NBS TECHNICAL NOTE 614

Longitudinal Magnetoresistance in Polar Semiconductors

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The magnetophonon effect due to resonant interactions of charge carriers with optical phonons in nondegenerate polar semiconductors is studied when several elastic scattering mechanisms are simultaneously active also. The simple model of the band structure, and the displaced Maxwellian distribution function are used. Analytical expressions for drift mobility are given for B = 0 and the quantum limit, and are compared with the results of other techniques where possible. The numerical calculations show that whenever magnetophonon oscillations are seen in the ohmic regime, maxima occur near the resonance magnetic fields for all lattice temperature and strengths of scattering mechanisms which compete with the optical phonon scattering. This is in contrast with typical experimental findings, in which minima usually (but not always) appear near the resonance fields for ohmic, longitudinal, conditions. The explanation of the minima must therefore be sought in the use of a distribution function in which carrier-carrier scattering is less important.

Key words: Magnetophonon effect; magnetoresistance; semiconductors; transport theory.

1. Introduction

This Technical Note is a back-up article for a much shorter paper being published elsewhere.[1] We here give a rather detailed exposition of the calculations only indicated in that paper, and provide some further discussion.

In earlier articles,[2,3] we developed the theory and showed the results of numerical computations of longitudinal magnetoresistance for the simple model of a semiconductor, using the displaced-Maxwellian distribution function for the carriers, and scattering of carriers by optical phonons via the deformation potential ("nonpolar")[2] and piezoelectric ("polar")[3] interactions. Scattering by acoustic phonons via the deformation potential was also included in the latter.[3] These computations showed oscillations in magnetoresistance, electron temperature, and acoustic gain, due to the "magnetophonon effect." For the longitudinal ohmic magnetoresistance, such oscillations were predicted theoretically in 1964,[4] and have since been seen in abundance experimentally.[5-16] However, the experimental observations in the longitudinal ohmic case have not appeared to bear out the theory in more than a cursory way.

The experiments typically show minima in the ohmic longitudinal magnetoresistance near the resonance fields, i.e., the magnetic fields for which the longitudinal optical (LO) phonon energy is an integral multiple of the cyclotron energy of the carrier. However, in certain materials[10,16] neither the maxima nor the minima lie close to the resonance points.

Gurevich and Firsov,[4] by solving the Boltzmann equation in an approximate way in the ohmic region, had shown that for pure polar LO scattering the extrema of the longitudinal magnetoresistance

oscillations would be maxima near the resonance fields. Our computations based on the displaced-Maxwellian also showed this for both the nonpolar[2] and polar[3] cases. Gurevich and Firsov[3] further deduced that when acoustic phonon scattering competed sufficiently strongly with the LO scattering, these maxima would convert to minima. They evaluated a parameter Γ which characterizes the relative strength of acoustic to optical phonon scattering. When Γ is greater than about unity, minima should be observed.[4] Experiments showing minima have been reported in which Γ has been estimated to be larger than unity,[12] supporting the Gurevich-Firsov theory. However, in other experiments showing minima, it appears that Γ is substantially less than unity[7,8] (see also section 3).

Further, it is not evident just how the maxima for pure optical phonon scattering will convert into minima with increasing strength of competing scattering mechanisms: Do the magnetoresistance extrema shift to higher or lower magnetic field, as might be inferred from the more polar materials,[10,16] or do the amplitudes of the extrema simply reverse their signs with relatively little shifting of the positions of the extrema?

These unresolved questions have prompted us to begin a more detailed theoretical study of this situation. We report here the results of an investigation in which the displaced-Maxwellian distribution function was used in a study of the longitudinal magnetoresistance

in polar semiconductors. Four scattering mechanisms were considered simultaneously in varying strengths: the polar optical phonon scattering mechanism, scattering of the carriers by acoustic phonons via both the deformation potential and piezoelectric mechanisms, and also scattering due to screened ionized impurities. Computer studies were made at several lattice temperatures. We have used the simple model of the band structure, that is, the model in which there is but a single isotropic, parabolic conduction band centered at k= 0, and we do not include Landau level broadening. This is the same model as used by Gurevich and Firsov.[4]

(If we had included nonparabolicity, which is an important requirement for the much-studied n-type InSb, the displaced-Maxwellian could not have been used. The same techniques used in deriving[17] the displaced-Maxwellian for a parabolic band result in a very much more complicated distribution function in the nonparabolic case.)

Our reason for choosing this particular distribution function is that it is the only one which at present seems to be amenable to extensive numerical computation for polar semiconductors. Use of the displaced-Maxwellian of course implies a good deal of carrier-carrier scattering which establishes the distribution. Conditions for the validity of this distribution are discussed by Conwell,[18] who references earlier work. Reference 2 contains further references to

work based on the displaced-Maxwellian. If it is felt that carrier-carrier scattering is not predominant in a given material, then of course the displaced-Maxwellian results should not be taken too literally.

In the next section, we establish the basic formulas corresponding to the model chosen. We derive formulas for ohmic drift mobilities for the various scattering mechanisms for both $B = 0$ and the quantum limit, that is, the limit in which only the lowest Landau level is considered to be occupied. These formulas are compared with ones obtained by solution of the Boltzmann equation, where possible. In section III we show several graphs of ohmic magnetoresistance at various scattering strengths and temperatures. We also exhibit some hot-electron magnetoresistance curves. Discussions of the results are given and some conclusions are drawn.

2. Theoretical Basis

The displaced-Maxwellian distribution function contains two parameters, which are the carrier drift velocity v_d and the carrier temperature $T_e = 1/k\beta_e$. These are determined self-consistently through two coupled equations which describe the constancy of the carrier momentum and energy in the steady state,[2] or in other words, which balance the drift and scattering terms of the first and second moments of the Boltzmann equation.

The scattering interactions for the three carrier-phonon mechanisms considered here may all be written in the form

$$V_\alpha = \sum \overline{q}_\alpha \, b_q \, e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}} + \text{Hermitian conjugate}, \qquad (1)$$

where b_q is the phonon destruction operator for mode $\underset{\sim}{q}$, and $\underset{\sim}{r}$ is the carrier position. The index α indicates a specific scattering mechanism, and polarization indices have been suppressed. The summation is over all phonon wave vectors $\underset{\sim}{q}$ in one or more polarization branches. The coupling coefficient $\overline{q}_\alpha$ is generally anisotropic, and it is customary to substitute an averaged quantity for it.

The scattering interaction between a carrier and N_I randomly distributed screened ionized impurities may be written[19]

$$V_{imp} = \pm b \sum_{\underset{\sim}{q},\underset{\sim}{R}} \frac{e^{i\underset{\sim}{q}\cdot(\underset{\sim}{r}-\underset{\sim}{R})}}{q^2 + r_s^{-2}} , \qquad (2)$$

where, in SI (or MKS) units,

$$b = e^2/\varkappa\varepsilon_o . \qquad (3)$$

Here $\varkappa$ is the dielectric constant and ε_o is the permittivity constant. The summation in eq (2) is over all impurity positions $\underset{\sim}{R}$ and Fourier components $\underset{\sim}{q}$. The screening length is r_s. An obvious extension of the work of Argyres and Adams[19] to the hot-electron regime yields the expression

$$q_s^2 \equiv r_s^{-2} = b\beta_e n_c , \tag{4}$$

where n_c is the carrier concentration.

Subsequent expressions are written for electrons, of charge $-e$ with $e > 0$. Drift mobility is defined by

$$v_d(E, B) = -\mu(E, B)\, E. \tag{5}$$

The magnetoresistance is

$$\frac{\Delta\rho}{\rho_o} = \frac{\rho(E, B)}{\rho(E, 0)} - 1 = \frac{n_c(E, 0)\mu(E, 0)}{n_c(E, B)\,\mu(E, B)} - 1 . \tag{6}$$

In this paper we shall ignore any E or B dependence of the carrier concentration n_c, so that under ohmic conditions, the magnetoresistance is calculated from the ratio of drift mobilities. For nonohmic constant-voltage conditions, the mobility ratio in eq (6) is calculated from the ratio of drift velocities, while for nonohmic constant-current (or constant drift velocity) the mobility ratio is given by $E(B)/E(0)$, where $E(B)$ is the B-dependent electric field across the sample.

We consider the $B = 0$ and $B \neq 0$ cases separately, treating the zero magnetic field case first.

