NBS TECHNICAL NOTE

414

Quantitative Methods for Management

G. Suzuki

U.S. DEPARTMENT OF COMMERCE
National Bureau of Standards

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NBS TECHNICAL NOTE 414

ISSUED MAY 23, 1967

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Washington, D.C., 20234

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QUANTITATIVE METHODS FOR MANAGEMENT

George Suzuki

An elementary treatment of some of the better known and widely used analytical methods in operations research/systems analysis. The material is presented in a manner which attempts to indicate why quantitative methods are useful in managerial decision-making situations. Some basic references are provided.

Key Words: Operations Research, Quantitative Analysis, Management, Decision-Making, Education.

1. Introduction

Quantitative methods are universally used in practically all management activities. Even the housewife in managing her household functions utilizes quantitative methods, either explicitly if she is systematically inclined, or implicitly in terms (for example) of intuitive planning factors for the weekly grocery purchase. Double entry accounting, which has an analytical foundation, is now a standard practice. Statistical quality control, scientific sampling, portrayal of information by graphs, cost accounting, and other such quantitative methods have wide acceptance. The usefulness of quantitative methods in general is unquestioned.

Since World War II, under the labels of Operations Research, Management Science and Systems Analysis, quantitative methods have been extended so that they can now be, at times, part of the mainstream of executive decision-making. That is, quantitative methods previously had been primarily used to provide data of various sorts which executives then integrated intuitively while performing their functions of organizing, planning and controlling. Recent developments in analytical methodology and the advent of electronic computers now permit quantitative methods to be utilized to provide answers when management asks "what would happen if" for highly complex situations. In certain situations, computers can be programmed with mathematical procedures to choose the best possible answer from all possible "what if's," thus mechanizing the rendering of decision. Most decision situations still require capable human judgments, and always will. However, the trend is towards facilitating the exercise of judgment by providing information of greater pertinence and smaller volume.

Managerial activities are inconceivable without numerical data of some sort, and numerical data imply or require quantitative methods for collection, processing, or analysis. Often the necessary methods for collection and processing of data involve only simple arithmetic, and "analysis" may only mean comparing one number with another. Trivial as they may be, these <u>are</u> quantitative methods. Even disregarding such elementary procedures, the list of quantitative methods useful to managers is a long one. To limit the discussion, the emphasis in this chapter will be on quantitative methods which directly involve those choices within the control of the decision-maker. Thus, accounting, which deals with systematic documentation and analysis of financial transactions, but not with the question of which transaction to engage in; or, forecasting methods, which extrapolate historical data, but do not directly include planned activities, will not be covered in this chapter. To a degree, the more traditional subjects are covered in other chapters of this book.

This chapter will describe how, and also why, quantitative methods are useful. Techniques will be mentioned, but explaining how to use them is beyond the scope of this chapter. References will be cited for those readers who wish to learn more about them.

In order not to extend the scope of the chapter unnecessarily, quantitative methods will be considered within the single broad context of managerial decision-making. Since planning, organizing, controlling and other managerial functions all involve decision-making, though with different time lags for implementation and with different immediate purposes, the general context of decision-making is appropriate for all managerial functions. Perhaps the managerial function which basically differs from the others is the decision as to whether a particular situation needs to be looked into, i.e., the recognition of a situation requiring a decision. Fundamentally, this also is a decision situation. However, quantitative methods of the sort to be considered here are not applicable to such situations, and so it will be assumed throughout that a decision situation in fact exists and has been identified. Beyond the identification of a decision situation, it is presumed further that there are objectives which, though not necessarily explicit, motivate resolving the situation with a "best" decision. The significance of these objectives to the decision-making process will be illustrated throughout this chapter.

The elements or components of the decision-making process will be explained briefly. This will provide the terminology to be used later in specifying in <u>which</u> among the many aspects of decision-making a particular quantitative method has proved useful.

2. The Decision-Making Process

When decision-making is viewed in the context of a few alternatives presented by a subordinate to his superior, perhaps with a recommendation for a choice, the process may appear to lack structure. All decision situations, however, involve certain factors which may be determined explicitly if the decision process is deliberate, or considered instinctively if the decision situation arises suddenly. These are: (1) the elements which are subject to choice, (2) the criteria by which to evaluate the alternatives, (3) the information relating the choice elements with the criteria, and (4) the means for making a choice.

Each element which is subject to choice is a variable. For example, a decision situation involving purchase of any of a number of items, from several sources of supply and in quantities that are not predetermined, has variables representing items, sources, and quantities. One alternative procurement plan has one set of "values" associated with each of the variables. The "value" to be assigned a variable can be numerical, such as 10,000 units of product A, or some qualitative identification such as company X. A schedule in a machine shop consists of what work is to be done, on which machines, by which workers, and at what times. The element of choice, or the variables, include the assignments of jobs to a machine, to a worker, and to time periods. The alternative schedules are the various particular assignments that might be made.

Decision situations invariably involve restrictions that limit the flexibility of (i.e., the range of choices available to) the choices available to the decision-maker. These restrictions or constraints may be physical, as with fixed amount of space or limited capacity of a machine; time-related as with a deadline; financial, as with a fixed amount of funds; or institutional, as with an organizational policy or legislative statute. An alternative is called feasible if the "values" assigned to the variables satisfy all restrictions. Clearly, the identification of all inviolable restrictions is an important part of the decision-making process.

The distinction between a constraint and a variable is often ambiguous, and factors which are meant to be variables are sometimes expressed as constraints or assumed to be constraints. For example, a real estate agent may be asked to locate a house for a prospective customer in another city. He is given information on the size, style, and other features desired and told that the price of the house should not exceed $25,000. Suppose the agent locates an outstanding buy, satisfying all of the requirements except that the price is a firm $25,050. The agent will probably assume that the restriction to $25,000 is really not rigid and will inform his client of this house. However, if the agent had interpreted the requirement to be a rigid constraint, the outstanding buy would not be a feasible alternative. Variables can sometimes operate as constraints in terms of the limited alternatives that might be presented to a decision-maker. For example, a project may be presented as a choice between a costly "crash" program and a less expensive "normal" program. If the two programs are feasible, there are probably other alternatives which are between the two and which may provide a more desirable solution. However, restriction to the two alternatives essentially implies that the choice of values between the two extremes are not permitted.

A decision involves the selection of a particular alternative, that is, the assignment of particular values to the variables. The evaluation which leads to the decision is not based on the alternatives, per se, but rather on the outcomes or consequences which would result from implementing the decision. Thus, the means for developing information on the outcomes represent an integral part of the decision-making process. Since the purpose of identifying the outcomes is to provide the basis for determining which alternative's outcome is most preferred, the outcomes should be characterized in terms which relate to the criteria pertinent to the decision-maker and which will facilitate his expression of preference. For example, if there are several alternative research projects, the outcomes might be specified in terms of the possible advancement of knowledge, the economic value of subsequent edploitation of the technical knowledge, and others, depending upon the criteria by which the outcomes are to be rated. It is important to note that the selection of the appropriate criteria requires not only pertinence to the objectives, but also the capability of being related to the choice variables, and the criteria together should encompass both benefits and disbenefits.

The means used by the decision-maker in arriving at his choice are obviously an important part of the decision-making process as they directly determine the kinds of information which he requires. The means for making a choice may appear to be straightforward, but in fact, there may be many available, depending upon the circumstances of the decision situation. Some quantitative procedures will be illustrated in later sections of this chapter.

3. Operations Research, Management Science, and Systems Analysis

With this description of the decision-making process, it may now be appropriate to comment briefly on the sometimes confusing labeling of quantitative methods under the headings of Operations Research (OR), Management Science (MS) and Systems Analysis (SA). These categories have so much in common that the distinctions are significant only for the purists. They all refer to the conscious formal application of logic and scientific objectivity to the problems of decision-making. The SA purists maintain that OR deals with decision situations which permit mathematical representation of the relationships between the controllable (choice) variables and a single unambiguous criterion of preference. That is, that OR deals with relatively simple systems. Systems Analysis is claimed by the purists to apply to complex situations involving vague objectives and many criteria, usually non-quantitative, so that the mathematical techniques so useful to simpler situations do not apply. The advocates of the OR label argue that because the pioneering applications performed under the OR label did deal with relatively unambiguous criteria and with situations that admitted optimization, this does not imply that OR is restricted to such situations. Those performing studies under the label of Management Science maintain that MS covers complex decision situations as well as the more simple situations ascribed by the SA purists to OR. Some who advocate the MS designation have associated OR with military and hardware systems, while some OR purists suggest that hardware systems are in the province of systems engineering.

The limited length of this chapter permits only examples dealing with relatively straight-forward decision situations, and hence the illustrations may be considered to be of the OR-type in the sense suggested by the SA purists.

The fundamentals pursued in this chapter, if not the examples, should satisfy the SA purists since facility in and understanding of the rationale of quantitative procedures provide the basis for describing and analyzing complex situations in a manner not generally possible without this knowledge.

4. Problems of Decision-Making

The task of decision-making, viewed in the context of the entire decision-making process, presents many problems. Each situation will have its own peculiar problems, but the following general categories are common:

1. Insufficient range of feasible alternatives.

2. Too many alternatives.

3. The outcomes of a possible action known only imperfectly, especially in terms pertinent to the objectives.

4. The possible range of outcomes known, but specific outcome uncertain.

5. Presence of many pertinent but conflicting criteria, rendering evaluation (choice) difficult; in addition, objectives may be vague.

Each of these problem categories will be discussed briefly; those to whose resolution quantitative methods can contribute significantly will be treated more fully in the succeeding sections.

