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NBS TECHNICAL NOTE 537

Nomographs for Use in the Fabrication and Testing of Ge(Li) Detectors

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CONTENTS

Nomographs for Use in the Fabrication and Testing of Ge(Li) Detectors

A. H. Sher

Six nomographs which can facilitate the fabrication and testing of lithium-drifted germanium gamma-ray detectors [Ge(Li) detectors] have been constructed which relate the following parameters: (1) time, temperature, applied bias, and drifted depth; (2) lithium mobility, crystal resistivity, and oxygen concentration; (3) area, capacitance, and drifted depth for planar Ge(Li) detectors; (4) drifted depth, length, and capacitance for coaxial Ge(Li) detectors; (5) total spectral resolution, system noise, and detector resolution; and, (6) detector resolution, gamma-ray energy, and effective Fano factor. The use of these nomographs is described and illustrative examples are given.

Key Words: Capacitance, detector resolution, effective Fano factor, Ge(Li) detector, lithium-ion drift, nomograph, oxygen concentration.

1. INTRODUCTION

During the fabrication of Ge(Li) detectors and in their testing, the need often arises to compare various sets of parameters or to estimate certain quantities from experimental data. By using nomographs, the need for repetitive or relatively complicated calculations can often be eliminated. To this end, nomographs relating the most commonly used detector parameters were constructed.

In constructing these nomographs, the aim was to provide charts which would be easy to use and interpret, and legible enough so that parameters could be manipulated with some degree of accuracy. Thus, five of the nomographs are in the form of alignment charts with parallel scales. The sixth nomograph, shown in figure 2, is in the form of a concurrency nomograph since construction of an alignment chart would have necessitated using scales for certain of the parameters which would not be easily readable or accurate over the range of interest. Nomography,[1][1] by A. S. Levins was a useful aid in preparing these charts; the interested reader is directed to this volume for assistance in constructing similar nomographs.

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[1] Figures in brackets indicate literature references given at the end of this paper.

2. FABRICATION OF Ge(Li) DETECTORS

During the lithium-ion drifting process in p-type germanium, it is often desirable after some elapsed period of time to obtain an estimate of the compensated or drifted depth. One reason might be to obtain an estimate of the oxygen concentration in the germanium crystal for a measured value of the lithium drift mobility to be used as one of the criteria for judging the economic or qualitative suitability of the germanium crystal for fabricating Ge(Li) detectors. The following nomographs may be used for the above purposes.

2.1 Nomograph Relating the Time, Temperature, Voltage, and Drifted Depth for Lithium-Ion Drifting in p-Type Germanium.

For lithium drifting in p-type Ge under ideal conditions (perfect material and control of variable parameters), the drifted depth W (cm) can be given by the following:

$$W = (2\mu_{Li}Vt)^{1/2} \quad (1)$$

where μ_{Li} is the lithium-ion drift mobility (cm^2/V-s), and is related to the temperature T (K), diffusion constant D (cm^2/s), and Boltzmann's constant k, by $\mu_{Li} = D/kT$, V is the applied bias (V), and t is the elapsed drift time (s). This nomograph is based on recent experimental measurements of μ_{Li} in germanium [2] over the temperature range used for most drifting. Assuming the drift behavior of the diode follows Equation 1, the nomograph (Fig. 1) can be used to estimate the *drifted depth* obtained after a certain period of drift time for a given applied bias and temperature. The *time* it will take to drift to a given depth as a function of the applied voltage and temperature of the drift (or any of the other drift parameters when related quantities are held constant) can also be determined from the nomograph.

As an example of the use of this nomograph, the drifted depth obtained for an applied bias of 1000 volts at a temperature of 15°C after 400 hours of drift can be found by connecting with a straight line the value of 1000 volts on the left-hand scale with a value of 15°C on the temperature scale.[2] The value of 400 hours on the time scale is then connected to the point of intersection on the unmarked (pivotal) scale and the value of drifted depth obtained is 0.8 cm. Conversely, by connecting the value of 0.8 cm on the drifted depth scale with the point of intersection on the unmarked scale, obtained from connecting the values of 1000 volts and 15°C, and extending this line to the time scale, the drifted depth of 0.8 cm is obtained after 400 hours of drift. Comparisons can also be obtained between sets of values of applied bias and temperature, in terms of drift rate, by

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[2] All further references to "connecting values" of various parameters on the nomograph scales shall be taken to mean using a straight line.

connecting the various values of temperature and bias under study and marking the intersection on the pivotal scale. The higher the intersection on this unmarked scale, the higher the drift rate.

