PB 151403

NBS

Technical Note No. 44

Boulder Laboratories

TABLES FOR THE STATISTICAL PREDICTION OF RADIO RAY BENDING AND ELEVATION ANGLE ERROR USING SURFACE VALUES OF THE REFRACTIVE INDEX

BY B.R. BEAN, B.A. CAHOON AND G.D. THAYER

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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March 16, 1960

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ABSTRACT

Radio ray bending, τ, and elevation angle error, ϵ, have been calculated for a wide range of meteorological conditions at 13 climatically diverse U. S. radiosonde stations. The parameters in the observed linear regression equations of τ and ϵ upon the surface value of the refractive index are given for heights of 0.1 to 70 kilometers and initial elevation angles of the ray from 0 to 900 milliradians.

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Consider Figure 1 where the geometry of radio ray refraction is shown. The angular bending of the radio ray, τ, is obtained from [1, 2]:

$$\tau = -\int_{n_1}^{n_2} \frac{\cot\theta}{n}\, dn \tag{1}$$

where n is the radio refractive index and θ is the local elevation angle. It is customary to evaluate (1) by assuming that n is spherically stratified. Snell's law is used to determine θ from

$$n_o r_o \cos\theta_o = nr\cos\theta, \tag{2}$$

or

$$\theta = \cos^{-1}\left(\frac{n_o r_o \cos\theta_o}{nr}\right) \tag{3}$$

where r is the radial distance from the center of the earth to the point under consideration. The zero subscript denotes the initial conditions of n, r, and θ; usually taken as those at the earth's surface. It is seen that the integral for τ is a function of the initial conditions and the distribution of n over the radio ray path. Even though τ is determined by the n structure over the entire radio path, the integral (1) becomes relatively insensitive to n-profile characteristics when θ_o is suitably large.

Schulkin [2] has shown that τ may be written

$$\tau = N_s \cdot 10^{-6} \cot \theta_o - \int_{(\cot \theta)_{N=o}}^{\cot \theta_o} N d(\cot \theta) \cdot 10^{-6} \tag{4}$$

where $N = (n - 1) 10^6$,

for the case of a radio ray passing completely through the earth's atmosphere. The notation N_s is used to denote the value of N at the earth's surface. For $\theta_o \geq 10^\circ$ the integral expression on the right hand side of (4) amounts to only a few per cent of the total, and thus the bending is expressed as a linear function of the initial conditions. This is a very useful observation since it means that τ may be determined from ordinary surface weather observations of temperature, pressure and humidity which are needed to determine n. This possibility has been explored recently by the present authors [3] with the conclusion that the total bending of a radio ray passing completely through the earth's atmosphere may be usefully expressed as a linear function of N_s for θ_o as small as 1 mr. The present report extends this conclusion by deriving the regression parameters for additional heights. Regression parameters are also derived for the prediction of elevation angle errors from N_s.

In this work a large sample of data was assembled, and regression lines of τ and ϵ upon N_s were determined. The "goodness-of-fit" of the data to the regression line was determined in each case by comparing the standard deviation of the raw data, say $s(\tau)$, (which is taken about the mean τ of the sample) with the standard deviation of the same data about the regression line. This last quantity is called the standard error of estimate, S.E. The percentage reduction in the uncertainty of estimating

τ by using the regression line rather than the mean of the raw data is thus given by

$$P = 100\left(\frac{S(\tau) - S\,E}{S(\tau)}\right).$$

As a general observation we note that a 50% reduction in error is accomplished by this method only when the bending is taken over at least 5 km and θ_o is equal to at least 20.0 mr. For the case of total atmospheric bending of the ray (h = 70 km) a 50% reduction is accomplished at θ_o of 5 mr ($\sim 0.3^o$) for both τ and ϵ.

The details of the n-profile sample used to derive the regression parameters are given in Appendix A while the regression parameters for bending are given in Appendix I and the regression parameters for elevation angles are given in Appendix II.

APPENDIX A

Characteristics of the Profile Sample

The following table is a listing of the 77 refractive index profiles used in the refraction study, giving pertinent profile characteristics, by stations. The three more important quantities are listed: the surface value of N, N_s; the initial gradient of N (with respect to height) to the first significant radiosonde level, $(dN/dh)_o$ (per km), and the drop in N over the first kilometer above the surface, ΔN.

Each station is listed, along with the Weather Bureau code number, and the characteristics of six profiles are given. The six profiles fall into six mutually exclusive profile types, as follows:

Type 1. Modified Ground Layer (MGL)

Superrefractive profiles having a gradient in excess of 100 N-units per km, but less than 156.9 N-units per km, in the initial layer, but no elevated layer with a gradient larger than 156.9 N-units per km.

Type 2. Maximum Surface (Max).

