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FIELD STRENGTH CALCULATIONS
FOR E.L.F. RADIO WAVES

BY JAMES R. WAIT AND NANCY F. CARTER



U. S. DEPARTMENT OF COMMERCE
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FIELD STRENGTH CALCULATIONS FOR E. L. F. RADIO WAVES

Abstract

The mode theory of propagation of electromagnetic waves at extremely low frequencies (1.0 to 3000 c/s) is briefly reviewed in this paper. Starting with the representation of the field as a sum of modes, approximate formulas are presented for the attenuation and phase constants. At the shorter distances, where the range is comparable to the wavelength, the spherical-earth mode series is best transformed to a series involving cylindrical wave functions. This latter form is used to evaluate the near field behavior of the various field components. The effect of the earth's magnetic field is also evaluated using a quasi-longitudinal approximation which is particularly appropriate for propagation in the magnetic meridian. In general it is indicated that if the gyro frequency is comparable or greater than the effective value of the collision frequency, the presence of the earth's magnetic field may be important for E. L. F. In this case the attenuation may be increased somewhat. The influence of a purely transverse magnetic field is also considered.

This technical note is actually a numerical supplement to the paper "Mode Theory and the Propagation of ELF Radio Waves," by J. R. Wait, J. Research N. B. S., 64D, July/Aug. 1960; however, for sake of completeness the relevant theory is briefly presented.

1. Introduction

The propagation of electromagnetic waves at extremely low frequencies (E. L. F.) has received considerable attention recently [1-5]. Unfortunately, the experimental data in this frequency range (1.0 c/s to 3000 c/s) is still rather limited [8-15]. There is little doubt, however, that the bulk of the energy is transferred via a waveguide mode. The bounding surfaces of this waveguide are the ground and the lower edge of the ionospheric E region.

An important feature of E. L. F. propagation is that the distance between source and observer may be comparable to the wavelength. For example, at 300 c/s the wavelength is 1000 km. In fact, many of the experiments are carried out at distances of this order and account must be taken of the near field effects.

It is the purpose of this note to present computations based on the mode theory as it applies to E. L. F. propagation. It is also hoped that these results will also be of assistance in interpretation of future experimental data.

II. Basic Theoretical Model

The assumed theoretical model is taken to be a homogeneous conducting spherical earth of radius a surrounded by a concentric conducting ionospheric shell of inner radius $a + h$. It is convenient to introduce the usual spherical coordinate system (r, θ, ϕ) . Using this model and assuming a vertical electric dipole source located at $\theta = 0, r = a$, the expression for the vertical electric field, E_r , is given by an expression of the following form [4, 16]

$$E_r = \frac{I ds \eta}{2kha^2} \sum_{n=0}^{\infty} \delta_n \frac{\nu(\nu+1)}{\sin \nu \pi} P_{\nu}(-\cos \theta) \quad (1)$$

where I - - average current in the source dipole;

ds - - length of source dipole;

η - - intrinsic impedance of air space $\cong 120\pi$ ohms;

$P_{\nu}(-\cos \theta)$ - - Legendre function of argument $-\cos \theta$ and
(complex) order ν ;

$\nu + \frac{1}{2} = ka S_n$ where S_n , for $n = 0, 1, 2, \dots$,

determined from the boundary condition and is
described below; and

$$\delta_n \cong \frac{1}{\sin 2kh C_n} \cong 1/2 \text{ for } n = 0$$

$$1 + \frac{2kh C_n}{2kh C_n} \cong 1 \text{ for } n = 1, 2, 3, \dots$$

The individual terms in the above series correspond to the waveguide modes. In the general case, the factor S_n is obtained from

the solution of a complicated transcendental equation which involves spherical Bessel functions of large argument and complex order. Certain aspects of this problem have been discussed by Schumann [1] and also the author [4] . For a homogeneous earth and a homogeneous ionosphere, both assumed isotropic, it has been shown that S_n may be approximated by [17]

$$S_n = \left\{ 1 - \left(\frac{\pi n}{kh} \right)^2 \frac{1}{4} \left[1 + \sqrt{1 + 4i \frac{\Delta kh}{(\pi n)^2}} \right]^2 \right\}^{\frac{1}{2}} \quad (2)$$

where $\Delta = \frac{1}{N_1} + \frac{1}{N_g}$

in terms of the refractive indices, N_g and N_1 , of the earth and the ionosphere, respectively. This equation is valid subject to the condition $|\Delta| kh \ll 1$. The sign of the radical is chosen in the above equation which makes the real part positive. The values of S_n then are located in the fourth quadrant of the complex plane. When $n=0$, the above simplifies to

$$S_0 = \left[1 - i \frac{\Delta}{kh} \right]^{\frac{1}{2}} \cong 1 - i \frac{\Delta}{2kh} \quad (3)$$

Now, since $|\Delta| kh \ll 1$, the radical in equation (1) can be expanded for $n > 0$ to yield

$$S_n = \left[1 - \left(\frac{\pi n}{kh} \right)^2 - i \frac{2\Delta}{kh} \right]^{\frac{1}{2}} \text{ for } n = 1, 2, 3 \dots \quad (4)$$

Furthermore, if in addition,

$$\frac{|\Delta|}{kh} \ll 1 - \left(\frac{\pi n}{kh} \right)^2$$

it is permissible to write, for all n ,

$$S_n \cong \left[1 - \left(\frac{\pi n}{kh} \right)^2 \right]^{\frac{1}{2}} - i \frac{\epsilon_n}{2kh} \Delta \left[1 - \left(\frac{\pi n}{kh} \right)^2 \right]^{-\frac{1}{2}} \quad (5)$$

where $\epsilon_0 = 1$, $\epsilon_n = 2$ (for $n = 1, 2, 3 \dots$). This latter equation is valid when the "waveguide modes" are not near cut-off and is in the form given by Schumann [2] . It is quite analogous to the standard results for the propagation constant in a rectangular waveguide with finitely conducting walls [18, 19] .

III. Earth-Flattening Approximation

The possibility that the curvature of the earth may be neglected is now investigated. To simplify field calculations at relatively short distances, it would seem desirable to transform the mode series to a form where the first term corresponds to the model of a flat earth and the succeeding terms are corrections for curvature. This approach has been used by Pekeris [20] and more recently by Koo and Katzin [21] in their investigations of microwave duct propagation. From their work, it may be shown that

$$\frac{P_\nu (-\cos \theta)}{\sin \nu \pi} \cong H_0^{(2)}(kS_n \rho) - \frac{1}{12} \left(\frac{\rho}{a} \right)^2 H_2^{(2)}(kS_n \rho) \quad (6)$$

+ terms in $(\rho/a)^4$, $(\rho/a)^6$, etc.,

where $\nu + 1/2 = kS_n \rho$ and $\rho = a\theta$. $H_0^{(2)}$ and $H_2^{(2)}$ are Hankel functions of the second kind of order zero and two, respectively. When the great circle distance ρ is reasonably small compared to the earth's radius a , only the first term in the expansion need be retained.

For convenience in what follows, it is desirable to express the field components as a ratio to the quantity

$$E_o = i (\eta/\lambda) I ds (e^{-ik\rho})/\rho, \quad (\eta \cong 120\pi);$$

E_o is the radiation field of the source at a distance ρ on a perfectly conducting ground. Thus, for both the source and the observer near the ground, it is not difficult to show that

$$E_z = W E_o$$

where $W \cong -i\pi \frac{\rho}{h} e^{ik\rho} \sum_{n=0}^{\infty} \delta_n S_n^2 H_0^{(2)}(kS_n \rho) \quad , \quad (7)$

$$E_\rho = -S E_o$$

$$S \cong \frac{\pi}{N_g} \frac{\rho}{h} e^{ik\rho} \sum_{n=0}^{\infty} \delta_n S_n E_1^{(2)}(kS_n \rho) \quad (8)$$

and $H_\phi = -T E_o/\eta$

$$\text{where } T \cong -\pi \frac{\rho}{h} e^{ik\rho} \sum_{n=0}^{\infty} \delta_n S_n H_1^{(2)}(kS_n \rho) = -N_g S \quad (9)$$

where terms containing $(\rho/a)^2, (\rho/a)^4$ have been neglected.

When $k\rho \gg 1$, corresponding to the "far-zone," the above expressions may be simplified since the Hankel functions may be replaced by the first term of their asymptotic expansion. This leads to the compact result

$$\begin{bmatrix} W \\ S \\ T \end{bmatrix} \cong \frac{(\rho/\lambda)^{\frac{1}{2}}}{(h/\lambda)} e^{i\left[\frac{2\pi\rho}{\lambda} - \frac{\pi}{4}\right]} \sum_{n=0}^{\infty} \delta_n \begin{bmatrix} S_n^{3/2} \\ -S_n^{1/2}/N_g \\ S_n^{1/2} \end{bmatrix} e^{-i2\pi S_n \rho/\lambda} \quad (10)$$

which is valid for $\rho \gg \lambda$. As expected, the ratio of W to T for a given mode is S_n which for low order or grazing modes is of the order of unity. The ratio of S to T, quite generally, is $-1/N_g$ which is very small compared to unity; in fact, it vanishes for a perfectly conducting ground as it must.

The excitation by a horizontal electric dipole may be treated in a similar manner. A complete discussion of the theory is given elsewhere [17]. The expression for the vertical field E_z of the horizontal dipole $I ds$ which is located at the origin and oriented in direction $\phi = 0$ may be written in the form

$$E_z = S E_0 \cos \phi \quad (11)$$

where S and E_0 are defined above. By using the reciprocity theorem, this result can also be deduced directly from equation (9). Comparing the above formula for E_z (for a horizontal dipole) with corresponding formula for E_z , for a vertical dipole, it may be easily shown that

$$\frac{E_z \text{ (for horizontal dipole)}}{E_z \text{ (for vertical dipole)}} \cong \frac{\cos \phi}{N_g} \left(1 - \frac{1}{N_g^2}\right)^{\frac{1}{2}} \cong \frac{\cos \phi}{N_g} \quad (12)$$

being valid in the asymptotic or far zone. In terms of the ground

conductivity σ_g and dielectric constant ϵ_g ,

$$\frac{1}{N_g} = \left(\frac{i \epsilon_o \omega}{\sigma_g + i \epsilon_g \omega} \right)^{\frac{1}{2}} \cong \left(\frac{i \epsilon_o \omega}{\sigma_g} \right)^{\frac{1}{2}} .$$

This is a small quantity.

The above formulas for W, S, and T are valid only if $\rho \ll a$. If the first curvature correction term is included, it is a simple matter to show, in the "far zone", that this amounts to multiplying the right-hand side by the factor

$$1 + \frac{1}{12} \left(\frac{\rho}{a} \right)^2 \text{ or } 1 + \frac{\theta^2}{12} .$$

Starting from the mode series for a spherical earth [17] and using the far field approximation for the Legendre function, it turns out that W, S, T have precisely the same form as in equation (10) except the factor $(\theta/\sin \theta)^{\frac{1}{2}}$ occurs in the right-hand side. Noting that

$$\left(\frac{\theta}{\sin \theta} \right)^{\frac{1}{2}} \cong 1 + \frac{\theta^2}{12} + \text{terms in } \theta^4, \theta^6, \text{ etc.}$$

it is apparent to a second order that the derived curvature correction is the same as from the Hankel function series.