2.1 B = 0

The complete Hamiltonian, with $\underset{\sim}{E}$ in the z-direction, is

$$H = p^2/2m^* + H_L + \sum_\alpha V_\alpha + eEz , \tag{7}$$

where $\underset{\sim}{p}$ is the electron momentum, m^* is its effective mass, H_L is the lattice Hamiltonian, and the interactions V_α have been defined above. The displaced-Maxwellian, normalized to unity is

$$\rho(\underset{\sim}{k}) = \Omega^{-1}(2\pi\beta_e\hbar^2/m^*)^{3/2} \exp\left[-\beta_e\hbar^2(\underset{\sim}{k}-\underline{\bar{k}})^2/2m^*\right] , \tag{8}$$

where Ω is the volume, and $\hbar\underline{\bar{k}}=m^*\underset{\sim}{v}_d$. The steady-state equations for electron momentum and energy are[2]

$$\text{momentum:} \quad eE = \sum_\alpha S_{m,\alpha} , \tag{9}$$

$$\text{energy:} \quad eE v_d = \sum_\alpha S_{e,\alpha} . \tag{10}$$

The left sides of these equations come from the drift term of the Boltzmann equation, and the right sides are the scattering terms. The summations are over all scattering mechanisms, and we shall not consider possible correlations between the various scattering processes. Also, we take the collisions with impurities to be perfectly elastic, so that the impurity scattering makes no contribution in eq (10). The scattering terms may be written[2]

$$S_{m,\alpha} = \hbar \sum_{\underset{\sim}{k},\underset{\sim}{k}'} (k'_z - k_z) W^\alpha_{\underset{\sim}{k}',\underset{\sim}{k}} \rho(\underset{\sim}{k}) , \tag{11}$$

$$S_{e,\alpha} = (\hbar^2/2m^*) \sum_{\underset{\sim}{k},\underset{\sim}{k}'} (k'^2 - k^2) W^\alpha_{\underset{\sim}{k}',\underset{\sim}{k}} \rho(\underset{\sim}{k}) . \tag{12}$$

The transition probability per unit time for electron-phonon scattering is

$$W^{\alpha}_{\underset{\sim}{k}',\underset{\sim}{k}} = \sum \rho(\{n_q\}) w_\alpha(\{n'_q\}\ \underset{\sim}{k}',\ \{n_q\}\ \underset{\sim}{k}) \quad . \tag{13}$$

The summation here is over all phonon numbers n_q and n'_q in the sets $\{n_q\}$ and $\{n'_q\}$. The lattice distribution function $\rho(\{n_q\})$ yields the Planck factor for the number $\bar{n}_{q,\alpha}$ of thermal phonons in mode $\underset{\sim}{q}$ and branch α:

$$\bar{n}_{q,\alpha} = [\exp(\beta_o \hbar \omega_{q,\alpha}) - 1]^{-1} \quad , \tag{14}$$

where $\omega_{q,\alpha}$ is the frequency of phonons of type α, and $T_L = 1/k\beta_o$ is the lattice temperature. Also,

$$w_\alpha = (2\pi/\hbar) |\langle \{n'_q\}\ \underset{\sim}{k}' | V_\alpha | \{n_q\}\ \underset{\sim}{k} \rangle|^2 \times$$

$$\delta [\mathcal{E}(\{n'_q\}\ \underset{\sim}{k}') - \mathcal{E}(\{n_q\}\ \underset{\sim}{k})] \quad . \tag{15}$$

The energy $\mathcal{E}$ is given by

$$\mathcal{E}(\{n_q\}\ \underset{\sim}{k}) = \frac{\hbar^2 k^2}{2m^*} + \sum_{\underset{\sim}{q}} \hbar \omega_{q,\alpha} (n_q + \frac{1}{2}) . \tag{16}$$

For ionized impurity scattering, the transition probability is simply

$$W_{\underset{\sim}{k}\,'\underset{\sim}{k}}^{imp} = (2\pi/\hbar)|<\underset{\sim}{k}'|V_{imp}|\underset{\sim}{k}>|^2\,\delta[\hbar^2(k^2-k'^2)/2m^*]\,. \qquad (17)$$

The momentum and energy scattering terms for carrier-phonon processes reduce to

$$S_{m,\alpha} = A\int_0^\infty dq\, q^2\,|\overline{q}_\alpha|^2\int_{-1}^{1} dx\, x F_\alpha(q,x), \qquad (18)$$

$$S_{e,\alpha} = A\int_0^\infty dq\, q\,\omega_{q,\alpha}\,|\overline{q}_\alpha|^2\int_{-1}^{1} dx\, F_\alpha(q,x), \qquad (19)$$

after integrating over carrier coordinates. Here

$$A = (2\pi\beta_e m^*)^{\frac{1}{2}}\,\Omega/4\pi^2\hbar\,, \qquad (20)$$

$$F_\alpha(q,x) = (\overline{n}_{q,\alpha}+1)\,e^{-\eta\alpha+} - \overline{n}_{q,\alpha}\,e^{-\eta\alpha-}\,, \qquad (21)$$

$$\eta_{\alpha\pm} = \frac{\beta_e\hbar^2}{2m^*}\left[\frac{q}{2} \pm \frac{m^*\omega_{q,\alpha}}{\hbar q} \mp \overline{k}\,x\right]^2\,. \qquad (22)$$

In these equations, $x = \cos\theta$ where θ is the angle between $\underset{\sim}{q}$ and $\underset{\sim}{E}$. If elasticity is assumed for the acoustic phonon processes, the term $m^*\omega_{q,\alpha}/\hbar q$ of eq (22) does not appear. However, in the numerical work the term causes no real complication, and so we have not assumed elasticity there. Neither have we assumed equipartition, but have used the full Planck factor, eq (19), in the numerical work. (Equipartition becomes increasingly inaccurate with increasing E.)

We now consider the four mechanisms separately in more detail.

2.1.1 Deformation potential, acoustic phonons.

The coupling coefficient can be written[20]

$$|\overline{q}_{d,ac}|^2 = E_1^2 \hbar q^2 / 2\rho\Omega\omega_q, \tag{23}$$

where E_1 is the deformation potential energy and ρ is the mass density. Only longitudinally polarized acoustic phonons are involved, and we use

$$\omega_q = \mu_\ell q, \tag{24}$$

where μ_ℓ is the longitudinal speed of sound averaged over directions.[21]

The E = B = 0 drift mobility expression is readily found to be

$$\mu_o = \frac{3e\rho\mu_\ell^2\hbar^4}{16\,E_1^2}\left(\frac{2\pi^3}{m^{*5}k^3T_L^3}\right)^{\frac{1}{2}}. \tag{25}$$

This is obtained from the ohmic limit of eqs (9) and (18), using elasticity and equipartition, and has been derived previously by Sladek[22] and Paranjape and deAlba.[23] The mobility as given by eq (25) is smaller than that obtained by solving the Boltzmann equation for deformation potential acoustic phonon scattering alone,[24] by the factor $9\pi/32$. The reason is that the displaced-Maxwellian implies carrier-carrier scattering, which further limits the mobility.

2.1.2 Piezoelectric, acoustic phonons.

For a given direction of propagation $\underset{\sim}{q}$, the contribution of a given polarization branch to $|q_{p,ac}|^2$ may be written[25]

$$K^2 b \hbar \omega_q / 2\Omega q^2, \tag{26}$$

where K is the dimensionless electromechanical coupling, and b is given by eq (3). Both quantities are directional dependent. We use an average value for the dielectric constant appearing in b. The expression (26) is summed over longitudinal and transverse modes, and averaged over directions[21] in the manner of Meijer and Polder.[26] We indicate the result symbolically here by

$$|\overline{q}_{p,ac}|^2 = B_p/\Omega q , \tag{27}$$

and give additional details regarding computation in section III.

The E = B = 0 drift mobility is found to be

$$\mu_o = \frac{3eu_{av}\hbar^3}{4B_p} \left(\frac{2\pi^3}{m^{*3} kT_L} \right)^{\frac{1}{2}} \tag{28}$$

in the elastic equipartition limit, a result given earlier by Sladek.[22] As in the preceding case, the result (28) differs from the Boltzmann equation result only in being smaller by the factor $9\pi/32$.

2.1.3 Polar, optical phonons.

Here we use[27]

$$|\overline{q}_{op}|^2 = 2\pi\hbar^2 eE_o/\Omega m^* q^2 . \tag{29}$$

E_o has units of electric field and is given by

$$E_o = \frac{m^* e\omega_o}{4\pi\varepsilon_o \hbar}\left(\frac{1}{\varkappa_\infty} - \frac{1}{\varkappa_o}\right), \tag{30}$$

where $\varkappa_o$ and $\varkappa_\infty$ are, respectively, the (directionally averaged) dielectric constants for zero and infinite frequencies, and ω_o is the longitudinal optical mode (LO) frequency at $k = 0$. We make the usual assumption that the LO frequencies at all k are equal to ω_o.

