NBS TECHNICAL NOTE

487

Considerations in Computing the Useful Frequency Range of Piezoelectric Accelerometers

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NBS TECHNICAL NOTE 487

ISSUED JULY 1969

Nat. Bur. Stand. (U.S.), Tech. Note 487, 20 pages (June 1969)
CODEN: NBTNA

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For sale by the Superintendent of Documents, Government Printing Office
Washington, D.C., 20402 - Price 30 cents

Contents

List of Tables

List of Figures

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Nathan Newman

This paper analyzes two lumped-parameter models for computing the usable frequency range of piezoelectric accelerometers. The analyses indicate why application of an electrical excitation to the piezoelectric element of a mounted pickup does not, in general, give the same result as application of a mechanical acceleration to the structure on which the pickup is mounted. Tabular results of the computations for various sets of parameters indicate those cases for which the electrical drive will give resonant frequency values within 2% of those for the pickup mounted on a vibrating structure. For these parameter sets, the electrical drive can be used as a reliable substitute.

Key words: Electrical excitation; lowest resonant frequency; mechanical excitation; piezoelectric accelerometer; usable frequency range.

1. Introduction

Piezoelectric accelerometers are often favored for measuring structural vibration because of their small size and mass, independence of external bias or excitation, and wide range of usable frequencies. Figure 1 shows a typical configuration, mounted on an aircraft panel. The small size generally means that the effect of the accelerometer on the structure's vibrational characteristics will be minimal. On the other hand, as will be shown, the structure very often does affect the vibrational characteristics of the attached accelerometer.

The sensitivity of an accelerometer (often called a pickup) is defined as the ratio of the output signal (voltage in this paper) to the acceleration of its base. The major resonant frequency of the accelerometer is the lowest frequency for which the sensitivity has a maximum. The usable frequency range of the accelerometer is generally taken as that region in which the sensitivity does not change significantly (viz. 5%) from the value found near 100 Hz when calibrated on a conventional shaker table. However, the upper limit of the range for which a particular accelerometer is usable when mounted on a particular structure is lower than that determined by the resonance of the accelerometer alone, when the mass of the accelerometer affects the motion of the structure. The properties of principal interest are the stiffness of attachment to, and the effective mass of the structure. For most applications, an accelerometer is not usable if the motion of the structure with the pickup attached is significantly different from the motion it would have without the pickup.

This paper analyzes two lumped-parameter models which represent experimental techniques often used to determine the resonant frequency of an accelerometer. The two experimental techniques analyzed are: the application of a sinusoidal acceleration of constant amplitude to the structure on which the pickup is mounted, and the application of a sinusoidal voltage of constant amplitude to the piezoelectric element. The results of the two techniques differ, because when a constant voltage is applied to the piezoelectric element, the motions of all the other elements of the mechanical system are determined by the mechanical parameters of the system, whereas application of a constant acceleration to the mounting structure imposes an overriding influence from outside the pickup which limits, to the other elements, the effect of these parameters. In both cases, the upper limit on the usable range of the pickup is set by the lowest resonance of the "pickup-structure" system under the driving conditions imposed.

2. Computations for Resonant Frequencies

The method of computation used to find the resonant frequencies is very considerably simplified by the use of lumped-constant models. The distributed masses are treated as though concentrated at single points and are designated m_1, m_2, and m_3 in Figure 2. Their interconnections are assumed to be massless and are represented by the spring constants k_a and k_s. The accelerometer is represented by a "seismic mass", m_1, linked to its base, m_2, by a spring, k_a. The accelerometer is attached to the test structure, m_3, by a spring, k_s.

