FREQUENCY STANDARDS AND CLOCKS: A TUTORIAL INTRODUCTION
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FREQUENCY STANDARDS AND CLOCKS:
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The topic of frequency standards and clocks is treated in a tutorial and non-mathematical way. The concepts of time, frequency, frequency stability, and accuracy are introduced. The general physical principles and design features of frequency standards and clocks are described. The design, performance, and limitations of quartz crystal oscillators and atomic devices (cesium, hydrogen, rubidium) are discussed in detail and critically compared for laboratory devices as well as for devices intended for field usage.

Key Words: Cesium beam; Clocks (atomic); Crystal oscillator; Frequency accuracy; Frequency stability; Frequency standards; Hydrogen maser; Quartz crystal; Rubidium gas cell; Timekeeping.

1. INTRODUCTION

Frequency standards and clocks: what do they have in common? A more complete answer will be given later. We note for the moment that most clocks and in particular the very accurate and precise ones are based on frequency standards. The reason for this is the intimate relationship between frequency (symbol \( \nu \), "nu") and time (symbol \( t \)). If we look at a series of events which are occurring in a somewhat regular fashion, e.g., the rise of the sun every morning, we can state how many of these events occur in a given time period: this number would be the frequency of this series of events. In our example we could say that the frequency of sunrises is \( \nu = 7 \) events per week.

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1 This paper was originally prepared for inclusion in a handbook on time and frequency for the Air Force.
or \( \nu = 365 \) events per year. "Events per week" or "events per year" would be called the unit which we used for our frequency number; this frequency number is different for different units. In our example we assumed that we know somehow what a week or a year is, i.e., we relied on some external definition for our unit of time.

We can now ask, what is the time between the events? The answer for our example is simple, one sunrise succeeds the other after \( t = \frac{1}{7} \) week or \( t = \frac{1}{365} \) year where we used "week" and "year" as two possible choices for our unit of time. We can rely on the regularity and precision of the occurrence of the events, in other words, on their precise periodicity. Hence, we could define: the unit of time is one week; one week is the time elapsed at the seventh sunrise following an initial sunrise.

We learned two things: (a) For periodic events, the time between the events \( t \) is related to the frequency \( \nu \) of their occurrence in the following simple way

\[
\nu = \frac{1}{t} \tag{1}
\]

and (b) that periodic events can be used to define time, i.e., the generator of the periodic events - the frequency standard - can be used as a clock. The frequency standard becomes a clock by the addition of a counting mechanism for the events.

In our example above, the frequency standard is the rotating earth. The time between recurring events is one day. This frequency standard served mankind for thousands of years and remained until very recently the source for the definition of time. The counting mechanism which made it a clock was the recording of years and days.
FREQUENCY = NUMBER OF EVENTS PER UNIT OF TIME

ACCUMULATED CLOCK TIME = \[ \frac{\text{TOTAL NUMBER OF EVENTS}}{\text{NUMBER OF EVENTS PER UNIT OF TIME}} \]

POSSIBLE DEFINITION:

UNIT OF TIME = A SPECIFIC NUMBER OF PERIODS OF A WELL-DEFINED EVENT GENERATOR

Fig. 1. Definition of time and frequency.
The needs to interpolate time from day to day, to get along for many days without celestial observations, and to more precisely measure time-intervals which are very much shorter than a day brought about the invention of clocks. Although there are other types of clocks like the sand-clock or the decay in radiation intensity of a radioactive substance, we shall confine ourselves to the discussion of clocks based on frequency standards.

The first clocks based on a frequency standard (a pendulum) were invented about 400 years ago. This type of clock is still most widely used today. The pendulum may be a suspended weight (gravitational pendulum) like in "grandfather" clocks or the balance (torsion pendulum) of modern wristwatches. The objects of our discussion in this report are today's most advanced frequency standards and clocks; however, a close look at our traditional clocks will show all the essential features which we will recognize again in our later discussion of quartz crystal and atomic clocks.

The pendulum in our clock is the frequency determining element. In order to arrive at a frequency standard the pendulum has to be set in motion and kept in motion: A source of energy is necessary together with means to transfer this energy to the frequency determining element. In a wristwatch this source of energy is typically the winding spring, and the energy is transferred by mechanical means which are controlled by the pendulum itself (feedback) in order to cause energy transfer in the proper amount at the proper time in synchronism with the movement of the pendulum. We now have a frequency standard; the tick frequency of its pendulum could be picked up acoustically, for example, and used as a standard frequency. This is actually being done commercially when adjusting the rate of a clock: the tick-frequency is compared to some (better) standard frequency. In order to arrive at a clock, a read-out mechanism is necessary which counts and accumulates the ticks (more
Fig. 2. Frequency standard and clock.
accurately: the time between the ticks) and displays the result. In our example of a wristwatch, this is accomplished by a suitably dimensioned set of gears and the moving hands on the clockface.

The unit of time today is the second (symbol s). Very much in analogy to our sunrise example, the second is defined in reference to a frequency determining element. Since 1967 by international agreement this "natural pendulum" is the cesium atom. One second is defined in the official wording as "the duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom". Accordingly, the frequency of the cesium pendulum is 9192631770 events per second (the cesium atom is a very rapidly oscillating pendulum). Following our eq (1) the unit of frequency is then defined as hertz (symbol Hz) which means the repetitive occurrence of one event per second (the use of "hertz" is preferred to the older term "cycle per second", cps).

2. BASIC CONCEPTS

2.1 Accuracy, Reproducibility, Stability

The performance of frequency standards is usually described in terms of accuracy, reproducibility, and stability. We will use these terms in the sense of the following definitions:

Accuracy: the degree of conformity of a measured and/or calculated value to some specified value or definition.

Reproducibility: the degree of agreement across a set of independent devices of the same design after adjustment of appropriate specified parameters in each device.
Stability: the frequency and/or time domain behavior\(^2\) of a process. In the time domain (i.e., the sampling time is the varied quantity) a frequently used measure of stability is the Allan variance (to be explained later) or its square root.

It is obvious from these definitions that frequency accuracy will be largely of interest in scientific measurements and in the evaluation and intercomparison of the most advanced devices, but of little or no interest to the average user of frequency standards. A good reproducibility is an asset in applications where it is of importance to rely on some degree of conformity of the output frequency of several devices. The characterization of the stability of a frequency standard is usually the most important information to the user. The frequency stability (symbol \(\sigma\), "sigma") of a frequency standard will depend on a variety of physical and electronic influences both internal and external to the device which cause frequency fluctuations. The frequency stability depends also on the exact measurement procedure which was used to measure the stability. We shall explain this in the following. Frequency stability can be measured by taking a reasonably large number of successive readings of an electronic counter which counts the frequency of the device to be evaluated. Each counter reading (in hertz) is obtained by sampling the counted frequency for some specified time, the sampling time (symbol \(\tau\), "tau"). This sampling time can usually be chosen by simply adjusting a knob on the counter; for example, a sampling time of 0.1s or 1s or 10s may be chosen.

\(^2\) In this paper, for tutorial purposes, only the time domain stability is used. We note, however, that for many scientific applications the frequency domain stability measure is more useful.
Everyone has had the experience that fluctuations tend to average out if observed long enough; however, this is not always so. Sigma will therefore usually depend on the sampling time of the measurement and tends to get smaller with longer sampling times; again, there are many exceptions to this.

It may be that the fluctuations at some later time are partially caused by, or depend to some degree on, the previous fluctuations. In this case, the actual value of $\sigma$ will also depend on the particular way in which the many counter readings are averaged and evaluated. Also, it will be of influence whether the counter starts counting again immediately after completion of the preceding count or if some time elapses ("dead-time") before counting commences again.

