE VOUTTR, ICASE=   6   2PART

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.025+005	0.000+000	4.271+000
Scale	0.000+000	1.025+005	0.000+000	4.271+000

Run 1972 May 03 (124) at 17:51:01   L1,L2= 1, 0   1024 Points, Freso   16

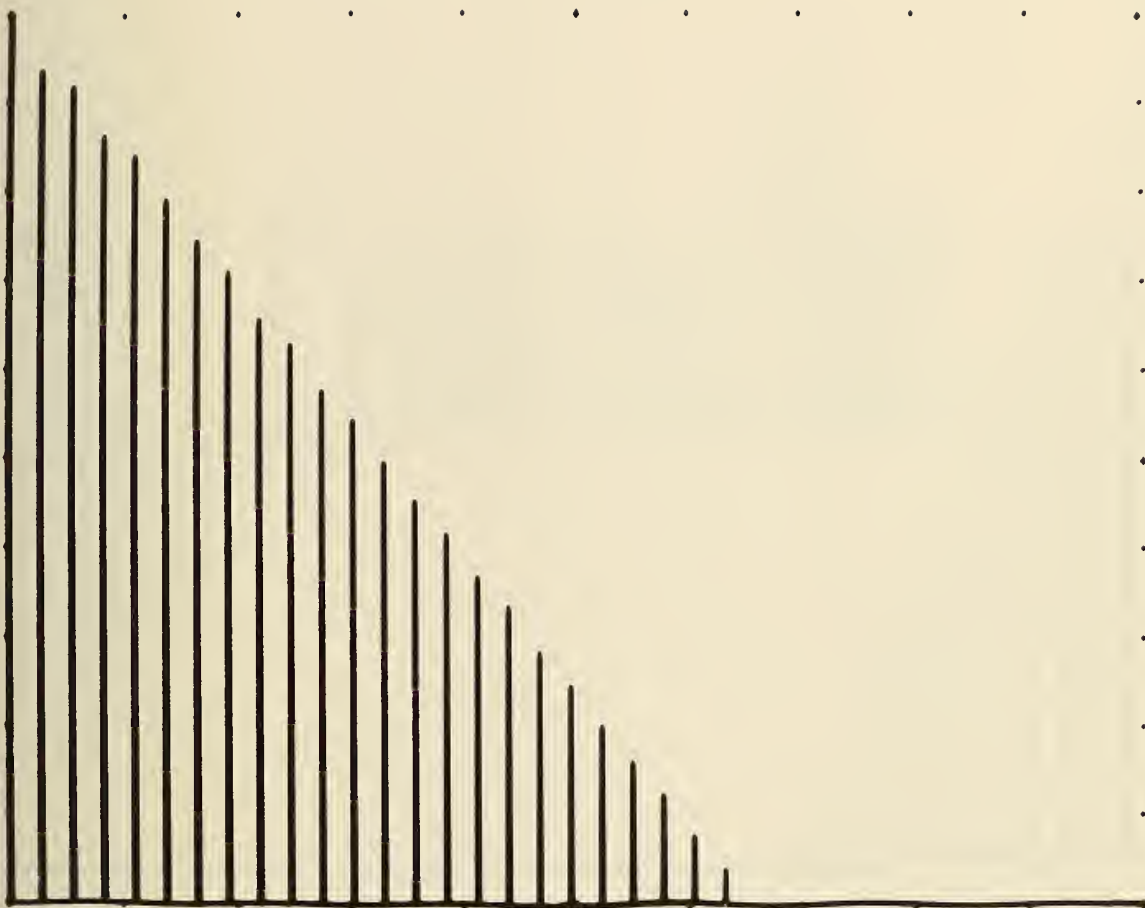Figure 14.   The second frequency block for the LOG_{10} voltage across the detector.

41

PHASE 12-VOUT TRAN ICASE= 6 2PART

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.023+003	-2.910-011	5.021-011
Scale	0.000+000	1.023+003	-2.910-011	5.021-011