The phonon scattering expressions (18) and (19) are not the simplest to use for optical phonon scattering. It is easier to integrate first on the phonon modes $\underset{\sim}{q}$, then over the angular parts of $\underset{\sim}{k}$, resulting in

$$S_{m,op} = \frac{2eE_o \overline{n}_o}{v}\left(\frac{\gamma}{\pi}\right)^{\frac{1}{2}} e^{-\gamma v^2} \int_o^\infty dx\, e^{-\gamma x^2} \left\{ e^{\gamma_o - \gamma \frac{x}{s^2}[xs + \ell n(x+s)]} \right.$$

$$\left. \times C(2\gamma vs) + \frac{1}{x}[xs - \ell n(x+s)]\, C(2\gamma v x) \right\}, \tag{31}$$

$$S_{e,op} = 4eE_o \left(\frac{2\hbar\omega_o\gamma^3}{m^*\pi}\right)^{\frac{1}{2}} \bar{n}_o e^{-\gamma v^2} \int_o^\infty dx\, e^{-\gamma x^2} x\ell n(x+s) \times$$

(32)

$$\left[e^{\gamma_o - \gamma} \frac{\sinh(2\gamma_v s)}{2\gamma_v s} - \frac{\sinh(2\gamma_v x)}{2\gamma_v x}\right],$$

where $s = (x^2+1)^{\frac{1}{2}}$, and

$$C(y) = \cosh y - y^{-1}\sinh y\ , \tag{33}$$

$$v = [m^*_v{}^2_d/2\hbar\omega_o]^{\frac{1}{2}}\ , \tag{34}$$

$$\bar{n}_o = [e^{\gamma_o} - 1]^{-1}\ , \tag{35}$$

$$\gamma_o = \beta_o\hbar\omega_o,\quad \gamma = \beta_e\hbar\omega_o \tag{36}$$

The E= B= 0 drift mobility is found to be

$$\mu_o = \frac{3}{E_o}\left(\frac{\pi\hbar\omega_o}{2m^*\gamma_o{}^3}\right)^{\frac{1}{2}} \frac{\sinh(\gamma_o/2)}{K_1(\gamma_o/2)}\ , \tag{37}$$

first derived by Stratton.[28] Properties of the modified Bessel function K_1 may be found in Ref. 29. The high temperature form of eq (37) is $9\pi/32$ times smaller than that deduced from the Boltzmann equation, as in the acoustic cases (equipartition assumed there is a "high"

temperature approximation). At lower temperatures ($\gamma_o > 0.36$) it is well known that the Boltzmann equation cannot be solved directly for the polar optical interaction because of the breakdown of the relaxation time approach. Howarth and Sondheimer[30] used a variational technique, which when corrected by the effective ionic charge in the manner of Ehrenreich,[31] gives precisely eq (37) in lowest order approximation.

2.1.4 Screened ionized impurities.

From eqs (2) - (4), (9), and (17), one can write

$$S_{m,imp} = \frac{-n_I b^2}{16 v_d} \left(\frac{2\beta_e}{\pi^3 m^*}\right)^{\frac{1}{2}} e^{-r^2} \times \tag{38}$$

$$\int_o^\infty dx\ x^{-1} [\ell n(1+x) - x/(1+x)] e^{-gx} C[2r(gx)^{\frac{1}{2}}] ,$$

where $n_I = N_I/\Omega$ is the impurity concentration,

$$g = \beta_e \epsilon_s / 4 , \tag{39}$$

$$\epsilon_s = \hbar^2 q_s^2 / 2m^* = \hbar^2 \beta_e n_c b / 2m^* , \tag{40}$$

$$r^2 = \beta_e m^* v_d^2 / 2 , \tag{41}$$

and C(y) is given by eq (33). In the ohmic region, the integral in eq (38) can be expressed in terms of the exponential integral[32] $E_1(g)$:

$$E_1(g) = \int_1^\infty dx\, e^{-gx}\, x^{-1} . \qquad (42)$$

The E= B= 0 drift mobility is

$$u_o = \frac{12e}{n_I b^2}\left(\frac{2\pi^3 k^3 T_L^3}{m^*}\right)^{\frac{1}{2}}\left[(1+g_o)e^{g_o}\, E_1(g_o)-1\right]^{-1} , \qquad (43)$$

where g_o is the $E \to 0$ form of g. Equation (43) is apparently new. Solution of the Boltzmann equation[19] in the ohmic limit yields a somewhat different integral than that to which eq (38) reduces, which cannot be expressed in terms of exponential integrals. However, for $g_o << 1$, which is typical in nondegenerate semiconductors, eq (43) may be simplified by using $E_1(g_o) \approx \ell n\,(1/g_o)$. Using $g_o << 1$ in the Boltzmann equation result,[19] one obtains the same expression as that to which eq (43) reduces, except that the drift mobility in the displaced Maxwellian formulation is $3\pi/32$ times smaller.

2.2 Finite Magnetic Field

In the familiar "Landau gauge," the Hamiltonian for $\underset{\sim}{E}$ and $\underset{\sim}{B}$ both in the z-direction may be written

$$H = [p_x^2 + p_z^2 + (p_y + m^* \omega_c x)^2]/2m^* + H_L + \sum_\alpha V_\alpha + eE_z , \qquad (44)$$

where

$$\omega_c = eB/m^* \qquad (45)$$

is the cyclotron frequency. The carrier energy in the plane wave states k_y, k_z, and Landau level $n = 0, 1, 2, \ldots$ is thus

$$\mathcal{E}_n(k_z) = \hbar^2 k_z^2/2m^* + \hbar\omega_c(n+\frac{1}{2}) . \tag{46}$$

We have associated the energy degeneracy with the y-direction, with N_y degrees of freedom. The displaced-Maxwellian may be written as the product $\rho_n \rho(k_y, k_z)$, where

$$\rho(k_y, k_z) = \frac{2\pi}{N_y L_z}\left(\frac{\beta_e \hbar^2}{2\pi m^*}\right)^{\frac{1}{2}} \exp\left[\frac{-\beta_e \hbar^2}{2m^*}(k_z-\bar{k})^2\right] , \tag{47}$$

$$\rho_n = (1-\exp(-\beta_e \hbar\omega_c)) \exp(-n\beta_e \hbar\omega_c) . \tag{48}$$

The distribution function is normalized such that

$$\sum_{n=0}^{\infty} \sum_{k_y} \int_{-\infty}^{\infty} dk_z \, \rho_n \rho(k_y, k_z) = 1, \tag{49}$$

and L_z is the length of the sample in the z-direction.

The matrix elements of $e^{i\vec{q}\cdot\vec{r}}$, needed for both phonon and impurity scattering, are

$$\langle n' k_y' k_z' | e^{i\vec{q}\cdot\vec{r}} | n k_y k_z \rangle = \delta_{k_y', k_y - q_y} \delta_{k_z', k_z - q_z} J_{n',n}(q_x, k_y+q_y, k_y), \tag{50}$$

where

$$J_{n',n}(q_x, k_y+q_y, k_y) = \int_{-\infty}^{\infty} dx\, e^{iq_x x}\, u_{n'}^{*}\left(\frac{x+\frac{k_y+q_y}{m^{*}\omega_c/\hbar}}{}\right) u_n\left(\frac{x+\frac{k_y}{m^{*}\omega_c/\hbar}}{}\right), \quad (51)$$

and $u_n(z)$ is the harmonic oscillator wave function

$$u_n(z) = \frac{i^n H_n(z)}{(2^n n!)^{\frac{1}{2}}} \left(\frac{m^{*}\omega_c}{\pi\hbar}\right)^{\frac{1}{4}} e^{-z^2/2}, \quad (52)$$

and $H_n(z)$ is the Hermite polynominal of order n. Now $J_{n',n}$ is a function only of λ, where

$$\lambda = \hbar q_{\perp}^{2}/2m^{*}\omega_c, \quad (53)$$

and $q_{\perp}^{2} = q_x + q_y^{2}$. One finds

$$|J_{n',n}|^2 = e^{-\lambda} f_{n',n}(\lambda), \quad (54)$$

where

$$f_{n',n}(\lambda) = n!n'!\lambda^{n'-n}\left[\sum_{s=o}^{n} \frac{(-\lambda)^s}{s!(n-s)!(n'-n+s)!}\right]^2, (n' \geq n). \quad (55)$$

If equipartition and elastic collisions are assumed for the deformation potential acoustic phonon scattering, the greatly simplifying result

$$\hbar(2\pi m^* \omega_c)^{-1} \int_{-\infty}^{\infty} dq_x \int_{-\infty}^{\infty} dq_y |J_{n',n}|^2 = \int_0^{\infty} d\lambda e^{-\lambda} f_{n',n}(\lambda) = 1 \quad (56)$$

can be used, as it can be with the nonpolar optical phonon scattering.[2] However, for the other scattering mechanisms considered here, this result cannot be used, and so, as with B = 0, we have not assumed equipartition or elastic collisions for the acoustic phonons, as no significant computational time-saving is thereby achieved.

