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U.S. DEPARTMENT OF COMMERCE / National Bureau of Standards

Optical Radiation Measurements:

Tables of Diffraction Losses

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[3] Located at Boulder, Colorado 80302.
[4] Part of the Center for Building Technology.

Tables of Diffraction Losses

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Preface

This is the eighth issue of a series of Technical Notes entitled OPTICAL RADIATION MEASUREMENTS. The series will consist primarily of reports of progress in, or details of, research conducted in radiometry and photometry in the Optical Radiation Section of the Heat Division.

The level of presentation in OPTICAL RADIATION MEASUREMENTS will be directed at a general technical audience. The equivalent of an undergraduate degree in engineering or physics, plus familiarity with the basic concepts of radiometry and photometry [e.g., G. Bauer, Measurement of Optical Radiations (Focal Press, London, New York, 1965)], should be sufficient for understanding the vast majority of material in this series. Occasionally a more specialized background will be required. Even in such instances, however, a careful reading of the assumptions, approximations, and final conclusions should permit the non-specialist to understand the gist of the argument if not the details.

At times, certain commercial materials and equipment will be identified in this series in order to adequately specify the experimental procedure. In no case does such identification imply recommendation or endorsement by the National Bureau of Standards, nor does it imply that the material or equipment identified is necessarily the best available for the purpose.

Any suggestions readers may have to improve the utility of this series are welcome.

Henry J. Kostkowski, Chief
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Tables of Diffraction Losses*

W. B. Fussell

Tables of diffraction losses are given for a range of typical experimental geometries for wavelengths from 0.2 to 100 micrometers. The scaling relationships for the diffraction losses for varying wavelengths and geometries are also given, and sample calculations are presented. General formulas are given for the diffraction losses; the formulas are derived from the Kirchhoff scalar paraxial diffraction theory. The accuracy of the tabulated values is estimated.

Key words: Diffraction; diffraction losses; Fresnel diffraction; Kirchhoff diffraction theory; photometry; radiometry; scalar diffraction theory.

1. Introduction

With the improved precision and accuracy of radiometric measurements, diffraction losses have become significant. It is useful, therefore, to compute and tabulate diffraction losses for a range of typical geometries and wavelengths. The Kirchhoff scalar paraxial diffraction theory is used to calculate these losses. This is an approximate model which evaluates the phase relationships over the diffracting aperture (see fig. 1) for each elemental source area, for a given detection point and wavelength; the resulting complex number is then integrated over the source area and the magnitude of the sum indicates the relative spectral irradiance at the given detection point, compared with other detection points on the detector area. The model assumes: a., all source points radiate independently (that is, incoherent radiation); b., there are no polarization effects (that is, no vector effects); c., off-axis angles are small, and hence obliquity effects can be neglected. (Section 6 outlines the derivation of the equations used to compute the tables.)

The mathematical formulas used to compute the diffraction losses are refinements of the basic Fraunhofer on-axis diffraction formula (see Blevin[1] [1]). The tabulated on-axis diffraction losses are estimated to be accurate to within 10% mathematically; the off-axis diffraction losses are estimated to be accurate to within 20%. (If the physical realities of an experiment differ from the assumptions of the Kirchhoff model, there will be additional errors besides those due to the mathematical approximations used to compute the tables; however, it is expected that

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*Supported in part by the Calibration Coordination Group of the Department of Defense.

[1] Figures in brackets indicate the literature references at the end of this paper.

most situations in radiometry and photometry will be within the regime of the Kirchhoff model. Blevin [1], for example, finds excellent experimental agreement with the Kirchhoff model.) Thus, if it is desired to calculate the off-axis diffraction loss for a given experiment to within 0.1% of the spectral irradiance at the detector, then the geometry of the experiment should be such that the tabulated diffraction loss is less then 0.5% of the spectral irradiance at the detector, since an error of 20% of 0.5% is equal to 0.1%.

The geometries and wavelengths selected for the diffraction loss tables are:

a., wavelengths from 0.2 to 100 micrometers;

b., source (or detector) diameters from 0.5 to 5 cm;

c., source (or detector)-to-aperture distances from 5 to 20 cm;

d., aperture diameters from 0.005 to 0.5 cm.

The geometry and terminology used in the diffraction loss tables is shown in figure 1.

In general, if the circumference of the circle produced by projecting the aperture from every point on the detector (the geometry in this report is assumed to be circularly symmetric in all cases), onto the plane of the source, lies within the source, then the radiation incident on the detector is proportional to the <u>source radiance</u> (less diffraction losses). On the other hand, if the circumference of the circle produced by projecting the aperture from every point of the source, onto the plane of the detector, lies within the detector, then the radiation incident on the detector is the <u>total source radiation through the aperture</u> (less diffraction losses). (The diffraction losses for a given configuration are identical, whether the source is treated as a detector and the detector as a source, or vice versa; this is sometimes a conceptual advantage in that it transforms a source radiance measurement into a total aperture radiation measurement.) In this report, the source radiance geometry will always be meant unless it is explicitly stated that the total aperture radiation geometry is under consideration.

For a given geometry, the diffraction loss in the plane of the detector is least on the axis; the diffraction loss increases steadily as the distance from the axis increases (see sec. 6). Therefore, the diffraction loss realized with a circular detector increases steadily as the detector radius increases. The diffraction losses listed in the following tables are for the on-axis case (the "point" detector), and also for the case of a detector that sees 90% of the diameter of the source (the radius of such a detector is designated x_{max}). These diffraction losses, designated $\bar{E}'_{min}$ and $\bar{E}'_{max}$ respectively, bracket the loss for detector radii between zero and x_{max} to within roughly ±20% for geometries where the aperture diameter is much less than the source diameter, and more accurately for ratios of the aperture diameter to the source diameter

larger than 0.1 (see the end of sec. 7).

2. Tables of Diffraction Losses as Functions of Wavelength and Geometry

Terminology:

λ is the wavelength in micrometers.

d is the source diameter in cm (or the detector diameter, for a total aperture radiation measurement; see sec. 1).

$\underline{b}$ is the source-aperture distance in cm.

D is the aperture diameter in cm.

v is the dimensionless quantity $\pi Dd(2\underline{b}\lambda)^{-1}$.

E'_{min} is the diffraction loss[1] for a point detector on-axis whose distance from the aperture is at least 10 times the source-aperture distance.

$\bar{E}'_{max}$ is the diffraction loss[1] averaged over the area of a circular detector of radius x_{max} (see sec. 1 and fig. 1); the radius x_{max} is defined to be that radius for which the field of view through the aperture covers the portion of the source disc whose diameter is 0.9 the source diameter; the distance of the detector from the aperture must be at least 10 times the source-aperture distance.

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[1]The diffraction losses E'_{min} and $\bar{E}'_{max}$ are given as a percentage of the irradiance that would be present at the detector in the absence of diffraction. The mathematical formulas used to compute E'_{min} and $\bar{E}'_{max}$ are given in section 6. Upper bounds for the errors in E'_{min} and $\bar{E}'_{max}$ can be computed using the dimensionless quantity v, and formulas for such computations are given in section 7.

$\underline{\lambda} = \underline{0.2}$ (μm)

$\underline{d} = \underline{0.5}$ (cm)

$\underline{b}$ (cm)	$\underline{D}$ = (cm)	.005	.01	.02	.05	.1	.2	.5
5	v	39.3	78.5	157	393	785	1570	3930
	(%)E'_{min}	1.62	0.81	0.41	0.16	0.08	.05	-
	(%)$\overline{E}'_{max}$	2.59	1.27	0.62	0.24	0.12	.06	-
10	v	19.6	39.3	78.5	196	393	785	1960
	E'_{min}	3.24	1.62	0.81	0.33	0.17	0.10	-
	$\overline{E}'_{max}$	5.19	2.55	1.24	0.47	0.23	0.13	-
20	v	9.82	19.6	39.3	98.2	196	393	982
	E'_{min}	6.49	3.24	1.62	0.66	0.34	0.19	-
	$\overline{E}'_{max}$	<18*	5.09	2.48	0.95	0.47	0.25	-

$\underline{d} = \underline{1}$ (cm)

$\underline{b}$ (cm)								
5	v	78.5	157	314	785	1570	3140	7850
	(%)E'_{min}	0.81	0.41	0.20	0.08	0.04	0.02	0.01
	(%)$\overline{E}'_{max}$	1.31	0.65	0.32	0.12	0.06	0.03	0.01
10	v	39.3	78.5	157	393	785	1570	3930
	E'_{min}	1.62	0.81	0.41	0.16	0.08	0.04	0.02
	$\overline{E}'_{max}$	2.62	1.30	0.64	0.25	0.12	0.06	0.03
20	v	19.6	39.3	78.5	196	393	785	1960
	E'_{min}	3.24	1.62	0.81	0.33	0.16	0.08	0.04
	$\overline{E}'_{max}$	5.24	2.59	1.27	0.49	0.24	0.12	0.06

*(Note: These values are upper bounds.)