The development of feasible alternatives depends upon many factors. Imagination, ingenuity, and technical knowledge are generally considered as being most important. However, the administrative process through which alternatives reach the executive decision-maker is also a significant factor. The decision-maker may be presented with two or three alternatives for a decision. His staff, however, may have considered many possibilities so that the few presented to the chief represent the staff's selection. The staff may have received information from the operating divisions representing those division's decisions as to what merits being transmitted up the chain of command. Thus, viewed in the context of the entire system, there exist many more possible alternatives than were even simultaneously compared as system-wide alternatives; the administrative process itself limiting the development of alternatives. When system-wide alternatives are compiled in this manner, the lower level objectives explicitly or implicitly involved in screening out many possibilities, and the coordinated treatment of the interfaces among the separate organizational units are important considerations. The decision-maker's objectives and his constraints also influence the development of alternatives in significant ways. For example, if a machine shop schedule is to be determined for one week, the alternatives are more restricted than if a longer period is involved so that the feasible alternatives might include procurement of additional personnel or even of new machinery. As mentioned before, the facility with which alternatives can be evaluated limits the number of alternatives that can be considered and thus inhibits development of the full spectrum of possible choices.

Quantitative methods are not a substitute for imagination, ingenuity, and technical know-how and hence do not contribute directly to the development or identification of novel approaches to solving a problem. That is, meagerness in the menu of identified alternatives due to lack of engineering developments, or to lack of technical and specialized knowledge, will find no remedy in the quantitative methods considered here. However, quantitative methods and their associated ways of formulating the decision situation can often expand the variety and the scope of alternatives that are brought into consideration. This is one of the primary functions of quantitative methods and will be discussed in a subsequent section.

The problem of too many alternatives is not often recognized. Where more alternatives exist than can be adequately considered, traditional administrative practices intervene to obscure the number and variety of the actual possibilities. The practice of hierarchical filtering mentioned before is an example; the top executive may believe there are only a few alternatives when in fact, the totality of system-wide possibilities is overwhelming. Another aspect is that each decision-maker has his own bag of techniques for decision-making. His conception of alternatives may be limited by his techniques. For example, scheduling production in a large factory may offer a tremendous number of possibilities. However, the scheduler may use a rather specific set of rules to arrive at a feasible schedule. If this is the only set of rules he knows, there may be only one possible schedule so far as he is concerned. Quantitative methods are admirably suited for problems with many alternatives, especially when employed in conjunction with electronic computers. Resource allocation problems, in general, have tremendous numbers of feasible alternatives. For quantitative methods to be most helpful, however, the decision criteria must be expressible in mathematical form (if only approximately). The use of quantitative methods for situations with many possible solutions will be considered in a subsequent section.

Perhaps the most difficult aspect of rational decision-making arises when the outcome of a possible action is not known (at least not very specifically) in terms which are pertinent to the objectives. Some direct consequences of the actions may be foreseeable, but the relationship of the observable consequences to the objectives may be obscured by largely unknown relationships. For example, if a nuclear attack is assumed, the direct consequences in terms of physical damages can be estimated, but the effects on broad national objectives cannot be estimated nearly as well. Clearly, the problems of identifying pertinent criteria and of relating action possibilities to these criteria are dominant in assessing alternative government programs. For this problem area, quantitative methods may not be directly applicable. However, mathematics provides an efficient language for representing and analyzing complex relationships and so may help in clarifying and identifying parts of the complex relationships between actions and objectives. Furthermore, the systematic and explicit formulation of the problem areas based on deliberate analysis can be useful. The Planning, Programming, and Budgeting System currently being promulgated for use in government agencies by the Bureau of the Budget and by the Program Planning System of the Department of Defense are based on this idea. Specific quantitative methods or techniques here are not yet available and hence are not covered in subsequent sections. However, this is an active area of study under such labels as Benefit-Cost (or Cost-Effectiveness) Analysis and Systems Analysis; and greater systematic knowledge and more powerful techniques are bound to develop.

Other problems in decision-making arise when the range of possible outcomes is known, but the specific outcome is uncertain. To illustrate with a simple example, if a coin is to be flipped, the possible outcomes (head or tail) are known, but not the outcome of a specific toss. In a similar manner, a prospective action might lead to any one of many possible outcomes, putting another dimension of difficulty into the evaluation of alternatives. Replacing the multiplicity of possible outcomes by some sort of "average" is one of the devices most commonly used to eliminate (in effect) such uncertainties. This practice can be very useful at times, but can also be misleading. There are many decision situations where uncertainty must be taken into account, and quantitative methods, especially those dealing with probability, provide a significant contribution. This problem area will be considered in the section under Uncertainty and Decision-Making.

The existence of (possibly conflicting) multiple criteria, in general, creates further problems in decision-making. The evaluation of multiple criteria situations is generally conceded to be the exclusive province of executive judgment. This is unlikely to change. However, as decision situations grow more complex and when multiple criteria situations are further complicated by many alternative possibilities, the powers of executive judgment are likely to be severly taxed. Some modest advances have been made in the use of quantitative methods in this area, and are discussed in the section on Utility Theory.

Frequently, lack of data underlies all of the problems mentioned. Methods for obtaining specific data, such as sampling, surveys, questionnaire administration, etc., are outside the scope of this chapter. However, it should be noted that routine data collection or management information systems ought to be developed on the basis of an explicit description of the decision-making process; the development of such a description may be enhanced by quantitative methods, some of which will be described in later sections.

The use of mathematical or logical models to represent relatively complex decision situations is a primary tool of the contemporary approach to assisting managerial decision-making. Therefore, before getting on to how specific quantitative methods may be helpful in decision situations, the idea of a mathematical model will be examined.

5. Mathematical Models

A mathematical model is an abstract representation of an object, process, or system by the use of symbols, functions, and relations. Some of the simplest mathematical models are used so widely that they are considered common sense. For example, if c is the cost per unit of some item, and a quantity X is to be purchased, then the total cost, T, is $T=cX$. This equation mathematically represents the cost consequence T in terms of the choice of quantity X and the system parameter c (cost per item). A more involved decision situation with constraints and interrelated variables would, of course, require a more elaborate and sophisticated representation, but basically the idea is the same.

Mathematical models can be categorized into three types depending on their purposes: (1) There are mathematical models which represent a theory or a hypothesis explaining some phenomenon or the behavior of a system. Physical laws such as the renowned $E=mc^2$, and Newton's law of gravitational attraction, which states that the force of attraction is proportional to the product of the masses and varies inversely as the square of the distance between the particles, $F=K(m_1m_2)/r^2$, are examples of mathematical models of this type. An example in the social sciences is the "Gravity Model," predicting the flow of traffic between two metropolitan areas in a manner similar to Newton's law of attraction. Lanchester's theory of combat and random processes represented by "classical" mathematical descriptions (e.g., Poisson or exponential distributions) are other examples. This type of mathematical model generally is not itself a representation of a decision-making situation, but may be utilized as part of such a representation, for example, the prediction of demand for some product. (2) A second type of mathematical model describes the relationship between the choice variables and the outcomes. For a particular set of choices and assumptions concerning the behavior of the recognized elements of the system, the mathematical model or equations yields the consequences. The simple equation, $T=cX$, at the beginning of the section is an example, and subjecting a miniature scaled aircraft to a wind tunnel test is a physical analog of this type of model. The scaled aircraft represents one alternative configuration of an airframe and the wind tunnel test yields performance characteristics which would be consequences of this configuration. A series of different configurations can be tested. Similarly, in a descriptive mathematical model, a series of different choices for variables and assumptions can be introduced and the outcomes associated with them determined. The decision-maker can then make a choice from among these outcomes. (3) The third type of mathematical model is an expansion of the second type in the sense that its information also permits quantitative derivation of the "best" choice. This requires the optimization model to have a single quantitative criterion of preference and to provide all feasible alternatives. Several examples of this type of model will be presented in later sections.

Mathematical models are either deterministic or probabilistic. A deterministic model assumes that each choice will lead to a fixed outcome. For the example cited earlier, if quantity X is purchased, the per item cost will be c and the total cost will be $T=cX$. In a probabilistic model, the actual cost c may be uncertain and hence the total cost is uncertain. The uncertainty is represented by probability concepts and measures.

Mathematical models are abstractions of reality and are never complete representations of the real situation. As in judgmental analysis, only the factors recognized as pertinent are identified and portrayed in the model. However, since reality is always complex, the mathematical model may be inadequate in some relationships and misrepresent others. Thus a mathematical model, like any other aid in analysis, must be tested to determine its reliability and accuracy.

6. Decision Situations with Many Alternatives

In some situations, the number of feasible alternatives is so enormous that though the relationship between choice variables and their consequences may be straightforward, even a powerful electronic computer may not be able to compute the consequences of each and every possible alternative. Intuitive decision-making under these circumstances might be able to consider only a handful of the possibilities. The odds are poor that the best alternative would happen to lie among these few.

To illustrate situations of this sort, consider an extension of a previously cited model. Suppose there are four possible sources for procurement of an item and these sources are located at different geographic locations. The purchases are to be delivered to three warehouses, each at a geographic location not necessarily the same as any of the sources. Purchase requirement is for 45,000 units, with 10,000 required at warehouse 1, 15,000 at warehouse 2, and 20,000 at warehouse 3. The four sources have 8,000, 12,000, 11,000, and 30,000 units, respectively, available for sale. The following table shows the quantities required at the warehouses, and the quantities available from the sources. The dollar amounts at the intersection between a source and a warehouse in the table is the cost of purchase of a unit from the source plus the cost of delivery of that one-unit from the source to the warehouse. The C's with their subscripts besides the costs are the symbolic representation of each of these costs.

		QUANTITY REQUIRED					
		10,000		15,000		20,000	
Quantity Available		Warehouse 1		Warehouse 2		Warehouse 3	
8,000	Source 1	C_1	$3.00	C_5	$3.00	C_9	$4.50
12,000	Source 2	C_2	4.80	C_6	3.20	C_{10}	5.00
11,000	Source 3	C_3	6.00	C_7	4.00	C_{11}	5.50
30,000	Source 4	C_4	5.20	C_8	4.10	C_{12}	6.00

The problem is to determine the sources from which 45,000 units are to be purchased so that the purchases and deliveries to the warehouse in the required quantities result in the least possible total cost.