The steady-state equations for carrier momentum and energy can be written in the forms eqs (9) and (10), respectively. For carrier-phonon interactions, the scattering terms are

$$S_{m,\alpha}(B) = -A \int_{-\infty}^{\infty} dq_z q_z |q_z|^{-1} \int_0^{\infty} dq_\perp q_\perp |\bar{q}_\alpha|^2 R_\alpha(q_\perp, q_z), \quad (57)$$

$$S_{e,\alpha}(B) = -A \int_{-\infty}^{\infty} dq_z |q_z|^{-1} \int_0^{\infty} dq_\perp q_\perp |\bar{q}_\alpha|^2 \omega_{q,\alpha} R_\alpha(q_\perp, q_z), \quad (58)$$

where A is given by eq (20), and

$$R_\alpha(q_\perp, q_z) = \sum_{n,n'=0}^{\infty} \rho_{n'} e^{-\lambda} f_{n',n}(\lambda) \left[(\bar{n}_{q,\alpha} + 1) e^{-\eta'_{\alpha+}} - \bar{n}_{q,\alpha} e^{-\eta'_{\alpha-}} \right], \quad (59)$$

$$\eta'_{\alpha\pm} = \frac{\beta_e \hbar^2}{2m^*} \left[\frac{q_z}{2} + \frac{m^* \omega_c (n-n')}{\hbar q_z} \pm \frac{m^* \omega_{q,\alpha}}{\hbar q_z} \mp \bar{k} \right]^2 . \quad (60)$$

For screened ionized impurities, the momentum scattering term may be written

$$S_{m,\,imp}(B)=\frac{-n_I b\hbar\omega_c\beta_e^2 e^{-r^2}}{4\pi^{3/2}}\int_o^\infty dy\, e^{-y^2}\int_o^\infty d\lambda e^{-\lambda}\left[R_{imp}(\lambda, y) + R_{imp}(\lambda,-y)\right], \tag{61}$$

where

$$R_{imp}(\lambda, y) = \sum_{n=0}^{\infty}\rho_n\left[\sum_{\ell=0}^{n} f_{n,\,n-\ell}(\lambda)\, c_\ell(\lambda, y)\sinh(2ry) + \sum_{\ell=1}^{\infty} f_{n,\,n+\ell}(\lambda)\exp(-\ell\beta_e\hbar\omega_c)\, c_\ell(\lambda, y)\sinh(2ry_\ell)\right], \tag{62}$$

$$y_\ell = (y^2+\ell\beta_e\hbar\omega_c)^{\frac{1}{2}}, \tag{63}$$

$$c_\ell(\lambda, y) = y_\ell^{-1}(y_\ell + y)\left[D(\lambda) + (y_\ell+y)^2\right]^{-2}, \tag{64}$$

$$D(\lambda) = \beta_e\hbar\omega_c\lambda + 4g, \tag{65}$$

and g and r are given in eqs (39) and (41), respectively.

We now look at some results in the quantum limit, which we define here to mean that the term $e^{-\beta e\hbar\omega_c}$ is negligible compared to unity, so that all carriers are in the lowest (n=0) Landau level.

2.2.1 Deformation potential, acoustic phonons.

The two acoustic phonon mechanisms can be treated together up to a point, by writing $|\bar{q}_\alpha|^2 = B_\alpha q^{p_\alpha}/\Omega$, where $p_\alpha = +1$ for the deformation potential interaction, and $p_\alpha = -1$ for the piezoelectric interaction. The momentum scattering term (57) may then be written, in the equipartition, elastic, ohmic, and quantum limits.

$$S_{m,\alpha}(QL) = \frac{-8AB_\alpha m^{*2} v_d \omega_c}{\hbar^3 \beta_o u_\alpha \Omega} \int_o^\infty dy\, e^{-y} \int_o^\infty d\lambda e^{-\lambda} q^{p_\alpha - 1} , \qquad (66)$$

and

$$q = \left[8m^* y/\beta_o \hbar^2 + 2m^* \omega_c \lambda/\hbar \right]^{\frac{1}{2}} . \qquad (67)$$

The ohmic drift mobility for deformation potential scattering is then

$$\mu_{QL} = \frac{\pi^3 e \hbar^3 \rho u_\ell^2}{E_1^2 \omega_c (2m^{*5} kT_L)^{\frac{1}{2}}} \qquad (68)$$

Comparing with the B = 0 case, eq (25), we see that

$$u_o/\mu_{QL} = 3\hbar\omega_c/8kT_L . \qquad (69)$$

Apart from a possible variation of carrier concentration with magnetic field, the longitudinal magnetoresistance, eq (6) thus increases

linearly with field in the ohmic, quantum limit. Equation (69) agrees with the Boltzmann equation result without carrier-carrier scattering,[19] except that a 9 replaces the 8 in eq (69).

2.2.2 Piezoelectric, acoustic phonons.

Using $p_\alpha = -1$, and [32]

$$\int_0^\infty dx\, e^{-ax} E_1(x) = a^{-1} \ell n(1 + a) \tag{70}$$

in eqs (66) and (67), one finds

$$\mu_{QL} = \frac{4\hbar^2 e u_{av}}{B_p \omega_c} \left(\frac{\pi^3 kT_L}{2m^{*3}}\right)^{\frac{1}{2}} \frac{c-1}{\ell n\, c} \tag{71}$$

where

$$c = \beta_o \hbar \omega_c / 4 . \tag{72}$$

Comparing with eq (28), we have

$$\frac{\mu_o}{\mu_{QL}} = \frac{3}{2} \frac{c\, \ell n\, c}{c-1} . \tag{73}$$

Thus the ohmic longitudinal magnetoresistance increases logarithmically with B at sufficiently high field, apart from any changes in carrier concentration. Except for the numerical factor, eq (73) agrees with

that obtained from the Boltzmann equation without carrier-carrier scattering,[33] for $c >> 1$.

2.2.3 Polar, optical phonons.

In the ohmic quantum limit, the momentum scattering term (57) becomes

$$S_{m,\,op}(QL) = \frac{-\hbar e E_o \omega_c v_d \, I(T_L, B)}{4 \sinh(\gamma_o/2)} \left(\frac{2m^*}{\pi k^3 T_L^{\,3}} \right)^{\frac{1}{2}} , \qquad (74)$$

where

$$I(T_L, B) = \int_o^\infty dx \, E_1(x) \exp\left[-(c-1)\, x - \gamma_o^{\,2}/(16\, cx)\right], \qquad (75)$$

and c is given in eq (72). The ohmic drift mobility is thus

$$\mu_{QL} = \left(\frac{\pi k^3 T_L^{\,3}}{2m^*} \right)^{\frac{1}{2}} \frac{4 \sinh(\gamma_o/2)}{E_o \hbar \omega_c \, I(T_L, B)} \quad . \qquad (76)$$

For c sufficiently greater than unity, it is evident that the term $\exp\left[-\gamma_o^{\,2}/(16cx)\right]$ in $I(T_L, B)$ makes no essential contribution to $I(T_L, B)$ apart from a temperature-dependent multiplicative factor. We thus write, using eq (70),

$$I(T_L, B) \approx r(\gamma_o)\,(c-1)^{-1} \ln c \; . \qquad (77)$$

That eq (77) is a good approximation can be seen from figure 1, where the exact integral, evaluated numerically, and eq (77) are plotted against c for three temperatures. The factor $r(\gamma_o)$ is there taken to be $\exp(-0.44\,\gamma_o)$, chosen strictly empirically. Thus, to a good approximation, we can write the drift mobility as

$$\mu_{QL} \approx \left(\frac{\pi k T_L}{2m^*}\right)^{\frac{1}{2}} \frac{c-1}{c \ln c} \frac{\sinh(\gamma_o/2)}{E_o\, r(\gamma_o)} , \qquad (76\,a)$$

and the ratio of mobilities as, with eq (37),

$$\frac{\mu_o}{\mu_{QL}} \approx \frac{3r(\gamma_o)}{\gamma_o K_1(\gamma_o/2)} \frac{c \ln c}{c-1} . \qquad (78)$$

The ohmic longitudinal magnetoresistance thus varies approximately logarithmically at sufficiently large B, apart from any carrier concentration variation.