$\underline{\lambda} = \underline{0.2}$ (μm)

$\underline{d} = \underline{2}$ (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
	v	157	314	628	1570	3140	6280	15700
5	(%)E'_{min}	0.41	0.20	0.10	0.04	0.02	0.01	0.00
	(%)$\overline{E}'_{max}$	0.66	0.33	0.16	0.06	0.03	0.01	0.01
	v	78.5	157	314	785	1570	3140	7850
10	E'_{min}	0.81	0.41	0.20	0.08	0.04	0.02	0.01
	$\overline{E}'_{max}$	1.32	0.66	0.32	0.13	0.06	0.03	0.01
	v	39.3	78.5	157	393	785	1570	3930
20	E'_{min}	1.62	0.81	0.41	0.16	0.08	0.04	0.02
	$\overline{E}'_{max}$	2.64	1.31	0.65	0.25	0.12	0.06	0.02

$\underline{d} = \underline{5}$ (cm)

b (cm)								
	v	393	785	1570	3930	7850	15700	39300
5	(%)E'_{min}	0.16	0.08	0.04	0.02	0.01	0.00	0.00
	(%)$\overline{E}'_{max}$	0.26	0.13	0.07	0.03	0.01	0.01	0.00
	v	196	393	785	1960	3930	7850	19600
10	E'_{min}	0.32	0.16	0.08	0.03	0.02	0.01	0.00
	$\overline{E}'_{max}$	0.53	0.26	0.13	0.05	0.03	0.01	0.00
	v	98.2	196	393	982	1960	3930	9820
20	E'_{min}	0.65	0.32	0.16	0.06	0.03	0.02	0.01
	$\overline{E}'_{max}$	1.06	0.53	0.26	0.10	0.05	0.02	0.01

λ = 0.5 (μm)

d = 0.5 (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
	v	15.7	31.4	62.8	157	314	628	1570
5	(%)E'_{min}	4.05	2.03	1.01	0.41	0.21	0.12	-
	(%)E'_{max}	6.48	3.18	1.55	0.59	0.29	0.16	-
	v	7.85	15.7	31.4	78.5	157	314	785
10	E'_{min}	8.11	4.05	2.03	0.82	0.42	0.24	-
	E'_{max}	<20*	6.37	3.10	1.19	0.59	0.32	-
	v	3.93	7.85	15.7	39.3	78.5	157	393
20	E'_{min}	-	8.11	4.06	1.64	0.84	0.48	-
	E'_{max}	-	<20*	6.19	2.37	1.17	0.64	-

d = 1 (cm)

b (cm)								
	v	31.4	62.8	126	314	628	1260	3140
5	(%)E'_{min}	2.03	1.01	0.51	0.20	0.10	0.05	0.03
	(%)E'_{max}	3.28	1.62	0.80	0.31	0.15	0.07	0.03
	v	15.7	31.4	62.8	157	314	628	1570
10	E'_{min}	4.05	2.03	1.01	0.41	0.20	0.11	0.05
	E'_{max}	6.55	3.24	1.59	0.61	0.30	0.15	0.07
	v	7.85	15.7	31.4	78.5	157	314	785
20	E'_{min}	8.11	4.05	2.03	0.81	0.41	0.21	0.11
	E'_{max}	<20*	6.48	3.18	1.23	0.59	0.29	0.14

*(Note: These values are upper bounds.)

λ = 0.5 (μm)

d = 2 (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
	v	62.8	126	251	628	1260	2510	6280
5	(%)E'_{min}	1.01	0.51	0.25	0.10	0.05	0.03	0.01
	(%)$\bar{E}'_{max}$	1.65	0.82	0.41	0.16	0.08	0.04	0.01
	v	31.4	62.8	126	314	628	1260	3140
10	E'_{min}	2.03	1.01	0.51	0.20	0.10	0.05	0.02
	$\bar{E}'_{max}$	3.30	1.64	0.81	0.32	0.15	0.07	0.03
	v	15.7	31.4	62.8	157	314	628	1570
20	E'_{min}	4.05	2.03	1.01	0.41	0.20	0.10	0.04
	$\bar{E}'_{max}$	6.59	3.28	1.62	0.63	0.31	0.15	0.06

d = 5 (cm)

b (cm)								
	v	157	314	628	1570	3140	6280	15700
5	(%)E'_{min}	0.41	0.20	0.10	0.04	0.02	0.01	0.00
	(%)$\bar{E}'_{max}$	0.66	0.33	0.16	0.06	0.03	0.02	0.01
	v	78.5	157	314	785	1570	3140	7850
10	E'_{min}	0.81	0.41	0.20	0.08	0.04	0.02	0.01
	$\bar{E}'_{max}$	1.32	0.66	0.33	0.13	0.06	0.03	0.01
	v	39.3	78.5	157	393	785	1570	3930
20	E'_{min}	1.62	0.81	0.41	0.16	0.08	0.04	0.02
	$\bar{E}'_{max}$	2.65	1.32	0.66	0.26	0.13	0.06	0.02

$\underline{\lambda} = \underline{1}$ (μm)

$\underline{d} = \underline{0.5}$ (cm)

$\underline{b}$ (cm)	$\underline{D}$ = (cm)	.005	.01	.02	.05	.1	.2	.5
	v	7.85	15.7	31.4	78.5	157	314	785
5	(%)E'_{min}	8.11	4.05	2.03	0.82	0.42	0.24	-
	(%)$\overline{E}'_{max}$	<20*	6.37	3.10	1.19	0.59	0.32	-
	v	3.93	7.85	15.7	39.3	78.5	157	393
10	E'_{min}	-	8.11	4.06	1.64	0.84	0.48	-
	$\overline{E}'_{max}$	-	<20*	6.19	2.37	1.17	0.64	-
	v	1.96	3.93	7.85	19.6	39.3	78.5	196
20	E'_{min}	-	-	8.12	3.28	1.69	0.96	-
	$\overline{E}'_{max}$	-	-	<20*	4.74	2.34	1.27	-

$\underline{d} = \underline{1}$ (cm)

$\underline{b}$ (cm)								
	v	15.7	31.4	62.8	157	314	628	1570
5	(%)E'_{min}	4.05	2.03	1.01	0.41	0.20	0.11	0.05
	(%)$\overline{E}'_{max}$	6.55	3.24	1.59	0.61	0.30	0.15	0.07
	v	7.85	15.7	31.4	78.5	157	314	785
10	E'_{min}	8.11	4.05	2.03	0.81	0.41	0.21	0.11
	$\overline{E}'_{max}$	<20*	6.48	3.18	1.23	0.59	0.29	0.14
	v	3.93	7.85	15.7	39.3	78.5	157	393
20	E'_{min}	-	8.11	4.05	1.63	0.82	0.42	0.22
	$\overline{E}'_{max}$	-	<20*	6.37	2.45	1.19	0.59	0.28

*(Note: These values are upper bounds.)

$\underline{\lambda} = \underline{1}$ (μm)

$\underline{d} = \underline{2}$ (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
	v	31.4	62.8	126	314	628	1260	3140
5	$(\%)E'_{min}$	2.03	1.01	0.51	0.20	0.10	0.05	0.02
	$(\%)\bar{E}'_{max}$	3.30	1.64	0.81	0.32	0.15	0.07	0.03
	v	15.7	31.4	62.8	157	314	628	1570
10	E'_{min}	4.05	2.03	1.01	0.41	0.20	0.10	0.04
	$\bar{E}'_{max}$	6.59	3.28	1.62	0.63	0.31	0.15	0.06
	v	7.85	15.7	31.4	78.5	157	314	785
20	E'_{min}	8.11	4.05	2.03	0.81	0.41	0.20	0.09
	$\bar{E}'_{max}$	<20*	6.55	3.24	1.26	0.61	0.30	0.12

$\underline{d} = \underline{5}$ (cm)

b (cm)								
	v	78.5	157	314	785	1570	3140	7850
5	$(\%)E'_{min}$	0.81	0.41	0.20	0.08	0.04	0.02	0.01
	$(\%)\bar{E}'_{max}$	1.32	0.66	0.33	0.13	0.06	0.03	0.01
	v	39.3	78.5	157	393	785	1570	3930
10	E'_{min}	1.62	0.81	0.41	0.16	0.08	0.04	0.02
	$\bar{E}'_{max}$	2.65	1.32	0.66	0.26	0.13	0.06	0.02
	v	19.6	39.3	78.5	196	393	785	1960
20	E'_{min}	3.24	1.62	0.81	0.32	0.16	0.08	0.03
	$\bar{E}'_{max}$	5.29	2.64	1.31	0.52	0.25	0.12	0.05

*(Note: These values are upper bounds.)

