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The Iron Storage Technique for Application in Frequency Standards

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The Ion Storage Technique for Application in Frequency Standards

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THE ION STORAGE TECHNIQUE FOR APPLICATION IN FREQUENCY STANDARDS

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Abstract

The basic technique of trapping ions in a quadrupole configuration driven by a radio frequency field is described, following closely a discussion originally given by H. G. Dehmelt. Relationships between the trap size, ion oscillation frequency, frequency and amplitude of the rf driving field, depth of the potential well, and the storage capacity are given, and a possible trap configuration is numerically evaluated. The effects of collisions, rf heating, and the loss of ions from the trap are briefly discussed. The second-order Doppler effect is identified as the most serious accuracy limitation, whereas electric and magnetic fields as well as ion collisions are not expected to impose any accuracy limitation worse than 10^{-14}. The frequency stability performance of an ion storage device is estimated to be of the same order as for conventional atomic frequency standards.

The ion storage technique thus is seen as a promising candidate for a primary frequency standard if further experimental research leads to a control of the second-order Doppler effect. Technical difficulties are anticipated also in the continuous generation of ions and in their preparation for interrogation.

Key Words: Figure of merit; Frequency stability; Frequency standard; Ion collisions; Ion storage; Ion trapping.

THE ION STORAGE TECHNIQUE FOR APPLICATION IN FREQUENCY STANDARDS

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1. INTRODUCTION

At present the most accurate frequency standard is the cesium beam tube. This fact is widely recognized, and is reflected in the international agreement of 1967 which bases the definition of the second on the transition frequency of cesium. Naturally there is a constant demand for a more accurate realization of this definition. In addition frequency standards of extreme precision are needed in applications such as navigation and communication systems, air traffic control systems, and various special military systems.

Looking ahead it therefore appears both necessary and desirable to find ways and means to meet these demands. The cesium beam tube is already at a very advanced stage of technical development. An accuracy capability of the order of 10^{-13} seems to be a limit for cesium beam tubes which will prove difficult to surpass.

Alternative approaches are being sought which hold the promise of significantly exceeding the accuracy performance of cesium beam tubes (Hellwig, to be published 1970). They include the hydrogen maser (Kleppner, et al., 1965), atomic and molecular beam tubes such as thallium (Beehler and Glaze, 1966) and hydrogen storage (Hellwig, 1970), infrared and optical devices such as the methane saturated absorption (Barger and Hall, 1969), and the ion storage technique (Dehmelt, 1967).

The purpose of this report is to outline the basic concept of the ion storage technique, its shortcomings and advantages, and its potential in terms of frequency stability and accuracy.

2. THE ION STORAGE TECHNIQUE

An atomic frequency standard is based ideally on the electric or magnetic dipole transition of an unperturbed free atom observed for an infinite length of time. The atomic cesium beam tube and the atomic hydrogen maser represent attempts toward realization of these ideal conditions. In the case of the cesium beam, the ideal of the free atom is realized to a high degree; however, the observation time, i.e., the time of flight through the microwave cavity, is fairly short. Interestingly, in the case of the hydrogen maser, the reverse is true. These two devices represent different compromises.

In contrast to this the ion storage technique holds the promise of combining the virtues of both the cesium tube and the hydrogen maser, and even exceeding their performances with respect to approaching the ideal conditions. Trapping times far in excess of one second have been achieved experimentally as reported by Fortson, Major, and Dehmelt (1966) and by Dehmelt (1967). In the following equations (1) through (24) of this paper a discussion of the trapping technique is given which relies strongly on the survey article by Dehmelt (1967) in connection with papers by Fischer (1959) and Wuerker, Shelton, and Langmuir (1959). The notation of Dehmelt (1967) is used. For a more detailed derivation of some of the following equations the reader is referred to Dehmelt's paper (1967).

An ion with mass m and charge e, placed in an oscillating, homogeneous electric field of amplitude E_0, will perform harmonic oscillations with the angular frequency Ω of the field

$$m\ddot{z} = e E_0 \cos \Omega t \ . \qquad (1)$$

A solution

$$z(t) = \bar{z} + \zeta(t) \tag{2}$$

may be found where $\bar{z}$ is the center of motion, and $\zeta(t)$ is determined by

$$\zeta(t) = -\zeta_0 \cos \Omega t \quad \text{with } \zeta_0 = \frac{e E_0}{m \Omega^2} . \tag{3}$$

The ions are confined to a region of dimension ζ_0 if their center of motion does not move. This form of trapping is unstable; any initial velocity remains as a drift, and spatial confinement is not achieved.

