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Quantifying Hazardous Microwave Fields: Analysis

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Quantifying Hazardous Microwave Fields: Analysis

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QUANTIFYING HAZARDOUS MICROWAVE FIELDS: ANALYSIS*

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ABSTRACT

The familiar power density radiation hazard standards are quite satisfactory for a field consisting of a single infinite traveling plane wave. However, for microwave and lower-frequency fields, hazards occur primarily in near fields which cannot be approximated as the aforementioned plane wave. Further, power density can be quite misleading or even meaningless as a measure of hazard in a near field. Thus, power density in a standing wave can be precisely zero, yet the hazard of such a field can be arbitrarily large. Similarly, a reactive field may present a considerable hazard, yet have zero time-average power density.

The major hazard from microwave and lower frequency radiation is believed to arise from dielectric heating of body tissues, and the heating of an isotropic medium is proportional to the sum of the squares of the absolute values of the electric field components $|E_X|^2 + |E_Y|^2 + |E_Z|^2$. Hence, electric field energy density is proposed for a radiation hazard standard.

Analytical limitations of various types of probes are considered and the advantages of a spherically-symmetric probe of lossy dielectric are discussed. For a rather general spherically-symmetric probe in an arbitrary field, both exact and approximate treatments are given for the calibration constant with full correction for the perturbation of the field by the probe. Conditions for a constant factor are also given.

Key Words: Hazards; Electromagnetic Radiation; Microwave Radiation; Radiation; Measurement; Standards; Energy Density; Probe.

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I. Inadequacy of Accepted Concepts and Standards.

For a linearly-polarized monochromatic infinite plane wave in a linear homogeneous isotropic medium, the magnitudes of the electric field, magnetic field, energy density,[1] and energy flow[1] (power density) are all readily measured and each of these magnitudes may be computed from any of the others without approximation, given the properties of the medium. Hence, any one of these magnitudes constitutes a valid index of such a field and, in fact, completely specifies the field apart from phase and the directions of propagation and polarization. Further, for such a wave in a lossless medium, only the phase varies with position or time. However, no such computation can be made for an arbitrary field. Rather, the ratios of these magnitudes can vary widely with the detailed nature of the field, i.e., depending upon detailed information seldom available in field monitoring or even in biological exposure experiments. For example, a standing wave formed from two of the preceding plane waves of equal amplitude and the same linear polarization but opposite direction has zero time-average energy flow (power density), yet the amplitudes of both the electric and magnetic fields are double those of the original waves at specific locations. Further, at distances from a source less than a small number of wavelengths (reactive zone), the time-average power flow may be zero, yet the electric and magnetic fields may be arbitrarily large.

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[1] Strictly speaking, the familiar expressions for both energy density and energy flow (in terms of the electric and magnetic fields) can neither be proved nor disproved [J.A. Stratton, Electromagnetic Theory, McGraw-Hill, N.Y., N.Y., 1941, pp, 110, 131 ff.]. However, the expressions for energy absorption have strong foundations and "direct" measurements of energy density and energy flow are usually really measurements of energy absorption. Because the hazard discussed here is indeed related to the hypothetical energy density, we will use the expression energy density without qualification.

The preceding complications are of little or no importance in the far field, i.e., for the usual practical sources, at distances from the source large compared to a^2/λ where λ is the wavelength and a is the greatest distance between points in the source, considering reflectors to be a part of the source. However, hazards and electromagnetic interference[2] (EMI) arising from microwave and lower frequencies tend to be important in the near rather than the far field and a large fraction of biological exposure experiments are carried out under near field conditions to achieve high power levels.

"Measurements" of near fields based upon far field concepts must be regarded as simplistic and are at best semi-quantitative, often not even qualitatively correct. For example, in certain regions, the field arising from a microwave oven in a stainless steel kitchen may increase significantly with increasing distance from the oven. Even for a high gain antenna carefully designed to give a uniform field, the energy density may vary by a factor of two in a region of nominal constancy outside the reactive zone. The near electric field amplitude of such a horn-lens antenna 100 wavelengths on an edge is shown in fig. 1; the horizontal dependence is nominally that of a square wave and all the structure shown is real, not noise. Due to the wavelengths involved, analogies between microwaves and optics, x-ray behavior, or gamma-ray behavior are commonly invalid and normally not as good as those with sound waves, since standing waves, resonances, and echoes occur with both microwaves and sound. Although workers in the field base their hazard measurements upon energy flow and commercial probes have energy flow scales, some of these probes respond primarily or partially to energy density per se.