Finally, electronic circuits will have a finite response time, e.g., they cannot follow fluctuations faster than some given rate. For example, our eye can not register light fluctuations which occur faster than about every $\frac{1}{10}$ of a second; using eq (1), we say that the eye has a frequency response of 10 Hz, or that its bandwidth is only 10 Hz, i.e. the eye can not follow frequencies higher than 10 Hz. In order to measure frequency stabilities for sampling times larger than some value $\tau$, we have to provide for an electronic frequency bandwidth which is larger than $\frac{1}{\tau}$.

We summarize: a recommended way of properly measuring and describing frequency stability is the following: (a) make sure that the frequency bandwidth of the total measuring set-up is larger than $\frac{1}{\tau_{\text{min}}}$ where $\tau_{\text{min}}$ is the smallest desired sampling time; (b) use a counter with a dead-time as small as possible; (c) take a sufficiently large number of readings at a given sampling time which is held constant and compute

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The dead-time should be less than the reciprocal bandwidth; if not, computation procedures exist to account for larger dead-times.
\[
\sigma = \sqrt{\frac{\text{addition of the squares of the differences between successive readings}}{2 \times \text{total number of differences used}}}
\]

(in the scientific literature this \( \sigma \) is called the square root of the Allan variance); (d) repeat (c) at other sampling times \( \tau \) and tabulate or plot \( \sigma \) as it depends on \( \tau \).

Commonly, \( \sigma \) will be given as a fractional value, i.e., the value obtained for the frequency stability is divided by the carrier frequency. For example, if a frequency stability of \( \sigma_{\delta \nu} = 10 \) Hz were measured at a carrier frequency of \( \nu = 5 \) MHz (MHz = megahertz = million Hz) then the fractional frequency stability would be

\[
\sigma_y = \sigma_{\delta \nu} / \nu = 2 \times 10^{-6}.
\]

We denote the kind of \( \sigma \) by a subscript, \( \delta \nu \) ("delta nu") referring to frequency fluctuation (measured in Hz) or \( \frac{\delta \nu}{\nu} = y \) referring to fractional frequency fluctuations (dimensionless). A stability of one part in a million is thus

\[
\sigma_y = 1 \times 10^{-6},
\]

and one part in a trillion is written as

\[
\sigma_y = 1 \times 10^{-12}.
\]

The common usage of the fractional frequency stability \( \sigma_y \) instead of the frequency stability \( \sigma_{\delta \nu} \) (given in Hz) has its good reasons. In almost all applications of frequency standards their nominal output frequency will be multiplied or divided, i.e., the standard frequency will be used to synthesize other frequencies. Also, frequency standards themselves will already synthesize several output frequencies, and different frequency standards will offer different output frequencies. In such a usage of frequency standards as well as in

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4 The counter readings can be taken in Hz; \( \sigma \) will then have the dimension of Hz.
the comparison of their performances it would be extremely inconvenient to state $\sigma_{\delta \nu}$ because this number (in Hz) will change with any alteration in frequency due to synthesis. This will not be so with $\sigma_y$ because $\delta \nu$ as well as $\nu$ will change in the same way in synthesizing procedures. Of course, we assume here that synthesis does not cause additional instabilities, an assumption which is mostly valid with today's electronic capabilities.

2.2 Time Accuracy

In clocks, time accuracy is of importance. In accordance with the definition for accuracy at the beginning of this section, the accuracy of a clock is the conformity of its reading (the date shown) with the reading of a standard time scale (a standard or reference date). A clock can be initially set (synchronized) with respect to a reference clock with some accuracy $\Delta t_0$. Its reading will then gradually deviate by an amount $\Delta t$ ("delta t") from the reading of the reference clock. This deviation, i.e., the accuracy of the clock, depends on the stability performance $\sigma$ of the frequency standard in the clock and on the fractional frequency offset $\frac{\Delta \nu}{\nu}$ of the clock with respect to the reference clock via

$$\Delta t = (t \times \frac{\Delta \nu}{\nu}) \pm t \times \sigma_y (\tau = t) + \Delta t_0,$$

where $\sigma_y (\tau = t)$ means the fractional frequency stability evaluated for a sampling time $\tau$ equal to the time $t$ which elapsed after the last setting of the clock. For example, a clock with a fractional frequency stability of $\sigma_y = 10^{-11}$ for sampling times of $10^5$ seconds (about 1 day) would be accurate to $\Delta t = \pm 10^5 \times 10^{-11} = \pm 1$ microsecond per day if $\Delta \nu$ is assumed to be zero. The same clock with a fractional frequency offset of $\Delta \nu/\nu = 10^{-9}$ would be accurate to $\Delta t = (100 \pm 1)$ microseconds after 1 day.

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The frequency offset $\Delta \nu$ is the frequency difference, measured in hertz, between the frequency standard (clock) and the reference.
Fig. 3. Relationships between clock accuracy, frequency stability, and frequency offset.
2.3 Clocks

In the Introduction we discussed that the addition to a frequency standard of a mechanism, which counts and accumulates and possibly displays the result, creates a clock.

This task can be performed by a frequency divider which, for example, derives a frequency of 60 Hz directly from a 5 MHz crystal oscillator. The 60 Hz voltage can be used to drive an electric clock similar to those driven by the 60 Hz power line frequency which we use at home or at work. Or, an additional electric pulse generator may be used which generates one very sharp electrical pulse per second. The time interval of 1 second between the pulses (corresponding to a frequency of 1 Hz) is directly derived from the output of our frequency standard. The pulses can be directly used in time comparisons with those of other, similar clocks; or a counter/accumulator can be driven by them.

3. FREQUENCY STANDARDS, GENERAL ASPECTS

3.1 Resonators

The performance of a frequency standard is to a considerable degree, but not exclusively, given by its frequency determining element. It determines the frequency by its resonance behavior. Some examples for resonance phenomena are (a) a rod, clamped only at one end, which can vibrate, (b) a block of solid material which can contract and expand and thus vibrate, (c) a capacitor-coil combination (tank circuit) in which the electric energy can oscillate back and forth between these two elements, (d) an antenna (a dipole) where the distribution of electric charges can oscillate back and forth, and (e) a coil in which an electric current can create a magnetic field which can oscillate between its two possible polarities (a magnetic dipole). All these devices have in common that they can vibrate or oscillate if they are excited. The method of excitation may be a mechanical pulse for (a) and (b) or an electrical pulse for (c) or a sudden surge of an electric field for (d) or of a magnetic field for (e). The devices exhibit a resonance, i.e.,
Fig. 4. Example of a clock system.
Fig. 5. Examples of resonators.
they are resonators with a well-defined frequency which is characteristic of the physical dimensions of the device: the length of the rod, the thickness of the block, the size of the capacitor and coil, the length of the antennas. Once excited, the oscillations will die out gradually with a decay time which is determined by the losses of the resonator. Some of these losses are internal friction as in our cases (a) and (b), and electrical resistance for (c) through (e); in any case, the oscillation energy is ultimately transformed into heat. If there were no losses, the oscillations would never stop; we would have an ideal resonator. The more losses, the faster the oscillations will decay and the resonance will be less pronounced. It is now obvious that we could use the decay time of the oscillations (symbol T) to describe the quality of our resonator. The larger T, the better the resonator.