$\underline{\lambda}$ = $\underline{2}$ (μm)

$\underline{d}$ = $\underline{0.5}$ (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
	v	3.93	7.85	15.7	39.3	78.5	157	393
5	(%)E'_{min}	-	8.11	4.06	1.64	0.84	0.48	-
	(%)E'_{max}	-	<20*	6.19	2.37	1.17	0.64	-
	v	1.96	3.93	7.85	19.6	39.3	78.5	196
10	E'_{min}	-	-	8.12	3.28	1.69	0.96	-
	E'_{max}	-	-	<20*	4.74	2.34	1.27	-
	v	0.982	1.96	3.93	9.82	19.6	39.3	98.2
20	E'_{min}	-	-	-	6.55	3.38	1.93	-
	E'_{max}	-	-	-	<18*	4.68	2.55	-

$\underline{d}$ = $\underline{1}$ (cm)

b (cm)								
	v	7.85	15.7	31.4	78.5	157	314	785
5	(%)E'_{min}	8.11	4.05	2.03	0.81	0.41	0.21	0.11
	(%)E'_{max}	<20*	6.48	3.18	1.23	0.59	0.29	0.14
	v	3.93	7.85	15.7	39.3	78.5	157	393
10	E'_{min}	-	8.11	4.05	1.63	0.82	0.42	0.22
	E'_{max}	-	<20*	6.37	2.45	1.19	0.59	0.28
	v	1.96	3.93	7.85	19.6	39.3	78.5	196
20	E'_{min}	-	-	8.11	3.25	1.64	0.84	0.43
	E'_{max}	-	-	<20*	4.90	2.37	1.17	0.56

*(Note: These values are upper bounds.)

$\underline{\lambda} = \underline{2}$ (μm)

$\underline{d} = \underline{2}$ (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
	v	15.7	31.4	62.8	157	314	628	1570
5	(%)E'_{min}	4.05	2.03	1.01	0.41	0.20	0.10	0.04
	(%)$\bar{E}'_{max}$	6.59	3.28	1.62	0.63	0.31	0.15	0.06
	v	7.85	15.7	31.4	78.5	157	314	785
10	E'_{min}	8.11	4.05	2.03	0.81	0.41	0.20	0.09
	$\bar{E}'_{max}$	<20*	6.55	3.24	1.26	0.61	0.30	0.12
	v	3.93	7.85	15.7	39.3	78.5	157	393
20	E'_{min}	-	8.11	4.05	1.62	0.81	0.41	0.17
	$\bar{E}'_{max}$	-	<20*	6.48	2.53	1.23	0.59	0.24

$\underline{d} = \underline{5}$ (cm)

b (cm)								
	v	39.3	78.5	157	393	785	1570	3930
5	(%)E'_{min}	1.62	0.81	0.41	0.16	0.08	0.04	0.02
	(%)$\bar{E}'_{max}$	2.65	1.32	0.66	0.26	0.13	0.06	0.02
	v	19.6	39.3	78.5	196	393	785	1960
10	E'_{min}	3.24	1.62	0.81	0.32	0.16	0.08	0.03
	$\bar{E}'_{max}$	5.29	2.64	1.31	0.52	0.25	0.12	0.05
	v	9.82	19.6	39.3	98.2	196	393	982
20	E'_{min}	6.48	3.24	1.62	0.65	0.32	0.16	0.07
	$\bar{E}'_{max}$	<18*	5.28	2.63	1.04	0.51	0.25	0.09

*(Note: These values are upper bounds.)

$\underline{\lambda} = \underline{5}$ (μm)

$\underline{d} = \underline{0.5}$ (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
	v	1.57	3.14	6.28	15.7	31.4	62.8	157
5	(%)E'_{min}	-	-	10.1	4.09	2.11	1.21	-
	(%)$\overline{E}'_{max}$	-	-	<23*	5.93	2.93	1.59	-
	v	0.785	1.57	3.14	7.85	15.7	31.4	78.5
10	E'_{min}	-	-	-	8.19	4.22	2.41	-
	$\overline{E}'_{max}$	-	-	-	<20*	5.85	3.19	-
	v	0.393	0.785	1.57	3.93	7.85	15.7	39.3
20	E'_{min}	-	-	-	-	8.44	4.82	-
	$\overline{E}'_{max}$	-	-	-	-	<20*	6.37	-

$\underline{d} = \underline{1}$ (cm)

b (cm)								
	v	3.14	6.28	12.6	31.4	62.8	126	314
5	(%)E'_{min}	-	10.1	5.07	2.03	1.02	0.53	0.27
	(%)$\overline{E}'_{max}$	-	<23*	7.96	3.06	1.48	0.73	0.35
	v	1.57	3.14	6.28	15.7	31.4	62.8	157
10	E'_{min}	-	-	10.1	4.06	2.05	1.06	0.54
	$\overline{E}'_{max}$	-	-	<23*	6.13	2.96	1.46	0.70
	v	0.785	1.57	3.14	7.85	15.7	31.4	78.5
20	E'_{min}	-	-	-	8.13	4.09	2.11	1.08
	$\overline{E}'_{max}$	-	-	-	<20*	5.93	2.93	1.39

*(Note: These values are upper bounds.)

$\underline{\lambda} = \underline{5}$ (μm)

$\underline{d} = \underline{2}$ (cm)

$\underline{b}$ (cm)	$\underline{D}$ = (cm)	.005	.01	.02	.05	.1	.2	.5
5	v	6.28	12.6	25.1	62.8	126	251	628
	(%)E'_{min}	10.1	5.07	2.53	1.01	0.51	0.26	0.11
	(%)$\overline{E}'_{max}$	<23*	8.19	4.05	1.58	0.77	0.37	0.15
10	v	3.14	6.28	12.6	31.4	62.8	126	314
	E'_{min}	-	10.1	5.07	2.03	1.02	0.51	0.22
	$\overline{E}'_{max}$	-	<23*	8.11	3.16	1.53	0.74	0.30
20	v	1.57	3.14	6.28	15.7	31.4	62.8	157
	E'_{min}	-	-	10.1	4.06	2.03	1.02	0.43
	$\overline{E}'_{max}$	-	-	<23*	6.32	3.06	1.48	0.59

$\underline{d} = \underline{5}$ (cm)

$\underline{b}$ (cm)								
5	v	15.7	31.4	62.8	157	314	628	1570
	(%)E'_{min}	4.05	2.03	1.01	0.41	0.20	0.10	0.04
	(%)$\overline{E}'_{max}$	6.61	3.30	1.64	0.65	0.32	0.15	0.06
10	v	7.85	15.7	31.4	78.5	157	314	785
	E'_{min}	8.11	4.05	2.03	0.81	0.41	0.20	0.08
	$\overline{E}'_{max}$	<20*	6.60	3.28	1.30	0.64	0.31	0.12
20	v	3.93	7.85	15.7	39.3	78.5	157	393
	E'_{min}	-	8.11	4.05	1.62	0.81	0.41	0.16
	E'_{max}	-	<20*	6.57	2.59	1.27	0.62	0.24

*(Note : These values are upper bounds.)

$\underline{\lambda} = \underline{10}$ (μm)

$\underline{d} = \underline{0.5}$ (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
5	v	0.785	1.57	3.14	7.85	15.7	31.4	78.5
	(%)E'_{min}	-	-	-	8.19	4.22	2.41	-
	(%)$\bar{E}'_{max}$	-	-	-	<20*	5.85	3.19	-
10	v	0.393	0.785	1.57	3.93	7.85	15.7	39.3
	E'_{min}	-	-	-	-	8.44	4.82	-
	$\bar{E}'_{max}$	-	-	-	-	<20*	6.37	-
20	v	0.196	0.393	0.785	1.96	3.93	7.85	19.6
	E'_{min}	-	-	-	-	-	9.65	-
	$\bar{E}'_{max}$	-	-	-	-	-	<20*	-

$\underline{d} = \underline{1}$ (cm)

b (cm)		.005	.01	.02	.05	.1	.2	.5
5	v	1.57	3.14	6.28	15.7	31.4	62.8	157
	(%)E'_{min}	-	-	10.1	4.06	2.05	1.06	0.54
	(%)$\bar{E}'_{max}$	-	-	<23*	6.13	2.96	1.46	0.70
10	v	0.785	1.57	3.14	7.85	15.7	31.4	78.5
	E'_{min}	-	-	-	8.13	4.09	2.11	1.08
	$\bar{E}'_{max}$	-	-	-	<20*	5.93	2.93	1.39
20	v	0.393	0.785	1.57	3.93	7.85	15.7	39.3
	E'_{min}	-	-	-	-	8.19	4.22	2.16
	$\bar{E}'_{max}$	-	-	-	-	<20*	5.85	2.78

*(Note : These values are upper bounds.)