IV. Distance and Frequency Dependence

Certain features of the mode series are best illustrated by a graphical plot of field strength vs. distance under various values of the parameters. Since $N_g \gg N_i$ it may be safely assumed for E. L. F. that $\Delta = 1/N_i$. Then on the assumption that the ionosphere is behaving like a conductor, it is possible to write

$$N_i = \left(1 - i \frac{\omega_r}{\omega} \right)^{\frac{1}{2}} \tag{13}$$

where

$$\omega_r = \frac{\omega_o^2}{\nu} = \frac{(\text{plasma frequency})^2}{\text{collision frequency}} .$$

The above relation for N_i is an approximation which is valid when the collision frequency ν between electrons and neutral ions is much less than the angular frequency ω . Since ν is of the order of 10^7 in the lower E region, this is certainly valid at E. L. F. The (angular) plasma frequency ω_o is given by

$$\omega_o^2 = \frac{e^2 N}{\epsilon_o m}$$

where N is the electron density, e and m are charge and mass of the electron, and ϵ_o is the dielectric constant of free space. ω_r , which is proportional to the ratio N/ν , is believed to be of the order of 10^5 in the D layer or the lower region of the E layer. At least this appears to be the effective value deduced from V. L. F. observations for highly oblique incidence [23]. At E. L. F. it appears that the waves are reflected at higher levels in the ionosphere and the effective value of ω_r is somewhat larger, being possibly of the order of 10^6 .

It should be noted in passing that ω_r may be replaced by σ_1/ϵ_o where σ_1 is the effective conductivity of the ionosphere and ϵ_o is the dielectric constant of free space.

To indicate the variation of the field as a function of distance, the magnitude and phase of T and W are shown in Figs. 1a to 4c for a range of values of ω_r and $h = 90$ km. The curves denoted $\omega_r = \infty$ correspond to a perfectly conducting ionosphere. These are presented for purposes of comparison with the set for the more reasonable values of ω_r . These are 2×10^6 and 5×10^5 . Actually, $|T|$ and $|W|$ are both divided by the square root of the distance ρ (in kilometers) and plotted since the curves become linear at larger distances. The slope of these linear parts of the curves is proportional to the attenuation rate (in db per 1000 km) as usually defined. It is seen immediately that at the shorter distances, the curves are no longer straight. This immediately indicates that considerable caution should be exercised in computing attenuation rates from spectral analyses of "sferics." For example, it is only

when the distance ρ exceeds about one-sixth of a wavelength, is it permissible to assume that $\log (|W|/\sqrt{\rho})$ or $\log (|E_z| \times \sqrt{\rho})$ vary in a linear manner with distance ρ . Similarly, the phase of W (or E_z) varies in a linear manner only when the distance exceeds about one-half wavelength. Similar remarks apply to the magnetic field variations with distance.

To shed further light on the nature of the E. L. F. fields in the waveguide, the radial impedance of the wave is now considered.

It is noted that

$$-\left. \frac{E_z}{\eta H_\phi} \right]_{z=0} = \frac{W}{T} \quad (14)$$

which is by definition the (normalized) impedance of the wave looking in the radial or ρ direction. First it should be noted that for $k\rho \gg 1$,

$$\frac{W}{T} \cong 1$$

and for $k\rho \ll 1$ and $\rho \ll h$,

$$\frac{W}{T} \cong \frac{1}{ik\rho} .$$

For the intermediate and interesting range, $k\rho$ is comparable to unity and ρ is somewhat greater than h . Using the numerical values of T and W mentioned above, the ratio $|W/T|$ and the phase (lag) defined by $\arg T - \arg W$ are plotted in Figs. 5a to 6c and as a function of distance for various frequencies. As before, $h = 90$ km and the same range of ω_r values are chosen.

The impedance ratio W/T is independent of the frequency spectrum of the source provided, of course, it may be represented by an equivalent vertical electric dipole. In fact, this complex ratio could be easily calculated from the frequency spectra of the waveforms of the vertical electric field and the horizontal magnetic field of an atmospheric. The observed variation of magnitude or phase of W/T as a function of frequency should then provide a basis for distance measuring. Such a scheme, while admittedly crude, requires only one receiving station equipped with a vertical whip and a loop antenna.

At the very short distances where ρ is of the order of the ionospheric reflecting height (i. e., 90 km), the mode series representation for the fields is very poorly convergent. However, alternate representations are available which converge very rapidly in this case [17]. For $\omega_r = \infty$, these are

$$T = \sum_{m=0}^{\infty} \epsilon_m \left(\frac{\rho}{r_m} \right)^2 \left(1 - \frac{i}{kr_m} \right) e^{-ik(r_m - \rho)} \quad (15)$$

and

$$W = \sum_{m=0}^{\infty} \epsilon_m \frac{\rho}{r_m} \left\{ \left[\left(1 - \frac{1}{(kr_m)^2} \right) - \frac{i}{kr_m} \right] - \left(\frac{2mh}{r_m} \right)^2 \left[\left(1 - \frac{3}{(kr_m)^2} \right) - \frac{3i}{kr_m} \right] \right\} e^{-ik(r_m - \rho)} \quad (16)$$

where $r_m = \left[\rho^2 + (2mh)^2 \right]^{\frac{1}{2}}$, $\epsilon_0 = 1$, $\epsilon_m = 2 (m \neq 0)$.

Individual terms in these series correspond to the contribution from images of the source dipole. These particular expressions are exact on the assumption of perfectly conducting planes. (i. e., $\omega_r = \infty$ and $\sigma_g = \infty$). It is known, however, from the mode calculations that at short distances (i. e., for $\rho < 100$ km), the numerical values of the functions T and W are only very slightly dependent on the finite value of ω_r and σ_g . Thus (15) and (16) are generally applicable.

To illustrate the nature of the fields at short ranges, the magnitude and phase of T and W are shown plotted in Figs. 7a to 8b. The solid curves correspond to equations (15) and (16) for $h = 90$ km. The dashed curves correspond to equations (15) and (16) when $h = \infty$; this is the same as taking only the first term in the series. At the very short ranges, it is seen that the solid and dashed curves merge together. Apparently in this region, the reflecting layer at 90 km can be ignored. At the greater ranges, the T and W functions both approach unity for the case of no reflecting layer (i. e., $h = \infty$).

Corrections for earth curvature resulting from diffraction would, of course, modify these dashed curves at ranges exceeding about 500 km. It may be concluded from the information plotted in Figs. 7 and 8 that only for extremely short ranges (i. e., less than about 30 km) is it permissible to neglect the presence of the ionospheric reflecting layer in field strength calculations at ELF.

V. Effect of the Earth's Magnetic Field

In the preceding it has been tacitly assumed that the ionosphere is behaving as an isotropic homogeneous conductor. In this case, the refractive index N_i of the ionosphere may be written [23]

$$N_i \cong \left[1 - i \frac{\omega_r}{\omega} \right]^{\frac{1}{2}} \cong \left(\frac{\omega_r}{i\omega} \right)^{\frac{1}{2}} \quad (17)$$

where

$$\omega_r = \frac{\omega_o^2}{\nu} = \frac{(\text{Plasma Frequency})^2}{(\text{Collision Frequency})}$$

When the earth's magnetic field is steeply dipping, it is appropriate to invoke the quasi-longitudinal approximation of Booker [23]. In this case the refractive index is double valued, one corresponds to the ordinary and the other the extra-ordinary, thus

$$N_i \cong \left[1 - i \frac{\Omega_r}{\omega} \exp(\pm i\tau) \right]^{\frac{1}{2}} \quad (18)$$

where

$$\tan \tau = \frac{\omega_L}{\nu} = \frac{\text{longitudinal component of gyro-frequency}}{\text{collision frequency}}$$

and

$$\Omega_r = \frac{\omega_o^2}{(\nu^2 + \omega_L^2)^{\frac{1}{2}}}$$

Using this model for the ionosphere, the reflection coefficient for a sharp boundary has been derived by Budden [24]. Adapting this result to E. L. F. it has been shown [17] that the formula for S_n has the

same form as the isotropic medium if Δ is defined by

$$\Delta = \frac{1}{N_g} + \left(\frac{i\omega}{\Omega_r} \right)^{\frac{1}{2}} \cos \tau/2 \quad . \quad (19)$$

Since $\Omega_r = \omega_r \cos \tau$ it is possible to write the results in the same form as the isotropic ionosphere if we set

$$\Delta \cong \frac{1}{N_g} + \frac{1}{(N_i)_{\text{eff.}}} \cong \frac{1}{(N_i)_{\text{eff.}}}$$

where

$$(N_i)_{\text{eff.}} = \left[\frac{(\omega_r)_{\text{eff.}}}{i\omega} \right]^{\frac{1}{2}} \quad .$$

$(N_i)_{\text{eff.}}$ and $(\omega_r)_{\text{eff.}}$ are the effective values of N_i and ω_r , respectively.

Specifically,

$$(\omega_r)_{\text{eff.}} \cong \omega_r \frac{\cos \tau}{(\cos \tau/2)^2} \quad . \quad (20)$$

In other words, the effective conductivity of the ionosphere is modified by the factor $\frac{\cos \tau}{(\cos \tau/2)^2}$ which varies from unity to zero as τ varies from 0 to $\pi/2$. Thus, the attenuation is increased as a result of a steeply dipping (or vertical) magnetic field. In fact

$$\frac{\text{Attenuation with magnetic field}}{\text{Attenuation without magnetic field}} \cong \frac{\cos (\tau/2)}{(\cos \tau)^{\frac{1}{2}}} \quad . \quad (21)$$

If $\tau \ll 1$ (i. e., $\omega_L \ll \nu$), this ratio becomes unity and the influence of the earth's magnetic field vanishes. At the level in the ionosphere where E. L. F. waves are reflected, it is expected that ω_L and ν are comparable, both being of the order of 10^6 . Assuming that they were actually equal, τ becomes 45° and the ratio of the attenuation rates is 1.05. Thus, it is only when ω_L is greater than ν does the earth's magnetic field appreciably influence the attenuation. The ratio is plotted in Fig. 9 as a function of $\tan \tau$ or ω_L/ν . This illustrates the situation clearly.

The preceding results are subject to the validity of Booker's quasi-longitudinal approximation of the Appleton-Hartree equation[23].

The validity of this approximation requires that

$$\frac{\omega_T^4}{4 \omega^2 \omega_L^2} \ll \left| \left(1 - \frac{\omega_o^2}{\omega^2} - i \frac{\nu}{\omega} \right)^2 \right| \quad (22)$$

where ω_L and ω_T are the longitudinal and transverse components of the (angular) gyro-frequency. Clearly this condition is violated when the transverse component of the earth's magnetic field is large such as for propagation along the magnetic equator. This case, however, has been considered by Barber and Crombie [25] who derived explicit results for the reflection coefficient at a sharply bounded ionosphere with a purely transverse magnetic field. Adapting their results to E. L. F. it is not difficult to show [17] that Δ is now given by

$$\Delta = \left(\frac{i\omega}{\omega_r} \right)^{\frac{1}{2}} X \quad (23)$$

$$X = \frac{\left(1 + \frac{i\omega}{\omega_r} \right)^{\frac{1}{2}} \left(1 + i \frac{\omega}{\omega_r} + i \frac{\omega_T^2}{\nu^2} \frac{\omega}{\omega_r} \right)^{\frac{1}{2}} - \frac{\omega_T}{\nu} \left(i \frac{\omega}{\omega_r} \right)^{\frac{1}{2}}}{\left(1 + i \frac{\omega}{\omega_r} \right)^2 - \frac{\omega_T^2}{\nu^2} \frac{\omega^2}{\omega_r^2}}$$

The real and imaginary parts of Δ are plotted in Figs. 10a and 10b as a function of the frequency parameter $5 \times 10^5 \times f/\omega_r$ where f is expressed in kc/s and ω_r is in (secs)⁻¹. The various values of ω_T/ν are shown in the curves; positive values of the ratio correspond to propagation from east-to-west while negative values correspond to propagation from west-to-east. Now the attenuation of the dominant mode in nepers per unit distances is given by $-kS_0$ where S_0 is given by equation (3).

Using the above results for Δ the attenuation coefficient α , expressed in db per 1000 km of path length, is plotted in Fig. 11a for $h = 90$ km. It is immediately evident that the attenuation is only slightly modified by the presence of a transverse magnetic field unless, of course, ω_T is considerably greater than ν . The phase velocity of the dominant zero-order mode, relative to free space, is

simply $1/\text{Re } S_0$. Denoting this dimensionless quantity by β it is plotted in Fig. 11b for $h = 90$ km. It is apparent that the presence of a transverse magnetic field does have an influence on the phase velocity.

