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Computer Solutions for Thermal–Acoustical Oscillations In Gas–Filled Tubes

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COMPUTER SOLUTIONS FOR THERMAL–ACOUSTICAL OSCILLATIONS IN GAS–FILLED TUBES

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COMPUTER SOLUTIONS FOR THERMAL-ACOUSTICAL OSCILLATIONS IN GAS-FILLED TUBES

M. T. Norton and R. C. Muhlenhaupt

A digital computer program to determine solutions for thermal-acoustical oscillations in gas-filled pipes is described. Using a typical temperature gradient and tube length from test data, the program calculates the effect of changes in heat transfer coefficient, friction factor, and pipe diameter. Details of the program are explained. A comparison is made between computation and one particular test datum point. Although the calculations are based on helium as the media, the program can accommodate any fluid treated as a perfect gas.

Key Words: Cryogenics, thermal oscillations, liquid helium

1. Introduction

When the open end of a pipe is at low temperature and the closed end is much warmer, thermal-acoustical oscillations often occur. These oscillations are common in cryogenic vent lines, pressure sensing lines, and liquid transfer lines. This study is an investigation of this usually undesirable phenomenon. The result is a digital computer program to predict frequencies and overpressure amplitudes from pipe diameter, temperature gradient, and gas properties. One test datum point is used to establish the relation between text book friction factors and friction level in an oscillating pipe.

Curves are presented showing the effect of pipe diameter, friction factor, and heat transfer coefficient on the magnitude of the oscillations.

Although the program is specifically for helium gas, minor changes can adapt it to other gases.

The calculations for frequency and amplitude are performed by quite different methods. The frequency depends on acoustic velocity,

and acoustic frequencies in general are essentially independent of amplitude. So it is assumed that in thermal oscillations, the frequency is independent of amplitude and, for a given gas, depends only on the temperature gradient. For convenience, when determining frequency, an initial pulse of pressure is assumed.

The amplitude depends on the frequency, but the method of calculating is not the same as that for frequency. In amplitude calculations various assumptions are made in order to approximate the shape of the pressure profile.

2. General Theory of Amplitude Calculations

There are infinite varieties of duct configurations which can produce oscillations when inserted into a cryogenic dewar. Of these the simplest and most common case is a constant diameter tube, closed at the warm end, and open at the cold end. This is the same as a dewar vent line with the shut off valve closed, or a pressure measurement line. It is well known that these are subject to thermal oscillations. The general theory applicable to this case is applicable to all cases.

The problem is not new nor is it confined to cryogenics. Glassblowers frequently experience it. A description of the phenomenon and a qualitative explanation is found on page 230 of Rayleigh (1945).

A qualitative explanation of thermal oscillations, fundamentally in agreement with Rayleigh, follows. During the pressure rise portion of an oscillation cycle there is movement of gas from the open towards the closed end. As a gas element travels, its pressure increases and, from adiabatic compression, its temperature rises. The wall temperature increases toward the closed end, and the gradient is steeper than the temperature gradient in the gas. Therefore, there is a temperature difference tending to transfer heat from the wall to the gas. Because heat

transfer requires time, first the pressure increases and then heat is transferred. After the pressure in the element has peaked, the process is reversed. Then there is a series of pressure drops and heat transfers out of the gas. When plotted on temperature-entropy coordinates this sequence of events has positive work area, a simplified version of which is shown on figure 1. Thus a gas element in contact with a wall, whose temperature is greater at the closed end, and whose gradient is steeper than the adiabatic gas temperature rise, does work on its surroundings. If each gas element is in contact with this type of temperature gradient then, clearly, work is done by the gas elements in the pipe. Conversely, any gas element in contact with a wall, whose temperature gradient is reversed or is less steep than adiabatic compression, will have work done on it. Furthermore, if all gas elements are in contact with a gradient such that work is done by all elements, it is clear that the motive power exists for continued oscillations. Likewise, if all gas elements are in contact with a gradient such that work is absorbed by all elements, it is clear that any accidental oscillations will quickly damp out. For the intermediate case, where the wall temperature gradient sometimes causes work to be done by, and sometimes on, the various gas elements, it is not clear if chance oscillations damp out or continue. A digital computer program can be used to sum up all the infinitesimals of work done in one cycle in all the gas elements in the pipe.

Thus, in an oscillation cycle, positive work is a necessary criterion for the existence of thermal oscillations. If the work summation is positive, any accidental oscillations build up to an equilibrium steady state condition or until heat transfer from the duct wall alters the temperature gradient, in which case the oscillations may cease. If the work summation is negative, any accidental oscillations quickly damp out.

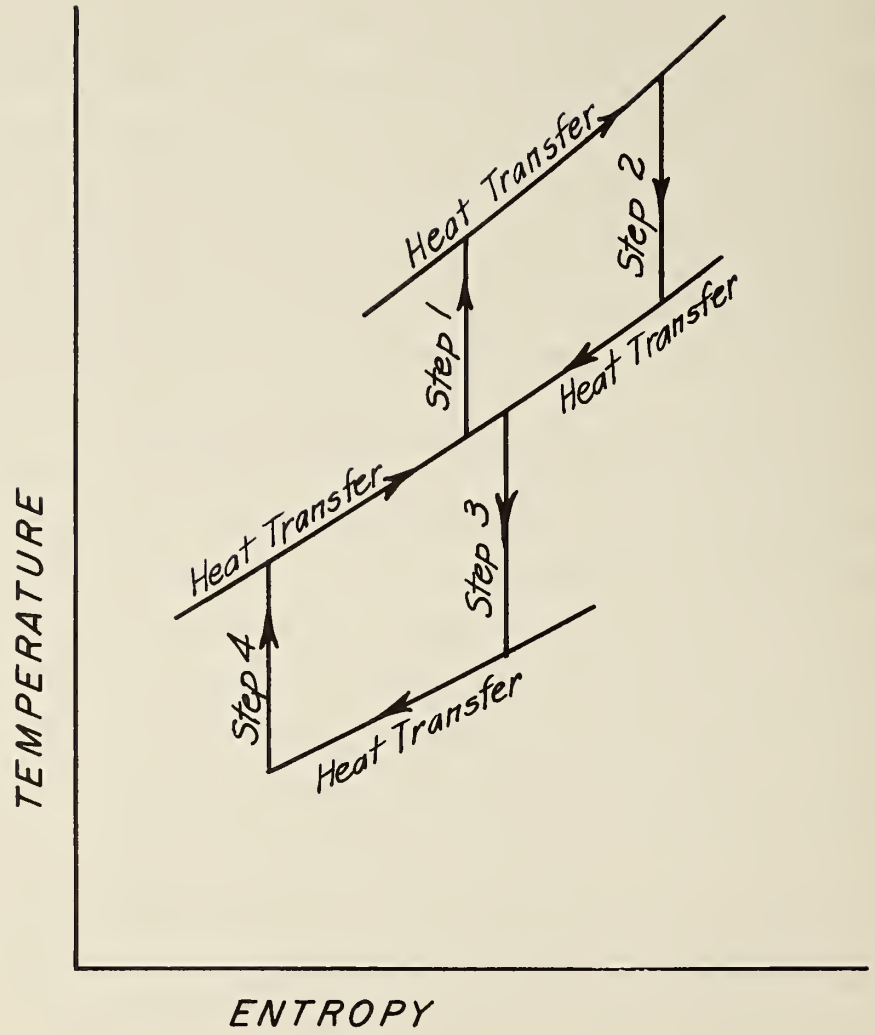

Figure 1. Work Cycle Of Simplified Thermal Oscillation Cycle

In case the work summation is positive, at equilibrium it must exactly balance losses. There are two types of losses, friction and kinetic energy, expelled into the dewar. Part of the friction loss is simply wall friction opposing the gas motion in either direction. Also, when the gas enters the pipe from the dewar there is turbulence created which is equivalent to additional wall friction. In this study the entrance loss will be handled as an additive to the wall friction.