An alternate way of measuring the resonance behavior is to use an external oscillator, to couple it to the resonator, and to sweep slowly the frequency of the external oscillator across the resonance. We will find again a resonance frequency at which the resonator will oscillate most pronounced, i.e., with the greatest intensity. On both sides of the resonance frequency, the response of the resonator will lessen until it ceases to respond. We can define a frequency interval around resonance in which the resonator response is relatively strong; we call it the resonance linewidth (symbol W). There is a simple relationship between W and T

\[ W = \frac{1}{T}. \]  

Equation (3) is only an approximation. Depending on the particular shape of the resonance curve we would have to insert a factor into this equation; however, this factor is never far from unity. T is defined as the decay to \( e^{-1} \) of the original amplitude. e is a number, \( e \approx 2.7 \).
Fig. 6. Decay time, linewidth, and Q-value of a resonator.
reciprocal fractional linewidth

\[ Q \equiv \frac{\nu}{W}. \] (4)

In frequency standards we obviously like to have large values for the \( Q \) of the frequency determining element. As an example, if \( Q = 10^6 \), then a fractional accuracy of \( 10^{-10} \) would imply that we can determine the center of our resonance curve to a small fraction (\( 10^{-4} \) or one hundredth of a percent) of its width. In the same example, a stability of \( 10^{-12} \) for some sampling time would correspond to an ability to keep the frequency to within \( 10^{-6} \) (one millionth) of the resonance linewidth around a given value. It is clear, therefore, that the frequency stability and accuracy of a frequency standard may be expected to become the better the higher the \( Q \)-value of the frequency determining element.

Many kinds of frequency determining elements have been and are being used in frequency standards. They can be grouped into three classes:

- mechanical resonators;
- electronic resonators;
- atomic resonators.

As far as mechanical resonators are concerned we will only discuss one group in detail, the quartz crystals. Other mechanical resonators like the pendulum and the tuning fork are of no importance in today's high performance frequency standards although they have been historically very important and are still widely used in low performance devices (e.g., in watches). For similar reasons we will also omit the discussion of electronic resonators like the tank circuits (our device (c) of above) and microwave cavities. Atomic resonators form the heart of our most accurate frequency standards and clocks and will, therefore, be extensively discussed.
3.2 Primary and Secondary Standards

At this point, we should briefly discuss the frequently used terms "primary frequency standard" and "secondary frequency standard". These terms should refer to the systems-use of the devices; any frequency standard, regardless of its accuracy or stability, can be a primary frequency standard, if it is used as the sole calibration reference for other frequency sources. A secondary frequency standard is a device which is occasionally calibrated against a primary frequency standard but operationally serves as the working reference for other frequency sources. The use of the terms "primary" and "secondary" to describe the performance and/or the design of a frequency standard itself is discouraged. These aspects can be described adequately and accurately by stating accuracy, reproducibility, and design features.

One class of frequency standards can be separated from the rest. We call those the evaluable frequency standards, which, by virtue of their basic physical operation as well as their design, allow the experimental evaluation of all known influences which might alter the output frequency. The accuracy of the evaluable frequency standard can thus be stated for the single device without reference to any other frequency standard. We will discuss it in more detail later in connection with cesium beam frequency standards.

4. QUARTZ CRYSTAL DEVICES

4.1 Quartz Crystal Resonator

The quartz crystal is a mechanical resonator much like our examples (a) and (b) of section 3.1. The resonator's oscillations have to be excited and sensed externally. In the case of a quartz crystal this is done by taking advantage of its piezoelectric properties. The piezoelectric effect is a special property of a certain class of crystals.
Fig. 7. Hierarchy of frequency standards.
Compression or dilatation of the crystal generates a voltage across the crystal, and conversely, the application of an external voltage across the crystal causes the crystal to expand or contract depending on the polarity of the voltage. A crystal is not a homogeneous medium but has certain preferred directions; thus, the piezoelectric effect has a directional dependence with respect to the orientation of the crystal. In order to take advantage of the piezoelectric effect one has to cut the crystal resonator from the crystal block in a well defined way with respect to the crystallographic directions. The raw material today is both natural quartz and synthetic quartz. The crystal is cut out of the raw crystal in the desired orientation with the aid of optical techniques which allow the determination of the crystallographic axes. The high precision final orientation of the cut and the tuning to the desired frequency is then done by grinding and etching under control of x-ray methods.

The quartz crystal can be cut and electrically excited in a variety of ways. The most common types of vibrations (modes) are the longitudinal and thickness modes, the flexure (bending) mode, the torsional mode, and the shear mode. In order to use the piezoelectric effect, electrodes have to be put on some of the crystal surfaces such that the desired mode is excited. The electrodes are typically created as extremely thin metallic coatings by vacuum evaporation of metals. Electric leads are attached to the electrodes, (e.g., by soldering), which usually also serve as the mounting support thus freely suspending the quartz crystal. In order to least perturb the mechanical vibrations of the crystal, the electrode-support leads are attached at points where no vibrational motion occurs (nodes). The crystal is usually encased and the enclosure is sometimes filled with a protective gas or is evacuated.
Fig. 8. The piezoelectric effect.
Fig. 9. Principal vibrational modes of quartz crystals.
A frequency standard can now be built by adding an electronic amplifier (energy transfer), feedback, and a power supply. We call such a device a quartz crystal oscillator. Its output frequency is determined by the quartz crystal resonator whose frequency in turn is determined by the physical dimensions of the crystal together with the properties of crystalline quartz. The resonance frequency thus depends on the orientation of the cut, the particular mode, and the dimensions of the crystal. As an example, we find that the resonance frequency for a longitudinal mode of vibration is approximately given by

\[ \nu_0 = 2.7 \times 10^3 \times \frac{1}{l}, \]

where \( l \) is the length of the crystal. Equation (5) is written such that the use of meters to express \( l \) will give the resonance frequency \( \nu_0 \) in hertz. Equation (5) allows us to estimate the size of crystals. For example, a 100 kHz crystal will have a length which is of the order of a few centimeters and a 10 MHz crystal will have a length (thickness) of just a few tenths of a millimeter. We see from this that the production of quartz crystals with resonance frequencies much above 10 MHz is hardly possible. However, one can excite resonators not only in their so-called fundamental mode (which we discussed so far) but also at multiples (overtones) of this fundamental resonance frequency. The best example for this is the violin string which also can be caused to oscillate at frequencies which are multiples of the fundamental frequency; the violinist depends on this. Quartz crystals which are designed for the excitation of multiples of their fundamental resonance are called overtone crystals.

4.2 Effects on the Crystal Resonator Frequency

Two deleterious effects, among others, are important in the design of crystals and crystal oscillators and limit their usefulness. The first is the temperature dependence of the quartz crystal resonance frequency, the second is a slow change of the resonance frequency as time goes on (drift).
Fig. 11. Quartz crystal oscillator.
Fig. 12. Fundamental and overtone resonance frequencies.
The temperature dependence is caused by a slight change in the elastic properties of the crystal with temperature. This can easily be imagined from the general behavior of matter that the packing density of atoms increases with lowering the temperature. However, certain cuts, i.e., certain crystallographic orientations of the crystal, minimize this effect over a rather wide range of temperatures, most notably the so-called "AT" and "GT" cuts. Temperature coefficients of less than one part in 100 million per degree (celsius or kelvin) are possible, in other words, the fractional frequency change will be less than $10^{-8}$ with one degree of temperature change. Nevertheless, this effect demands certain precautions in the design of a crystal oscillator if very high frequency stabilities over longer times (hours or days) are desired and/or if large environmental temperature fluctuations are to be tolerated. Hence, crystals are enclosed in electronically regulated ovens which maintain a constant temperature; in certain crystal oscillators this is done to better than $\frac{1}{1000}$ of a degree.