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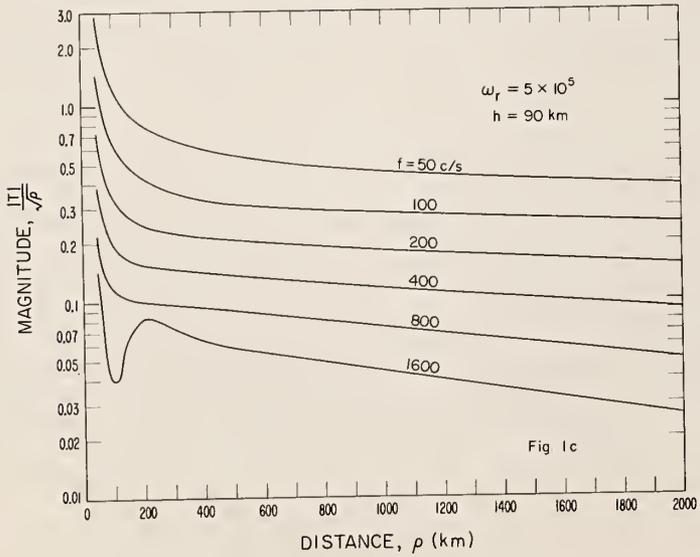
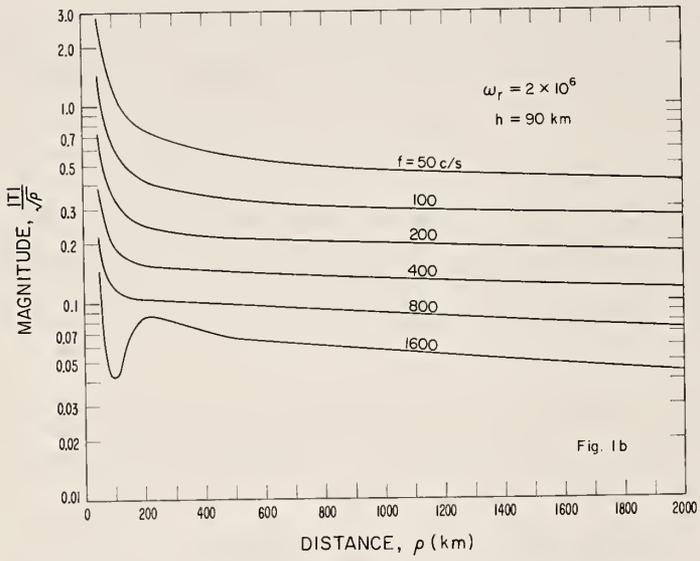
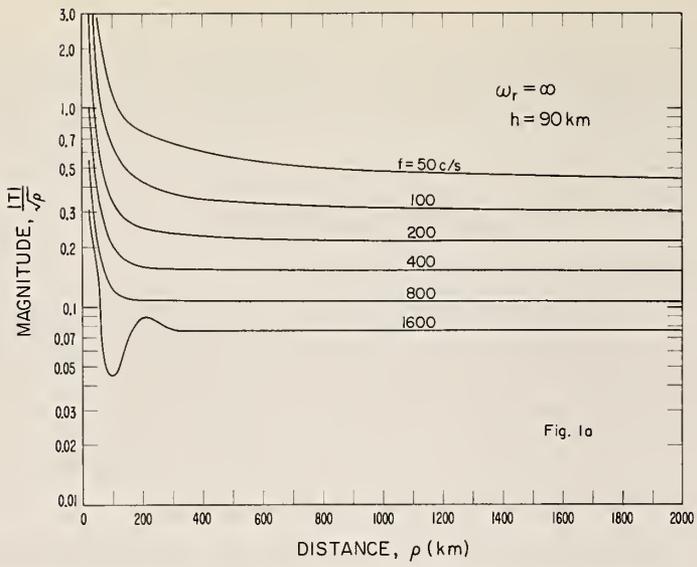
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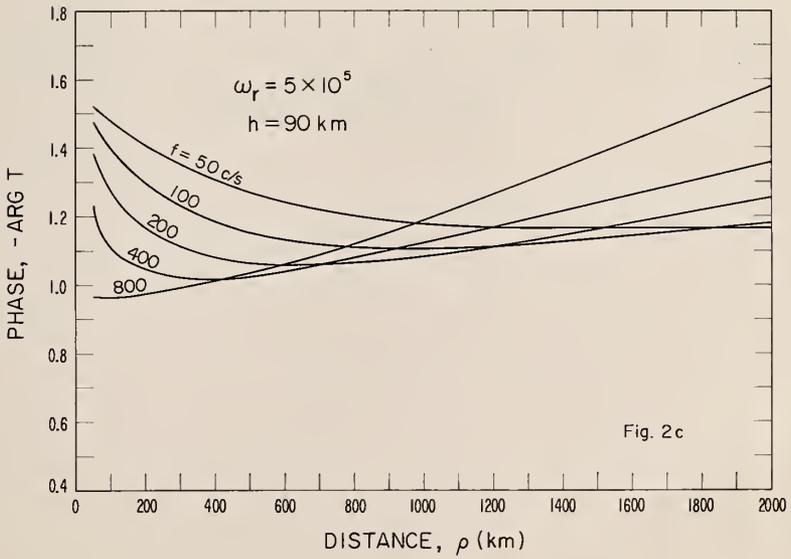
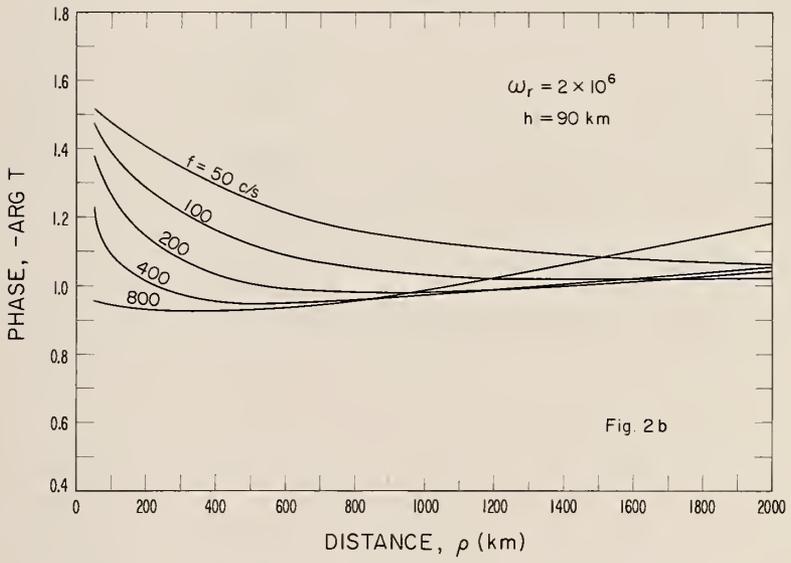
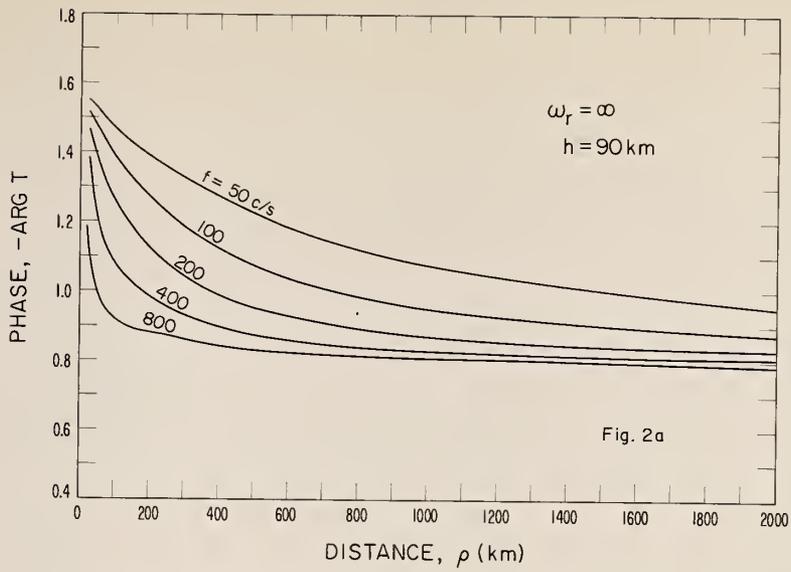
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Figure Captions

- Figs. 1a, b, c, The normalized magnitude of the magnetic field as a function of distance from the source (ρ is expressed in kilometers in both ordinate and abscissa).
- Figs. 2a, b, c, The normalized phase (lag), in radians, of the magnetic field.
- Figs. 3a, b, c, The normalized magnitude of the electric field as a function of distance from the source.
- Figs. 4a, b, c, The normalized phase (lag) of the electric field.