Run 1972 May 03 (124) at 17:51:02   L1,L2= 1, 0   1024 Points, Frame   17

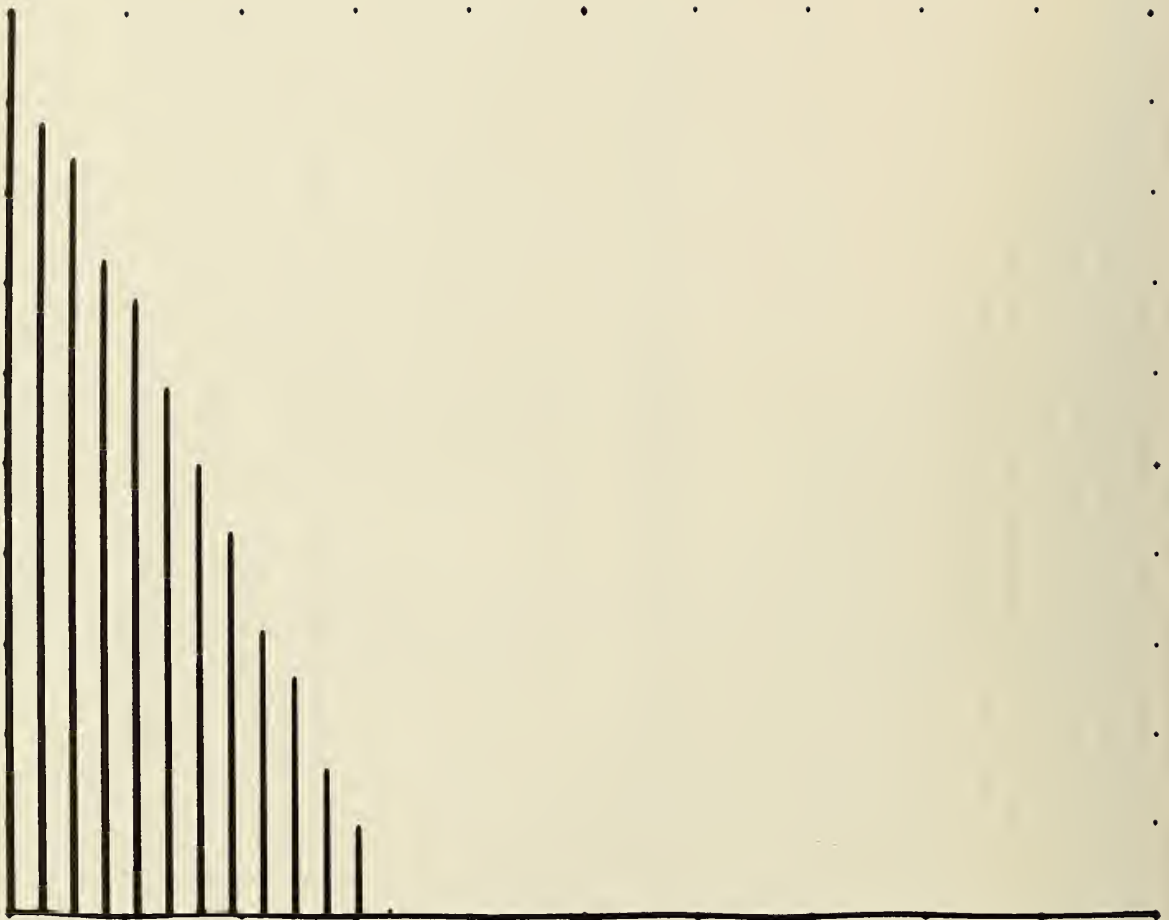Figure 15.   The second frequency block for the relative phase.

42

AMPLITUDE VPHTR, ICASE=   6   2PART

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.025+005	0.000+000	4.271+000
Scale	0.000+000	1.025+005	0.000+000	4.271+000

Run 1972 May 05 (124) at 17:51:05   L1,L2= 1, 0   1024 Points, Frame   18

Figure 16.   The second frequency block for the LOG_{10} voltage across
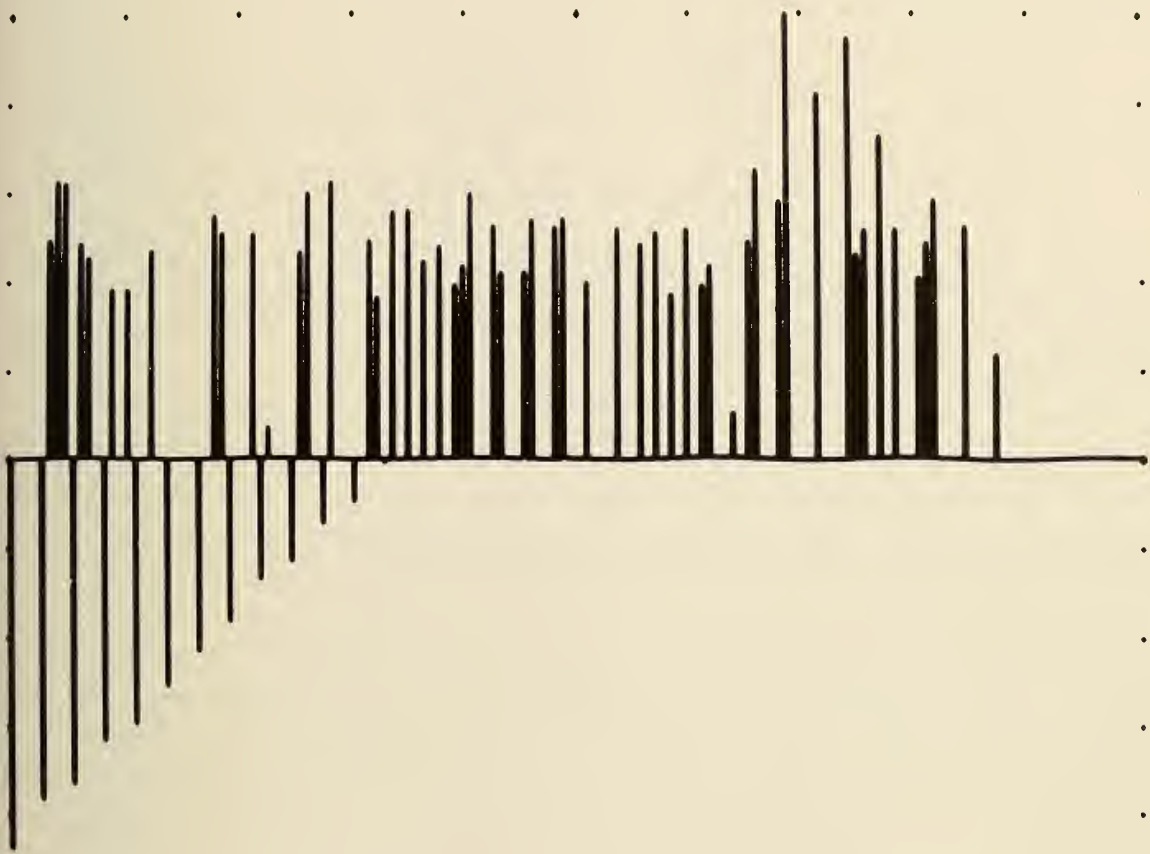   junction without z_2.