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[2] Analytical considerations of electromagnetic interference parallel those of radiation hazards to a considerable extent. Thus, most of this paper also applies to EMI.

II. Exposure Measurements in Waveguide and Coaxial Lines.

Perhaps the simplest non-far field situation applicable for exposure experiments with tissues or small animals is inside rectangular waveguide or coaxial line. Here the walls and inner conductor may be a copper mesh, fine compared to a wavelength. For waveguide, the largest inside dimension should be well between one-half and one wavelength so that one and only one mode will propagate. For lower frequencies, rectangular coaxial line such as that used in a Stark cell and shown in fig. 2 may be used; here the longest dimension should be significantly less than half a wavelength to prevent the propagation of more than one mode. If the tissue and its surrounding culture medium constitute a transverse slab which completely fills the cross-section of the guide or coaxial line and has a uniform complex dielectric constant, the field inside the sample may be readily computed from the dielectric constant and the input power. The field varies with the position in the cross-section and, if the slab is not thin compared to a waveguide, often with the longitudinal position as well. However, the longitudinal variation may be minimized by filling the guide with a medium having essentially the same complex dielectric constant and magnetic susceptibility as the slab, minimizing reflections. (In this case, the dimensions of the guide should be related to the wavelength in the medium and the medium should extend far enough on both sides of the tissue slab to insure a pure mode.) Further, attention may be confined to a region of uniform field. In the case of the Stark cell, high fields may be generated where the septum approaches the outside wall; to avoid contamination of the sample being studied by decomposition products from regions of high field, the septum should not be close to the wall and/or the portion of the sample being studied should be separated from the portion in the high field by a thin impervious dielectric. For a whole animal (invariably of irregular shape and varying dielectric constant), the situation is more complicated in any near field; however, in principle, reflection effects could also be minimized by immersion in a medium with a similar dielectric constant provided that gaps were small compared to a wavelength, that the medium did not cause too great an attenuation, and that thermal effects, respiration, etc. were properly handled.

III. Energy Density as a Measure of Hazard.

Since a field with initially zero energy flow (power density) may be responsible for arbitrarily great hazard to a subject later placed in it or arbitrarily great EMI to an electronic device placed in it, power density is not a suitable index of either hazard or EMI caused by a near field. Preferably the index should be closely related to the hazard and be easy to measure. Electrical energy density is proposed for the index. Since biological materials are essentially non-magnetic, they interact with an electromagnetic field primarily only because of their dipole moments (more strictly, the dipole transition moments), absorbing energy quanta at a rate proportional (for a linear medium) to the electrical energy density in the tissue. Although the quanta are much too small to cause the breaking of a chemical bond, a molecule which has absorbed one or more quanta has an activation energy which may well cause the molecule to be significantly more reactive in a general sense, say for exchange reactions involving the breaking "bonds" between compounds in the protoplasm and loosely held water. If desired, this may be considered to be extremely localized heating. Thus, electrical energy density should be a reasonable hazard index for both thermal and non-thermal effects including resonant absorption by specific molecules.

It is expected that the hazard to each portion of the body will be associated with its own "constant" of proportionality and that this "constant" will vary with frequency, perhaps sharply. There may well be other effects associated, e.g., with polarization of the field or nonlinearity of the tissue (say in pulsed fields). However, such effects can be studied as deviations from those predicted from energy density measurements assuming a linear isotropic medium.

For EMI in a near field, energy flow has the same disadvantages and energy density has many of the same advantages as for hazards.

IV. Measurement of Energy Density.

The preceding energy density discussion is based upon the energy density within the given tissue, not upon that of the field into which the animal or person might enter. In principle, probes might be implanted in an animal or either the animal or person might be simulated in terms of the complex dielectric constant and magnetic susceptibility as a function of position. Since these procedures would be inconvenient for monitoring, the relation between the energy density inside an animal or person to that inside the probe must be considered (including perturbation of the field by the probe, person, or animal) and used in probe design. General principles will be discussed first, then illustrated with an exact treatment of a possible probe.

In principle, a probe should simulate the body or a given electronic instrument in its extraction of electrical energy from the field. Since the body makes only a slight perturbation in a field of hard x-rays or gamma rays, so should a probe used to measure them, i.e., the probe should be analogous to a high resistance voltmeter rather than a shorting ammeter which gives a reading strongly dependent upon the internal impedance of the source. In contrast, from the shortwave radio region through the ultraviolet, the body absorbs almost all the incident energy apart from that which it reflects. (The penetration depth[3] increases steadily from 3000 GHz to 3 MHz with a depth of 10 cm for 30 MHz and 52 cm for 3 MHz

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[3] Skin depth as defined in electromagnetics.

