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Optical Radiation Measurements:

Approximate Theory of the Photometric Integrating Sphere

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Preface

This is the seventh issue of a series of Technical Notes entitled OPTICAL RADIATION MEASUREMENTS. The series will consist primarily of reports of progress in, or details of, research conducted in radiometry and photometry in the Optical Radiation Section of the Heat Division.

The level of presentation in OPTICAL RADIATION MEASUREMENTS will be directed at a general technical audience. The equivalent of an undergraduate degree in engineering or physics, plus familiarity with the basic concepts of radiometry and photometry [e.g., G. Bauer, Measurement of Optical Radiations (Focal Press, London, New York, 1965)], should be sufficient for understanding the vast majority of material in this series. Occasionally a more specialized background will be required. Even in such instances, however, a careful reading of the assumptions, approximations, and final conclusions should permit the non-specialist to understand the gist of the argument if not the details.

At times, certain commercial materials and equipment will be identified in this series in order to adequately specify the experimental procedure. In no case does such identification imply recommendation or endorsement by the National Bureau of Standards, nor does it imply that the material or equipment identified is necessarily the best available for the purpose.

Any suggestions readers may have to improve the utility of this series are welcome.

Henry J. Kostkowski, Chief
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Approximate Theory of the Photometric Integrating Sphere

W. B. Fussell

An approximate mathematical theory of the photometric integrating sphere is developed. The analysis is accurate to the first order in the ratio of the baffle area to the sphere wall area. The sphere is assumed to be occupied by a circular baffle and a spherical lamp; the centers of the baffle and the lamp lie on a diameter of the sphere. The surfaces of the sphere and the baffle are assumed to reflect in a uniformly diffuse manner. The lamp is assumed to absorb a fraction of the radiation incident upon it, and to transmit (or specularly reflect) the remainder. The luminance distribution at the sphere window is derived for a general source input at any point of the sphere wall. A model lamp illuminance distribution is assumed, and a formula for the fractional error in comparing the total luminous fluxes of two lamps in the integrating sphere, is derived. The physical significance of the formula is described.

Keywords: Illuminance distribution; integrating sphere; lamp comparisons; photometric accuracy; photometry; total luminous flux.

1. Introduction

The photometric integrating sphere was invented by R. Ulbricht, who made the first sphere in 1900 [1][1]. The usefulness of the integrating sphere for comparing the total luminous fluxes[2] of lamps was quickly recognized, and many large spheres were soon constructed [3].

In 1915 a large integrating sphere of 2.2-meter (88-inch) diameter was constructed at NBS by Rosa and Taylor [4]. They compared total luminous flux values obtained with this sphere for several incandescent

[1] Figures in square brackets are literature references at the end of the text.

[2] The nomenclature of this paper is that of the International Lighting Vocabulary (1970) [2].

lamp-reflector combinations, against corresponding values obtained by distribution photometry, and found differences ranging from 0 to 1.6% in magnitude. These differences are within acceptable limits for NBS values of total luminous flux, even today. (The current NBS goal is an accuracy of $2\frac{1}{2}\%$ for reported values of total luminous flux, for incandescent lamps.)

Integrating spheres are now universally used in photometric laboratories to compare the total luminous fluxes of lamps of all types. NBS for example, uses at present two 2-meter (79-inch) diameter spheres, one 76-cm (30-inch) sphere, and one 30-cm (12-inch) sphere, to compare the total luminous fluxes of incandescent lamps emitting from 10,000 lumens to 6 lumens.

The purpose of the analysis presented in this paper is: (1), to pinpoint the major sources of error in comparing the total luminous fluxes of lamps with the integrating sphere; (2), to estimate the magnitude of the errors in the present NBS techniques for comparing the total luminous fluxes of incandescent lamps with the integrating sphere; (3), to show how the errors described in (1) and (2) may be minimized.

2. Current NBS Techniques for Comparing the Total Luminous Fluxes of Lamps with the Integrating Sphere

Figure 1 shows the cross section of a typical integrating sphere with the baffle and a lamp inside the sphere. The lamp (whose total luminous flux is to be compared with that of another lamp) is located at the center of the sphere; the baffle is placed between the sphere window and the lamp so that it completely screens the lamp from the window. The baffle is usually placed roughly 1/3 the sphere radius from the lamp, since calculations have shown that this distance will approximately minimize the error in comparing the total luminous fluxes of two lamps [4]. Usually, the substitution technique is employed; that is, if the total flux of lamp A is to be compared with that of lamp B, the detector output is measured with <u>only</u> lamp A in the sphere, and then with only lamp B in the sphere.

The correction for the difference between the absorptances of lamps A and B is found approximately by Helwig's method [5], which uses an auxiliary lamp placed in the sphere near the wall. Lamps A and B are successively placed in the sphere with the auxiliary lamp (which is shielded so that none of its flux strikes either the sphere window, or A or B, directly). The lamps A and B are <u>unlit</u>, and the correction for the difference between the absorptances of A and B is approximately the <u>ratio</u> of the detector outputs produced by the auxiliary lamp when first A, and then B, is placed in the sphere.

The diameter of the integrating sphere used to measure the total luminous flux of a lamp, depends upon the magnitude of the lamp's flux output. As will be shown later, if the sphere is too large, the detector signal will be too small, if the sphere is too small, the sphere error will be too large. At NBS, lamps emitting between 270 and 10,000 lumens are measured in a 2-meter diameter sphere; lamps of smaller flux output are measured in either the 76-cm or the 30-cm sphere.

At present, only tungsten-filament incandescent lamps are calibrated for total luminous flux on a regular basis at NBS [6].

The internal surfaces of the NBS integrating spheres (and the surfaces of the baffles) are coated with paints of high reflectance which reflect in an approximately uniformly diffuse manner. (A surface which reflects - or emits - in a uniformly diffuse manner is defined as a surface which has the same luminance in all directions.) One 2-meter diameter sphere is painted with a thick coating of barium sulfate; the other 2-meter sphere - and the 76-cm and 30-cm spheres - is coated with Burch sphere paint (Burch Paint Mfg. Co., 10609 Briggs Road, Cleveland, Ohio 44111). The spectral reflectance of Burch sphere paint varies with wavelength, falling off towards the blue end of the visible spectrum [7]. (The spectral reflectance of barium sulfate is more uniform over the visible spectrum than that of Burch sphere paint [8].) The net effect of this varying spectral reflectance is to reduce the color temperature of the illumination at the detector from that of the lamp in the sphere, by a large amount. (For example, the color temperature of the illumination at the detector, produced by a 2856 K lamp in the sphere, is reduced by 700 K - including the effect of the sphere window [9].)

This reduction in the color temperature of the illumination at the detector from that of the lamp, due to the decreasing spectral reflectance of Burch sphere paint towards the blue end of the visible spectrum, produces an error in comparing the total luminous fluxes of two lamps with <u>different</u> color temperatures. To compensate for this effect, a blue filter (Corning 5900 glass; Corning Glass Works, Corning, N.Y. 14830) is used to make the net spectral transmittance of the sphere (from the lamp to the detector) approximately independent of wavelength. Calculations show that the spectral reflectance of an integrating sphere coated with Burch sphere paint can be neutralized sufficiently by this technique, to reduce the error due to this cause, in comparing lamps having color temperatures of 2360 K and 3000 K, to less than 0.2%.

3. Elementary Sphere Analysis

Elementary integrating sphere analysis has been presented in the literature [10]. The first step is to compute the luminance distribution in an empty sphere.

3.1. Empty Sphere Luminance Distribution

Refer to figure 2 and let α be the colatitude angle of a source point $P(\alpha)$ on the sphere wall; let θ be the colatitude angle of a detection point on the sphere wall. (The azimuth angles of the source point and the detection point are ignored, since the sphere wall reflectance is assumed to be spatially uniform, and hence the sphere is symmetric about the polar axis.)