A different solution to the temperature problem is the so-called temperature compensated crystal oscillator or TCXO. An additional frequency determining element in the oscillator, which can essentially be just a small capacitor (see our example (c) of sec. 3.1), gives the opportunity to tune the oscillator over a limited range by varying this additional element. If a temperature sensor is added which causes a change in this capacitor one can adjust the response in such a way that the change in resonance frequency of the crystal resonator is just compensated by a suitable adjustment of the added capacitor. Bimetal springs have been used which mechanically change the setting of the capacitor much in the same way as the automatic choke in many automobiles works. Today, capacitors whose value changes with an applied voltage (varactors) are used; the applied voltage is derived from a temperature sensing circuit. The TCXO thus does not necessarily
require further temperature control by an oven. However, we see the drawback of this approach. In adding a further frequency determining element, the crystal resonator has to relinquish a corresponding part of its control on the output frequency of the whole oscillator. We, therefore, realize that the stability performance of a TCXO will degrade the more, the wider the temperature range of compensation is made. The long-term stability (days) of TCXO's is therefore below that of crystals with a good oven control. We find TCXO's in small, usually portable units of relatively low performance, e.g., for applications where frequency stabilities from day to day and frequency changes over some tens of degrees of temperature of not better than $10^{-9}$ are needed.

The drift, or aging, is a common behavior of all crystal oscillators. It is a nearly linear (uniform) change in resonance frequency with time, which frequently is negative (i.e., the resonance frequency decreases). A frequency decrease could be interpreted as an increase in the crystal size according to eq (5). Many physical mechanisms have been considered as the cause: contamination of the surfaces (deposition of foreign material); changes in the electrodes or the metallic plating; reformation of loose (from grinding and etching) surface material; changes in the internal crystal structure; etc.; all of this possibly caused or enhanced by the vibrating motion of the oscillating crystal. Careful fabrication and electrode design combined with clean vacuum enclosures have led over the years to a reduction of the aging to about $10^{-11}$ per day and better for the best crystals. This aging corresponds to $10^{-11}$ fractional thickness change per day, as we can see from eq (5). For a 5 MHz crystal with a thickness of a little less than a millimeter this aging corresponds to an absolute thickness change of only $10^{-11}$ of a millimeter or less than $\frac{1}{10}$ of one percent of the diameter of an atom. It seems surprising that mechanical resonators can be built which change their dimension by so little.
Two more effects on crystal resonators are to be considered. One is its relative sensitivity to gravitational forces and acceleration: frequency changes will occur because of the stresses in the crystal caused by these forces. This influence depends on the direction of the force relative to the crystallographic axes and thus can be minimized for certain orientations. The magnitude of the effect is typically of the order of $10^{-9}$ for accelerations corresponding to the earth's gravitation.

The other effect is related to intermittent operation. If a crystal oscillator is turned off and, after some time, put back into operation it will not oscillate immediately at the original frequency but will exhibit first a "warm-up" due to temperature stabilization of the crystal resonator and its oven and then for some time (as long as many days) a large but diminishing drift until it reaches its previous aging performance. The frequency at which it will then operate might also be substantially different (as compared to its stability and aging performance) from its frequency before the interruption.

4.3 Quartz Crystal Oscillator Performance

Crystal resonators have $Q$-values which are typically in the range from $Q = 10^6$ to almost $Q = 10^7$. These are very high $Q$-values as compared to most other resonators except, most notably, atomic resonators.

These high $Q$-values are an essential prerequisite for the excellent stability performance of crystal oscillators. The best presently available devices show stabilities of a few parts in $10^{13}$ for sampling times from one second to a day. There is some experimental evidence that some crystal resonators may perform better and that limitations are primarily caused by noise in the electronic components in the oscillator circuits. This noise (flicker noise) may possibly be reduced by a special selection of low noise components (transistors, capacitors, etc.) and by some special circuit design. Thus there is a reasonable chance that the stability may reach values of better
Fig. 13. Frequency stability of the better quartz crystal oscillators.

Fractional Frequency Stability

Sampling Time (Seconds)
than $10^{-13}$ for sampling times of seconds to hours. For times shorter than one second, the stability is often determined by additive noise in the output amplifiers and can then be reduced by a (crystal) filter in the output. The long-term stability beyond several hours sampling time is determined by the aging and by external influences such as line voltage variations, temperature fluctuations, etc. A specification of accuracy is not very well possible with crystal oscillators. Without any frequency calibration they possibly can be fabricated via thickness-determination (not very practical) to about $10^{-6}$ frequency accuracy. If they are calibrated against a high accuracy frequency standard they maintain this calibration (accuracy) according to their long-term stability performance, e.g., a crystal with the low aging of $10^{-11}$ per day will be accurate to a few parts in $10^8$ for the duration of a year.

We realize, therefore, that crystal oscillators require calibration, which may be rather frequent depending on the requirements. The frequency adjustments are being made with a small added capacitor in much the same way as we discussed before in connection with the TCXO. The most stable ones with the lowest aging rate cost approximately $2000, have a volume of a few thousand cubic centimeters, and require a power of about 10 watts. They have an elaborate crystal oven for temperature control, well-designed electronics, and usually several output frequencies which are derived from the oscillator frequency with the aid of frequency dividers and multipliers. These high performance devices presently use 5-MHz or 2.5-MHz crystal resonators with Q-values of a few million.
Crystal oscillators which are cheaper and/or smaller are available in an immense variety of designs at certain sacrifices in frequency stability and/or environmental insensitivity. Costs can go down to below $100, sizes to a few cubic centimeters, and power requirements to less than 0.1 watt. The reliability of crystal oscillators is usually not limited by the crystal, the mean time between failure (MTBF) being that of any electronic circuit of equivalent sophistication.

5. ATOMIC RESONANCE DEVICES

5.1 Atomic Resonators

The basis of commercially available atomic frequency standards are resonances in atoms at microwave frequencies (range 1 to 100 GHz, GHz = gigahertz = billion hertz). Resonances in molecules and resonances at higher frequencies in the infrared and visible regions have been studied for frequency standard applications but have not yet led to practical devices; however, their potential is highly promising (see sec. 8). We will confine ourselves, therefore, to the discussion of microwave resonances in atoms. We may picture such a resonance as a little magnetic dipole antenna as was discussed in our example (e) in section 3.1. We will typically deal with a great many of these dipole antennas (atomic resonators), and we can separate them into two kinds: (1) receiving antennas, which will absorb energy from a field at their resonance frequency, much like our TV antenna at home, and (2) transmitting antennas which will radiate energy at their resonance frequency in a fashion analogous to the action of the transmitting antenna of a broadcasting station. A physicist might say that the atoms which act like a receiving antenna are in the "lower state" and those acting like a

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7 Of course, this is a simplification.
transmitting antenna are in the "upper state". In a natural ensemble of a great many atoms (a gas), we will find that the total number of upper state atoms is nearly equal to the total number of lower state atoms. This has an important consequence: if this gas is placed in an external magnetic field, which oscillates at the atomic resonance frequency, all atoms may resonate; however, nearly half of the atoms receive (absorb) energy from the field, the other half emit (add) energy of an equivalent amount to the field. It is obvious that the net effect is almost zero; the gas as a whole acts as if it has almost no resonance although the individual atoms each may resonate. From this we see that in order to observe the atomic resonance we have to change somehow the relative amounts of the two kinds of atoms; the upper or the lower state has to be in the majority. The way in which this is done determines the design of an atomic resonance device, which is the frequency determining element in our atomic frequency standard.