2.1. Mechanical Drive

Suppose a sinusoidally varying mechanical force, $F \cos \omega t$, is imposed on m_3, from outside the system (Figure 3), so that its position, x_3, is $A_3 \cos \omega t$. The resultant motions of m_1 and m_2 (i.e. x_1 and x_2) are now considered as the frequency of the drive force is varied. After transient effects die away, the equations describing the motion of m_1 and m_2 under the dynamic forces are:

$$m_1 \ddot{x}_1 + k_a(x_1 - x_2) = 0 \qquad (1)$$

$$m_2 \ddot{x}_2 + k_a(x_2 - x_1) + k_s(x_2 - x_3) = 0 \qquad (2)$$

The resonance sought is the lowest value of ω at which a maximum of $(x_1 - x_2)$ occurs. When the system is in dynamic equilibrium and resonance is approached from below, the motions will be at the drive frequency and in phase.

$$x_1 = A_1 \cos \omega t \quad (3)$$

$$x_2 = A_2 \cos \omega t \quad (4)$$

$$\ddot{x}_1 = - A_1\omega^2 \cos \omega t \quad (3a)$$

$$\ddot{x}_2 = - A_2\omega^2 \cos \omega t \quad (4a)$$

By sutstitution in (1) and (2), we obtain

$$(k_a - m_1\omega^2)\, A_1 \qquad - k_a A_2 = 0 \quad (5)$$

$$- k_a A_1 + (k_s + k_a - m_2\omega^2)\, A_2 = k_s A_3 \quad (6)$$

Resonance occurs when $A_1 - A_2$ (and therefore A_1 and A_2 individually) has a maximum value. There is no problem due to phase considerations because resonance is approached from below and, with no damping, x_1 and x_2 can be considered to be in phase with the motion of the driving element i.e. with x_3. The solution for A_1 and A_2 each has the determinant of its coefficients in the denominator. Thus maximum values of A_1 and A_2 [and consequently $(x_1 - x_2)$] occur when this determinant vanishes. An equation in the resonant frequency ω (explicitly in ω^2) results:

$$(k_a - m_1\omega^2)\, (k_a + k_s - m_2\omega^2) - k_a^2 = 0 \quad (7)$$

Rewritten as a quadratic in ω^2,

$$\omega^4 - [k_a (\frac{1}{m_1} + \frac{1}{m_2}) + k_s \frac{1}{m_2}]\, \omega^2 + \frac{k_a k_s}{m_1 m_2} = 0 \quad (8)$$

At this point, calculations are simplified by introducing non-dimensional parameters a, b, and r where

$a \equiv m_2/m_1$

$b \equiv m_3/m_1$ (b is not used in this section)

$r \equiv k_s/k_a$

In addition, if we use

$\omega_o^2 \equiv k_a/m_1$ (or $\omega_o = \sqrt{k_a/m_1}$)

we obtain equations (8a) and (8b):

$$\omega^4 - (1 + \frac{1}{a} + \frac{r}{a})\, \omega_o^2\, \omega^2 + \frac{r}{a}\, \omega_o^4 = 0 \quad (8a)$$

$$(\frac{\omega^2}{\omega_o^2})^2 - (1 + \frac{1}{a} + \frac{r}{a})\, \frac{\omega^2}{\omega_o^2} + \frac{r}{a} = 0 \quad (8b)$$

Note that the use of the non-dimensional relative frequency, ω/ω_o, as well as the non-dimensional parameters a and r, make Equation (8b) non-dimensional. Conforming with the verbal description of ω/ω_o, a and b may be referred to as relative masses and r, as a relative spring constant.

The relative resonant frequency is given by:

$$\frac{\omega^2}{\omega_o^2} = \frac{[1 + \frac{1}{a}(1 + r)] - \sqrt{[1 + \frac{1}{a}(1 + r)]^2 - \frac{4r}{a}}}{2} \qquad (9)$$

Although the solution (9) is explicitly for ω^2/ω_o^2, it will be referred to as the solution for the relative resonance, ω/ω_o, as well without specifically stating, each time, that the square root process is involved. Only the negative sign is used for the radical because the lower resonance is sought. Since the motion of the structure m_3 is forced, the value of m_3 does not affect the resonance and thus b does not appear in (9).