Although this is a small problem, an attempt to obtain the least cost solution by trial and error methods will reveal the many awards which are possible. For a larger problem, for example, with 10 sources and 25 warehouses, the number of different, but feasible purchase-delivery combinations may be so enormous that it would not be practical for even the most powerful computer to obtain a solution by "one-by-one" examination methods.

The mathematical representation, or model, for this problem will be illustrated. Let $X_1, X_2, \ldots X_{12}$ represent the decision variables. Thus X_1 represents the quantity, if any, to be obtained from source 1 and delivered to warehouse 1. Let the subscripts of X correspond to those of the respective C's in the above table. The model showing the relationship between the decision variables and the total cost consequence is

$$\text{Total Cost} = C_1X_1 + C_2X_2 + C_3X_3 + \ldots + C_{12}X_{12}.$$

The decision variables, the X's, have constraints. $X_1 + X_5 + X_9$ must not exceed 8,000 units, the maximum quantity available from source 1. There are similar restrictions for the other sources. $X_1 + X_2 + X_3 + X_4$ must equal 10,000, the quantity required at warehouse 1, and similarly for the other warehouses. Thus, the alternatives are represented by the values of the X's which satisfy the following constraints:

None of the X's can be negative, and

$$\left.\begin{aligned} X_1 + X_2 + X_3 + X_4 &= 10{,}000 \\ X_5 + X_6 + X_7 + X_8 &= 15{,}000 \\ X_9 + X_{10} + X_{11} + X_{12} &= 20{,}000 \end{aligned}\right\} \quad \text{warehouse requirements}$$

$$\left.\begin{aligned} X_1 + X_5 + X_9 &\leq^* 8{,}000 \\ X_2 + X_6 + X_{10} &\leq 12{,}000 \\ X_3 + X_7 + X_{11} &\leq 11{,}000 \\ X_4 + X_8 + X_{12} &\leq 30{,}000 \end{aligned}\right\} \quad \text{quantities available}$$

The "solution" or decision to this problem is shown in the following table:

	warehouse 1	warehouse 2	warehouse 3	Quantity Purchased
source 1	$x_1 = 0$	$x_5 = 0$	$x_9 = 8{,}000$	8,000
source 2	$x_2 = 0$	$x_6 = 11{,}000$	$x_{10} = 1{,}000$	12,000
source 3	$x_3 = 0$	$x_7 = 0$	$x_{11} = 11{,}000$	11,000
source 4	$x_4 = 10{,}000$	$x_8 = 4{,}000$	$x_{12} = 0$	14,000

* $\leq$ is read "less than or equal to."

If, as in this case, the unit cost (purchase and delivery) is constant for each source-warehouse pair, or at least does not decrease with increasing quantity (quantity discount), the situation can be modelled or formulated as a Linear Program. This particular type of linear programming problem is called a "Transportation" problem and is so distinguished because simpler solution methods are available than for the general linear program. The very same mathematical representation could apply to a decision situation involving personnel instead of procurement sources, and job vacancies instead of warehouses; transportation, per se, need not be involved at all. "Transportation" problems occur frequently and constitute one of the most frequent applications of linear programming. Analytical means are available to obtain their solutions, so that it is not necessary to carry out an exhaustive one-by-one examination of all possible alternatives.

Since linear programming represents a powerful analytical technique, permitting the selection of the best from among an enormous number of possible alternatives, another illustration will be cited, this time without the mathematical model. Suppose a firm has three different machines, the labor and material to produce four different products, and an order for these products which must be shipped in a week. The order is for 100 pounds of product A, at least 30 pounds of product B, and any combination of products C and D so long as the total for the two products does not exceed 200 pounds. Product A requires 10 minutes per pound on machine 1 and 15 minutes per pound on machine 2. Product A can also be made by using only machine 3, in which case it takes 30 minutes per pound. Similar information exists for the other products and is tabulated below:

Time Required On Each Machine By Various Processes
In Minutes Per Pound

	Product A process 1	Product A process 2	Product B process 1	Product B process 2	Product B process 3	Product C (only 1 process)	Product D (only 1 process)
Machine 1	10	--	35	--	--	--	--
Machine 2	15	--	--	--	15	10	--
Machine 3	--	30	--	40	20	--	15
Profit per pound	$ 4	$ 3	$ 5	$ 1	$ 5	$ 3	$ 2

The profits for products A and B depend upon the process used. The profits per pound for the products and the processes are shown on the bottom line. The sequence of machines on process 1 for product A and process 3 for product B are immaterial and there are 40 hours of time available. Since time is limited, the firm cannot use the most profitable process for each of the products. In this situation, an alternative is a specification of the quantity of each product to be produced by each process. The number of possible alternatives is very large, and the reader may wish to discover for himself the difficulty of finding, without using advanced mathematics, a decision which represents the most profitable mix of products and processes. The reader will find that he will not know what the best mix is until he has examined all possibilities. Mathematical methods provide a criterion which identifies the best mix when it is encountered without an exhaustive search of remaining possibilities.

Mathematical Programming, which includes linear programming as one special case, is characterized by a situation which has (1) two or more activities (controllable variables); (2) restrictions on the extent, or levels, at which the activities can be carried out; and (3) a quantitative criterion of preference. In the previous example, there were seven activities, namely the product-process pairs. The restrictions on these activities were time availability, the quantities specified by the order, and the implicit assumption that only three machines were available. If additional machines could have been obtained at extra cost per unit, additional variables representing the possible utilization of the extra cost machines could have been included in the model. The criterion of preference was the total profit. Since mathematical programming often deals with decision situations where limited resources (money, material, time, labor, equipment, and facilities) to be allocated among the activities, these problems are generally called allocation problems.

Linear programming is a special type of mathematical programming where (1) the amount of each resource required per unit of activity is constant regardless of the level of the activity, and (2) the magnitude of the criterion unit associated with each unit of activity is constant. That is, all of the relationships representing the decision situation can be expressed as linear equations.

Inventory problems illustrate another type of decision situation where many possible decisions exist, and where the choice of the best decision may be facilitated by the use of a mathematical model. Inventory problems are concerned with acquisition of stock in anticipation of future demand. A principal motive for holding inventory is that it is less expensive and more convenient to obtain a relatively large quantity at one time than to make smaller or separate purchases for each requirement. Other motives for holding inventories are anticipation of price rises or of temporary unavailability of supply. Whatever the motive for holding inventory, there are reasons for limiting its amount, such as the cost of storage, deterioration and obsolescence of items, possible decrease in price, and better uses for the money involved. The problem or decision is concerned with establishing procurement rules to provide the best balance between the advantages and the disadvantages in holding inventory.

Consider a situation where the demand is constant for each time period and known to be "d" units; procurement costs "r" dollars for each order plus "c" dollars per unit procured; and maintaining inventory costs "h" per cent per time period on the value of items held in stock. If many small orders are placed, storage cost may be small, but the cost of ordering will be large, since it costs "r" dollars for each order. Conversely, if few orders are placed for large quantities, the ordering cost will be small, but the storage cost will be large. It is possible to obtain the procurement quantity which will result in the least total cost by many trials and errors; but a mathematical model and a little calculus will provide a solution with much less effort. If x is the order size (quantity per order), then there will be d/x orders for each period of time and the cost of these orders will be rd/x+cd. Demand is assumed to be uniform over the time period, so that the stock can be considered to be held for half the time period and the per period cost of holding inventory is h (cx/2). The total cost each period, in terms of the order quantity x, is represented by the model:

$$\text{Total cost} = r\,\frac{d}{x} + cd + h(cx/2)$$

The value for x which yields the least total cost is

$$x = \sqrt{2\ rd/hc}$$

This is known as the formula for "economic ordering quantity" (EOQ) or "economic lot size," and is widely used. It can also be applied as an "economic production quantity" formula, where r represents the set-up cost per production "run."

It may have occurred to the reader that the decision situations to which the economic ordering quantity formula can be applied are rather limited. Demand is rarely known in advance and is unlikely to be uniform, and delivery may be uncertain. Thus there are possibilities of stock-outs, and if shortages are regarded as costly, as for certain items in military operations, then their disbenefits must be taken into account. To accommodate more realistic and varied situations, a large body of quantitative methods for determining good inventory policy decisions has been developed. Much of the extensive literature on inventory theory has dealt with developing procurement rules for individual inventory items. In recent years there have been increasing efforts devoted to larger systems, which include interrelated demands, redistribution of stock among several supply facilities as an alternative to new procurement, and systems involving echelons or hierarchies of supply facilities. In military logistics, shortages of different items can have widely varying consequences, and so the concept of "military worth" of individual supply items has been receiving greater emphasis. Some quantitative aspects of more complex inventory situations will be discussed later.

7. Uncertainty and Decision-Making

For the decision situations considered so far, it has been assumed that the relationships between the decision variables and the outcome are known in advance, or at least that the outcome can be estimated with acceptable accuracy. Thus for the model: Total Cost = $C_1X_1 + C_2X_2 + C_3X_3$ for procurement from three possible sources, it was assumed that the prices were firm and that if quantities X_1, X_2, and X_3 were purchased from the respective sources, the total cost would invariably be the sum indicated by the equation.

In reality, the outcomes of proposed actions are generally uncertain. This uncertainty may arise for many reasons. For example, factors which are not within the control of the decision-maker can influence the outcome. These can be of two types: (1) competitive*, where there is purposeful action

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* The case of competitive uncertainty, called Game Theory, will be considered later.

such as by an "opponent" (e.g., a business competitor or military enemy) and (2) non-competitive, where there is no deliberate opposition (e.g., the effects of the weather). Factors which are normally predictable by the decision-maker may deviate from expected behavior, e.g., the assignment of a person to a job turning out badly because of unexpected illness, or the purchase of material proving unfortunate because it was one of the few defective lots which (by chance) pass the quality controls without detection. When the outcomes of each of the possible alternatives can vary widely, depending upon the nature and relative importance of the uncontrollable or non-considered factors, the decision-maker must explicitly take these uncertainties into account. Clearly characterization of possible uncertainties is an important aspect of decision-making.