2.2.4 Screened ionized impurities.

The momentum scattering term (61) reduces to

$$S_{m,\,imp}(QL) = \frac{-n_I b^2 e^{-r^2}}{2\pi^{3/2}\hbar\omega_c} \int_o^\infty dy\, e^{-\gamma^2} \sinh(2ry) z^{-1} e^z E_2(z) , \qquad (79)$$

in the quantum limit, where

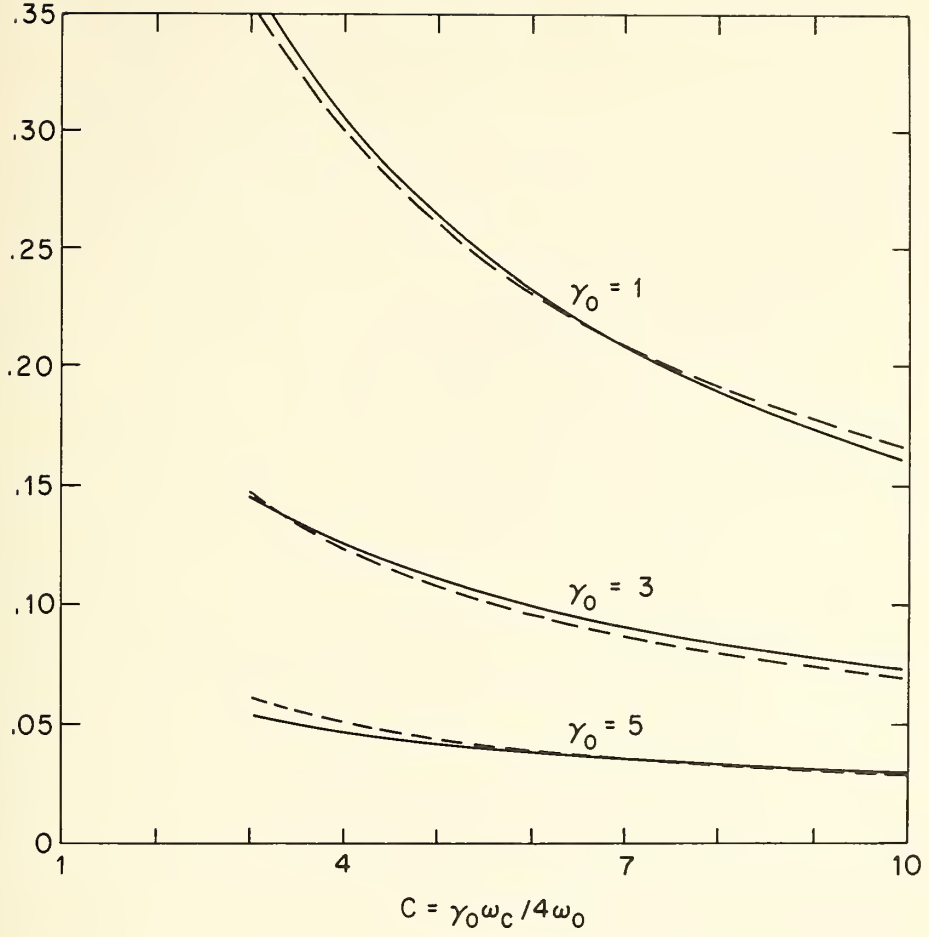

Figure 1. Comparison of the integral $I(T_L, B)$ defined in eq (75) (solid line) to the approximate form given in eq (77) (dashed line).

$$z = 4(g+y^2)/\beta_e \hbar \omega_c , \tag{80}$$

r is given by eq (41), and $E_2(z)$ is the second-order exponential integral.[32] In the ohmic limit, the drift mobility is found to be

$$\mu_{QL} = \frac{8\sqrt{2}\, e g_o (\pi k T_L)^{\frac{3}{2}}}{n_I b^2 m^{*\frac{1}{2}} [F(g_o) - F(g_o/c)]} \frac{c-1}{c} , \tag{81}$$

where

$$F(x) = x\, e^x\, E_1(x) . \tag{82}$$

With eq (43), the ratio of B = 0 to quantum limit mobilities is

$$\frac{\mu_o}{\mu_{QL}} = \frac{3c}{2(c-1)} \frac{F(g_o) - F(g_o/c)}{(1+g_o) F(g_o) - g_o} . \tag{83}$$

The longitudinal magnetoresistance due to ionized impurities thus saturates at sufficiently high magnetic field. The Boltzmann equation result of Argyres and Adams,[19] which omits carrier-carrier interaction, also shows this, but with a different saturation value. For example, if $g_o << 1$, and $c \gtrsim 1$, then using $E_1(x) \approx -\gamma_E - \ell n\, x$ where $\gamma_E = 0.5772 \ldots$ is Euler's constant, one obtains from eq (83)

$$\frac{\mu_o}{\mu_{QL}} = \frac{3}{2} \frac{\gamma_E + \ell n\, g_o + (c-1)^{-1} \ell n\, c}{\gamma_E + \ell n g_o + 1} \quad , \tag{83 a}$$

to a good approximation. The result of Argyres and Adams[19] reduces to $2[\ell n(3/g_o)-1]^{-1}(1 + 3/c)^{-1}$.

3. Numerical Results

In this section we exhibit some results of our numerical work. For this, it is necessary of course to choose values for the parameters such as m^*, ω_o, E_1, etc. We have elected to use those which seem to be representative of n-InSb, which is probably the most extensively studied material to date regarding magnetophonon effects. However, it should be kept in mind that InSb has a very nonparabolic central conduction band, whereas our model assumes a parabolic conduction band. Further, the displaced-Maxwellian distribution function implies a degree of carrier-carrier scattering which probably exceeds that in most samples studied so far. Thus, the numerical work presented here should not be understood as an attempt to describe n-InSb. Instead, this study was undertaken to determine the extent to which temperature and elastic scattering could shift the magnetophonon structure, within the displaced-Maxwellian formulation.

Figures 2, 3, and 4 show curves of ohmic magnetoresistance versus magnetic field (expressed as ω_c/ω_o) for three temperatures corresponding to $\gamma_o = \theta/T_L = \hbar\omega_o/kT_L = 14.0$, 3.65, and 2.60, respectively. With $\theta = 281$ K,[11] these values correspond to lattice temperatures of 20 K, 77 K, and 108 K. We have also used $E_o = 470$ V/cm, and $m^* = 0.0159\, m_o$, which is the value of the so-called magnetophonon mass in n-InSb.[11] The value of the deformation potential E_1 has been uncertain, although Tsidilkovskii and Demchuk[34] have recently made a strong argument for a value of about 30 eV. They reference much previous work, but omit the work of Nill and McWhorter[35] (4.5 ev), Whalen and Westgate[36] (16.2 eV), and Martin and Mead[37] (16 eV or higher). In the figures we have used several values for E_1. Also, we use the Nill and McWhorter value of 0.06 coul/m^2 for the piezoelectric constant e_{14}, which is close to that of Whalen and Westgate. The piezoelectric coupling coefficient for acoustic phonon scattering is obtained in the manner of Meijer and Polder[26] by considering all polarization modes and averaging over directions. The result is

