NBS TECHNICAL NOTE 594-10

U.S. DEPARTMENT OF COMMERCE / National Bureau of Standards

Optical Radiation Measurements:

The NBS 20-, 60-, and 85- Degree Specular Gloss Scales

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[2] Part of the Center for Radiation Research.
[3] Located at Boulder, Colorado 80302.
[4] Part of the Center for Building Technology.

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Preface

This is the tenth issue of a series of Technical Notes entitled OPTICAL RADIATION MEASUREMENTS. The series will consist primarily of reports of progress in, or details of, research conducted in radiometry, photometry, and spectrophotometry in the Optical Radiation Section of the Heat Division.

The level of presentation in OPTICAL RADIATION MEASUREMENTS will be directed at a general technical audience. The equivalent of an undergraduate degree in engineering or physics, plus familiarity with the basic concepts of radiometry and photometry [e.g., G. Bauer, Measurement of Optical Radiations (Focal Press, London, New York, 1965)], should be sufficient for understanding the vast majority of material in this series. Occasionally a more specialized background will be required. Even in such instances, however, a careful reading of the assumptions, approximations, and final conclusions should permit a non-specialist to understand the gist of the argument if not the details.

At times, certain commercial materials and equipment will be identified in this series in order to adequately specify the experimental procedure. In no case does such identification imply recommendation or endorsement by the National Bureau of Standards, nor does it imply that the material or equipment identified is necessarily the best available for the purpose.

Any suggestions readers may have to improve the utility of this series are welcome.

Henry J. Kostkowski, Chief
Optical Radiation Section
National Bureau of Standards

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

The NBS 20-, 60-, and 85- Degree Specular Gloss Scales

Jack J. Hsia

The 20-, 60-, and 85- degree specular gloss scales are established at NBS with an accuracy of about one half gloss unit. This is one of many programs of the spectrophotometry group of the Optical Radiation Section of NBS directed toward improving the accuracy and assurance of spectrophotometric measurements made throughout the scientific and industrial community.

The specular gloss scales are established through an unique technique employing polarized light flux both parallel and perpendicular to the plane of incidence. General calibration equations are derived. NBS instrumentation and measurement procedures are discribed. Instrument calibration and error analyses are performed. Some of the analyses can also be applied to reflectance measurements in general.

Key Words: Accuracy; appearance; error analysis; gloss; photometry; reflectance; scattering; spectrophotometry; specular gloss.

1. Introduction

The fundamental purpose of the spectrophotometry group of the Optical Radiation Section of NBS is to improve the accuracy and usefulness of spectrophotometric measurements. The purpose of this Technical Note is to present the technique and instruments employed at NBS to maintain the 20-, 60-, and 85- degree specular gloss scales according to the ASTM method. This technique and some of the analyses will be used in our future work on specular gloss and reflectance measurements in general.

Specular gloss is just one of many aspects of appearance of material surfaces [1, 2]+, but it is not within the scope of this Technical Note to discuss the physiological and psychological aspects concerning our sensations of gloss. We will treat gloss as a special type of reflectance*. Sections 2 and 3 provide the definition and calibration equations. Section 4 deals with the refractive index determination of the primary working standard. Details of the NBS instrument and the measurement procedures are given in section 5 and 6. Calibrations of instrument and error analyses based on theory and calibration results are given in detail in section 7. Finally, in the summary the error budget is presented to give the estimated uncertainty of measurements for different geometries.

2. Definition of Specular Gloss

According to the ASTM Method [3], specular gloss is defined as the ratio of the luminous flux reflected from, to that incident on, a specimen at the specular direction for specified solid angles and system spectral response. To set the specular gloss scale, polished black glass with a refractive index of 1.567 is assigned a value of 100 for each of the three geometries, 20, 60, and 85 degree [3].

+Figures in brackets indicate the literature reference at end of this technical note.

*Earlier works on gloss by former and present members of the NBS staff such as Dr. D.B. Judd, Mr. R.S. Hunter, Mr. H.K. Hammond, III, and Mr. I. Nimeroff, have led to the establishment of the present ASTM method [3].

3. Calibration Equations for Specular Gloss

General Equations will be derived for the gloss and luminous flux ratio using an unpolarized light source according to the definition. If the light source is not completely unpolarized, separate measurements should be made with the light source polarized parallel and perpendicular to the plane of incidence. A calibration equation for the luminous flux ratio is derived to permit converting the measured quantities with polarized components to gloss for an unpolarized light source.

3.1 Defining Measurement

The method and mathematical notations of NBS Technical Note 594-9 [4] will be used in the development of calibration equations. In this method, each point in the space is assigned to either the instrument space, the sample space, or the boundary surface between the two spaces, and the measurement is defined on the boundary surface along.

According to paragraph 6 of the ASTM Test Method [3], the sample surface should have good planarity. Therefore, it is sensible to designate the boundary surface as a plane parallel to the average surface and tangent to it at its farthest point toward the instrument space. The gloss value of sample x, g_x can be written as

$$g_x = F_{xs}\, g_s \tag{3.1}$$

where g_s is the gloss value of the standard. F_{xs} is the luminuous flux ratio which is expressed as

$$F_{xs} = \frac{\int^5 L\, S_x\, R\, \vec{U}\cdot d\vec{A}\, d\Omega\, \vec{u}\cdot d\vec{a}\, d\omega\, d\lambda}{\int^5 L\, S_s\, R\, \vec{U}\cdot d\vec{A}\, d\Omega\, \vec{u}\cdot d\vec{a}\, d\omega\, d\lambda}\ ^{*} \tag{3.2}$$

where L is the radiance input; S_x and S_s are the scattering functions of the sample and of the standard; R is the relative responsivity of the receiver; $\vec{U}$ and $\vec{u}$ are the direction vectors of travel of the incident and emergent radiation; $d\Omega$ and $d\omega$ are elements of solid angle oriented in the directions $\vec{U}$ and $\vec{u}$; $d\vec{A}$ and $d\vec{a}$ are elements of area through which the energy passes onto the sample and emerges from the sample; and $d\lambda$ is the element of wavelength.

Paragraph 4.4 of the ASTM method provides that the spectral characteristics of the source-filter-photoreceiver combination shall yield the CIE source C weighted for CIE luminous efficiency function. The spectral characteristic of the radiance input L is represented by the source C relative weighting function C_λ and the relative responsivity R with the luminous response V_λ. Since the ASTM method makes no mention of the distribution of L and R with respect to points on the boundary plane, it is logical to assume an ideal, namely that L is non-zero over a finite area and that R is uniform over the entire boundary. The specimen area to be measured is not specified, but this creates no problem if the gloss of the sample is uniform.

Figure 1 shows a schematic diagram of the source and receiver apertures. BOb forms the plane of incidence.+ $d\vec{A}$, $d\vec{a}$ are normal to the sample surface and are in the plane of incidence. Plane XY is perpendicular to OB and plane xy is perpendicular to Ob and the

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*the symbol $\int^n$ indicates integration with respect to n variables.

+Plane of measurement.