5.2 State Selection

We will now discuss the two most important methods to accomplish what is called state selection: the change in the relative numbers of the two kinds of atoms.

(a) Spatial state selection: This method relies on an actual sorting procedure where the two atomic states are sorted into different directions in space. One of the states can then be used, the other is discarded. As an example, an actual system may produce an atomic gas by heating the substance in an oven to a suitable temperature. The atoms leave the oven through a hole and form an atomic beam in an adjacent vacuum chamber. The atomic beam is then passed through a rather strong magnet.

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8 There is a peculiarity with atoms: an atom changes from the upper to the lower state upon emission of a well-defined amount of energy, and, correspondingly, an atom changes from the lower to the upper state after receiving an equal amount of energy at the atomic resonance frequency.
Fig. 14. Spatial state selection.
which causes the separation of the beam into two beams, each of them containing only one kind of atoms. We remember that our picture of an atom is that of a little magnetic dipole antenna. The magnet exerts a force on these magnetic dipoles, which acts in opposite directions for the two different atomic states (we note this without further explanation).

(b) Optical state selection: This technique takes advantage of the fact that the atoms have more than just one resonance. Other resonances typically correspond to infrared or visible (light) frequencies. We can excite one of these resonances by shining intense light of the corresponding frequency on the atoms. If the light is filtered carefully, very monochromatic (one color, i.e., a very well-defined frequency) light is obtained. If the light frequency is chosen properly, only one kind of the atoms will resonate. At the same time, this light frequency is too far away from the corresponding light resonance of the other kind of atoms. The light resonance can thus "remove" one kind of the atoms and provide for the desired majority of the other kind.

5.3 Interrogation of the Atoms

We are now almost in a position to assemble our atomic frequency standard. We still need some means to observe the atomic resonance. A microwave cavity is typically used. Such a cavity has microwave resonances which are determined by its physical size. The (electrical) losses of this cavity are given by the electrical conductivity of the cavity material. Such a cavity may have the shape of a cylinder or that of a box. It resonates in a way quite similar to the working of an organ-pipe in the case of acoustic waves. The best known example of a microwave cavity is the microwave oven where food is cooked or fried by placing it into the resonance field of a cavity. In order to observe the atomic resonance we have to place the state selected atoms inside of this cavity.
Fig. 15. Optical state selection.

Equal amount of the two kinds of atoms

State selected through action of the light

Light beam

Gaseous cell

Reflector-mirror

Filter

Lamp
and subject them for some specified time to a microwave signal at their resonance frequency. The microwave signal will change the relative number of atoms in the two states, e.g., if all atoms were initially in the upper state we find some in the lower state after the microwave signal acted upon them for some time (see footnote 8). If the frequency of the external microwave signal is exactly at the atomic resonance frequency, this transfer of atoms from one state to the other reaches a maximum. The center of the atomic resonance is thus found by monitoring the number of atoms in one of the two states and by varying the microwave frequency until the number of atoms in the lower state reaches a maximum, or the number of atoms in the upper state a minimum. A proportional electric signal can be derived which is fed back to the oscillator generating the microwave signal. Thus an automatic servo can be built which keeps the oscillator (usually a crystal oscillator) locked to the atomic resonance.

5.4 Signal Detection

The detection of the effect of the microwave signal on the atoms can be done in basically three ways, two of which relate to the two methods of state selection.

(a) Atom detection: The atoms which leave the cavity as an atomic beam are passed through the field of a magnet which spatially sorts the two states. An atom detector is placed to intercept one of these states. The output of the detector thus indicates the number of atoms in the upper (or lower) state.

(b) Optical detection: If the atoms are optically state selected, a detector for light (photo-detector) can be placed in a position such that the light which has interacted with the atoms is detected. Since the light removes atoms from one of the states, its intensity will change if atoms which resonate at the light frequency are added. Such an addition takes place
Fig. 16. Atom detection.
Fig. 17. Optical detection.
as a result of the microwave signal which, in effect, transfers atoms from one state (with no light interaction) to the other (taking part in the light resonance). The light at the photo-detector therefore is a measure of the number of atoms in one of the states.

(c) Microwave detection: In this type of detection, the microwave signal is transmitted through the cavity to a microwave detector. Since the atoms will either add energy to the signal (if initially in the upper state) or subtract energy (if initially in the lower state), the microwave power level at the detector is a measure of the number of atoms changing state.

5.5 Atomic Frequency Standard

The analogy to our mechanical frequency standard, the wrist watch, which we discussed in section 1, may now be drawn: the frequency determining element is the atomic resonator which consists of a cavity containing the state selected atoms, and some means of detecting the change in the number of atoms in the two states. A control signal, related to the number of atoms which changed their state due to the action of the microwave signal, is fed back to the (crystal) oscillator. The oscillator and the associated frequency multiplier or synthesizer\(^9\) correspond to the energy transferring means in our wrist watch (see sec. 1).

Some power supply has to provide the energy necessary to drive the oscillator, multiplier, and possibly the atom state selector. Very high Q-values can be achieved with atomic resonators. To understand this, we have to recall our eqs (3) and (4).

\(^9\)As we discussed before, good crystal oscillators are available at frequencies of several MHz; the atomic resonances are at GHz frequencies. The crystal oscillator frequency therefore has to be multiplied by a factor of about 1000.
Fig. 18. Microwave detection.
Fig. 19. Atomic frequency standard.
The Q is related to the time T, which describes the average duration of oscillations of the resonator. In atomic resonators we find two chief causes for the termination of oscillations. The first one is collisions of the resonating atoms with each other and with surrounding walls; each collision usually terminates the oscillation. The second one is a rather obvious cause: the atoms may simply leave the region of microwave signal interaction. In an atomic beam apparatus, the atoms enter the cavity, traverse the cavity in a certain time, and then leave the cavity. As an example, let us assume that we have a cavity of 1 meter length which is traversed by atoms with an average speed of 100 meters per second (a typical value). We assume an atomic resonance frequency of 10 GHz. The interaction time T is then \( \frac{1}{100} \) second, the linewidth 100 Hz (from eq (3)), and the Q-value is \( 10^8 \) (from eq (4)). We arrived at a Q-value which is considerably better than that of a quartz crystal oscillator!

5.6 Effects on the Atomic Resonator Frequency

Before we discuss effects which might cause changes in the output frequency of an atomic standard, we should make a very important statement: The atomic resonance frequency itself is given to us by nature, it will not drift or age. Hence, atomic resonators with Q-values of \( 10^8 \) or higher may be expected to have accuracies of one part in \( 10^8 \) or better because we will not be able to pull the resonance frequency further away than the linewidth of the resonance.

In the following, we are not exhaustive but list only the major perturbing effects:
(a) **Noise:** Random noise in the crystal oscillator, the detector, the microwave cavity, and the frequency synthesizer will cause corresponding fluctuations of the output frequency.

(b) **Cavity pulling:** The microwave cavity is itself a resonator. Thus we have an additional frequency-determining element besides the atoms. It influences the output frequency by pulling the combined resonance frequency to a value which usually lies between the resonances of the atom and cavity. This necessitates the tuning of the cavity to the atomic resonance. The requirements for tuning will be relaxed if the Q-value of the atomic resonance is as high as possible and the cavity-Q as low as possible.