In one pressure cycle each element will undergo heat transfer and will do some positive or negative work. As each element slides along the wall or into the duct, there is frictional return of work to heat in the gas element. After summing up for all elements in one cycle, the excess of positive work done over frictional work must equal the kinetic energy ejected into the dewar during one cycle.

This criterion, that the net work done after subtracting friction must equal the kinetic energy ejected, is used in an iterative calculation to determine the pressure amplitude of helium gas in a given pipe with a given temperature gradient. The method used is to estimate the pressure amplitude and calculate for all gas elements in the duct, the work done and the friction and kinetic energy ejected. If the net work exceeds the kinetic energy, the estimate is low. If the net work is less than the kinetic energy, the estimate is high. With a few iterations it is possible to determine the pressure amplitude for which the net work equals the kinetic energy ejected. This is the calculated pressure.

Figure 2 illustrates the method used. The particular pipe diameter under investigation is .3175 cm (1/8 inch). In this illustration, pipe friction is calculated using friction factors that remain constant during each run, and heat transfer is computed using unmodified heat transfer coefficients from McAdams (1954). When the overpressure estimate is low, the ratio of ejected kinetic energy to work done is less than unity. When

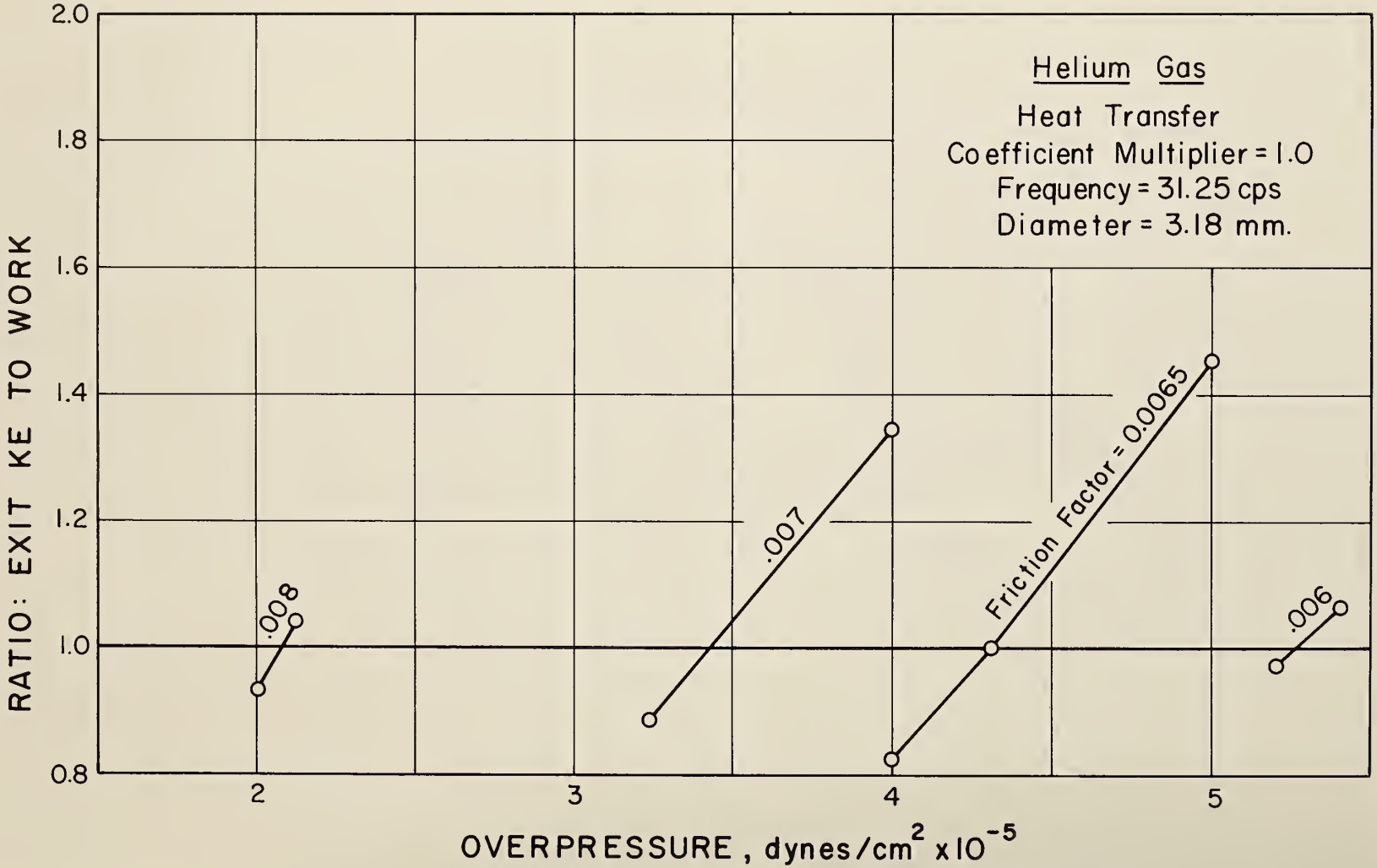

Figure 2. Typical Amplitude Determination

the estimate is high,the ratio exceeds unity. Numerical interpolation helps determine the final calculated overpressure.

3. General Theory of Frequency Calculations

Except for temperature gradient and heat transfer effects, the pressure pulses of thermal oscillations are the same as those of water hammer. As described by Rich (1951), one common case of water hammer involves pressure pulses traveling in a pipe with its open end in a large reservoir and its closed end at a valve. In that case the initial condition is liquid flowing in a long pipe from a reservoir through a valve to equipment of some sort or to discharge. If the valve is suddenly closed, there is a sequence of four steps which cyclically repeat themselves until damped out by friction. These steps are as follows.

Step one starts just after the valve closure. Liquid velocity in the immediate vicinity of the closure is transformed into pressure. This pressure pulse travels to the open reservoir end at sonic speed. When it arrives at the reservoir, the pipe is momentarily filled with stationary liquid whose pressure is higher than that in the reservoir. The amount of pressure rise in step one is given by

$$\Delta P = C\rho |U|$$

where

ΔP = pressure rise - - - - - - - dynes/cm^2

C = sonic velocity - - - - - - - cm/sec

ρ = fluid density - - - - - - - - g/cm^3

$|U|$ = magnitude of fluid velocity- cm/sec.