Use of this nomograph is valid only if the drift behavior of the diodes follows Equation 1. It has been shown that for many germanium crystals, the lithium drift behavior does not follow this ideal case [3]. Loss of mobile lithium ions during drift can cause an apparent decrease in the drift rate; in these cases [3] the drifted depth can usually be obtained by using the following equation:

$$W = [\mu_{Li} V\tau(1 - e^{-2t/\tau})]^{1/2} \tag{2}$$

where τ is the lithium loss time constant (s). When $\tau >> t$, Equation 2 reduces to Equation 1. Thus the nomograph (Fig. 1) should only be used to estimate the maximum drifted depth to be obtained when loss of lithium during drift does not significantly affect drift rate.

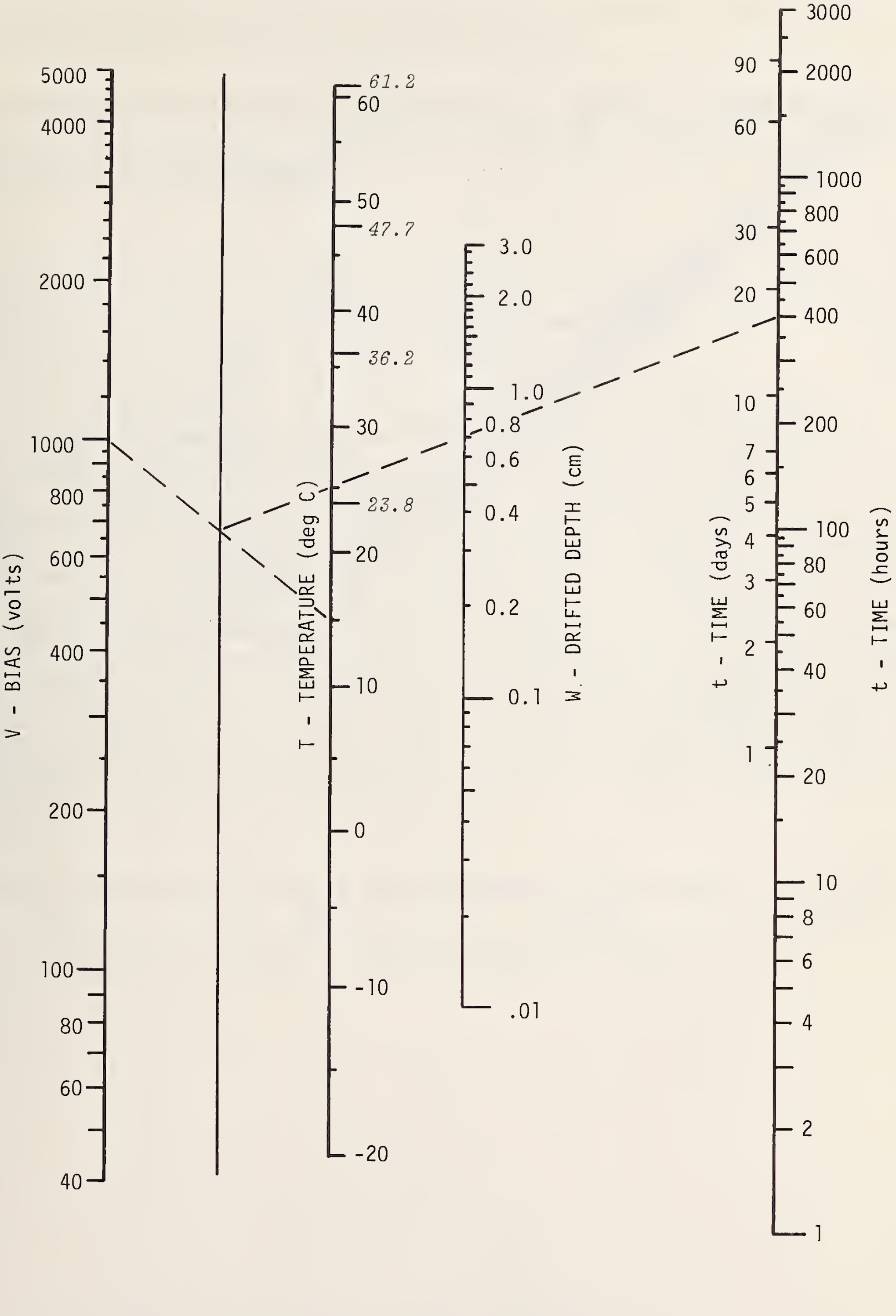

Fig. 1 Nomograph relating time (t), temperature (T), applied bias (V), and drifted depth (W).