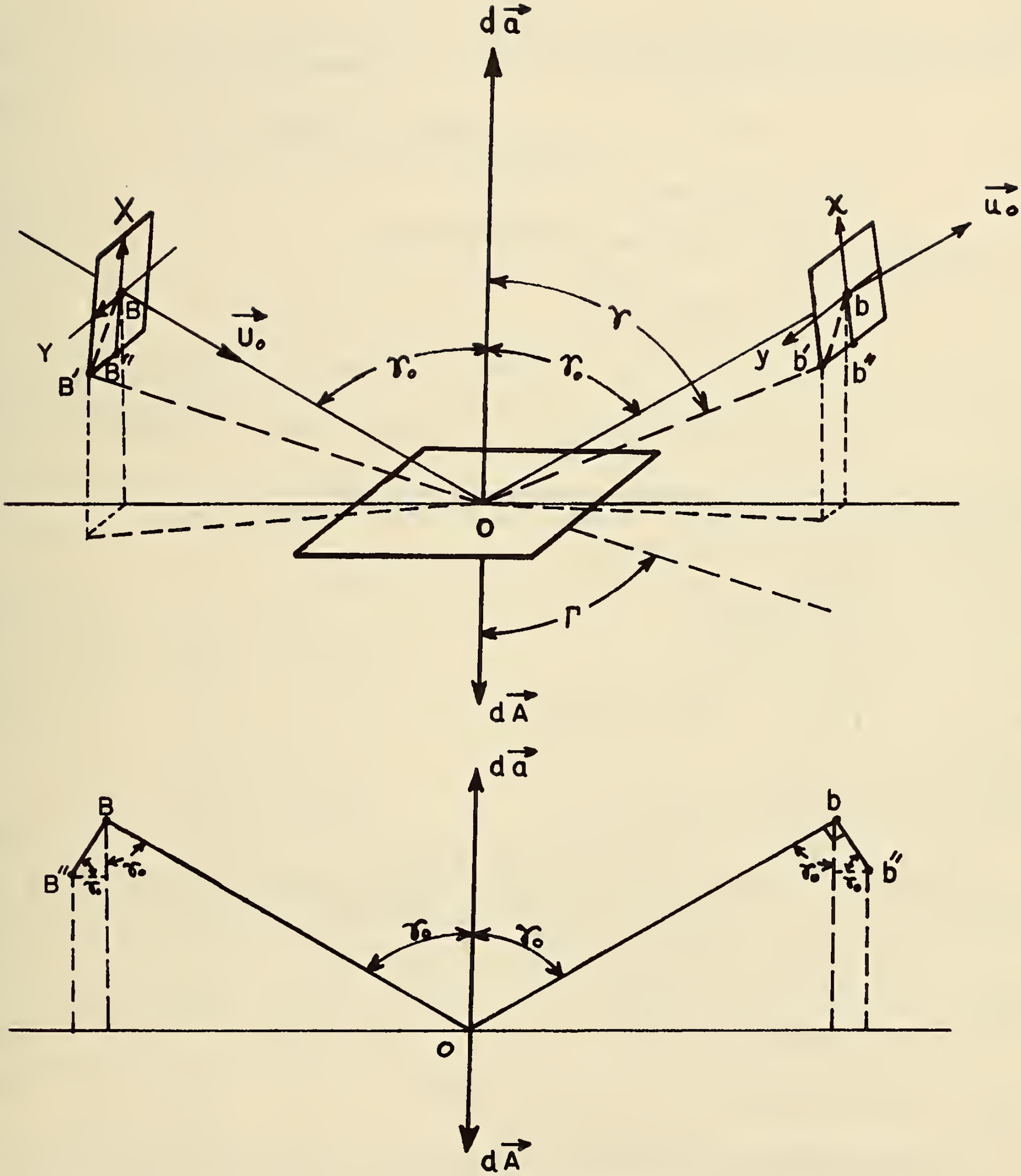

Fig. 1 A diagram describing the coordinates of the source and receiver apertures

points B and b are the origins (0,0) in (X,Y) and (x,y), respectively. The X and x axes lie in the plane of incidence. (X,Y) and (x,y) are scaled so that OB=1 and Ob=1, respectively. The geometric characteristics of L and R are indicated by the name "specular gloss" with specular angle γ_o. The angular limitations described in paragraph 4.2 and table 1 of the ASTM method require that L=0 outside of the source aperture and that R=0 outside of the receiver aperture.

Thus

$L=C_\lambda$ for $\gamma_o-\Delta\Gamma_1 < \tan^{-1} X < \gamma_o + \Delta\Gamma_1$ in the plane of incidence

and $o \leq |\tan^{-1} Y| < \Delta\Gamma_2$ from the plane of incidence

over a finite area

and zero elsewhere where Γ is the angle of incidence, and

$R=V_\lambda$ over the entire boundary

for $\gamma_o-\Delta\gamma_1 < \tan^{-1} x < \gamma_o + \Delta\gamma_1$ in the plane of incidence

and $o \leq |\tan^{-1} y| < \Delta\gamma_2$ from the plane of incidence

and is zero at other angles. Where γ is the angle of viewing.

Equation (3.2) can be rewritten as

$$F_{xs} = \frac{\int C_\lambda D_x V_\lambda \, d\lambda}{\int C_\lambda D_s V_\lambda \, d\lambda} \tag{3.5}$$

Where $$D_{x \text{ or } s} = \int^4 S_{x \text{ or } s} \vec{U}\cdot d\vec{A} \; u\cdot d\vec{a} \; d\Omega \, d\omega \tag{3.6}$$

Equation (3.6) can further be expressed as

$$D_{x \text{ or } s} = \int^4 S_{x \text{ or } s} \cos\Gamma \, dA \cos\gamma \, da \, d\Omega \, d\omega \tag{3.7}$$

where

$$\cos\Gamma = (\cos\gamma_o + X \sin\gamma_o)(1 + X^2 + Y^2)^{-\frac{1}{2}} \tag{3.8}$$

$$d\Omega = dXdY(1+X^2 + Y^2)^{-3/2} \tag{3.9}$$

$$\cos\gamma = (\cos\gamma_o + x \sin\gamma_o)(1 + x^2 + y^2)^{-\frac{1}{2}} \tag{3.10}$$

$$d\omega = dxdy(1 + x^2 + y^2)^{-3/2} \tag{3.11}$$

Equation (3.5) implies the following assumptions for a defined measurement:

a. The experimental conditions are the same for both the sample and primary standard, that is, no change in source, filters, aperture sizes, angles of incidence and viewing, detector and gains.

b. The detector system response is linear.

c. The source is unpolarized.

3.2 Partially Polarized Source

Since the measured gloss is sensitive to source polarization, when a partially polarized light source is used, separate measurements should be made with light polarized parallel and perpendicular to the plane of incidence. The gloss value is then computed in such a way that fulfills the requirement that the light source is completely unpolarized.

An unpolarized light can be assumed to consist of two equal components: a component, $C_{\lambda 1}$, polarized parallel to the plane of incidence, and a component, $C_{\lambda 2}$, polarized perpendicular to the plane of incidence. For a detection system insensitive to polarization, equation (3.5) can be rewritten as

$$F_{xs} = \frac{\int C_{\lambda 1} D_{x1} V_\lambda \, d\lambda + \int C_{\lambda 2} D_{x2} V_\lambda \, d\lambda}{\int C_{\lambda 1} D_{s1} V_\lambda \, d\lambda + \int C_{\lambda 2} D_{s2} V_\lambda \, d\lambda} \tag{3.12}$$

Since $C_{\lambda 1} = C_{\lambda 2}$, equation (3.12) can be simplified to

$$F_{xs} = (\bar{D}_{x1} + \bar{D}_{x2})/(\bar{D}_{s1} + \bar{D}_{s2}) \tag{3.13}$$

$$\text{where } \bar{D} = \frac{\int C_\lambda D \, V_\lambda \, d\lambda}{\int C_\lambda V_\lambda \, d\lambda} \tag{3.14}$$

For a partially polarized source, C'_λ, the receiver signals, I, for sample and standard measured with light polarized parallel and perpendicular to the plane of incidence, are:

$$I_{x1} = K \int C'_{\lambda 1} D_{x1} V_\lambda \, d\lambda \tag{3.15}$$

$$I_{x2} = K \int C'_{\lambda 2} D_{x2} V_\lambda \, d\lambda \tag{3.16}$$

$$I_{s1} = K \int C'_{\lambda 1} D_{s1} V_\lambda \, d\lambda \tag{3.17}$$

$$I_{s2} = K \int C'_{\lambda 2} D_{s2} V_\lambda \, d\lambda \tag{3.18}$$

where K is the instrument constant. But $C'_{\lambda 1} \neq C'_{\lambda 2}$, since the light source is partially polarized. $C'_{\lambda 1}$ and $C'_{\lambda 2}$ can be expressed in terms of $C_{\lambda 1}$ and $C_{\lambda 2}$

$$C'_{\lambda 1} = \ell \, C_{\lambda 1} \tag{3.19}$$

and

$$C'_{\lambda 2} = m \, C_{\lambda 2} \tag{3.20}$$

where ℓ and m are proportionality constant that have been measured and found to be roughly independent of wavelength.

The ratio of the V_λ weighted irradiance of parallel-polarized light to that of perpendicular-polarized light is called the polarization ratio, α,

$$\alpha = \frac{\int C'_{\lambda 1} V_\lambda \, d\lambda}{\int C'_{\lambda 2} V_\lambda \, d\lambda} \tag{3.21}$$

Inserting equations (3.19) and (3.20) into equation (3.21) and using the relation $C_{\lambda 1} = C_{\lambda 2}$, the expression for α reduces to

$$\alpha = \ell/m \quad \text{or} \quad \ell = \alpha\, m \tag{3.22}$$

Using relations (3.19), (3.20), (3.22) and (3.14) in equations (3.15) to (3.18), we have

$$\bar{D}_{x1} = I_{x1}/(K\,\alpha\, m\int C_{\lambda 1}\, V_\lambda\, d\lambda) \tag{3.23}$$

$$\bar{D}_{x2} = I_{x2}/(K\, m\int C_{\lambda 2}\, V_\lambda\, d\lambda) \tag{3.24}$$

$$\bar{D}_{s1} = I_{s1}/(K\,\alpha\, m\int C_{\lambda 1}\, V_\lambda\, d\lambda) \tag{3.25}$$

$$\bar{D}_{s2} = I_{s2}/(K\, m\int C_{\lambda 2}\, V_\lambda\, d\lambda) \tag{3.26}$$

where $C_{\lambda 1} = C_{\lambda 2}$.

Combining equations (3.13), (3.23), (3.24), (3.25) and (3.26), the luminous flux ratio can be expressed by the measured quantities I_{x1}, I_{x2}, I_{s1}, I_{s2}, and α:

$$F_{xs} = \frac{I_{x1} + \alpha\, I_{x2}}{I_{s1} + \alpha\, I_{s2}} \tag{3.27}$$

Thus the gloss value of sample x can be written as

$$g_x = \frac{I_{x1} + \alpha\, I_{x2}}{I_{s1} + \alpha\, I_{s2}} \cdot g_s \tag{3.28}$$

where g_s is the gloss value of the standard.