Profile with the highest value of N_s found for that station over period of record, but <u>not</u> having an initial gradient in excess of 100 N-units per km, nor any elevated layer with a gradient in excess of 156.9 N-units per km.

Type 3. Minimum Linear (Lin).

Profiles having a minimum value of N_s for that station, but <u>not</u> having any layer with a gradient in excess of 60 N-units per km.

Type 4. Elevated Duct (ED).

Profiles having at least one elevated layer with a gradient of over 156.9 N-units per km, but with an initial layer having a gradient less than 100 N-units per km.

Type 5. Surface Duct (Duct).

Profiles having an initial layer with a gradient in excess of 156.9 N-units per km.

Type 6. Combined Gradient (Comb).

Profiles having an initial layer with a gradient between 100 and 156.9 N-units per km, and at least one elevated layer with a gradient in excess of 156.9 N-units per km. A combination of superrefraction (type 1) and elevated duct (type 4).

Note that no profile fulfilling the requirements of the type 6 definition could be found for Ely, Nev. Also, the definitions for types 2, 3 and 4 had to be somewhat modified in order to include Truk, Caroline Islands (the modification being to allow initial gradients of above 100 for type 2 and 4 and gradients up to 80 for type 3).

Station	Type	N_s	$(dN/dh)_o$	ΔN
Bismarck, N. D.	MGL	322.5	-132	-58
24011	Max	350.5	-59	-44
	Lin	295.0	-36	-35
	Duct	341.5	-715	-82
	ED	314.5	-39	-40
	Comb	318.0	-141	-48
Columbia, Mo.	MGL	367.0	-145	-56
13983	Max	383.0	-38	-73
	Lin	309.5	-37	-36
	Duct	365.0	-472	-74
	ED	307.5	-41	-34
	Comb	346.0	-141	-56
Denver, Colorado	MGL	266.0	-135	-44
23062	Max	289.5	-39	-39
	Lin	237.0	-22	-23
	Duct	254.0	-197	-46
	ED	278.0	-58	-65
	Comb	286.0	-134	-56
Ely, Nevada	MGL	280.0	-127	-38
23154	Max	280.0	-46	-42
	Lin	249.5	-28	-30
	Duct	267.0	-246	-49
	ED	255.0	-47	-30
	Comb	None found		
Joliet, Illinois	MGL	390.5	-129	-78
14834	Max	395.0	-58	-97
	Lin	309.5	-36	-36
	Duct	337.0	-451	-54
	ED	320.0	-32	-48
	Comb	340.0	-142	-60

Station	Type	N_s	$(dN/dh)_o$	ΔN
Miami, Florida 12839	MGL	376.5	-150	-59
	Max	391.5	-37	-60
	Lin	333.6	-24	-39
	Duct	379.5	-194	-66
	ED	360.0	-65	-54
	Comb	358.5	-134	-55
Portland, Maine 14764	MGL	357.5	-146	-80
	Max	375.0	-64	-75
	Lin	315.0	-37	-39
	Duct	347.5	-322	-68
	ED	345.0	-68	-56
	Comb	342.0	-133	-46
San Antonio, Texas 12921	MGL	366.0	-150	-65
	Max	377.5	-53	-54
	Lin	301.5	-42	-36
	Duct	375.5	-181	-61
	ED	335.5	-21	-37
	Comb	359.0	-136	-44
Santa Maria, Calif. 23236	MGL	340.0	-145	-52
	Max	343.0	-37	-51
	Lin	323.5	-45	-42
	Duct	337.0	-197	-48
	ED	339.5	-48	-74
	Comb	330.0	-129	-79
Tatoosh, Wash. 24240	MGL	336.5	-128	-68
	Max	343.0	-45	-46
	Lin	315.0	-38	-38
	Duct	337.0	-253	-70
	ED	326.0	-42	-51
	Comb	323.0	-132	-57

Station	Type	N_s	$(dN/dh)_o$	ΔN
Washington, D. C.	MGL	344.0	-144	-48
13743	Max	391.0	-82	-68
	Lin	297.5	-29	-30
	Duct	336.0	-378	-65
	ED	350.0	-57	-51
	Comb	370.5	-144	-74
Truk Island	MGL	388.5	-138	-62
99999	Max	400.0	-103	-70
	Lin	383.5	-75	-64
	Duct	402.5	-330	-72
	ED	373.5	-115	-52
	Comb	393.5	-142	-64
Fairbanks, Alaska	MGL	313.1	-140	-47
26411	Max	343.5	-48	-48
	Lin	291.0	-35	-34
	Duct	305.0	-400	-34
	ED	328.0	-43	-62
	Comb	307.0	-147	-34

TABLE I

Values of b and a in the regression equation $\tau = bN_s + a$, where:

τ is the total bending in milliradians to the height h,

N_s the value of the radio refractivity at the earth's surface,

b the change in total bending per unit change in N_s,

a is the zero intercept,

r is the correlation coefficient,

S E is the standard error of prediction using the regression line,

S_τ is the standard deviation of the calculated values of the total bending,

$$P = 100 \left[\frac{S_\tau - SE}{S_\tau} \right].$$

Table I-A, h = 0.1 km.