POWER I2VOUT SSTR, ICASE=   6   2PART

	X - Min	Max	Y - Min	Max
Function	0.000+000	1.023+003	0.000+000	4.349+000
Scale	0.000+000	1.023+003	0.000+000	4.349+000

Run 1972 May 03 (124) at 17:51:03   L1,L2= 1, 0   1024 Points, Frame   19

Figure 17.   The second 128 lines for LOG_{10} power into detector.

POWER I1VOUT SSTR, ICASE=   6   2PART

	X - Min	Mas	Y - Min	Max
Function	0.000+000	1.025+003	-4.350+000	4.962+000
Scale	0.000+000	1.025+003	-4.962+000	4.962+000

Run 1972 May 05 (126) at 17:51:04   L1,L2= 1, 0   1024 Points, Frame 20

Figure 18. The second 128 lines for LOG_{10} power out of junction.
(Noise pulses are seen as energy sources here.)

45

(Here we have compared the length of the 8th harmonic to the 16th harmonic which is given as full scale of (4.410). We take the absolute value in this effort. The minus sign is simply a code to tell us that the power is out instead of in. The 132th harmonic has value $(5.5 \times (4.350)/6.2 = 3.85)$. The 133rd harmonic has value $(3.5 \times (4.963)/7.1 = 2.45)$. The above values are extracted from the plots and the given scales in the plots. To complete the determination of the power we need to look at the conversion table of figure 19 to find that the positive 4th harmonic is reduced by -2, the negative 8th is reduced by -6, the positive 133rd is reduced by -24, and the negative 132 is reduced by -16. Thus the power is given as

Harmonic	Power
4	$10^{3.918}$
8	$-10^{-2.66}$
132	$-10^{-12.15}$
133	$10^{-21.55}$

To scale these numbers to watts just multiply them by the 4.8830×10^{-11}. Notice that the noise signal in the 133 harmonic is approximately 9 decades less than the corresponding non-noise signal of the 132 harmonic.

It should be obvious that if one wants extensive detailed tables of the various spectra, it is better to put in appropriate print statements into the programs. Our purpose has been mainly qualitative visual information of the power spectra. Figure 19 has a table of power and other information at selected harmonics. These numbers are in the dimensionless form used in the calculation. To convert to watts, volts, amperes, etc., it is necessary to use the constants given in figure 5.

We finish this section by describing the remaining details shown in figure 19. The first row of data gives the total steady state power across the junction without the Z_2 part (POWJ), the total steady state power across the full junction (POW1), the power across the detector (POW2), the power put in by the applied voltages (POWIN), and finally the effective impedance KO. The next block of six different row pairs give the harmonic number to be applied to the row pair information (KEY), the applied voltages (B), the net initial phase at s = 0 (PHC), the power in (POWIN), the power across the junction (POW1), and the power across the detector (POW2). The second row of a given row pair has the dimensionless values for the complex number impedances of that frequency. The frequencies chosen to be printed out are d.c., the direct beat frequency, the lower applied r.f. frequency, the upper applied r.f. frequency, the highest frequency used as a consequence of the number of data points computed, and finally the rotation frequency.

The remaining undiscussed information has the net d.c. voltage across, and the current through the junction without the Z_2 term, the data summarizing the error controls used in the program (EP1, EP2, EP3) and finally the last computed P(α) set EP4(I6) . Please look at the program to see what these errors limits mean in detail.

6. THE SUMMARY

In this report we are documenting the computer program, the mathematical techniques and approximations used, and the procedures for use and interpretation of the computer program. The

```
POWJ,POW1,POW2,POWIN,KW.
    0 705-005    0 705-005    0 710-004   9 551-007    9 125-002
KEY.D.PHC.POWIN.POW1.POW2.
21.22.29
    0    9 250-004  -6 762-000   9 356-006   4 145-002    9 120-005
    9 125-002   0 000-000   0 000-000   0 000-000   9 125-005   0 000-000
    1   0 000-000   0 000-000   0 000-000   5 750-020   4 305-020
    9 125-002   0 000-000   0 000-000   0 000-000   9 125-002   0 000-000
    4   2 400-005  -0 000-000   9 216-007   0 200-005   7 521-004
    9 125-002   0 000-000   0 000-000   0 000-000   9 125-005   0 000-000
    9  -0 000-000  -0 000-000   0 000-000  -1 000-010   1 502-021
    9 125-002   0 000-000   0 000-000   0 000-000   9 125-205   0 000-000
 1024   0 000-000   0 000-000   0 000-000  -4 765-022   1 340-029
    9 125-002   0 000-000   0 000-000   0 200-000   9 125-005   0 000-000
    4   0 000-000   0 000-000   0 000-000   0 200-005   7 521-004
    9 125-002   0 000-000   0 000-000   0 000-000   9 125-005   0 000-002
```

```
VJUN(VOLTS), IJUN(AMPERES)
       9 00000-004      4 04761-005
```

```
ERROR LIMITS EP1.EP2.EP3
EP4(6)
       1 000-000    2 560-006    1 002-005
    0 765-010   0 000-000   0 000-000   2 000-000   3 300-020   0 000-000
```

```
SCALE EXP ONLY DC SET THEN MIN.MAX SET FOR EACH GRAPH
-7 -7  0  -7 -7 -7
    0   0   0   0   0   0   0   0  -9 -15 -20 -20 -21 -21 -21 -20
    0   0   0   0   0   0   0   0  -4  -9 -15 -15 -15 -15 -15 -15
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0  -4  -9 -15 -15 -15 -15 -15 -15
    0   0   0   0   0   0   0   0  -0 -15 -51  90 -51 -92 -51 -90
   -6 -16 -24 -25 -25 -26 -25 -24  -2 -24 -25 -25 -25 -25 -25 -25
```

Figure 19. Scaling information for a particular steady state run.