4. Refractive Index Determination of the Primary Working Standard

The primary working standard is a highly polished, plane, black glass. Its refractive index for sodium light at 589 nm was determined by means of the Abbe total- reflectometer which has a glass hemisphere with a refractive index of 1.7971 for sodium light. The method consists of directly measuring the angle of total reflection at the plane surface of the hemisphere, on which the black glass is laid. Intervening is a thin liquid film of refractive index higher than that of the black glass, in order to avoid any chance of total reflection before the black glass surface is reached. A sodium light was used to illuminate from the hemispherical side, and a telescope attached to a rotatable arm with an angular index was used to view the bright and dark fields and determine the critical angle, ϕ_c. The refractive index of the black glass is equal to 1.7971 sin ϕ_c.

The total-reflectometer was first calibrated using NBS #38 refractive index standard with a value of 1.522727 at 25°C. This standard is in the shape of a triangular prism with one surface highly polished. Its refractive index measured on the total-reflectometer at 25°C is 1.5228. Thus the accuracy of the total-reflectometer in measuring refractive index around 1.52 is 0.0001.

The refractive index of the black glass standard (NBS NO. 600315-B) is determined to be 1.5269 ± 0.0001. The primary working standard is assigned, according to the ASTM method [3], gloss values of 89.2, 93.6, and 99.4 for the 20, 60 and 85 degree geometries, respectively, (See Appendix B).

The uncertainty of ± 0.0001 in refractive index will contribute a systematic error in the measurement of a high gloss sample of no more than 0.027, 0.016, and 0.002 gloss unit for specular angles of 20, 60, and 85 degrees, respectively. The luminous flux ratio F_{xs} is smaller for a low gloss sample. Thus, according to equation (3.1), an even smaller error would result in the low gloss sample by this uncertainty in the primary working standard.

5. Instrument

The NBS goniophotometer is used for our gloss measurements. The design geometry is that of a monoplane goniophotometer with a fixed source unit, and with specimen and receiver units each rotatable about an axis normal to the plane of incidence. A schematic drawing of the goniophotometer is shown in Figure 2.

5.1 Light Source

The source contains a 100-watt incandescent lamp (rated for 6.6A) with a singly-coiled tungsten filament. The current in the tungsten lamp is regulated by a power supply using external sensing (0.01% regulation).

The lamp filament is imaged onto a precision source aperture through an achromatic condenser lens. With another achromatic lens (f.l. 192 mm), the flux that leaves the source aperture is collimated into a 50-mm diameter beam. Either an iris diaphragm or an adjustable rectangular diaphragm is used to limit the size and the shape of the collimated beam. Filters can be used to change the spectral distribution. A rotatable, glass-laminated plastic polarizer is used to provide linearly polarized light. A shutter is mounted after the polarizer.

5.2 Specimen Mount

The specimen mount is on a large rotatable table (480 mm in diameter) engraved with an angle scale. It will hold specimens that range in size up to 100 mm by 200 mm rectangular and up to 10 mm thick. The table can be rotated manually or by using a stepping motor.

5.3 Receiver

The receiver optical components are enclosed in a light-tight baffle tube, at the front of which is placed an iris diaphragm. The acromatic collector lens (f.l. 508 mm), 70 mm diameter, is placed behind the iris diaphragm. Receiver apertures of different sizes may be mounted at the focal point of the collector lens to restrict the solid angle within which the flux from the specimen is collected.

A thirty-cm diameter sphere coated with $BaSO_4$ powder is placed directly behind the receiver aperture. The detector, mounted in a housing attached to the sphere, is an 11-stage, end-on photomultiplier with an S-20 response. The high voltage power supply is voltage regulated to 0.001% and the anode current is measured by using a preamplifier and a picoammeter, with the voltage output from the picoammeter measured by a digital voltmeter.

5.4 Data Acquisition and Data Processing

Data are obtained with a data acquisition and control system which consists of a teletypewriter, logic circuits, and a timer. The control system has start and stop codes, codes for recording data, and codes for sequentially actuating the electrical switches. The angles of incidence and viewing and the shutter can be programmed with a punched paper tape, or a minicomputer, allowing data to be taken automatically.

5.5 Equipment

Appendix A[5] contains more detailed information of apparatus employed and gives manufacturer and model number.

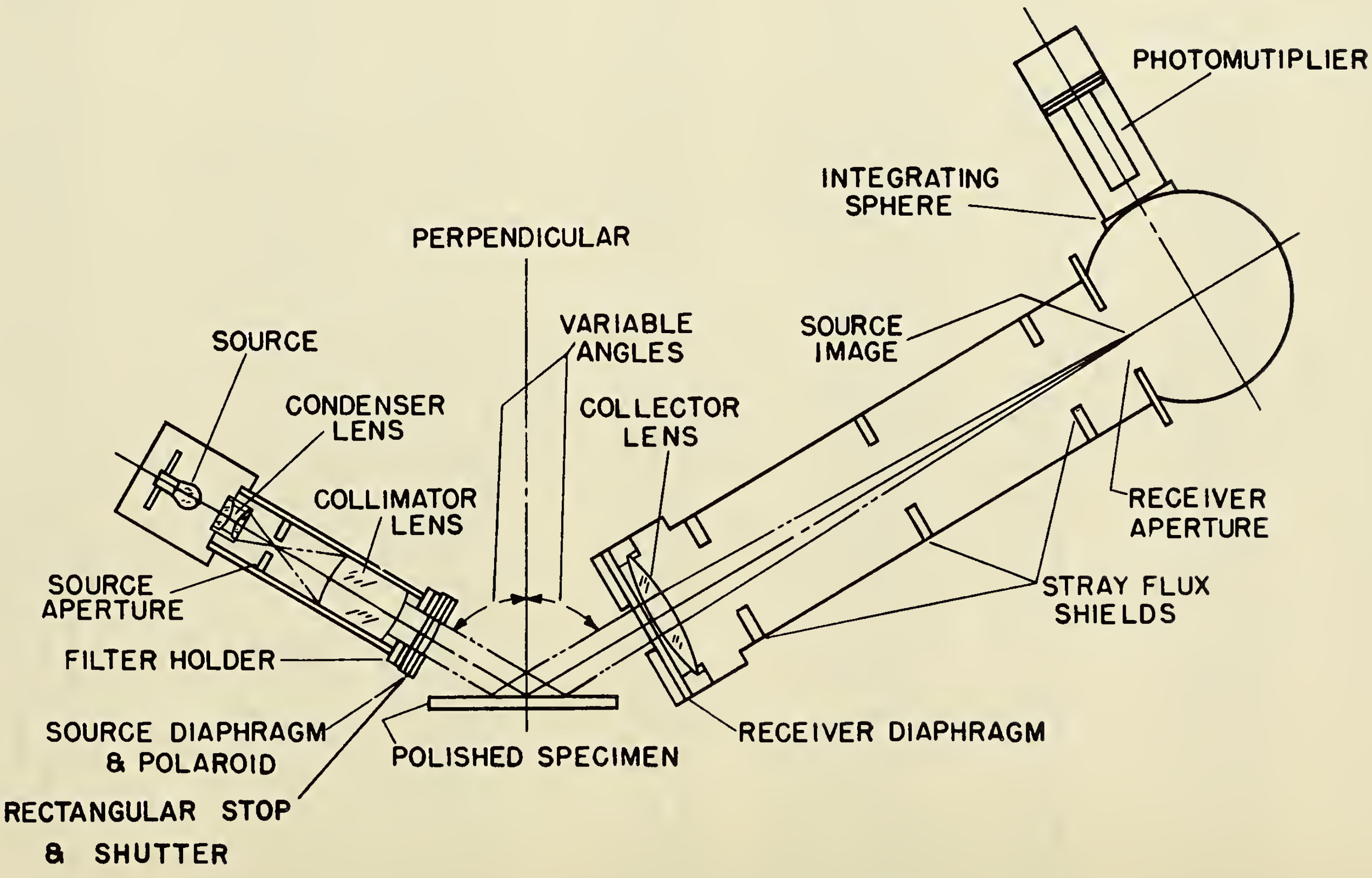

Fig. 2 NBS Goniophotometer

6. Procedure of Measurement

The polarization ratio, α, is defined in section 3.2 as the ratio of the V_λ weighted irradiance of the light polarized parallel to that polarized perpendicular to the plane of incidence. Polarization ratios of the light source are determined before and after each set of measurements.