[H.P. Schwan, "Survey of Microwave Absorption Characteristics of Body Tissues", in Proc. Second Tri-Service Conference on Biological Effects of Microwave Energy, July 8-10, 1958. ASTIA Document AD 131 477]. Hence, a probe designed for this important frequency range should also absorb most of the non-reflected energy and probably have a low reflectivity as well [H.P. Schwan, "Radiation Biology, Medical Applications, and Radiation Hazards", in Microwave Power Engineering, vol 2, edited by Ernest C. Okress, Academic Press, New York, 1968]. (However, in some circumstances, say in continuous monitoring of a radar beam, it is impractical to use a probe which absorbs large quantities of energy.)

A related phenomenon concerns the magnetic field associated with the electric field. In absorption of a quantum of radiation, both magnetic and electric energy are involved. Further, both the initial electric and magnetic fields and the fields remaining after absorption are related by Maxwell's equations so that absorption of all the electric energy requires absorption of all the magnetic energy. For an infinite plane wave in a vacuum, the magnetic and electric energies are equal. However, the ratio of the energies may vary widely in a near field, say from the predominately magnetic field near an electromagnet to the predominantly electric field between plates of a large capacitor. Thus, a probe which properly simulates the body gives readings partially dependent upon the magnetic field, although the probability of quantum absorption is essentially independent of the magnetic field. For EMI measurements, a probe which reacts directly to the magnetic field may be desirable and is discussed in Section V on probe design.

Any device which measures a single complex voltage or its magnitude may be regarded as a device which measures a single constituent of a field, i.e., the coefficient of a single basis function from the infinite series of basis functions used to represent the field. Thus, an ideal dipole measures the coefficient of one of the three electric dipole constituents of the field and completely neglects all magnetic multipoles and all but one of the electric multipole constituents of the field, regardless of their strengths. This limitation does not apply (in a practical sense) to a dipole in a linearly polarized plane wave only because the probe has a fixed calibration factor (which may be accurately computed); however, even in an infinite plane wave, a dipole oriented to give maximum amplitude can give an error of a factor of two in the energy density, depending upon whether the field is linearly or circularly polarized. A probe based upon losses (say, resulting in heating or pressure rise) does not have these limitations and so is suitable for direct measurement of total electric energy density; such a probe measures the energy density and adds all components without cancellation, all of the contributions being positive. For any electric (or magnetic) dipolar field, a probe which sums the squares of the absolute values of the voltages from three identical perpendicular dipoles can give an accurate result independent of probe orientation. Note that even an infinitesimal probe need not measure the field at a point, e.g., a quadrupolar probe; thus, a tiny dipole and a tiny quadrupole measure unrelated quantities.

A probe comparable in size to a wavelength will show resonances much the same as a half-wave dipole, i.e., the calibration constant will be a rather strong function of frequency, less so if the medium is lossy.

For convenience, a probe should not require orientation nor discriminate on the basis of direction of the incoming waves. It should be noted that no voltage probe has these properties, but that an energy density probe with spherical symmetry does. A general theory of such a probe is given in Section V, including a procedure for a priori computation of its calibration "constant", i.e., the ratio of the energy density of the field prior to the introduction of the probe to the energy density in the probe. No calibration constant can be computed for any probe without some knowledge or assumptions concerning the spatial dependence of the field, but an energy density probe with spherical symmetry requires a minimum of knowledge -- much less than is required for a voltage probe.

For pulse power, the analysis is significantly more complicated. The present analysis has been modified to include pulse effects but will not be reported here.

V. Detailed Design of a Possible Probe.

Since previous studies have involved assumptions and approximations inappropriate for near fields, a detailed study is carefully carried out for a mathematically-manageable probe. The study reveals complications which can arise and illustrates some of the general statements of the preceding portions of this paper.

Consider a medium which is linear, isotropic, piecewise homogeneous, source-free, and subject to Ohm's law. For every homogeneous region, every solution of Maxwell's equations is solenoidal and is a solution of the vector Helmholtz equation. For a single frequency and a medium which has spherical symmetry, the complex electric field $\underline{E}$ and magnetic field $\underline{H}$ are conveniently expressed as a linear combination of Hansen's $\underline{m}$ and $\underline{n}$ functions [Stratton, p. 416]. (The time-dependent factor $e^{-i\omega t}$, where ω is the angular frequency, is assumed but suppressed.) Every such function is a solution of Maxwell's equations [Stratton, p. 394], and the set of functions is presumably complete [Stratton, p. 393] for the problem at hand. (The $\underline{L}$ and $\underline{l}$ functions are not solutions of Maxwell's equations.)