47

results using this program are to be shown in other reports.

Here the discussion is directed toward the procedures used to check the correctness of the computer program. The strict d. c. cases have been selectively compared with the Stewart[1] voltage—current model. The r. f. applied voltage case has been selectively compared with the Fack, et al,[4] model and their reported results. Both cases agree within the accuracy of the techniques and the graphs of the data reported in those papers. The final check has been a comparison with the voltage driven model. Again the results are in agreement. The integral equation part has not been checked against any other model or results since there are no such published results. In terms of the computer program, the program has been checked to be sure that the D is correctly generated. The only remaining part for possible programming errors is in the integral equation section in subroutine PART. The behavior and visual check shows no obvious errors. A detailed quantitative test is not known.

The most important numerical conclusion was to establish that it is very necessary to fix the φ_a (o) and to allow the V_a (o) to be adjusted when one wants to pin the rotation frequency. If one wants to use only time domain information, then one should use the much simpler programs such as used by Fack, Stewart, and is used in the appendix and fix b_o instead of the rotation frequency. If one is interested in high order harmonic mixing, and detailed harmonic spectra, then this program is best. If noise effects on the system of equations are of interest, then it is necessary to use the time sequence procedures and to allow selected Fourier transforms on the various time series. That analysis will be discussed in another report.

Because the analysis has been done for discrete uniformly spaced time points, we have discrete induced frequencies that are rationally related to each other. The consequences of this situation are additional steps induced in the d. c. current-voltage plots. The excursion of those steps is reduced by increasing the number of time points used. To get an idea on the range of any d. c. step for a given rotation frequency, it is necessary to make at least four calculations. First we chose a particular PHA(1) ≡ φ_o and compute the B_1 for that case. If the program converges, then we have one point for that case. To get a convergence case, we use the transient program with the chosen B_1 and deduce an adequate PHA(1). The steady state program just improves the accuracy of the results. Second, we compute a $B_\pm$ with PHA(1) now $\varphi_o \pm \frac{\pi}{2}$. At least one of these will converge. If both converge, then $(B_+ + B_-)/2$ is the center and $|(B_+ - B_-)|$ is the range of the step. For example, if the $B_+(B_-)$ does not converge because the phase location denies steady state solution, then it is necessary to compute $\varphi_o + \frac{\pi}{4}$, $(\varphi_o - \frac{\pi}{4})$ case to get the $B_{+\frac{1}{2}}$, $(B_{-\frac{1}{2}})$ value. We assume that the range of a step is defined by the following simple formulae-- B = X + R sin θ, where X is the center of the step, R is the range and θ is the phase relation relative to the center of step. Normally θ is not equal to φ_o and normally the formulae is not strictly accurate. It is only true if the dominate processes creating a step are due to a single frequencing mixing. For estimating the range's order of magnitude, we can accept this approximate formulae to get the range and center of the step for the case considered, namely $B_- (B_+)$, B_1 respectively are given by

$$B_1 = X + R \sin \theta$$

$$B_{\mp \frac{1}{2}} = X + R \sin (\theta \mp \pi/4)$$

$$B_{\mp} = X + R \sin (\theta \mp \pi/2)$$

or

$$X = (B_1 + B_{\mp} - \sqrt{2}\ B_{\mp \frac{1}{2}})/(2 - \sqrt{2})$$

$$R = \left[(B_1 - X)^2 + (B_{\mp} - X)^2 \right]^{\frac{1}{2}}$$

$$\frac{B_1 - X}{R} = \sin\theta \text{ or } \theta \text{ is gotten to give center.}$$ For more accurate analysis, it is necessary to do empirical work.

7. APPENDIX

In this section we discuss briefly a simpler computer program called a transient program that can be used to get improved estimates for the effects of r.f. and d.c. applied voltages to the Josephson equations without the operator D. The equations used are closely related to those given in references 1 and 4. We use the trapezoidal spline method with variable step size so that the accuracy per step is kept uniform throughout. Figure 20 gives the Fortran listing for this program. This program simply generates a time series of selected points. This data can then be used to estimate the average rotation frequency, N_1, and to fix a PHA(1), VA(1), and a U.