However, if the oscillating homogeneous field is replaced by an oscillating inhomogeneous field, we can create conditions which allow spatial trapping due to the fact that a resultant force $\bar{F}$ will act on the center of motion. Without proof it is noted that the force $\bar{F}$ may be derived from an artificially defined pseudo-potential ψ

$$\frac{1}{e} \bar{F} = - \text{grad } \psi , \tag{4}$$

where ψ relates to the oscillating field by

$$\psi = \frac{e E_0^2(\bar{x}, \bar{y}, \bar{z})}{4 m \Omega^2} . \tag{5}$$

In order to effect trapping, we must require a force which always points to some center of confinement. Recalling the problems encountered in dipole optics, e.g., in the beam optics of an ammonia maser, we are led to a spatial quadrupole configuration as shown in figure 1. A hyperbolic ring R of radius r_0 is closed at both sides by hyperbolic end caps C_1 and C_2 separated by a distance $2 z_0$. The center of the trap is given by $(r, z) = (0, 0)$ in cylindrical coordinates. The electric potential of this

trap is described by

$$\phi = A(r^2 - 2z^2), \tag{6}$$

where the oscillating field causes

$$A = A_0 \cos \Omega t \ . \tag{7}$$

The electric field is -grad ϕ, and using eq (5) one obtains at the center of motion $(\bar{z}, \bar{r})$

$$\psi(\bar{r}, \bar{z}) = \frac{e A_0^2}{m \Omega^2} (\bar{r}^2 + 4\bar{z}^2). \tag{8}$$

The ions are confined with their center of motion in a potential well $\psi(r, z)$ as shown in figure 2. The depth of the potential well in the z direction (r = 0) is

$$\bar{D} = 4 z_0^2 \frac{e A_0^2}{m \Omega^2} \tag{9}$$

and in the r direction (z = 0)

$$\bar{D}' = \bar{D}\left(\frac{r_0}{2 z_0}\right)^2 \ . \tag{10}$$

From figure 2 and eqs (8) to (10) it is apparent that the center of motion $\bar{z}$ itself will not be at rest. It will oscillate harmonically in the r and z directions with the frequencies $\bar{\omega}_r$ and $\bar{\omega}_z$ according to the potential curves of figure 2. Inspection of eq (10) shows that the wells will have

identical depth for the configuration $r_0 = 2 z_0$. This case shall be assumed for all further discussion.* It warrants $\bar{\omega}_z = \bar{\omega}_r$, and thus prevents transfer of energy between the r-motion and the z-motion which could lead to escape from the trap (collision with the electrodes).

The oscillation of the center of motion may be written

$$\bar{z} = \bar{z}_0 \cos \bar{\omega}_z t \tag{11}$$

where $\bar{\omega}_z$ can be determined from

$$e \bar{D} = \frac{1}{2} m z_0^2 \bar{\omega}_z^2 . \tag{12}$$

The actual motion of the ion is a superposition of two oscillations, (a) the harmonic oscillation of $\bar{z}$ as described by eq (11), and (b) the oscillation $\zeta(t)$ according to eq (3) but with the important addition that the amplitude is no longer a constant but $\zeta_0 = \zeta_0(\bar{z}) = \zeta_0(t)$. We may combine these equations with eq (6) to determine the electric field strength. We obtain for the complete motion (in one dimension) $z(t) = \bar{z} + \zeta(t)$

$$z = (1 + \sqrt{2} \frac{\bar{\omega}_z}{\Omega} \cos \Omega t) \bar{z}_0 \cos \bar{\omega}_z t . \tag{13}$$

Without proof we note (Dehmelt, 1967) that these results (the validity of the pseudopotential approach) depend on the condition

$$\bar{\omega}_z << \Omega . \tag{14}$$

*Effective symmetrizing may also be achieved in the general case $r_0 \neq 2 z_0$ by simply adding a dc bias to the oscillating voltage.

Wuerker (1959) has noted that trapping is possible as far down as $\Omega > 2\bar{\omega}_z$, however, it is advisable to chose values of $\Omega > 10\bar{\omega}_z$ and to avoid any harmonic relationship.