Let μ and ϵ be the complex permeability and permittivity and $k = \omega(\mu\epsilon)^{\frac{1}{2}}$ be the propagation constant. As shown in Fig. 3, consider a probe with instrumentation from radius R = 0 to a, a shield S from R = a to b, a lossy material L from R = b to c, and a dielectric D used for thermal insulation from R = c to d, immersed in a medium A, say air. If no internal instrumentation is used, a and b would be zero. Both the media and the propagation constants are indicated by the upper case Latin letters as shown in the diagram. The corresponding lower case Greek letters are used for the permeabilities. For an electrical probe, the permeabilities will ordinarily be essentially that of free space.

In the expression for the electric field, let M_{smnb} and N_{smnb} be the coefficients of the ratios of $\underline{m}_{smnb}$ and $\underline{n}_{smnb}$, respectively, to the square root of $n(n+1)(1+f)2\pi(n+m)!/(2n+1)(n-m)!$, where f is unity for m = 0 but otherwise zero [Stratton, pp. 417-418]. The subscript s is used to denote the symmetry, either even (e) or odd (o). The subscript b indicates the kind of the spherical Bessel function, either the first (j) or second (y) kind. (The y's are not needed for the central region of the probe (R = 0 to the first discontinuity) nor for the field in the absence of the probe.) The corresponding coefficients for $i\omega\mu\underline{H}/k$ are N_{smnb} and M_{smnb} respectively (note reversal) [Stratton, p. 394].

Given the coefficients in any one region of the probe or the region outside the probe, the coefficients of the other regions can be obtained with the aid of boundary conditions [Stratton, pp. 483-4, (1a), (5), (10)]. Because of symmetry, coefficients with different s, m, or n are independent, as are the M and N coefficients. Thus, for given s, m, and n, the four M (or four N) coefficients corresponding to the two kinds of Bessel functions on the two sides of a boundary are related by two equations. Suppressing the s, m, and n subscripts and adding a subscript to indicate the medium, the following equations are obtained:

$$(N_{jA} j_A + N_{yA} y_A)\ A/\alpha = (N_{jD} j_D + N_{yD} y_D)\ D/\delta \tag{1}$$

$$(N_{jA} \partial_{jA} + N_{yA} \partial_{yA})/A = (N_{jD} \partial_{jD} + N_{yD} \partial_{yD})/D \tag{2}$$

$$M_{jA} j_A + M_{yA} y_A = M_{jD} j_D + M_{yD} y_D \tag{3}$$

$$(M_{jA} \partial_{jA} + M_{yA} \partial_{yA})/\alpha = (M_{jD} \partial_{jD} + M_{yD} \partial_{yD})/\delta \tag{4}$$

for the A-D interface, where $j_A = j_n(AR)$, $y_A = y_n(AR)$, and $\partial_{yD} = \frac{\partial}{\partial R}\left\{R j_n(DR)\right\}$, all here evaluated for $R = d$.

The coefficients of the field in the presence of the probe may be related to those for the field in the absence of the probe ("unperturbed field") by an integral equation, assuming that the perturbation of the field does not affect the distribution of the sources. (For a similar case, see Morse and Feshbach, Methods of Theoretical Physics, McGraw-Hill Book Co., N.Y., N.Y., 1953, pp. 1897, 1875.) Because of orthogonalities, the equation separates into a number of equations, each involving only M or N coefficients and a single set of s, m, and n values. Thus, a knowledge of the coefficients in any part of the probe, surrounding space, or in the unperturbed field permits determination of all the other coefficients, given the properties of the probe; further, determination of the coefficients splits into separate problems as indicated.