It is assumed throughout this paper that the sphere wall and the baffle surfaces reflect in a uniformly diffuse manner. This means that the luminance of an element of surface area is independent of the direction from which it is viewed.

(The luminance, in a given direction, of an element of surface area, is defined to be the quotient of the luminous flux leaving the surface through an element of solid angle in the given direction, divided by the product of the element of solid angle and the projection of the element of surface area in the given direction. Symbolically, the luminance L is defined as

$$L = \lim_{\substack{\delta\sigma \to 0 \\ \delta A \to 0}} \left[(\delta^2\phi)/(\delta\sigma \cdot \delta A \cos\theta)\right],$$

where $\delta^2\phi$ is the element of luminous flux leaving the element of surface area δA, through the element of solid angle $\delta\sigma$, in a direction inclined at angle θ to the normal to the element of surface area.)

Let the reflectance of the sphere wall be r, and let the luminance of the sphere wall at D(θ), produced by a 1 lumen source input at P(α), be denoted L(α,θ). The 1 lumen source input at P(α) produces a secondary source of luminous intensity r/π candelas at P(α); this secondary source illuminates the wall of the sphere with a <u>uniform</u> illuminance of $r/(4\pi R^2)$ lux (neglecting multiple reflections). To demonstrate that this illuminance <u>is</u> uniform, refer to figure 2 and consider the chord PD. The length of PD, denoted $\overline{PD}$, is $2R\sin(\beta/2)$; the angle between the normal to the sphere at P(α) and the chord PD is $\Sigma = \pi/2 - \beta/2$; this is also the angle between the normal to the sphere at D(θ) and the chord PD. Thus the illuminance at D(θ) produced by the 1 lumen source input at P(α), denoted E(α,θ), is given by

$$E(\alpha,\theta) = (r/\pi)(\overline{PD})^{-2}\cos^2\Sigma;$$

since

$$\overline{PD}^2 = 4R^2\cos^2\Sigma,$$

the equation for E(α,θ) becomes

$$E(\alpha,\theta) = r(4\pi R^2)^{-1},$$

and so it is seen that E(α,θ) is indpendent of the positions of P(α) and D(θ), as desired.

Thus the illuminance at any point on the sphere wall, produced by a source input at any other point on the wall, is independent of the positions of the source point and the detection point, when the reflectance of the sphere wall is spatially uniform and uniformly diffuse. Stated in another way, the viewfactor of unit area on the sphere, as viewed from any point on the sphere, is constant and equal to $1/(4\pi R^2)$. (The viewfactor of a surface A_1, as viewed from surface A_2, is defined as the fraction of the total flux emitted by A_2 which is incident upon A_1, assuming that A_2 radiates in a uniformly diffuse manner. The concept of viewfactor is also useful in radiometry and heat exchange.)

The constant viewfactor of any area of the sphere, as viewed from any point of the sphere, is the <u>basic property</u> which makes the empty sphere, with walls of spatially uniform and uniformly diffuse reflectance, a perfect flux integrator.

Now it is necessary to consider the interreflections in the sphere. It is seen that the uniform illuminance over the wall of the sphere, E (dropping the functional dependence notation), produced by the 1 lumen source input at $P(\alpha)$, in turn produces a luminance distribution L' which is also uniform. It is clear that

$$L' = rE/\pi = r^2/(4\pi^2R^2).$$

The uniform luminance distribution L' in turn produces a uniform illuminance over the sphere wall, E', which is given by

$$E' = 2\pi(4R^2)^{-1}\int_0^\pi d\theta' \sin\theta' L'R^2;$$

this reduces to

$$E' = r^2/(4\pi R^2),$$

which is equivalent to

$$E' = rE.$$

Thus it is seen that the uniform illuminance in the sphere produced by the n-th reflection from the wall, E_n, is related to the <u>initial</u> illuminance from the source point, E, by the equation,

$$E_n = r^n E.$$

Therefore the <u>total</u> illuminance in the sphere, E_{TOT}, is also uniform and is given by

$$E_{TOT} = E \sum_{n=0}^{\infty} r^n,$$

which reduces to

$$E_{TOT} = E/(1-r).$$

Consider now instead of a point source input, the input illuminance distribution $E_{IN}(\alpha)$. The total input flux, ϕ^*_{IN}, is

$$\phi^*_{IN} = 2\pi R^2 \int_0^{\pi} d\alpha' \sin\alpha' E_{IN}(\alpha').$$

The total illuminance over the sphere wall produced by $E_{IN}(\alpha)$ is therefore (<u>excluding</u> the <u>input</u> illuminance)

$$E_{TOT} = r(1-r)^{-1} \int_0^{\pi} d\alpha' \sin\alpha' E_{IN}(\alpha') 2^{-1}.$$

In terms of ϕ^*_{IN}, it is seen from the preceding two equations that

$$E_{TOT} = \phi^*_{IN} r/[4\pi R^2(1-r)].$$

This is the <u>basic integrating sphere equation</u>; it states that the illuminance inside an empty sphere, with walls whose reflectance is spatially uniform and uniformly diffuse, is proportional to the total source luminous flux in any region of the sphere which is not directly illuminated by the source.

Note that in this simple model, E_{TOT} becomes very large as the wall reflectance r approaches 1; note also that E_{TOT} decreases rapidly with increasing sphere radius R.

A necessary condition on E_{TOT} is that it satisfy flux conservation; that is, the total flux absorbed at the wall of the sphere, ϕ^*_{OUT}, must equal the source input flux, ϕ^*_{IN}. To show that E_{TOT} satisfies flux conservation, note that

$$\phi^*_{OUT} = E_{TOT} 4\pi R^2(1-r)/r = \phi^*_{IN}.$$

3.2. Introduction of the Baffle

Consider a source point $P(\alpha)$ which is screened from a detection point $D(\theta')$ by an infinitesimal baffle. The effect of such a baffle is to eliminate the initial illuminance due to the source point $P(\alpha)$ at $D(\theta')$. Thus the total illuminance distribution in the sphere is no longer uniform, and it is now a function of θ as well as α. It is seen that, at a detection point $D(\theta)$ which is not screened from the source point by the baffle, the total illuminance (now denoted $E(\theta)_{TOT}$ since it is a function of θ) for a 1 lumen source input, is

$$E(\theta)_{TOT} = r/[4\pi R^2(1-r)],$$

identical to the illuminance equation for the empty sphere given above. However, for the screened detection point $D(\theta')$, it is clear that the total illuminance, $E(\theta')_{TOT}$, for a 1 lumen source input, is

$$E(\theta')_{TOT} = E(\theta)_{TOT} - r/(4\pi R^2),$$

since the illuminance from the source point which is screened by the baffle from the detection point is $r/(4\pi R^2)$. Thus one arrives at the result,

$$E(\theta')_{TOT} = r^2/[4\pi R^2(1-r)].$$

The basic effect, therefore, of even an infinitesimal baffle is to screen every point on the sphere from another point on the sphere; if a detection point is screened from a source point, the total illuminance at the detection point due to the screened source point, is equal to the total <u>unscreened</u> illuminance due to the source point <u>multiplied</u> by the sphere-wall reflectance r.

Consider now the effect of a baffle of finite size. The baffle clearly screens an area of the sphere wall from each point on the wall; the screened area associated with each point is defined by the projection of the baffle boundary from the point to the sphere wall. For a source point $P(\alpha)$ and a detection point $D(\theta')$ screened by the baffle, the major effect of the baffle is the screening of the initial illuminance from $P(\alpha)$ at $D(\theta')$. This effect is termed a "zero-order" effect, since it is independent of the baffle area. For an accurate "first order" analysis ("first order" as used in this paper means the ratio of the baffle area to the total sphere-wall area, a quantity assumed to be much less than 1), several other effects must be considered (such as the flux reflected from baffle surfaces, and the screening of sphere-wall areas which are not source areas). These effects, however, vary as the baffle area and thus are excluded from the elementary analysis of this section.