Basic to the treatment of uncertainty is the associated variability in the result of an action. Obviously, if the result cannot vary significantly, then uncertainty does not matter. One should keep in mind that uncertainty is a relative concept, in the sense that if the possible variability is trivial or insignificant then the uncertainty can be ignored. Also basic to the characterization of uncertainty is some knowledge of the likelihood with which each of the possible outcomes will result from an event or an action. Information on these likelihoods may be derived from experiments and from experiences with similar situations. The measures of probability is the usual way of denoting the likelihood of an outcome. Probability is generally stated as a percentage or as a decimal between zero and one, and represents the relative frequency with which each of the possible outcomes might occur. Thus, from experiments, it is known that the probability of a head on a flip of a coin is 50% or 0.50. From rainfall records, the probability of getting 2.0 - 2.5 inches of rain in August in city "A" is 84.7% or 0.847. In many situations, experiments may not be possible and experience may not provide precise estimates. However, some intuitive feelings of the likelihoods of the possible outcomes might be expressed, despite one's feeling of inadequacy about translating such feelings into quantitative terms. Such probabilities are called subjective probabilities, and permit incorporating more information than does the "neutral in the face of total ignorance" assumption that assumes that all outcomes are equally likely.

The list of each possible outcome together with its associated probability of occurrence is called a Probability Distribution (or a Frequency Distribution) and represents a useful way of characterizing the uncertainty of results. Thus, heads-1/2, tails-1/2, is the probability distribution of the outcome of a flip of a fair coin. The following is an example of a probability distribution of the demand for a hypothetical product.

Demand in Thousands	Probability of occurrence
0 - 9	.05
10 - 19	.10
20 - 29	.28
30 - 39	.25
40 - 49	.20
50 or more	.12

The well-known "normal" or "bell-shaped" distribution is a type of probability distribution which appears to "fit" many actual situations.

A table to show a probability distribution (as above) is cumbersome and hence certain numerical measures have been devised to represent the extent of variability. The most important of these are the Variance and the Standard Deviation. The Standard Deviation is the square root of the Variance. The Variance is the average of the squared differences between each of the values and the average (arithmetic mean) of all the values. Thus, if the values do not vary much, they do not differ much from the average and the Variance would be small.

An illustrative problem involving probabilities will now be presented. Suppose the probability distribution of daily demand for a perishable item is estimated from past experience to be as follows:

Demand for (units):	10	11	12	13	14	15	16	17	18	19	20
Probability:	.02	.04	.07	.18	.20	.16	.12	.10	.07	.03	.01
Cumulative Probability:	.02	.06	.13	.31	.51	.67	.79	.89	.96	.99	1.00

The item can be purchased for $5 per unit and can be sold for $10 per unit. Purchases are made at the beginning of the day and all unsold items can be returned at $3 (loss of $2). On any day, how many units ought to be purchased? Suppose that 14 units are purchased. From the probability distribution, it is seen that there is a probability of .02 that 10 will be sold for an income of $50. The 4 unsold units with loss of $8, give a net income of $42. There is a probability of .18 that 13 will be demanded for an income of $65, and one unsold unit with loss of $2, for net income of $63. If 14 units are to be purchased, all of the possible outcomes are as follows:

Demand for 10, net revenue $42, with probability .02

Demand for 11, net revenue $49, with probability .04

Demand for 12, net revenue $56, with probability .07

Demand for 13, net revenue $63, with probability .18

Demand for 14 or more, net revenue $70, with probability .69

If 15 items are purchased, then the outcome possibilities are as follows:

Demand for 10, net revenue $40, with probability .02

Demand for 11, net revenue $47, with probability .04

Demand for 12, net revenue $54, with probability .07

Demand for 13, net revenue $61, with probability .18

Demand for 14, net revenue $68, with probability .20

Demand for 15 or more, net revenue $75, with probability .49

Similar computations can be made for all of the alternative quantities of purchase. What quantity should be purchased?

Comparisons of the uncertain outcomes of the alternative decisions can be facilitated by the use of two further concepts associated with uncertain situations, Expectation and the Expected Value. Expectation (or Mathematical Expectation) is the product of the probability with the value of the corresponding outcome. In the previous example, if 15 items are purchased, the Expectation associated with a demand of 10 items is (.02)($40) = $8. This corresponds to the familiar concept of a fair bet. For example, if the probability of a head or a tail on the flip of a coin in one-half, then the return for a proper call is twice the amount bet. If a wheel of fortune has 100 numbers and the return is a prize worth $5, the Expectation for any number selected in advance is one hundredth of $5, or 5 cents. The Expected Value is the sum of the Expectations for all of the possible outcomes, and corresponds to the concept of the average (arithmetic mean). In the coin example, if one dollar is received if a head turns up and a loss of a dollar if a tail results, the Expected Value or average return is $(1/2)(1) + (1/2)(-1) = 0$. The term Expected Value relates to the long run with many trials and not to the outcome of any one trial. For the probability distribution of the demand in the previous problem, the Expected Value for the number of units demanded is:

$$(.02)(10)+(.04)(11)+(.07)(12)+(.18)(13)+(.20)(14)+(.16)(15)+(.12)(16)+(.03)(19)+(.01)(20)=14.67$$

or, in more familiar terms, the average demand is 14.67 units per day.

The reader is undoubtedly familiar with the computation of the average (arithmetic mean), that is, adding all of the individual values and dividing by the number of values. If each of the values can be considered to be an equally likely result, the probability for each value is one divided by the number of values and the average is thus computed as the sum of the Expectations.

For the same example, if 15 items are purchased, the Expected Value of the net income (Expected net income) is:

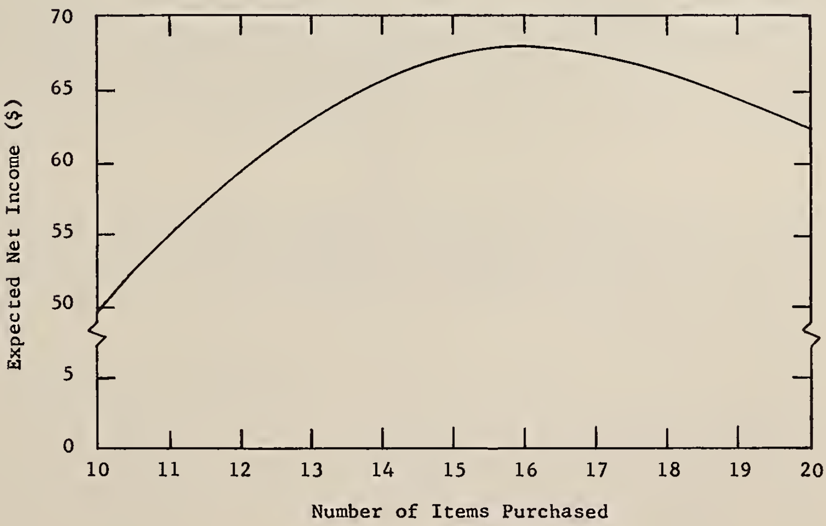
$$.02(\$40) + .04(\$47) + .07(\$54) + .18(\$61) + .20(\$68) + .49(\$75) = \$67.79$$

The Expected net income for each of the possible purchases is similarly found to be as follows:

Number purchased:	10	11	12	13	14	15	16	17
Expected net income (in $):	50.0	54.86	59.44	63.53	66.36	67.79	68.10	67.57

Number purchased:	18	19	20
Expected net income (in $):	66.41	64.62	62.69

The relationship between quantity purchased and expected net income is graphed below:



If the criterion for preference among the alternative possibilities is the largest Expected return, then the decision should be to purchase 16 items each day.* Although the Expected return with a purchase plan for 16 items per day is $68.10, the actual demand can be as low as 10, in which case the return is $38.00, resulting from 10 sold for $50, and 6 unsold for a loss of $12. Purchase of 10 each day will yield a constant return of $50 per day since demand is at least 10 every day. With a purchase of 16 per day, the Expected net income is higher, but the net income per day will have greater variability, varying from $38 to $80. There may be individuals who would prefer a lower Expected value in order to have less variability, that is, less risk.

If the purchases are to be made on many days, as implied in this example, the long run average will conform to the Expected value much as flipping a coin many times will result in fairly even numbers of heads and tails. Suppose, however, that a decision must be made for one day only. A gambler might want to purchase 20 items since he could obtain, with luck, the highest possible income of $100. The very conservative person might purchase 10 items since this assures him of $50. Another might purchase 14 items since a demand of 14 has the highest probability of occurrence. A purchase of 16 would give the highest Expected income. These possible criteria have been given names. The gambler who purchases 20 items is said to follow a Maximax criterion. The conservative who would purchase 10 items will follow a Minimax criterion. The one who purchases 14 items follows a Maximum Likelihood criterion, and the one who purchases according to the maximum expected value follows the Bayes' Decision Rule. The choice of criterion is one of personal inclination towards risk and of personal need for a specific outcome.

In the example of uncertain demand just cited, it was noted that the average demand per day is 14.65 or, in round numbers, 15 per day. The Expected values for the various purchase plans show that purchase of 15 per day does not, in the long run, yield the largest income, perhaps contrary to "common sense." Computations will show that if the salvage value of unsold stock is greater than the $3 assumed in the example, then using the average demand to determine the purchase quantity will be an even poorer plan relative to the best plan. Thus, the use of the probability distribution as a more complete description of the uncertain situation yields better decisions.