λ = 10 (μm)

d = 2 (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
	v	3.14	6.28	12.6	31.4	62.8	126	314
5	(%)E'_{min}	-	10.1	5.07	2.03	1.02	0.51	0.22
	(%)E'_{max}	-	<23*	8.11	3.16	1.53	0.74	0.30
	v	1.57	3.14	6.28	15.7	31.4	62.8	157
10	E'_{min}	-	-	10.1	4.06	2.03	1.02	0.43
	E'_{max}	-	-	<23*	6.32	3.06	1.48	0.59
	v	0.785	1.57	3.14	7.85	15.7	31.4	78.5
20	E'_{min}	-	-	-	8.11	4.06	2.05	0.86
	E'_{max}	-	-	-	<20*	6.13	2.96	1.18

d = 5 (cm)

b (cm)								
	v	7.85	15.7	31.4	78.5	157	314	785
5	(%)E'_{min}	8.11	4.05	2.03	0.81	0.41	0.21	0.08
	(%)E'_{max}	<20*	6.60	3.28	1.30	0.64	0.31	0.12
	v	3.93	7.85	15.7	39.3	78.5	157	393
10	E'_{min}	-	8.11	4.05	1.62	0.81	0.41	0.16
	E'_{max}	-	<20*	6.57	2.59	1.27	0.62	0.24
	v	1.96	3.93	7.85	19.6	39.3	78.5	196
20	E'_{min}	-	-	8.11	3.24	1.62	0.81	0.33
	E'_{max}	-	-	<20*	5.19	2.55	1.24	0.47

*(Note : These values are upper bounds.)

$\underline{\lambda} = \underline{20}$ (μm)

$\underline{d} = \underline{0.5}$ (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
5	v	0.393	0.785	1.57	3.93	7.85	15.7	39.3
	$(\%)E'_{min}$	-	-	-	-	8.44	4.82	-
	$(\%)\bar{E}'_{max}$	-	-	-	-	<20*	6.37	-
10	v	0.196	0.393	0.785	1.96	3.93	7.85	19.6
	E'_{min}	-	-	-	-	-	9.65	-
	$\bar{E}'_{max}$	-	-	-	-	-	<20*	-
20	v	0.098	0.196	0.393	0.982	1.96	3.93	9.82
	E'_{min}	-	-	-	-	-	-	-
	$\bar{E}'_{max}$	-	-	-	-	-	-	-

$\underline{d} = \underline{1}$ (cm)

b (cm)								
5	v	0.785	1.57	3.14	7.85	15.7	31.4	78.5
	$(\%)E'_{min}$	-	-	-	8.13	4.09	2.11	1.08
	$(\%)\bar{E}'_{max}$	-	-	-	<20*	5.93	2.93	1.39
10	v	0.393	0.785	1.57	3.93	7.85	15.7	39.3
	E'_{min}	-	-	-	-	8.19	4.22	2.16
	$\bar{E}'_{max}$	-	-	-	-	<20*	5.85	2.78
20	v	0.196	0.393	0.785	1.96	3.93	7.85	19.6
	E'_{min}	-	-	-	-	-	8.44	4.32
	$\bar{E}'_{max}$	-	-	-	-	-	<20*	5.56

*(Note : These values are upper bounds.)

$\underline{\lambda} = \underline{20}$ (μm)

$\underline{d} = \underline{2}$ (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
5	v	1.57	3.14	6.28	15.7	31.4	62.8	157
	(%)E'_{min}	-	-	10.1	4.06	2.03	1.02	0.43
	(%)$\bar{E}'_{max}$	-	-	<23*	6.32	3.06	1.48	0.59
10	v	0.785	1.57	3.14	7.85	15.7	31.4	78.5
	E'_{min}	-	-	-	8.11	4.06	2.05	0.86
	$\bar{E}'_{max}$	-	-	-	<20*	6.13	2.96	1.18
20	v	0.393	0.785	1.57	3.93	7.85	15.7	39.3
	E'_{min}	-	-	-	-	8.13	4.09	1.73
	$\bar{E}'_{max}$	-	-	-	-	<20*	5.93	2.36

$\underline{d} = \underline{5}$ (cm)

b (cm)								
5	v	3.93	7.85	15.7	39.3	78.5	157	393
	(%)E'_{min}	-	8.11	4.05	1.62	0.81	0.41	0.16
	(%)$\bar{E}'_{max}$	-	<20*	6.57	2.59	1.27	0.62	0.24
10	v	1.96	3.93	7.85	19.6	39.3	78.5	196
	E'_{min}	-	-	8.11	3.24	1.62	0.81	0.33
	$\bar{E}'_{max}$	-	-	<20*	5.19	2.55	1.24	0.47
20	v	0.982	1.96	3.93	9.82	19.6	39.3	98.2
	E'_{min}	-	-	-	6.49	3.24	1.62	0.66
	$\bar{E}'_{max}$	-	-	-	<18*	5.09	2.48	0.95

*(Note : These values are upper bounds.)

λ = 50 (μm)

d = 0.5 (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
5	v	0.157	0.314	0.628	1.57	3.14	6.28	15.7
	(%)E'_{min}	-	-	-	-	-	12.1	-
	(%)$\overline{E}'_{max}$	-	-	-	-	-	<23*	-
10	v	0.079	0.157	0.314	0.785	1.57	3.14	7.85
	E'_{min}	-	-	-	-	-	-	-
	$\overline{E}'_{max}$	-	-	-	-	-	-	-
20	v	0.039	0.079	0.157	0.393	0.785	1.57	3.93
	E'_{min}	-	-	-	-	-	-	-
	$\overline{E}'_{max}$	-	-	-	-	-	-	-

d = 1 (cm)

b (cm)								
5	v	0.314	0.628	1.26	3.14	6.28	12.6	31.4
	(%)E'_{min}	-	-	-	-	10.2	5.28	2.70
	(%)$\overline{E}'_{max}$	-	-	-	-	<23*	7.32	3.48
10	v	0.157	0.314	0.628	1.57	3.14	6.28	15.7
	E'_{min}	-	-	-	-	-	10.6	5.40
	$\overline{E}'_{max}$	-	-	-	-	-	<23*	6.95
20	v	0.079	0.157	0.314	0.785	1.57	3.14	7.85
	E'_{min}	-	-	-	-	-	-	10.8
	$\overline{E}'_{max}$	-	-	-	-	-	-	<20*

*(Note : These values are upper bounds.)

λ = 50 (μm)

d = 2 (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
5	v	0.628	1.26	2.51	6.28	12.6	25.1	62.8
	(%)E'_{min}	-	-	-	10.1	5.08	2.56	1.08
	(%)$\bar{E}'_{max}$	-	-	-	<23*	7.66	3.70	1.48
10	v	0.314	0.628	1.26	3.14	6.28	12.6	31.4
	E'_{min}	-	-	-	-	10.2	5.12	2.16
	$\bar{E}'_{max}$	-	-	-	-	<23*	7.41	2.96
20	v	0.157	0.314	0.628	1.57	3.14	6.28	15.7
	E'_{min}	-	-	-	-	-	10.2	4.32
	$\bar{E}'_{max}$	-	-	-	-	-	<23*	5.91

d = 5 (cm)

b (cm)								
5	v	1.57	3.14	6.28	15.7	31.4	62.8	157
	(%)E'_{min}	-	-	10.1	4.05	2.03	1.01	0.41
	(%)$\bar{E}'_{max}$	-	-	<23*	6.48	3.18	1.55	0.59
10	v	0.785	1.57	3.14	7.85	15.7	31.4	78.5
	E'_{min}	-	-	-	8.11	4.05	2.03	0.82
	$\bar{E}'_{max}$	-	-	-	<20*	6.37	3.10	1.19
20	v	0.393	0.785	1.57	3.93	7.85	15.7	39.3
	E'_{min}	-	-	-	-	8.11	4.06	1.64
	$\bar{E}'_{max}$	-	-	-	-	<20*	6.19	2.37

*(Note : These values are upper bounds.)