Earlier in this section ω_o was introduced as an abbreviated symbol for $\sqrt{k_a/m_1}$. The choice of this symbol was not fortuitous. The quantity $\sqrt{k_a/m_1}$ is generally accepted as the undamped natural frequency for a single mass driven by a single spring whose other end attaches to a relatively very large mass. Throughout the paper, ω_o, is used as a reference frequency so that division of ω by ω_o gives the non-dimensional quantity, relative resonant frequency.

The use of relative parameters has a twofold purpose. The initial benefit is the simplification of the mathematical manipulations in the solution of a differential equation. However, its more significant worth perhaps lies in the potential of scaling one set of solutions of ω/ω_o into many. An example of the scaling procedure is given in the section on "limiting and special cases".

2.2. Electrical Drive

The case of electrical drive is shown in Figure 4, with the applied force and displacements indicated. A sinusoidal voltage applied to the piezoelectric element of the pickup generates a force, F cos ωt, across k_a. The equations of motion expressing forces acting on the three masses are:

$$m_1 \ddot{x}_1 + k_a (x_1 - x_2) = F \cos \omega t \qquad (10)$$

$$m_2 \ddot{x}_2 + k_a (x_2 - x_1) + k_s (x_2 - x_3) = -F \cos \omega t \qquad (11)$$

$$m_3 \ddot{x}_3 + k_s (x_3 - x_2) = 0 \qquad (12)$$

At dynamic equilibrium below resonance the motions and accelerations are:

$$x_1 = B_1 \cos \omega t \qquad (13)$$

$$x_2 = B_2 \cos \omega t \qquad (14)$$

$$x_3 = B_3 \cos \omega t \qquad (15)$$

and

$$\ddot{x}_1 = -B_1\omega^2 \cos \omega t \qquad (13a)$$

$$\ddot{x}_2 = -B_2\omega^2 \cos \omega t \qquad (14a)$$

$$\ddot{x}_3 = -B_3\omega^2 \cos \omega t \qquad (15a)$$

By substitution in the equations of motion,

$$(k_a - m_1\omega^2)\, B_1 - k_a\, B_2 = F \qquad (16)$$

$$k_a\, B_1 + (k_a + k_s - m_2\omega^2)\, B_2 - k_s\, B_3 = -F \qquad (17)$$

$$k_s\, B_2 + (k_s - m_3\omega^2)\, B_3 = 0 \qquad (18)$$

and resonances occur when the determinant of the coefficients vanishes.

Setting the determinant equal to zero, we get:

$$(k_a - m_1\omega^2)\,(k_a + k_s - m_2\omega^2)\,(k_s - m_3\omega^2) - k_a^2\,(k_s - m_3\omega^2) - k_s^2\,(k_a - m_1\omega^2) = 0 \qquad (19)$$

Performing the indicated multiplication, and removing the factor ω^2 which corresponds to non-oscillatory motion

$$\omega^4 - [k_a\,(\frac{1}{m_1} + \frac{1}{m_2}) + k_s\,(\frac{1}{m_2} + \frac{1}{m_3})]\,\omega^2 + k_a k_s\,(\frac{1}{m_1 m_2} + \frac{1}{m_2 m_3} + \frac{1}{m_3 m_1}) = 0 \qquad (20)$$

Converting to relative parameters, as was done in equation (8) and solving:

$$\frac{\omega^2}{\omega_0^2} = \frac{[1 + \frac{1}{a}(1+r) + \frac{r}{b}] - \sqrt{[1 + \frac{1}{a}(1+r) + \frac{r}{b}]^2 - \frac{4r}{a} - \frac{4r}{b}(1+\frac{1}{a})}}{2} \qquad (21)$$

This is a general solution for ω^2/ω_0^2 and includes the previous solution, equation (9), as a special case for which b is very large, (i.e. $m_3 >> m_1$).