The time-average energy density is given by $(\underline{E} \cdot \overline{\underline{E}}\ \mathrm{Re}\ \epsilon + \underline{H} \cdot \overline{\underline{H}}\ \mathrm{Re}\ \mu/4$, where the overline indicates that the complex conjugate is to be taken and the symbols $\underline{E}$ and $\underline{H}$ represent peak values. The integral of the time-average energy density over the polar and azimuthal angles θ and ϕ is simplified by the fact that the orthonormalities between the $\underline{m}$'s and $\underline{n}$'s given by Stratton [pp. 417-418] also apply for the Hermitian dot products (including those for solutions involving different kinds of Bessel functions if each square of a Bessel function in Stratton's expressions is replaced by the product of the first Bessel function by the complex conjugate of the second, both of the given order. For any medium, the integral of the time-average electric energy density over θ and ϕ is given by

$$\frac{\operatorname{Re}\,\epsilon}{4}\sum_{s,m,n}[\,|M_{smnj}\,j_n + M_{smny}\,y_n|^2 + \frac{n+1}{2n+1}|N_{smnj}\,j_{n-1} + N_{smny}\,y_{n-1}$$

$$+\frac{n}{2n+1}|N_{smnj}\,j_{n+1} + N_{smny}\,y_{n+1}|^2\,] \qquad (6)$$

and the integral of the time-average magnetic energy density is given by the expression obtained by interchanging the symbols M and N and replacing Re ϵ by β. Here $\beta = |k/\omega\mu|^2 \operatorname{Re}\mu = |\epsilon/\mu|\operatorname{Re}\mu$; hence, for real μ, $\beta = |\epsilon|$. The Bessel functions of course have the argument kR. The losses in a medium are given by an expression identical to that for the energy except that Re ϵ is replaced by 2σ and Re μ is replaced by 2χ, where σ is the electrical conductivity and χ is the analogous magnetic quantity,both for the given frequency. Thus, Re ϵ and β are associated with the electrical and magnetic energy density, while σ and χ are associated with the electrical and magnetic losses respectively The possibility of a magnetic probe will be discussed later.

Neither the unperturbed field nor, in the absence of internal instrumentation, the field in L involves y functions. Therefore, the time-average angular integrals of energy density in the unperturbed field and the loss in L depend only upon the sums

$$\sum_{s,m}|M_{smnj}|^2 \text{ and } \sum_{s,m}|N_{smnj}|^2 ,$$

not the individual coefficients. Further, the relations between th coefficients for the unperturbed field and those for L are independent of both s and m, so that specification of the sums for the unperturbed field determines the sums for L. Thus, neglecting losses in other regions, the calibration constant of the probe is independent of any changes in the unperturbed field which do not change the sums. A wide range applicability of the calibration "constant" is, of course, important for any probe, whether the calibration is computed or measured; such a range seems to be characteristic of the proposed probe.

It is, of course, convenient to be able to use and calibrate a probe in a plane wave and also to be able to specify the class of fields for which the calibration constant is applicable. For a plane wave travelling in the z direction with its electric vector in the x direction, the M_{olnj}'s are proportional to $i^n((2n+1)(n+1)!/n(n+1)(n-1)!)^{\frac{1}{2}}$, the N_{elnj}'s proportional to $-i$ times these quantities, and all the other coefficients zero [Stratton, p. 419]. Assuming that $\sum_{s,m} |M_{smnj}|^2 = \sum_{s,m} |N_{smnj}|^2$ for each n requires that the electric and magnetic energies of the unperturbed field be equal if $\mathrm{Re}\ \epsilon = \beta$. Assuming in addition the sums to be zero for $n = 0$ but proportional to $(2n+1)(n+1)!/n(n+1)(n-1)!$ for other n's permits one to compute a calibration constant (given the propagation constants and susceptibilities), valid for a large class of fields, including plane waves as a special case. (The calibration constant can, of course, also be obtained by direct measurement.) As will be discussed later, these assumptions may be relaxed to a degree for a small probe with low dielectric constants.

Ordinarily the lossy material of the probe would have a complex permittivity and essentially a real permeability and so provide a measure of the electric field.

It would be desirable to obtain an independent measure of the magnetic field, and this would be possible if a suitable material could be found, i.e., one which is linear and isotropic with a complex permeability but fairly small dielectric loss tangent. Such materials do exist for small fields (low on the hysteresis curve), but only exploratory work could determine whether a practical probe could be developed. The preceding analysis would apply for both kinds of probes, and provide information concerning the relative magnitudes of the M and N sums.

For a probe small compared to each of the $1/|k|$'s, the Bessel functions may be approximated by $j_n(z) \cong z^n/(2n+1)!!$ and $y_n \cong -(2n-1)!!/z^{n+1}$, where $(2n+1)!!$ is the product of the odd integers up to and including $2n+1$. Hence, for the lower microwave frequencies and below, the preceding expressions are significantly simplified. Further, because practical fields have a limited amount of fine structure, at least the higher M and N coefficients tend to decrease with increasing n, much like those for a plane wave. As a result, only the lower terms need be used for a probe tiny compared to both the scale of the field structure and a wavelength. (The wavelength condition is not required for X- or γ-rays, where the wavelength is small compared to the dimensions of the probe, introducing averaging effects).