Of considerable importance for the actual experiment is the determination of the required rf voltage amplitude V_0. The amplitude may be determined from eq (6)

$$V_0 = |\phi(z = z_0, r = 0) - \phi(z = 0, r = r_0)| = A_0(r_0^2 + 2z_0^2) . \quad (15)$$

With A_0 from eq (9) and $\sqrt{\frac{m}{e}}$ from eq (12) we may write

$$V_0 = \sqrt{2}\left[1 + \frac{1}{2}\left(\frac{r_0}{z_0}\right)^2\right]\frac{\Omega}{\bar{\omega}_z}\bar{D} . \quad (16)$$

For $r_0 = 2z_0$ eq (16) simplifies to

$$V_0 = 3\sqrt{2}\,\frac{\Omega}{\bar{\omega}_z}\,\bar{D} . \quad (17)$$

Assuming $\Omega \approx 10\bar{\omega}_z$ the required rf voltage amplitude is about 40 times the depth of the potential well.

3. STORAGE CAPACITY

The effective storage volume V may be approximated by a sphere of radius z_0,

$$V = \frac{4}{3} \pi z_0^3 \quad . \tag{18}$$

The maximum possible charge density ρ_{max} can be found from the Laplace equation

$$\Delta\psi = 4\pi \rho_{max} \quad . \tag{19}$$

Using eq (8) we calculate for the maximum ion density, $n_{max} = \frac{1}{e} \rho_{max}$,

$$n_{max} = \frac{3 \bar{D}}{4\pi e z_0^2} \quad . \tag{20}$$

The total number of ions which can ultimately be stored is then

$$N_{max} = \frac{z_0 \bar{D}}{e} \quad . \tag{21}$$

For a typical trap $\bar{D} \approx 10$ V and $z_0 \approx 1$ cm we obtain $N_{max} \approx 10^8$.

4. COLLISIONS, HEATING, AND ESCAPE FROM THE TRAP

The movement of the trapped ions is not of the random, Brownian nature but is ordered according to eq (13), resembling spatial Lissajou patterns (Wuerker, et al., 1959). However, the distribution of the amplitudes of the individual ion motions, e.g., their energies, will be random. The energy of one ion is given by the time average of its kinetic energy $\langle \frac{1}{2} m \dot{z}^2 \rangle$. The mean energy $\overline{W}$ of one ion in the trapped ion cloud corresponds to an effective ion temperature T_i which may be defined by ($k \equiv$ Boltzmann constant)

$$3 k T_i \equiv \overline{W}. \tag{22}$$

We note that the formal temperature T_i depends on various processes related to the trapping and may greatly differ from room temperature. The ions will collide with each other transferring motional energy. The time between collisions τ_c can be obtained from a formula for a uniform plasma given by McDonald (1957)

$$\tau_c = \frac{C \sqrt{A} \left(\frac{k T_i}{e} \right)^{3/2}}{n Z^4 \ln \Lambda} \tag{23}$$

where the constant C has the value $0.78 \times 10^7 \frac{s}{V^{3/2} cm^3}$. The symbols have the following meaning: A and Z $\equiv$ atomic weight and ion charge, n $\equiv$ ion density, and $\Lambda \equiv$ ratio of the cutoff distance for Coulomb interaction to that of closest approach. With A = Z = 1 (protons), $n = n_{max}$, and $\Lambda = z_0 / r_{min} \approx z_0 n_{max}^{1/3}$ one obtains for a typical trap ($\overline{D} \approx 10$ V, $z_0 \approx 1$ cm) and for $T_i \approx$ room temperature the value $\tau_c \approx 1$ ms.*

*Note that according to eq (23) for Coulomb type interaction, τ_c decreases as the temperature decreases.

This result indicates that ion-ion collisions occur rather frequently in a trap filled to its capacity. Certainly τ_c is much shorter than the storage time of ions.

We therefore assume that the ion energies are distributed according to a Maxwell function. Ions in the high energy "tail" of the Maxwell distribution are capable of leaving the trap. The loss rate $1/\tau_B$ of ions can be calculated to be

$$\frac{1}{\tau_B} = \frac{1}{2\tau_c}\left(\frac{e\bar{D}}{kT_i}\right)^2 e^{-e\bar{D}/kT_i} . \qquad (24)$$

This loss of ions represents not only a decrease in ion density but also a heat loss to the trapped ion cloud such that the mean ion energy decays as $\exp(-t/\tau_B)$.