3. Resonant Frequency Solutions for Mechanical vs. Electrical Drive

The equations (9) and (21) lead to solutions for ω^2/ω_o^2 (and therefrom ω/ω_o) which are in general different, but not necessarily significantly so. The relative resonant frequency ω/ω_o, directly converts to the resonant frequency, ω, for any given $\omega_o = \sqrt{k_a/m_1}$.

In order to distinguish between the values of ω/ω_o, they will be identified by $\omega(a,r)/\omega_o$, the relative resonant frequency for the mechanical drive and by $\omega(a,b,r)/\omega_o$, the relative resonant frequency for the electrical drive. Abbreviated descriptive designations are RRF-MD and RRF-ED, respectively. The latter, which is a function of "b", as well as of "a" and "r", indicates the presence of a structure which responds to the electrical drive rather than to other forces imposed on it.

Ultimately, we would like to express the departure of the RRF-ED from the RRF-MD as a percentage of the RRF-MD. This is the percent deviation of the relative resonant frequency using the electrical rather than the mechanical drive. This is designated in Table 2 of PD of RF, (see Equation 23). In terms of the two values of ω/ω_o,

$$\text{PD of RRF} = \frac{\omega(a,b,r)/\omega_o - \omega(a,r)/\omega_o}{\omega(a,r)/\omega_o}\ 100\% \qquad (22)$$

Since ω_o is constant for any particular set (a,r), it follows that:

$$\text{PD of RRF} = \frac{\omega(a,b,r) - \omega(a,r)}{\omega(a,r)}\ 100\% = \text{PD of RF} \qquad (23)$$

The expression in the middle is thus the percent deviation of the resonant frequency, of electrical relative to mechanical drive.

Table 1 applies to the case of mechanical drive, as computed from Equation (9). It lists $\omega(a,r)/\omega_o$ for a number of representative pairs of values (a,r). It will be found that the value of $r = \infty$ makes $\omega/\omega_o = 1$, whatever the value of a. This corresponds to the accelerometer base m_2 rigidly attached to a very large mass m_3, so that m_1 and k_a are the only resonance-determining parameters.

The relative resonant frequency for an electrical drive, $\omega(a,b,r)/\omega_o$ is given in Table 2, as determined by calculations using Equation (21). Because of the additional parameter, b, Table 2 is much larger than Table 1 and is presented in four parts. The final column, headed PD of RF, gives a numerical indication of how far apart the results of the two computations are, for any particular parameter set, (a,b,r).

The parameter sets, (a,b,r), for which results are given in Table 2, correspond to a rather wide range of mass and spring constant ratios.

However, the PD of RF do not fall into a readily predictable pattern, and from Table 2 it is not easy to determine the resonant frequencies and parameters (a,b,r) that combine to give a particular value of PD of RF. Table 3 attempts to partially fill this gap. Two levels of PD (percent deviation) were arbitrarily selected; the one 2% (or less), tolerable, the other 10% (or more), intolerable. Herein, "tolerable" means that results could be accepted without any resonant frequency correction being required; "intolerable" means that results (PD of RF) point to a significant difference between $\omega(a,b,r)$ and $\omega(a,r)$. Thus the electrical drive can make the accelerometer appear to have a higher resonant frequency.

4. Some Limiting or Special Cases

Equation (21) provides the general solution for the relative resonant frequency, ω/ω_o. Unless $m_3 >> m_1$, $\omega \equiv \omega(a,b,r)$; if m_3 is infinite, $\omega \equiv \omega(a,r)$. Some interesting examples arise for certain specific values of one or more of the parameters.