7.1 The Transient Equation

Here we show the equations used in the program and describe how they relate to equation 11. The transient equations are:

$$\frac{d\varphi}{ds} = V$$

$$\frac{dV}{ds} = -\nu_T V - a \sin\varphi + \overline{B} + \overline{B}_L \cos(N_L s + \theta_L) + \overline{B}_U \cos(N_U s + \theta_U) \tag{A1}$$

Here

$$\varphi = \varphi_a + \varphi_b + \varphi_c \ ,$$

where

$$\frac{d\varphi_b}{ds} = V_b, \quad \frac{dV_b}{ds} = -\nu_T V_b + \overline{B}_L \cos(N_L s + \theta_L) + \overline{B}_U \cos(N_U s + \theta_U) \ ,$$

and φ_c is the initial value of φ_a at $s = 0$; thus here $\varphi_a = 0$ at $s = 0$. We note that $\overline{B} = b_o/K_0$, $\overline{B}_L = b_L/K_0$, and $\overline{B}_U = b_U/K_0$. The best method to fix a, ν_T, and K_0 is to use the main program first for a given C, R, Z_i, a_1, J, etc., with a d.c. case. We then use this program called the transient calculation for the various $\overline{B}_L$, $\overline{B}_U$, θ_L, θ_U cases. Please note that the scale used for b_L, b_U necessarily gives different phases and amplitudes in the main program if the $K(N_L)$, $K(N_U)$ are different from K_0. When the programs are used, this difference needs to be considered. Before we look at the input and output of this program, we briefly consider the criteria for the step size. It simply restricts the absolute change in φ to be less than STEP/(3V1-V2) and the absolute change in V to be less than STEP*a/$\left[\nu_T(3Q1-Q2)\right]$. The STEP is the control accuracy for this allowed change per step; the V1 and V2 are the last two v's used; the Q1 and Q2 are the last two q's used. The q is $\frac{dV}{ds}$.

49

7.2 The Use of the Transient Program

This program is organized under the assumption that a given computer run will fix the r.f. and selected control parameters and will sweep the applied d.c. current B over many values. Thus the first read statement has a parameter which fixed the range of s. The initial V, and PH has no direct correspondence to the expected output VA and PHA, hence they are used to just start the calculation. The IPRINT defines how many steps are skipped before the usual print mode takes place. Figure 21 shows the flow of input data. Figure 22 gives an example of printed output data. The first three rows of printed output simply gives the summary of the input data use in this particular computer run. The list of names S, VA, etc., are the labels for the six columns of numbers that follow the Warning statement. The S is the time variable. The VA is the φ_a with the average steady state slope of the ramp term not removed. PH is the total phase φ. PHB is the r.f. phase including the initial PHC. VR is the integral of V; it is used to estimate the slope of the ramp. PHA is the φ_a phase with the ramp term still present. To illustrate how to get the necessary initial data for the main program from this transient program, we run through the sample output to get the proper numbers. Please note that the printed output far exceeds the necessary information for these variables. It may be possible to suppress all printing except the few numbers of interest. The current calculation assumes that the transients are unimportant after S exceeds 2π, hence we let the computation only run about one more 2π interval. This last interval is used to estimate the ramp and the needed initial PHA(1), VA(1), and U in the main program. Note that P16, V16, and BI are respectively used as the input variables for this initial data. In our sample output data we have the following needed data:

S	VA	PHB	VR	PHA
6.28245	8.665	-.756	20.74	21.486
12.57033	8.933	-.645	46.03	46.66

$B = 102$, and $\nu_T = 32$. We use subscript 1 and 2 to label each data now. We chose these points because we want PHB to be nearly the same in both this program and in the main program. If you change to other THL and THU then you can consider other data points. Keep in mind that it is not necessary to have accurate values. The purpose is to get a guess that is at least accurate to 10 to 20% so that the circle of convergence for the quasilinear procedure will operate effectively in the main program and so that some information is available to estimate potential effects due to spiking. We can then choose the necessary accuracy in the main calculation. The ramp slope is estimated as