The probability distribution of demand for inventory items (for example, spare parts) is utilized extensively in the modelling of decision situations related to inventory control. It is also used to represent the uncertainties related to failure of equipment and the scheduling of maintenance. Probability distributions are also central to Replacement Theory. Replacement Theory, which is closely related to inventory and maintenance control, deals with best practices concerning when to replace equipment based on cost of new equipment and cost of retaining, fixing, and overhauling existing equipment. Probability distributions are also utilized in Queueing Theory, which will be described in the next section.

The treatment of uncertainty so far has been concerned with situations where uncertainty arises from lack of knowledge about behavior of relevant factors which are indifferent to the decision-maker's objectives. Uncertainty arising from weather, from variabilities among material, machines, and workers, from reactions to advertising, from certain behavioral patterns among homogeneous groups of people, and from random throws of a pair of dice or spins of a roulette wheel are examples. Uncertainty also can arise from lack of knowledge concerning the behavior of competitors who are not passive about the decision-maker's objectives and who can take purposeful actions to prevent outcomes desired by a competing decision-maker.

Game Theory is the study of decision-making in situations with competing "players" with conflicting objectives. In Game Theory situations, the consequences of one competitor's action depend in part upon the action of his opponents. For example, in weapons system planning, in political campaigns, and often in business planning, competitors' possible actions and counteractions are important factors in arriving at decisions. Clearly, decision-making in such situations calls for different considerations than when uncertainty arises from unmotivated sources.

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*The mathematical model of this problem can be expressed as:

$$\text{Expected income} = rx\int_{x}^{\infty} f(y)\,dy + \int_{0}^{x} [ry + s(x-y)]\, f(y)\,dy - cx$$

where: x is the number of items to be purchased per day
r is the selling price per unit
c is the cost per unit
s is the salvage value per unit
f(y) is the probability distribution of demand

This mathematical expression for expected net income can be solved for the maximum expected value without computing the expected income for each purchase possibility.

Some fundamentals of game theory can be demonstrated with the following simple game: Player A has 3 possible actions and player B also has 3 possible actions. Each player must select his action without knowledge of what action the other player will choose; however, each player knows in advance what the consequence will be for each combination of actions that might be selected by himself and his competitor. For each possible action A might take, B has 3 possible actions, and so there are 9 possible pairs of actions and 9 consequences. These consequences are termed "payoffs," since in the usual context of game theory, the consequence is what B pays A. The consequences of the 9 possible combinations of actions are displayed in this "payoff" table:

		B's Actions		
		1	2	3
A's Actions	1	4	-2	0
	2	2	2	1
	3	-3	-1	0

This means that if A takes his action 1 and B also takes his action 1, then B must pay A 4 units of something of value. If A takes action 1 and B chooses action 2, then A must pay B 2 units. Where the entry is zero, neither pays the other. The payoff entries in the matrix are what A gains or loses if A and B should take the actions identified on the margins.

Consider the game being played many times. The maximum that A can get is 4 units. However, if A takes action 1 to try to get his maximum, and B takes action 2, then A must lose 2 units. Similarly, the most B can get is 3, but if B takes action 1 to try to get his 3 units and A takes action 1, B would lose 4 units. A reasonable decision for B is to take action 3, since he can then limit his loss to at most 1 unit, and for A to take his action 2 since he can be assured of gaining at least 1 unit. There is not much incentive for either to stray from these actions, for if one strays and the other does not, the one who strays will be in a worse position.

Some game situations involve a "mixed strategy." That is, assuming that the game is played many times, each player should try to vary his actions, utilizing a mixed set of actions in order to optimize the consequences to him. The game of matching coins is an example. In this game, both players independently choose a side of a coin and then the choices are displayed. A must match B, which means that if both players had chosen the same side of the coin, then B must pay A one unit. If the sides chosen are unlike, then A must pay B one unit. The payoff matrix is:

		B	
		Heads	Tails
A	Heads	1	-1
	Tails	-1	1

If this is to be played many times, neither player would benefit from persisting in one choice since the other could easily take advantage of it. If one competitor should favor one side of a coin even slightly more than the other, his competitor (if he detects this bias) can gain an advantage. Thus, the most favorable strategy for each player is to randomize his selections so that 1/2 of the time heads is shown and tails the other half. This is an example of a mixed strategy, i.e., the actions are varied from game to game, with the choice of action selected randomly within the best relative frequency of actions so that the opposing player cannot anticipate the outcome.

The two simple illustrations cited fall into the category called zero-sum, two-person games, since two competitors are involved and what is gained by one is lost by the other. Zero-sum, n-person games are those where there are more than two competitors involved and gains equal losses. Non-zero-sum games are those where there are two or more competitors and where gains do not equal losses. Most business situations are non-zero-sum games in that one firm's losses are not necessarily another company's gains. All competitors may gain, but by different amounts. Behavioral theory in non-zero-sum games is much more difficult since two or more competitors can form coalitions and conspire against other competitors, or can offer payments or bribes to others in order to secure more favorable actions.

Game Theory in its current state of development cannot be considered as a practical tool in the sense of exhibiting numerous successful applications to real decision situations. However, it is widely recognized that the concepts involved in the theory are often helpful in clarifying and enhancing the judgment of those engaged in such decision situations. The capability for describing complex decision situations so as to identify the significant elements and relationships, even without specific mathematical functions or equations, is useful in resolving complex decision situations. As stated earlier, facility with and understanding of quantitative methodology provide a strong background for this capability.

8. Quantitative Methods Without Optimization

The previous section illustrated the use of quantitative methods to determine the optimal or best alternative action or strategy among many possibilities. In many decision situations, it may not be possible to develop quantitative methods that can be used to yield the optimum choice. For example, there may not be a well defined, quantitatively measurable criterion for indicating preferences among the alternatives; or even if such a criterion existed, the relationships among the decision variables and the criterion may be so involved that mathematical methods may not be available to solve for the optimum. In these cases, quantitative methods may still be useful, not to determine the best solution, but to yield answers to specific questions of the type "what if?" A wind tunnel test of one airframe model does not provide an optimal airframe design. The wind tunnel test provides an economical answer to the question, "If an airframe is designed with these configurations, what might its performance characteristics be?" Similarly, models of decision situations can be developed which economically yield estimated consequences of specific alternatives. If the model is carefully developed, it may be possible to test a variety of possible alternatives under a variety of environmental conditions.

These models are called descriptive or predictive models. The purpose is to predict the consequences of selected actions or policies, but in order to accomplish this, the model must accurately describe the behavior of the pertinent system elements with the decision variables as they might be implemented. Descriptive models can employ analytical, simulation, or gaming techniques, or combinations of them, for solution.

9. Descriptive Mathematical Models

Descriptive (or predictive) models that can be represented mathematically are employed in a variety of decision situations and may vary widely in form, compactness of representation, and method of solution. Unfortunately, there is no adequate classification scheme of these models that permits an easy overview of their respective ranges of usefulness.

Decision problems involving queues or waiting lines, and/or sequencing of activities, are quite general and often mathematically tractable. Since quantitative methods for these problem areas have had extensive use, they will be illustrated briefly.

10. Queueing Theory

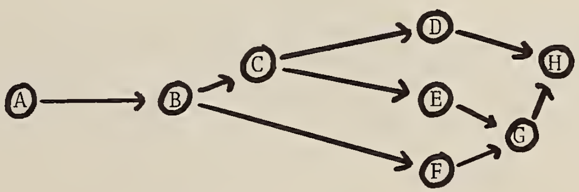
Whenever a customer, arriving for some service, finds all the facilities in use and decides to wait, a queue is formed. Since waiting is annoying, a queue is usually an unsatisfactory situation. However, the elimination or reduction of queues requires additional service facilities which would increase the cost of the service. Hence, the amount of time that customers would be expected to wait and their willingness to wait are important elements in designing and operating service facilities. Service facilities to which the theory can be applied include such things as cafeterias, traffic lights, telephone facilities, tool booths, supermarket checkout counters, and airport runways. Queueing Theory in the broad sense is concerned with decision-making where the length of time a customer must wait is an important consideration, and it attempts to provide information on queueing characteristics for particular service and customer arrival configurations.

Queueing situations are characterized by the time pattern of customer arrivals (input distribution), customer behavior in a queue, the manner in which customers are selected for service (queue discipline), time required for service (service-time distribution), and departure. If customers arrive at fixed intervals of time and the service time is constant for each customer, the resulting situation is clear. There will be no queue (or at most one customer waiting) if the service time is less than the time interval between arrivals, while the queue will grow constantly if the service time is greater than the arrival interval. Generally both arrival intervals and service times are variable (uncertain), and the question is: "For a particular input distribution, service time distribution, number of service channels, and queue discipline, what will be the average customer waiting time or the average number of customers in the waiting line?" The important factors which determine the queue characteristics are the uncertainties as to customer arrivals and service times. Even if the average service time is less than the average time interval between arrivals, so long as some customers can occasionally require extremely lengthy service, the service facilities may need to be different from when the average service time is the same but subject to less variation. If the average service time is equal to the average time between arrivals, one might expect that there would be no queue. Queueing theory, however, can be used to demonstrate that this is not the case if service time and arrivals are uncertain. A queue will form even if the average service rate is less than the average arrival rate. A service facility should have an average service time smaller than the average time between arrivals of customers and occasionally idle service facility is a necessary part of efficient customer service.

Mathematical models of queueing situations are generally expressible only in terms of rather complex equations and hence will not be illustrated here. There is, however, an extensive literature on the subject dealing with situations (among others) involving customers leaving or not entering queues, jockeying among queues, different types of input and service distributions, different queue disciplines (random choice, first come first served, customers with priorities, etc.), multiple channels, sequence of service, and batch arrivals. Queueing models are descriptive and provide information on expected queue characteristics for particular situations. They do not directly determine what the service facilities should be. However, like Game Theory and other quantitative methods, Queueing Theory provides information and understanding which should lead to better decisions without the disadvantages of experimenting with the actual facilities.