θ_o	$\overline{N}_s$	$\overline{\tau}$	r	b	a	S.E	S_τ	P
0.0	334.0	7.3077	0.2665	0.0479	-8.7011	6.7277	6.9801	3.6160
1.0	334.0	4.4555	0.2785	0.0257	-4.1217	3.4363	3.5778	3.9549
2.0	334.0	3.0547	0.2881	0.0162	-2.3732	2.0960	2.1888	4.2398
5.0	334.0	1.5038	0.3048	0.0073	-0.9181	0.8792	0.9231	4.7557
10.0	334.6	1.1519	0.1915	0.0053	-0.6085	1.0551	1.0749	1.8420
20.0	334.6	0.5604	0.2070	0.0025	-0.2639	0.4555	0.4656	2.1692
52.4	334.6	0.2129	0.2100	0.0009	-0.0973	0.1688	0.1727	2.2583
100.0	334.6	0.1111	0.2105	0.0005	-0.0507	0.0879	0.0899	2.2247
200.0	334.6	0.0550	0.2105	0.0002	-0.0250	0.0435	0.0445	2.2472
400.0	334.6	0.0264	0.2107	0.0001	-0.0120	0.0208	0.0213	2.3474
900.0	334.6	0.0088	0.2108	0.00004	-0.0040	0.0070	0.0072	2.7778

Table I-B, h = 0.2 km.

θ_o	$\overline{N}_s$	$\overline{\tau}$	r	b	a	S.E	S_τ	P
0.0	334.0	8.9492	0.2849	0.05801	-10.4261	7.5726	7.9001	4.1455
1.0	334.0	5.9957	0.2979	0.0348	-5.6431	4.3330	4.5391	4.5405
2.0	334.0	4.3994	0.3104	0.0239	-3.5707	2.8357	2.9831	4.9412
5.0	334.0	2.3844	0.3415	0.0117	-1.5287	1.2512	1.3312	6.0096
10.0	334.6	1.6872	0.2306	0.0073	-0.7436	1.1990	1.2322	2.6944
20.0	334.6	0.8368	0.2550	0.0035	-0.3184	0.5122	0.5297	3.3038
52.4	334.6	0.3198	0.2604	0.0013	-0.1162	0.1890	0.1958	3.4729
100.0	334.6	0.1671	0.2610	0.0007	-0.0603	0.0983	0.1019	3.5329
200.0	334.6	0.0827	0.2613	0.0003	-0.0299	0.0486	0.0504	3.5714
400.0	334.6	0.0397	0.2613	0.0002	-0.0143	0.0233	0.0241	3.3195
900.0	334.6	0.0133	0.2604	0.00005	-0.0047	0.0078	0.0081	3.7037

Table I-C, h = 0.5 km.

θ_o	$\bar{N}_s$	$\bar{\tau}$	r	b	a	S.E	S_τ	P
0.0	333.1	10.9591	0.3615	0.0769	-14.6443	7.6170	8.1694	6.7618
1.0	333.1	7.9386	0.3997	0.0510	-9.0567	4.4954	4.9042	8.3357
2.0	333.1	6.2579	0.4369	0.0384	-6.5408	3.0395	3.3790	10.0474
5.0	333.1	3.9386	0.5205	0.0228	-3.6605	1.4376	1.6837	14.6166
10.0	333.8	2.7795	0.3933	0.0140	-1.9055	1.2733	1.3850	8.0650
20.0	333.8	1.4731	0.4563	0.0071	-0.8926	0.5365	0.6029	11.0134
52.4	333.8	0.5573	0.4731	0.0027	-0.3308	0.1966	0.2232	11.9176
100.0	333.8	0.3026	0.4753	0.0014	-0.1721	0.1022	0.1161	11.9724
200.0	333.8	0.1499	0.4760	0.0007	-0.0851	0.0505	0.0574	12.0209
400.0	333.8	0.0719	0.4761	0.0003	-0.0408	0.0242	0.0275	12.0000
900.0	333.8	0.0241	0.4764	0.0001	-0.0137	0.0081	0.0092	11.9565