If $k_s = 0$, there is no attachment to the structure and we have the familiar case of the unmounted accelerometer. If the base is driven mechanically, $\omega = \omega_o$ or $\omega/\omega_o = 1$. If the drive is electrical $\omega(a) = \omega_o\sqrt{1 + 1/a}$ or $\omega(a)/\omega_o = \sqrt{1 + 1/a}$. In terms of the fundamental masses, $\omega(a)/\omega_o = \sqrt{1 + m_1/m_2}$. The percent deviation of resonant frequency (PD of RF) is to a first approximation (1/2a) 100% [using the equivalent of Equations (22) and (23)].

In a similar manner, if the accelerometer base is rigidly fastened to a structure, m_2 and m_3 combine and $\omega/\omega_o = 1$ for the mechanical drive and $\omega(a,b)/\omega_o = \sqrt{1 + 1/(a+b)}$. In terms of the fundamental masses, $\omega(a,b)/\omega_o = \sqrt{1 + m_1/(m_2 + m_3)}$. Again to a first approximation PD of RF is [1/2(a+b)] 100%. If the mass ratio, "a" or "b", (or "a" for the unmounted accelerometer) is large (i.e. >> 1), then the PD of RF becomes small and both test methods give essentially the same results. Of course, if "a" or "a+b" for the respective examples are sufficiently small, the arbitrary limit of 10% for PD of RF will be exceeded and the resonant frequency difference will be "intolerable".

An illustrative example, sometimes used in texts, considers three equal masses (a = b = 1) interconnected by two like springs (r = 1). Treating this problem by the two methods, we find that direct substitution in Equation (9) results in $\omega^2(a,r)/\omega_o^2 = (3-\sqrt{5})/2$ so that $\omega(a,r)/\omega_o = (\sqrt{5}-1)/2$ (or 0.618). Substitution in Equation (21) gives $\omega^2(a,b,r)/\omega_o^2 = 1$ so that $\omega(a,b,r)/\omega_o = 1$. PD of RF = [(1-.618)/.618] 100% or 61.8%, which is quite "intolerable". However, these relative parameters are not likely to be encountered in practical situations, and the example is merely intended to show how large a spread of resonant frequency determinations one might find (without an unreasonable search for wild parameter values). This particular example corresponds to the first row in Table 2 and uses the first row in Table 1.

Scaling is a procedure by which one solution set, $\omega(a,b,r)/\omega_0$, can be cascaded into many. Although ω/ω_0 remains fixed during the process, ω itself can change.

Let α and β be two non-zero but not necessarily unequal constants. Now let

$$\begin{aligned} m_1' &= \alpha\, m_1 \\ m_2' &= \alpha\, m_2 \\ m_3' &= \alpha\, m_3 \\ k_a' &= \beta\, k_a \\ k_s' &= \beta\, k_s \end{aligned} \tag{24}$$

Note that the system constants (m_1', m_2', m_3', k_a', k_s'), leave the parameter set, (a,b,r), unchanged. Accordingly, ω/ω_0 is unchanged even though the specific system constants were changed, allowing that both α and β were not unity. If β and α are equal, neither ω/ω_0 nor ω is changed. However, if β is different from α, then ω_0, and therefore ω, are each changed by the factor $\sqrt{\beta/\alpha}$.

5. Summary and Conclusion

Laboratory experience has demonstrated that the lowest resonant frequency of a mounted accelerometer, as found by applying a sinusoidal acceleration to the structure on which it is mounted, often differs from the resonance found by applying a sinusoidal voltage to its piezoelectric element. Significant differences do exist, and the test method used appears to be responsible. Magnitudes of differences are tabulated in terms of the system parameters. Lumped constant models, while not exact representations of the physical system, do permit an approximate solution of the problem.

Two specific physical models (mechanical and electrical drive) have general solutions given in Equations (9) and (21), respectively. Using various parameter sets (m_2/m_1 and k_s/k_a), Equation (9) gives the relative resonant frequency for the mechanical drive. Equation (21) does the same for the electrical drive. Results for the former are given in Table 1; those for the latter in Table 2. Table 2 also shows (in the last column) the important quantity, PD of RF. This column, in essence, summarizes the tolerability of substituting the electrical for the mechanical drive method.