$$\tilde{N}_1 = (VR_2 - VR_1)/(s_2 - s_1) + 4.0$$

The initial data is thus estimated as

$$BI = B - \nu_T * \tilde{N}_1 = -26$$

$$V16 = (VA_1 + VA_2)/2. - \tilde{N}_1 = 4.8$$

$$P16 = (PHA_1 - \tilde{N}_1 * 2\pi) + (PHA_2 - \tilde{N}_1 * 4\pi) / 2.$$

$$= (PHA_1 + PHA_2/2. - \tilde{N}_1 * 3\pi = -3.626 .$$

50

```
               PROGRAM JOSPA
C              TO GET INTIAL WAVE SHAPE FOR STARTING GUESSES IN JOSPHC
C              PHDOT=V  $ VDOT=B-V*ANUT-A*SINF(PH)+BL*COSF(NL*S+THL)+
C              BU*COSF(NU*S+THU)
               COMMON /1/ B,ANUT,BL,THL,BU,THU,NL,NU,A,STEP,STEP1
               READ 1,RANGE,V,PH,BL,BU,THL,THU,STEP,ANUT,A,IPRINT,NL,NU
70             READ 1,B
1              FORMAT(5E12.3/5E12.3/3I10)
               IF(EOF,60)71,72
71             CALL EXIT
72             PRINT 2,B,V,PH,BL,BU,THU,THL,STEP,ANUT,A,RANGE,IPRINT ,NL,NU
2              FORMAT(* B,V,PH,BL,BU/THU,THL,STEP,ANUT,A,RANGE/IPRINT ,NL,NU *
1              /5E12.3/6E12.3/3I10/* S,VA,PH,PHB, VR,PHA * //)
               PRINT 99
99             FORMAT(* WARNING THE VA AND PHA HAVE NOT HAD THE RAMP TERMS REMOVE
1D *)
               J=0 $STEP1=STEP*A/ANUT$IF(ABSF(V)-1.)26,26,25
26             H=STEP/100. $GO TO 270
25             H=STEP/(100.*ABSF(V))
270            PHC=PH        $ S=0.
               I1=-2$RANG1=RANGE-7.
27             I1=I1+1 $ IF(S-RANGE)4,4,70
40             J=J-1         $IF(J)41,41,3
41             IF(S-RANG1)3,417,417
417            IF(S)410,410,412
410            IF(NL*NU)70,70,415
415            X1=-BL/(NL*SQRTF(NL*NL+ANUT*ANUT))
               X2=-BU/(NU*SQRTF(NU*NU+ANUT*ANUT))
               APH1=ATANF(ANUT/NL)   +THL  $X3=-X1*NL
               APH2=ATANF(ANUT/NU)   +THU  $ X4=-X2*NU
412            S8=NL*S+APH1  $ S9=NU*S +APH2
               PHB=X1*COSF(S8)+X2*COSF(S9)
               VB=X3*SINF(S8)+X4*SINF(S9)
               VA=V-VB
               PHA=PH-PHB          -PHC
413            PRINT 42,S,VA,PH,PHB,VR ,PHA $J=IPRINT
3              CONTINUE  $ GO TO 27
42             FORMAT(7E15.8)
4              IF(I1)14,15,16
14             V2=V $ PH2=PH $S=S2=0. $ CALL START(PH,S,QX) $QT=QX-ANUT*V $GO TO
1  410
15             PHY=PH2+H*V2 $CALL COM(PH,V,Q,S,PH2,V2,QX,0.,H,PHY)
               VR=V*H
               QT=Q-ANUT*V $ PH1=PH$ V1=V $Q2=QX $Q1=Q $S1=S $GO TO 40
16             CALL PROJ(PHY,H1,H,V2,V1,Q2,Q1,PH1)
               CALL COM(PH,V,Q,S,PH1,V1,Q1,S1,H1,PHY) $QT=Q-ANUT*V
               DL1=PH-PH1 $DL2=PH1-PH2
               Q2=Q1   $V2=V1 $PH2=PH1 $S2=S1
               VR=VR+V*H1
               Q1=Q $ V1=V $ PH1=PH $S1=S $H=H1 $IF(DL1*DL2)412,412,40
               END




               SUBROUTINE COM(PH,V,Q,S,PHX,VX,QX,SX,HX,PHY)
               COMMON /1/ B,ANUT,BL,THL,BU,THU,NL,NU,A,STEP,STEP1
               DATA(ERO=1.E-5)
               H=HX
               H3=H*ANUT/2. $H4=1.-H3 $H5=1.+H3 $S=SX+HX
               CALL QR(0,QY,QZ,PHY) $ H6=(H*H*.25)/H5 $H9=H4/H5 $H10=H/(2.*H5)
               H7=1./(1.-H6*QZ) $H8=VX*H/H5+PHX+H6*QX $QZ1=QF(S) $QY=QY+QZ1
               IV=5
4              IV=IV-1 $ IF(IV)5,6,6
6              DEL=(H8-PHY+H6*QY)*H7
               PHY=PHY+DEL $CALL QR(1,QY,0.,PHY) $QY=QY+QZ1
               IF(ABSF(DEL)-ERO)8,4,4
5              PRINT 7,S $CALL EXIT
7              FORMAT(* NO CONVERGENCE *,E12.3)
8              PH=PHY $Q=QY $V=(H9*VX+H10*(QY+QX) )    $RETURN $END
```

Figure 20. The transient Fortran program listing.

51

```
      SUBROUTINE PROJ(PHY,H1,H,V2,V1,Q2,Q1,PH1)
      COMMON /1/ B,ANUT,BL,THL,BU,THU,NL,NU,A,STEP,STEP1
      H1A=STEP/ABSF(3.*V1-V2)
      H1B=STEP1/ABSF(3.*Q1-Q2)
      H1=MIN1F(H1A,H1B)
      PHY=PH1+H1*(V1+(V1-V2)*H1/(2.*H))          $RETURN $END


      SUBROUTINE START(PHX,SX,QX)
      COMMON /1/ B,ANUT,BL,THL,BU,THU,NL,NU,A,STEP,STEP1
      CALL QR(1,QX,0.,PHX) $QX=QX+QF(SX) $RETURN $END


      FUNCTION QF(Z)
      COMMON /1/ B,ANUT,BL,THL,BU,THU,NL,NU,A,STEP,STEP1
      QF=B+BL*COSF(NL*Z+THL)+BU*COSF(NU*Z+THU) $RETURN $END


      SUBROUTINE QR(I,QA,QB,X)
      COMMON /1/ B,ANUT,BL,THL,BU,THU,NL,NU,A,STEP,STEP1
      IF(I)1,1,3
1     QB=-A*COSF(X)
3     QA=-A*SINF(X) $RETURN$END
```

Figure 20. (Continued)

52

```
PROGRAM JOSPA

    READ 1, RANGE, ETC.

70 READ 1, B
IF(EOF) EXIT
IF(S-RANGE)4,4,70

4 | COMPLETE
  | PRINT
```

```
SUBROUTINES CALLED

    COM
    PROJ
    START
    QF
    QR
```

Figure 21.   The flow chart for Josephson transient program.

```
B,V,PH,BL,BU/THU,THL,STEP,ANUT,A/RANGE,IPRINT ,NL,NU
 1.020+002    1.000+000   -3.142+000    7.680+002   -0.000+000
-0.000+000   -0.000+000    1.000-001    3.200+001    2.560+002    1.300+001
   20         4
              5
S,VA,PH,PHB, VR,PHA
```