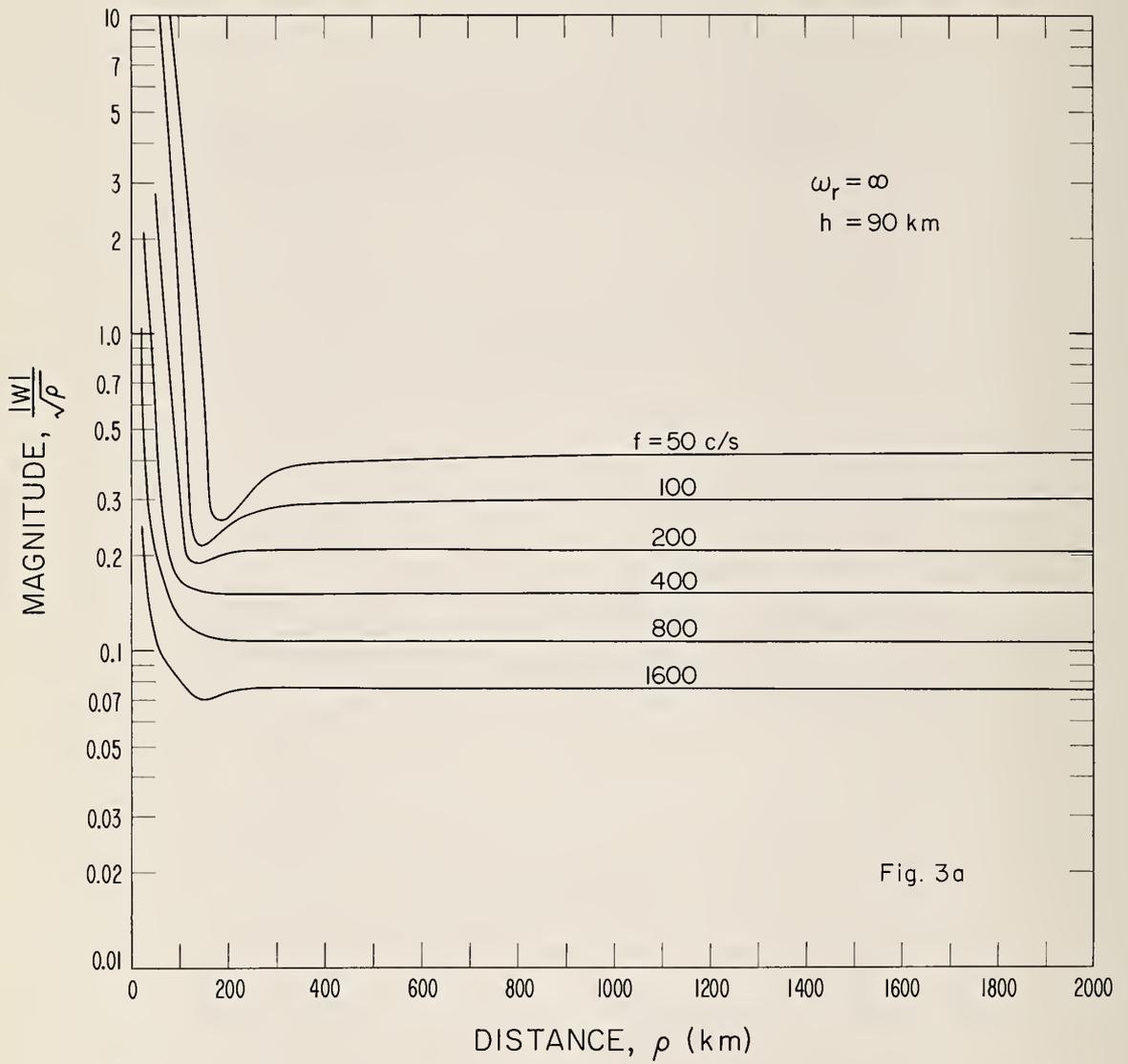
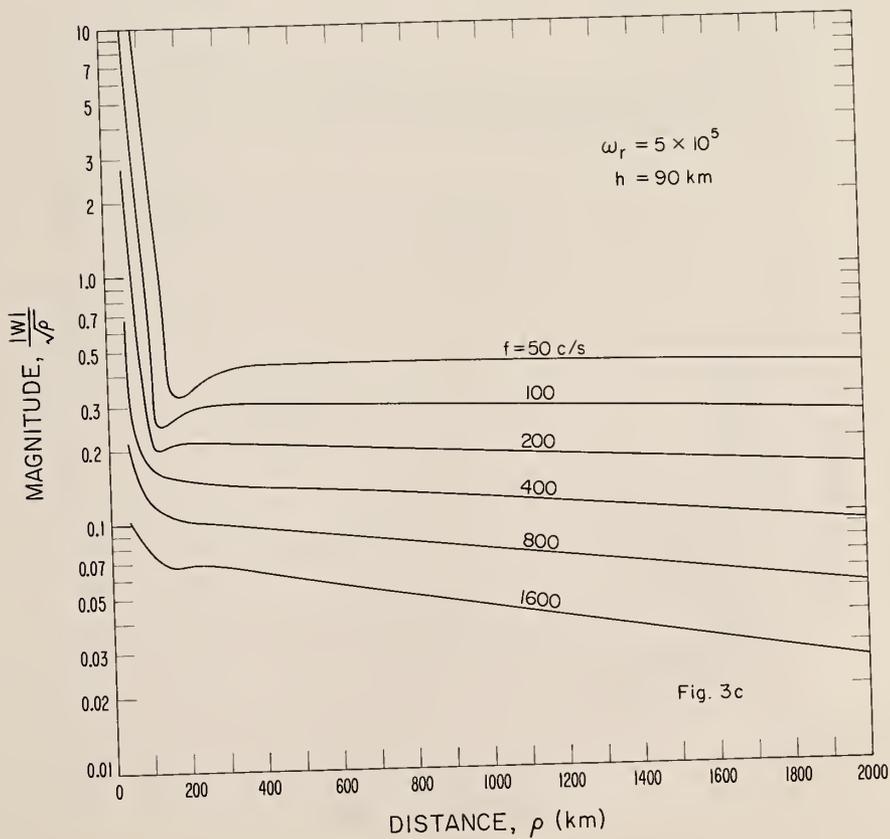
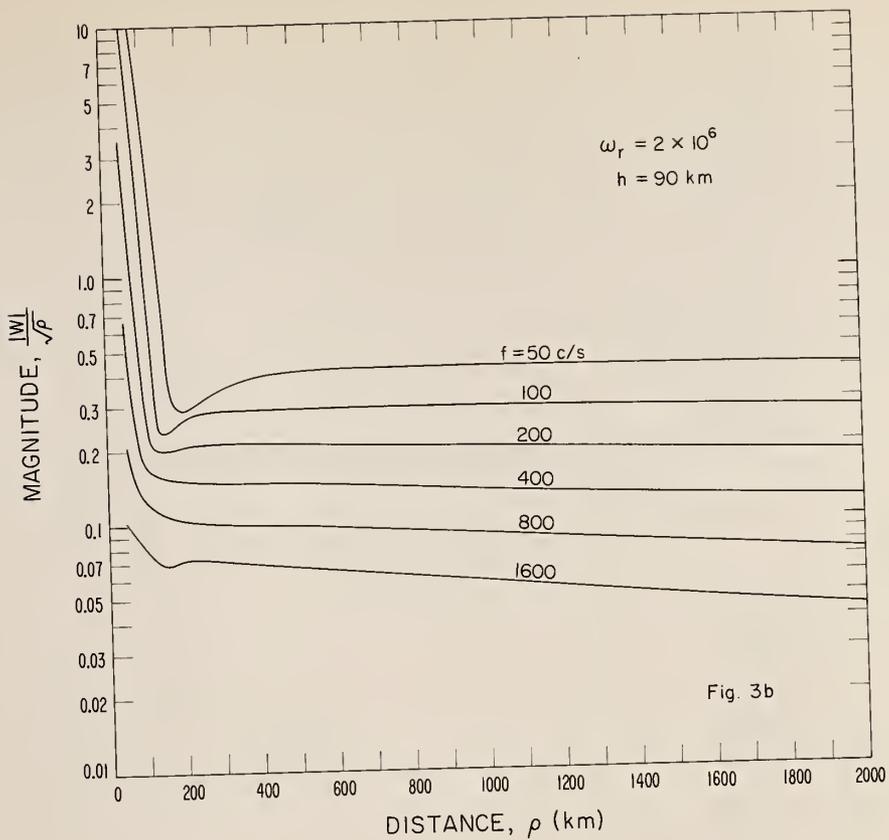
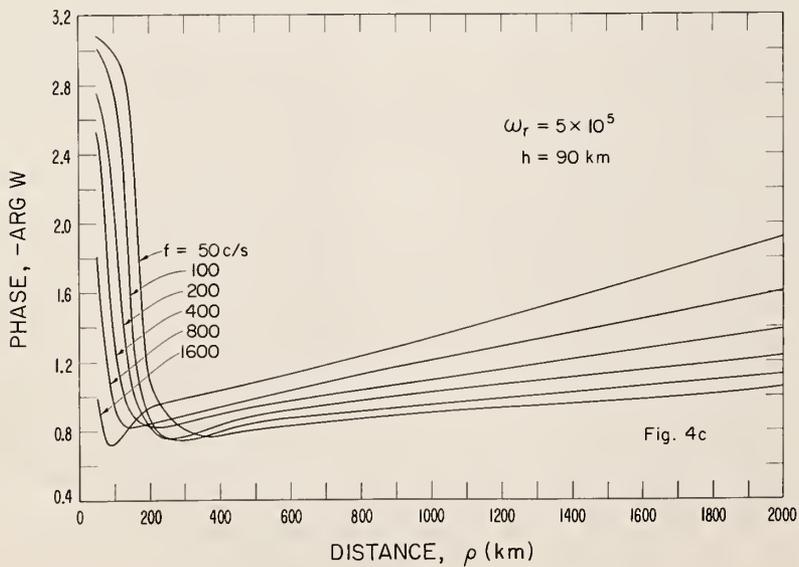
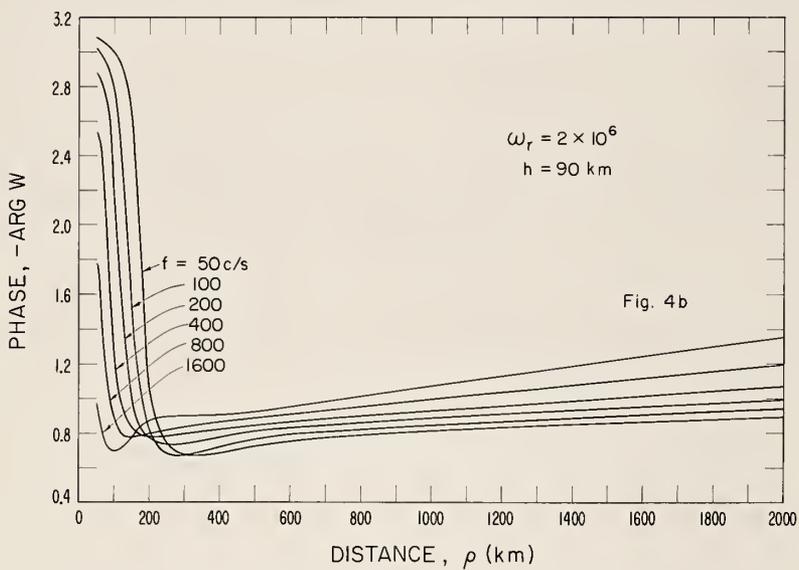
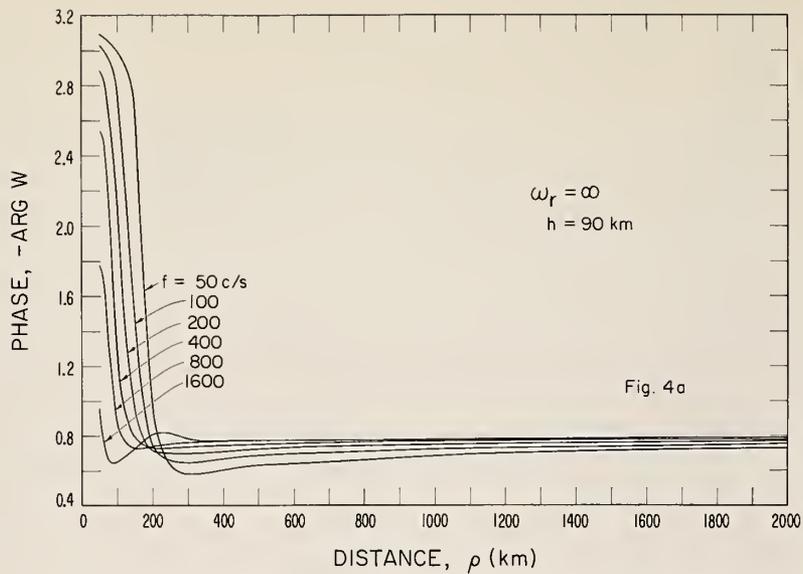
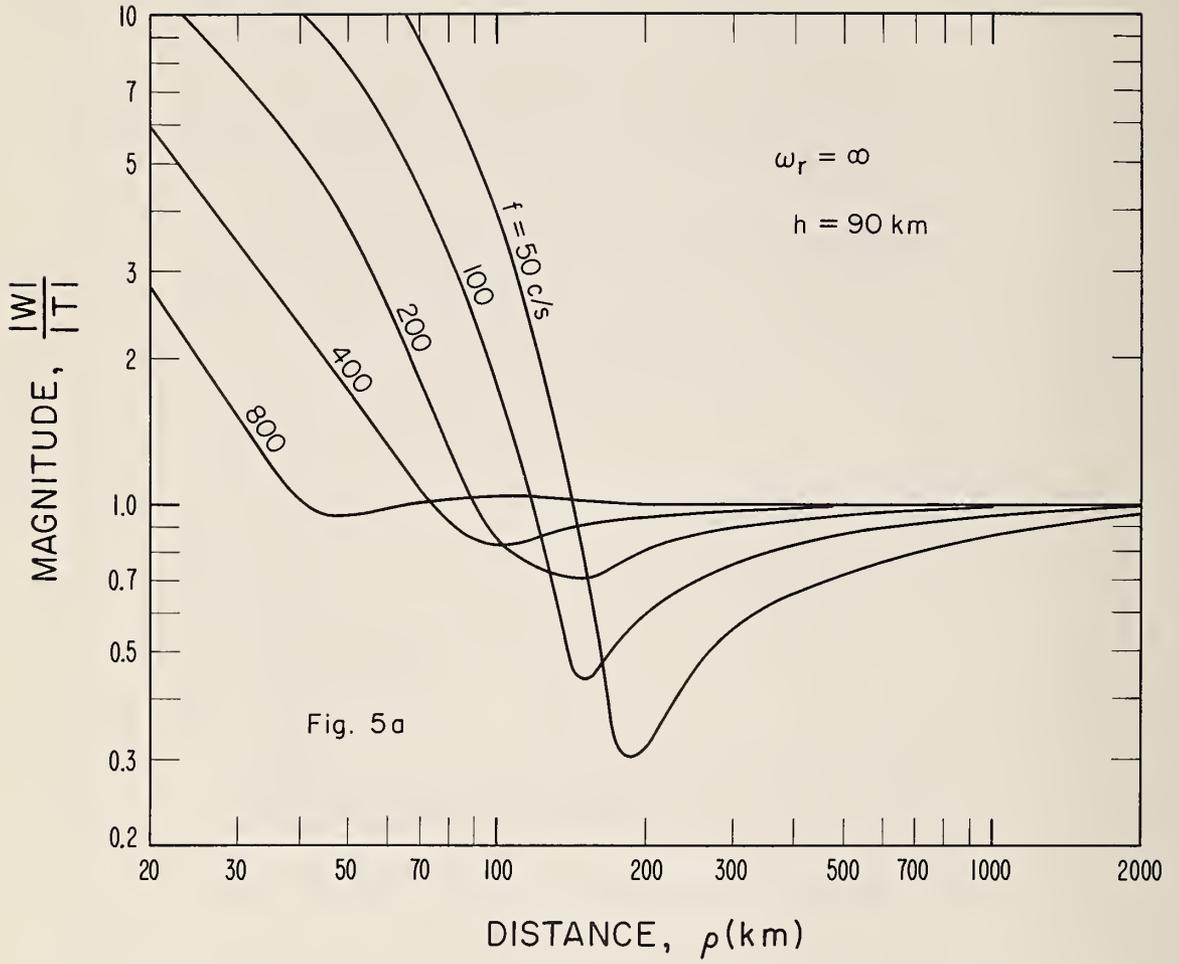