$\underline{\lambda} = \underline{100}$ (μm)

$\underline{d} = \underline{0.5}$ (cm)

$\underline{b}$ (cm)	$\underline{D}$ = (cm)	.005	.01	.02	.05	.1	.2	.5
5	v	0.079	0.157	0.314	0.785	1.57	3.14	7.85
	(%)E'_{min}	-	-	-	-	-	-	-
	(%)$\bar{E}'_{max}$	-	-	-	-	-	-	-
10	v	0.039	0.079	0.157	0.393	0.785	1.57	3.93
	E'_{min}	-	-	-	-	-	-	-
	$\bar{E}'_{max}$	-	-	-	-	-	-	-
20	v	0.020	0.039	0.079	0.196	0.393	0.785	1.96
	E'_{min}	-	-	-	-	-	-	-
	$\bar{E}'_{max}$	-	-	-	-	-	-	-

$\underline{d} = \underline{1}$ (cm)

$\underline{b}$ (cm)								
5	v	0.157	0.314	0.628	1.57	3.14	6.28	15.7
	(%)E'_{min}	-	-	-	-	-	10.6	5.40
	(%)$\bar{E}'_{max}$	-	-	-	-	-	<23*	6.95
10	v	0.079	0.157	0.314	0.785	1.57	3.14	7.85
	E'_{min}	-	-	-	-	-	-	10.8
	$\bar{E}'_{max}$	-	-	-	-	-	-	<20*
20	v	0.039	0.079	0.157	0.393	0.785	1.57	3.93
	E'_{min}	-	-	-	-	-	-	-
	$\bar{E}'_{max}$	-	-	-	-	-	-	-

*(Note : These values are upper bounds.)

$\underline{\lambda} = \underline{100}$ (μm)

$\underline{d} = \underline{2}$ (cm)

b (cm)	D = (cm)	.005	.01	.02	.05	.1	.2	.5
5	v	0.314	0.628	1.26	3.14	6.28	12.6	31.4
	(%)E'_{min}	-	-	-	-	10.2	5.12	2.16
	(%)$\bar{E}'_{max}$	-	-	-	-	<23*	<16*	2.96
10	v	0.157	0.314	0.628	1.57	3.14	6.28	15.7
	E'_{min}	-	-	-	-	-	10.2	4.32
	$\bar{E}'_{max}$	-	-	-	-	-	<23*	5.91
20	v	0.079	0.157	0.314	0.785	1.57	3.14	7.85
	E'_{min}	-	-	-	-	-	-	8.65
	$\bar{E}'_{max}$	-	-	-	-	-	-	<20*

$\underline{d} = \underline{5}$ (cm)

b (cm)								
5	v	0.785	1.57	3.14	7.85	15.7	31.4	78.5
	(%)E'_{min}	-	-	-	8.11	4.05	2.03	0.82
	(%)E'_{max}	-	-	-	<20*	6.37	3.10	1.19
10	v	0.393	0.785	1.57	3.93	7.85	15.7	39.3
	E'_{min}	-	-	-	-	8.11	4.06	1.64
	E'_{max}	-	-	-	-	<20*	6.19	2.37
20	v	0.196	0.393	0.785	1.96	3.93	7.85	19.6
	E'_{min}	-	-	-	-	-	8.12	3.28
	E'_{max}	-	-	-	-	-	<20*	4.74

*(Note: These values are upper bounds.)

3. Scaling the Diffraction Loss Tables

The range of the diffraction loss tables can be extended by using scaling relationships (provided that the errors in the formulas used to compute the diffraction losses do not become excessive in the extended range). It is clear from the formulas in section 6 for the quantities E'_{min} and $\bar{E}'_{max}$ given in the tables, that the scaling relationships are simple only if the ratio of the aperture diameter to the source diameter, Dd^{-1}, is constant. Subject to this condition, the scaling relationships are as follows:

Quantity	Scales as (constant Dd^{-1})
E'_{min}, $\bar{E}'_{max}$	λ, $\underline{b}$, D^{-2} (or d^{-2}) ;
Upper Bounds	$\lambda^{0.5}$, $\underline{b}^{0.5}$, D^{-1} (or d^{-1}).

(The upper bounds are the quantity denoted $E(v,v,0)$ in sec. 6.)

4. Effective Wavelengths to Use in the Diffraction Loss Tables

The effective wavelength, λ^{-}, for computing the diffraction loss for a given experimental geometry with a source of spectral radiance distribution $S_\lambda(\lambda)$, is defined to be that wavelength which yields the average spectral diffraction loss when substituted into the approximate Kirchhoff scalar paraxial model (see sec. 1). Furthermore, if the approximate formula for the effective diffraction loss for a circular detector, eq (12) of section 6, is valid (see sec. 7 for a discussion of the mathematical errors in the formulas used in this report), then the diffraction loss scales proportionally to the wavelength, and an explicit equation for λ^{-} can be derived in the form,

$$\lambda^{-} = \frac{\int_{\lambda 1}^{\lambda 2} \lambda d\lambda S_\lambda(\lambda)}{\int_{\lambda 1}^{\lambda 2} d\lambda S_\lambda(\lambda)} ,$$

where $\lambda 1$ and $\lambda 2$ are the short- and longwavelength limits to $S_\lambda(\lambda)$.

If $S_\lambda(\lambda)$ is the Planck blackbody spectral radiance function [2], denoted $L_\lambda(\lambda,T)$ at temperature T, then λ^{-} can be related to the temperature by approximate equation (derived by Blevin[1]),

$$\lambda^{-} = 5324/T \text{ (micrometers)}, \tag{1}$$

if T is in degrees Kelvin. Thus λ^{-} is about $1.84\lambda_{max}$, the wavelength of maximum spectral radiance for a blackbody at temperature T.

The effective wavelength, $\lambda^{=}$, for computing the luminous diffraction loss for a source of spectral radiance distribution $S_\lambda(\lambda)$, is given by the equation,

$$\bar{\bar{\lambda}} = \frac{\int_{\lambda 1}^{\lambda 2} \lambda d\lambda V(\lambda) S_\lambda(\lambda)}{\int_{\lambda 1}^{\lambda 2} d\lambda V(\lambda) S_\lambda(\lambda)},$$

where $V(\lambda)$ is the spectral luminous efficiency function for photopic vision and $\lambda 1$ and $\lambda 2$ are the limits of the visible spectrum [3]. If $S_\lambda(\lambda)$ is the Planck blackbody spectral radiance function, then Blevin [1] has shown that $\bar{\bar{\lambda}}$ is 0.572 micrometers for a blackbody temperature of 2856 K (CIE Illuminant A).

5. Sample Diffraction Loss Calculations

5.1. Sample Diffraction Loss Calculations for a Simple Case

A simple example is the following: Compute the average diffraction loss over the face of a circular detector which views a 500 K blackbody through a small aperture. The geometry is that of a source radiance measurement (see fig. 1). The diameter of the blackbody aperture (d) is 1 cm; the distance from the blackbody aperture to the diffracting aperture (b) is 5 cm; the diameter of the diffracting aperture (D) is 0.1 cm; the distance from the diffracting aperture to the detector (a) is 60 cm; the detector diameter ($2x_o$) is 5 cm. Since the blackbody temperature is 500 K, eq (1) shows that the effective diffraction wavelength $\bar{\lambda}$ is 10.6 micrometers. Referring to the diffraction loss tables (sec. 2), it is seen that the tabulated wavelength closest to 10.6 micrometers is 10; at this wavelength, and at d = 1 cm, b = 5 cm, D = 0.1 cm, the on-axis diffraction loss E'_{min} (for a at least 10b, a condition which is met by this example) is found to be 2.05%; the corresponding area-average diffraction loss over the face of a detector of radius x_{max}, $\bar{E}'_{max}$, is found to be 2.96%. From the formula for X_{max},

$$x_{max} = 0.5[(0.9d-D)\underline{ab^{-1}} - D], \qquad (2)$$

it is found that x_{max} is 4.8 cm; the detector radius is given above as 2.5 cm. Denoting the desired average diffraction loss over the face of the detector by the symbol $\bar{E}'$, it is reasonable to interpolate between E'_{min} and $\bar{E}'_{max}$ by the following area-weighting formula:

$$\bar{E}' = E'_{min} + [\bar{E}'_{max} - E'_{min}](x_o/x_{max})^2. \qquad (3)$$

Thus $\bar{E}'$ is found to be 2.30% at the wavelength of 10 micrometers; the scaling table in section 3 shows that both E'_{min} and $\bar{E}'_{max}$ scale proportionally to the wavelength, so the desired value of $\bar{E}'$ at a wavelength of 10.6 micrometers is therefore obtained by multiplying 2.30% by the ratio 10.6/10 = 1.06 to get <u>2.44</u>%.

5.2. Sample Diffraction Loss Calculations for a Complex Case

Figure 2 shows the essential geometry of a circularly symmetric source-radiometer system currently in use at NBS. The source S is a

blackbody whose temperature is roughly 300 K; the source aperture SA limits the radiating area; the radiometer aperture RA defines the solid angle in which radiation is received from SA; the radiometer cavity RC collects the radiation transmitted through RA. It is desired to calculate the diffraction loss for radiation from S transmitted through SA and RA to RC.

This is really a 2-step diffraction problem; the total diffraction loss DL is obtained from both:

a., the diffraction loss for radiation from S transmitted through SA to RA, denoted DL_a, and;

b., the diffraction loss for radiation from SA transmitted through RA to RC, denoted DL_b.