Step two starts just when the overpressure wave of step one reaches the reservoir. The overpressure at the open end drops to reservoir pressure. In the process it acquires a velocity equal to its

original velocity but in the opposite direction. This reverse velocity and reservoir pressure wave travels from the reservoir to the closure at sonic speed. When it arrives at the closed end the pipe is momentarily filled with liquid at reservoir pressure, whose velocity is everywhere the reverse of the original.

Step three starts as soon as the reverse velocity wave arrives at the closed end. Liquid velocity in the immediate vicinity of the closure is transformed into pressure drop. This reduced pressure pulse travels to the open reservoir end at sonic speed. When it arrives, the pipe is momentarily filled with stationary liquid whose pressure is lower than that in the reservoir. The amount of pressure drop is the same as the pressure rise of step one.

Step four starts just when the underpressure wave of step three reaches the reservoir. The underpressure at the open end rises to reservoir pressure and acquires the original velocity. This pressure and velocity travels from the reservoir to the closure at sonic speed. Momentarily at the end of step four the liquid pressure and velocity are the same as they were at the beginning of step one.

After step four the process repeats itself until friction damps out the oscillations.

Except for the effects of heat transfer and gas temperature gradient, the process is similar for thermal oscillations. Because of liquid evaporation or other causes, a small pressure rise occurs in the dewar. Momentarily the tube is filled with stationary gas whose pressure is lower than that in the dewar. This is the same situation that exists in the pipe in the water hammer case at the end of step three. Again ignoring the effects of heat transfer and gas temperature gradient, the thermal oscillations follow steps four, one, two, three of the water hammer case.

If thermal oscillations followed the foregoing sequence of events exactly, a comparatively simple theory of heat transfer from the warm regions of the tube to the dewar could be derived. Even though the actual case is more complicated, an examination of the simple case is instructive.

During step four every particle of gas in the tube is pressurized a small amount and travels a short distance into a warmer region of the tube. Some heat is transferred after pressurization. During step one the gas is pressurized once more and again moves into a still warmer region. More heat is transferred. During step two the gas is depressurized and moves into a cooler region. Some heat is transferred. Once equilibrium has been obtained the final condition at the end of step three and the original condition at the start of step four are identical.

This chain of events is shown on temperature, entropy coordinates on figure 1.

A fluid which follows the path shown must do work on the surroundings. Part of the work done is that required to overcome wall and internal gas friction to maintain the process. The balance of the work done is expended in expelling gas from the tube to the dewar. This gas expelled into the dewar contains the energy that increases the liquid evaporation rate.

If the behavior of a thermal oscillation were exactly the same as that of water hammer, the frequency could be determined by dividing the tube into regions of known temperature, determining the average sonic velocity in each region, and calculating the time required for a sonic pulse to travel from one end of the tube to the other. The period of the oscillation by this method would be the time required for the pulse to traverse the tube four times, and the frequency would be the reciprocal of the period. This method was applied to the data of Bannister (1965). As is shown in table I, the method is not sufficiently accurate.

TABLE I

Using Temperatures of Figure 2

Position Inches	Length cm	Temp. °K	Sonic Vel. cm/sec	Time sec
0 - 8	20.3	4.3	12.2×10^3	1.664×10^{-3}
8 - 16	20.3	6	14.4×10^3	1.411×10^{-3}
16 - 24	20.3	12	20.38×10^3	$.999 \times 10^{-3}$
24 - 32	20.3	90	55.8×10^3	$.364 \times 10^{-3}$
32 - 40	20.3	242	91.5×10^3	$.222 \times 10^{-3}$
40 - 48	20.3	295	101.0×10^3	$.201 \times 10^{-3}$
Time for one traverse				4.861×10^{-3} sec
Period				0.01944 sec
Frequency calculated				51.4 cps
Frequency measured				34 cps

Since the calculated frequency of table I does not agree with measurement, it is evident that the method is inadequate. The reason is simply that a temperature gradient exists in the pipe and causes pressure wave attenuations and reflections. Therefore, the peak pressure at any point is a summation of multitudinous reflections and does not necessarily advance at sonic velocity, even though the first warning of a pressure pulse does travel at sonic speed. To calculate frequency it is found convenient to divide the pipe into a large number of constant temperature regions, assume the initiation of a small pulse in the dewar, and calculate the resulting pressure versus time throughout the pipe from equations derived in Appendix A.

In the derivations, all pressures are considered as absolute. However, since the derived expressions show pressure differences, it is often possible in the digital computer program to ignore the absolute pressure level and to consider only over or underpressure. Of course, in determining gas densities, the absolute pressure must be used.

These derived relationships have been programmed for calculation on a high speed digital computer. As a first operation the program divides the pipe into gas elements such that each has an equal sonic traverse time. The number of these elements may be varied. The length of time required for sound to traverse one tube length is one calculation time. Temperature gradients are obtained from Bannister (1965). The original pressure level is an input variable. The amplitude of the assumed pressure pulse is arbitrarily assumed to be 1000 dynes/cm^2, although other values may be used. For as many calculation times as desired, the program calculates and prints gas velocities and pressures for each of the N elements. By examination of the printout it is possible to determine the pulse frequency. The program presented herein determines the period between each of the first 10 pressure minimums thereby obtaining 9 periods, and 9 corresponding frequencies. The average of these 9 frequencies is then considered to be the true frequency. Minor modifications can increase the number of individual periods and frequencies averaged.

4. Description of Amplitude Calculations

For amplitude calculations some assumptions were made as to the nature of the pressure oscillations. It was noticed during frequency calculations that even a step pressure input resulted in almost sinusoidal pressure variation at the closed end. Furthermore, all peak pressures tended to occur at the same time, although the amplitude varied from end to end. From oscilloscope observations it has been noticed that the oscillations tended towards sinusoidal. Therefore, it is assumed that at any position in the pipe the pressure oscillation is sinusoidal. The end to end variation of the maximum overpressure is obtained from a simple cosine curve and the fact that at the open end there is a pressure node at which the maximum overpressure is always zero.

For amplitude determinations the duct is divided into a large number of elements, all of which have the same length. To account for gas entering and leaving the pipe an additional quantity of elements is assumed to exist in the dewar. Thus, there are N elements in the pipe and M elements altogether. The period of one cycle is divided into 360 degrees. One time interval is taken to be a small fraction of the total. Usually, a 5 degree time interval is selected as being small enough for accuracy without requiring excessive computer cost.

Because of pressure change in the element nearest the closed end, element one, its midpoint will oscillate back and forth in the duct. Its temperature will change due to compression and heat transfer. The point on the wall in contact with the midpoint will change with the movement, resulting in temperature difference and heat transfer. There also will be density and position changes that are somewhat different from an adiabatic case.