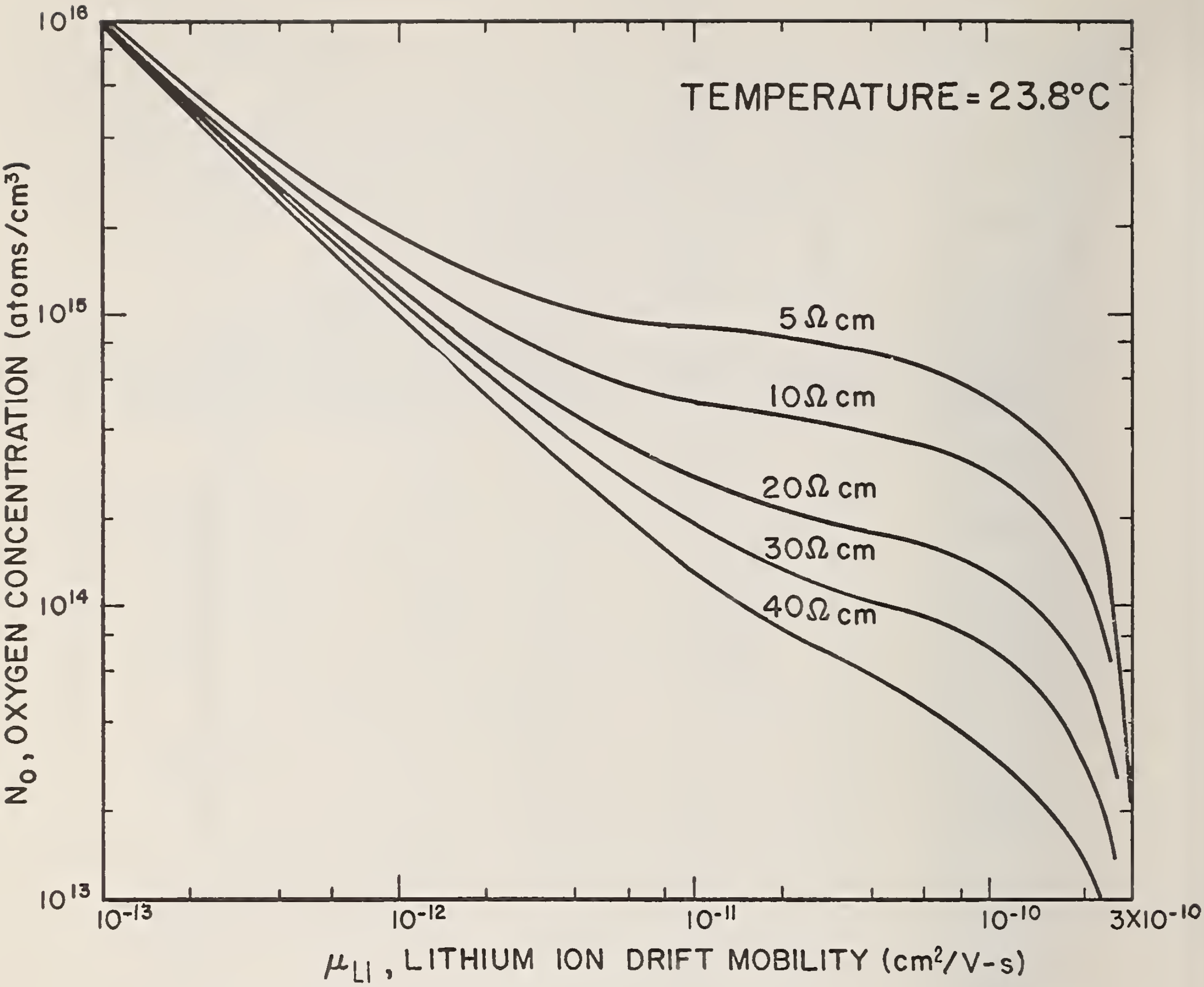

Fig. 2 Concurrency nomograph relating lithium-ion drift mobility (μ_{Li}), resistivity of single-crystal, p-type germanium (Ω-cm), and oxygen impurity concentration (N_o).