3.3. Effects of Absorbing Objects

The effects of introducing absorbing objects into the sphere are of two types: first, the screening effect which was treated in an elementary fashion above, and second, the absorption effect. This effect depends upon the area of the absorbing object, and the absorptance of the object. An elementary expression for the absorption effect can be derived from the flux conservation requirement discussed in section 3.1.

Consider a source point $P(\alpha)$ in an empty sphere which generates the uniform illuminance E_{TOT} over the wall of the sphere. If an absorbing object of area A and absorptance $\underline{\lambda}$ is introduced into the sphere, then the illuminance in the sphere is no longer uniform, due to the screening effect of the object. If the object is assumed to be opaque, with reflecting surfaces of absorptance $\underline{\lambda}$ (at any angle of incidence), then the illuminance at the detection point $D(\theta)$, denoted $E_{TOT}(\alpha,\theta,\underline{\lambda})$, is given approximately by

$$E_{TOT}(\alpha,\theta,\underline{\lambda}) = E_{TOT}\frac{[1-(1-r)S(\alpha,\theta)]}{[1+\underline{\lambda}A/(4\pi R^2(1-r))]},$$

where $S(\alpha,\theta)$ is the screening function of the object. This function is defined to be 1 if the object intercepts the line connecting the source point $P(\alpha)$ and the detection point $D(\theta)$, and 0 otherwise.

If the object is assumed to be transparent, with non-reflecting surfaces, and to absorb the fraction $\underline{\lambda}$ of the incident flux which strikes it from any direction, then the illuminance at the detection point is given approximately by

$$E_{TOT}(\alpha,\theta,\underline{\lambda}) = \frac{[1-\underline{\lambda}(1-r)S(\alpha,\theta)]}{[1+\underline{\lambda}A/(4\pi R^2(1-r))]}.$$

4. Analysis: Sphere with Coaxial Disc Baffle

4.1. First Order Luminance at the Sphere Window as a Function of Source Point Coordinates

In figure 3, $\underline{a}$ is the radius of the disc baffle B; the baffle axis goes through the centers of the sphere and the lamp L; the plane of the baffle is a distance $\underline{h}$ from the center of the sphere; the center of the lamp is a distance $\underline{l}$ from the center of the sphere ($\underline{l}$ is measured in the opposite sense from $\underline{h}$); the radius of the lamp is $\underline{b}$, and R is the radius of the sphere. The position of the source point $P(\alpha)$ is defined by the colatitude angle α; α'' is the colatitude angle of the projection on the sphere wall of the center of the lamp, from $P(\alpha)$; $\underline{\alpha}$ is the colatitude angle of the projection on the sphere wall of the center of the baffle, from $P(\alpha)$. The detection point is now taken to be the sphere window W at the south pole of the sphere (that is, at $\theta=\pi$). The luminance of the sphere wall at the window, produced by a 1 lumen source input at $P(\alpha)$, is denoted $L_1(\alpha,\pi,\underline{\lambda})$; $\underline{\lambda}$ is the absorptance of the lamp. The surface of the baffle visible from the lamp is labeled the "north" surface, and has reflectance r_N. The surface of the baffle visible from the sphere window is labeled the "south" surface, and has reflectance r_S. The lamp is assumed to be transparent and non-reflecting, and to absorb the fraction $\underline{\lambda}$ of radiation incident upon it from any direction.

The viewfactor of the baffle from the point $P(\alpha)$ is defined as the fraction of the luminous flux reflected from $P(\alpha)$ which strikes the baffle; this viewfactor is denoted $F_{BP}(\alpha)$. Similarly, the viewfactor of the lamp from $P(\alpha)$ is denoted $F_{LP}(\alpha)$. The viewfactors of the baffle and the lamp from the detection point at the sphere window, are denoted

$F_{BD}(\pi)$ and $F_{LD}(\pi)$, respectively.

At this point it is assumed (as suggested by Safwat [11]) that the baffle radius $\underline{a}$ is much less than the sphere radius R; it is also assumed that the radius of the lamp $\underline{b}$ is sufficiently small to ensure that no luminous flux from the lamp reaches the sphere window directly (this is the "screening condition"); thus the lamp radius $\underline{b}$ is also much less than the sphere radius R. On the basis of these assumptions, therefore, it is reasonable to compute the luminance produced at the sphere window by a 1 lumen source input at $P(\alpha)$, $L_1(\alpha,\pi,\underline{\lambda})$, to terms of the <u>first order</u> in the ratio of the area of the baffle to the sphere-wall area, and to neglect terms of higher order in this ratio. Thus the first order luminance at the sphere window produced by a 1 lumen source input at $P(\alpha)$ is found to be (note that this is the luminance <u>within</u> the sphere at the window):

$\underline{0 < \alpha < 2\delta}$,

$$L_1(\alpha,\pi,\underline{\lambda}) = r^2(4\pi^2R^2G)^{-1} \times\left[1-(1-r_S)F_{BD}(\pi)-(1-r_N)F_{BP}(\alpha)-\underline{\lambda}F_{LP}(\alpha)\right]; \qquad (1)$$

$\underline{2\delta < \alpha < \gamma}$,

$$L_1(\alpha,\pi,\underline{\lambda}) = r(4\pi^2R^2G)^{-1} \times\left[1-r(1-r_S)F_{BD}(\pi)-r(1-r_N)F_{BP}(\alpha) +r(2-r_S-r_N)\underline{a}'^2 4^{-1}+r\underline{\lambda}(\underline{b}'^2-F_{LP}(\alpha))\right]; \qquad (2)$$

$\underline{\gamma < \alpha < \pi}$,

$$L_1(\alpha,\pi,\underline{\lambda}) = r(4\pi^2R^2G)^{-1}$$

$$\times[1-r(1-r_S)F_{BD}(\pi)-r(1-r_S)F_{BP}(\alpha)$$

$$+r(2-r_S-r_N)\underline{a}'^2 4^{-1}+r\underline{\lambda}(\underline{b}'^2-F_{LP}(\alpha))$$

$$+4(1-r)r_S F_{BD}(\pi)F_{BP}(\alpha)\underline{a}'^{-2}]. \qquad (3)$$

In eqs (1)-(3), the quantity G is

$$G = [(1-r)+r(2-r_S-r_N)\underline{a}'^2 4^{-1}+r\underline{\lambda}\underline{b}'^2],$$

and

$$\underline{a}' = \underline{a}/R, \ \underline{b}' = \underline{b}/R;$$

in addition, referring again to figure 3, it is seen that δ is the half angle subtended by the baffle at the sphere window W, while γ is the colatitude angle of the baffle plane.

The physical significance of the various terms in eqs (1)-(3) is as follows:

a. The term "$+rr_N F_{BP}(\alpha)$" represents luminous flux from the source point which strikes the north surface of the baffle, is reflected to the sphere wall, and then is reflected to the sphere window.

b. The term "$-rF_{BD}(\pi)$" represents luminous flux from the sphere wall (excluding the direct flux from the source point) which is screened from the sphere window by the baffle.

c. The term "$-rF_{BP}(\alpha)$" represents luminous flux from the source point which is screened from the sphere wall by the baffle, and is thus screened <u>indirectly</u> from the sphere window. (Note that the terms "a." and "c." represent positive and negative effects, at the window, of the <u>same</u> flux; for example, if r_N=1, then term "a." <u>cancels</u> term "c.".)

d. The term "$-r(r_S+r_N)\underline{a}'^2 4^{-1}$", together with the <u>same term</u> in the quantity G in the denominator, represents the luminous flux from the sphere wall (excluding the direct flux from the source point) which strikes the north and the south surfaces of the baffle, is reflected to the sphere wall, and then is reflected to the sphere window.