Table I-D, h = 1.0 km

θ_o	$\bar{N}_s$	$\bar{\tau}$	r	b	a	S.E	S_τ	P
0.0	334.0	12.8881	0.3936	0.0840	-15.1802	7.6151	8.2839	8.0735
1.0	334.0	9.9030	0.4620	0.0607	-10.3739	4.5217	5.0985	11.3131
2.0	334.0	8.2213	0.5238	0.04918	-8.2066	3.1040	3.6438	14.8142
5.0	334.0	5.7851	0.6348	0.0337	-5.4816	1.5931	2.0619	22.7363
10.0	334.6	4.2571	0.5718	0.0224	-3.2378	1.2574	1.5326	17.9564
20.0	334.6	2.4573	0.6598	0.0124	-1.6959	0.5531	0.7360	24.8505
52.4	334.6	1.0040	0.6823	0.0049	-0.6495	0.2071	0.2833	26.8973
100.0	334.6	0.5297	0.6851	0.0026	-0.3388	0.1080	0.1482	27.1255
200.0	334.6	0.2629	0.6859	0.0013	-0.1676	0.0534	0.0734	27.2480
400.0	334.6	0.1262	0.6860	0.0006	-0.0803	0.0256	0.0352	27.2727
900.0	334.6	0.0423	0.6864	0.0002	-0.0270	0.0086	0.0118	27.1186

Table I-E, h = 2.0

θ_o	$\bar{N}_s$	$\bar{\tau}$	r	b	a	S.E	S_τ	P
0.0	334.0	15.1568	0.4524	0.0985	-17.7584	7.5391	8.4535	10.8168
1.0	334.0	12.1678	0.5490	0.0752	-12.9451	4.4420	5.3145	16.4173
2.0	334.0	10.4745	0.6316	0.0636	-10.7566	3.0277	3.9053	22.4720
5.0	334.0	7.9624	0.7707	0.0475	-7.8969	1.5234	2.3907	36.2781
10.0	334.6	6.1731	0.7634	0.0345	-5.3712	1.1421	1.7681	35.4052
20.0	334.6	3.9063	0.8515	0.02111	-3.1571	0.5086	0.9699	47.5616
52.4	334.6	1.7023	0.8668	0.0089	-1.2770	0.2003	0.4018	50.1493
100.0	334.6	0.9089	0.8679	0.0047	-0.6705	0.1057	0.2128	50.3289
200.0	334.6	0.4529	0.8681	0.0023	-0.3323	0.0524	0.1057	50.4257
400.0	334.6	0.2175	0.8682	0.0011	-0.1593	0.0252	0.0507	50.2959
900.0	334.6	0.0730	0.8684	0.0004	-0.0535	0.0084	0.0170	50.5882

Table I-F, h = 5.0

θ_o	$\bar{N}_s$	$\bar{\tau}$	r	b	a	S.E	S_τ	P
0.0	334.0	18.0620	0.4962	0.1115	-19.1704	7.5676	8.7166	13.1817
1.0	334.0	15.0710	0.6101	0.0881	-14.3543	4.4401	5.6037	20.7649
2.0	334.0	13.3714	0.7030	0.0764	-12.1589	3.0001	4.2194	28.8975
5.0	334.0	10.8162	0.8504	0.0601	-9.2514	1.4422	2.7415	47.3938
10.0	334.6	8.8939	0.8674	0.0464	-6.6445	1.0420	2.0943	50.2459
20.0	334.6	6.2299	0.9484	0.0308	-4.0706	0.4028	1.2699	68.2810
52.4	334.6	3.0324	0.9674	0.0139	-1.6236	0.1426	0.5627	74.6579
100.0	334.6	1.6670	0.9695	0.0075	-0.8348	0.0739	0.3017	75.5055
200.0	334.6	0.8388	0.9701	0.0037	-0.4098	0.0365	0.1505	75.7475
400.0	334.6	0.4039	0.9702	0.0018	-0.1960	0.0175	0.0723	75.7953
900.0	334.6	0.1357	0.9703	0.0006	-0.0658	0.0059	0.0243	75.7202

Table I-G, h = 10.0 km

θ_o	$\overline{N}_s$	$\overline{\tau}$	r	b	a	S.E	S_τ	P
0.0	334.0	19.8159	0.5099	0.1149	-18.5627	7.5227	8.7448	13.9752
1.0	334.0	16.8244	0.6290	0.0915	-13.7469	4.3895	5.6464	22.2602
2.0	334.0	15.1232	0.7250	0.0799	-11.5514	2.9443	4.2746	31.1210
5.0	334.0	12.5574	0.8734	0.0635	-8.6434	1.3733	2.8201	51.3031
10.0	334.6	10.5936	0.8950	0.0498	-6.0729	0.9713	2.1773	55.3897
20.0	334.6	7.8054	0.9723	0.0338	-3.5012	0.3179	1.3596	76.6181
52.4	334.6	4.1243	0.9907	0.0157	-1.1441	0.0844	0.6217	86.4243
100.0	334.6	2.3435	0.9927	0.0085	-0.5084	0.0406	0.3359	87.9131
200.0	334.6	1.1950	0.9931	0.0043	-0.2310	0.0197	0.1679	88.2668
400.0	334.6	0.5776	0.9932	0.0020	-0.1078	0.0094	0.0807	88.3519
900.0	334.6	0.1942	0.9932	0.0007	-0.0359	0.0032	0.0271	88.1919