2.2 Nomograph Relating the Lithium-Ion Drift Mobility, Resistivity, and Oxygen Concentration of a Germanium Crystal.

This concurrency nomograph[3] relates the lithium-ion drift mobility μ_{Li} (cm^2/V-s), to the sample resistivity (ohm-cm), and oxygen concentration N_o (cm^{-3}), at 23.8°C, through the following equation [4]:

$$N_o = n(1 - f) + C(f^{-1} - 1) \qquad (3)$$

where n is the acceptor concentration (cm^{-3}) [5], C is the dissociation constant for the lithium-oxygen complex LiO^+ (3.5×10^{12} cm^{-3} [6]), and f is the ratio of the observed lithium mobility to the maximum lithium mobility at 23.8°C (taken as 3.04×10^{10} cm^2/V-s [2]).

This nomograph (Fig. 2) is used in the following way: for an experimentally measured value of lithium mobility (see Reference 2) of 2×10^{-10} cm^2/V-s, for example, and a specimen resistivity of 10 ohm-cm, the oxygen concentration is found to be approximately 1.2×10^{14} atoms/cm^3. The important effect of the sample resistivity, i.e., acceptor concentration for p-type crystals, on the lithium mobility for a given oxygen concentration is illustrated. For example, at an oxygen concentration of 1.2×10^{14} atoms/cm^3, the lithium-ion mobility in a 40 ohm-cm, p-type germanium crystal would be more than an order of magnitude lower than the mobility in a 10 ohm-cm crystal. As the oxygen concentration of germanium crystals increases above approximately 10^{15} atoms/cm^3, the variation of the acceptor concentration has less effect on lithium mobility. Thus, the usual assumption of an inverse linear relationship between oxygen concentration and lithium-ion drift mobility in germanium is found to be invalid [3].

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[3] This nomograph has been previously published in Reference 3.