(Note that the <u>net</u> effect of the terms "$-r(r_S+r_N)\underline{a}'^2 4^{-1}$" in the numerator <u>and</u> the denominator is <u>positive</u>. For example, if the quantity (1-r) is much larger than the magnitude of the remaining terms in G, then G^{-1} may be written to first order accuracy as

$$G^{-1} \approx (1-r)^{-1}[1-r(2-r_S-r_N)\underline{a}'^2 4^{-1}(1-r)^{-1}-r\lambda\underline{b}'^2(1-r)^{-1}].$$

When this expansion of G^{-1} is multiplied the numerator, the term "$-r(r_S+r_N)\underline{a}'^2 4^{-1}$" in the numerator combines with the term "$+r(r_S+r_N)\underline{a}'^2 4^{-1}(1-r)^{-1}$" in the expansion of G^{-1} to yield the <u>net</u> term "$+r^2(r_S+r_N)\underline{a}'^2 4^{-1}(1-r)^{-1}$" in the first order expansion of the product.)

e. The term "$+r\underline{a}'^2 2^{-1}$", together with the same term in the quantity G in the denominator, represents the luminous flux from the sphere wall (excluding the direct flux from the source point) which is screened from the sphere wall by the baffle, and is thus screened <u>indirectly</u> from the sphere window.

f. The term "$+r\lambda\underline{b}'^2$", together with the same term in the quantity G in the denominator, represents the luminous flux from the sphere wall (excluding the direct flux from the source point) which is partly screened (since the lamp absorptance $\underline{\lambda}$ is less than 1) from the sphere wall by the lamp, and is thus screened indirectly from the sphere window.

g. The term "$-r\underline{\lambda}F_{LP}(\alpha)$" represents luminous flux from the source point which is partly screened from the sphere wall by the lamp, and is thus screened indirectly from the sphere window.

h. The term "$+rr_S F_{BD}(\pi)$" represents luminous flux from the sphere wall (excluding the direct flux from the source point) which strikes the south surface of the baffle, and is reflected directly to the sphere window.

i. The term "$+rr_S F_{BP}(\alpha)$" has the same physical significance as term "a.", "$+rr_N F_{BP}(\alpha)$", except that the source point now illuminates the south surface of the baffle.

j. The term "$+4(1-r)r_S F_{BD}(\pi)F_{BP}(\alpha)\underline{a}'^{-2}$" represents luminous flux from the source point which strikes the south surface of the baffle, and then is reflected directly to the sphere window.

Referring again to figure 3, it is convenient to denote the intersections of the baffle axis with the sphere as the "north" and "south" poles; the north pole is marked N in figure 3, the south pole is marked W, since the sphere window is located at this point. The photometric detector D (in the typical photometric integrating sphere) is located at the sphere window, but is placed outside the sphere; the detector views the luminous flux transmitted through the translucent (diffusely transmitting and reflecting) window. (Filters required to match the spectral responsivity of the sphere-detector system to the C.I.E. photopic luminosity function [12], are placed between the detector and the sphere window.)

4.2. Lamp Flux Reflected from the Baffle

In figure 3, it is seen that if 1 lumen is incident upon the north surface of the baffle, the resulting luminance at the sphere window, denoted L_{1B}, is

$$L_{1B} = r_N 2\int_0^{\gamma} d\theta' \sin\theta' F_{BD}(\theta') L_1(\theta',\pi,\underline{\lambda})\underline{a}'^{-2} - \underline{\lambda} r_N F_{LB} \bar{L}_1(0,\pi,\underline{\lambda}).$$

In this equation, F_{LB} is the viewfactor of the lamp as viewed from the baffle, $L_1(\theta',\pi,\underline{\lambda})$ is the luminance produced at the sphere window by a 1 lumen source input to the sphere wall at a point whose colatitude angle is θ', and $\bar{L}_1(0,\pi,\underline{\lambda})$ is the average luminance produced at the sphere window, by a 1 lumen source input which illuminates uniformly the area of the sphere wall defined by the projection of the lamp from the center of the baffle, onto the sphere wall. (The preceding equation for L_{1B} is approximate, and valid to first order only. Note that F_{LB} is of first order in $\underline{a}'^2$, and that for an extended source distribution, the fraction of the total source flux emitted by the baffle can reasonably be assumed to be also of first order; thus for an extended source distribution, the term involving F_{LB} becomes of second order and hence may be discarded in a first order analysis.)

4.3. Extended Source Distributions

If the source illuminance on the sphere wall is denoted $E(\alpha)$, the total lamp flux ϕ_L^* (lumens) is given by the equation,

$$\phi_L^* = 2\pi R^2 \int_0^{\pi} d\theta' \sin\theta' E(\theta') + \phi_{LB}^*(1-r_N), \tag{4}$$

where ϕ_{LB}^* is the lamp flux which strikes the baffle. The luminance L_T

produced at the sphere window by this lamp flux is therefore

$$L_T = 2\pi R^2 \int_0^\pi d\theta' \sin\theta' E(\theta') L_1(\theta',\pi,\underline{\lambda}).$$

It is useful to define the ratio of the luminance at the sphere window to the lamp flux which produces that luminance; denote this ratio k, so that

$$k = L_T/\phi_L^*.$$

It is seen from the preceding equations that k is given in terms of $E(\alpha)$ by the equation,

$$k = \frac{2\pi R^2 \int_0^\pi d\theta' \sin\theta' E(\theta') L_1(\theta',\pi,\underline{\lambda})}{2\pi R^2 \int_0^\pi d\theta' \sin\theta' E(\theta') + \phi_{LB}^*(1-r_N)}. \tag{5}$$

It is convenient now to normalize the lamp flux to 1 lumen, for comparing lamps of different illuminance distributions. Then eq (5) becomes,

$$k = 2\pi R^2 \int_0^\pi d\theta' \sin\theta' E'(\theta') L_1(\theta',\pi,\underline{\lambda}),$$

where $E'(\theta')$ is the normalized source illuminance distribution on the wall of the sphere.

Consider now two different lamps, A and B, whose total luminous flux ratio is desired. The the fractional error $\delta k'$ in the ratio of the sphere-window luminances, L_{TB}/L_{TA}, compared with the true total luminous flux ratio, ϕ_{LB}^*/ϕ_{LA}^*, is defined,

$$\delta k' = (k_B - k_A)/k_A.$$

If the normalized illuminance distribution of lamp A is $E'_A(\theta')$, and that of lamp B is $E'_B(\theta')$, then it is seen that

$$\delta k' = \frac{\int_0^\pi d\theta' \sin\theta' \left[E'_B(\theta')L_{1B}(\theta',\pi,\underline{\lambda}_B) - E'_A(\theta')L_{1A}(\theta',\pi,\underline{\lambda}_A)\right]}{\int_0^\pi d\theta' \sin\theta' E'_A(\theta')L_{1A}(\theta',\pi,\underline{\lambda}_A)}. \qquad (6)$$

4.4. Extended Source Distributions: Approximate Model

To compute the fractional error in the ratio of the sphere-window luminances of two different lamps, with respect to the ratio of the total luminous fluxes of the lamps, one may of course use experimental lamp-illuminance distribution data, if these are available. Another approach is to use a model distribution to approximate the true lamp illuminance distribution. A useful one-parameter model distribution is the cardioid,

$$E'(\theta) = (1+g(\theta)\cos\theta)/(4\pi R^2);$$

the parameter $g(\theta)$ is assumed to be a random variable such that

$$\int_0^\pi d\theta' \sin\theta' g(\theta')\cos\theta' = 0. \qquad (7)$$

If the above cardioid lamp illuminance distribution is substituted into eq (4) for the total lamp flux ϕ^*_L, then it is seen that the total lamp flux is <u>independent</u> of $g(\theta)$, provided it satisfies eq (7) and that the lamp flux which strikes the baffle, ϕ^*_{LB}, is also independent of $g(\theta)$.