To illustrate how a set of data are obtained, the procedure for three sample measurements will be discussed which will demonstrate how measurements are made on from one to three samples. A set of measurements consisting of 35 readings, with 25-second intervals, is recorded on a punched paper tape according to the sequence:

0,I0,0,I1,0,I2,0,I3,	1
0,I0,0,I3,0,I2,0,I1,0,I0	2
0,I1,0,I2,0,I3,0,I0,0,	2
I3,0,I2,0,I1,0,I0,0	1

All readings in lines marked '1' are obtained with the source polarized parallel to the plane of incidence, and in lines marked '2' with the source polarized perpendicular to the plane of incidence. '0' is a dark-current reading. 'I0', 'I1', 'I2', and 'I3' are the readings obtained when the black glass standard, sample 1, sample 2, or sample 3 respectively are in the sample holder. These data are then processed by computer as follows:

(a) The dark currents on both sides of each I0, I1, I2, and I3 are averaged and subtracted from these readings, yielding a sequence of 17 adjusted signal currents.

I(0),I(1), I(2), I(3),	1
I(0), I(3), I(2), I(1), I(0), I(1), I(2), I(3), I(0)	2
I(3), I(2), I(1), I(0)	1

(b) The averages $I(0)_1$, $I(1)_1$, $I(2)_1$, $I(3)_1$, $I(0)_2$, $I(1)_2$, $I(2)_2$, and $I(3)_2$ together with the polarization ratio, α are then used to compute the gloss value of the samples, g(i), by the following equation according to equation (3.28):

$$g(i) = \left[\frac{I(i)_1 + \alpha I(i)_2}{I(0)_1 + \alpha I(0)_2}\right] g(0) \qquad (6.1)$$

where i = 1, 2, & 3 and g(0) is the assigned gloss value of the black glass, primary working standard at the measured angle.

7. Calibration of Instrument and Error Analyses

In this section instrument calibration and error analyses will be performed for spectral condition, polarization, receiver solid angle, specular angle, linearity, and random error. Under each subsection the following format is used: 1. deriving mathematics of error analysis to find out what parameter we need to know, 2. describing measurements of parameters, that is calibration of instrument components, and 3. indicating results and contribution to error budget.

7.1 Spectral Condition

7.1.1 Derivation of Mathematics

From equations (3.1) and (3.5), the gloss value of sample x can be expressed as

$$g_x = \frac{\int C_\lambda D_x V_\lambda d\lambda}{\int C_\lambda D_s V_\lambda d\lambda} g_s \qquad (7.1)$$

Let $f(y)$ $(=C_\lambda V_\lambda)$ be the normalized CIE source C - luminous efficiency combination function and y be the rescaled wavelength from 0 to 1 for 400 to 700 nm. Equation (7.1) can thus be written as

$$g_x = \frac{\int_0^1 f(y)\, D_x\, dy}{\int_0^1 f(y)\, D_s\, dy} g_s \qquad (7.2)$$

and

$$\int_0^1 f(y)\, dy = 1 \qquad (7.3)$$

Systematic error would be introduced if the experimental spectral condition deviates from the above. In such case, the gloss value obtained for sample x is

$$g_x' = \frac{\int_0^1 f'(y)\, D_x\, dy}{\int_0^1 f'(y)\, D_s\, dy} \cdot g_s \qquad (7.4)$$

and the systematic error introduced will be

$$\Delta g = g_x' - g_x \qquad (7.5)$$

As will be indicated in the next subsection, the difference between function $f'(y)$ and $f(y)$ of this experiment can well be expressed as a sine function

$$f'(y) = f(y) + h \sin(2\pi y) \qquad (7.6)$$

where h is the proportionality constant.

Assuming D_x and D_s are quadratic functions of y, where D_x and D_s in this case are the reflectance functions, equation (7.2) and (7.4) can be rewritten as

$$g_x = \frac{\int_0^1 f(y)[a' + b'y + c'y^2]\, dy}{\int_0^1 f(y)[a + by + cy^2]\, dy} \cdot g_s \qquad (7.7)$$

and

$$g_x' = \frac{\int_0^1 [f(y) + h \sin 2\pi y][a' + b'y + c'y^2]\, dy}{\int_0^1 [f(y) + h \sin 2\pi y][a + by + cy^2]\, dy} \cdot g_s \qquad (7.8)$$

Upon integration, equation (7.8) is simplified to

$$g_x' = \frac{\left[\int_0^1 f(y)(a' + b'y + c'y^2)\, dy\right] - \frac{h}{2\pi}(b' + c')}{\left[\int_0^1 f(y)(a + by + cy^2)\, dy\right] - \frac{h}{2\pi}(b + c)} \cdot g_s \qquad (7.9)$$

Inserting equations (7.7) and (7.9) into equation (7.5), we have the systematic error Δg expressed by the following equation

$$\Delta g = \frac{Uv - Vu}{V(V-v)} \cdot g_s \tag{7.10}$$

where

$$U = \int_0^1 f(y)(a' + b'y + c'y^2)dy \tag{7.11}$$

$$V = \int_0^1 f(y)(a + by + cy^2)dy \tag{7.12}$$

$$u = h(b' + c') /2\pi \tag{7.13}$$

$$v = h(b + c) /2\pi \tag{7.14}$$

7.1.2 Calibration of Instrument

In order to evaluate the spectral condition of the instrument, measurements are made of the color temperature of the source, the relative spectral response of the receiver, and the spectral transmittance of selected filters to obtain the composite source-filter-receiver spectral condition.

a. Color temperature of the source unit.

The source unit consists of the tungsten lamp, the condenser lens, the collimation lens, and the polarizer. The lamp current is set at 6.50 amperes. A freshly pressed MgO tablet is placed in the specimen mount oriented to yield a 45-degree angle of incidence. A red to blue ratio photometer is used to determine the color temperature [6] by sighting on the MgO tablet. The spectral reflectance of the freshly pressed MgO is non-selective within 1% with respect to wavelengths in the visible region. Thus the use of an MgO diffuser will have no significant effect on the measurement of color temperature. The color temperature of the unit was determined to be 2856 K ± 30 K.

b. The relative spectral response of the receiver.

The relative spectral transmittance of the collector lens and the twelve-inch diameter sphere was measured by using a small sphere over the detector as the detection unit.

The relative spectral response of the detector alone was measured on a photodetector spectral response console. The console has a 3.5 nanometer spectral resolution and an uncertainty of ± 3 percent of the peak response. In this measurement, the detector is irradiated diffusely by light reflected from a pressed MgO tablet. The detector anode current is adjusted to about 1×10^{-7}A, approximately the level used when making gloss measurements. The relative spectral response of the detector is determined by the ratio of its output to that of a spectrally non-selective thermopile at each wavelength. A reference thermopile is used in conjunction with an electronic servo system to maintain an approximately constant flux at the test port of the instrument regardless of its wavelength. The test port flux is measured with a large area standard thermopile at frequent intervals to check the calibration of the instrument.

c. Source-filter-receiver spectral condition.

With the color temperature of the source unit and the response of the receiver unit measured, a set of filters is then selected such that the spectral condition of the source-filter-receiver combination closely resembles that of the CIE luminous efficiency with CIE source C. The comparison is given in figure 3.

7.1.3 Contribution to Error Budget

The proportionality constant h, first introduced in equation (7.6), is determined to be approximately 0.13 from figure 3. Fitting the measured 20-deg. specular spectral reflectance values of the black glass and a ceramic tile having a 20- deg. gloss of 60 with quadratic function in y as defined preceding equation (7.2), one has

$$a = 0.0455, \quad b = -0.0047, \quad c = 0.0022, \text{ and}$$
$$a' = 0.0300, \quad b' = 0.0009, \quad c' = -0.0002.$$

For a first approximation, U and V in equations (7.11) and (7.12) can be reduced to a' and a respectively. Using a standard with $g_s = 89.2$ and all the quadratic parameters to calculate u and v, the systematic error Δg is computed to be -0.07 gloss unit. The error will be smaller for lower gloss specimens.

7.2 Polarization Condition

7.2.1 Derivation of Mathematics

Refer to equation (3.28). Let the polarization ratio, α, be the only variable. Taking the derivative of g_x, we have the error in gloss value caused by the uncertainty in determining this ratio as follows

$$\Delta g_x = g_x \left(\frac{I_{x2}}{I_x} - \frac{I_{s2}}{I_s} \right) \Delta\alpha \qquad (7.15)$$

where

$$I_x = I_{x1} + \alpha I_{x2} \qquad (7.16)$$

and

$$I_s = I_{s1} + \alpha I_{s2} \qquad (7.17)$$

Uncertainty in the polarization ratio can be caused by the measurement imprecision and imperfect polarization. Imprecision of measurement uncertainty is due to the noise in the measurement system. The effect of an imperfect polarizer will be discussed below.