11. Network Analysis

Network modelling, i.e., the graphical display of relationships among activities, is frequently utilized to describe the sequence relationships among activities. An illustration of a simple network is as follows:



The node A may be the start of an activity, followed by tasks or events B,C,D,E,F,G, and H may be final completion. The arrows indicate the direction of flow or the necessary sequence of tasks.

Besides the values of a graphic display and of the explicit identification of the necessary activities, tasks, or events, networks can also be utilized as quantitative models. In PERT (Program Review and Evaluation Technique), each node represents an identifiable event or milestone and the arrows represent tasks. With each task are associated estimates of the time required to perform the task and the possible variation in the task time. These estimates are often indicated by 3 estimates of time, a pessimistic estimate which indicates a reasonable maximum, an optimistic estimate which is a reasonable minimum, and the most likely time. Based on the most likely times for each of the tasks, the most likely completion time is developed. The longest "path" through the network is called the "critical path." The activities on the critical path are the ones to emphasize in control, since all of the other paths have slack in the sense that their total times are less than those on the critical path. The estimated variabilities of the task times are used to determine the estimated variance of the completion time. Knowledge of the critical path and amount of slack in other paths may permit reallocation of manpower or other resources so as to shorten completion time or to provide greater safety margins in completions of tasks which are tightly scheduled. The reallocation is performed manually with PERT. However, variants have been developed in which under certain conditions the adjustments are determined using linear programming methods.

The network illustrated may also represent a network of roads for one direction of traffic. For example, cars travelling from A to B may branch onto two roads, either going to C or to F. By observing the proportion of cars taking each branching possibility in the network, and interpreting these proportions as probabilities, it is possible to predict the probably flow of cars through the road network. If each branching probability (called Transition Probability) is independent of the prior route taken, then the model can be represented as a Markov Chain and there are efficient mathematical procedures for obtaining certain information about flows through the network. Let $p_{AB},p_{BC},p_{BF},p_{CD},p_{CE},p_{EF},p_{FG},p_{DH},p_{GH}$ be the set of probabilities that a car at the node represented by the first subscript will take the route leading to the node of the second subscript. $p_{AB},p_{DH},p_{EG},p_{FG}$, and p_{GH} are all equal to 1 since there are no branching possibilities at A,D,E,F, and G. These transition probabilities can be represented as a matrix as follows:

	B	C	D	E	F	G	H
A	1	0	0	0	0	0	0
B	0	p_{BC}	0	0	p_{BF}	0	0
C	0	0	p_{CD}	p_{CE}	0	0	0
D	0	0	0	0	0	0	1
E	0	0	0	0	0	1	0
F	0	0	0	0	0	1	0
G	0	0	0	0	0	0	1

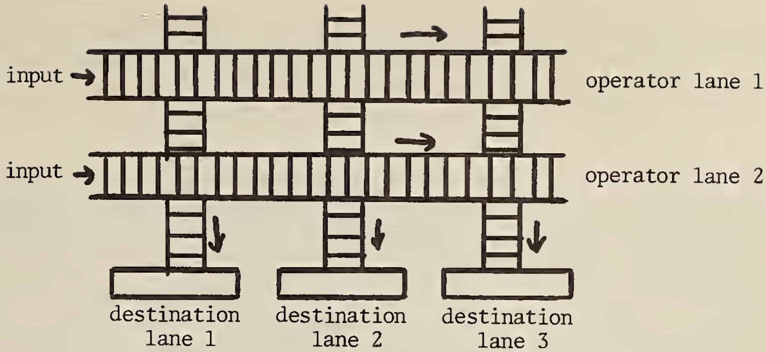
The rules of the branch of mathematics called "matrix algebra" can be used to determine the expected relative frequency that each node will be entered and other descriptive characteristics of the flow through the network. The importance of having such a routine and readily-automated solution method is especially evident for large networks.

12. Simulation

Representing a decision situation by a mathematical model requires that the relationships among the identified system elements be portrayed by mathematical functions, equations, and formal structures that operate under the rules of mathematics. For many decision situations, the mathematically tractable forms may be poor approximations of reality and their use may yield inadequate results. For example, in a queueing situation, the customer arrival pattern may be totally unlike any that can be represented by a mathematically tractable arrival distribution. Or, the servicing practice may involve additional personnel assisting at certain times according to rules which may be difficult to represent mathematically.

The advent of the large electronic digital computer has permitted the development of Simulation models which are not "solved" by the usual mathematical methods. The Simulation models are, as the name implies, simulations of some specified process or operation. Thus, the computer "runs through" the sequence of activities guided by conditions and rules including those which are the object of the query, maintaining the necessary records, and if necessary, performing calculations of the sort involved in solving mathematical models. Highly complex systems that would be difficult or impossible to represent and solve mathematically can be programmed and simulated on an electronic computer. Because electronic computers operate at fantastic speeds, uncertain situations can be sampled many times as if the same basic situation was repeated many times to generate a distribution of potential consequences. Also, many time periods of activities can be simulated within a very short time. The simulation model essentially provides management with an experimental laboratory in which new policies, equipment, schedules and changes in other decision variables can be introduced, the operation under these conditions simulated, and the consequences estimated.

The basic ideas of a computer simulation will be illustrated for a particular problem related to the design of a package sorting machine. The machine includes a number of conveyor belts (operator lanes) moving in the same direction. A worker (one for each lane) places packages with destination codes on the operator lane. The operator lanes pass over a number of other conveyor belts moving at right angles to the operator lanes. These conveyor belts are "destination lanes." A package moves along the operator lane, until it comes to the appropriate destination lane where the package is mechanically transferred to the destination lane and proceeds to a bin. The conveyors of both the operator and destination lanes are continuous series of containers (moving containers) so that a package is placed in a container and is transferred to a container and each container can have only one package. The schematic arrangement is shown on the next page:



In designing the machine, there is the problem of what to do when a package on an operator lane arrives at a destination lane and the destination lane already has a package from a prior operator lane. Since all belts are moving continuously at uniform speed, either the package must continue on the operator lane and be recycled through again, or there can be provision for limited storage at each intersection to act as a buffer. If the buffer storage were large enough, there would be no need to recycle. Limited buffer storage would require some recycling. Since buffer space is costly, there is a problem in deciding what capacity of the buffer storage should be designed into the machine.

The simulation may be performed in several ways, but only one will be illustrated. For simplicity, it is assumed that the movements of all of the belts are synchronized, and also the movement of the package in the beffer storage, so that it is always possible to transfer a package from operator lane to destination belt if space is available. It is assumed that transit through the buffer storage is immediate. Although the machine will have many destination lanes, the analysis can be made on the basis of one destination lane, a specified number of operator lanes, and a buffer storage of a particular size for each intersection.

To perform the simulation, it is necessary to know the arrival pattern of packages to the particular destination. Assume that an analysis of destinations shows that 1/6 of the packages are for this destination and that the packages for this destination are randomly mixed with packages to other destinations. Since the belts are synchronized and move at uniform speed, the time unit can be taken as the time required for movement of the conveyor one container length and time can then be considered to proceed in discrete units.*

The simulation consists of a series of cycles with each cycle taking one unit of time. Each cycle involves the following activities which the computer is programmed to imitate:

1. Suppose there is a "package" on some operator lane, and it has just reached a certain destination belt. A query is made whether the package shall drop off at this belt or proceed on to other destinations. If the simulation were to be done manually, a six-sided die with one face identified as the destination could be tossed. If the destination face turns up, the package is regarded as meant for this destination. If some other face turns up, the package is destined for some other destination. On the computer, the throw of the die can be simulated by generating random 4 digit numbers.** The random number is compared to 1667 (since 1/6 is 16.67 percent). If the random number is 1667 or less, then this is like the destination face of a die turning up and the package is for the destination belt. If the number is greater than 1667, the package proceeds on the operator belt.

2. If the package is for this destination, the content of the buffer storage is examined. If the maximum number is already in the buffer, the new arrival cannot enter and is noted as being recycled. A count is maintained for packages that are recycled. If the content of the buffer is less than the maximum, the content is increased by one (the new arrival).

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*Or the conveyor can be thought of a moving injerks, one container length at a time.

**There are many ways of doing this. For example, a randomly selected 10 digit number is squared and the middle four digits of the product is selected. The middle 10 digit number is again squared to determine the second 4 digit number. This process can be continued. This "mid-square" procedure is not recommended for obtaining long series of numbers, since long-term patterns arise which spoil the desired "randomness."

3. If the buffer storage contains one or more packages, the container (on the destination belt) into which the package will be discharged from the buffer is checked to determine whether it already contains a package. If it is empty, a package is "transferred" to the container by adding one to the container and subtracting one from the buffer. If the container already has a "package," then a package from the buffer cannot be transferred and the buffer content remains the same.

4. The content of each destination container is shifted forward to the next container. This simulates the movement of the destination belt one container distance. The destination container directly fed from the first operator lane (no prior operator lane), after shifting its content forward, will always get a zero content to replace it since the containers coming to the first operator will always be empty.

5. A new cycle begins by returning to step 1. A count is made for each cycle and after a predetermined number of cycles, the process ends and the percentage of packages recycled on each operator lane is obtained.*

This simulation may then be repeated for another buffer storage size, of if the buffer storage capacities at each operator lane can be different, then a different configuration of capacities can be introduced. Also, the proportion of packages to the particular destination can be varied in order to obtain relationships between arrival densities, buffer sizes, and recycling.

A simulation need not be performed on a computer. However, even relatively simple simulations can require a great many individual steps and involve keeping tract of voluminous records. The use of computers provides the possibility of "experiencing" long runs of package processing in a matter of minutes or even seconds. Also, the computer program could probably be easily modified to try different package arrival patterns. The particular problem described can also be treated analytically, but the analytical procedures would be beyond the scope of this chapter.