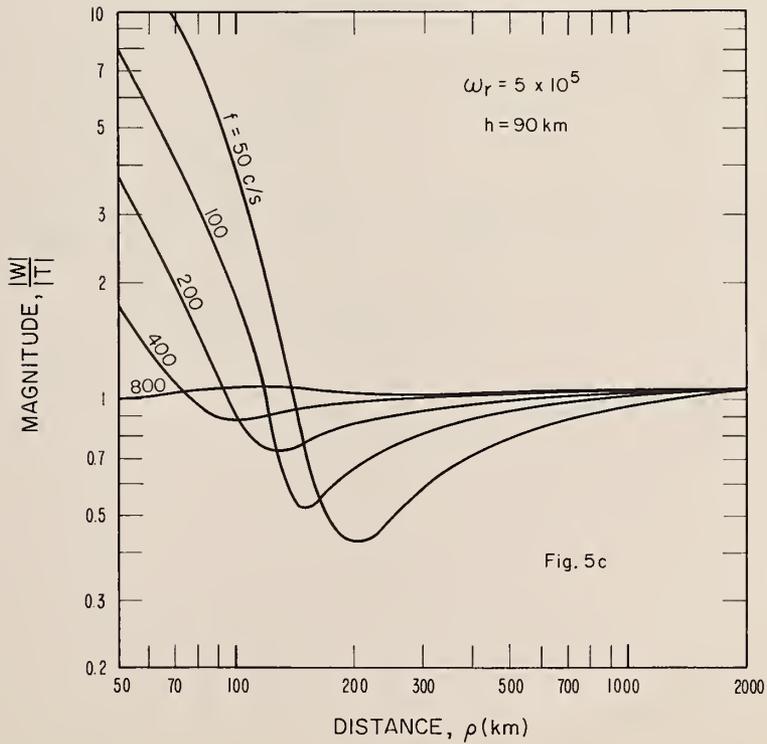
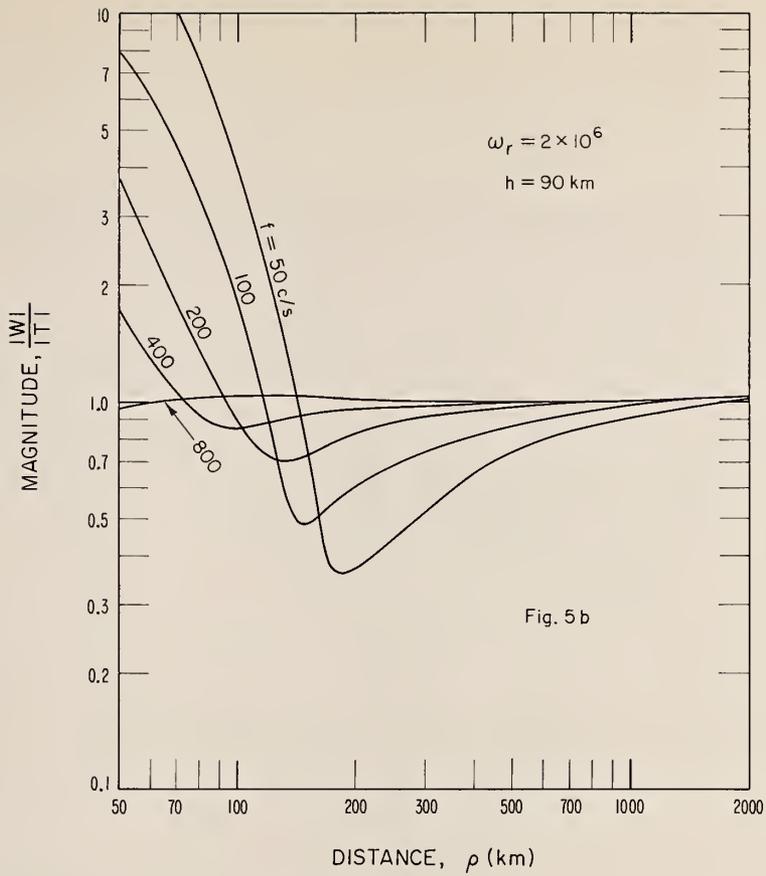
Fig. 3a

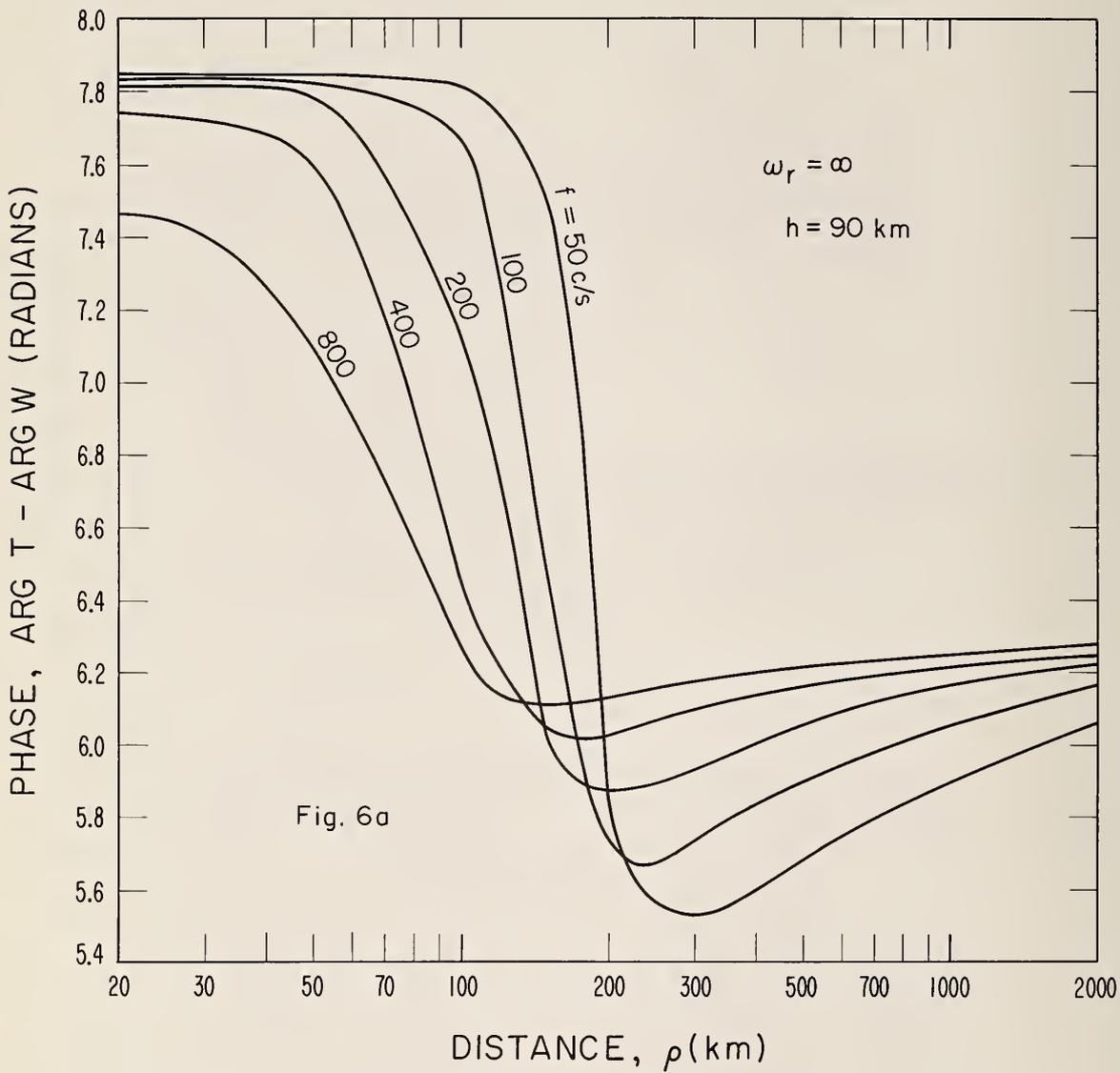


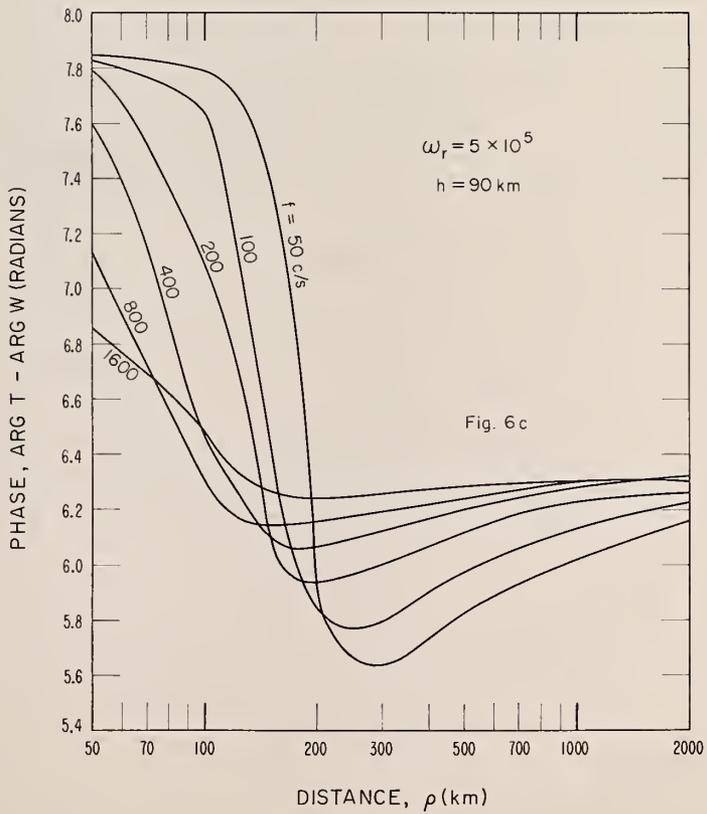
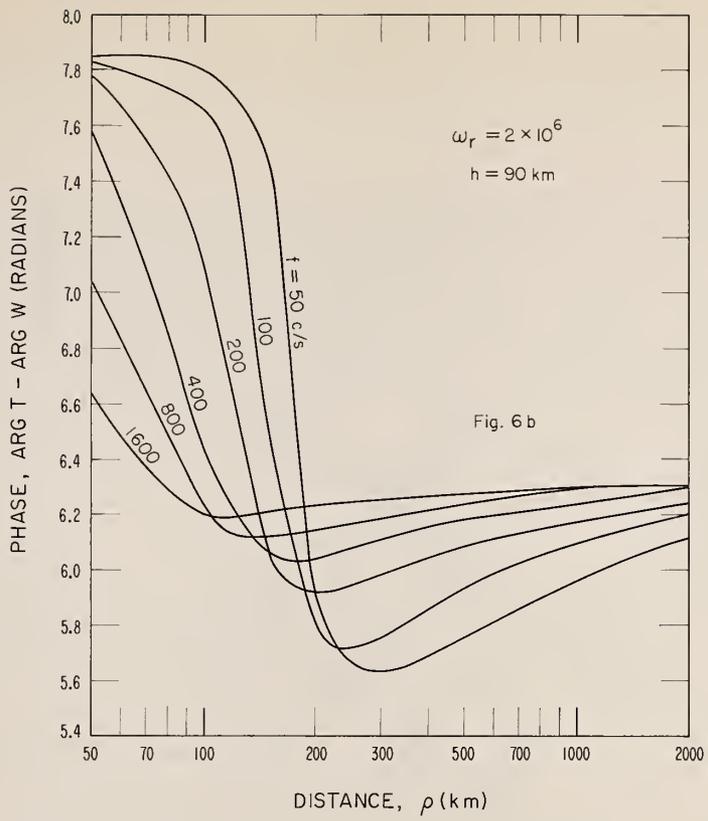