Assume now that the two different lamps, A and B, have the following parameters: lamp A is characterized by $\underline{\lambda}_A$, $\underline{b}_A$, and $g_A(\theta)$; lamp B is characterized by $\underline{\lambda}_B$, $\underline{b}_B$, and $g_B(\theta)$. Now substitute these values into eq (6) for $\delta k'$, refer to eqs (1)-(3) for $L_1(\theta,\pi,\underline{\lambda})$, and use the normalization condition (that is, the total flux from both lamp A and lamp B is 1 lumen) to obtain the approximation,

$$\delta k' \approx G_B^{-1}\left[r(\underline{\lambda}_A\underline{b}_A'^2-\underline{\lambda}_B\underline{b}_B'^2)+(1-r)^2\int_0^{2\delta}d\theta'\sin\theta'\cos\theta'(g_A(\theta')-g_B(\theta'))2^{-1}\right], \quad (8)$$

where

$$G_B = \left[(1-r)+r(2-r_S-r_N)\underline{a}'^2 4^{-1}+r\underline{\lambda}_B\underline{b}_B'^2\right],$$

and it has been assumed further that lamps A and B have similar illuminance distributions in the sense that the average value of the difference between their normalized fluxes, over any area of the sphere, is never larger in magnitude than a first order quantity.

It is helpful to consider the physical significance of the terms in eq (8) at this point. The first term,

$$r(\underline{\lambda}_A\underline{b}_A'^2-\underline{\lambda}_B\underline{b}_B'^2)/G_B,$$

represents the difference between the fraction of its total luminous flux absorbed by lamp A, and the fraction of its total luminous flux absorbed by lamp B. This is seen from the fact that the flux absorbed by the sphere wall is proportional to $4\pi R^2(1-r)$, the flux absorbed by lamp A is proportional to $4\pi r\underline{\lambda}_A\underline{b}_A^2$, the flux absorbed by lamp B is proportional to $4\pi r\underline{\lambda}_B\underline{b}_B^2$, and the flux absorbed by the baffle is proportional to $\pi r\underline{a}^2(2-r_S-r_N)$.

The second term,

$$(1-r)^2G_B^{-1}\int_0^{2\delta}d\theta'\sin\theta'\cos\theta'(g_A(\theta')-g_B(\theta'))2^{-1},$$

represents the difference between the fraction of its total luminous flux absorbed by the screened region of the sphere wall for lamp A, and the fraction of its total luminous flux absorbed by the screened region of the sphere wall for lamp B.

It should be noted that this analysis of the physical significance of $\delta k'$ (for sources of similar illuminance distributions) is <u>independent</u> of the positions of the lamps and the baffles within the sphere, provided that the first order approximation is valid; that is, the lamps and the baffles may have any position within the sphere for which the lamps are screened from the sphere window and from each other, and for which all the interaction viewfactors (the viewfactors of the baffles and the lamps as viewed from the sphere wall and from each other) are of the first order. If these conditions are fulfilled, then the sphere error $\delta k'$ in comparing the total luminous fluxes of lamps A and B is given to first order accuracy by the formula,

$$\begin{aligned}\delta k' \approx \Big[&(\text{Fractional Flux Absorbed by A}) \\ &-(\text{Fractional Flux Absorbed by B}) \\ &+(\text{Fractional Screened Flux of A Absorbed}) \\ &-(\text{Fractional Screened Flux of B Absorbed})\Big]. \qquad (9)\end{aligned}$$

Eq (9) is <u>not</u> valid, of course, if the illuminance distributions of lamps A and B differ widely.

In most practical cases, the sphere-wall absorptance is much larger than the lamp and the baffle absorptances; this means that

$$(1-r) \gg \left[r(2-r_S-r_N)\underline{a}'^2 4^{-1}+r\underline{\lambda}\underline{b}'^2\right].$$

If this inequality holds, then $\delta k'$ becomes approximately,

$$\delta k' \approx \left[r(\underline{\lambda}_A\underline{b}_A'^2-\underline{\lambda}_B\underline{b}_B'^2)(1-r)^{-1}+(1-r)\int_0^{2\delta}d\theta'\sin\theta'\cos\theta'(g_A(\theta')-g_B(\theta'))2^{-1}\right]. \quad (10)$$

Now define an average value of $(g_A(\theta)-g_B(\theta))$ over the screened region of the sphere wall by the formula,

$$\overline{(g_A(\theta)-g_B(\theta))}_{SC} = (2\sin^2\delta)^{-1}\int_0^{2\delta}d\theta'\sin\theta'\cos\theta'(g_A(\theta')-g_B(\theta')).$$

Then eq (10) for $\delta k'$ can be written,

$$\delta k' \approx \left[r(\underline{\lambda}_A\underline{b}_A'^2-\underline{\lambda}_B\underline{b}_B'^2)(1-r)^{-1}+(1-r)\overline{(g_A(\theta)-g_B(\theta))}_{SC}\sin^2\delta\right];$$

this formula assumes that the magnitude and the sign of the average value of $(g_A(\theta)-g_B(\theta))$ over the screened region of the sphere wall are known; usually this information is not available, and it is reasonable to consider $\overline{(g_A(\theta)-g_B(\theta))}_{SC}$ as a random variable. In this case, the two components of $\delta k'$ should be added in quadrature, so that the formula for $\delta k'$ becomes,

$$\delta k' \approx \left[(r(\underline{\lambda}_A\underline{b}_A'^2-\underline{\lambda}_B\underline{b}_B'^2)(1-r)^{-1})^2+((1-r)\overline{(g_A(\theta)-g_B(\theta))}_{SC}\sin^2\delta)^2\right]. \quad (11)$$

4.5. Optimum Baffle and Lamp Positions to Minimize the Error in Measuring Total Luminous Flux

Referring to figure 3 and eq (9), it is seen that the sphere error $\delta k'$ can be reduced by decreasing the magnitude of the fractional screened fluxes of lamps A and B. It is also clear from figure 3 that the fractional screened flux is a function of the positions of the baffle and the lamp within the sphere. Note particularly that in the general case, where a fraction of the lamp flux is directly incident upon the north surface of the baffle, the fractional screened flux includes that incident upon the baffle. If the lamp illuminance distribution is spherically uniform, then the fractional screened flux is equal to the sum of:

(1), the fractional solid angle (that is, the fraction of a complete sphere) subtended by the projection of the baffle from the sphere window onto the sphere wall, at the lamp center; plus (2), the fractional solid angle subtended by the baffle at the lamp center. (A more realistic situation is a spherically uniform lamp illuminance distribution up to a limiting colatitude angle which may - or may not - include part of the baffle.) Thus it is desired to find the baffle and lamp positions which will minimize the sum of (1) and (2) above; this is equivalent to minimizing the total screened solid angle, as viewed from the lamp center. It can be shown that this total screened solid angle is approximately equal to $\pi \underline{a}'^2[(\underline{h}'+\underline{l}')^{-2}+4(1-\underline{h}')^{-2}(1-\underline{l}')^{-2}]$, where $\underline{h}'=\underline{h}/R$ and $\underline{l}'=\underline{l}/R$.

The positions of the baffle and the lamp which minimize the preceding expression for the total screened solid angle, viewed from the lamp center, are found to be

$$\underline{h}' = \underline{l}' = 0.212.$$

Thus the optimum positions of the baffle and the lamp when the lamp illuminance distribution is spherically uniform, are: (1), the baffle placed 0.212 of the sphere radius from the center of the sphere, and 0.788 of the sphere radius from the sphere window; and (2), the lamp center placed 0.212 of the sphere radius from the center of the sphere, and 1.212 of the sphere radius from the sphere window; both the baffle and the lamp are assumed to be coaxial with the sphere axis which passes through the center of the sphere window.