A problem which is sufficiently simple to permit description in a short amount of space may convey the impression that simulation (as also other quantitative methods described) is applicable to only trivial situations or to situations where experience mught provide an adequate basis for decision. The purpose of simulation, of course, is to provide information about situations which are so complex as to defy satiafactory intuitive analysis. Thus there have been extensive simulations performed of a variety of highly complex situations, such as a large part of a military supply system, scheduling of aircraft maintenance for a major airline company, passenger transportation in the Washington-Boston Corridor, and many others.

Simulation of complex situations is generally an expensive operation, mainly due to the requirements for large amounts of data generated by the specificity of activities and their relationships. This is also true for most mathematical models, whether descriptive or optimizing.

13. Gaming

Gaming or Games (War Games, Management Games) refer to simulations where human participants are actively involved and play specific decision-making roles. A computer may be used to determine those decisions' consequences, which are reported to the appropriate participants who then act on the basis of this feedback information on the results of their own and other participants' decisions. Gaming differs from Game Theory in that Game Theory is concerned with how competitors should act in order to maximize their own gains. A Game Theory situation can be "gamed" with actual participants to determine how people really do act. In gaming, the human participants are assigned roles, such as president of a firm or company commander of a military force, and are provided rules, resources available, objectives, alternatives, and other information. Generally, the participants are competitive groups or at least groups with different and possibly conflicting objectives. As in purely computer simulation, time can be compressed and years of "experience" can be played in relatively short periods of time.

The difference between gaming and simulation is that in simulation, one set of levels for the decision variables is traced through to its consequences. In gaming, the participants make decisions for which the consequences are obtained through an analytical model or possibly simulated, and these consequences, communicated back to the participants, lead to the next group of decisions.

In most games, the participants are scored based on the results of their actions, and they attempt to maximize their respective scores. The ability of games to capture the full realism of actual

*Since a package is assumed to be able to move from operator lane through the buffer to the destination container in one cycle, the perceptive reader may have noticed that the sequence described in step 2 with the maximum number already in the buffer is not correct when the destination container is empty. This inconsistency is retained for ease of explanation.

operations, especially in terms of the consequences of actions, is often questionable. However, war games are used extensively in the military, and much useful information has been obtained through such exercises.

14. Indicators of Preference

The identification of the best choice among feasible alternatives is a simple matter if the criterion for choice can be measured numerically, and if preference is a matter of selecting the largest value (e.g., maximum profit) or the smallest value (e.g., minimum costs). The pertinent factors related to performance, however, often involve qualitative factors, or factors which have natural or derived numerical measures but which are not comparable, such as time and costs, or number of attack submarines and number of strategic bombers. Under these conditions, it becomes very difficult to insure that decisions are consistent with organization objectives, and that portions of them entrusted to different decision-makers are treated on a consistent basis. Utility Theory is concerned with this problem, and attempts to develop methods and understanding which will improve the judgmental process of the decision-maker. The ultimate goal is the translation of the pertinent criteria into a single numerical scale which reflects the decision-maker's preferences for all combinations and levels of the criteria involved. The present state of development of Utility Theory is far from realizing such objectives; hence the discussion here will deal with certain rudimentary ideas, and with problems rather than solutions.

Preferences among outcomes can be expressed quantitatively in two basic forms. The simpler form is merely to express the order of preference - which one is preferred over another. The more difficult form is to express how much one is preferred over another. If the decision circumstances are such that it is feasible to rank-order the preferences for all alternatives, then this is sufficient to establish the decision (just choose the top-ranking alternative). There are circumstances, however, when even ordering the alternatives is difficult and it is desirable to find ways to facilitate the process. Situations involving many possible alternatives but a single numerical criterion have already been considered. Beyond this are situations where several qualitative or non-additive quantitative factors are involved, together with many possible alternatives. Even when there are relatively few alternatives, policy statements which provide consistent ordering guidelines to subordinates' decision-making involve further complications.

Let items A,B,C,D, and E (items could be people, jobs, programs, equipment, etc.) be arranged in preference order, with A being the most preferred and E the least preferred. If the decision calls for the selection of one item, then the preference order is sufficient to determine the choice. Suppose, however, the five items are five possible projects which are expected to have the following costs:

Project A - $100,000
Project B - $200,000
Project C - $600,000
Project D - $300,000
Project E - $500,000

and the total budget is assumed to be $1,100,000.* There are many combinations of projects which are possible within the budget limitations. Among them are:

(1) A,B, and C at a total cost of $900,000

(2) B,C, and D at a total cost of $1,100,000

(3) A,B,D, and E at a total cost of $1,100,000

It is seen that the preference order placed on the individual projects is not sufficient to indicate a clear choice among the combinations of projects. Although C is preferred to either D or E individually, D and E together may be preferred to C, in which case A,B,D, and E is preferred to A,B, and C. If C is preferred to having D and E, or if C plus $200,000 to spend for something else if preferred to D and E, then (1) is preferred to (3). If the budget limitation were $100,000, then (1) would be a clear choice. However, in general, in order for a clear choice to be evident, preferences must be indicated for feasible combinations of items rather than for specific items. This point is more clearly illustrated in the next example, in which there are two factors comprising the basis for preference.

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*This example and the one that follows are taken from G. Mellon, "An Approach to a General Theory of Priorities: An Outline of Problems and Methods." Econometric Research Program, Princeton University, Research Memorandum No. 42, 30 July 1962.

Assume that there are three different jobs, an also three individuals each of whom can fill any of the three jobs. Suppose the jobs are ranked in order of importance: job 1 is considered the most important, job 2 is the next, and job 3 is considered the least important. Similarly, individual 1 is ranked as the most desirable, individual 2 is the next, followed by individual 3 ranked lowest. The possible assignments of individuals to jobs are shown as

		jobs		
		1	2	3
	1	A	B	C
Individual	2	D	E	F
	3	G	H	I

A indicates that individual 1 is assigned to job 1, B means individual 1 assigned to job 2, etc. From the rankings of the individuals and the jobs, A is clearly the highest ranked combination, and I is the lowest ranked combination. What can be said about the seven other combinations B,C,D,E,F,G, and H? The following statements are unambiguous:

A>B>C
D>E>F
G>H>I

A>D>G
B>E>H
C>F>I

> is read "is preferred to."

There is no basis, however, to order B and D,G,E, and C or H and F. Some might say that A,E,I is the best assignment plan, but on the basis of the information available, there is no reason to choose A,E,I over, Say, D,B,I. A may be preferred over D, but E is not preferred over B, thus A,E,I is not a clear choice over D,B,I.

It is evident that when several independent attributes are involved in decision-making and each attribute is priority - or preference-rated, these orderings are not sufficient to determine a complete ordering of the combinations of attributes. If priority-rated attributes or criteria are provided as guidelines to subordinates, then the latter can encounter circumstances in which they must impose their own opinions to arrive at a decision. It is not difficult to imagine that one subordinate might interpret D>B, another subordinate will interpret B>D and each will act accordingly.

When many criteria are involved, each with its separate priority structure, the existence of these priority structures may collectively provide only minimal information or guidelines for decision-making. In a practical situation, there may be heirarchies of authorities imposing priorities; there may be functional priorities; there may be project priorities; there may be time priorities, etc. The existence of these separate priorities or preferences may provide little or no guidance as to which of two particular alternatives should be regarded as preferable.

One of the most common devices to arrive at an ordering in the multiple criteria priority situation is to impose a priority order on the criteria. This device is called a Lexicographic Ordering. An alphabetical ordering is an example of a lexicographic ordering. The first letter position is overriding over the second position which is overriding over the third position, etc. Thus, a word beginning with A is placed before a word beginning with B, hence AZ is placed before BA. If the letters in the first position are the same, then the second position becomes dominant, etc., thus Ac is placed before Ad.

In the previous example concerning jobs and people, a lexicographic ordering is justified if the job category can be considered as overriding the individual category. In this ordering, job 1 with any individual has precedence over job 2 with any individual, which in turn has precedence over job 3 with any individual. Thus, the ordering is

A>D>G>B>E>H>C>F>I

and provides a complete ordering of the job-individual combinations. The ordering of the assignment plans is

AEI>AHF>DBI>DHC>GBF>GEC.

Although lexicographic orderings have certain advantages, they often impose stronger priorities than are really desired. Depending upon the number of classes within a highly rated category, it is not difficult to imagine decreasing marginal returns of low priority classes. Insisting that the lowest priority class of the highest priority category have precedence over the highest class of the second priority category may reduce the overall effectiveness significantly. A familiar example is the situation in which a casual request by the boss which has extremely low priority so far as he is concerned, is accorded top priority by his subordinates and overrides much more significant activities.

Due to the convenience of numerical values, they are often used to represent preferences. When numerical values are used to convey the preference order, the ordinal characteristics of the numerical values are utilized. Thus, if the highest preference item is assigned a value 100, the next highest 99, and so on, so that the higher the number, the greater the preference, then the numbers have no more significance than the use of alphabetic characters to denote order. The same order can also be indicated by .0085, .0084, etc. Furthermore, when numbers are used to represent only the order of preference, arithmetic operations on the numbers may be misleading since the usual arithmetic operations may not be applicable ot ordinal numbers. For example, if A is the most preferred and is assigned a value 10, B is assigned 8 and C is assigned 3, then the values for B and C added gives 11, but the fact that 11 is larger than 10 does not necessarily mean that B and C together would be preferred to A.

All numbers representing preference order can be multiplied by a constant value and a constant value can be added to each without altering the preference order. If the values for A,B, and C above are all multiplied by 10 and 15 added, then the results, 115, 95 and 45, still retain the previous order.