Initial approximate temperatures and densities are assumed. After heat transfer has been calculated repeatedly around one complete cycle, conditions are different than initially assumed. However, after a sufficient number of calculation cycles, equilibrium is obtained. Then the position of the cold end of element one is a series of points varying in a cycle, but invariant from one cycle to the next.

This final series of positions of the cold side of element one is the permanent series of positions of the warm side of element two. Initial approximate temperatures and densities are assumed in element two followed by the same type of heat transfer calculations that were performed for element one. After numerous cycles of calculations, a series of positions is obtained for the cold side of element two. These are the permanent series of positions of the warm side of element three.

In general this same series of calculations is performed for each of the M elements. In each time interval the heat transfer, DQ, and the infinitesimal work done, DWK, are computed. The summation of these quantities during the final cycle gives the net heat transfer into,and the net work done by,the I'th element, Q(I) and WORK(I). Because of the oscillation, gas is continually entering and leaving the duct. As a result there are three types of elements. One type, like element number one, is entirely inside the duct both at the beginning and at the end of a time interval. A second type is partly inside the duct and partly in the dewar, either at the beginning or at the end of the time interval, or both. The third type is out of the duct and inside the dewar at all times.

For the first type, the calculations, though involved, are relatively straightforward. For the third type, practically no calculations are needed. For the second type it is necessary to divide the gas element into a section inside the pipe and a section outside the pipe, at the beginning and at the end of the time interval. As a result there are three routes through the amplitude program, one for each type of calculation.

Using Fortran notation the cosine variation of overpressure with distance along the pipe is expressed as,

PX(I) = PM*COSF(PI/2. *(XAVG(2)-XX(1)/XL)

where

PM = Maximum overpressure at closed end - dynes/cm^2

PI = 3.1415926536

XAVG(2) = Position of element center - cm

XL = Length of duct - cm

PX(I) = Maximum overpressure in element (I) at position XAVG(2) - dynes/cm^2

XX(1) = Position of closed end of duct - cm.

The above relationship holds if (XAVG(2)-XX(1)) is less than XL. Otherwise PX(I) becomes zero.

The momentary overpressure at the end of a given time interval in the cycle is

$$P(2) = PX(I)*SIN2$$

where

P(2) = overpressure at end of interval - dynes/cm^2

SIN2 = the sine function of the fraction of the cycle completed at the end of the interval.

One of the primary variables calculated during each time interval is the infinitesimal work done. The obvious way to make this calculation is to multiply the average pressure during the interval by the volume increase during the interval. In Fortran notation this is

$$DWK = PRESS*(VOL(2)-VOL(1))$$

where

DWK = infinitesimal work done in the interval - ergs

PRESS = average absolute pressure in the interval - dynes/cm^2

VOL(2) = element volume at end of the interval - cm^3

VOL(1) = element volume at beginning of the interval - cm^3.

An instability in the program was finally traced to this method of computing work. An equivalent expression, derived in Appendix B, was finally used. The equivalent method produced infinitesimal work terms that at equilibrium were the same as those of the original method with an error less than one part in 5000, and the instability was largely eliminated. This method of calculation, in Fortran notation, is

DWK = (XK-1.)/XK*DQ-VOLM*(P(2)-P(1))/XK-DFR/XK

where

XK = specific heat ratio

DQ = infinitesimal quantity of heat transferred in a time interval - ergs

VOLM = average element volume during interval - cm^3

P(2) = element overpressure at end of time interval - $dynes/cm^3$

P(1) = element overpressure at beginning of interval - $dynes/cm^3$

DFR = infinitesimal frictional energy loss during interval - ergs.

Kinetic energy is calculated in two ways. One method is to determine the infinitesimal kinetic energy in a given element every time the velocity is towards the dewar, that is to say when it is positive. This is done with the equation

DKE = 0.5*(VEL*AF*ROE*DT)*VEL*VEL

where

DKE = infinitesimal kinetic energy - ergs

VEL = gas velocity - cm/sec

AF = pipe cross section area - cm^2

ROE = average density - g/cm^3

DT = time interval - sec.

The terms in the parentheses are the mass of gas moving past a point during the interval, so the equation is the usual one in which kinetic energy is one half the product of the mass and the velocity squared.

By summing up the DKE for each time interval, the dewar bound kinetic energy in one cycle for each element, ZKE(I), is found. The

kinetic energy of the N'th or M'th elements, ZKE(N) or ZKE(M), should be representative of the kinetic energy expelled into the dewar.

A more correct value of the kinetic energy expelled into the dewar is that at the exit from the duct. During one cycle, during each time interval, some one element is partly in and partly out of the duct. By summing the infinitesimal amount of kinetic energy of these elements, a close value of the kinetic energy expelled into the tank is obtained. The equation used is

$$EKE(J) = 0.5*(VEL*AF*ROE*DT)*VEL*VEL$$

where

EKE(J) = infinitesimal kinetic energy at the duct exit during the J'th time interval - ergs.

Kinetic energy is obtained in the program by both methods. Approximately the same summations are obtained from each method. The values obtained from summation of EKE(J) are used because they more closely reflect actual operation.

The program for calculating amplitude is called THM LOS. Since this program does nothing except call the subroutines, READ DATA, ELEMENT, and AMPLITUD, there is no need for discussion of it, but there follows a detail description of the subroutines.

5. Detail of Subroutine READ DATA in Amplitude Calculations

This subroutine is used to read information into the program and to calculate certain derived quantities. Most of the input information is printed out to show the calculation conditions. In the following description no explanation is made of Fortran statements which merely have to do with the mechanics of the digital computer program such as COMMON, TYPE REAL, or FORMAT statements. Similarly no explanation

is made of statements which themselves are merely explanations. Subroutine READ DATA is given in Appendix E.

In line 5 a number of quantities are read which have the following meanings:

PSTART	=	dewar pressure - dynes/cm^2
DIAM	=	the duct diameter. (This is read in inches and converted to centimeters)
FRF	=	Fanning friction factor
FACTOR	=	a multiplier used if desired to alter text book heat transfer coefficients
AVGFREQ	=	the frequency of the oscillations - cycles/sec
PM	=	the assumed maximum overpressure - dynes/cm^2
N	=	the number of elements in the duct
NJJ	=	the maximum number of iterations permissible
NFR	=	a multiplier used if desired to alter text book friction factors
JD	=	the number of degrees in one time interval. (JD can have the values 1, 2, 3, 4, 5, 6, 8, 9, 10. The value 5 has usually been used. One cycle is 360 degrees, so JD is a fraction of a cycle).

Since the program determines the oscillations that result from a given temperature gradient, temperature versus duct positions are needed. This can be from test data, or from an arbitrary gradient to determine its oscillation characteristics. In line 10 the temperature, TMP, in degrees Kelvin is read for each of 32 duct positions, S, which are read in inches.

Viscosity of the gas, VIS, in micropoise is read at 16 temperatures, TMRVIS, in degrees Kelvin at line 15.

Thermal conductivity, CONGAS, in milliwatts/cm-°K is read at 12 temperatures, TCONGAS, in degrees Kelvin at line 20.