The solutions to Equations (9) and (21) give relative resonant frequency rather than resonant frequency itself. The conversion to ω involves only direct multiplication by $\sqrt{k_a/m_1}$. It is most fortunate that the percent deviation is the same for the resonant frequency as for the

Tech Note [illegible]

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7-69

Library of Congress 69 [2]

Erofeev, I͡Urii Nikolaevich.

Работа заторможенных релаксационных генераторов при малой скважности выходных импульсов. Под ред. Ю. А. Мантейфеля. Москва, "Сов. радио," 1968.

208 p. with diagrs. 20 cm. 0.50

(USSR 68–12942)

At head of title: Ю. Н. Ерофеев.
Bibliography: p. 205–[206]

1. Oscillators, Transistor. 2. Pulse circuits. I. Title.
Title romanized: Rabota zatormozhennykh relaksat͡sionnykh generatorov.

relative resonant frequency and the "PD of RF" is used as the column loading in Tables 2 and 3.

Table 3 selects values of PD of RF from Table 2 and regroups them. A 2% (or less) PD of RF is listed as "tolerable" and a 10% (or more) PD of RF is listed as "intolerable" and must not be ignored.

Analysis indicates that for the mechanical drive (except for $k_s >> k_a$, for which $\omega = \omega_0$) $\omega < \omega_0$. The situation is different for the electrical drive. It develops that for

$$k_s/k_a < m_3/m_1 \qquad \omega < \omega_0 \ ;$$

$$k_s/k_a = m_3/m_1 \qquad \omega = \omega_0 \ ;$$

$$k_s/k_a > m_3/m_1 \qquad \omega = \omega_0 \ .$$

All three relationships apply regardless of the value of m_2/m_1. It is interesting to note that when $k_s/k_a = m_3/m_1$, $\omega = \omega_0$ no matter what the value of m_2/m_1.

In using the tables, the uncertainty of both ω/ω_0 and PD of RF as computed depends on the adequacy of the lumped parameter representation and on the accuracy with which the values of the parameters can be assigned. However, the analysis and tabulations in this paper will permit for improved confidence in the resonant frequency found by the use of an electrical drive.

Grateful acknowledgment is tendered to Seymour Edelman for his constructive and clarifying suggestions during the preparation of this paper.

Table 1 - Lowest Relative Resonant Frequency,* ω/ω_o, for Mechanical Drive

(see Fig. 3 for configuration)

$a = m_2/m_1$	$r = k_s/k_a$	ω/ω_o
1	1	.618
1	2	.765
1	4	.874
1	16	.968
1	∞	1.000
2	1	.541
2	2	.707
2	4	.848
2	8	.929
2	∞	1.000
4	1	.437
4	2	.600
4	4	.781
4	16	.962
4	∞	1.000
16	1	.243
16	4	.481
16	16	.877
16	64	.925
16	∞	1.000

*The lowest resonant frequency, ω, is equal to $\omega_o \cdot \frac{\omega}{\omega_o}$. Since $\omega_o = \sqrt{k_a/m_1}$, its value is not specified.

Table 2.1 - Lowest Relative Resonant Frequency, ω/ω_o, and the Percent Deviation of the Resonant Frequency, PD of RF, for Electrical Drive

(see Fig. 4 for configuration)

$b = m_3/m_1$	$a = m_2/m_1$	$r = k_s/k_a$	ω/ω_o	PD of RF
1	1	1	1.000	61.8%
2			.848	37.2
4			.745	20.5
8			.683	10.5
16			.648	4.8
∞			.618	0.0
1	1	2	1.126	47.2
2			1.000	30.7
4			.902	17.9
8			.837	9.4
16			.840	5.1
∞			.765	0.0
1	1	4	1.179	34.9
2			1.083	23.9
4			1.000	14.4
8			.944	8.0
16			.908	3.9
∞			.874	0.0
1	1	16	1.214	25.4
4			1.078	11.4
16			1.000	3.3
64			.976	0.8
∞			.968	0.0
1	1	∞	1.224	22.4
2			1.154	15.4
4			1.095	9.5
8			1.054	5.4
∞			1.000	0.0
0	1	any	1.414	