VI. Summary and Conclusions.

The existing standards for microwave radiation hazards are based upon far-field concepts which may have little or even no validity in near fields. However, hazards occur largely in near fields, and even many dosimetry experiments are carried out in near fields. Because of the great complexity possible in a near field, a probe should, in principle, simulate the individual who might be subject to the possible hazard; however, such a probe is not feasible. Electric energy density provides a reasonable index of a large class of microwave radiation hazards and is simpler to measure in a near field than is energy flow. Analytical aspects of near field measurements are discussed in general, and design of a possible probe described in detail. So that near-field dosimetry experiments are meaningful and reproducible, great care must be used in field measurements and describing conditions which determine the field in the subject.

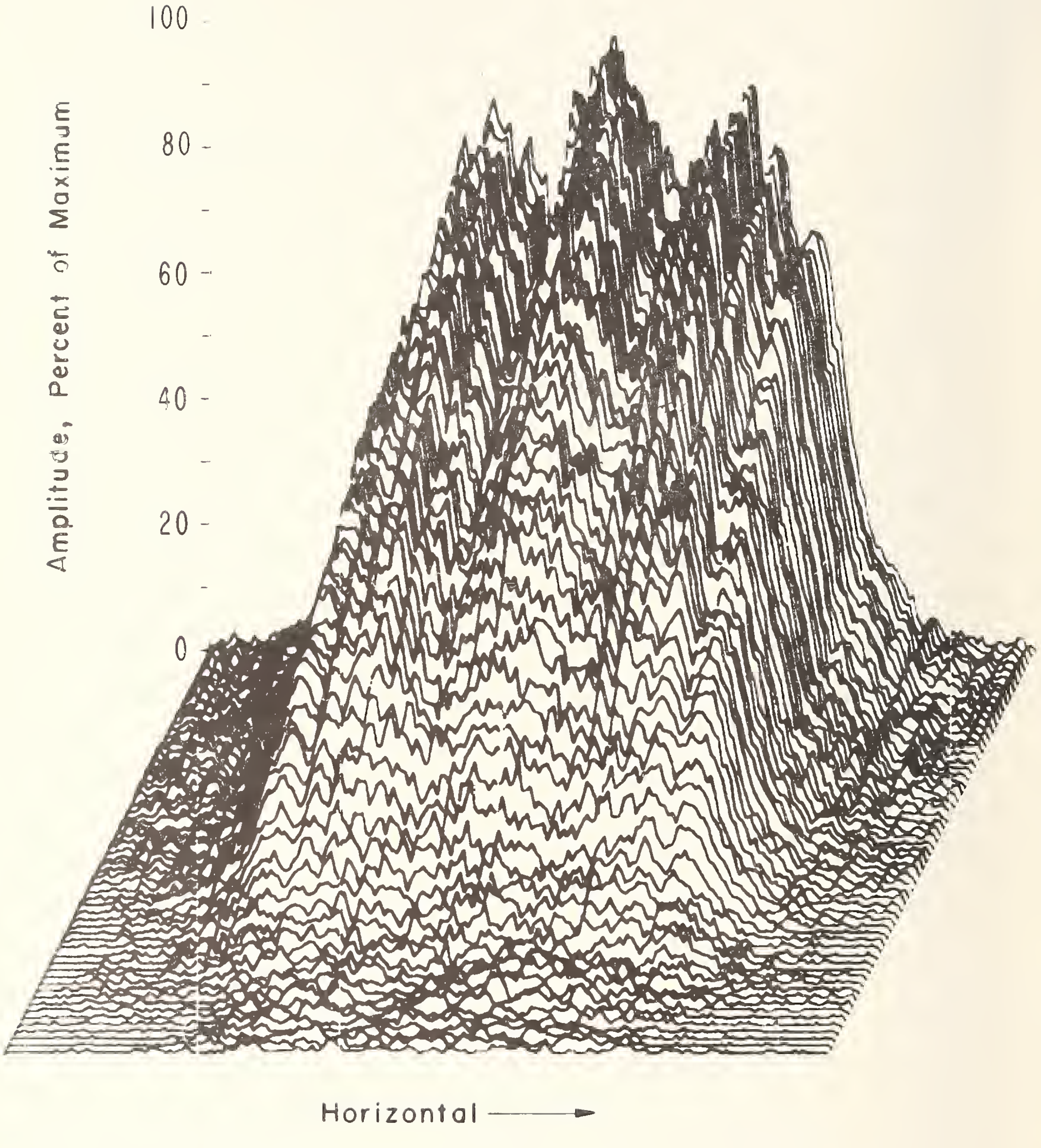

Figure 1. Amplitude Variation Near a Horn-Lens Antenna.