When the polarizer leaks in the direction normal to the intended direction of polarization, systematic error will be introduced in α. Define the transmittance ratio of the polarizer, β, as the ratio of the transmittance of the polarizer in the direction normal to the polarization direction to that in the polarization direction. β is assumed to be independent of wavelength. Under this condition, the signals obtained by the instrument are

$$I'_{x1} = K\int C'_{\lambda 1} \; D_{x1} \; V_\lambda d\lambda + K\int \beta \; C'_{\lambda 2} \; D_{x2} \; d\lambda \qquad (7.18)$$

$$I'_{x2} = K\int C'_{\lambda 2} \; D_{x2} \; V_\lambda d\lambda + K\int \beta \; C'_{\lambda 1} \; D_{x1} \; d\lambda \qquad (7.19)$$

and similarly, for I'_{s1} and I'_{s2} with x replaced by s in equations (7.18) and (7.19). In terms of the expressions by equations (3.15) and (3.16), the above two equations can be simplified to

$$I'_{x1} = I_{x1} + \beta I_{x2} \qquad (7.20)$$

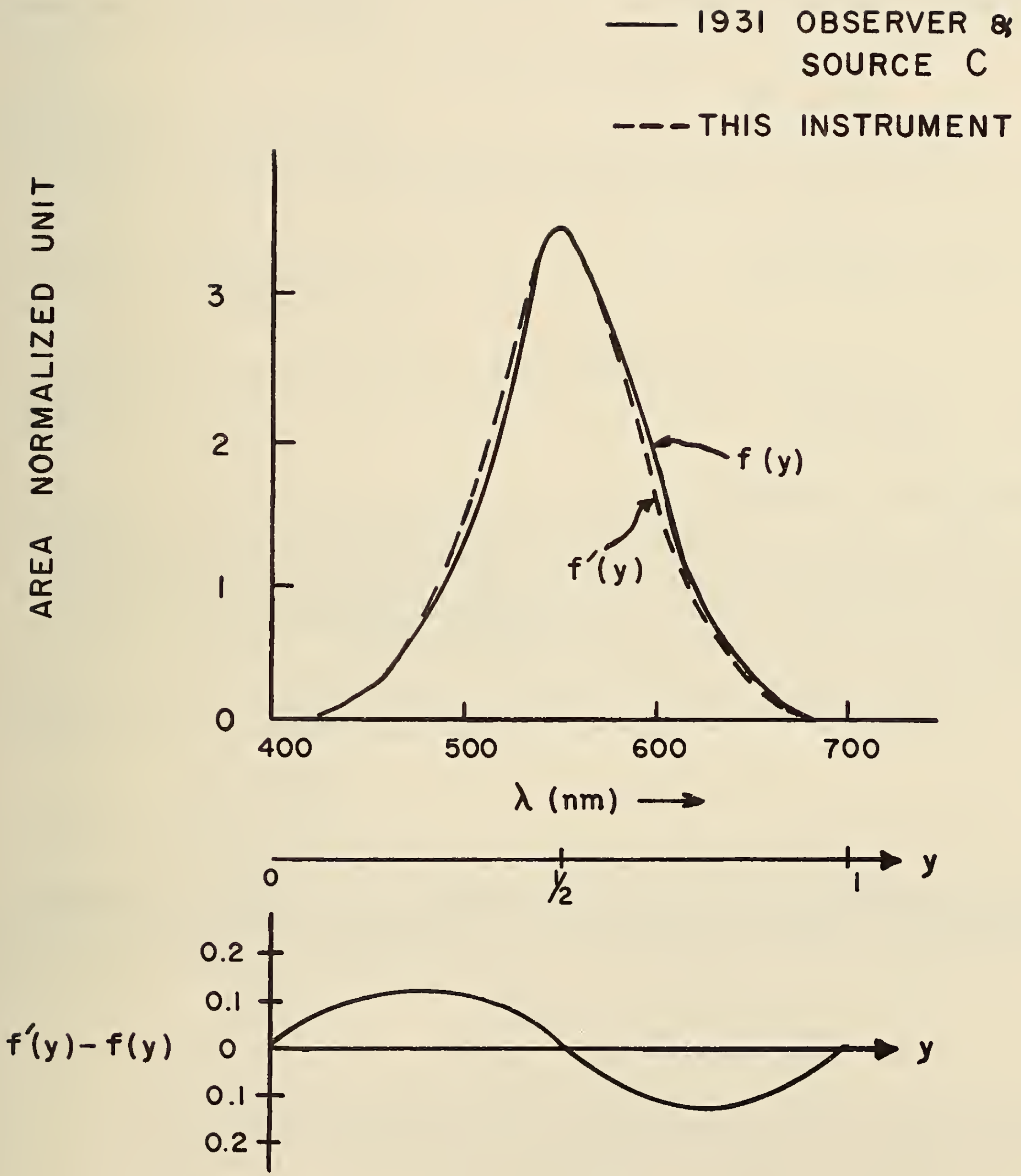

Fig. 3 Comparison between the spectral condition specified by the ASTM method and that of the NBS instrument

$$I'_{x2} = I_{x2} + \beta I_{x1} \tag{7.21}$$

and similarly,

$$I'_{s1} = I_{s1} + \beta I_{s2} \tag{7.22}$$

$$I'_{s2} = I_{s2} + \beta I_{s1} \tag{7.23}$$

The luminous flux ratio obtained by using an imperfect polarizer will be calculated as

$$F'_{xs} = \frac{I'_{x1} + \alpha I'_{x2}}{I'_{s1} + \alpha I'_{s2}} \tag{7.24}$$

Combining equations (7.20), (7.21), (7.22), (7.23), and (7.24), we have:

$$F'_{xs} = \frac{I_{x1} + \alpha' I_{x2}}{I_{s1} + \alpha' I_{s2}} \tag{7.25}$$

where

$$\alpha' = \alpha\left(\frac{1 + \beta/\alpha}{1 + \beta\alpha}\right) \tag{7.26}$$

7.2.2 Calibration of instrument

The leakage of the polarizer, installed in the instrument, for radiation polarized normal to the intended direction of polarization on the source unit was evaluated. Another polarizer made with the same material as that of the polarizer to be determined is placed on the receiver unit. Signals with the polarizing directions of the two polarizers parallel (P_1) and crossed (P_2) to each other are measured:

$$P_1 = T^2 + \alpha^2 T_\ell^{\,2} \tag{7.27}$$

and

$$P_2 = (1 + \alpha)\, TT_\ell \tag{7.28}$$

where for each polarizer, T and T_ℓ are the transmittance in the polarization direction and in the direction normal to it, respectively. Solving equations (7.27) and (7.28), the transmittance ratio of the polarizer can be calculated as

$$\beta \simeq \frac{T_\ell}{T} = \frac{\left(P_1 + \eta P_2\right)^{\frac{1}{2}} - \left(P_1 - \eta P_2\right)^{\frac{1}{2}}}{\alpha\left[\left(P_1 + \eta P_2\right)^{\frac{1}{2}} + \left(P_1 - \eta P_2\right)^{\frac{1}{2}}\right]} \tag{7.29}$$

Where $\eta = 2\alpha/(1 + \alpha)$. Since $P_2 << P_1$, equation (7.29) can be reduced to

$$\beta \simeq \frac{1}{(1 + \alpha)} \frac{P_2}{P_1} \tag{7.30}$$

The transmittance ratio of the polarizer, β was determined to be 0.0035.

The polarization ratio α of the source unit is approximately 0.9 with an uncertainty of about 0.001 due to the noise of the measurement system.

7.2.3 Contribution to error budget

From equation (7.26), using the values of α and β, $\alpha' = 1.00007\,\alpha$ is obtained. Thus the total uncertainty, due to both the imperfect polarizer and measurement noise, of the polarization ratio α is 0.00107. The systematic error in the gloss value caused by this uncertainty, using equation (7.15), is computed to be much less than 0.01 gloss unit for both low and high gloss samples.

7.3 Receiver Solid Angle

7.3.1 Derivation of mathematics

Referring to equation (7.1), the gloss value of sample x is expressed as

$$g_x = \frac{\int C_\lambda D_x V_\lambda \, d\lambda}{\int C_\lambda D_s V_\lambda \, d\lambda} \cdot g_s \tag{7.31}$$

For simplicity, assuming D_x and D_s are independent of λ, one has

$$g_x = \frac{D_x}{D_s} \cdot g_s \tag{7.32}$$

From equation (3.7), the above equation can be expressed as

$$g_x = \frac{\int^4 S_x \cos\Gamma \, dA \cos\gamma \, da \, d\omega \, d\Omega}{\int^4 S_s \cos\Gamma \, dA \cos\gamma \, da \, d\omega \, d\Omega} \cdot g_s \tag{7.33}$$

When the solid angles subtended by the source and detector are small, cosΓ and cosγ are nearly constant. Assuming the light is uniformly distributed on the sample and the sample surface is homogeneous, equation (7.33) can further be reduced to

$$g_x = \frac{\int^2 S_x \, d\omega \, d\Omega}{\int^2 S_s \, d\omega \, d\Omega} \cdot g_s \tag{7.34}$$

The scattering function from the polished black glass is represented by S_s. Since the reflected beam size is much smaller than that of the receiver aperture, a small deviation in the receiver aperture dimension from that specified by the ASTM method will not change the integration in the denominator. Equation (7.34) can therefore be expressed as,

$$g_x = K \int^2 S_x \, d\omega \, d\Omega \tag{7.35}$$

The characteristic surface scattering function of a sample can generally be expressed by a normal distribution function. The empirical evidence is shown in figure 4. The normalized reflectances of a roughened black glass are given as a function of angles of viewing with 1.2 deg. circular receiver aperture. The angle of incidence is fixed at 60 degrees from normal. Some theoretical considerations of scattering and reflecting properties of surfaces are given in reference [9].