If the lamp position is fixed, and only the baffle position is varied, then to every lamp position corresponds a baffle position which minimizes the total screened solid angle; for a spherically uniform lamp illuminance distribution, the optimum values of $\underline{h}'$ and $\underline{l}'$ are related by the equation,

$$\underline{h}' = [(1-\underline{l}')^{2/3}-4^{1/3}]/[(1-\underline{l}')^{2/3}+4^{1/3}].$$

If, for example, $\underline{l}'=0$ (the lamp at the center of the sphere), then the optimum value of $\underline{h}'$ is 0.386.

Now assume that the lamp has a spherically uniform illuminance distribution up to a limiting colatitude angle which <u>excludes</u> the baffle; in other words, <u>no</u> lamp flux strikes the baffle directly. Then it is found that the optimum baffle and lamp positions are

$$\underline{h}' = \underline{l}' = 0;$$

that is, the baffle and the lamp should both be located at the center of the sphere. (Note that the optimum values of $\underline{h}'$ and $\underline{l}'$ derived above for a coaxial baffle and lamp configuration, <u>also</u> hold for baffle and lamp positions along any chord of the sphere emanating from the sphere window, <u>except</u> that $\underline{h}'$ and $\underline{l}'$ must be replaced by $\underline{h}'\cos\Sigma$ and $\underline{l}'\cos\Sigma$, respectively, and these displacements along the chord are referenced to the chord center, which is a distance $R\cos\Sigma$ from the sphere window; here Σ is the angle between the chord and the sphere axis coaxial with the sphere window.)

4.6. Upper Bound on the Fractional Sphere Error

Eq (11) shows that: (1), if two lamps have the same fractional absorptance, the dominant term in the error in comparing their total luminous fluxes in an integrating sphere (if they have similar illuminance distributions) is the difference in their fractional screened fluxes; (2), if two lamps have the same fractional screened flux, the dominant term in the error in comparing their total luminous fluxes is the difference in their fractional absorptances.

It is useful to estimate an upper bound for the fractional sphere error in present NBS techniques for comparing the total luminous fluxes of 1000-watt incandescent lamps in a 2-meter integrating sphere [9]. The geometry employed is as follows: the baffle is placed 0.55 meter from the sphere window and is shaped roughly like a projection of the 1000-watt lamps; the baffle dimensions are 0.15 meter by 0.23 meter (the equivalent radius is 0.093 meter, to yield the same area); the lamps are separately placed at the center of the sphere for total flux measurement (that is, only one lamp is in the sphere at the time of measurement); the radius of the lamps is roughly 0.08 meter. The reflectance of the sphere wall and of the baffle surfaces is assumed to be 0.90 [7]. To proceed further, it is necessary to introduce the following reasonable assumptions: (1), the difference in the absorptance $\underline{\lambda}$ of the lamps is less than 0.05; (2), the magnitude of the quantity $\overline{(g_A(\theta)-g_B(\theta))}_{SC}$ is less than 0.5 (that is, the fractional screened fluxes of the lamps differ by less than half the fractional screened flux of a lamp of spherically uniform illuminance distribution).

With the preceding assumptions, eq (11) for the fractional sphere error $\delta k'$ yields 0.29% for the component of the sphere error due to the difference in fractional lamp absorptance, and 0.13% for the component of the sphere error due to the difference in fractional screened flux. Adding these two components in quadrature, as indicated in eq (11), yields an estimated upper bound of 0.32% for the fractional sphere error in present NBS techniques for comparing the total luminous fluxes of 1000-watt incandescent lamps in a 2-meter integrating sphere.

4.7. Factors Neglected

The preceding analysis has neglected several factors and their effects on the sphere error in comparing the total luminous fluxes of lamps in the integrating sphere. These factors are: (1), varying spectral reflectance of the sphere and the baffle coatings, (2), varying spectral absorptance of the lamp, (3), spatial variations in the sphere-wall reflectance, (4), specular component of the sphere-wall reflectance, (5), reflectance of the lamp, (6), deviations of the sphere-window transmittance from the cosine law, (7), detector signal-to-noise considerations, (8), deviations of the sphere wall from perfect sphericity.

Referring to eq (11), it is seen that the effect of increasing the spectral reflectance of the sphere wall is to increase the fractional fluxes absorbed by the lamps (whose total luminous fluxes are to be compared), and to decrease the absorption of the fractional screened fluxes of the lamps. It is also seen from eq (11) that the effect of increasing the spectral absorptances of the lamps is to increase the fractional fluxes absorbed by the lamps.

Theoretically, if the spectral distributions in the visible are given of, (1), the spectral reflectance $r(\lambda)$ of the sphere wall (λ is the wavelength), (2), the spectral absorptances $\underline{\lambda}_A(\lambda)$ and $\underline{\lambda}_B(\lambda)$ of the lamps A and B, (3), the spectral transmittance $t(\lambda)$ of the sphere window, and (4), the spectral luminous fluxes of the lamps, $J'_A(\lambda)$ and $J'_B(\lambda)$, normalized to unit luminous flux, then the average sphere error over the visible spectrum, $\overline{\delta k'}$, can be computed.

The approximate formula for $\overline{\delta k'}$ is (accurate to first order, and assuming that lamps A and B have similar spectral luminous flux distributions):

$$\overline{\delta k'} \approx \Big(\Big[\int_{\lambda 1}^{\lambda 2} d\lambda'(r(\lambda')t(\lambda')(1-r(\lambda'))^{-1}(J_B'(\lambda')-J_A'(\lambda'))\Big]^2$$

$$+\Big[\int_{\lambda 1}^{\lambda 2} d\lambda' r^2(\lambda')t(\lambda')(1-r(\lambda'))^{-2}J_A'(\lambda')(\underline{\lambda}_A(\lambda')\underline{b}_A'^2-\underline{\lambda}_B(\lambda')\underline{b}_B'^2)\Big]^2$$

$$+\Big[\int_{\lambda 1}^{\lambda 2} d\lambda' r(\lambda')t(\lambda')J_A'(\lambda')\sin^2\delta\overline{(g_A(\theta)-g_B(\theta))}_{SC}\Big]^2\Big)^{0.5}$$

$$X\Big(\int_{\lambda 1}^{\lambda 2} d\lambda' r(\lambda')t(\lambda')(1-r(\lambda'))^{-1}\Big)^{-1}. \qquad (12)$$

Eq (12) is analogous to eq (11) for $\delta k'$ in that the error components are treated as random variables and hence are added in quadrature to obtain the total sphere error $\overline{\delta k'}$; λ_1 and λ_2 are the limits of the visible spectrum (approximately 0.38-0.78 micrometer [12]); $J_A'(\lambda)$ and $J_B'(\lambda)$ are normalized so that

$$\int_{\lambda 1}^{\lambda 2} d\lambda' J_A'(\lambda') = \int_{\lambda 1}^{\lambda 2} d\lambda' J_B'(\lambda') = 1.$$

Spatial variations in the reflectance of the sphere wall may be a significant source of error. Experimental work is needed to establish the magnitude of this effect.

The specular component of the sphere-wall reflectance is estimated to produce a second order sphere error in comparing the total luminous fluxes of two lamps of similar illuminance distributions; hence this error can be neglected in a first order analysis. (To ensure the validity of this assumption, the baffle must be of adequate size to make the maximum angle of reflection, from the portions of the sphere wall directly illuminated by the source, to the sphere window, less than the critical angle at which specularity effects become important.)

The effect of luminous flux reflected from the lamp bulb is estimated to be negligible for extended sources.

The deviations of the sphere window from cosine-law transmittance (that is, from uniformly diffuse transmittance) are probably a significant source of error, except where the lamps compared have very similar illuminance distributions. Probably the best procedure (if the illuminance at the sphere window is high enough to provide an adequate signal-to-noise ratio at the detector) is to remove the window and replace it with a small auxiliary integrating sphere; the inner surface of this auxiliary sphere is then viewed by the detector.