$$B_p = 2b^2 e_{14}^2 \hbar Q/\rho e^2 \quad , \tag{84}$$

where B_p appears in eq (27). The averaged quantity Q is

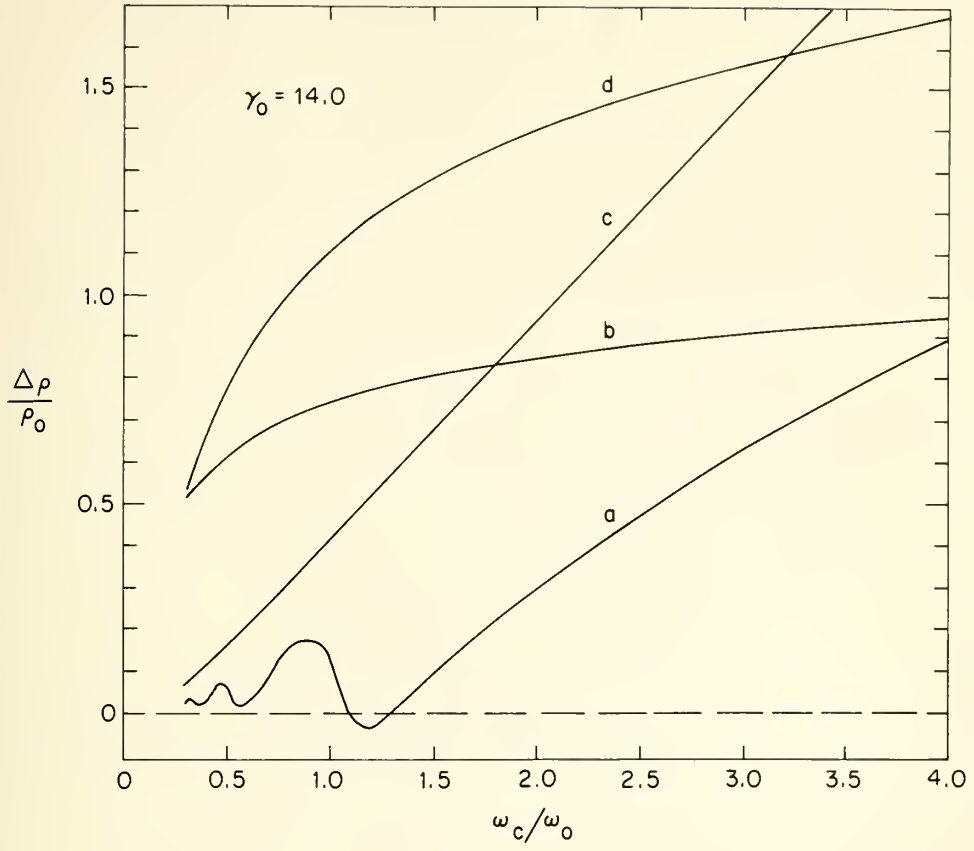

Figure 2. Ohmic magnetoresistance at $\gamma_o = 14.0$ (20 K for InSb). Values of E_1 (eV), e_{14} (coul/m^2), impurity concentration (cm^{-3}), and Γ are, respectively: (a) 0, 0, 0, 0; (b) 4.5, 0.06, 4×10^{13}, 0.27; (c) 16.2, 0, 0, 0.01; (d) 16.2, 0, 10^{15}, 1.56.

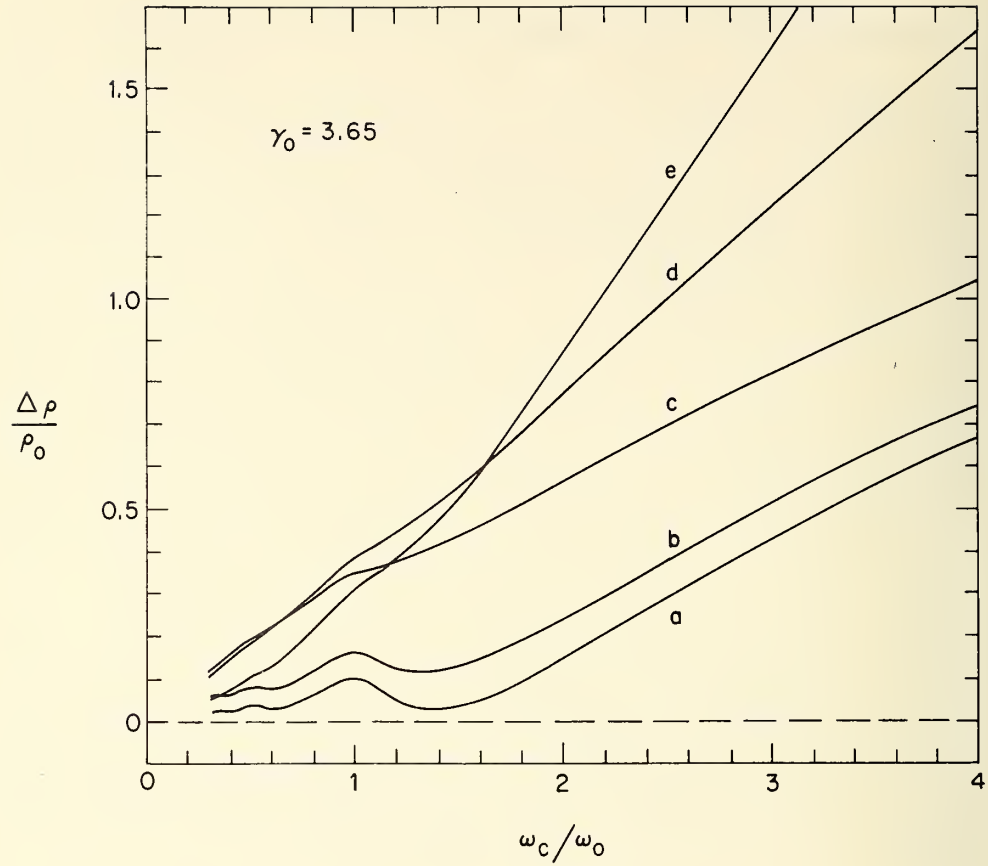

Figure 3. Ohmic magnetoresistance at $\gamma_{o_2} = 3.65$ (77 K for InSb). Values of E_1 (eV), e_{14} (coul/m^2), impurity concentration (cm^{-3}), and Γ are, respectively: (a) 0, 0, 0, 0,; (b) 4.5, 0.06, 4 x 10^{13}, 0.04; (c) 16.2, 0, 10^{15}, 0.43; (d) 30, 0.06, 10^{15}, 0.58; (e) 0.58; (e) 30, 0.06, 4 x 10^{13}, 0.23.

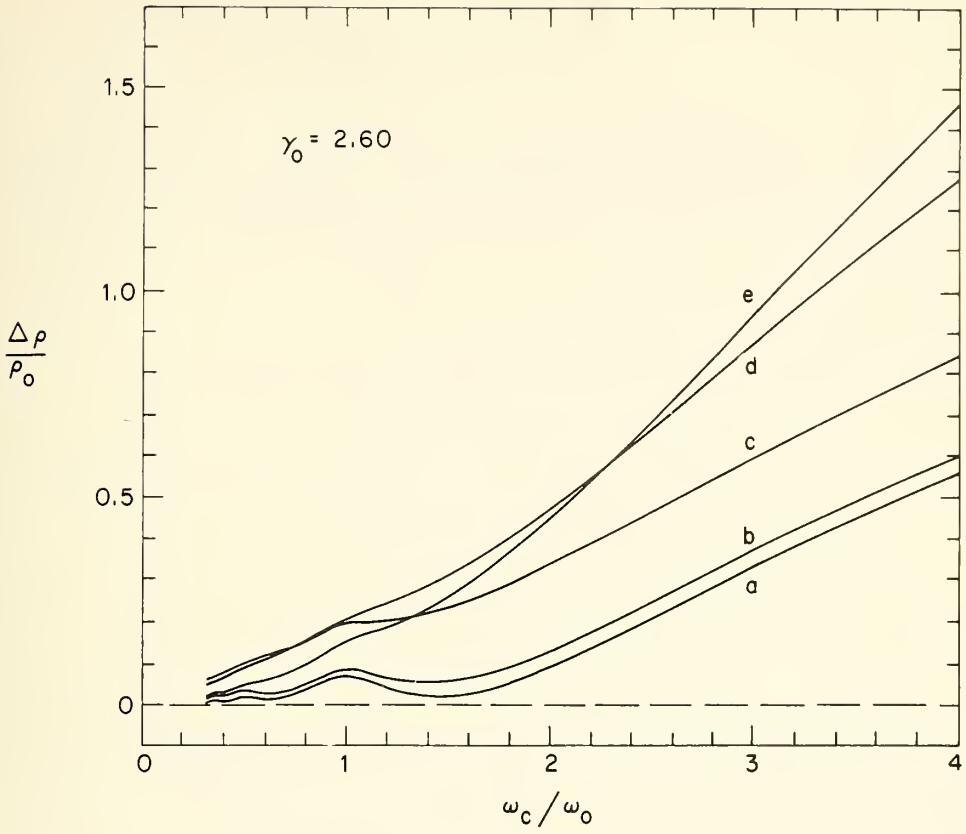

Figure 4. Ohmic magnetoresistance at $\gamma_o = 2.60$ (108 K for InSb). Values of E_1 (eV), e_{14} (coul/m^2), impurity concentration (cm^{-3}), and Γ are, respectively: (a) 0, 0, 0, 0; (b) 4.5, 0.06, 4×10^{13}, 0.03; (c) 16.2, 0, 10^{15}, 0.31; (d) 30, 0.06, 10^{15}, 0.51; (e) 30, 0.06, 4×10^{13}, 0.30.