In Appendix C, a general equation is set up for estimating the uncertainties in gloss caused by deviation from the specification of the aperture sizes and misalignment.

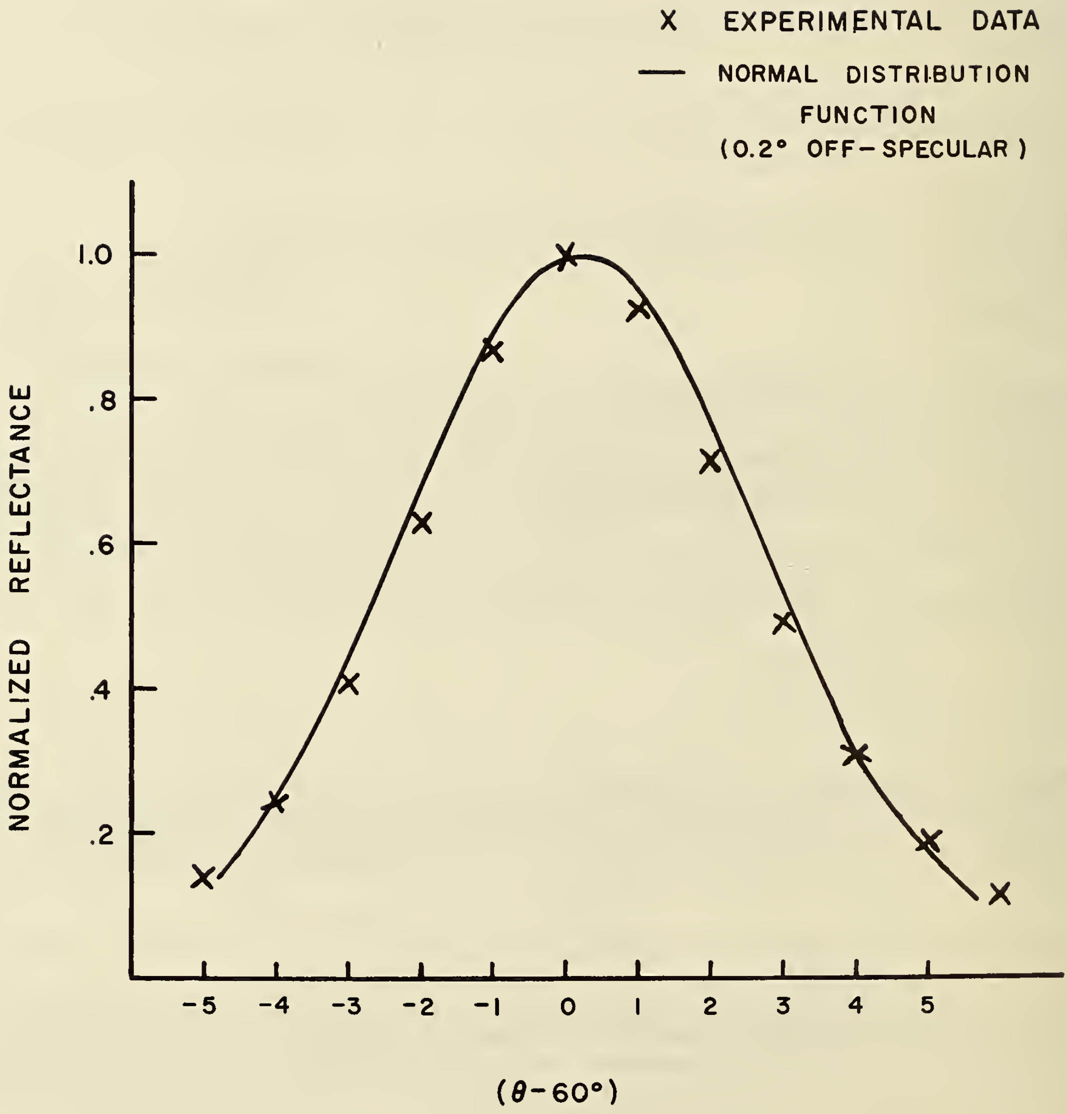

Fig. 4 Comparison of the relative reflectance near the specular angle and the normal distribution function

Finite sizes of source and receiver apertures are considered. Also included in the general equation is the displacement of the angular peak location of the reflectance curve designated as the off-specular peak in reference [10].

7.3.2 Calibration of instrument

Due to the monoplane design, the goniophotometer was only used to measure Θ_h, the angular dimensions of receiver apertures in the plane of incidence. The widths of the receiver apertures in and perpendicular to the plane of incidence were measured with a micrometer. The angular dimensions of the receiver apertures perpendicular to the plane of incidence were calculated.

For the determination of Θ_h, a source aperture subtending 0.75 degree in the plane of incidence was used. The light beam was adjusted to a width of 5 mm in the plane of incidence. The receiver aperture was scanned to determine the angle between the points at which the signal dropped to half its center value. This is defined as the angular width of the receiver aperture. For example, to calibrate the receiver aperture with 4.4 degree nominal dimension, first the receiver unit is lined up with the light source. This sets the receiver position at 0 degree. The high voltage power supply was then adjusted to 2.600 volts DC on the digital voltmeter. The receiver was scanned around +2.2 degree and -2.2 degree, and the reading recorded. With an interpolation method, the two angular positions yielding 1.300 volts DC were determined. The sum of the absolute values of the two angular positions is the angular width of this receiver aperture in the plane of incidence.

The respective widths of the receiver aperture x,y in and perpendicular to the plane of incidence were measured by using an optical scanning device. The angular dimensions of the receiver aperture perpendicular to the plane of incidence, Θ_v, was then calculated using the following equation

$$\Theta_v = 2 \tan^{-1}[(\frac{y}{x}) \tan (\frac{\Theta_h}{2})] \text{ [rad]}. \qquad (7.36)$$

The results of the calibration of the receiver aperture used for 20, 60, and 85 degree gloss measurements are presented in Table 1. Table 1 also contains the ASTM specified values and allowable tolerances, the measured values and uncertainties, and the deviation of the measured value from the specified value.

7.3.3 Contribution to error budget

Using the general equations derived in Appendix C and measured data of receiver apertures in table 2, the error in gloss value caused by the deviation of the receiver apertures of this instrument from that specified by the ASTM method is shown in table 2 which also includes off-specular angle designating the displacement of the angular peak location of the reflectance curve from the specular angle.

7.4 Specular Angle

7.4.1 Derivation of mathematics

In section 7.3.1 and Appendix C, a general equation is set up for estimating the uncertainties in gloss caused by misalignment of the source and detector, that is, the deviations of the angles of incidence and viewing from that specified by the ASTM method. Also included in the general equation is the displacement of the angular peak location of the reflectance curve designated as the off-specular peak in reference [10].

7.4.2 Calibration of instrument

The source and receiver units as well as specimen mount are first aligned. The optical axis of the receiver unit is in line with the optical axis of the source unit. When both units are set at zero degree across the turntable, the common axis ajoins, but

Table 1. Characterization of Receiver Apertures

	In plane of measurement (Full Angle)			Perpendicular to plane of measurement (Full Angle)		
	ASTM Spec.	NBS Measured	NBS Uncertainty	ASTM Spec.	NBS Measured	NBS Uncertainty
20- deg. Receptor	1.8°	1.81°	±0.01°	3.6°	3.613°	±0.01°
tolerance	±0.05°			±0.1°		
Deviation from ASTM		0.01°			0.013°	
60- deg. Receptor	4.4°	4.43°	±0.01°	11.7°	11.703°	±0.01°
tolerance	±0.1°			±0.2°		
Deviation from ASTM		0.03°			0.003°	
85- deg. Receptor	4.0°	4.01°	±0.01°	6.0°	6.000°	±0.01°
tolerance	±0.3°			±0.3°		
Deviation from ASTM		0.01°			0.000°	

Table 2. The error in gloss unit caused by deviation from the ASTM Specification of the receiver apertures

		NBS Goniophotometer deviation from ASTM Spec.		Gloss		
Degree	Off-Specular angle (deg.)	Δa (deg.)	Δb (deg.)	30	50	70
20	0	0.01	0.013	0.07	0.07	0.06
60	0	0.03	0.003	0.10	0.11	0.11
60	0.5	0.03	0.003	0.11	0.12	0.12
85	0	0.01	0	0.04	0.04	0.04
85	0.5	0.01	0	0.04	0.04	0.04
85	1	0.01	0	0.04	0.05	0.05

is perpendicular to, the axis of rotation of the turntable.

The angular displacement scale of the turntable is checked for accuracy from 0° to ± 85°. A standard angle block accurate to ±5 seconds is used together with an auxiliary collimator and a sighting telescope. A standard angle block with two mirror-like surfaces is secured on the specimen mount. The angle between these two surfaces is 95 degrees (compliment of 85 degrees). With the turntable set at 0°, the auxiliary collimator is adjusted in order to view the reflected light from the angle block through the telescope cross hair and the reading recorded. The turntable is then set at 85 degrees. The above sighting procedure is repeated. The difference between these two readings is the deviation from the 85 degree setting. The angular setting accuracy thus determined is within 0.02 degree.