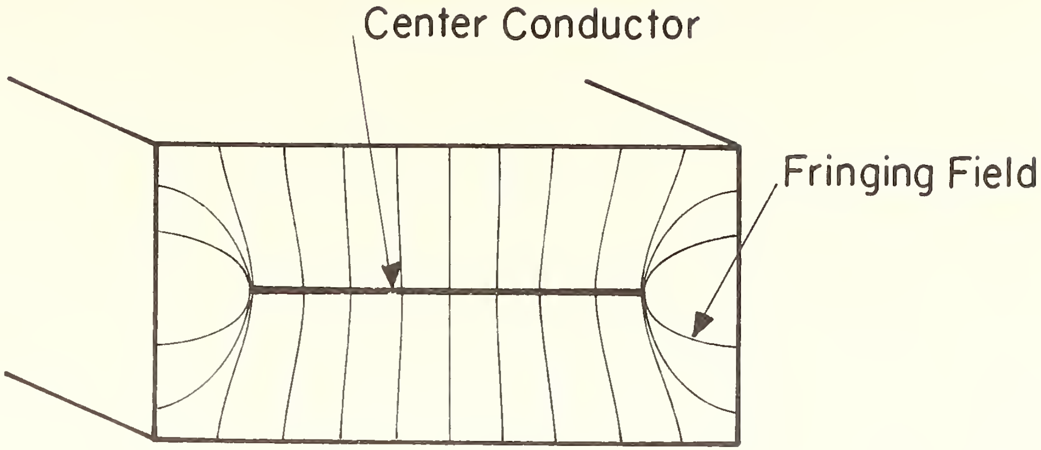

Figure 2a. Cross Section of Stark Cell.