Table I-H, h = 20.0 km

θ_o	$\overline{N}_s$	$\overline{\tau}$	r	b	a	S.E	S_τ	P
0.0	334.0	20.9521	0.5155	0.1165	-17.9573	7.5131	8.7682	14.3142
1.0	334.0	17.9604	0.6367	0.0931	-13.1413	4.3763	5.6752	22.8873
2.0	334.0	16.2588	0.7336	0.0814	-10.9463	2.9281	4.3085	32.0390
5.0	334.0	13.6897	0.8815	0.0651	-8.0397	1.3521	2.8638	52.7865
10.0	334.6	11.7166	0.9028	0.0514	-5.4747	0.9573	2.2263	57.0004
20.0	334.6	8.8856	0.9785	0.0353	-2.9228	0.2909	1.4109	79.3820
52.4	334.6	4.9843	0.9968	0.0169	-0.6738	0.0535	0.6637	91.9391
100.0	334.6	2.9329	0.9984	0.0093	-0.1802	0.0203	0.3645	94.4307
200.0	334.6	1.5226	0.9986	0.0047	-0.0467	0.0096	0.1837	94.7741
400.0	334.6	0.7401	0.9986	0.0023	-0.0161	0.0046	0.0885	94.8023
900.0	334.6	0.2492	0.9986	0.0008	-0.0048	0.0016	0.0297	94.6128

Table I-I, h = 70.0 km

θ_o	$\bar{N}_s$	$\bar{\tau}$	r	b	a	S.E	S_τ	P
0.0	334.0	21.1795	0.5174	0.1170	-17.9071	7.5113	8.7773	14.4236
1.0	334.0	18.1878	0.6391	0.0936	-13.0912	4.3738	5.6864	23.0831
2.0	334.0	16.4861	0.7361	0.0820	-10.8960	2.9251	4.3216	32.3144
5.0	334.0	13.9167	0.8837	0.6558	-7.9895	1.3481	2.8800	53.1910
10.0	334.6	11.9426	0.9051	0.0519	-5.4209	0.9539	2.2430	57.4721
20.0	334.6	9.1072	0.9797	0.0358	-2.8696	0.2862	1.4292	79.9748
52.4	334.6	5.1787	0.9979	0.0173	-0.6246	0.0445	0.6800	93.4559
100.0	334.6	3.0817	0.9997	0.0096	-0.1402	0.0095	0.3768	97.4788
200.0	334.6	1.6126	1.0000	0.0048	-0.0212	0.0013	0.1910	99.3194
400.0	334.6	0.7862	1.0000	0.0024	-0.0027	0.0002	0.0922	99.7831
900.0	334.6	0.2650	1.0000	0.0008	-0.0002	0.0001	0.0310	99.6774

TABLE II

Values of m and ℓ in the regression equation $\epsilon = mN_s + \ell$, where:

ϵ is the elevation angle error in milliradians to the height h,

N_s the value of the radio refractivity at the earth's surface,

m the change in elevation angle error per unit change in N_s,

ℓ is the zero intercept,

r is the correlation coefficient,

SE is the standard error of prediction using the regression line,

S_ϵ is the standard deviation of the calculated values of the elevation angle error,

$$P = 100 \left[\frac{S_T - SE}{S_T} \right].$$

Table II-A, h = 0.1 km

θ_o	$\bar{N}_s$	$\bar{\epsilon}$	r	m	ℓ	S.E	S_ϵ	P
0.0	334.0	3.7064	0.2637	0.0243	-4.3954	3.4429	3.5692	3.5386
1.0	334.0	2.2716	0.2760	0.0131	-2.1055	1.7712	1.8428	3.8854
2.0	334.0	1.5598	0.2855	0.0083	-1.2185	1.0837	1.1307	4.1567
5.0	334.0	0.7682	0.3017	0.0037	-0.4741	0.4561	0.4784	4.6614
10.0	334.6	0.6233	0.1609	0.0027	-0.2834	0.6501	0.6587	1.3056
20.0	334.6	0.3005	0.1777	0.0012	-0.1175	0.2706	0.2750	1.6000
52.4	334.6	0.1140	0.1805	0.0005	-0.0426	0.0998	0.1014	1.5779
100.0	334.6	0.0595	0.1824	0.0002	-0.0227	0.0518	0.0527	1.7078
200.0	334.6	0.0294	0.1815	0.0001	-0.0110	0.0256	0.0260	1.5385
400.0	334.6	0.0142	0.1844	0.0001	-0.0055	0.0123	0.0125	1.6000