7.4.3 Contribution to error budget

For 85-degree gloss measurement, a 0.02 degree misalignment in incident angle and viewing angle will each yield a change of about 0.1 gloss unit and less than 0.03 gloss unit for 20- and 60- degree gloss measurements.

7.5 Linearity

7.5.1 Derivation of mathematics

Refer to equation (3.28). Non-linearity in the measurement system will cause error in gloss value determination. The error can be evaluated by the following equation:

$$\Delta g_x = g_x \left[\frac{\Delta I_{x1} + \alpha \Delta I_{x2}}{I_{x1} + \alpha I_{x2}} - \frac{\Delta I_{s1} + \alpha \Delta I_{s2}}{I_{s1} + \alpha I_{s2}} \right] \tag{7.37}$$

where ΔI_{x1}, ΔI_{x2}, ΔI_{s1}, and ΔI_{s2} are non-linearity errors.

7.5.2 Calibration of instrument

The linearity of the detector and electronic system is measured by using the light-addition method with a double-aperture apparatus [7], that is, the output of the detector system when irradiated by two light beams simultaneously is compared to the sum of the outputs obtained when the detector is irradiated by each light beam separately. The non-linearity is less than 0.05 percent of the full-scale reading. The results are presented in table 3 together with the standard deviation of the non-linearity determination.

7.5.3 Contribution to error budget

Departure from a true non-linear response is corrected in determination of gloss values by using the values in table 3. Note that the standard deviation of the linearity correction is less than 0.008% thus making a negligible contribution to the random error in gloss measurement.

7.6 Random Error

The measurement procedure described in section 6 is used to minimize the effect of linear drift.

The random error dg_x of the measured gloss g_x is given by

$$dg_x = g_x \left[\left(\frac{dI_{x1}}{I_x} \right)^2 + \alpha^2 \left(\frac{dI_{x2}}{I_x} \right)^2 + \left(\frac{dI_{s1}}{I_s} \right)^2 + \alpha^2 \left(\frac{dI_{s2}}{I_s} \right)^2 \right]^{1/2} \tag{7.38}$$

where $I_x = I_{x1} + \alpha I_{x2}$ and $I_s = I_{s1} + \alpha I_{s2}$. dI_{x1}, dI_{x2}, dI_{s1}, and dI_{s2} are evaluated from measurements. Random error amounts to 0.14, 0.10, and 0.06 gloss unit for samples with gloss values of 70, 50, and 30, respectively.

Table 3. Linearity determination of detector and electronic system

I_a(%)	ΔI(%)	SD(ΔI) %
10	-.0303	.0022
20	-.0476	.0033
30	-.0541	.0039
40	-.0524	.0047
50	-.0446	.0059
60	-.0333	.0069
70	-.0208	.0073
80	-.0095	.0066
90	-.0018	.0044
100	0.	0.

where

I_a is the measured radiant flux in percent of maximum

ΔI is the amount of the linearity correction required to obtain the true radiant flux in percent of maximum, $I = I_a + \Delta I$

SD(ΔI) is the standard deviation of ΔI.

8. Summary

In this technical note the NBS procedure for measuring gloss according to ASTM D523 has been analyzed to determine the accuracy with which the measurements can be made.

All the components of errors discussed above are summarized below:

Component of error	Range of error (Gloss Unit)
1. Spectral condition	0.04 - 0.08
2. Polarization condition	<0.01
3. Receiver solid angle	0.07 - 0.12
4. Specular angle	0.03 - 0.20
5. Linearity	<0.01
6. Random error	0.06 - 0.14

The overall uncertainties of the 20-, 60-, and 85- degree gloss measurement geometries are computed as the sum of the component errors.

Geometry (deg.)	Uncertainty at Selected Gloss Levels (Gloss Unit)		
	70	50	30
20	0.5	0.5	0.4
60	0.5	0.5	0.3
85	0.7	0.6	0.5

The NBS procedure utilizes a light source polarized parallel and perpendicular to the plane of incidence. Calibration equations are derived to convert the measured data to gloss unit according to the definition. The refractive index of the primary working standard has been determined. Instrument components are described and calibration results are presented. A detailed error analysis has been performed.

Using the ASTM allowed tolerance for receiver apertures and the general equations derived in Appendix C, the equivalent allowed tolerance in gloss is about one and half units for 85- degree gloss measurements, or an allowed range of three gloss unit which is rather large (see Appendix D). Perhaps this ASTM allowed tolerance for 85- degree gloss measurement should be given further studies. The development of intermediate gloss standards with good planarity and gloss uniformity is also one of the continuous efforts in NBS directed toward improving the accuracy and measurement assurance of gloss measurement in the industry. Progress in this and other efforts by NBS to improve gloss measurements in the field will be reported in future technical notes.

The author thanks Dr. William H. Venable, Jr. for many stimulating discussions concerning this Technical Note. Mr. Wiley A. Hall determined the color temperature of the source unit and Mr. Robert J. Bruening measured the relative spectral response of the detector. The angular setting of the turntable was calibrated by Mr. Burnice N. Norden. The author acknowledges Mr. Kenneth L. Eckerle's assistance in the Linearity determination using double-aperture method. Appreciation is extended to Mr. Irving Malitson for the loan of a refractive index standard and help in refractive index determination of the primary gloss working standard, and to Mr. William B. Fussell for helpful consultation on the mathematical analyses in Appendix C.

9. Appendices

A. Equipment*

SOURCE UNIT

KEPCO regulated DC power supply
model KS 18-50M-6956, Ser. c-33685

Leeds & Northrup shunts 0.1 ohm, 15 amp; 0.5 ohm, 10 amp.

DIGITEC (United System Corp.) DC voltmeters
model 202N, ser. 8104c
model 251, ser. 1553

*Certain commercial instruments or materials are identified here in order to specify adequately the procedure. In no case does such identification imply endorsement or evaluation by the National Bureau of Standards.

DETECTOR UNIT

John Fluke MFG. Co. Inc. high voltage DC power supply
model 413D, ser. 157.

DATA ACQUISITION UNIT

Keithley Instruments high speed picoammeter
model 417, ser. 33339 with model 4170 input head.
using 10^{-7} V scale.

DANA digital voltmeter model 5403, ser. 542131A
accessory module 020. using DC mode, 10 V range.

COMPUMETRICS control data acquisition system.

Industrial Timer Corp. timer (model Cu-A).

Teletype teletypewriter.

B. Calculation of gloss value with known index of refraction of a smooth dielectric surface

According to the ASTM method [3], the gloss value of the primary working standard at specular angle γ_o, $g_s(\gamma_o)$, can be obtained as follows with known refractive index n_1.

$$g_s(\gamma_o) = \frac{\rho(\gamma_o,\ n=n_1)}{\rho(\gamma_o,\ n=1.567)} \cdot 100 \tag{B.1}$$

Where $\rho(\gamma_o,n)$ is the specular reflectance determined by the Fresnel's equation:

$$\rho(\gamma_o,n) = \tfrac{1}{2}\left[\frac{\sin^2(\gamma_o - \gamma')}{\sin^2(\gamma_o + \gamma')} + \frac{\tan^2(\gamma_o - \gamma')}{\tan^2(\gamma_o + \gamma')}\right] \tag{B.2}$$

and

$$\gamma' = \sin^{-1}\left(\frac{\sin\gamma_o}{n}\right) \tag{B.3}$$

C. Details of mathematical derivations of errors caused by size difference and misalignment of source and receiver apertures

Refering to equation (7.32), the gloss value of sample x is expressed as

$$g_x = \frac{D_x}{D_s} g_s \tag{7.32}$$

The error in gloss value caused either by incorrect dimensions or misalignments of source and receiver apertures, can be determined from the derivative of equation (7.32):

$$\frac{dg_x}{g_x} = \frac{dD_x}{D_x} - \frac{dD_s}{D_s} \tag{C.1}$$

Refer to equation (3.7). For small subtended solid angles of source and receiver apertures, uniform illumination, and a homogeneous sample surface, D_x and D_s can be expressed as

$$D_s = \int^2 S_s \, d\omega d\Omega \tag{C.2}$$

$$D_x = \int^{2} S_x \, d\omega d\Omega \tag{C.3}$$

Where S_s and S_x are the scattering functions of the primary standard and the sample, respectively.

The scattering function S_s of the primary standard is almost a delta function. A small deviation of the receiver aperture size from that specified, or misalignment of source and receiver aperture, will not change the value of D_s except due to index of refraction.