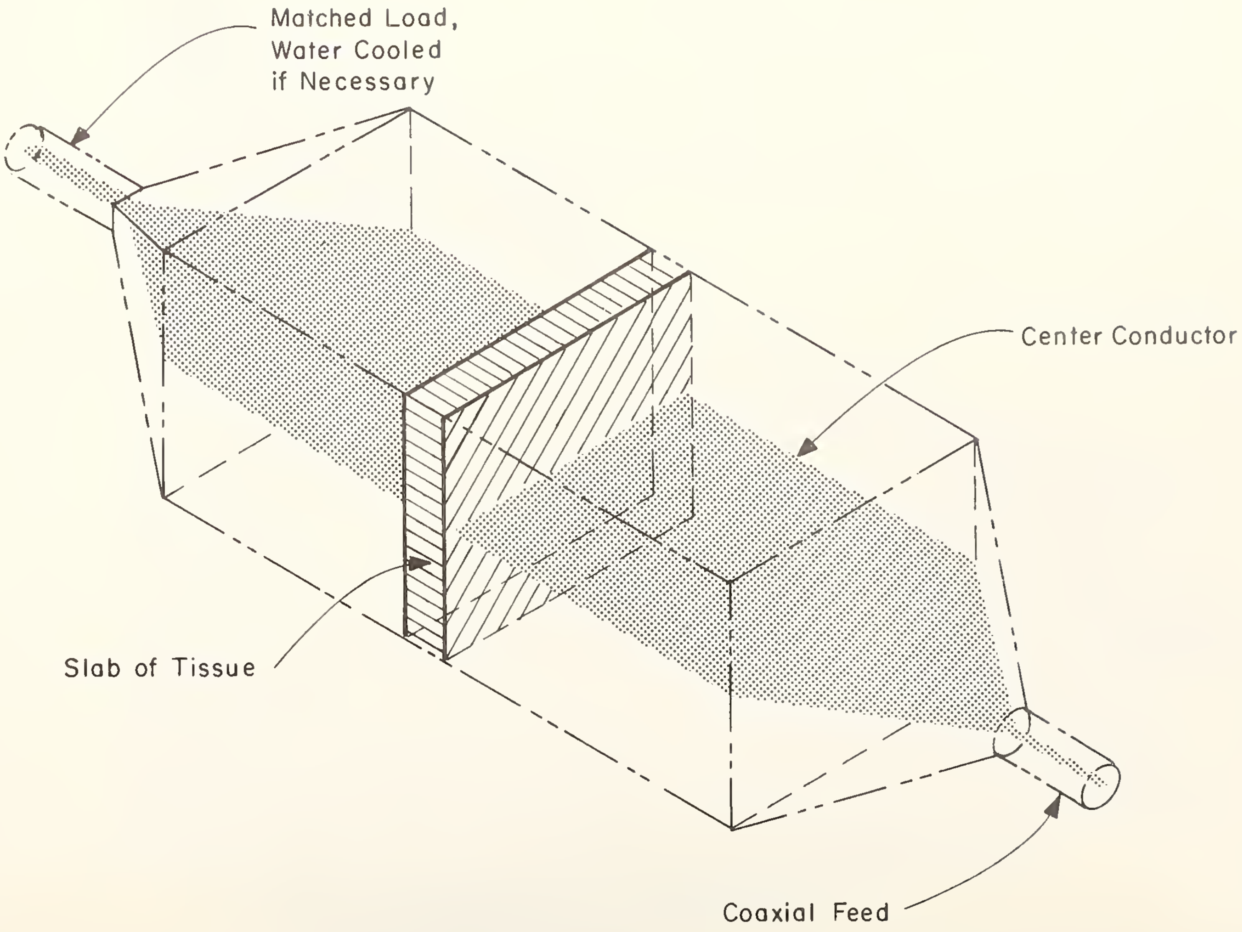

Figure 2b. General Plan of Stark Cell.

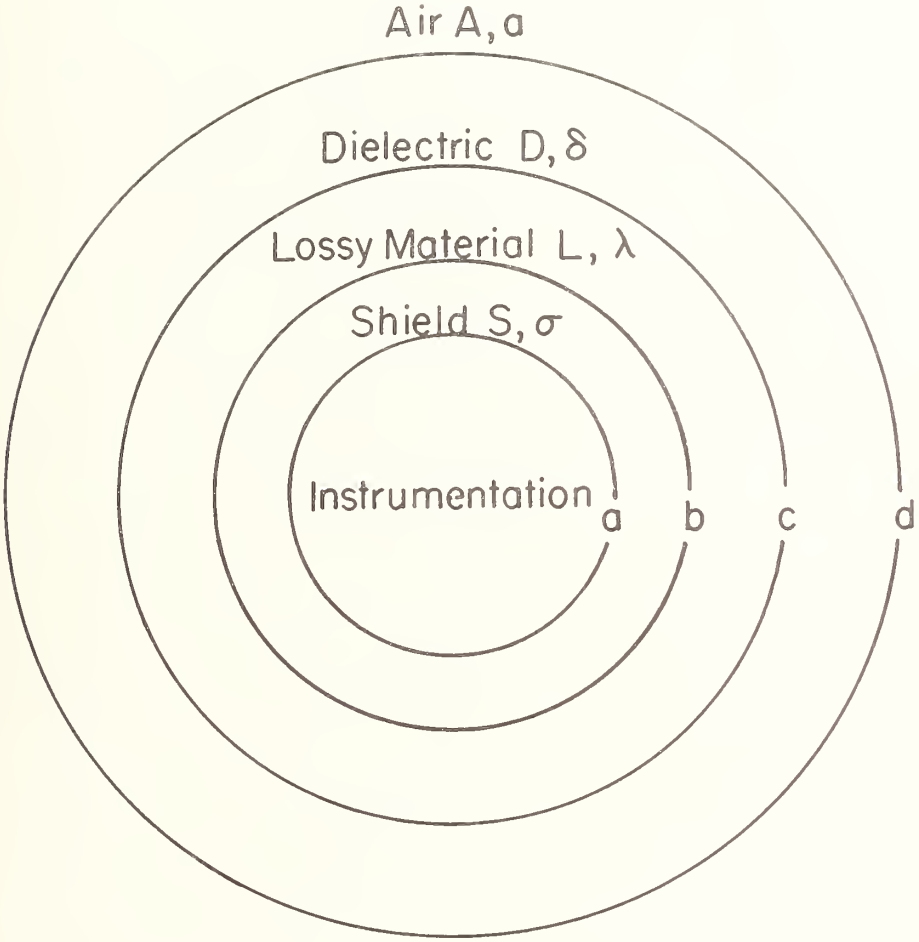

Figure 3. Cross Section of a Spherical Energy-Density Probe.

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