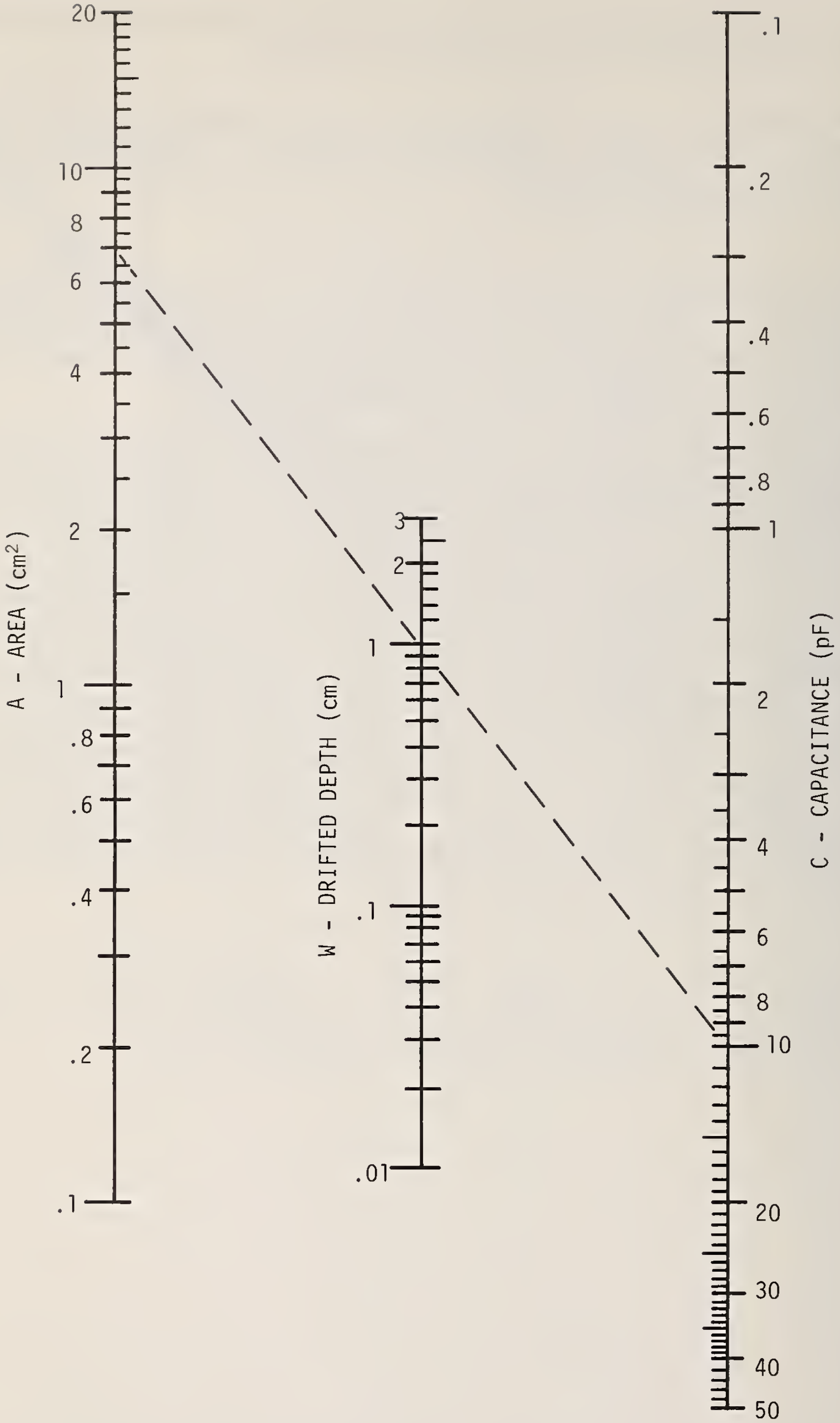

Fig. 3 Nomograph relating diode junction area (A), capacitance (C), and drifted depth (W) for planar Ge(Li) detectors.

3. TESTING AND EVALUATION OF Ge(Li) DETECTORS

The relationships of capacitance and size of both planar and coaxial Ge(Li) detectors and detector resolution and system noise are described by the following nomographs.

3.1 Nomograph Relating the Junction Area, Capacitance, and Drifted Depth for Planar Ge(Li) Detectors

This chart relates the junction area A (cm^2), capacitance C (pF), and drifted depth W (cm), of a Ge(Li) diode under reverse bias through the following equation:

$$W = \kappa A/(36\pi C) = 1.41(A/C). \qquad (4)$$

If the value of the detector area is connected with the value of the capacitance, the drifted depth is given by the intersection of the line on the center scale as shown in Fig. 3. For example, a diode with an area of 7 cm^2 and a capacitance of 10 pF would have a drifted depth of approximately 1 cm. Conversely, the nomograph may also be used to determine the expected capacitance of a detector with known drifted depth and junction area.