The characteristic scattering function of a sample can generally be expressed by a normal distribution function of angular displacement from the specular angle. (See figure 4 and reference 9). Consider the rectangular source aperture (2A, 2B) and receiver aperture (2a, 2b). For small subtended solid angles of source and receiver apertures 2A and 2a are approximately equal to the angular dimensions in radian in the plane of incidence while 2B and 2b are angular dimensions perpendicular to the plane of incidence and $1 + x^2 + y^2 \simeq 1$ and $1 + X^2 + Y^2 \simeq 1$. From equations (3.9) and (3.11), equation (C.3) can be written as

$$D_x = \int_{-B}^{B} \int_{-A}^{A} \int_{-b}^{b} \int_{-a}^{a} S_x \, dx dy dX dY \tag{C.4}$$

Where $S_x = \exp\{-1/2[(x-X-d)^2 + (y-Y)^2]\}$ (C.4a)

and d is the off-specular angle in the plane of incidence.

Four different cases will be considered:

(i) Size error in receiver aperture: the dimension of the receiver aperture is ($2a + 2\Delta a$, $2b + 2\Delta b$) and the receiver aperture is centered on the mirror image of the source axis.

$$D_x = \int_{-B}^{B} \int_{-A}^{A} \int_{-b-\Delta b}^{b+\Delta b} \int_{-a-\Delta a}^{a+\Delta a} S_x \, dx dy dX dY \tag{C.5}$$

$$dD_x = D_x' - D_x \tag{C.6}$$

$$dD_s = 0 \tag{C.7}$$

(ii) Size error in source aperture: the dimension of the source aperture is ($2A + 2\Delta A$,$2B+2\Delta B$) and the source aperture is centered on the specular angle.

$$D_x' = \int_{-B-\Delta B}^{B+\Delta B} \int_{-A-\Delta A}^{A+\Delta A} \int_{-b}^{b} \int_{-a}^{a} S_x \, dx dy dX dY \tag{C.8}$$

$$dD_x = D_x' - D_x \tag{C.9}$$

For a uniform illumination D_s is proportional to the solid angle subtended by the source aperture. Thus

$$\frac{dD_s}{D_s} = \frac{(2A + 2\Delta A)(2B + 2\Delta B) - (2A)(2B)}{(2A)(2B)} = \frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta A \cdot \Delta B}{A \cdot B} \tag{C.10}$$

(iii) The receiver aperture is misaligned Δa and Δb.

$$D_x' = \int_{-B}^{B} \int_{-A}^{A} \int_{-b-\Delta b}^{b+\Delta b} \int_{-a+\Delta a}^{a+\Delta a} S_x dxdydXdY \tag{C.11}$$

$$dD_x = D_x' - D_x \tag{C.12}$$

$$dD_s = 0 \tag{C.13}$$

(iv) The source aperture is misaligned ΔA and ΔB.

$$D_x' = \int_{-B+\Delta B}^{B+\Delta B} \int_{-A+\Delta A}^{A+\Delta A} \int_{-b}^{b} \int_{-a}^{a} S_x \; dxdydXdY \tag{C.14}$$

$$dD_x = D_x' - D_x \tag{C.15}$$

$$dD_s = 0 \tag{C.16}$$

Using the definition of error function and equations (C.1) and (C.4a), the error in gloss value caused by the above four cases can be evaluated considering both first and second orders.

(i) Receiver aperture (size error)

$$\frac{dg_x}{g_x} = \frac{1}{M_1}\left[R_b \cdot \Delta a + \sqrt{\frac{2}{\pi}} \cdot E_A \frac{(\Delta a)^2}{2}\right] + \frac{1}{M_2}\left[2R_c \cdot \Delta b + 2\sqrt{\frac{2}{\pi}} E_c \frac{(\Delta b)^2}{2}\right]$$

$$+ \frac{2}{M_1 M_2}\left[R_b \cdot R_c \cdot \Delta a \cdot \Delta b\right] \tag{C.17}$$

(ii) Source aperture (size error)

$$\frac{dg_x}{g_x} = \frac{1}{M_1}\left[R_A \Delta A + \sqrt{\frac{2}{\pi}} E_A \frac{(\Delta A)^2}{2}\right] + \frac{1}{M_2}\left[2R_D \Delta B + 2\sqrt{\frac{2}{\pi}} E_c \frac{(\Delta B)^2}{2}\right]$$

$$+ \frac{2}{M_1 M_2}\left[R_A \cdot R_D \cdot \Delta A \cdot \Delta B\right] - \left[\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta A \cdot \Delta B}{A \cdot B}\right] \quad \text{(C.18}$$

(iii) Receiver aperture (misalignment)

$$\frac{dg_x}{g_x} = \frac{1}{M_1}\left[R_B \; \Delta a + \sqrt{\frac{2}{\pi}} E_A \frac{(\Delta a)^2}{2}\right] + \frac{1}{M_2}\left[2\sqrt{\frac{2}{\pi}} E_c \frac{(\Delta b)^2}{2}\right] \tag{C.19}$$

(iv) Source aperture (misalignment)

$$\frac{dg_x}{g_x} = \frac{1}{M_1}\left[-R_B \; \Delta A + \sqrt{\frac{2}{\pi}} E_A \frac{(\Delta A)^2}{2}\right] + \frac{1}{M_2}\left[2\sqrt{\frac{2}{\pi}} E_c \frac{(\Delta B)^2}{2}\right] \tag{C.20}$$

$$g_x = \frac{g_s}{D_s} \frac{\pi}{2} M_1 M_2 \qquad (C.21)$$

where

$$M_1 = \sqrt{\frac{2}{\pi}} E_A + (u + A)R_1 - (u - A)R_2 + (v + A)R_3 - (v - A)R_4$$

$$M_2 = 2\sqrt{\frac{2}{\pi}} E_C + 2(b + B)R_5 - 2(b - B)R_6$$

$$R_A = R_1 + R_2 + R_3 + R_4 \qquad E_A = E_1 - E_2 + E_3 - E_4$$

$$R_B = R_1 - R_2 - R_3 + R_4$$

$$R_b = R_1 - R_2 + R_3 - R_4$$

$$R_C = R_5 - R_6 \qquad E_C = E_5 - E_6$$

$$R_D = R_5 + R_6$$

$$R_1 = \mathrm{erf}\left(\frac{u + A}{\sqrt{2}}\right) \qquad E_1 = \exp\left[-\tfrac{1}{2}(u + A)^2\right]$$

$$R_2 = \mathrm{erf}\left(\frac{u - A}{\sqrt{2}}\right) \qquad E_2 = \exp\left[-\tfrac{1}{2}(u - A)^2\right]$$

$$R_3 = \mathrm{erf}\left(\frac{v + A}{\sqrt{2}}\right) \qquad E_3 = \exp\left[-\tfrac{1}{2}(v + A)^2\right]$$

$$R_4 = \mathrm{erf}\left(\frac{v - A}{\sqrt{2}}\right) \qquad E_4 = \exp\left[-\tfrac{1}{2}(v - A)^2\right]$$

$$R_5 = \mathrm{erf}\left(\frac{b + B}{\sqrt{2}}\right) \qquad E_5 = \exp\left[-\tfrac{1}{2}(b + B)^2\right]$$

$$R_6 = \mathrm{erf}\left(\frac{b - B}{\sqrt{2}}\right) \qquad E_6 = \exp\left[-\tfrac{1}{2}(b - B)^2\right]$$

$$u = a - d \qquad v = a + d$$

For each geometry, "a" value is determined to give a maximum error in gloss value when g_x is 50 gloss unit. Using equation (C.21), the ratio of g_s/D_s is also determined. For simulation of higher or lower gloss sample, the limit of integration "a" (also b, A, and B) is scaled which is equivalent to changing the characteristic scattering function.

According to the ASTM method b/a, A/a, and B/a are 2.0000, 0.4167, and 0.8333 respectively for 20° gloss measurement; 2.6667, 0.1705, and 0.6818 respectively for 60° gloss measurement; and 1.5000, 0.1875, and 0.7500 respectively for 85° gloss measurement. Thus b, A, and B are determined once "a" is selected.

D. The error in gloss unit based on the allowed tolerance of the receiver aperture and the general equations derived in Appendix C

ASTM allowed tolerance of receiver Aperture				Gloss Unit		
Degree	Off-Specular Angle (deg.)	Δa (deg.)	Δb (deg.)	30	50	70
20	0	0.05	0.10	0.41	0.42	0.38
60	0	0.10	0.20	0.35	0.38	0.34
60	0.5	0.10	0.20	0.35	0.39	0.37
85	0	0.30	0.30	1.39	1.46	1.29
85	0.5	0.30	0.30	1.42	1.53	1.41
85	1	0.30	0.30	1.51	1.74	1.79

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15. SUPPLEMENTARY NOTES

16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

The 20-, 60-, and 85- degree specular gloss scales are established at NBS with an accuracy of about one half gloss unit. This is one of many programs of the spectrophotometry group of the Optical Radiation Section of NBS directed toward improving the accuracy and assurance of spectrophotometric measurements made throughout the scientific and industrial community.

The specular gloss scales are established through an unique technique employing polarized light flux both parallel and perpendicular to the plane of incidence. General calibration equations are derived. NBS instrumentation and measurement procedures are described. Instrument calibration and error analyses are performed. Some of the analyses can also be applied to reflectance measurements in general.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

Accuracy; appearance; error analysis; gloss; photometry; reflectance; scattering; spectrophotometry; specular gloss.

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