Thus the total diffraction loss, denoted DL, is

$$DL = DL_a + DL_b - DL_a DL_b,$$

since the diffraction loss at the detector is given as a percentage of the irradiance that would be present in the absence of diffraction.

The effective diffraction wavelength $\bar{\lambda}$ for this problem is found from the given temperature of 300 K and eq (1) of section 4 to be 17.7 micrometers. It is clear that the quantity of interest in computing DL_a is the effective radiance of SA, compared with the radiance of S. On the other hand, the quantity of interest in computing DL_b is the fraction of the total radiation from SA, transmitted through RA, which is collected by RC. Therefore, in computing DL_b it is necessary to treat RC as the source and SA as the detector, since (as explained in sec. 1) the diffraction loss formulas and the tables in this report all refer to the source radiance measurement geometry, and not to the total aperture radiation measurement geometry.

Referring to figure 2, and using the terminology of the tables, it is seen that the essential parameters for computing DL_a and DL_b are:

DL_a: $\bar{\lambda}$ = 17.7 micrometers, d = 0.2 cm, $\underline{b}$ = 0.35 cm, D = 0.05 cm,

$\underline{a}$ = 17.1 cm, x_o = 0.575 (source radiance measurement, see fig. 3);

DL_b: $\bar{\lambda}$ = 17.7 micrometers, d = 2.0 cm, $\underline{b}$ = 8.0 cm, D = 1.15 cm,

$\underline{a}$ = 17.1 cm, x_o = 0.025 cm (total aperture radiation measurement, see fig. 3).

To compute DL_a, note that d = 0.2 cm is smaller than 0.5 cm, the smallest tabulated value for d; therefore it is necessary to multiply d and D (to keep the ratio Dd^{-1} constant) by a scaling factor β to use the tables. Let d' = βd be the scaled d and D' = βD be the scaled D; if

β = 10, then d' = 2.0 cm and D' = 0.5 cm, which are tabulated values.

Furthermore, b = 0.35 cm is much smaller than 5.0 cm, the smallest tabulated value for b; therefore it is necessary to multiply b by a scaling factor α to use the tables. Let b' = αb be the scaled b; if α = 14.3, then b' = 5.0 cm, a tabulated value.

In addition, the effective wavelength λ^- = 17.7 micrometers is not a tabulated wavelength. Therefore λ^- is multiplied by a scaling factor θ to use the tables. Let $\lambda^{-\prime} = \theta\lambda^-$ be the scaled λ^-; if θ = 1.13, then $\lambda^{-\prime}$ = 20 micrometers, a tabulated value.

Next compute x_{max} from eq (2) of section 5.1 to get x_{max} = 3.15 cm, so that x_o/x_{max} = 0.18.

Referring to the diffraction loss tables for the values of $\bar{E}'_{min}$ and $\bar{E}'_{max}$ for the scaled parameters, $\lambda^{-\prime}$ = 20 micrometers, d' = 2.0 cm, D' = 0.5 cm, b' = 5.0 cm (note that the condition that a be at least 10b is met for the geometry of DL_a), it is found that $\bar{E}'_{min}$ = 0.43% and $\bar{E}'_{max}$ = 0.59% for the scaled parameters. To interpolate between $\bar{E}'_{min}$ and $\bar{E}'_{max}$ to obtain the desired average diffraction loss $\bar{E}'$ over the radiometer aperture RA, for the scaled parameters, refer to eq (3) of section 5.1 and substitute the preceding values of $\bar{E}'_{min}$, $\bar{E}'_{max}$, and x_o/x_{max} into eq (3). The resulting value of $\bar{E}'$ = 0.435% for the scaled parameters is essentially equal to $\bar{E}'_{min}$.

The scaling process must now be reversed to obtain DL_a, the diffraction loss for the original unscaled parameters. Referring to the scaling table in section 3, it is seen that

$$DL_a = \bar{E}'(\text{scaled})\beta^2\alpha^{-1}\theta^{-1},$$

or DL_a = 6.19$\bar{E}'$(scaled) = 2.69%. This value is very close to that calculated for DL_a from the more accurate formula, eq (12) of section 6, 2.70%.

Unfortunately DL_b cannot be obtained from the tables in their present form. The values of $\bar{E}'_{min}$ and $\bar{E}'_{max}$ in the tables are computed by assuming that the detector-aperture distance a is much greater than the source-aperture distance b. This clearly does not hold for the geometry of DL_b (see fig. 2), since a = 17.1 cm (as explained above, since this is a total aperture radiation measurement, the source is treated as the detector and vice versa) and b = 8.0 cm, and therefore a = 2.14b and the condition for the validity of the tables that a be at least 10b is not met.

In addition, since Dd^{-1} = 0.575 for the geometry of DL_b, the values of d and D cannot be scaled to fit the tables because Dd^{-1} = 0.5 is the largest value tabulated, and Dd^{-1} must be held constant in scaling.

In a situation of this kind, it is necessary to return to the general formula given in section 6, eq (12), for the average diffraction

loss over the detector disc. This formula is not subject to the restriction of the tables, that the aperture-detector distance $\underline{a}$ be at least 10 times the source-aperture distance $\underline{b}$. In eq (12) the average diffraction loss over the detector disc is denoted $\bar{E}(u,v,w_o)$, where u, v, and w_o are dimensionless functions of the geometry and wavelength given by eqs (5), (6), and (8), respectively. Substituting the values given above for DL_b into eqs (5), (6), and (8), u, v, and w_o are computed and then substituted into eq (12) for $\bar{E}(u,v,w_o)$ to get DL_b = 0.87%.

Finally, therefore, the total diffraction loss, DL, from the source S to the radiometer cavity RC, is found to be DL = 3.55%.

6. Formulas Used for Computing the Diffraction Loss Tables

The formulas used in computing the diffraction loss tables are derived from the basic Fresnel-Kirchhoff diffraction formula as given, for example, in Born and Wolf[4].

For a source radiance measurement (as shown in fig. 3), the diffraction losses increase with increasing detector radius x_o. It is felt that a reasonable upper bound for the detector radius, for a source radiance measurement, is defined by the condition that the detector field of view not extend beyond the inner portion of the source disc whose radius is 0.9 of the source radius. If this upper bound is denoted x_{max}, it is seen from figure 3 that x_{max} is given by eq (2) of section 5.1. The minimum source diameter, for a source radiance measurement, is that which makes x_{max} = 0; thus the minimum source diameter is $[D(1 + \underline{ba}^{-1})/0.9]$.

For a total aperture radiation measurement, the diffraction losses <u>decrease</u> with increasing detector radius. It is felt that a reasonable lower bound for the detector radius, for a total aperture radiation measurement, is defined by the condition that all the source radiation which passes through the aperture - except for diffraction losses - be incident upon that portion of the detector disc whose radius is 0.9 of the detector radius. If this lower bound is denoted x_{min}, it is seen from figure 3 that

$$x_{min} = 0.5[(d+D)(0.9)^{-1}\underline{ab}^{-1} + D].$$

Following the analysis of Blevin[1], it is found that the on-axis diffraction loss at the center of the detector, denoted E(u,v,0), for the source radiance geometry, is approximately

$$E(u,v,0) = \pi^{-1}[(v-u)^{-1} + (v+u)^{-1}], \tag{4}$$

where u and v are the dimensionless quantities,

$$u = \pi D^2(\underline{a}^{-1} + \underline{b}^{-1})(2\lambda)^{-1}, \tag{5}$$

$$v = \pi Dd(2\underline{b}\lambda)^{-1}. \tag{6}$$

If the aperture-detector distance $\underline{a}$ is much greater than the source-aperture distance $\underline{b}$, then u is approximately

$$u_{min} = \pi D^2(2\underline{b}\lambda)^{-1} = vDd^{-1}; \qquad (7)$$

note that u_{min} is independent of $\underline{a}$.

Now let $E(u,v,w_o)$ denote the diffraction loss at the off-axis point in the detector plane whose radius is x_o (see fig. 3), and define a third dimensionless quantity,

$$w_o = \pi D x_o(\underline{a}\lambda)^{-1}. \qquad (8)$$

Blevin [1] has shown that $E(u,v,w_o)$ is approximately

$$E(u,v,w_o) = \pi^{-2}\int_0^{\pi} d\theta\,[(-u + w_o\cos\theta + (v^2-w_o^2\sin^2\theta)^{0.5})^{-1} + (u + w_o\cos\theta + (v^2-w_o^2\sin^2\theta)^{0.5})^{-1}], \qquad (9)$$

and that $E(u,v,w_o)$ increases steadily from the minimum value $E(u,v,0)$ as the radius of the off-axis detection point increases from $x = 0$ (on axis) out to the radius of the rim of the detector (which is also labeled x_o in fig. 3).