Viscosity and conductivity are obtained from Johnson (1960) either directly or by extrapolation.

To determine natural convective heat transfer coefficients, a table of Nusselt number versus the Grashof-Prandtl product is needed. The $\log_{10}$ of the Nusselt number, ZNU, is read at 16 values of the $\log_{10}$ of the Grashof-Prandtl product, GRPR, in line 25. Figure 7-10 from McAdams (1954) supplied 13 of these values. The two largest values used, not obtainable from figure 7-10, were obtained from figure 7-7 of the same reference. Since the Nusselt number goes to zero if the Grashof-Prandtl product goes to zero, it was desired to approximate this condition. To do so the $\log_{10}$ of the Nusselt number is set to -37 when the $\log_{10}$ of the Grashof-Prandtl product is -37.

In lines 30 and 40012 the duct positions, S, and the duct diameter, DIAM, are converted from inches to centimeters.

In lines 35 and 40011 the viscosity, VIS, is converted from micropoise to poise.

In lines 40 and 40010 the conductivity, CONGAS, is converted from mw/cm-°K to erg/sec-cm-°K.

In line 45 the molecular weight of helium, WT, the universal gas constant, RU, and the value for PI are entered. From these the helium gas constant, R, and the cross section area of the duct, AF, are calculated. In line 50 the specific heat ratio, XK, is entered, and the specific heats, CPG and CVG, are calculated. In line 55 the temperature of the gas in the dewar just outside the duct, TDEW, is entered and the density, RHODEW, is calculated. The quantity, M, is also calculated. Without oscillations there are N gas elements in the duct. With oscillations,

elements or portions of elements enter the duct from the dewar and enter the dewar from the duct. To keep track of all these elements an additional quantity of elements is considered. Thus, M represents a number of elements some of which are always in the duct, some are partly in and partly out of the duct, and some are always in the dewar.

The remainder of the subroutine READ DATA is used for printing out the input information. Computational units are printed except in the case of viscosity, VIS, where convenience and space cause it to be printed in micropoise rather than poise.

The input value of the Fanning friction factor, FRF, is not printed in READ DATA. As explained later, the input value is subject to change. The range of values actually used is printed from subroutine AMPLITUD.

6. Detail of Subroutine ELEMENT in Amplitude Calculations

The subroutine ELEMENT, when used with amplitude calculations, divides the duct into N elements and M elements total in the duct and dewar. All elements are of the same length, DXX. The position of each boundary, XX, the wall temperature at each boundary, TEMP, the initial average temperature of each element, TMG, the initial average density of each element, RHO, the mass of gas in each element, GM, the initial mid-point of each element, XMID, are also calculated in ELEMENT and printed. These are all initial or non-oscillating conditions. This sub-routine, ELEMENT, is given in Appendix F.

In line 5 the warm boundary temperature of the first element, TEMP(1), is set equal to the temperature read in at that point, TMP(1). Similarly the warm boundary, XX(1), is set equal to S(1). The quantities NP1 and MP1, needed later, are set equal to N+1 and M+1. Line 15 sets the cold end boundary, XX(I+1), of elements 1 through N. Lines 20 through 33 compute the temperature, TEMP(I+1), corresponding to each position,

XX(I+1). Lines 35 and 40 calculate the average initial element temperature, TMG(I), the average initial density, RHO(I), the initial average position, XMID(I), and the mass of gas in each element, GM(I). Lines 45 through 60 compute the same quantities for elements N+1 through M. The remainder of the program prints out the values of these quantities.

7. Detail of Subroutine HEAT TR

When called by subroutine AMPLITUD, subroutine HEAT TR calculates the friction factor, FRF, the Reynolds number, REYNO, the heat transfer coefficient by forced convection, HHT1, the Grashof-Prandtl product, GP, the heat transfer coefficient by natural convection, HHT2, the total heat transfer coefficient, HHT, and the heat transferred, dQ. When it is desired not to calculate the friction factor but to use a constant input friction factor, the subroutine omits calculation of friction factors. Subroutine HEAT TR is given in Appendix G.

In line 1, if the variable, ITESTFRF, has been set equal to unity in subroutine AMPLITUD, only friction factors are desired and the program jumps to line 386.

Lines 2 through 3 calculate the momentary conductivity, CONG, of a gas element from the momentary element temperature, TAVG, by linear interpolation in tables which have been read in the program.

Lines 4 through 5 calculate the momentary viscosity, VISC, of the gas element from the momentary element temperature, TAVG, also by linear interpolation.

At line 6, if the viscosity is zero, an error exists and the program is terminated.

In line 8 the Reynolds number, REYNO, is calculated using variables determined in AMPLITUD. Since a Reynolds number is always positive, its absolute value is used.

In line 10 the lowest possible value of the Reynolds number is set to 0. 001 to avoid the possibility of division by zero. Furthermore, when the Reynolds number is less than 0. 001 forced convection is usually negligible. Friction may be appreciable, but this assumption merely forces the friction factor to a single high value when the Reynolds number would be below 0. 001.

In line 22, if the conductivity of the gas is negative or zero, an error exists and the program terminates.

In line 25 the Prandtl number is calculated. If it is zero or negative, an error exists and line 26 initiates program termination.

In line 30, if the momentary gas temperature, TAVG, is negative or zero, an error exists and the program terminates.

Line 31 calculates the Nusselt number, ZNUSSELT, by equation (9-10a) of McAdams(1954). From this, line 32 determines the forced convective heat transfer factor, HHT1. The multiplier, FACTOR, is an input quantity used to determine the effect on oscillations of reduced or increased heat transfer factors.

Line 33 determines the heat driving temperature difference, THETA. If it is zero, the program skips unnecessary calculations and goes to line 385.

Lines 35 through 42 calculate the $\log_{10}$ of the momentary Grashof-Prandtl product, GPL. If the Grashof-Prandtl product, GP, is negative, an error has occurred and the program is sent to line 410 and terminated. If it is zero the program goes to line 380 and skips unnecessary calculations.

Lines 43 through 370 determine the $\log_{10}$ of the free convective Nusselt number, ZNUL, and line 375 determines the corresponding Nusselt number, ZNUS.

In line 380 the free convective heat transfer coefficient, HHT2, is determined. The multiplier, FACTOR, is an input quantity used to determine the effect of increased or decreased heat transfer factors on oscillations. The overall heat transfer coefficient, HHT, is obtained by simple addition of forced and natural heat transfer coefficients.

In line 385 the heat transfer coefficient, HHT, is multiplied by the heat transfer area, AREA, the temperature difference, THETA, and the time increment, DT, to obtain the infinitesimal heat transferred, DQ, for the element in the time increment.

Lines 386 through 395 have to do with friction factor. In line 386 the minimum value of the Reynolds number is limited to 0.001.

In line 2386 the decision is made whether or not to calculate the friction factor. The program has the option of using a constant input friction factor or of using a modified text book friction factor. If the value of the friction factor during input is zero, then in subroutine AMPLITUD the variable, IFRF, is set to zero. Alternately, if the input value of the friction factor is not zero, the variable, IFRF, is set to unity. Thus, line 2386 causes calculation of a friction factor if the original input friction factor is zero. Otherwise, it sends the program to line 99999 and makes no friction factor calculations.