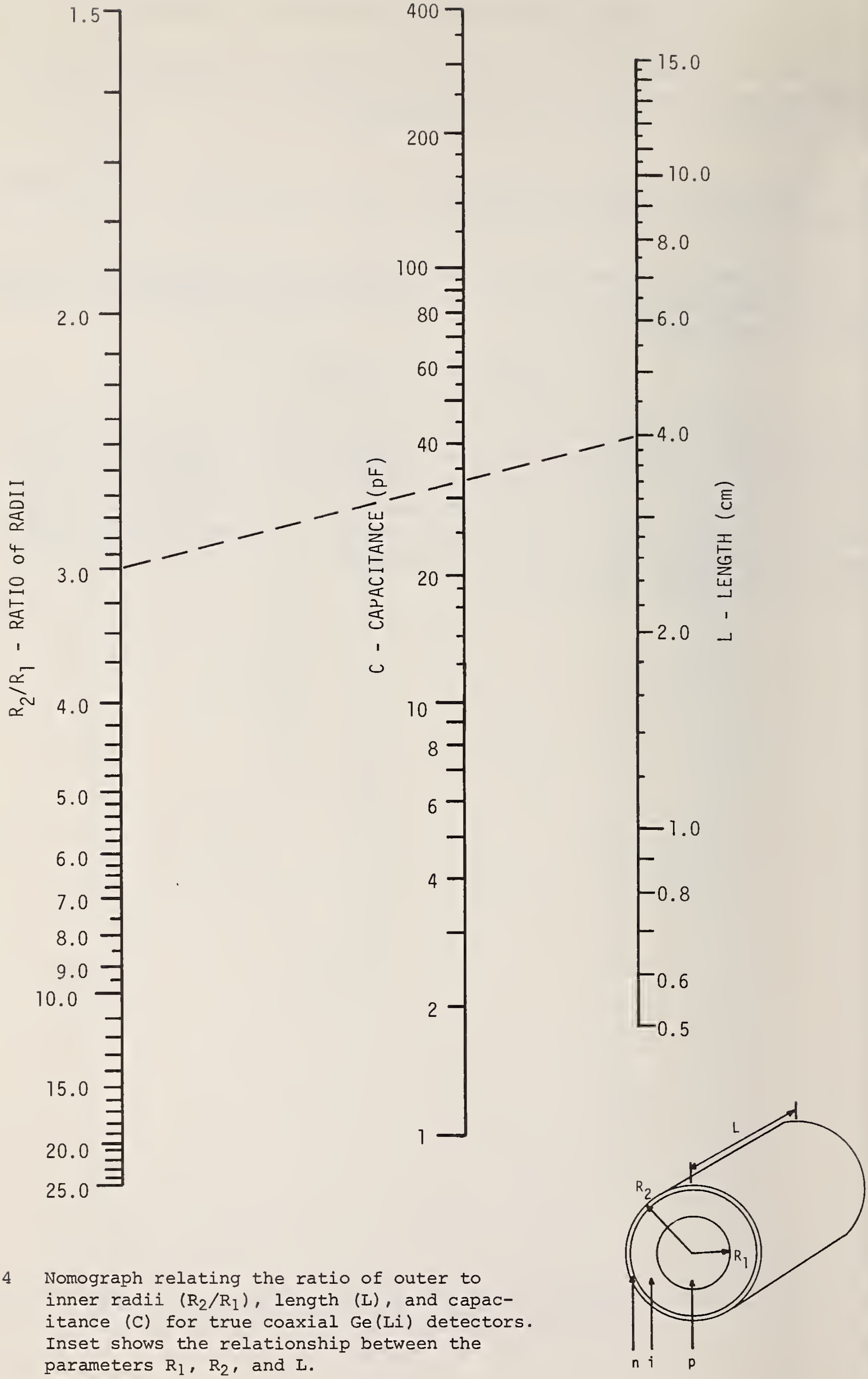

Fig. 4 Nomograph relating the ratio of outer to inner radii (R_2/R_1), length (L), and capacitance (C) for true coaxial Ge(Li) detectors. Inset shows the relationship between the parameters R_1, R_2, and L.

3.2 Nomograph Relating the Length, Ratio of Outer to Inner Radii, and Capacitance for Coaxial Ge(Li) Detectors

True or double-open-ended coaxial, cylindrical, Ge(Li) detectors with length L (cm), outer radius R_2, and inner radius R_1, can be related to the capacitance C (pF), through the following equation:

$$C = [1.41(2\pi L)]/[\ln(R_2/R_1)]. \qquad (5)$$

The inset in Fig. 4 shows the relationship of these parameters to the coaxial detector geometry. The difference ($R_2 - R_1$) is the drifted depth. Consider a crystal where R_2 is 1.5 cm and R_1 is 0.5 cm; therefore, $R_2/R_1 = 3$, and for L = 4 cm, the expected detector capacitance is 32 pF. Conversely, for a known detector capacitance, crystal length, and known radius R_2, the chart may be used to obtain a value R_1 by finding the ratio R_2/R_1; the drifted depth may then be estimated from the difference ($R_2 - R_1$).