This type of cooling is, of course, not very practical. A very promising method of extracting energy from the ions has been applied successfully by Church and Dehmelt (1969). The method follows directly from eq (13). An external lossy resonance circuit is coupled to the micromotion frequency $\bar{\omega}_z$ of the ions thereby damping their oscillation. Such cooling can be made very effective but is in competition with significant heating processes of which the predominant one is collision heating. Ions will collide with parent atoms or background atoms and transfer energy to them which is subsequently dissipated in randomizing collisions to the ion cloud. The ion, supposed to be at rest after such a collision (as an approximation), will pick up kinetic energy according to eq (13) and figure 2. This energy has to be supplied by the trapping rf field. The net effect is an energy delivery from the rf field to the ion cloud. In the case of collisions with parent atoms, charge exchange also has to be considered.

This collision heating can of course be avoided by working in a very good vacuum, or it can be reduced and even reversed (cooling) by selecting heavy ions and a background gas of light atoms in which case the light atoms will quickly transport energy away from the ion cloud. In order to get a feeling for the vacuum which is necessary to avoid collisions we may write down approximate, numerical relationships between the collision time τ_r and the particle density n and the pressure p

$$\tau_r = C_1 \frac{1}{p}; \quad n = C_2 p \ , \tag{25}$$

where the coefficients are approximately $C_1 \approx 10^{-5}$ Ns/m^2, $C_2 \approx 3 \times 10^{20}$ N^{-1}m^{-1}. For example a vacuum of 10^{-8} N/m^2 $\left(1 \text{ N/m}^2 = 7.5 \times 10^{-3} \text{ torr}\right)$ corresponds to a particle density of 3×10^{12} particles/m^3 and a collision time of 10^3 s, the actual values depending somewhat on the kind of gas under consideration. To obtain efficient cooling from a background gas of light atoms we must require that the collision time is much shorter than the storage time. This condition somewhat contradicts the free particle approach because a more efficient collisional cooling can be expected to introduce also larger perturbations.

5. EXAMPLE OF AN ION STORAGE DEVICE

For the design of an ion storage device the following parameters may be regarded as given: the ratio $\Omega/\bar{\omega}_z$, the dimensions r_0 and z_0, and the maximum number of ions to be stored N_{max}. One has to know the required depth $\bar{D}$ of the potential well, the amplitude of the rf voltage, V_0, and the oscillation frequency $\bar{\omega}_z$.

As before we shall assume $r_0 = 2 z_0$. We then obtain from eqs (12), (17), and (21) the following relationships

$$\bar{D} = \frac{e N_{max}}{z_0} ,$$

$$V_0 = 3\sqrt{2}\,\frac{\Omega}{\bar{\omega}_z}\,\bar{D} , \tag{26}$$

$$\bar{\omega}_z = \sqrt{2\,\frac{e}{m}\,\bar{D}}\,\frac{1}{z_0} ,$$

or, rewritten in terms of the given parameters,

$$\bar{D} = \frac{e N_{max}}{z_0} ,$$

$$V_0 = 3\sqrt{2}\, e\,\frac{\Omega}{\bar{\omega}_z}\,\frac{N_{max}}{z_0} , \tag{27}$$

$$\bar{\omega}_z = e\sqrt{\frac{2}{m}}\,\frac{\sqrt{N_{max}}}{\sqrt{z_0^{\,3}}} .$$

It is most interesting to note that the voltage amplitude V_0 required to store a given number of ions decreases as the trap size increases.* The same is valid for $\bar{D}$ and $\bar{\omega}_z$. This is rather important because lower voltages and larger trap sizes appear desirable from the engineering point of view. An actual trap configuration shall be calculated using eq (27). The parameters are chosen as follows:

$$\frac{\Omega}{\bar{\omega}_z} \approx 10; \quad z_0 = \frac{r_0}{2} = 10 \text{ cm}; \quad N_{max} = 10^8.$$

This leads (for He^4 ions) to

$$\bar{D} = 1.4 \text{ V}; \quad V_0 = 60 \text{ V}; \quad \frac{\bar{\omega}_z}{2\pi} \approx 80 \text{ kHz}.$$

*For a given ion density n_{max}, V_0 increases with the square of the trap size.