If the Reynolds number is less than 1000 in line 387, the friction factor is obtained from the usual laminar equation, but multiplied by NFR. The quantity, NFR, is an input multiplier used to explore the effect of increased or reduced friction. If the Reynolds number is less than 1000 in line 388, the friction factor has already been obtained, and control returns to AMPLITUD.

If the Reynolds number is greater than or equal to 1000, lines 389 through 395 calculate the friction factor by an equation (6-8e) of

McAdams (1954). The multiplier NFR is used as in laminar flow. The method is iterative. A value for friction factor is assumed in lines 389 and 390. A better estimate is found in line 392. If this estimate disagrees with the previous estimate by an appreciable amount the program returns to line 390 and iterates again. Otherwise, the calculation of friction factor is complete.

Lines 400 through 420 are used in case of error. In the event of an unstable calculation, TAVG, CONG, or GP can become negative. If this occurs, there is a print-out to aid in diagnosis and the quantity, DT, is set equal to zero, which causes termination of the program.

8. Detail of Subroutine AMPLITUD

This subroutine is the fundamental part of the program. All the other subroutines are for the purpose of helping subroutine AMPLITUD. As an element changes pressure, its density changes, heat is transferred from or to the wall, work is done by the element, friction is experienced, and at the open end, kinetic energy is lost from the pipe and ejected into the dewar. From an assumed value of the maximum overpressure at the closed end, AMPLITUD computes the behavior of each element in turn from 1 to M. Subroutine AMPLITUD is given in Appendix H.

The line before line 1 determines whether or not the program will use input or calculated friction factors. If the input value of friction factor, FRF, is zero,the parameter, IFRF, is set equal to zero. If the input value is different from zero, the parameter, IFRF, is set equal to unity. Then, if the input friction factor is zero, IFRF is zero, and subroutine HEAT TR computes friction factors for each element in each time interval. Alternately, if the input friction factor is any desired non-zero quantity, IFRF is unity, and subroutine HEAT TR leaves the friction factor unchanged at its input value.

Lines 1 through 40009 cause the printing of a representative table of friction factors used in the program versus Reynolds number. Where a constant friction factor is used, this same friction factor shows for each Reynolds number. Where a calculated friction factor is used, the friction factor calculated for the corresponding Reynolds number is printed. The friction factor printed includes the multiplier NFR for the calculated case. When the input friction factor is used, NFR is inoperative.

Lines 5 through 25 set the values of certain constants needed during calculations or set initial values for variables to insure proper operation of the program. The most important of these is the time interval, DT, calculated in line 10. The time of one interval is the time for one cycle times the fraction of a cycle used. Since the time for one interval is the reciprocal of the frequency, AVG FREQ, and since the cycle fraction is JD/360, in Fortran notation, the time interval is given by

$$DT = JD/360./AVG\ FREQ.$$

Line 30 is a DO LOOP ending in line 40000 which causes the calculation of all quantities needed for each of the elements 1 through M.

Lines 31 through 55, for each gas element in turn, set initial values for variables to insure proper operation.

In the DO LOOP from line 60 to line 40002, the calculation for each element is caused to repeat as many times as needed. The maximum number of repetitions is the quantity NJJ, an input quantity. The value of NJJ, in general,is kept high enough to insure that conditions calculated for the final cycle are substantially the same as for the previous one. The quantity, IQ, is used to indicate the adequacy of this convergence. Later in the program, if cycle convergence is adequate, IQ is increased by 1. If this happens twice, the accuracy is sufficient and the program proceeds to the next element.

In line 61, the maximum number of cycles, JJ MAX(I), used for the gas element I is determined. Since this quantity is printed, it is possible to see if the input quantity NJJ is adequate.

In line 65 the quantity QI is set equal to Q(I) and WORK I to WORK(I) for later use in determining adequacy of convergence. Later the sum of all infinitesimal heat transfer to one gas element, Q(I), and the sum of the work terms in one gas element, WORK(I), is obtained. At that time QI and WORK I become the heat and work summations for the previous cycle and a comparison is then possible.

Other calculations in lines 65 and 70 set initial values to insure proper program operation.

Line 80, a DO LOOP ending in line 40003, causes calculations for one cycle to occur for the element already selected in line 30. It sets the variable J equal to the portion of a cycle, expressed in degrees, completed at the end of this time interval. The first time interval is zero to JD degrees (If JD is 5 then the interval is 0-5 degrees.) and so on until the cycle is completed at 360 degrees. Because of line 80, the input variable JD must be divisible into 360. Also computed in line 80 is the sine function of the angular fraction of the cycle completed at the end of the time interval.

The quantity, JLJD, refers to the time (or angular fraction of the cycle) at the beginning of the interval just as the quantity, J, refers to it at the end of the interval. In line 85 it is calculated. At the beginning of the cycle J = JD, JLJD = 360, but at any later interval when J > JD, JLJD = J-JD.

In lines 90 through 105 numerous variables at the beginning of the current interval are set equal to the same variable at the end of the previous interval. Thus, RO(1), the gas density at the beginning of the interval, is set equal to RO(2), the density at the end of the previous interval. Later,

a value of the density at the end of the current interval is determined, which then becomes RO(2). The same is true for the volume of the element, VOL, the average position of the element, XAVG, the overpressure of the element, P, the absolute pressure of the element, PRES, the mass of the gas of an element that is in the pipe, G, and the mass of an element that is in the dewar, GID2 and GID1.

Line 110 determines if a gas element is entirely outside the pipe and in the dewar by means of the variable, X. Variable, X, is similar to XX. XX is computed in ELEMENT and is the non-oscillating or initial boundary of the various elements. Thus, XX(1) was set equal to input position S(1) and is the closed end of the pipe. XX(2) is the position of the boundary between elements 1 and 2. XX(I+1) is the boundary between elements I and I+1. The final boundary in the pipe is XX(N+1) which is equal to S(32). Also in line 25, to save space, the variable, XF, was set equal to XX(N+1) and represents the boundary between the pipe and the dewar. Since XX is invariant with time it is single subscripted. X varies with time as well as with the element being considered and, therefore, is double subscripted. Thus X(1, J) refers to the warm side of a given element at time J, and X(2, J) refers to the cold side at time J. To conserve storage, the first subscript is always 1 or 2. This necessitates a change when shifting from one element to the next. The value of X for the previous element that was the cold side becomes the value for the warm side of the next element. This shift is accomplished in line 35 as the program progresses from one element to the next. For element one, the initial values of X for all time increments are set equal to XX(1). So control shifts from line 110 to line 690 whenever the warm side position of the element is either at the end of the pipe or protruding into the dewar.

If the element is partly or wholly in the pipe at the beginning or end of the interval, the calculation continues to line 120.

In line 120, to insure proper operation of the program, two heat transfer parameters are set to zero.