Referring to eq (4) for $E(u,v,0)$, it is seen that if the aperture-detector distance $\underline{a}$ is much greater than the source-aperture distance $\underline{b}$, then $E(u,v,0)$ is approximately $E(u_{min},v,0)$, denoted E'_{min}, and

$$E'_{min} = 2(\pi v)^{-1}(1 - D^2d^{-2})^{-1}. \qquad (10)$$

Note that E'_{min} is independent of $\underline{a}$ and that it is the minimum value of $E(u,v,0)$, considered as a function of $\underline{a}$, since $E(u,v,0)$ decreases steadily to E'_{min} as $\underline{a}$ increases (assuming the other parameters are held constant).

Referring to eq (9) for $E(u,v,w_o)$, it is seen that the average diffraction loss over the surface of a detector of radius x_o, denoted $\bar{E}(u,v,w_o)$, is given by the formula,

$$\bar{E}(u,v,w_o) = w_o^{-2}\int_0^{w_o} 2w\,dw\,E(u,v,w). \qquad (11)$$

Since the off-axis diffraction loss $E(u,v,w_o)$ increases steadily as the detection point moves away from the axis, it is clear that the average diffraction loss over the disc of radius x_o also increases steadily as x_o increases. Thus $\bar{E}(u,v,w_o)$ attains its maximum value, considered as a function of x_o, for a detector radius of x_{max}.

If the detector plane is now moved towards the aperture, holding the source diameter d, the source-aperture distance $\underline{b}$, and the aperture

diameter D constant, then a point will be reached for which $x_{max} = 0$. The value of $\underline{a}$ at this point is denoted $\underline{a}_{min}$ (see fig. 3), and it is seen that

$$\underline{a}_{min} = \underline{b}D(0.9d - D)^{-1}.$$

It can be shown that the maximum value of $\bar{E}(u,v,w_{max})$, the average diffraction loss over the detector disc of radius x_{max}, considered as a function of the aperture-detector distance $\underline{a}$, occurs at $\underline{a}_{min}$; it can also be shown that

$$w_{max} + u = 0.9v,$$

where w_{max} is defined as

$$w_{max} = \pi D x_{max}(\underline{a}\lambda)^{-1}.$$

Note that the preceding analysis assumes that the detector radius is x_{max}, and that the radius varies with x_{max} as the detector is moved towards the aperture.

Now consider the behavior of the average diffraction loss over the disc of radius x_{max} as the detector plane is moved away from the aperture. It is clear that u will approach its minimum value u_{min} asymptotically in this case, and that consequently w_{max} will correspondingly approach its maximum value, considered as a function of $\underline{a}$, which is $v(0.9 - Dd^{-1})$. It can be shown that the average diffraction loss over the disc of radius x_{max}, $\bar{E}(u,v,w_{max})$, attains its minimum value, considered as a function of $\underline{a}$, when $\underline{a}$ is much larger than $\underline{b}$ (and hence u is approximately equal to u_{min} and w_{max} is approximately equal to $v(0.9 - Dd^{-1})$). This minimum value of $\bar{E}(u,v,w_{max})$ is denoted $\bar{E}'_{max}$ and is independent of $\underline{a}$.

Steel, De, and Bell [5] have derived a very useful and compact approximate formula for $\bar{E}(u,v,w_o)$,

$$\bar{E}(u,v,w_o) = (2\pi w_o)^{-1} \ln \frac{[(v+w_o)^2 - u^2]}{[(v-w_o)^2 - u^2]}. \quad (12)$$

Thus $\bar{E}'_{max}$ can be expressed approximately by the formula,

$$\bar{E}'_{max} = [2\pi v(0.9-Dd^{-1})]^{-1} \ln \left[19 \frac{(1.9-2Dd^{-1})}{(0.1+2Dd^{-1})}\right]. \quad (13)$$

7. Estimated Accuracy of the Diffraction Loss Tables

Steel, De, and Bell [5] show that an upper bound for the fractional error in E'_{min}, as computed from eq (10) of section 6, is $(2v)^{-1}$; thus E'_{min} is not given in the tables for values of v less than 5, in order to limit the estimated error in the tabulated values to less than 10% (of the value).

Similarly, Steel, De, and Bell [5] show that an upper bound for the fractional error in $\bar{E}'_{max}$, as computed from eq (13) of section 6, is $0.06 + 1.6v^{-1}$; thus $\bar{E}'_{max}$ is not given in the tables for values of v less than 12, in order to limit the estimated error in the tabulated values to less than 20% (of the value). However, an upper bound for $\bar{E}'_{max}$ is given for values of v between 6 and 12; this upper bound is the diffraction loss for the point on the axis at which the rim of the aperture appears to coincide with the rim of the source; in other words, the field of view through the aperture from this point coincides with the source disc (in fig. 3, this point is the distance $\underline{a}_o$ from the aperture). Blevin [1] shows that the diffraction loss for this point, denoted E(v,v,0), is given by the approximate formula,

$$E(v,v,0) = (\pi v)^{-0.5}.$$

Note that the ratio $[\bar{E}'_{max}/E'_{min}]$ depends on Dd^{-1} only, and varies approximately as follows:

Dd^{-1}	$\bar{E}'_{max}/E'_{min}$
0.0	1.64
0.1	1.45
0.2	1.39
0.3	1.35
0.4	1.32
0.5	1.29.

Thus $\bar{E}'_{max}$ and E'_{min} bracket the diffraction loss, for detector radii between zero and x_{max}, to within ±20% roughly for the worst case, $Dd^{-1} = 0$, and more accurately for larger values of Dd^{-1}.

Furthermore, if the Kirchhoff scalar paraxial model does not accurately represent the physical behavior of the experimental situation, then the diffraction loss values in the tables will contain an additional error besides those due to the mathematical approximations used to compute the tables.

A good criterion for the validity of the Kirchhoff model, as Stratton [6] points out, is that the diameter of the diffracting aperture must be much larger than the wavelength. In the terminology of figure 3, if

$$D/\lambda > 10,$$

then the Kirchhoff model is held to be valid; if this condition does not hold, then the exact vector model may be required.

8. References

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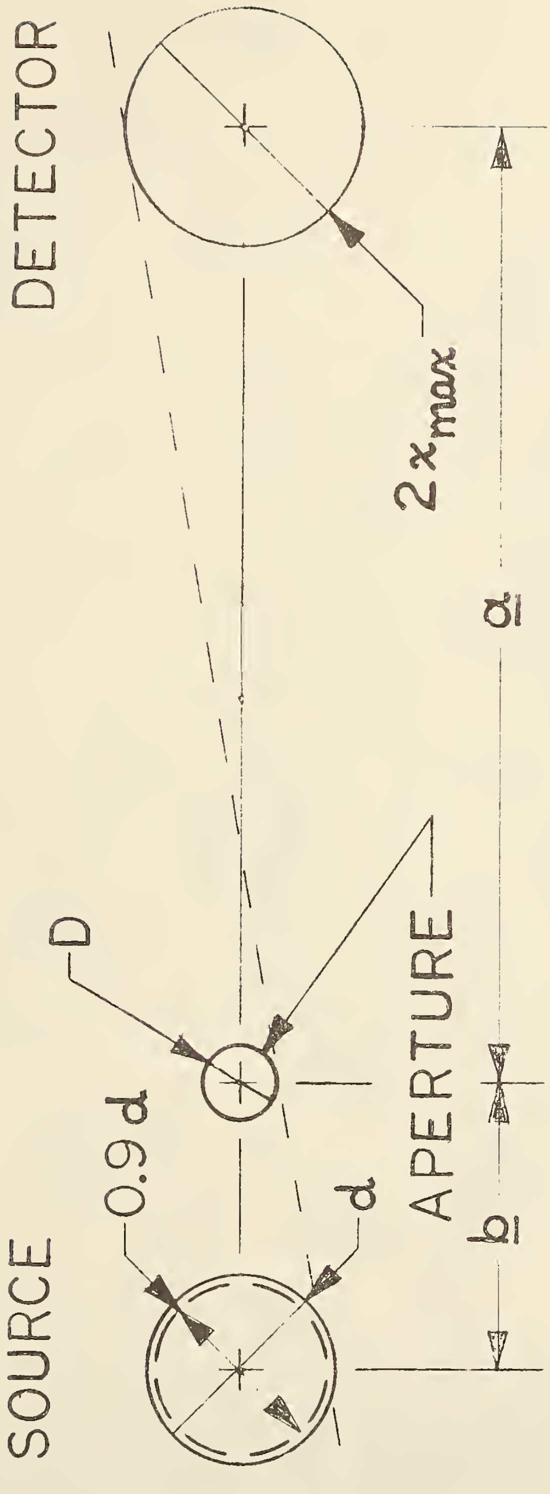

Figure 1. Diffraction geometry used in the diffraction loss tables.