Assigning ordinal numbers to represent preferences may be satisfactory for particular decision situations where the outcomes are certain, but can be very misleading for uncertain situations. For example, suppose the outcome of alternative A is uncertain so that the probability of an outcome with a preference order of 10 is 80%, while there is a 20% chance that the outcome will have a preference order of 5. These hypothetically possible outcomes and those for alternatives B and C are tabled below.

PROBABILITY OF OUTCOME WITH SPECIFIC PREFERENCE ORDER

	10	9	8	7	6	5	4	3	2
A	.80					.20			
B		.10	.60	.20		.10			
C					.10	.10	.20	.50	.10

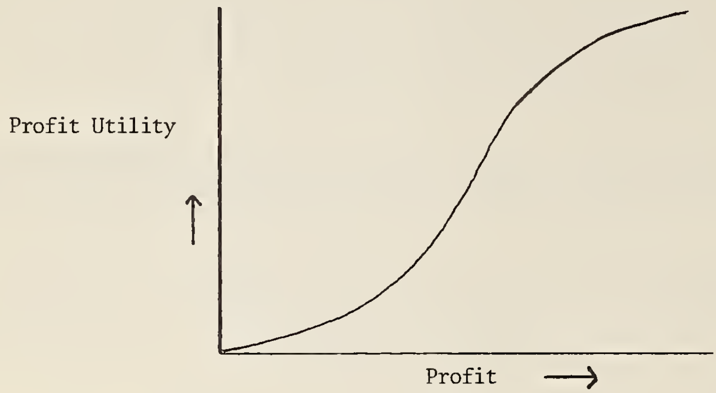
Using the expected value concept described before, the expected values of the preference orders of the 3 alternatives are A:9.0, B:7.6, C:3.6. Suppose that any outcome below preference order 6 is very undesirable. This might be reflected by a different numbering system but which still retains the same preference order as before for the outcomes.

	100	99	98	97	96	5	4	3	2
A	.80					.20			
B		.10	.60	.20		.10			
C					.10	.10	.20	.50	.10

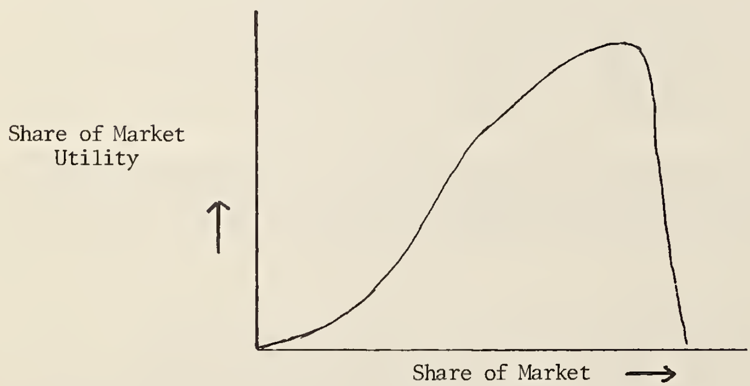
The expected values of the preference orders of the 3 alternatives are now A:81, B:98.6, C:12.6 and B has the largest expected value. This example is perhaps extreme; however, it illustrates that assigning arbitrary numbers that reflect only order of preference can in uncertain situations suggest mathematical operations leading to meaningless results.

When numerical values are determined in such a way as to represent strength or degree of preference, then the numbers are said to have cardinal properties or represent Cardinal Utility. Numerical preference values developed in this way can be manipulated arithmetically since the rules of arithmetic apply to cardinal numbers. The determination of Cardinal Utility with any precision is readily seen to be a difficult, if not impossible task, and even approximations require care. However, numerical values are used so frequently (by non-mathematicians) to represent Cardinal Utility and are misused so widely in this context, that some discussion seems warranted.

Even certain natural quantitative measures may not have precise cardinal properties in terms of Utility. Amounts of money provide an example. For an individual without any funds, the first $100 may have much greater significance or Utility than the second $100. Thus $200 may not have twice the Utility value of $100. As another example, to a person who desperately needs at least $1000, $1000 may have much more than twice the Utility value of $500. If amounts of money, as in net profit, gross sales, or costs represent the desired preference criterion, then the lack of cardinality is not a hindrance since money still has ordinal qualities in that more is preferred to less (for cost the reverse is of course true). However, if quantity of money is only one among several criteria of preference, then the lack of cardinality may be important. For example, the decision-maker's overall preference criterion may be based on some combination of profit and share of the market. The "Utility" curve for profit for the decision-maker may be of the form:



and the "Utility" curve for share of the market may have the form:



The shape of the second curve might, for example, reflect the idea that too large a share of the market would invite anti-trust actions. Thus for this decision-maker, an increment of the share of the market at the lower end of the market share scale is relatively more desirable than an increment of profit at the lower end of the profit scale. The same increments at the higher end of both scales show the reverse to be true.

Although still viewed as experimental for practical applications, perhaps the best known procedure for developing a Cardinal Utility scale derived from a decision-maker's indication of preferences is a procedure developed by von Neuman and Morgenstern. Suppose the possible outcomes are arranged in preference order, from the most preferred, A, to the least preferred, say N. Thus, A>B>C>...>N. The ends of the scale are arbitrarily determined, say A=100, and N=0, and it is desired to assign numerical values to B,C,... between 100 and 0 which will reflect the relative strength of preference of all the possible outcomes. Obviously, if the outcome A is a sure thing, A would be preferred to all other outcomes. The decision-maker is asked a series of questions starting with the following. Suppose you have two choices, (1) the outcome B with certainty, or (2) a gamble, with outcome A having probability 1/2 and outcome N having probability 1/2 (i.e., on a flip of a coin, a head means outcome A and a tail means the least desirable outcome, N). If the decision-maker chooses (1) (B with certainty), then the same question is asked with a larger probability for A and a correspondingly smaller probability for N. Since A with certainty is the most preferred outcome, it seems reasonable that as the probability for A gets closer to certainty, the gamble will become more desirable. Presumably, then, there will be a probability value for which the decision-maker will be indifferent between the gamble for A or N, and the certainty of B. If in the initial question the gamble (1/2 for A, 1/2 for N) is preferred, then it again seems reasonable that as the probability for A gets closer to zero (N becomes more certain), the gamble becomes less desirable. There must again, then, be a value for the probability for which the gamble and the certainty of B will be indifferent. If the probability of A at the point of indifference is p, then Utility value for B = p (utility value for A) + (1-p) (utility value for N) or Utility value for B = p (100) + (-p) (0). Similar series of questions are asked for each of the possible outcomes.

This procedure and others of similar nature can be applied to developing preference (utility) values for sets of possible outcomes and also for the quantification of a particular criterion. If several criteria are involved in a decision situation, then this type of procedure can be further utilized to develop the "weights" for combining the criteria into a single quantitative scale. To illustrate this last point, suppose the decision on an inventory plan is to be made on the basis of two criteria, say, cost and the number of times shortages might occur. The preference values for various combinations of cost and shortages might be obtained by some methods such as the von Neuman - Morgenstern method. Then from these values, a "curve" might be fitted by statistical methods to obtain the relative weights implicit to the two criteria by the preference values inferred for the combinations of the two criteria.

The reader is cautioned that the proper assignment of numerical values to reflect cardinal preferences requires careful analysis of the conditions underlying the decision situation. Since these values represent management policy in the most direct manner, careful construction is a minimum requirement.

Even though quantitative representation of preference may be difficult to determine with precision when the outcomes involve many factors, the use of numerical values has the important feature of making explicit the particular rationality involved in choice making. This is particularly important for repetitive decision-making, since experience will provide information by which errors and inconsistencies can be systematically corrected, and numerical "preference numbers" can be induced gradually to converge to satisfactory values, a difficult task when the rationality is hidden in intuitive judgment.

15. Conclusion

Two rather extreme views of mathematical methods, both unfortunate, are often held by managers. At one extreme, there is a tendency for managers to be overwhelmed by the seeming sophistication of certain quantitative methods and to accept their results unquestioningly. The basic concepts of any mathematical model are capable of being explained to one who understands the decision situation, including the assumptions implicit in the mathematical structure. The method of solution, like a doctor's prescription, need not be explained, but the manager should demand a careful explanation of the model. This is important because quantitative methods can be badly misused. A mathematical model is a more or less accurate and useful representation of reality, and reality is always complex. Unfortunately, reality is not only complex, but also rarely conforms directly to textbook models, and even textbook models can appear to be highly sophisticated to an unquestioning manager. This attitude, besides possibly failing to detect that a proposed model does not adequately represent the decision situation, can also lead to overselling of quantitative methods so that complicated approaches are used where calculations on the back of an envelope might suffice.

The opposite extreme view is the one held by the manager who denies any usefulness for quantitative methods. He claims, often quite correctly, that there are so many factors involved in the real decision situation that a mathematical model cannot possibly involve them all. The lack of precise representation of reality is a weakness which is not unique to quantitative methods, although the formalism of mathematics bares the lack more clearly. No one can ever consider all of the related factors which may be relevant to a decision situation. Some degree of abstraction is always necessary. Thus, lack of perfect realizm is not reasonable ground for criticism. The advantage of quantitative methods is their ability to consider more factors than can be reasonably handled by the human mind, however capable. Where precisely applicable, methods such as linear programming are superior to intuitive solution; however, most uses of quantitative methods are attempts only to <u>facilitate</u> the judgmental process.

The purpose of this chapter has been to expose the reader to some recent developments in quantitative methods and through them to attempt to provide an understanding of why and how quantitative methods can be useful to management. The details of how these methods can be applied to specific situations must be obtained from specialized sources, which can be found in the voluminous literature of this field.

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Contributions by Dr. Alan J. Goldman, Applied Mathematics Division, Dr. Joseph D. Crumlish, and others of the Technical Analysis Division in the preparation of the final draft are gratefully acknowledged.

☆ U.S. GOVERNMENT PRINTING OFFICE: 1967—251-027/113

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