The program now calculates heat transfer from the wall to the gas. Lines 240 through 490 do this for elements wholly in the pipe. Lines 240 through 300 plus lines 500 through 2689 do it for elements partly in the pipe and partly in the dewar.

The heat transfer calculation is necessarily iterative. From the previous calculations on other elements, the warm end position of the element is known. Then from an approximate initial volume of the element its approximate average position is found. Heat is transferred as if this were its real average position. A more accurate density and volume are calculated. From this new volume a new average position is found and the heat transfer is recalculated. The process is repeated until the heat transfer calculation stabilizes.

In line 240, the variable, KOUNT, has 1 added to it in order to print out later the number of heat transfer iterations needed for convergence. KOUNT was initially set to zero in line 105 so that its value always represents the number of heat transfer iterations for a given element and time interval during the last cycle iteration.

In line 240, the parameter, DQ1, is set equal to AQ, the heat transfer from the previous iteration, for convergence test purposes. Note: AQ and DQ are essentially the same.

In line 241, the maximum value of KOUNT for the given element, KOUNT MAX(I), is found. KOUNT MAX(I) is set to zero in line 65 so that the final KOUNT MAX(I) is for the final cycle.

Line 245 sets the position of the cold end of the given element equal to the warm end position plus the length of the element. This position varies slightly with each KOUNT because the heat transfer changes the volume of the element. That is, VOL(2) is the element volume at the end of the time interval. Dividing by the cross section area, AF, gives the length. But VOL(2) is not known absolutely until after the heat transfer and work terms are evaluated. Initially, an approximation for VOL(2) is found in line 45. Successive calculations make it more accurate. During the final heat transfer iteration of the final cycle the cold side position of the given element at time J, X(2, J), is firmly established.

Line 250 determines the average position of the element at the end of the time interval, J, by averaging the cold and warm end positions.

Line 255 establishes the gas velocity during the interval. It divides the motion of the average position by the time interval.

Line 260 determines the average gas temperature during the interval. For an initial approximation, line 35 sets the instantaneous temperatures during a cycle to the gas temperature found in ELEMENT for the given gas element. During iteration a more accurate value is found for the temperature at the end of the interval, T(J), and, therefore, for the average temperature, TAVG. The temperature determined at the end of the previous interval establishes the temperature at the beginning of the current interval, T(JLJD).

Lines 265 and 270 calculate the maximum overpressure occurring at the end of the time interval when the average position is at XAVG(2). Line 275 multiplies this maximum overpressure by the sine of the angular position to obtain the instantaneous overpressure at the end of the interval. The corresponding absolute pressure, PRES(2), is obtained by adding dewar pressure to the overpressure.

Line 280 obtains the average absolute pressure, PRESS, and the average density, ROE, during the interval by averaging conditions at the beginning and end of the interval.

Line 300 transfers calculations to line 500 if the element protrudes into the dewar at the beginning or at the end of the interval.

If the element is entirely in the pipe, line 305 calculates its average volume, VOLM, by dividing the element mass, GM(I), by its average density, ROE.

Line 405 calculates the average position, AVGX, of the element during the interval by taking the average of the average positions at the beginning and end of the interval.

Lines 410 through 420 calculate the wall temperature, TWI, corresponding to the average position of the element during the interval.

Line 425 calls the subroutine HEAT TR. The quantity ITEST FRF is set to zero so that heat transfer will be calculated. Had it been set to unity, only friction factor information would have been returned.

Sometimes the program is unstable and troubles occur in subroutine HEAT TR. If this occurs, DT is set to zero in the subroutine and line 430 causes termination of the program.

Line 435 calculates the infinitesimal friction work, DFR, done in the interval by a formula derived in Appendix C. Since the friction is always a positive number, the absolute value is used.

Line 440 calculates the infinitesimal net work, DWK, after subtracting friction work, by a formula derived in Appendix B.

Line 455 calculates the internal energy change of the element, QW, from the first law of thermodynamics by subtracting the net work, DWK,

from the heat input, DQ, returned from HEAT TR. The quantity AQ, used later, is set equal to DQ.

The gas temperature at the end of the time interval, T(J), is calculated in line 460. To the temperature at the beginning, T(JLJD), is added the temperature rise computed by dividing the internal energy increase, QW, by the mass of gas in the element, GM(I), and the specific heat, CVG.

Lines 465 and 470 calculate the element density and volume at the end of the interval.

During the first iteration of a heat transfer calculation in an interval, it is not possible for equilibrium to have been established, so line 475 forces the program back to line 240 for at least one more iteration.

Sometimes an undue number of heat transfer iterations occur and it is necessary to proceed without complete convergence. If there have been more than 10 attempts, KOUNT will exceed 10, and line 480 sends the program to line 700 to continue on with the program. This seldom happens. If it does, the quantity, KOUNT MAX(I), calculated in line 241 exceeds 10 and shows in the print-out.

A test for heat transfer convergence is in line 485. If the heat transfer computed is appreciably different from the previous computation, the program returns to line 240 and tries again. To avoid division by zero the quantity AQ, if zero, is changed to 0.000001.

If the program gets to line 490, it has completed heat transfer and work calculations for this time interval of this element, and the calculations have been for an element entirely in the pipe. It now skips calculations designed for elements partly or wholly in the dewar and control is shifted to line 700.

If the element is partly in the dewar and partly in the pipe, either at the beginning or the end of the interval, control is transferred to line 500 by line 300. For heat transfer purposes it is necessary to determine the length of the element touching the wall. Four quantities are defined, C1, C2, W1, and W2. These are the positions of the cold side at the interval beginning, the cold side position at the interval end, and the warm side at the beginning and end. Depending on when and by how much the element protrudes into the dewar, these quantities may be the sides of the element or the ends of the pipe. Lines 500 through 530 determine these positions.

Lines 550 and 551 calculate the infinitesimal amount of kinetic energy ejected into the dewar during this time interval.

Line 600 calculates the heat transfer area at the start, AREA1, at the end, AREA2, and the average area, AREA.

Line 605 calculates the average position of the element for heat transfer purposes.

Lines 610 through 620 determine the wall temperature, TWI, at the average element position.

Line 625 determines heat transfer into the element by calling subroutine HEAT TR. The quantity ITEST FRF is set equal to zero so that HEAT TR returns heat transfer as well as friction factor information.

Line 630 terminates the program if a calculation instability is found in HEAT TR.

Line 635 calculates the infinitesimal friction work done during the interval by a formula derived in Appendix C.

Line 639 calculates the average volume of the element in the pipe. Since it is the volume in the pipe rather than the entire element volume that is desired, the quantities G(1) and G(2) are used for the mass of gas instead of GM(I).

Line 640 calculates the net work done in the interval by a formula derived in Appendix B.

Line 655 calculates the internal energy change in the element.

Line 670 calculates the gas volume of the given element that is in the pipe, VOLIP2, the mass of this gas, G(2), and the mass of gas of this element in the dewar, GID2, at the end of the interval.