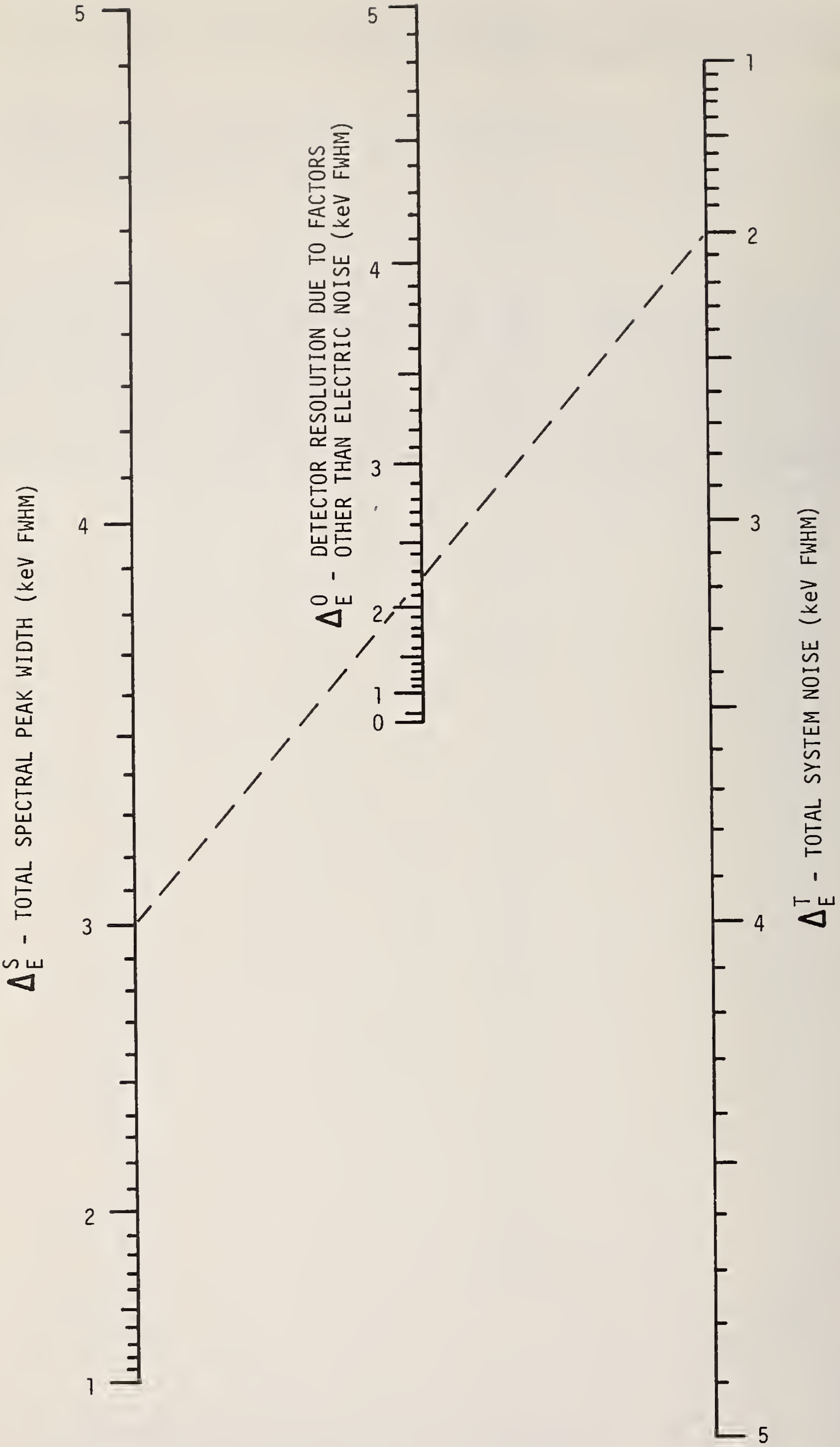

Fig. 5 Nomograph relating total spectral peak width (Δ_E^S), total system noise (Δ_E^T), and detector resolution due to all factors other than electric noise (Δ_E^O).

3.3 Nomograph Relating Total System Spectral Resolution, Total Noise Contribution, and Detector Contribution Due to all Factors Other Than Electrical Noise.

The total system noise Δ_E^T (keV FWHM), the detector contribution due to all factors other than electrical noise Δ_E^O (keV FWHM), and the total system spectral peak width Δ_E^S (keV FWHM), are related through the following equation [7]:

$$\Delta_E^O = [(\Delta_E^S)^2 - (\Delta_E^T)^2]^{1/2}. \qquad (6)$$

The total system spectral resolution, Δ_E^S, is obtained by measuring the full width of the pulse-height spectral peak at half the maximum height (FWHM); the total noise contribution, Δ_E^T, is obtained by measuring the FWHM of a peak generated by a precision pulser with the detector connected into the system as described in Reference 6. As an example of the use of this nomograph (Fig. 5) consider the case when the total system spectral resolution is 3 keV FWHM and the total noise contribution is 2 keV; the detector contribution due to all factors other than electrical noise from the detector and amplifier is 2.25 keV FWHM. If the width of a pulser peak is found from a measurement in which the detector is replaced by an equivalent capacitor, then the right-hand scale is the amplifier noise, Δ_E^A, and the value obtained from the center scale is the *detector* contribution, Δ_E^D [7].