Table 2.2 - Lowest Relative Resonant Frequency, ω/ω_o, and the Percent Deviation of the Resonant Frequency, PD of RF, for Electrical Drive

$b = m_3/m_1$	$a = m_2/m_1$	$r = k_s/k_a$	ω/ω_o	PD of RF
2	2	1	.833	54.0%
4			.707	30.7
8			.631	16.7
∞			.541	0.0
2	2	2	1.000	41.4
4			.890	25.8
8			.811	14.7
∞			.707	0.0
2	2	4	1.073	26.5
4			1.000	17.9
8			.939	10.7
∞			.848	0.0
2	2	8	1.098	18.2
4			1.045	11.5
8			1.000	7.6
16			.969	4.3
∞			.929	0.0
2	2	∞	1.118	11.8
4			1.080	8.0
8			1.049	4.9
16			1.022	2.2
∞			1.000	0.0
0	2	any	1.224	

Table 2.3 - Lowest Relative Resonant Frequency, ω/ω_o, and the Percent Deviation of the Resonant Frequency, PD of RF, for Electrical Drive

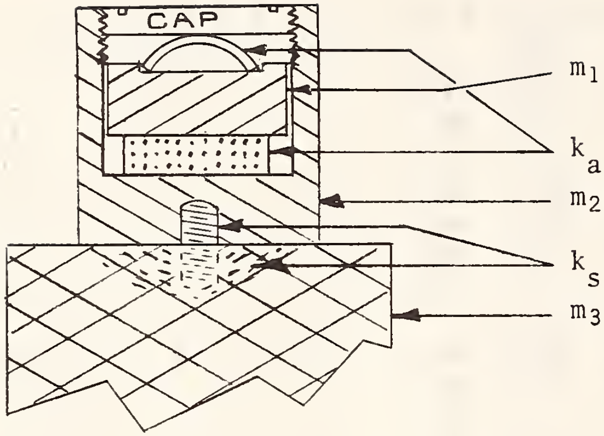
$b = m_3/m_1$	$a = m_2/m_1$	$r = k_s/k_a$	ω/ω_o	PD of RF
4	4	1	.651	49.0%
8			.555	27.1
∞			.437	0.0
4	4	2	.931	55.2
8			.753	25.5
16			.682	13.7
∞			.600	0.0
4	4	4	1.000	28.1
8			.927	18.2
16			.866	10.9
∞			.781	0.0
4	4	16	1.033	7.4
8			1.012	5.2
16			1.000	4.0
80			.970	0.8
∞			.962	0.0
4	4	∞	1.061	6.1
8			1.041	4.1
16			1.025	2.5
96			1.005	0.5
∞			1.000	0.0
0	4	any	1.118	

Table 2.4 - Lowest Relative Resonant Frequency, ω/ω_o, and the Percent Deviation of the Resonant Frequency, PD of RF, for Electrical Drive

$b = m_3/m_1$	$a = m_2/m_1$	$r = k_s/k_a$	ω/ω_o	PD of RF
16	16	1	.347	42.9%
32			.300	23.5
64			.272	12.0
∞			.243	0.0
16	16	4	.688	43.0
64			.521	8.3
∞			.481	0.0
16	16	16	1.000	14.0
64			.943	7.5
∞			.877	0.0
16	16	64	1.082	17.0
64			1.000	8.1
∞			.925	0.0
16	16	∞	1.015	1.5
64			1.006	0.6
∞			1.000	0.0
0	16	any	1.031	

Table 3 - Parameter Sets, (a,b,r), Corresponding to 2% and 10% Levels of Percent Deviation of Resonant Frequency, PD of RF