$$Q = \left(\frac{4}{3u_1} + \frac{6}{4u_2} \right) / 13 \ , \tag{85}$$

where u_1 and u_2 are longitudinal and transverse sound velocities for propagation in the (1, 1, 1) and (1, 1, 0) directions, respectively, given by

$$\rho u_1^2 = \frac{1}{3} (c_{11} + 2c_{12} + 4c_{44} + 4e_{14}^2/\varkappa\varepsilon_o) \ , \tag{86}$$

$$\rho u_2^2 = c_{44} + e_{14}^2/\varkappa\varepsilon_o \ . \tag{87}$$

The elastic constants are given the values $c_{11} = 6.66$, $c_{12} = 3.35$, and $c_{44} = 3.14 \times 10^{10}$ nt/m^2. We also chose $\varkappa = 17.8$ and $\rho = 5.78$ gm/cm^3. The calculated values of u_1 and u_2 are 3900 and 2300 m/sec, respectively, and the grand average u_{av} over directions and polarizations is 2800 m/sec, which is used in $\omega_q = u_{av} q$. For the deformation potential scattering we used $u_\ell = 3700$ m/sec, which is the average longitudinal sound velocity.

In evaluating the screening terms g and g_o, we have set the carrier concentration equal to the impurity concentration. Finally, we have summed over as many Landau levels as necessary to achieve about 0.1% accuracy. In some cases, as many as 14 levels were included.

We have calculated the Gurevich-Firsov[4] Γ (see the discussion in the Introduction) for each of the curves in figures 2-4. Extending their result to include all scattering mechanisms which compete with the carrier-optical phonon scattering, and assuming that the respective resistivities are simply additive, we obtain

$$\Gamma = \frac{4}{3}\,\omega_c \left(\frac{\hbar}{\pi\,\omega_o kT_L}\right)^{\frac{1}{2}} \sum_\alpha \rho_\alpha \,. \tag{88}$$

The three dimensionless resistivities ρ_α in the present case are

$$\rho(\text{def. pot., ac.}) = \frac{16E_1^{\,2}m^{*2}}{3eE_o\hbar^4\rho u_\ell^{\,2}} \left(\frac{\hbar\omega_o k^3 T_L^{\,3}}{\pi^3}\right)^{\frac{1}{2}}, \tag{89}$$

$$\rho(\text{piezo., ac.}) = \frac{8m^{*}b^2 e_{14}^{\,2} Q}{3e^3 E_o \hbar^2 \rho u_{av}} \left(\frac{\hbar\omega_o kT_L}{\pi^3}\right)^{\frac{1}{2}}, \tag{90}$$

$$\rho(\text{imp.}) = \frac{n_I b^2}{12eE_o} \left(\frac{\hbar\omega_o}{\pi^3 k^3 T_L^{\,3}}\right)^{\frac{1}{2}} [(1+g_o)e^{g_o} E_1(g_o) - 1]. \tag{91}$$

Γ-values were calculated at the first resonance point, that is at $\omega_c = \omega_o$. As is seen from the figure captions, Γ is typically below unity even in the most favorable cases. Thus according to Gurevich-Firsov theory, resonance point maxima, or perhaps simply very small amplitudes of oscillation, are to be expected in n-InSb. However,

as pointed out in the introduction, magnetophonon oscillations in n-InSb have always shown minima near the resonance points.

In each of figures 2-4, we show curves for pure optical phonon scattering ($\Gamma = 0$), labelled (a), which show the magnetophonon oscillations most clearly. To be noted are the facts that the resonance extrema are maxima, as in the Gurevich-Firsov theory,[4] but that the maxima lie somewhat to the low-field side of the resonance points, defined by $\omega_c = \omega_o/N$, $N = 1, 2, \ldots$ The shift is greatest at low temperatures and low N-values. The pure polar optical curve at $\gamma_o = 14$ also shows the near-logarithmic behavior at large B, discussed in the preceding section.

The principal result of adding other scattering mechanisms to the polar optical scattering is to diminish and finally obliterate the magnetophonon oscillations, as seen in the figures. In all cases we have examined, the oscillations have vanished before Γ has reached unity. One also sees from the figures that there is no significant "phase shift," due to these other mechanisms, moving the minima toward the resonance points, which possibility was considered in the introduction.

In figures 5 and 6, we exhibit the results of some hot-electron calculations, obtained by solving simultaneously the carrier momentum and energy steady-state equations. A "constant-current" (actually,

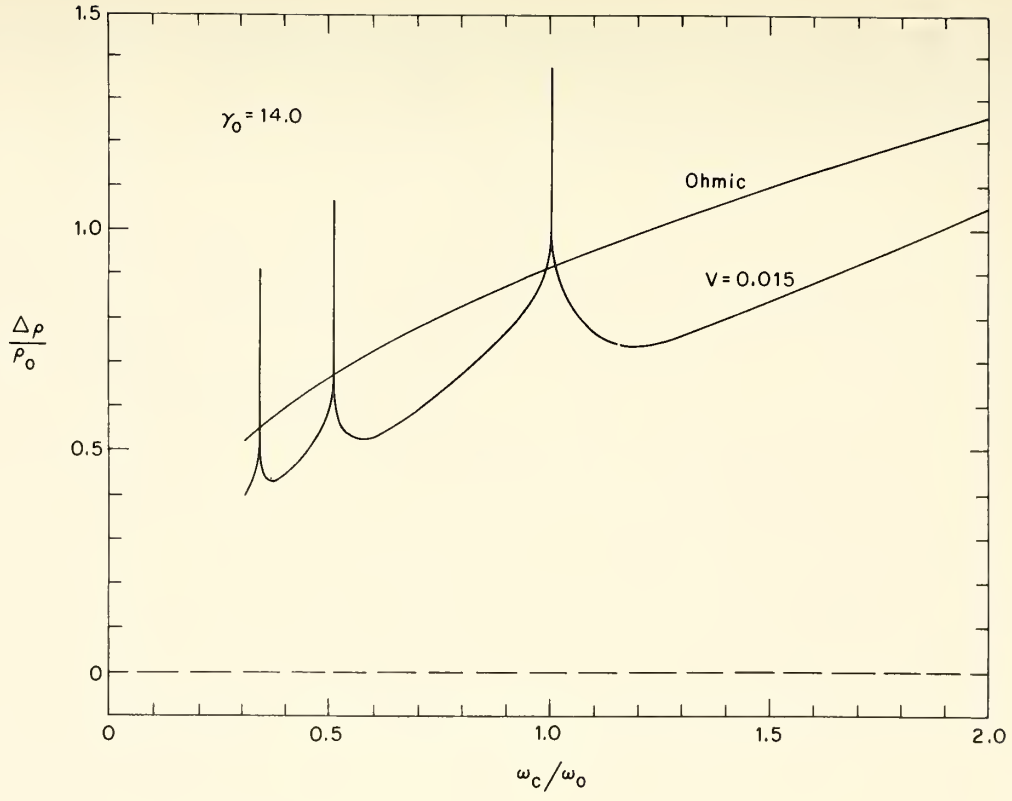

Figure 5. Ohmic and hot-electron magnetoresistance at $\gamma_o = 14.0$. $E_1 = 16.2$ eV; $e_{14} = 0.06$ coul/m^2; impurity concentration $= 4 \times 10^{13}$ cm^{-3}.