Table II-B, h = 0.2 km

θ_o	$\bar{N}_s$	$\bar{\epsilon}$	r	m	ℓ	S.E	S_ϵ	P
0.0	334.0	4.8189	0.2739	0.0310	-5.5364	4.2249	4.3929	3.8244
1.0	334.0	3.2877	0.2838	0.0191	-3.0776	2.4984	2.6055	4.1105
2.0	334.0	2.4324	0.2927	0.0132	-1.9685	1.6702	1.7467	4.3797
5.0	334.0	1.3252	0.3146	0.0065	-0.8506	0.7628	0.8036	5.0772
10.0	334.6	1.0273	0.1919	0.0044	-0.4623	0.8906	0.9075	1.8623
20.0	334.6	0.5048	0.2125	0.0021	-0.1941	0.3757	0.3845	2.2887
52.4	334.6	0.1925	0.2168	0.0008	-0.0705	0.1385	0.1419	2.3961
100.0	334.6	0.1006	0.2175	0.0004	-0.0367	0.0721	0.0739	2.4357
200.0	334.6	0.0498	0.2170	0.0002	-0.0179	0.0356	0.0365	2.4658
400.0	334.6	0.0239	0.2170	0.0001	-0.0085	0.0170	0.0174	2.2989

Table II-C, h = 0.5 km

θ_o	$\overline{N}_s$	$\overline{\epsilon}$	r	m	ℓ	S.E	S_ϵ	P
0.0	333.1	6.2802	0.3255	0.0430	-8.0534	4.8023	5.0789	5.4461
1.0	333.1	4.6053	0.3457	0.0289	-5.0198	3.0131	3.2111	6.1661
2.0	333.1	3.6281	0.3661	0.0216	-3.5807	2.1131	2.2708	6.9447
5.0	333.1	2.2499	0.4197	0.0125	-1.9306	1.0427	1.1488	9.2357
10.0	333.8	1.6990	0.2887	0.0083	-1.0741	1.0693	1.1168	4.2532
20.0	333.8	0.8853	0.3304	0.0041	-0.4899	0.4568	0.4840	5.6198
52.4	333.8	0.3449	0.3418	0.0016	-0.1803	0.1679	0.1787	6.0436
100.0	333.8	0.1806	0.3430	0.0008	-0.0934	0.0872	0.0929	6.1356
200.0	333.8	0.0894	0.3439	0.0004	-0.0463	0.0431	0.0459	6.1002
400.0	333.8	0.0429	0.3408	0.0002	-0.0218	0.0208	0.0221	5.8824

Table II-D, h = 1.0 km

θ_o	$\overline{N}_s$	$\overline{\epsilon}$	r	m	ℓ	S.E	S_ϵ	P
0.0	334.0	7.5072	0.3371	0.0472	-8.2579	5.1154	5.4334	5.8527
1.0	334.0	5.7577	0.3772	0.0339	-5.5583	3.2281	3.4856	7.3875
2.0	334.0	4.7279	0.4180	0.0270	-4.2916	2.2775	2.5070	9.1544
5.0	334.0	3.2343	0.5150	0.0179	-2.7591	1.1590	1.3521	14.2815
10.0	334.6	2.4855	0.3983	0.0123	-1.6158	1.1044	1.2041	8.2800
20.0	334.6	1.4036	0.4831	0.0066	-0.8154	0.4703	0.5371	12.4372
52.4	334.6	0.5689	0.5129	0.0026	-0.3070	0.1714	0.1997	14.1713
100.0	334.6	0.2998	0.5167	0.0014	-0.1595	0.0890	0.1039	14.3407
200.0	334.6	0.1488	0.5177	0.0007	-0.0787	0.0440	0.0514	14.3969
400.0	334.6	0.0714	0.5180	0.0003	-0.0379	0.0211	0.0247	14.5749

Table II-E, h = 2.0 km

θ_o	$\overline{N}_s$	$\overline{\epsilon}$	r	m	ℓ	S.E	S_ϵ	P
0.0	334.0	9.0572	0.3795	0.0567	-9.8908	5.3659	5.7999	7.4829
1.0	334.0	7.1992	0.4413	0.0427	-7.0696	3.3704	3.7560	10.2662
2.0	334.0	6.1064	0.5020	0.0355	-5.7509	2.3735	2.7443	13.5116
5.0	334.0	4.5071	0.6351	0.0258	-4.1251	1.2198	1.5791	22.7535
10.0	334.6	3.5774	0.5642	0.0188	-2.7184	1.0773	1.3048	17.4356
20.0	334.6	2.2144	0.7007	0.0113	-1.5650	0.4499	0.6306	28.6552
52.4	334.6	0.9557	0.7531	0.0047	-0.6256	0.1615	0.2455	34.2159
100.0	334.6	0.5095	0.7608	0.0025	-0.3279	0.0835	0.1287	35.1204
200.0	334.6	0.2538	0.7633	0.0012	-0.1627	0.0412	0.0638	35.4232
400.0	334.6	0.1219	0.7643	0.0006	-0.0784	0.0198	0.0306	35.2941