WARNING THE VA AND PHA HAVE NOT HAD THE RAMP TERMS REMOVED

```
0.0000000+000  -2.2637692+000  -3.14159000+000  -7.3846538-001   0.0000000+000   7.3846538-001
5.2366890-001   9.0583331+000   4.0314262+000    5.4848116+000    7.1363491+000   1.6882046+000
1.2644935+000  -4.7236083+000   4.7243774+000   -5.6085878+000   -1.6033596+000   4.2174011+000
1.9589183+000  -4.1678109+000   7.6273521+000    5.8749734+000    1.0721074+001   4.8939686+000
2.6702531+000   1.0387663+000  -1.2688871+000    5.3893344+000    1.8615323+000   7.2620372+000
3.5625798+000  -6.9947028-001   1.3206977+001    5.9533194+000    1.6308559+001   1.0352479+001
4.2394952+000   1.0525912+001   4.9693708+000   -5.3735448+000    8.1041234+000   1.3484057+001
5.1330417+000  -6.5643708-001   1.9486012+001    5.9532742+000    2.2591354+001   1.6674329+001
5.8101493+000   1.0525274+001   1.1252328+001   -5.3720415+000    1.4388901+001   1.9765960+001
6.0004458+000   4.6213363+000   1.2722212+001   -5.6598327+000    1.5868677+001   2.1523635+001
6.0211319+000   2.8073161+000   1.2971203+001   -5.4877908+000    1.6117684+001   2.1600584+001
6.0419940+000   9.8052400-001   1.3222137+001   -5.2762563+000    1.6368603+001   2.1639983+001
6.0627470+000  -7.1037167-001   1.3471485+001   -5.0293933+000    1.6617962+001   2.1642468+001
6.0829914+000  -2.1343034+000   1.3716442+001   -4.7551903+000    1.6863001+001   2.1613222+001
6.1023350+000  -3.2083703+000   1.3955407+001   -4.4640450+000    1.7102137+001   2.1561043+001
6.1204866+000  -3.9013495+000   1.4187906+001   -4.1665132+000    1.7334889+001   2.1496009+001
6.1372894+000  -4.2241216+000   1.4414201+001   -3.8714845+000    1.7561502+001   2.1427275+001
6.1527079+000  -4.2072859+000   1.4634326+001   -3.5853490+000    1.7782588+001   2.1361865+001
6.1667936+000  -3.8945574+000   1.4850361+001   -3.3120005+000    1.7998905+001   2.1304452+001
6.1796514+000  -3.3321376+000   1.5062822+001   -3.0532830+000    1.8211253+001   2.1257695+001
6.1914116+000  -2.5653672+000   1.5271621+001   -2.8095617+000    1.8420445+001   2.1222773+001
6.2022123+000  -1.6370191+000   1.5478055+001   -2.5802386+000    1.8627255+001   2.1199883+001
6.2121879+000  -5.8684359-001   1.5682049+001   -2.3641448+000    1.8832465+001   2.1188639+001
6.2214641+000   5.4822635-001   1.5886340+001   -2.1598177+000    1.9036842+001   2.1188348+001
6.2301557+000   1.7335386+000   1.6090933+001   -1.9656647+000    1.9241154+001   2.1198187+001
6.2383666+000   2.9360329+000   1.6295663+001   -1.7800612+000    1.9446182+001   2.1217314+001
6.2461916+000   4.1234844+000   1.6501337+001   -1.6013935+000    1.9652730+001   2.1244921+001
6.2537177+000   5.2637185+000   1.6710502+001   -1.4280685+000    1.9861643+001   2.1280261+001
6.2610268+000   6.3238052+000   1.6922562+001   -1.2584993+000    2.0073826+001   2.1322651+001
6.2681984+000   7.2692387+000   1.7138801+001   -1.0910737+000    2.0290258+001   2.1371465+001
6.2753124+000   8.0631135+000   1.7360405+001   -9.2410704-001    2.0512024+001   2.1426102+001
```

Figure 22. Sample printout from the transient program.

Note that only integer $\tilde{N}_1$ can be used in the main program and that the choice of B in the transient program does not necessarily correspond to such integers.

8. ACKNOWLEDGMENTS

We wish to thank Stephen Jarvis and Clark Hamilton for useful discussions.

9. REFERENCES

1. W. C. Stewart, App. Phys. Letters <u>12</u>, 8 (1968), pp. 277-280.
2. <u>Theory and Application of Spline Functions</u>, Ed. T. Greville, Academic Press (1969). An Introduction to the Application of Spline Functions to Initial Value Problems, Frank R. Loscalzo, pp. 37-64.
3. A. Miele, "Modified Quasilinearization Method for Solving Nonlinear, Two Point Boundary Value Problems," (Rice University, 1970), Clearinghouse Reports N71-1779.
4. H. Fack, V. Kose, and H.-J. Schrader, "Calculation of Characteristic Curves for Point-Contact-Josephson-Elements Subjected to High Frequency Radiation," Messtechnik (February 1971) pp. 31-38.