(c) **Microwave spectrum:** If the exciting microwave signal has not a symmetric but an asymmetric distribution of frequencies\(^{10}\), a frequency pulling occurs which conceptually is related to the mechanism of cavity pulling. By careful design in the electronics, this effect can be made negligible.

(d) **Collisions:** Collisions between the atoms, and between the atoms and the walls of a vessel (gas cell) in which the atoms may be contained, will not only shorten the duration of the oscillation but also cause a frequency shift. Obviously, these effects can be minimized by having low atom densities and no walls, if possible.

(e) **Doppler effect:** The apparent change in frequency if the emitter moves relative to an observer is called Doppler effect. Everybody knows this for acoustic waves from the experience of an approaching, whistling locomotive or from automobile racing. Here, the moving objects are the oscillating atoms and the observer is the microwave signal will never be just one single frequency but will have a certain distribution of frequencies (spectrum).
cavity. The Doppler effect can be highly reduced by choosing a particular direction of the atomic beam with respect to the direction of the oscillating magnetic microwave field in the cavity; in our acoustic example, this corresponds to the passing of the whistle at some large distance instead of having the whistle move directly towards the observer. An alternate way of reducing the Doppler effect is the containment of atoms in a region small compared to the wavelength of the microwave radiation, e.g., by placing a gas cell inside of the cavity.

(f) **Magnetic field:** Of the effects which we discuss here, this is the only one which directly affects the atomic resonance. We remember that we used a small magnetic dipole antenna as the model for the atomic resonance. If this antenna is placed in an external static magnetic field (as that of a permanent magnet) the tuning of the antenna changes, i.e., the atomic resonance frequency changes. This necessitates magnetic shielding, which is a characteristic design feature of all presently used atomic frequency standards. The shielding is usually quite elaborate and reduces the external magnetic fields, e.g., the earth's magnetic field, to 1% or less of its external value. This residual magnetic field can then be measured quite precisely by using the atomic resonance itself. In fact, the associated measurement precision is so good that magnetic field effects do not seriously impair the accuracy of atomic frequency standards. However, fluctuations of the external magnetic field, or the movement of the device through a varying field, e.g., as in a relocation, may perturb the frequency of the atomic standard.
Generally speaking, one tries to minimize all of the effects listed above, and to keep their influence as stable with time as possible. Such an approach suffices for most applications\textsuperscript{11}. For laboratory standards, however, where frequency accuracy is the primary purpose, all effects must be evaluated in a series of experiments, and reevaluations must be done occasionally in order to detect changes with time.

6. AVAILABLE ATOMIC FREQUENCY STANDARDS

We will now discuss the design and performance of the three types of atomic frequency standards which are currently in operational use.

6.1 Cesium Beam Frequency Standard

The atomic resonance is at 9192 631 770 Hz. The standard is based on the atomic beam method using spatial state selection and atom detection. An oven contains the cesium metal. If heated to about 100\textdegree C, enough cesium gas will be produced to form an atomic beam which leaves the oven through one or many channels into a vacuum chamber. This chamber is evacuated to a pressure of less than 10\textsuperscript{-9} of atmospheric pressure. The beam traverses first the state selecting magnet, then the microwave cavity\textsuperscript{12} where an external microwave signal acts on the beam, and finally reaches the atom detector after passing another state selecting magnet. The atom detector is simply a wire (typically

\textsuperscript{11}The magnetic effect is an exception. The residual magnetic field inside of the shields must be evaluated and possibly reset any time after changes in the external field occur, e.g., after moving the device. The magnetic shields may also have to be demagnetized.

\textsuperscript{12}Typically, a cavity with separated interrogation regions is used. This design offers certain advantages over a simple cavity of a length equivalent to the separation of the two regions.
Fig. 20. Cesium beam frequency standard.
Fig. 21. Frequency stability of commercial cesium beam frequency standards.

a) typical performance
b) typical performance of recently available high beam intensity tubes

Individual units may perform slightly worse or better than shown. Modern laboratory standards perform still better.
made from tungsten) which is heated to about 900° C by passing an electric current through it. Cesium atoms which impinge on this wire (which is biased with a few volts dc) become electrically charged (ionized) and can be collected on an auxiliary electrode. The stream of electrically charged atoms at this electrode represents an electric current which is amplified, detected, and fed into the feedback network.

The speed of the atoms and the length of the cavity determine the Q-value of the atomic resonator. Typical atom speeds are 100 meters per second. In commercial devices which have to be reasonably small, the cavity is about 0.2 meter long; the corresponding interaction time T is two thousandths of a second. From eqs (3) and (4), we calculate a linewidth of a few hundred hertz and a Q-value of a few $10^7$. In laboratory devices one can go to very long cavities. Cavities of up to 4 meters long are used, leading to Q-values of a few $10^8$. The fractional frequency stability of laboratory and commercial devices can reach parts in $10^{-14}$ at sampling times of less than one hour to days. The short-term frequency stability is limited by fluctuations in the atomic beam intensity which are basic and unavoidable ("shot noise"). These fluctuations affect the frequency stability less if more intense atomic beams are used. This approach which is becoming available in both commercial and laboratory devices, improves the stability (compare sec. 7). In contrast to commercial devices, the laboratory standards are designed to allow a more complete and easier evaluation of all effects on the frequency. Cesium standards are used extensively where its high reproducibility and long-term stability (sampling times of more than a day) are needed. For most applications, cesium standards need not be calibrated. They are the work horses in most of today's standard frequency and time services. The laboratory standard serves to realize the definition of the second (see sec. 1).
6.2 Rubidium Gas Cell Frequency Standard

The atomic resonance is at 6834 682 608 Hz. The standard is based on the gas cell method using optical state selection and optical detection. The gas cell contains rubidium gas at a pressure of only about $10^{-9}$ of atmospheric pressure. In order to reduce the effect of collisions among the rubidium atoms, an inert buffer gas (e.g., argon) is introduced into the cell at a pressure of about 1/1000 of atmospheric pressure. This allows lifetimes of the rubidium atom oscillations (the oscillation lifetime $T$ is still limited by atom collisions) of about 1/100 second. From eqs (3) and (4) we calculate a corresponding linewidth of about 100 Hz and a $Q$-value of several $10^7$. Atomic collisions as well as the simultaneous action of the light and the microwave signals on the same atom cause frequency shifts of the order of $10^{-9}$. These frequency shifts depend strongly on the composition, temperature, and pressure of the buffer gas and on the intensity of the light. As a result, rubidium gas cells vary in their resonance frequency by as much as $10^{-9}$ depending on the particular setting of the frequency shifting parameters during manufacturing. Since these influences cannot be expected to stay unchanged as time goes on, rubidium standards need not only initial calibration but also recalibrations because they exhibit a frequency drift, i.e., an aging analogous to crystal oscillators. The stability performance of rubidium standards is nevertheless quite spectacular. At one second sampling time they display a stability of better than $10^{-11}$ and perform near the $10^{-13}$ level for sampling times of up to a day. For longer times the frequency stability is spoiled by the frequency drift which is typically $1 \times 10^{-11}$ per month (much less than the drift of crystal oscillators). Commercial devices with the above mentioned performance are available as well as rubidium standards of lower performance at significantly reduced costs.

50
Fig. 22. Rubidium gas cell frequency standard.
Fig. 23. Frequency Stability of commercial rubidium gas cell frequency standards.
Rubidium standards are used where excellent medium term stability (minutes to a day) is needed, where its reduced costs, size, and weight, as compared to cesium standards, are important, and where a crystal oscillator with its more frequent needs for recalibrations and its greater environmental sensitivity does not suffice.