Table II-F, h = 5.0 km

θ_o	$\overline{N}_s$	$\overline{\epsilon}$	r	m	ℓ	S.E	S_ϵ	P
0.0	334.0	11.4413	0.4324	0.0704	-12.0857	5.6994	6.3209	9.8325
1.0	334.0	9.4276	0.5186	0.0554	-9.0682	3.5425	4.1432	14.4985
2.0	334.0	8.2509	0.5983	0.0477	-7.6679	2.4766	3.0909	19.8745
5.0	334.0	6.5231	0.7543	0.0372	-5.9085	1.2570	1.9147	34.3500
10.0	334.6	5.4083	0.7424	0.0290	-4.3100	1.0255	1.5306	33.0001
20.0	334.6	3.7306	0.8789	0.0194	-2.7570	0.4118	0.8631	52.2883
52.4	334.6	1.8030	0.9248	0.0089	-1.1721	0.1431	0.3761	61.9516
100.0	334.6	0.9898	0.9316	0.0048	-0.6154	0.0733	0.2015	63.6228
200.0	334.6	0.4978	0.9336	0.0024	-0.3043	0.0360	0.1004	64.1434
400.0	334.6	0.2396	0.9339	0.0012	-0.1460	0.0173	0.0483	64.1822

Table II-G, h = 10.0 km

θ_o	$\bar{N}_s$	$\bar{\epsilon}$	r	m	ℓ	S.E	S_ϵ	P
0.0	334.0	13.3409	0.4617	0.0797	-13.2953	5.9448	6.7020	11.2981
1.0	334.0	11.1993	0.5602	0.0637	-10.0839	3.6560	4.4137	17.1670
2.0	334.0	9.9541	0.6475	0.0555	-8.5935	2.5360	3.3278	23.7935
5.0	334.0	8.1208	0.8057	0.0443	-6.6912	1.2652	2.1359	40.7650
10.0	334.6	6.8842	0.8125	0.0353	-4.9338	0.9913	1.7006	41.7088
20.0	334.6	5.0188	0.9301	0.0245	-3.1637	0.3779	1.0286	63.2607
52.4	334.6	2.6400	0.9679	0.0118	-1.3002	0.1195	0.4759	74.8897
100.0	334.6	1.4988	0.9742	0.0065	-0.6607	0.0584	0.2592	77.4691
200.0	334.6	0.7641	0.9763	0.0032	-0.3208	0.0281	0.1299	78.3680
400.0	334.6	0.3693	0.9766	0.0016	-0.1533	0.0135	0.0626	78.4345

Table II-H, h = 20.0 km

θ_o	$\bar{N}_s$	$\bar{\epsilon}$	r	m	ℓ	S.E	S_ϵ	P
0.0	334.0	15.1600	0.4805	0.0874	-14.0318	6.1895	7.0577	12.3015
1.0	334.0	12.8858	0.5866	0.0703	-10.5942	3.7658	4.6500	19.0151
2.0	334.0	11.5687	0.6780	0.0616	-8.9972	2.5905	3.5241	26.4919
5.0	334.0	9.6224	0.8351	0.0495	-6.9273	1.2665	2.3024	44.9922
10.0	334.6	8.2658	0.8497	0.0397	-5.0189	0.9637	1.8279	47.2783
20.0	334.6	6.2329	0.9538	0.0278	-3.0650	0.3423	1.1397	69.9658
52.4	334.6	3.4978	0.9861	0.0136	-1.0682	0.0899	0.5414	83.3949
100.0	334.6	2.0613	0.9917	0.0076	-0.4672	0.0384	0.2981	87.1184
200.0	334.6	1.0712	0.9934	0.0038	-0.2067	0.0172	0.1504	88.5638
400.0	334.6	0.5209	0.9939	0.0018	-0.0947	0.0080	0.0724	88.9503