6. FREQUENCY ACCURACY

Since no actual device is discussed in this report, all possible uncertainties due to the method of interrogating the stored ions are excluded, such as cavity pulling, external magnetic fields, microwave spectrum, etc. Only those effects will be discussed which relate directly to the storage process.

6.1. Stark Effect

The Stark effect on a hydrogen-like atom has been calculated by C. Schwartz (1959). The fractional frequency shift of the hyperfine transition is given by

$$\frac{\Delta\nu}{\nu} = - \frac{14.91\,\varepsilon^2 a_0^4}{e^2} E^2 , \tag{28}$$

where E = electric field, a_0 = Bohr radius, e = electron charge, and ε = absolute permittivity. We may rewrite eq (28)

$$\frac{\Delta\nu}{\nu} = C E^2 , \tag{29}$$

with $C \approx -3.5 \times 10^{-21}$ cm^2/V^2. For the previously discussed trap configuration, where electric fields of the order of 10 V/cm are used, the corresponding Stark effect is clearly negligible. This feature, the near absence of shifts due to the containing fields, is the most interesting advantage of the ion storage method. Other known storage methods such as the coated storage bulb (hydrogen) or the buffer gas cell (rubidium) introduce fractional frequency shifts of 10^{-11} and more.

6.2. Ion-Ion Collisions (Pressure Shift)

The repulsive electrostatic force of the ions provides for collision parameters which are large enough to prevent van der Waals interaction. The only remaining interaction is a Coulomb interaction leading to Stark effect perturbation of the energy levels analogous to the one described in the preceding section 6.1. A rough estimate of the effect can be given by calculating the electric field experienced by one ion at the distance r_0 from another similar ion, where r_0 is the mean separation of ions in the trap. We can calculated r_0 from eq (20) using the relation

$$r_0 \approx n_{max}^{-1/3} \quad . \tag{30}$$

The corresponding average electric field seen by the ions is

$$E_0 = \frac{e}{r_0^2} \approx e\, n_{max}^{2/3} \quad . \tag{31}$$

With densities of the order of 10^6 ions/cm^3 we obtain $E_0 \approx 2 \times 10^{-2}$V/cm. According to eq (29) such fields will cause totally negligible effect.

6.3. Second-Order Doppler Shift

The second-order Doppler frequency shift is given by

$$\frac{\Delta\nu}{\nu} = -\frac{1}{2}\left(\frac{v}{c}\right)^2 , \tag{32}$$

where v = ion velocity and c = velocity of light. This frequency shift can not be compensated, it can be reduced only by lowering the ion velocity, that is, by ion cooling. However, we are not primarily concerned with the magnitude of the total shift, but with the uncertainty associated with this correction. The frequency accuracy of the ion storage device therefore depends critically on the ability to control and/or measure the mean ion velocity.

6.4. Shifts Due to Magnetic Fields

Various sources for magnetic fields exist in the ion storage device. (1) The driving electric rf field has a magnetic component. (2) The movement of ions corresponds to electric currents which cause magnetic fields. (3) An ion moving through the containing electric field experiences a motional magnetic field which is proportional to the vector product of the velocity vector with the electric field vector.

All these frequency shifts shall not be discussed in detail. They can be expected to cause only very small effects. In fact, as H. Schüssler and his colleagues have concluded (Schüssler, Fortson, and Dehmelt, 1969), they give rise to an uncertainty of less than 10^{-15}.

7. FREQUENCY STABILITY

The fractional frequency stability of a quantum electronic frequency standard may conveniently be written as

$$\sigma = \frac{K}{M} \frac{1}{\sqrt{\tau}} \tag{33}$$

The symbols in eq (33) have the following meaning: τ is the measurement time interval, the constant K contains the device characteristics, and M is a basic figure of merit* given by

$$M = \sqrt{n_s}\, T_r \tag{34}$$

where $T_r \equiv$ relaxation time, and $n_s \equiv$ flux of signal atoms. As an example we have for a beam tube (Cutler and Searle, 1966; Lacey, Helgesson, and Holloway, 1966)

$$K_T = \frac{0.388}{\sqrt{2\pi\nu}}$$

and for a maser oscillator (Shimoda, Wang, and Townes, 1956; Cutler and Searle, 1966)

$$K_M = \frac{1}{2\sqrt{2\pi\nu}} \sqrt{\frac{kT}{h\nu}} .$$

This illustrates the fact that for the same figure of merit M the operation of a device as a beam tube is superior to its use as a maser by the factor** (Hellwig, 1970)

$$\sqrt{2} \sqrt{\frac{kT}{h\nu}} .$$

*This M differs from the usual definition in the literature of a "figure of merit."