6.3 Atomic Hydrogen Maser

The atomic resonance frequency is at 1420 405 752 Hz. The standard is based on the atomic beam method using spatial state selection and microwave detection. The beam source is a radio frequency gas discharge in molecular hydrogen which produces atomic hydrogen with high efficiency. The atomic hydrogen beam leaves the source through one or many channels into a vacuum chamber. The beam then traverses a state selecting magnet and enters a storage bulb in the microwave cavity. The storage bulb is made from quartz glass which has low electric losses and thus does not spoil significantly the cavity-Q. The storage bulb is evacuated to a pressure of less than $10^{-11}$ of atmospheric pressure. Its inner walls are lined with a fluorocarbon coating, a substance similar to the non-stick coating in cookware. This coating allows many collisions of the hydrogen atoms with the walls without significantly disturbing the oscillations of the atoms. The underlying physical mechanisms are not yet fully understood. The storage bulb is typically 0.15 meters in diameter and dimensioned in such a way as to keep hydrogen atoms for a time of about one second.

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13 Maser is an acronym meaning microwave amplification by stimulated emission of radiation.

14 All natural hydrogen gas is composed of hydrogen molecules; each hydrogen molecule is formed by chemical bonding of two hydrogen atoms.
Then they leave the bulb and thus leave also the microwave cavity. From eqs (3) and (4) we calculate a linewidth of about 1 Hz and a $Q$-value of about $10^9$, the highest $Q$-value in all presently used atomic standards.\textsuperscript{15} If the intensity of the hydrogen beam, which consists only of upper state atoms (emitting atoms), is sufficiently large and if the cavity losses are sufficiently low, self-oscillation will start in the cavity; i.e., the maser itself will generate a microwave signal. We have a maser-oscillator with an output frequency directly derived from the atomic resonance. A crystal oscillator can be locked to this frequency by frequency comparison techniques. As compared to the cesium standard, the hydrogen maser is not quite as accurate because of experimental difficulties in the evaluation of the frequency shift due to the collisions of the hydrogen atoms with the fluorocarbon surface of the storage bulb. In order to obtain self-oscillation the cavity-$Q$ must be relatively high. Thus cavity pulling is relatively strong, and it limits the long-term stability (over times longer than several days) to values not better than those of cesium standards. In the region from a few seconds to a day the hydrogen maser has the best stability of all existing standards. Its application is rather limited to uses where these stabilities are critical and where a rather bulky device is no handicap. Unlike cesium and rubidium standards, hydrogen masers have not yet been evaluated under adverse environmental conditions.\textsuperscript{16} The number of hydrogen masers in use is very small compared to the numbers of cesium beam devices and of rubidium devices.

\textsuperscript{15}Higher $Q$-values are realized in experimental devices such as methane stabilized helium neon lasers and ion storage devices as well as in microwave cavities which are cooled to the superconducting state (compare sec. 8).

\textsuperscript{16}Relevant data are being accumulated at the time of this writing in connection with a program to use a hydrogen maser in a space probe in order to gain scientific data on the general theory of relativity. This program is sponsored by NASA and is carried out by the Smithsonian Astrophysical Observatory.
Fig. 24. Hydrogen maser oscillator.
Fig. 25. Frequency stability of a hydrogen maser oscillator. The curve is a composite of the best documented performances of several differently designed devices. However, there is sufficient evidence to indicate that a device could be built which performs as indicated. Programs are under way towards this goal.
7. SUMMARY AND COMPARISON OF AVAILABLE FREQUENCY STANDARDS

Table 1 gives a summary of the performance characteristics of the frequency standards discussed in sections 4 and 6. Individual units may give better (or worse!) performance than indicated by the numbers in the table. The table may be used to compare the four types of devices.

<table>
<thead>
<tr>
<th>Device</th>
<th>Resonance Frequency</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crystal Oscillator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TCXO</td>
<td>Manufactured throughout the kHz and MHz regions</td>
<td>10^5 to 10^6</td>
</tr>
<tr>
<td>Oven controlled</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High quality type (oven controlled)*</td>
<td>2.5 MHz and 5 MHz typically</td>
<td>a few 10^6</td>
</tr>
<tr>
<td>Cesium Beam</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small commercial</td>
<td></td>
<td>10^7</td>
</tr>
<tr>
<td>Typical commercial**</td>
<td>9192 631 770 Hz</td>
<td>a few 10^7</td>
</tr>
<tr>
<td>Laboratory type</td>
<td></td>
<td>a few 10^6</td>
</tr>
<tr>
<td>Rubidium Gas Cell</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small commercial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commercial</td>
<td>6834 682 608 Hz</td>
<td>a few 10^7</td>
</tr>
<tr>
<td>Hydrogen Maser Oscillator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laboratory type ***</td>
<td>1420 405 752 Hz</td>
<td>10^9</td>
</tr>
</tbody>
</table>

* Values for selected units in parentheses.
** Values for the most recently developed high performance devices in parentheses.
*** The design of some units approaches a commercial design.
Table 1 (contd.)

<table>
<thead>
<tr>
<th>Principal causes for long-term instability</th>
<th>Fractional frequency stability*</th>
<th>Time between resynchronization for timekeeping of**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>one second</td>
<td>one hour</td>
</tr>
<tr>
<td>Aging of crystal resonator and electronic components; environmental effects</td>
<td>Cannot be specifically stated (typically ranging from $10^{-8}$ to $10^{-9}$)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1 \times 10^{-11}$</td>
<td>$1 \times 10^{-10}$</td>
</tr>
<tr>
<td></td>
<td>$(2 \times 10^{-13})$</td>
<td>$(2 \times 10^{-13})$</td>
</tr>
<tr>
<td></td>
<td>$1 \times 10^{-12}$</td>
<td>$1 \times 10^{-12}$</td>
</tr>
<tr>
<td>Aging of components, environmental effects; some units continue to improve in stability without apparent limitation</td>
<td>$2 \times 10^{-11}$</td>
<td>$4 \times 10^{-13}$</td>
</tr>
<tr>
<td></td>
<td>$5 \times 10^{-11}$</td>
<td>$7 \times 10^{-13}$</td>
</tr>
<tr>
<td></td>
<td>$(6 \times 10^{-12})$</td>
<td>$(1 \times 10^{-13})$</td>
</tr>
<tr>
<td></td>
<td>$(1 \times 10^{-12})$</td>
<td>$2 \times 10^{-14}$</td>
</tr>
<tr>
<td>Aging of light source, filter, and gas cell, environmental effects</td>
<td>$2 \times 10^{-11}$</td>
<td>$6 \times 10^{-13}$</td>
</tr>
<tr>
<td>Cavity pulling environmental effects</td>
<td>$5 \times 10^{-12}$</td>
<td>$3 \times 10^{-13}$</td>
</tr>
</tbody>
</table>

* The data can also be regarded as the time required between recalibrations to achieve the (calibrated) time accuracy which corresponds to the stated stability figure (assuming no frequency offset). It should be noted that aging or drift (if "well-behaved") does not limit the systems usage of a device since it can be measured and predicted.