Detector signal-to-noise considerations are important in limiting the maximum sphere diameter which can be used to compare lamps of a given flux output. At NBS, for example, the 2-meter diameter integrating sphere is not used to measure total luminous fluxes of less than 200 lumens [7]. Thus to optimize the accuracy of measurements of total luminous flux, the sphere diameter should be roughly that which equalizes the magnitudes of the computed sphere error $\overline{\delta k'}$, and the measurement error due to detector noise.

The effect of deviations of the sphere wall from perfect sphericity, on the sphere error in comparing two lamps of similar illuminance distributions, can be shown to be of second order (and hence negligible) for deviations which consist of flattened portions of first order area (that is, the area of each flattened portion is comparable with the baffle area).

5. Conclusions

The approximate mathematical theory of the photometric integrating sphere, developed in section 4, leads to the following conclusions:

For typical sphere parameters, the fractional sphere error in comparing the total luminous fluxes of two incandescent lamps is probably less than 0.2% <u>if</u>: (1), the viewfactor of the baffle from the sphere window is less than 0.025; (2), the lamp flux directly incident upon the baffle is negligible; (3), the lamps compared have similar illuminance distributions; (4), the lamp dimensions are similar; (5), the lamp absorptances differ by less than 0.02; (6), the total flux of each lamp is measured with <u>only</u> that lamp in the sphere (the screening effect and the absorptance of the other lamp introduce errors if it is also present in the sphere).

6. References

[1] Ulbricht, R., Die bestimmung der mittleren raümlichen licht-intensität durch nur eine messung, Elektrotechnische Zeit. 21, 595 (1900).

[2] International Lighting Vocabulary (Bureau Central de la CIE, Paris, 1970).

[3] Ulbricht, R., Das Kugelphotometer, pp. 1-12 (R. Oldenbourg, Munich and Berlin).

[4] Rosa, E. B., and Taylor, A. H., Theory, construction, and use of the photometric integrating sphere, Sci. Papers Natl. Bur. Stand. (U.S.) 18, 281 (Aug. 1922).

[5] Keitz, H. A., Light Calculations and Measurements, p. 345 (MacMillan and Co., New York, 1971).

[6] Natl. Bur. Stand. (U.S.), Tech. News Bull. 56, 153 (June 1972).

[7] Private communication from V. I. Burns of the NBS Optical Radiation Section.

[8] McSparron, D., Mohan, K., Raybold, R., Saunders, R., and Zalewski, E., Spectroradiometry and Conventional Photometry, an Interlaboratory Comparison, Natl. Bur. Stand. (U.S.), Tech. Note 559, p. 186 (Nov. 1970).

[9] Burns, V. I., and McSparron, D. A., Photometric Calibration Procedures, Natl. Bur. Stand. (U.S.), Tech. Note 594-3, p. 21 (Nov. 1972).

[10] For a typical presentation see: Walsh, J. W. T., Photometry, chap.7 (Dover Publications, Inc., New York, 1958).

[11] Safwat, H. H., Effect of centrally located samples in the integrating sphere, J. Opt. Soc. Am. 60, 534 (April 1970).

[12] American Institute of Physics Handbook, 3rd ed., chap. 6, p. 184 (McGraw-Hill Book Co., New York, 1972).

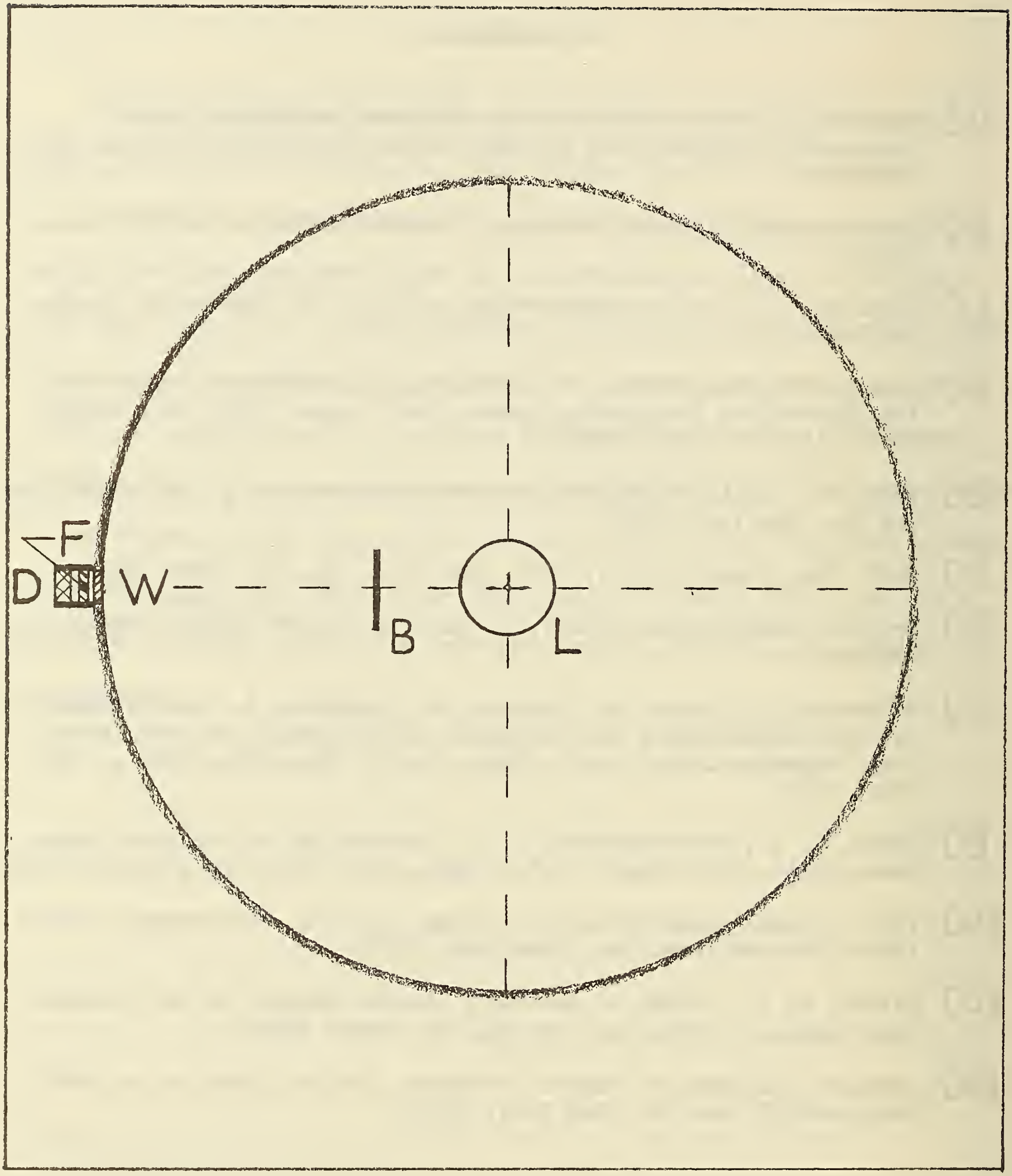

Figure 1. Cross section of typical integrating sphere: B-baffle, D-detector, F-filter, L-lamp, W-window.