**This is only valid for frequencies $\nu < kT/h$.

The flux of signal atoms and the relaxation time which make up our basic figure of merit are not the only important parameters in determining frequency stability. The principle which is used in the interrogation of the atoms is also important and is reflected in the value of K. Using the same value of M, the ion storage device might be operated as a beam tube, as a maser, or using some other method. Its actual stability performance will then depend on the corresponding value of K.

Nevertheless it is of interest to compare the basic figure of merit of the ion storage device with those of existing devices. In order to do this, eq (34) will be rewritten introducing $\eta = n_s T_r$, the number of signal atoms during the period T_r.

$$M = \sqrt{\eta T_r} . \tag{35}$$

We shall now compare the cesium beam tube, a hydrogen storage device, and the methane saturated absorption device with the ion storage device. Two alternative assumptions are made: The optimistic assumption is based on a 100 percent use of the stored ions (every one of the stored ions can be excited and interrogated), the pessimistic one on values actually realized in the experiment.

Table I

Figures of Merit for Different Methods

	Cesium beam	Hydrogen storage	Methane saturated absorption	Ion storage Optimistic	Ion storage Pessimistic
η	10^6	10^{12}	10^5	10^8	10^3
T_r(s)	10^{-2}	1	10^{-5}	10^2	10
M($s^{1/2}$)	10^2	10^6	1	10^5	10^2

It can be concluded that the basic stability performance of the ion storage device may be expected to be somewhere between the cesium beam and the hydrogen storage devices.

8. CONCLUSIONS

We discussed the principles of ion trapping and some of the problem areas involved. It was concluded that the trapping process itself does not contribute significantly to frequency uncertainties. In fact the total influence of the involved static and dynamic electric and magnetic fields on the transition frequency was predicted to cause not more than a fractional frequency bias of 10^{-14}.

It was shown that the action of the trapping rf field can be described by a potential well with a finite depth and capacity. Typically the maximum capacity corresponds to a trapping of 10^8 ions. However, in order to avoid "boiling over" with its associated loss of ions and change of temperature, the trap should not be filled to capacity. An interesting feature is the decrease in the required driving voltage with an increase in trap size for a constant total number of trapped ions. The trap size is, of course, limited due to other requirements related to the microwave interrogation of the trapped ions.

The movement of the trapped ions which is correlated with the trapping rf field causes a second-order Doppler shift of the frequency which depends on the mean square velocity of the ions. Cooling of the ions, e.g., by coupling their motion to an external lossy circuit, is possible; however, various heating processes occur in the trap which are not well understood. The lowest effective temperature attained so far in an experiment was 900 K (Church and Dehmelt, 1969). The constancy of this temperature is poor, and its value cannot yet be measured very precisely. An optimistic assumption for the accuracy in temperature determination would be ±50 K. This leads to a second-order Doppler shift uncertainty of approximately $\pm 2 \times 10^{-11}$ for helium ions and $\pm 1 \times 10^{-13}$ for mercury ions.

In order to reduce the main source of heating, the collision processes, an excellent vacuum has to be maintained, preferably below 10^{-8} N/m^2 ($\approx 10^{-10}$ torr). This somewhat contradicts the requirement for efficient ionization of atoms and preparation of the ions for interrogation where higher particle densities would be desirable.

A distinct problem area is the interrogation of the ions, or in other words, the design of an operating frequency standard (Dehmelt, 1969). The discussion of these aspects could be the subject of a separate report. Here we shall only mention that present techniques of ionization and excitation are based on collisions with electrons and a polarized atomic beam (e.g., cesium), respectively. Obviously these techniques allow only a pulsed operation. Although this does not preclude a working frequency standard, continuous operation is highly desirable. A solution might be optical pumping for the excitation and chemical reactions for the ionization where neutral particles drift into the trap region with subsequent reactions leading to ions. However the efficiency of such methods appears to be fairly low.