Table II-I, h = 70.0 km

θ_o	$\bar{N}_s$	$\bar{\epsilon}$	r	m	ℓ	S.E	S_ϵ	P
0.0	334.0	17.6781	0.4999	0.0985	-15.2202	6.6214	7.6453	13.3925
1.0	334.0	15.1699	0.6137	0.0794	-11.3649	3.9661	5.0233	21.0459
2.0	334.0	13.7249	0.7083	0.0698	-9.5814	2.6984	3.8227	29.4111
5.0	334.0	11.5706	0.8617	0.0563	-7.2363	1.2867	2.5357	49.2566
10.0	334.6	10.0018	0.8811	0.0451	-5.0975	0.9476	2.0036	52.7051
20.0	334.6	7.6843	0.9701	0.0317	-2.9121	0.3102	1.2772	75.7125
52.4	334.6	4.4705	0.9948	0.0158	-0.8032	0.0630	0.6198	89.8354
100.0	334.6	2.7082	0.9983	0.0089	-0.2595	0.0202	0.3476	94.1887
200.0	334.6	1.4354	0.9992	0.0045	-0.0822	0.0070	0.1776	96.0586
400.0	334.6	0.7035	0.9995	0.0022	-0.0319	0.0028	0.0860	96.7442

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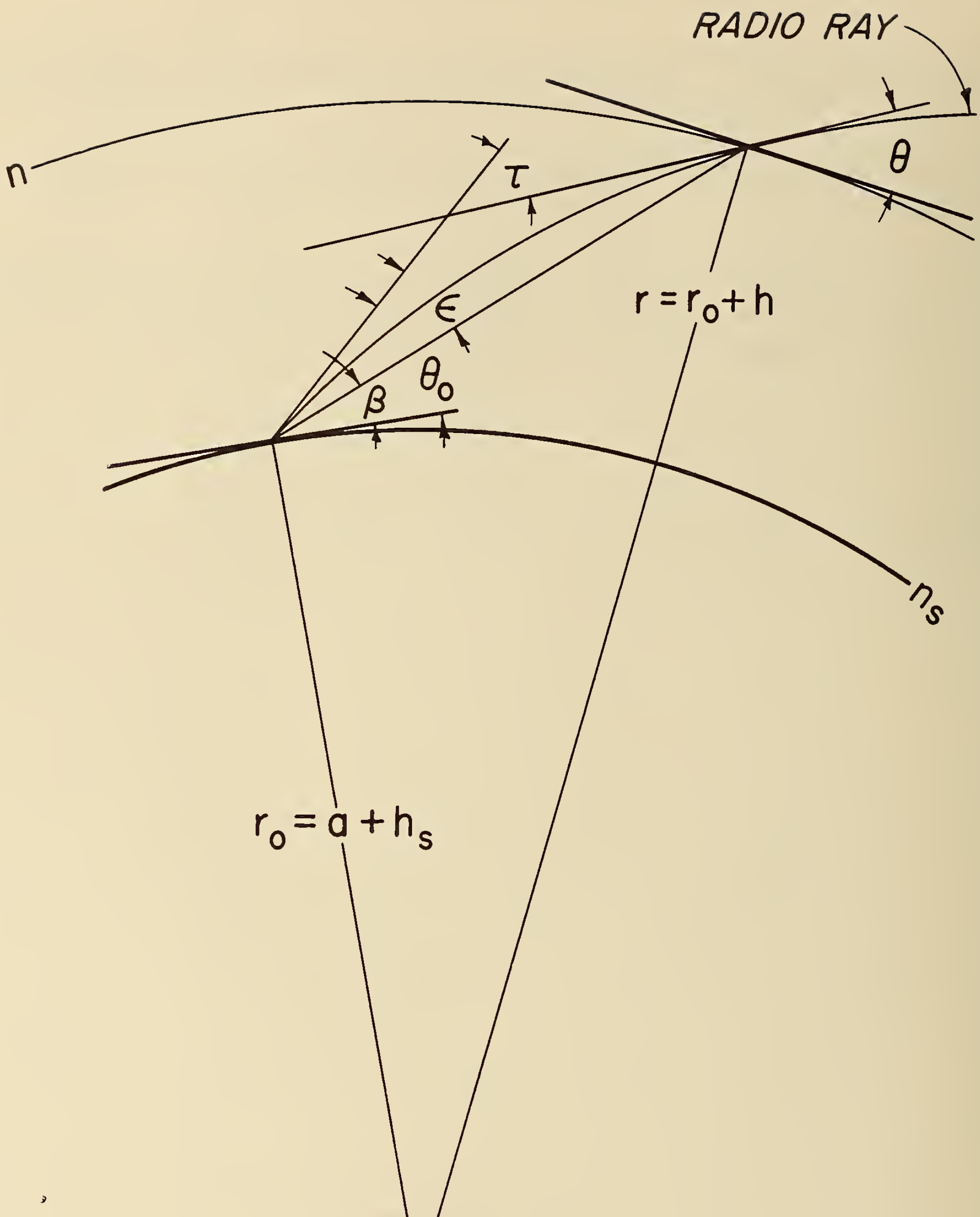

Figure 1

U.S. DEPARTMENT OF COMMERCE
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