** Based on the assumption that data and equipment are properly utilized.
Table 1 (contd.)

<table>
<thead>
<tr>
<th>Fractional Frequency Drift (Aging)</th>
<th>Fractional Frequency Reproducibility</th>
<th>Fractional Frequency Accuracy **</th>
<th>Principal Environmental Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-9}$ per day</td>
<td></td>
<td></td>
<td>Accelerations (g-forces, shock, vibration), temperature changes, ionizing radiation, changes in crystal drive level.</td>
</tr>
<tr>
<td>$5 \times 10^{-10}$ per day</td>
<td></td>
<td>$5 \times 10^{-11}$</td>
<td>Strong magnetic fields*, strong accelerations, large temperature changes</td>
</tr>
<tr>
<td>$1 \times 10^{-11}$ per day (5 \times 10^{-13})</td>
<td>As manufactured worse than $10^{-6}$</td>
<td>$2 \times 10^{-11}$</td>
<td>Strong magnetic fields*, temperature changes, atmospheric pressure</td>
</tr>
<tr>
<td>None detectable with resolution of $1 \times 10^{-12}$ per year. (In some units drift of as much as $1 \times 10^{-12}$ per year may occur.)</td>
<td>$1 \times 10^{-11}$</td>
<td>$2 \times 10^{-11}$</td>
<td>Strong magnetic fields*, temperature changes, atmospheric pressure</td>
</tr>
<tr>
<td>$1 \times 10^{-11}$ per month</td>
<td>$1 \times 10^{-10}$</td>
<td>$5 \times 10^{-10}$</td>
<td>Strong magnetic fields*, temperature changes, atmospheric pressure</td>
</tr>
<tr>
<td>None detectable with resolution of $1 \times 10^{-12}$ per year</td>
<td>$5 \times 10^{-13}$</td>
<td>$1 \times 10^{-12}$</td>
<td>Strong magnetic fields*, temperature changes</td>
</tr>
</tbody>
</table>

* As compared to earth magnetic field.

** The figures given for hydrogen and all cesium standards are likely to improve in the future.
<table>
<thead>
<tr>
<th>Behavior under intermittent operation</th>
<th>Reliability</th>
<th>Size* vol. in cm³</th>
<th>Weight* in kg</th>
<th>Power demand* in watts</th>
<th>Estimated selling price in k$* (for frequency standard)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast warm-up</td>
<td>Frequency may have systematic offset after operation is resumed</td>
<td>Not limited by crystal resonator</td>
<td>&gt;10</td>
<td>&gt;0.1</td>
<td>&gt;0.1</td>
</tr>
<tr>
<td>Warm-up requires min. to hours</td>
<td></td>
<td></td>
<td>10,000</td>
<td>5-10</td>
<td>15</td>
</tr>
<tr>
<td>Warm-up in minutes, no frequency offsets will occur</td>
<td>Life of the cesium resonator (beam tube) is at least 3 yrs. (continuous operation)</td>
<td>10,000</td>
<td>16</td>
<td>30</td>
<td>12</td>
</tr>
<tr>
<td>Warm-up in minutes, no frequency offsets will occur</td>
<td>Lifetime of the cesium resonator is about 1 yr. (continuous operation)</td>
<td>30,000</td>
<td>30</td>
<td>40</td>
<td>15</td>
</tr>
<tr>
<td>Warm-up in minutes, no frequency offsets will occur</td>
<td>Life of the rubidium resonator (optical pkg.) is at least 3 yrs. (cont. operation)</td>
<td>1,000</td>
<td>1</td>
<td>12</td>
<td>5</td>
</tr>
<tr>
<td>Warm-up in hours, no frequency offsets will occur</td>
<td>Insufficient no. of units. Some have operated continuously for much longer than 1 yr.</td>
<td>20,000</td>
<td>10-20</td>
<td>35</td>
<td>7.5</td>
</tr>
<tr>
<td>Warm-up in hours, no frequency offsets will occur</td>
<td></td>
<td>300,000</td>
<td>200</td>
<td>40</td>
<td>60</td>
</tr>
</tbody>
</table>

* These figures are rounded to one significant digit. For single unit purchase only.
8. TRENDS

All devices which were discussed in this report have improved over the past years in one or more of the following aspects: reliability, size, weight, stability performance, and environmental insensitivity. A considerable amount of effort is still being expended to go further in this direction; however, it would be inappropriate here to try to estimate the improvement of performance specifications which might (or might not) be achieved in the future.

There are several other devices, designs, and concepts which have been studied or are being investigated with some potential for future use in frequency standards. Most of these are summarized in Table 2.
<table>
<thead>
<tr>
<th>Resonator</th>
<th>Resonance</th>
<th>Q (incl. projected values)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ammonia absorption</td>
<td>Around 22 GHz</td>
<td>$10^8$ to $10^9$</td>
<td>Sensitive to electric fields; insensitive to external magnetic fields.</td>
</tr>
<tr>
<td>Ammonia maser oscillator</td>
<td>Around 22 GHz</td>
<td>A few $10^7$</td>
<td>First &quot;atomic&quot; clock (1948) relatively low Q</td>
</tr>
<tr>
<td>Thallium atomic beam</td>
<td>21 GHz</td>
<td>$10^7$ to $10^9$</td>
<td>Insensitive to external magnetic fields, detection problems.</td>
</tr>
<tr>
<td>Barium oxide molecular beam</td>
<td>19 GHz</td>
<td>$10^7$ to $10^9$</td>
<td>Sensitive to electric fields. Insensitive to external magnetic fields, detection problems.</td>
</tr>
<tr>
<td>Hydrogen atomic beam</td>
<td>1.4 GHz</td>
<td>$10^8$ to $10^9$</td>
<td>Storage principle and microwave detection (no self oscillations), or storage principle and atom detection, or straight beam (as in cesium) and atom detection. Detection problems.</td>
</tr>
<tr>
<td>Ion storage</td>
<td>In the GHz region depending on chosen ion</td>
<td>$10^9$ to $10^{10}$</td>
<td>Storage in electric and magnetic fields (no walls!). Problems with storing a sufficient number of ions. Helium and mercury ions have been used.</td>
</tr>
<tr>
<td>Saturated molecular absorption</td>
<td>In the THz region* depending on chosen molecule</td>
<td>$10^9$ to $10^{11}$</td>
<td>Exciting oscillator is a laser. Small and simple design. Problems in usage because of very high frequency (infrared and visible radiation). Methane and iodine are two of the several molecules being used.</td>
</tr>
<tr>
<td>Rubidium maser oscillator</td>
<td>6.8 GHz</td>
<td>A few $10^7$</td>
<td>Similar to rubidium gas cell with similar limitations, but self-oscillations with microwave detection.</td>
</tr>
<tr>
<td>Superconducting microwave cavity</td>
<td>In the GHz region</td>
<td>$10^9$ to $10^{12}$</td>
<td>Technology problems. Very low accuracy but good stability to be expected.</td>
</tr>
</tbody>
</table>

* THz = one trillion hertz = $10^{12}$ Hz.
9. LITERATURE

The following references are a limited selection. They are intended as a guide for those who wish to penetrate the subject in greater depth.

A. Introduction

B. Basic Concepts

C. Frequency Standards, General Aspects
(1) Most general physics textbooks.

D. Quartz Crystal Devices
E. Atomic Resonance Devices


F. Available Atomic Frequency Standards


G. Trends


H. General Scientific Overview

ACKNOWLEDGMENTS

The author wishes to thank Byron E. Blair for his assistance in setting up the basic outline of this paper, and Donald Halford and David W. Allan for their assistance in the preparation of this paper.
The topic of frequency standards and clocks is treated in a tutorial and non-mathematical way. The concepts of time, frequency stability, and accuracy are introduced. The general physical principles and design features of frequency standards and clocks are described. The design, performance, and limitations of quartz crystal oscillators and atomic devices (cesium, hydrogen, rubidium) are discussed in detail and critically compared for laboratory devices as well as for devices intended for field usage.
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