FORM NBS-114A (1-71)

U.S. DEPT. OF COMM. BIBLIOGRAPHIC DATA SHEET	1. PUBLICATION OR REPORT NO. NBS TN-627	2. Gov't Accession No.	3. Recipient's Accession No.
4. TITLE AND SUBTITLE Computation of Spectral Data for a Josephson Junction Circuit			5. Publication Date November 1972
			6. Performing Organization Code
7. AUTHOR(S) E. G. Johnson, Jr. and D. G. McDonald			8. Performing Organization
9. PERFORMING ORGANIZATION NAME AND ADDRESS NATIONAL BUREAU OF STANDARDS, Boulder Labs DEPARTMENT OF COMMERCE Boulder, CO 80302			10. Project/Task/Work Unit N 2710164
			11. Contract/Grant No.
12. Sponsoring Organization Name and Address Same as 9 above.			13. Type of Report & Period Covered Final (4/72-10/72)
			14. Sponsoring Agency Code

15. SUPPLEMENTARY NOTES

16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

A computer program has been developed to study power flow between differen
frequency channels in a Josephson junction circuit. This paper discusses the
mathematical assumptions used to get such results. They are the trapezoidal
approximation from spline theory and the use of a finite range of frequencies
characterize the frequency spectrum. This paper describes the program and
provides the FORTRAN listing, flow charts, and discusses how to use the
program. A discussion of possible sources of errors is also included.

17. KEY WORDS (Alphabetical order, separated by semicolons)
Differential equation; fast Fourier transform; Josephson junction; nonlinear
integral-differential equation; spline theory.

| 18. AVAILABILITY STATEMENT

[X] UNLIMITED.

[ ] FOR OFFICIAL DISTRIBUTION. DO NOT RELEASE
TO NTIS. | 19. SECURITY CLASS
(THIS REPORT)

UNCLASSIFIED | 21. NO. OF PA
63 |
| | 20. SECURITY CLASS
(THIS PAGE)

UNCLASSIFIED | 22. Price
60¢ |

NBS TECHNICAL PUBLICATIONS

PERIODICALS

JOURNAL OF RESEARCH reports National Bureau of Standards research and development in physics, mathematics, and chemistry. Comprehensive scientific papers give complete details of the work, including laboratory data, experimental procedures, and theoretical and mathematical analyses. Illustrated with photographs, drawings, and charts. Includes listings of other NBS papers as issued.

Published in two sections, available separately:

• Physics and Chemistry

Papers of interest primarily to scientists working in these fields. This section covers a broad range of physical and chemical research, with major emphasis on standards of physical measurement, fundamental constants, and properties of matter. Issued six times a year. Annual subscription: Domestic, $9.50; $2.25 additional for foreign mailing.

• Mathematical Sciences

Studies and compilations designed mainly for the mathematician and theoretical physicist. Topics in mathematical statistics, theory of experiment design, numerical analysis, theoretical physics and chemistry, logical design and programming of computers and computer systems. Short numerical tables. Issued quarterly. Annual subscription: Domestic, $5.00; $1.25 additional for foreign mailing.

TECHNICAL NEWS BULLETIN

The best single source of information concerning the Bureau's measurement, research, developmental, cooperative, and publication activities, this monthly publication is designed for the industry-oriented individual whose daily work involves intimate contact with science and technology—*for engineers, chemists, physicists, research managers, product-development managers, and company executives.* Includes listing of all NBS papers as issued. Annual subscription: Domestic, $3.00; $1.00 additional for foreign mailing.

Bibliographic Subscription Services

The following current-awareness and literature-survey bibliographies are issued periodically by the Bureau: Cryogenic Data Center Current Awareness Service (weekly), Liquefied Natural Gas (quarterly), Superconducting Devices and Materials (quarterly), and Electromagnetic Metrology Current Awareness Service (monthly). Available only from NBS Boulder Laboratories. Ordering and cost information may be obtained from the Program Information Office, National Bureau of Standards, Boulder, Colorado 80302.

NONPERIODICALS

Applied Mathematics Series. Mathematical tables, manuals, and studies.

Building Science Series. Research results, test methods, and performance criteria of building materials, components, systems, and structures.

Handbooks. Recommended codes of engineering and industrial practice (including safety codes) developed in cooperation with interested industries, professional organizations, and regulatory bodies.

Special Publications. Proceedings of NBS conferences, bibliographies, annual reports, wall charts, pamphlets, etc.

Monographs. Major contributions to the technical literature on various subjects related to the Bureau's scientific and technical activities.

National Standard Reference Data Series. NSRDS provides quantitative data on the physical and chemical properties of materials, compiled from the world's literature and critically evaluated.

Product Standards. Provide requirements for sizes, types, quality, and methods for testing various industrial products. These standards are developed cooperatively with interested Government and industry groups and provide the basis for common understanding of product characteristics for both buyers and sellers. Their use is voluntary.

Technical Notes. This series consists of communications and reports (covering both other-agency and NBS-sponsored work) of limited or transitory interest.

Federal Information Processing Standards Publications. This series is the official publication within the Federal Government for information on standards adopted and promulgated under the Public Law 89–306, and Bureau of the Budget Circular A–86 entitled, Standardization of Data Elements and Codes in Data Systems.

Consumer Information Series. Practical information, based on NBS research and experience, covering areas of interest to the consumer. Easily understandable language and illustrations provide useful background knowledge for shopping in today's technological marketplace.

CATALOGS OF NBS PUBLICATIONS

NBS Special Publication 305, Publications of the NBS. 1966-1967. When ordering, include Catalog No. C13.10:305. Price $2.00; 50 cents additional for foreign mailing.

NBS Special Publication 305, Supplement 1, Publications of the NBS, 1968-1969. When ordering, include Catalog No. C13.10:305/Suppl. 1. Price $4.50; $1.25 additional for foreign mailing.

NBS Special Publication 305, Supplement 2, Publications of the NBS, 1970. When ordering, include Catalog No. C13.10:305/Suppl. 2. Price $3.25; 85 cents additional for foreign mailing.

Order NBS publications (except Bibliographic Subscription Services) from: Superintendent of Documents, Government Printing Office, Washington, D.C. 20402.