PD of RF = 2%			PD of RF = 10%		
$r = k_s/k_a$	$a = m_2/m_1$	$b = m_3/m_1$	$r = k_s/k_a$	$a = m_2/m_1$	$b = m_3/m_1$
16	16	375	16	16	34
	12	130		12	9
				10	4
	8	64		8	3
	4	36		4	3.4
	2	30		2	3.7
	1	26		1	4.6
4	16	420	4	16	80
	8	225		8	43
	4	125		4	24
	2	75		2	14
	1	50		1	9
1	16	425	1	16	81



m_1 - seismic mass

k_a - piezoelectric crystal compliance

m_2 - accelerometer base mass

k_s - attachment spring constant - combination of mounting stud and compliance of attachment region

m_3 - attachment region mass

Figure 1: A Typical Configuration of a Piezoelectric Accelerometer Mounted on an Aircraft Panel

m_1 - seismic mass

m_2 - accelerometer base

m_3 - attachment region of structure

k_a - attachment spring constant, seismic mass to base

k_s - attachment spring constant, accelerometer base to structure

Figure 2: Lumped Parameter Model of a Mounted Accelerometer

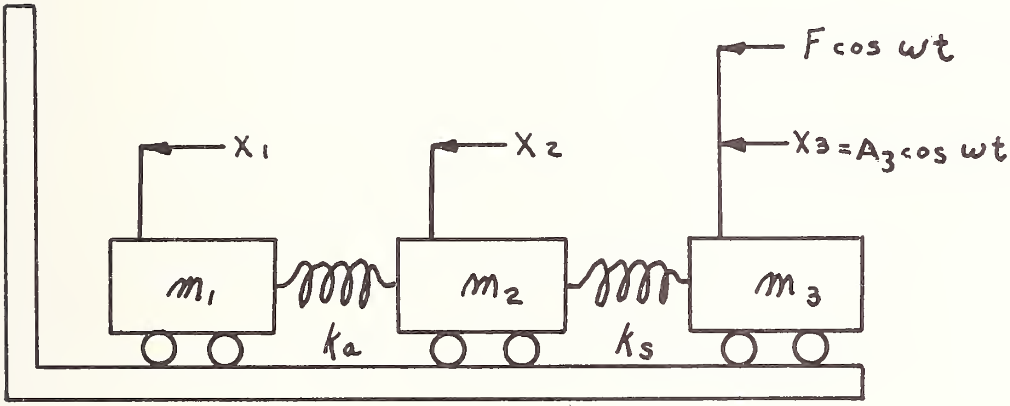


Figure 3: Mechanical Drive: Applied Force and Displacements

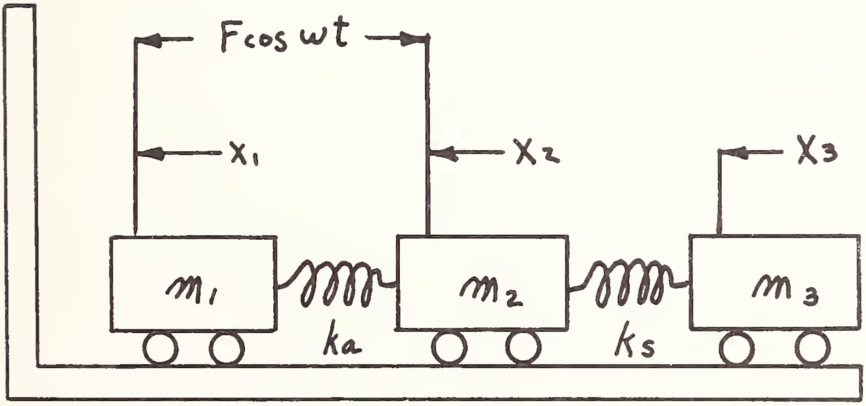


Figure 4: Electrical Drive: Applied Force and Displacements

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