In summary, one concludes that the main advantage of ion storage is the ability to realize long interaction times without perturbing the microwave transition significantly as compared to other storage techniques like the coated bulb for atomic hydrogen where a moderate amount of perturbation occurs. As was pointed out in the preceding section 7, however, the short term frequency stability cannot be expected to be very high; but good slave oscillators might render this aspect unimportant. The most severe problems are those of the determination of the effective temperature (second-order Doppler shift) and its constancy, and the techniques to be used for creating and interrogating the ions.

Solutions are to be found in fairly extensive research programs, the outcome of which can not yet be predicted.

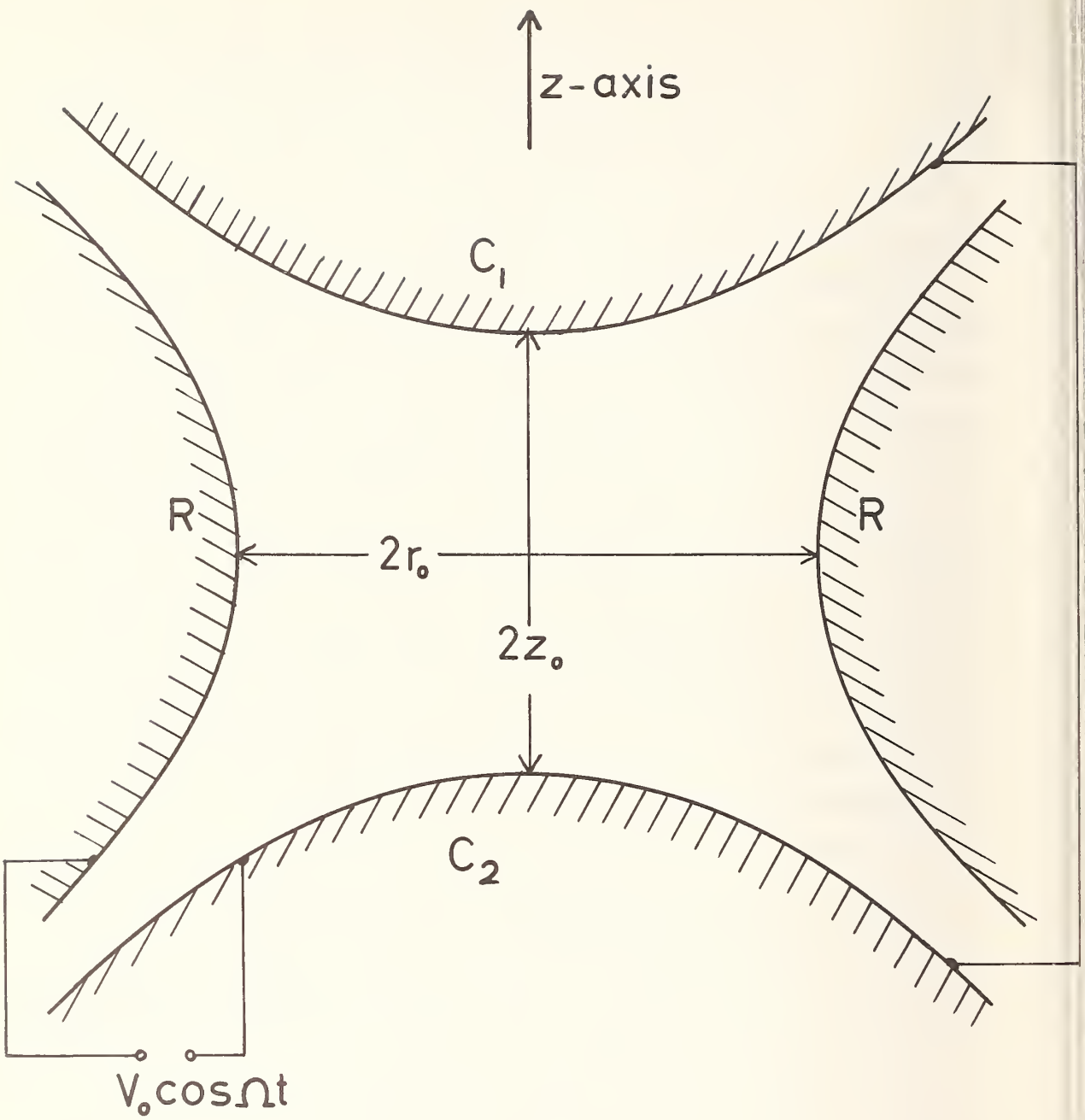

Figure 1 Schematic configuration of an ion trap

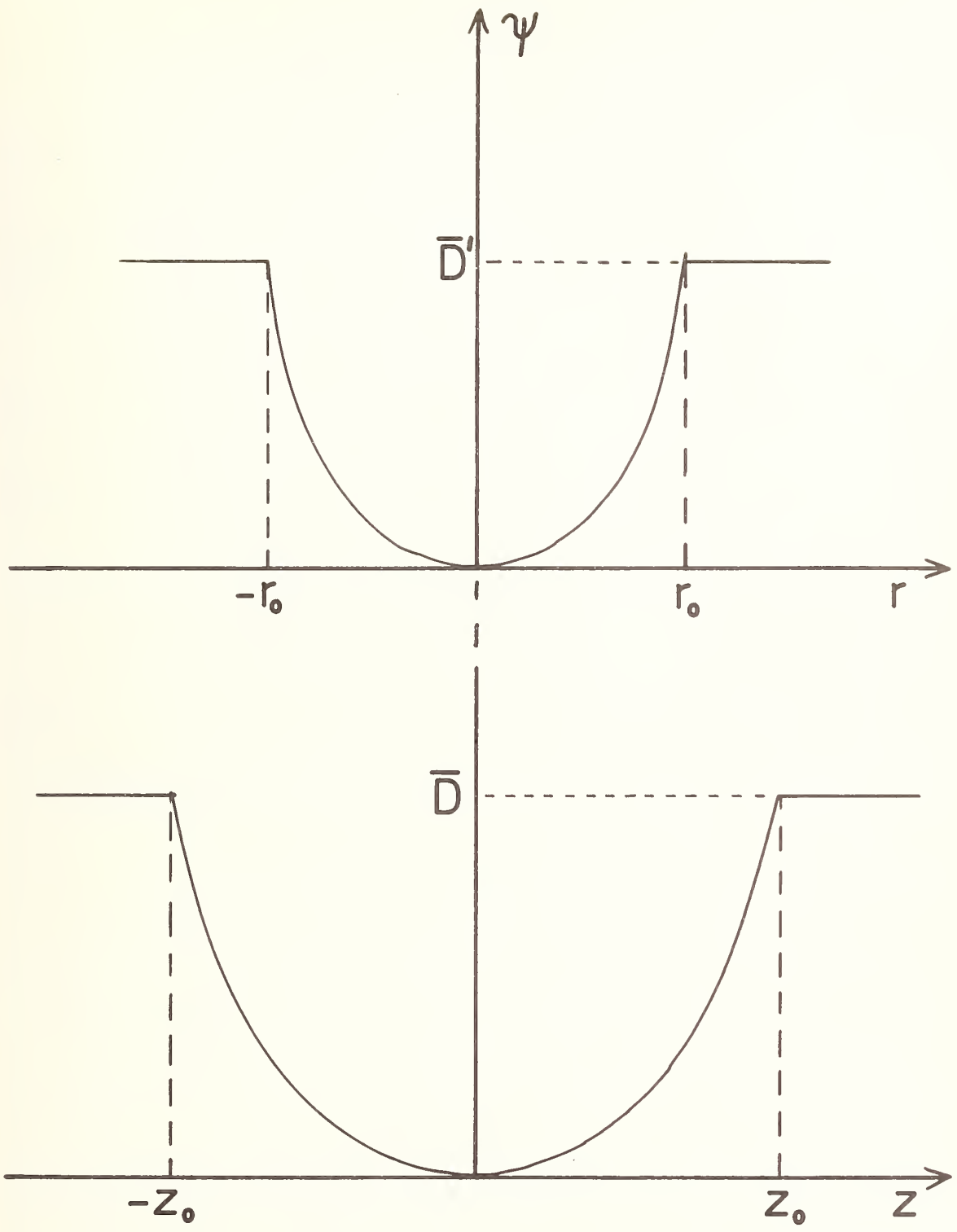

Figure 2 Potential curves of the ion trap (cylindrical coordinates)

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