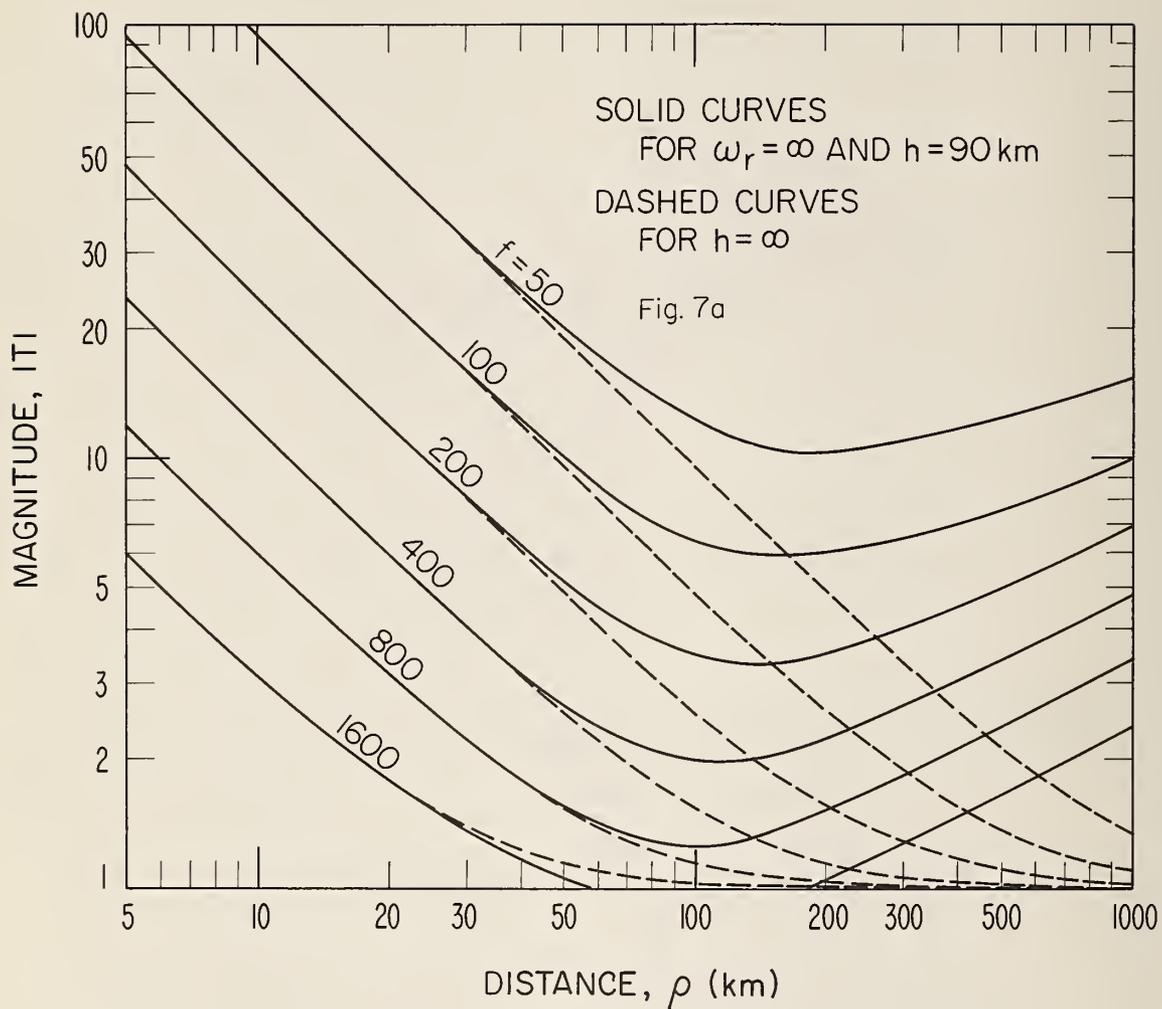
- Figs. 5a, b, c The magnitude of the radial wave impedance.
- Figs. 6a, b, c The phase (lag) of the radial wave impedance.
- Figs. 7a, b The magnitude and phase of the normalized magnetic field at short distances. The broken curves correspond to no reflecting layer.
- Figs. 8a, b The magnitude and phase of the normalized electric field at short distances.