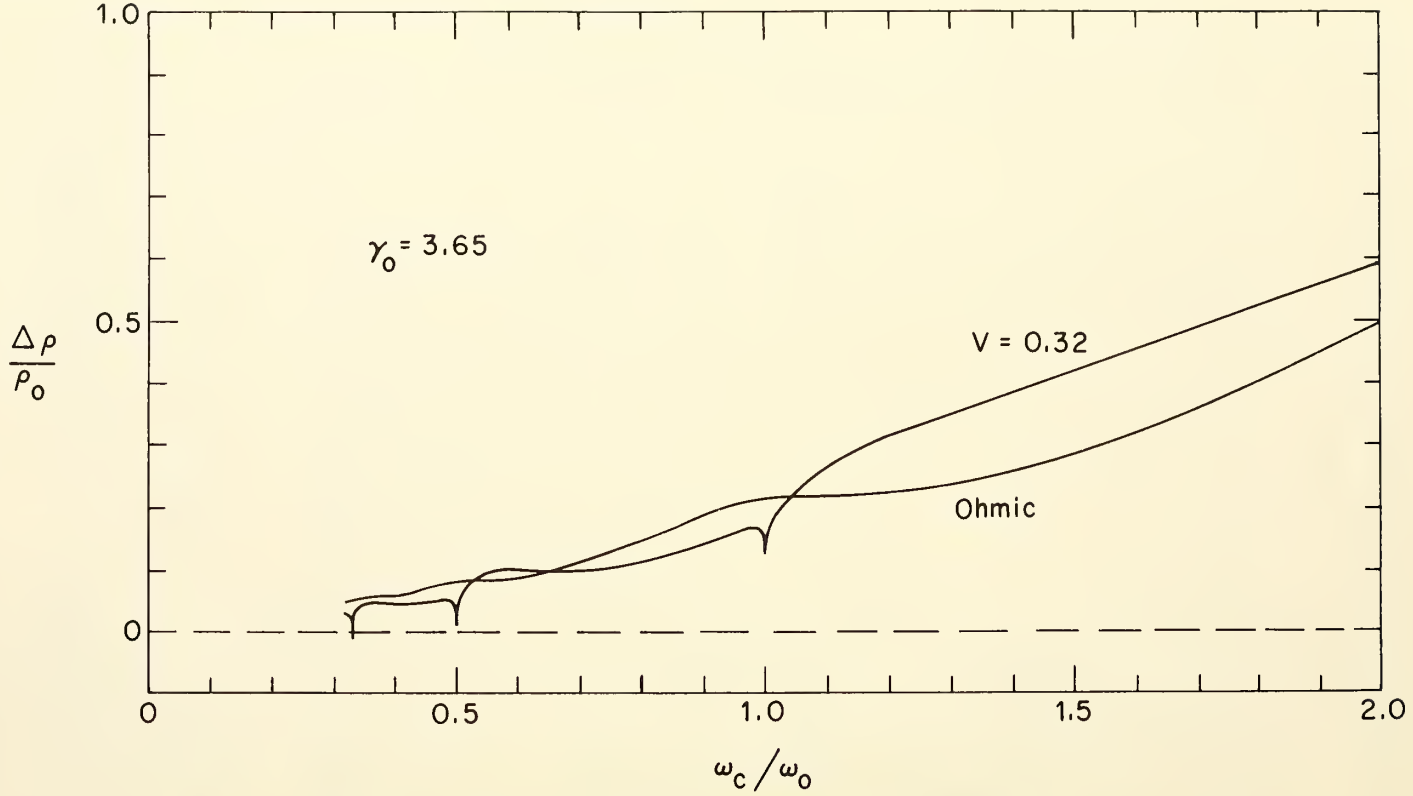

Figure 6. Ohmic and hot-electron magnetoresistance at $\gamma_o = 3.65$. $E_1 = 16.2$ eV; $e_{14} = 0.06$ coul/m^2; impurity concentration $= 4 \times 10^{13}$ cm^{-3}.

constant drift velocity) magnetoresistance curve is shown in each figure. The ohmic magnetoresistance is also shown for comparison. The dimensionless drift velocity v in the figures is defined in eq (34). With the n-InSb parameters given earlier, figure 5 corresponds to a temperature of 20 K, and v= 0.015 corresponds to $v_d = 0.11 \times 10^5$ m/sec, or E = 0.56 V/cm at B = 0. In figure 6, T_L = 77 K, and v= 0.32 corresponds to $v_d = 2.34 \times 10^5$ m/sec, or E = 42 V/cm at B = 0.

The most noticeable features of figures 5 and 6 are the sharp cusps precisely at the resonance points.[2, 3] These are energy-transfer effects superimposed on the background of momentum scattering which comprises the ohmic curves. Nonparabolicity of the conduction band and Landau level broadening would tend to round off the cusps. Figure 5, showing resonance maxima, is interesting in that it shows how the magnetophonon structure, absent in the ohmic case, can be brought out by application of nonohmic electric field or current. This effect has been exploited by Stradling and Wood,[38, 39] and by Aksel'rod et. al.[40] In figure 5 the scattering is dominated by ionized impurities, the resistivity of which varies essentially at $T_e^{-3/2}$ at both B = 0 and the quantum limit. Since the carrier temperature decreases to the lattice temperature at the resonance points,[2, 3] the magnetoresistance increases. Figure 6 shows resonance cusps which are minima. At this temperature, the optical phonon scattering dominates both the acoustic

phonon and impurity scattering. The minima are the result of the polar optical phonon scattering itself.

Low temperature, hot-electron longitudinal magnetoresistance in GaAs,[38] CdTe,[41] and InAs[42] do show maxima, in agreement with figure 5. However, InSb is an exception.[39, 40] The reason for this is not clear, although Yamada and Kurosawa[43] argued that a deviation from the Maxwellian distribution function could occur under hot-electron conditions and could account for the minima. Other, physical, arguments can also be advanced[39], to explain, in part, the minima. Further investigation is in order on this point.

In summary, the displaced-Maxwellian distribution function together with the simple model of the band structure cannot explain the observed longitudinal magnetoresistance resonance minima which frequently occur under ohmic conditions. This is in spite of the fact that the B = 0 and quantum limit mobilities largely agree, except for numerical factors, whether computed from the displaced-Maxwellian, or the Boltzmann equation without carrier-carrier scattering. It does not seem likely that the simple band structure assumed (parabolic and isotropic) is the principal reason for this failure, since apart from InSb, the semiconductors studied do not deviate greatly from the simple model (subsidiary valleys are surely not important at ohmic conditions), and yet exhibit near-resonance minima, or in

some cases an ambiguous situation. More likely, the lack of agreement lies with the displaced-Maxwellian itself. Several factors point to this. First of all, the original work of Gurevich and Firsov,[4] based on an approximate but direct solution of the Boltzmann equation, implies this. The materials studied experimentally have usually been fairly pure ($\leq 10^{15}$ cm^{-3} impurity concentration), whereas the displaced-Maxwellian implies a carrier concentration usually greater than about 10^{15} cm^{-3} (this figure is quite uncertain; it is a function of the strengths of all the carrier-phonon and carrier-impurity interactions, as well as of temperature, and electric and magnetic field strengths[18]). Further, the work of Yamada and Kurosawa[43] is a further indication of the inadequacy of the displaced-Maxwellian for the usual experimental situations. Nonohmic magnetophonon results of the displaced-Maxwellian, however, can be qualitatively satisfactory, as noted above and also previously.[3]

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21. The procedure to be used in averaging out the anisotropies of acoustic phonon propagation is not unique. For example, for deformation potential acoustic phonon scattering, and assuming equipartition ($\bar{n}_q \approx 1/\beta_o \hbar\omega_q$), one should average the quantity $u_\ell^{-2}(q)$ in eq (18) and $u_\ell^{-1}(q)$ in eq (19). However, at large values of E, equipartition becomes inaccurate. Further, $u_\alpha(q)$ also appears in the exponents, eqs (21) and (22). The fact of near elasticity for acoustic phonon collisions minimizes this complication. The "best" method would be to average the quantity $\bar{n}_q/u_\ell(q)$ in eq (18), and $\bar{n}_q$ in eq (19) for the defomation potential case, and still different quantities for the piezoelectric case, but this is extremely impractical computationally. We have adopted the much simpler, and usual, procedure of using a directionally averaged $u_\ell(q)$ wherever it occurs with the deformation potential

interaction, and a slightly more sophisticated procedure (see sections II. 1. 2 and III) for the piezoelectric case. Different averaging procedures typically give results differing by at most a few percent.

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The magnetophonon effect due to resonant interactions of charge carriers with optical phonons in nondegenerate polar semiconductors is studied when several elastic scattering mechanisms are simultaneously active also. The simple model of the band structure, and the displaced Maxwellian distribution function are used. Analytical expressions for drift mobility are given for B = 0 and the quantum limit, and are compared with the results of other techniques where possible. The numerical calculations show that whenever magnetophonon oscillations are seen in the ohmic regime, maxima occur near the resonance magnetic fields for all lattice temperature and strengths of scattering mechanisms which compete with the optical phonon scattering. This is in contrast with typical experimental findings, in which minima usually (but not always) appear near the resonance fields for ohmic, longitudinal, conditions. The explanation of the minima must therefore be sought in the use of a distribution function in which carrier-carrier scattering is less important.

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