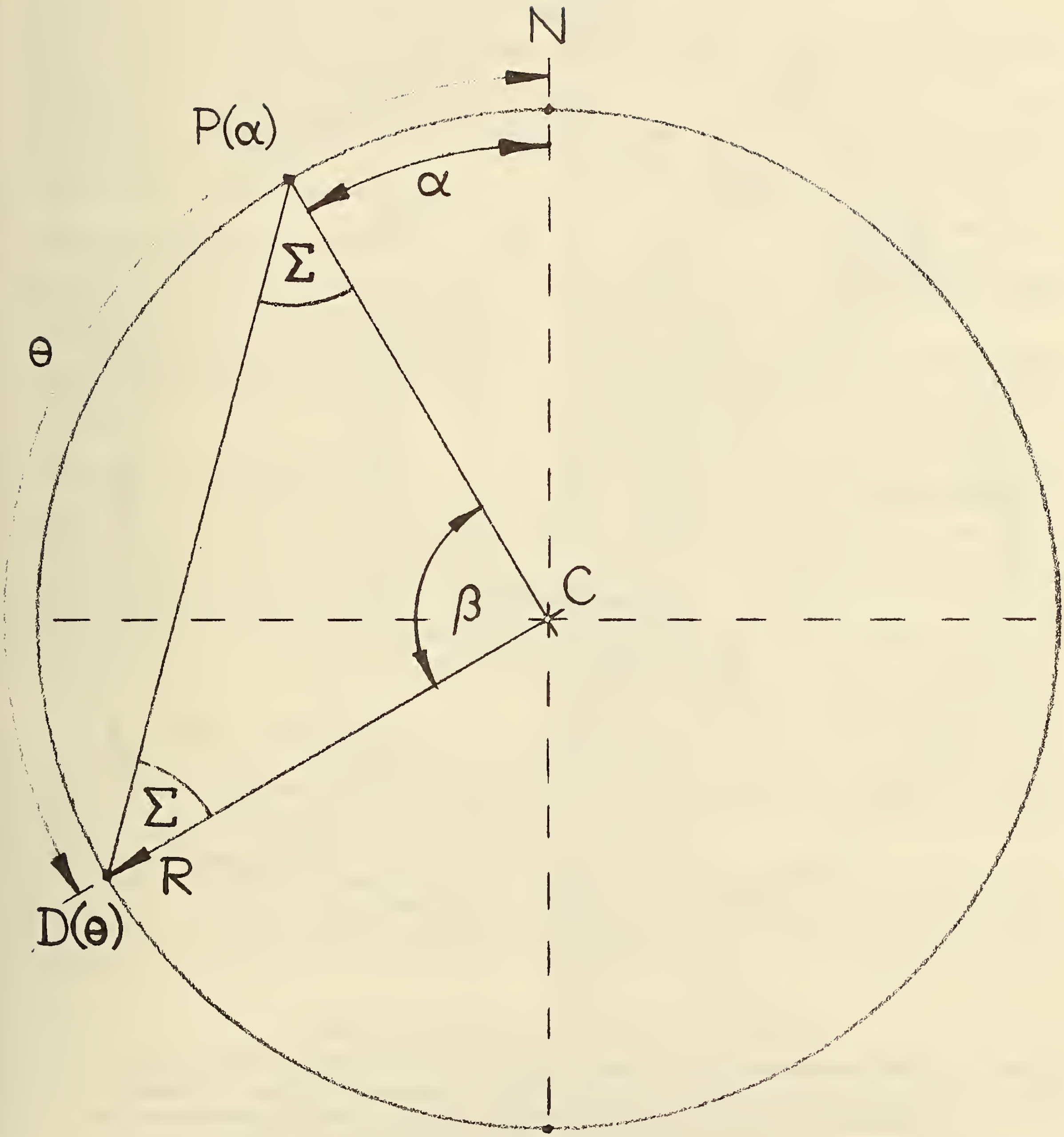

Figure 2. Cross section of empty sphere; viewfactor geometry.

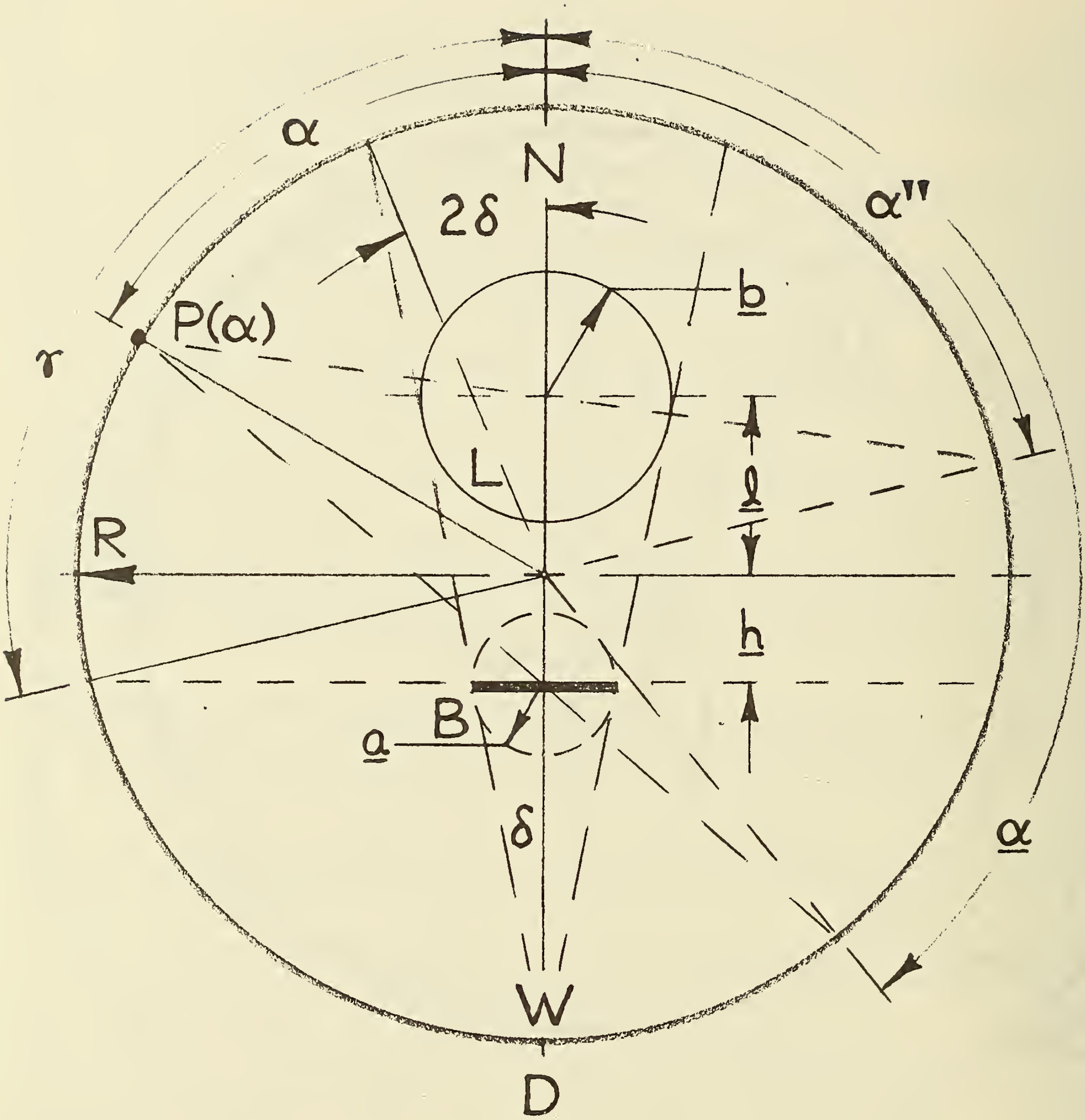

Figure 3. Integrating sphere geometry: B-baffle, D-detector, L-lamp, N-north pole, P(α)-source point, R-sphere radius, W-sphere window; a-baffle radius, b-lamp radius, h-displacement of baffle south of sphere equator, l-displacement of lamp north of sphere equator; α-colatitude angle of projection of baffle center from P(α) to opposite sphere wall, α"-colatitude angle of projection of lamp center from P(α) to opposite sphere wall, δ-half angle subtended by baffle at window, γ-colatitude angle of baffle plane intersection with sphere.

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