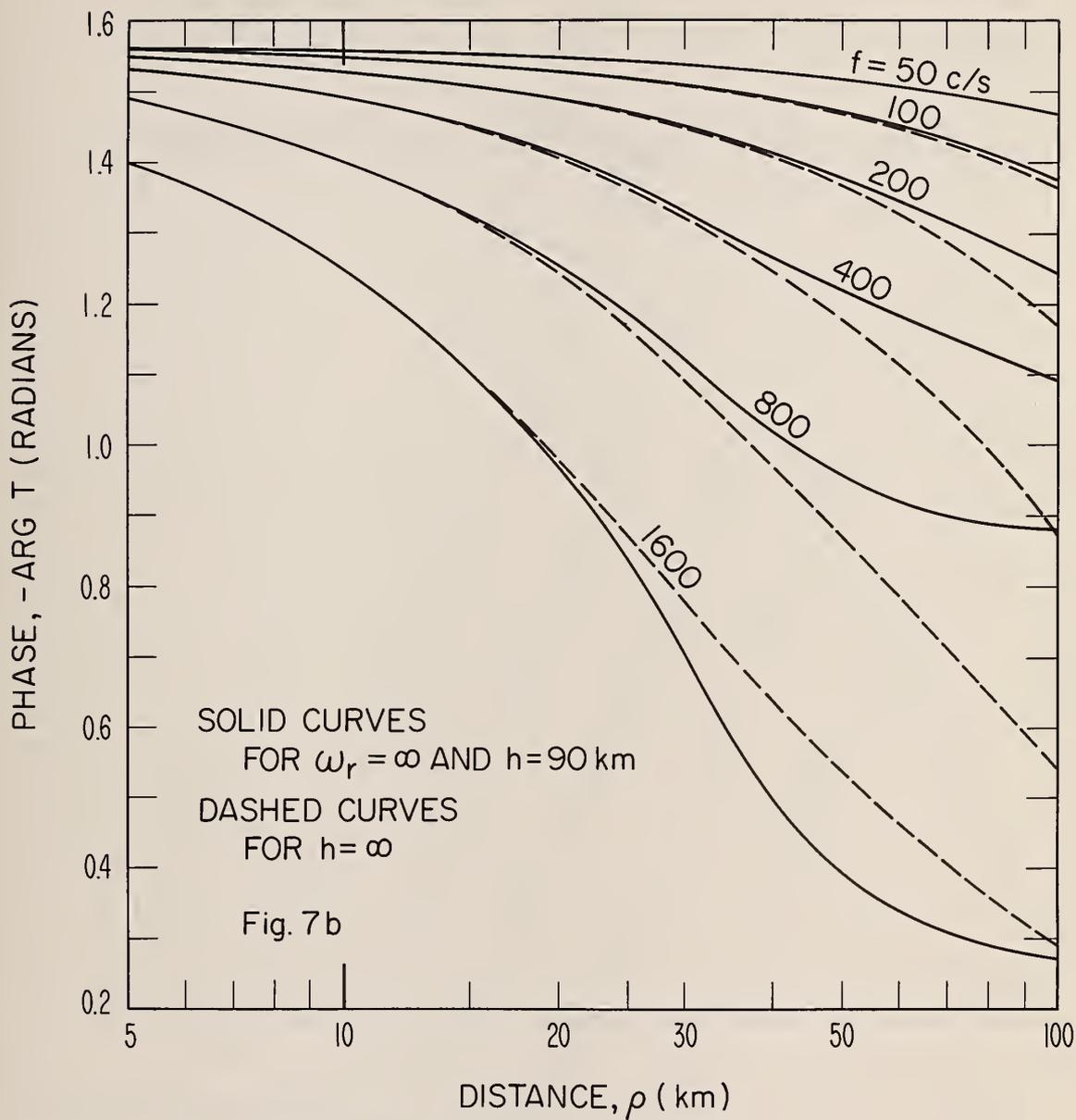


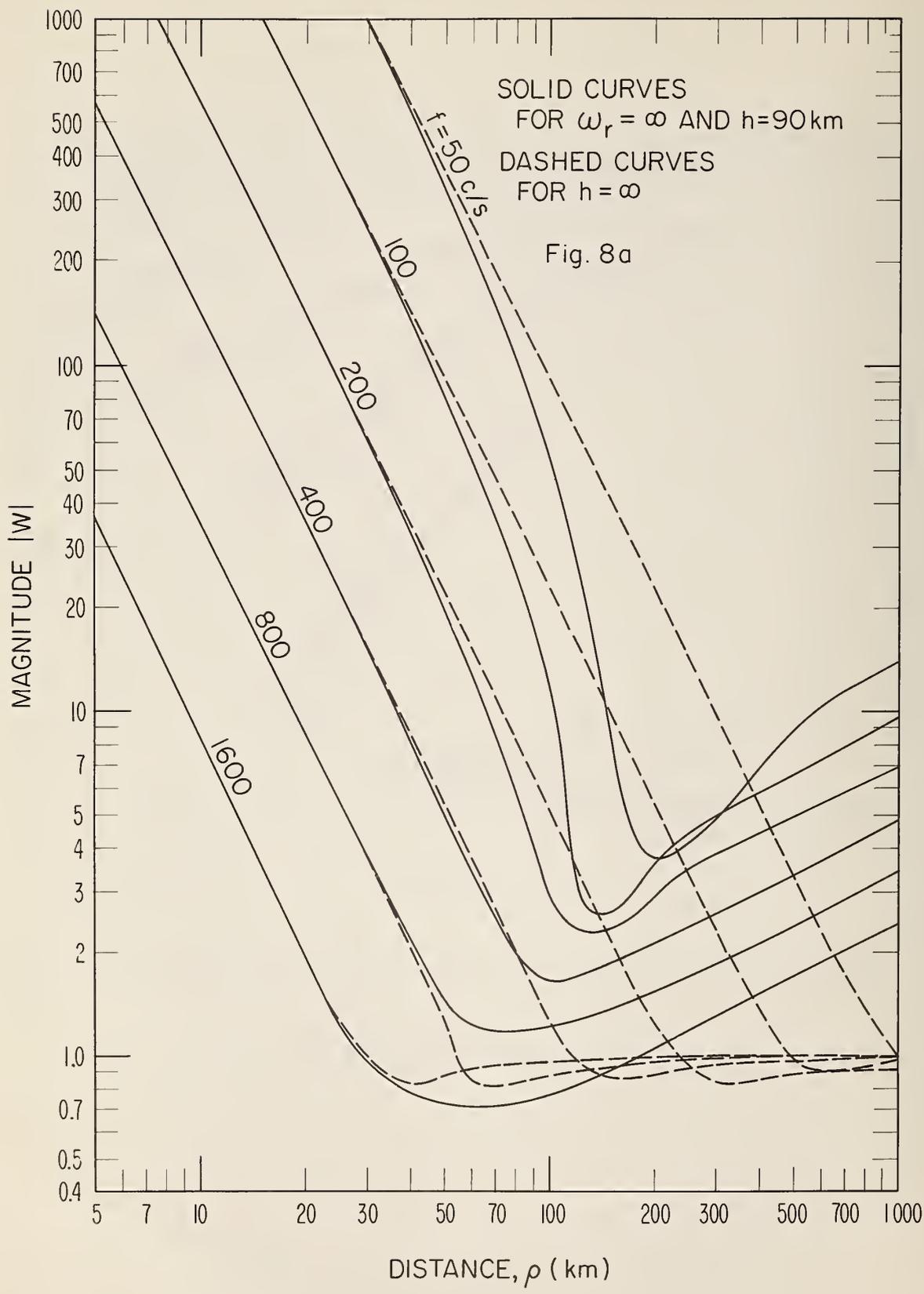


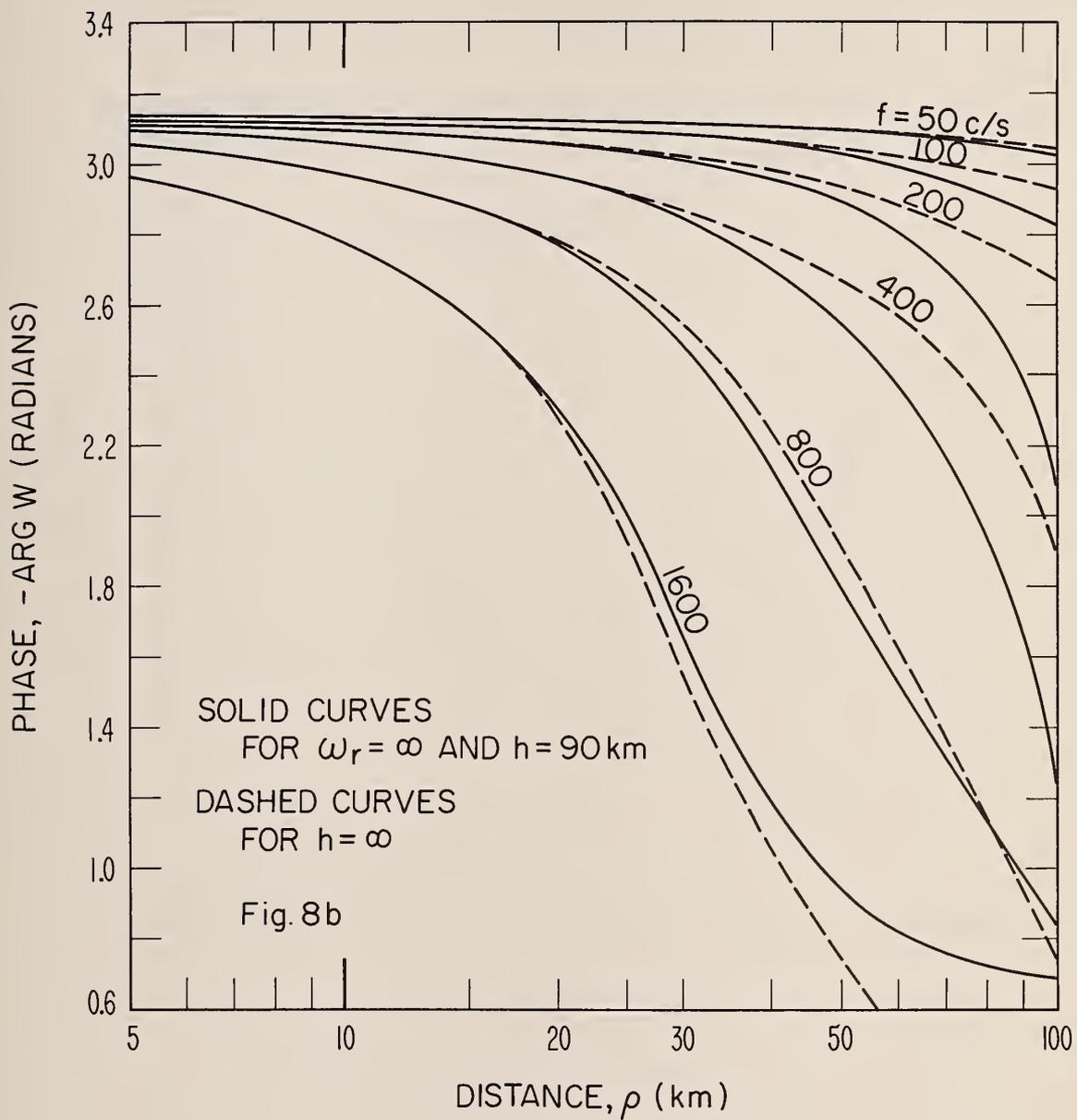




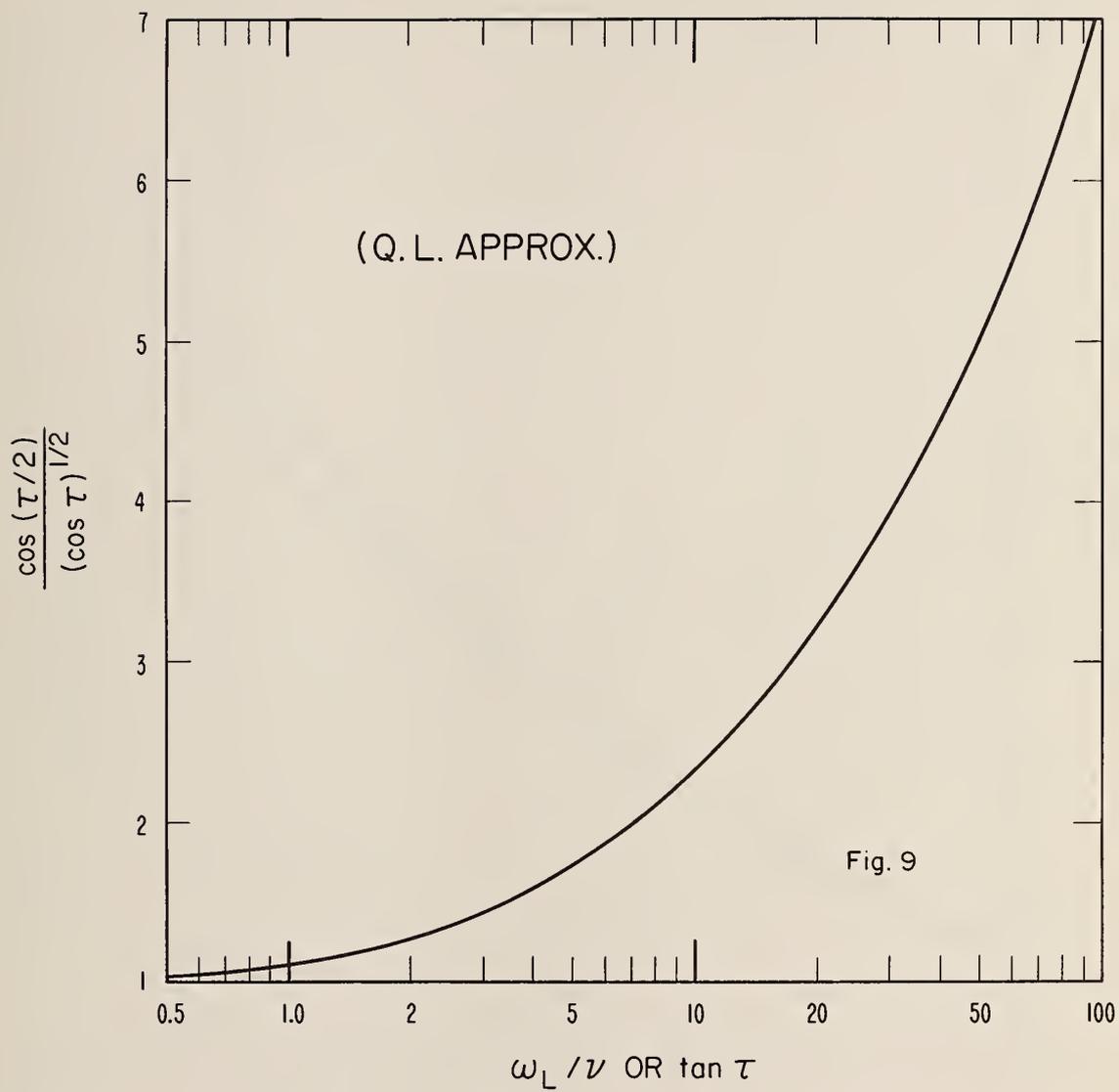


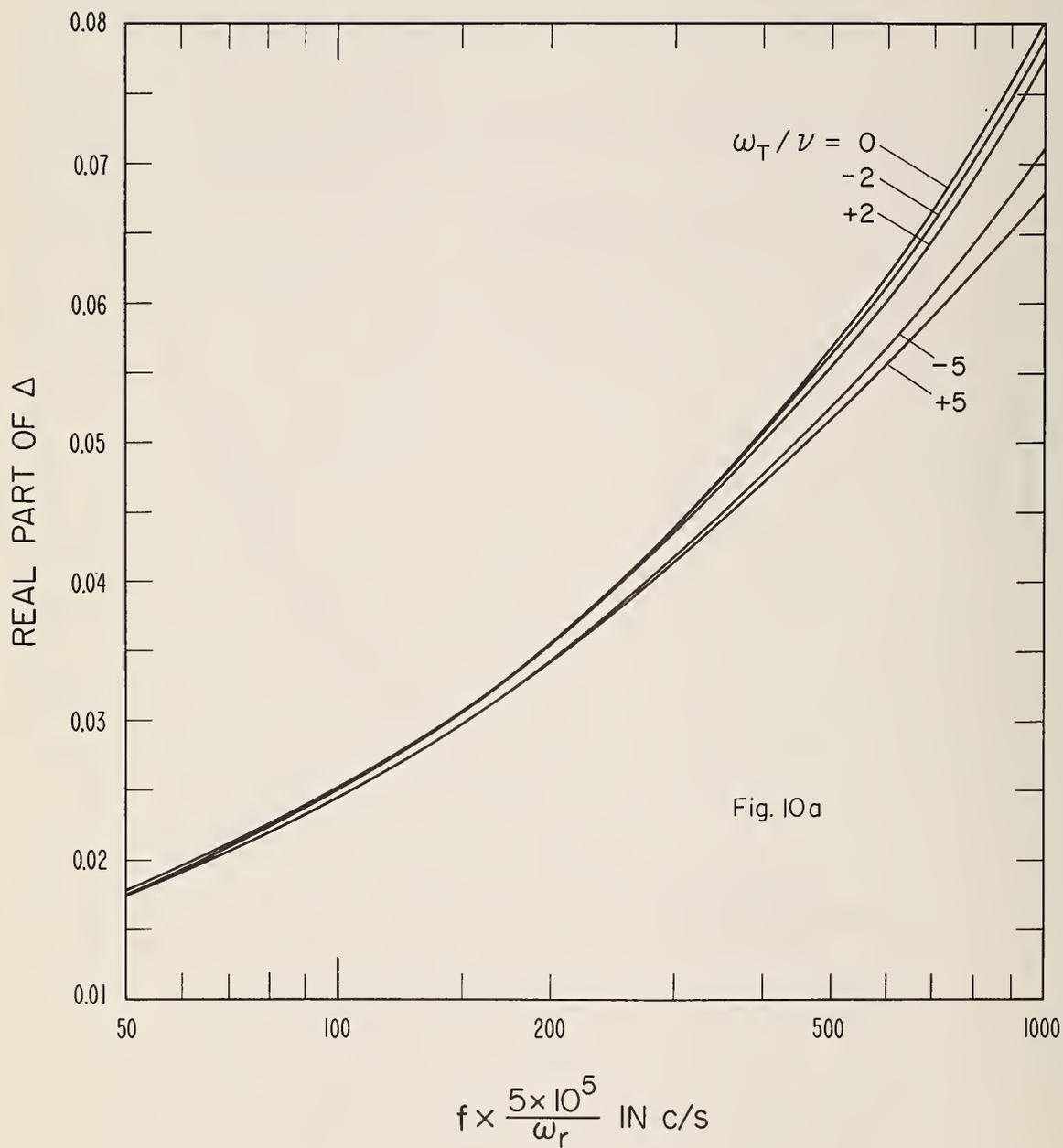






- Fig. 9 The earth's -magnetic-field-correction to the attenuation factor for propagation in the magnetic meridian.
- Figs. 10a, b The real and imaginary parts of the factor Δ as a function of normalized frequency (for a purely transverse magnetic field).
- Figs. 11a, b Attenuation and relative phase velocity for a purely transverse field.