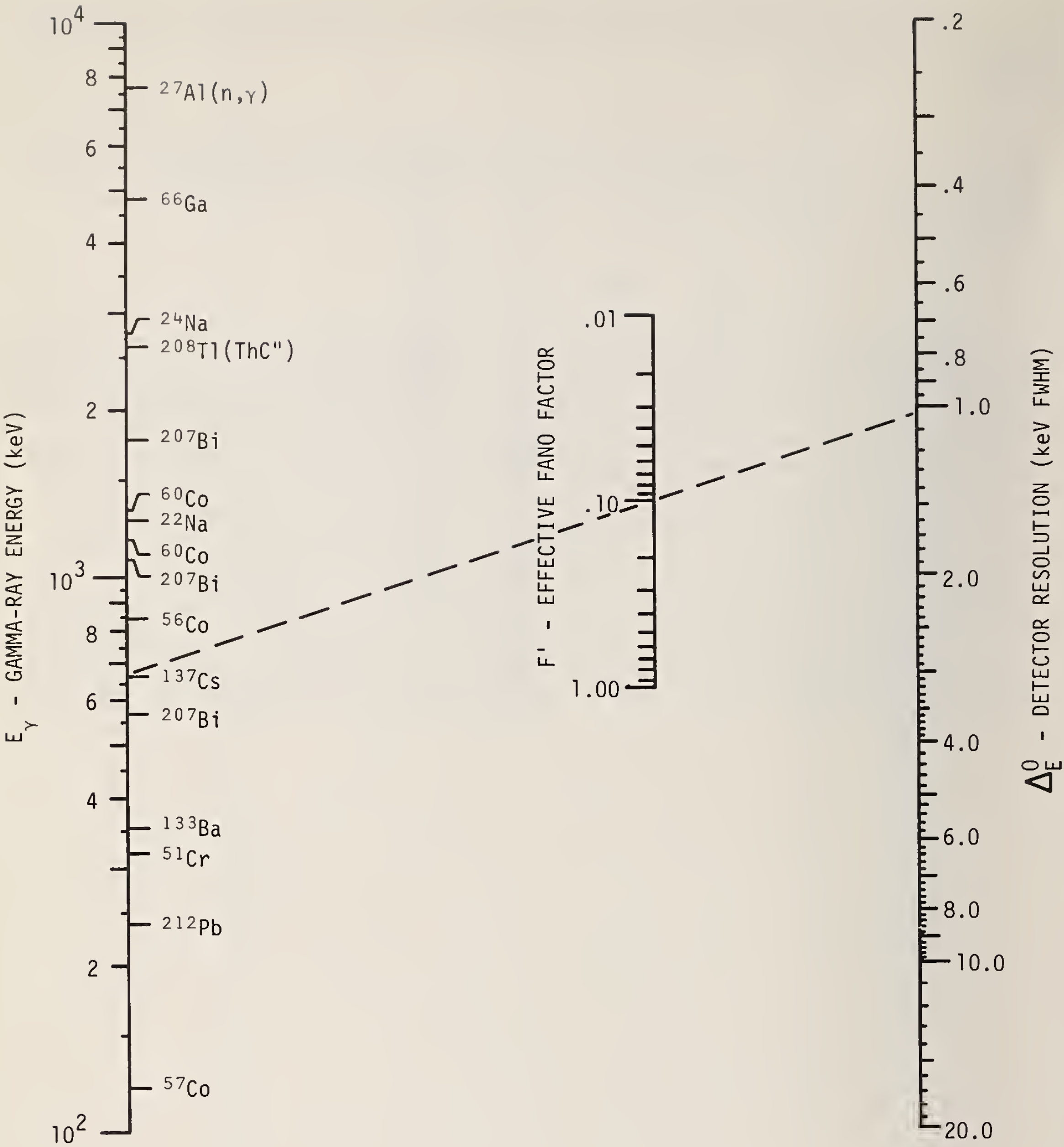

Fig. 6 Nomograph relating gamma-ray energy (E_γ), detector resolution due to all factors other than electric noise (Δ_E^0), and effective Fano factor (F').

3.4 Nomograph Relating Gamma-Ray Energy, Detector Contribution Due to all Factors Other Than Electric Noise, and Effective Fano Factor.

The gamma-ray energy E_γ, the detector contribution due to all factors other than electrical noise Δ_E^O (keV FWHM), and the effective Fano factor F', are related by the following equation:

$$F' = [\Delta_E^O/2.355(E_\gamma \varepsilon)^{1/2}]^2, \qquad (7)$$

where ε = 2.97 eV/ion pair [8]. For this nomograph (Fig. 6), the example given shows that for the 662-keV gamma-ray of ^{137}Cs and for Δ_E^O of about 1 keV FWHM, the effective Fano factor is 0.1. Assuming Poisson statistics, the nomograph may also be employed to obtain the statistical width, Δ_E, for a particular gamma-ray spectral peak by connecting the value of gamma-ray energy and F' = 1, and extending the straight line to the right-hand scale to obtain the statistical width. Discussions on recent measurements of the effective Fano factor may be found in References 9 through 11.

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