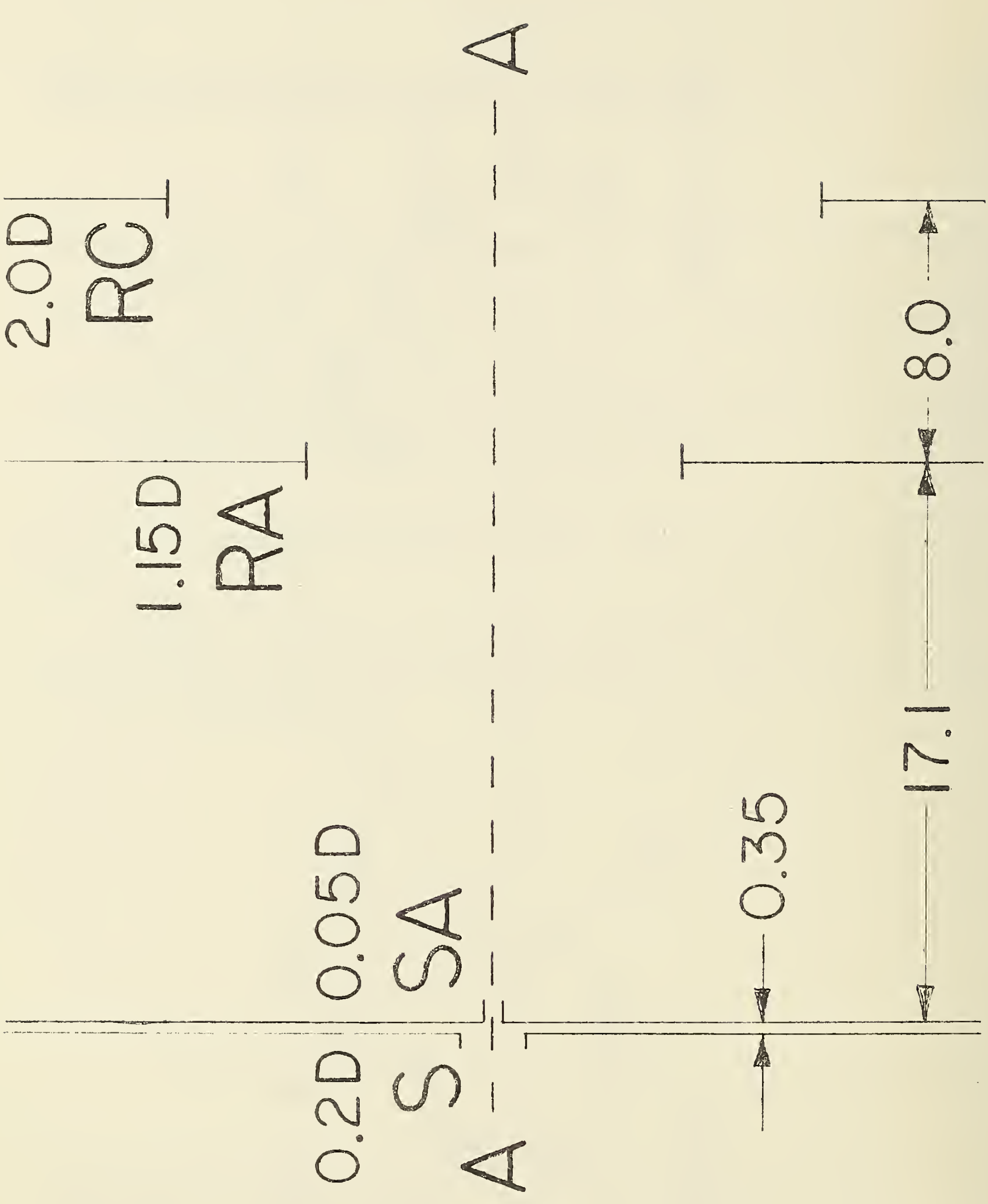

Figure 2. Diffraction geometry for the sample diffraction loss calculations; complex case: A-A, optical axis; RA, radiometer aperture; RC, radiometer cavity; S, 300 K blackbody source; SA, defining aperture for S (distances in cms; aperture diameters magnified 10 times with respect to distances along the axis).

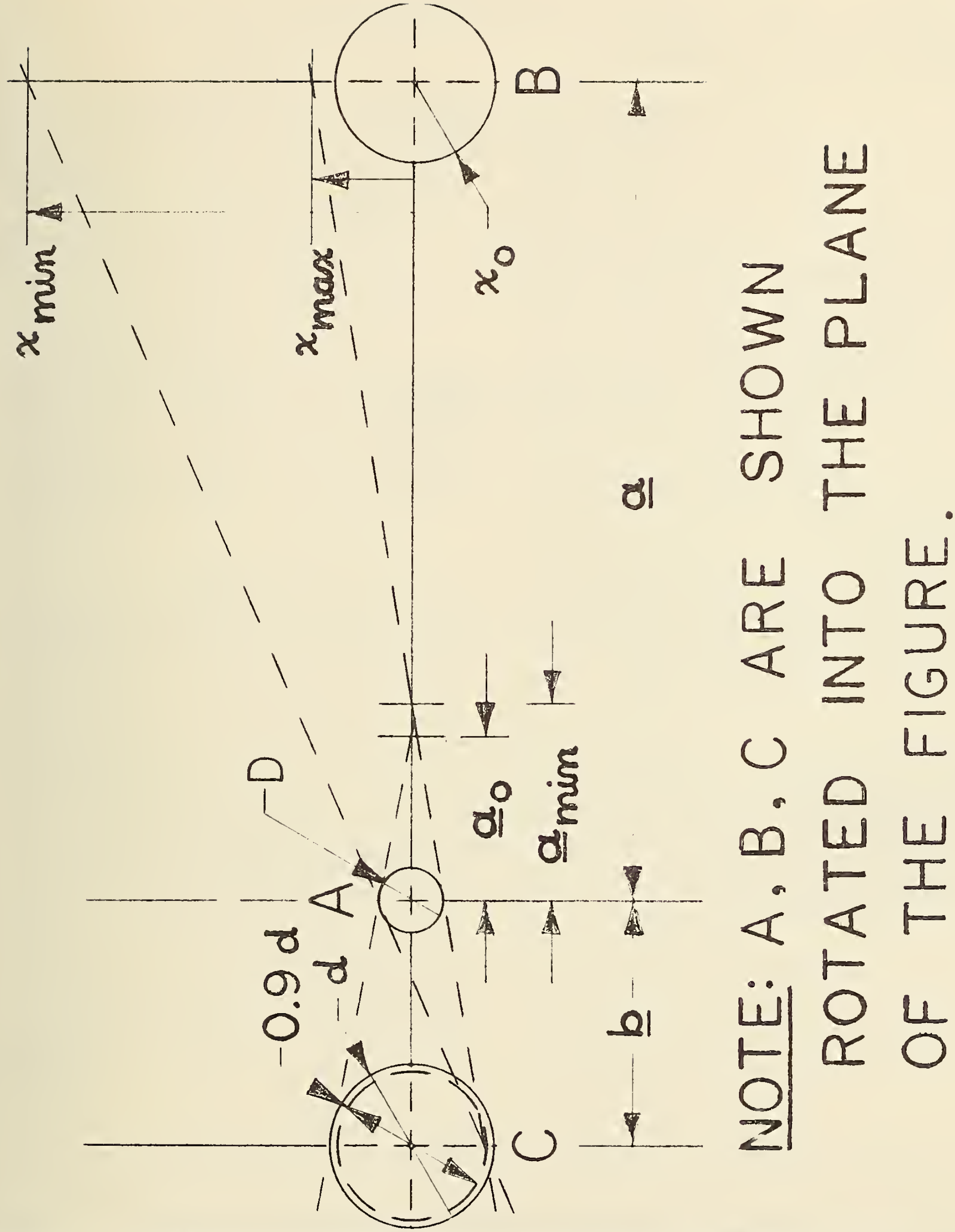

Figure 3. Diffraction geometry used in the general diffraction loss formulas: A, aperture; B, detector; C, source; d, source diameter; D, aperture diameter; x_o, detector radius; x_{max}, maximum detector radius for a source radiance measurement; x_{min}, minimum detector radius for a total aperture radiation measurement; $\underline{a}$, aperture-detector distance; $\underline{a}_{min}$, minimum aperture-detector distance; $\underline{a}_o$, aperture-detector distance for computing an upper bound for the diffraction loss; $\underline{b}$, source-aperture distance.

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16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

Tables of diffraction losses are given for a range of typical experimental geometries for wavelengths from 0.2 to 100 micrometers. The scaling relationships for the diffraction losses for varying wavelengths and geometries are also given, and sample calculations are presented. General formulas are given for the diffraction losses; the formulas are derived from the Kirchhoff scalar paraxial diffraction theory. The accuracy of the tabulated values is estimated.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

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