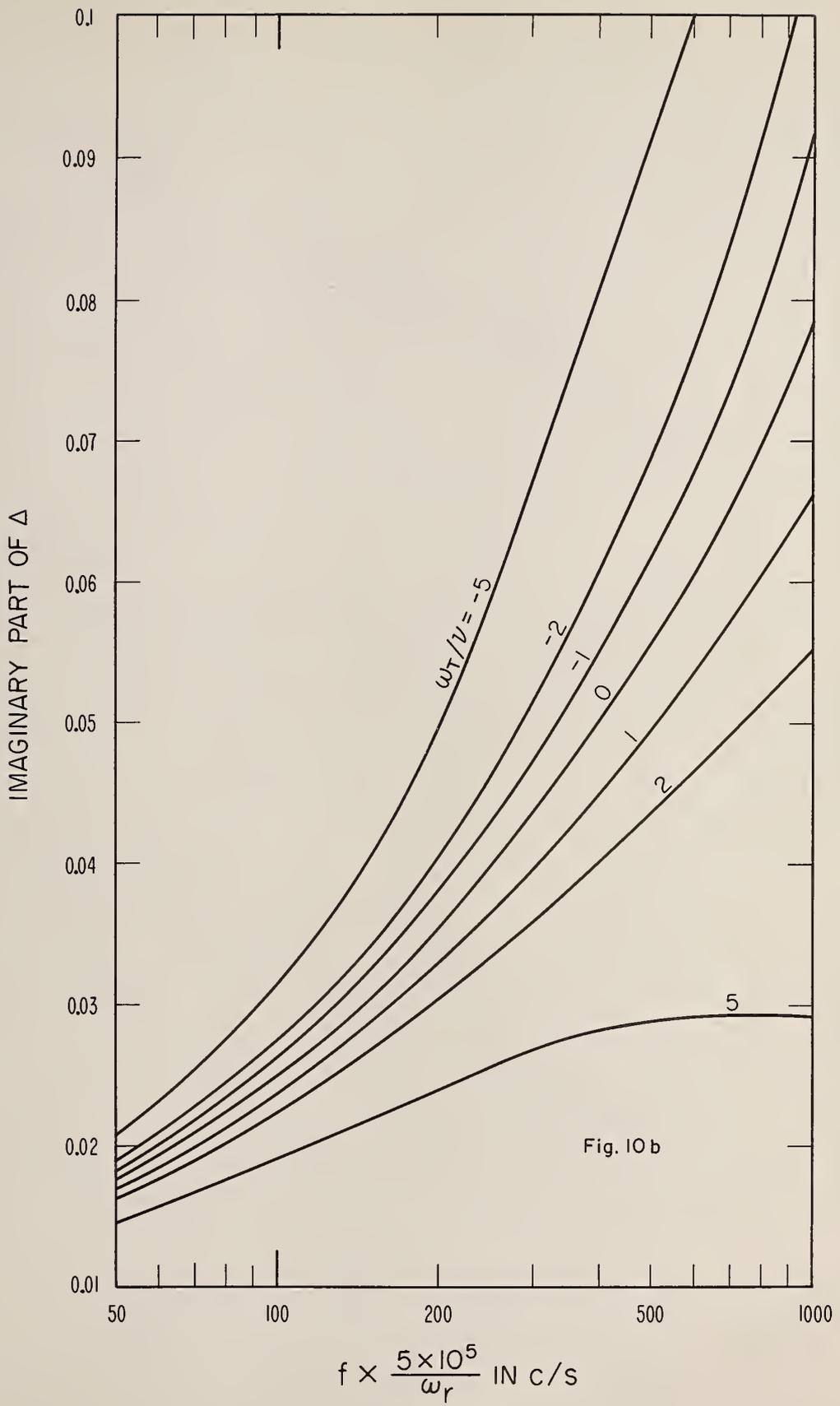
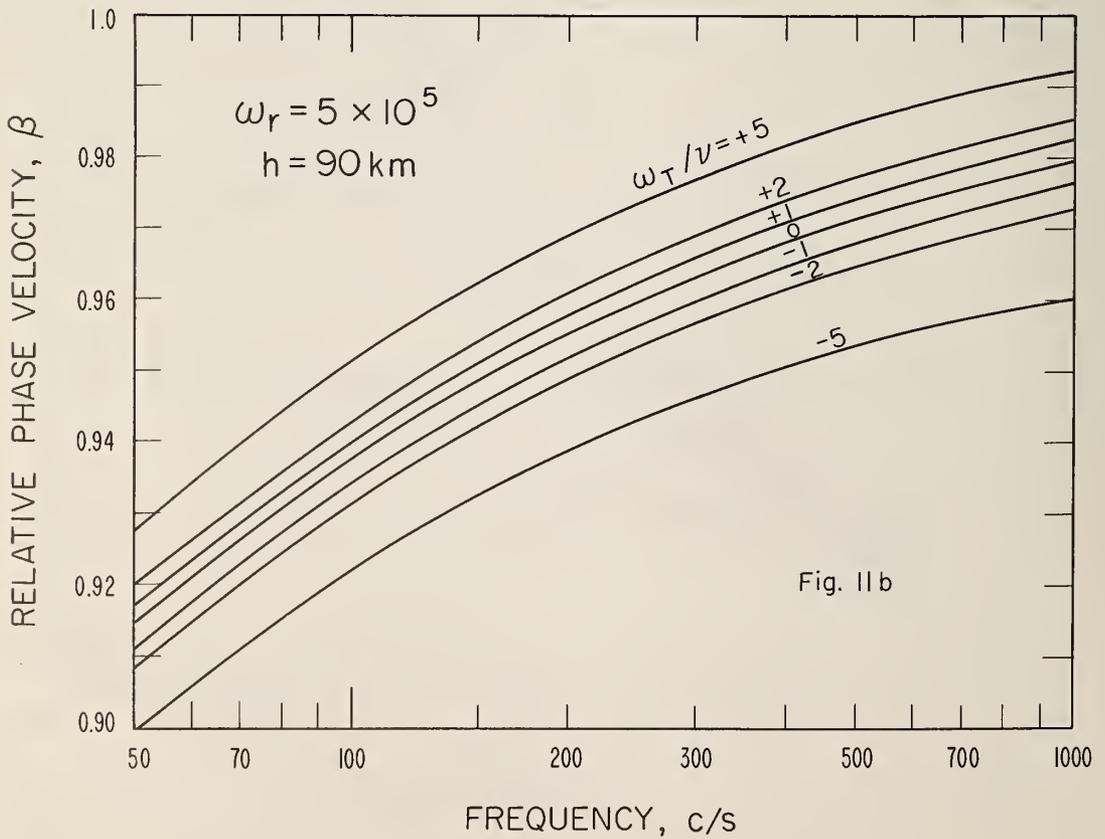
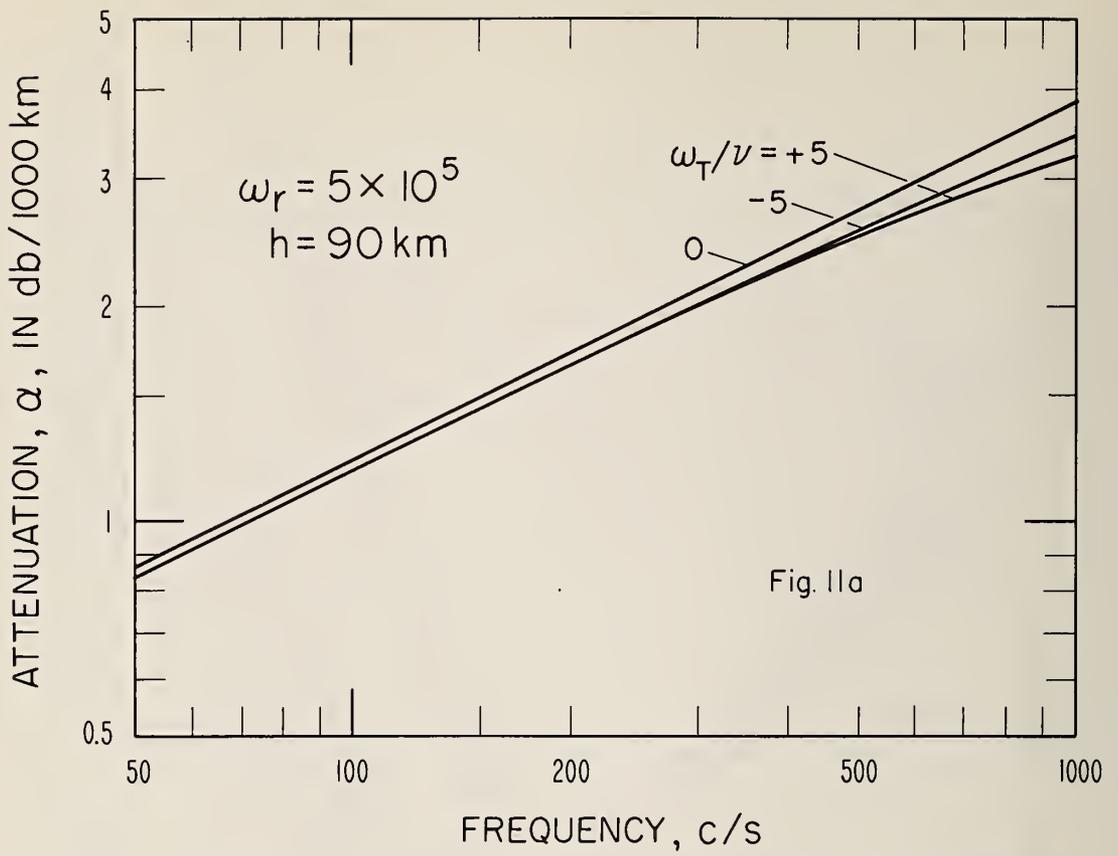


Fig. 10 b





THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

WASHINGTON, D.C.

Electricity and Electronics. Resistance and Reactance. Electron Devices. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Photographic Technology. Length. Engineering Metrology.

Heat. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology. Molecular Kinetics. Free Radicals Research.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Neutron Physics. Radiation Theory. Radioactivity. X-rays. High Energy Radiation. Nucleonic Instrumentation. Radiological Equipment.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Molecular Structure and Properties of Gases. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Air Conditioning, Heating, and Refrigeration. Floor, Roof, and Wall Coverings. Codes and Safety Standards. Heat Transfer. Concreting Materials.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analog Systems. Application Engineering.

• Office of Basic Instrumentation.

• Office of Weights and Measures.

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships. VHF Research. Radio Warning Services. Airglow and Aurora. Radio Astronomy and Arctic Propagation.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Research. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation Obstacles Engineering. Radio-Meteorology. Lower Atmosphere Physics.

Radio Standards. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Electronic Calibration Center. Microwave Physics. Microwave Circuit Standards.

Radio Communication and Systems. Low Frequency and Very Low Frequency Research. High Frequency and Very High Frequency Research. Ultra High Frequency and Super High Frequency Research. Modulation Research. Antenna Research. Navigation Systems. Systems Analysis. Field Operations.



NBS