Lines 674 and 675 calculate the density of the fraction of the gas element that is in the dewar at the end of the interval, ROID2. This quantity is used in line 676 to determine the volume of the dewar fraction of the gas element, VOLID2, and the total volume of the element, VOL(2). Note: The calculations of lines 674 through 676 possibly are not completely rigorous, but they do not affect the results obtained using kinetic energies, EKE.

Lines 680 through 687 calculate the temperature of that portion of the gas element that is in the pipe at the interval end. Three routines are necessary, one for flow out of the pipe, one for flow into it, and one special case where there is no gas in the pipe at the end of the interval. It should be noted that T(J) is always the temperature at time J of the gas fraction in the pipe.

Line 688 computes the density of the gas fraction in the pipe at the interval end.

In lines 688 through 689 criteria similar to those of lines 475 through 485 are used to test heat transfer convergence.

At line 2689, the program skips calculations used for elements that are entirely in the dewar.

Lines 690 through 697 are for the case of the gas element entirely in the dewar. No computations are really needed. Those that are made are for the purpose of maintaining consistency of printout.

At line 700 all calculations have been performed on the gas element for the current time interval. In line 701, the infinitesimal work, DWK, just calculated, is added to the work term for the element so that at the end of the cycle the term, WORK(I), will represent the total work done by the element in the interval. Similar steps are taken to obtain the total heat input to the element, Q(I), and the total friction in the element, FR(I). In lines 703 and 704 the dewar bound kinetic energy of the element, ZKE(I), is computed.

If the convergence from cycle to cycle is not yet adequate, the quantity IQ will be zero and detail printing is not desired so line 705 sends the program to line 40003 for return to line 80 to advance to the next time interval, or if a cycle has been completed, to line 723.

If only one cycle has been computed for the element, line 723 transfers control to line 40002 which returns control to line 60 to start a second cycle.

Lines 724 through 750 concern cycle convergence, printing, and program termination.

There are three criteria for cycle convergence. For the elements that are wholly in the pipe, the heat received by the element must equal the net work done by the element. For all elements at equilibrium, the heat received by an element in a calculation cycle must equal the heat received in the previous calculation cycle. The work done in a calculation cycle must equal that done in the previous one. These three criteria are calculated in the form of three ratios, QLWOQ, DELQIOQI, and DELWKOWK. They are defined as follows. QLWOQ is the difference between the heat and

work, divided by the heat input. DELQIOQI is the difference between this cycle's and the previous cycle's heat, divided by this cycles heat transfer. DELWKOWK is the same applied to work done. In all three criteria the absolute value is used.

If the heat transfer and the work done are both zero, the criterion is satisfied and line 725 sends the program to line 750.

Line 726 avoids division by zero.

Line 730 and 735 calculate the three criteria.

If the values of any two of the three criteria are less than 0.0001 in lines 738 through 740, the program is sent to line 750.

The quantity NJJL2 was calculated in line 6 and is equal to two less than the input quantity NJJ, which limits the number of cycles that can be calculated. If the number of cycles is as great as NJJL2, it will be desired to print detailed information before proceeding to the next element. The program is therefore sent to line 750 by line 742.

Line 750 adds 1 to the variable, IQ. Then during the next cycle various details will be printed. If cycle convergency has occured twice, the variable IQ increases to two and detailed printing occurs again. When IQ reaches three, cycle convergency is undoubtedly satisfactory and line 60 causes the program to proceed to the next element.

Line 40002 is the end of the cycle or JJ iterative DO LOOP. Line 40000 ends the element DO LOOP. Most of the remainder of the subroutine concerns printing. There are however a few calculations remaining.

In lines 760 through 40001 the work done in all elements is summed as is the heat transfer and the friction. There are now two types of kinetic energy that have been calculated. In each element the infinitesimals have been summed to find the total dewar bound kinetic energy, ZKE(I), in

element I. The kinetic energies in either the N'th or M'th element, ZKE(N) or ZKE(M), could serve as an approximation of the kinetic energy ejected into the dewar.

The kinetic energy has also been calculated for each time interval in line 550 and summed up in line 780 as SUMEKE. The quantities ZKE(N), ZKE(M), and SUMEKE are all printed and found to agree closely.

The purpose of the program is to compare the sum of work done by all the elements with the kinetic energy ejected into the dewar. The best comparison is a ratio of the kinetic energy to the work. Lines 785 and 787 compute three such ratios. The quantity EKEOSW is SUMEKE/ SUM WORK; ZKEMOSW is ZKE(M)/SUM WORK and ZKENOSW is ZKE(N)/ SUM WORK. Because it is based on the most reliable method the quantity EKEOSW is used.

9. Detail of Subroutine READ DATA in Frequency Calculations

The subroutine READ DATA for frequency calculations is very similar to the one for amplitude. The difference is that much of the input needed for amplitude is unnecessary for frequency. The input quantities have the same meaning in each case. Subroutine READ DATA for frequency is given in Appendix I.

10. Detail of Subroutine ELEMENT in Frequency Calculations

The subroutine ELEMENT, when used with frequency calculations, divides the duct into N elements. The length of each element is adjusted so that it takes sound the same length of time to cross each. Thus, the length of each element divided by its velocity of sound is the same, and the quantity, DXXOVC, is the same for each element. The same quantities are computed and printed for each element as they were for amplitude calculations. Additionally the velocity of sound for each element, C(I), is computed and printed.

Line 10 sets the temperature of the warm end boundary of element 1, TEMP(1), to the input value of the warm end of the pipe, TMP(1). It also sets the position of this boundary, XX(1), equal to the input position of the warm end of the pipe, S(1). In addition line 10 computes the value of NP1 = N+1, which is needed later.

Line 20 is an estimate made of the quantity DXXOVC. This estimate is the length of the whole pipe, S(32) - S(1), divided by the number of elements, N, and also divided by the velocity of sound in the median element, SQRTF(XK*R*TMP(16)).

Lines 24 through 40002, for each element in turn, calculate the sonic velocity in an iterative routine, and determine the boundary positions of the element. Then lines 47 through 49 test to see if the cold end of the N'th element is in its proper position. If not an adjustment is made to the quantity, DXXOVC, and the program returns to line 24.

Thus, line 25 estimates the sonic velocity of the element, C(I), using the warm side temperature TEMP(I). Line 28 calculates the position of the cold side. Lines 29 through 33 calculate the temperature of the cold side. In line 37, the average temperature of the element is computed. Then the tentative sonic velocity of the element, CCC, is computed using the average element temperature. Line 38 tests to see if the tentative and estimated velocities are nearly the same. If they are not, line 39 re-estimates the sonic velocity to be the tentative value and the program returns to line 27. If they are nearly the same, line 43 establishes the sonic velocity as being the tentative value and the program proceeds to the next element.

At line 47 the positions of all the elements have been computed. In line 47 a test is made to see if the cold side position of the N'th element closely coincides with the position of the dewar end of the pipe. If this is

not sufficiently close, the quantity, DXXOVC, is adjusted in line 49 and the program returns to line 24 repetitively until it is sufficiently close after a number of iterations. When it is sufficiently close the program goes to line 50 and a number of quantities are computed and printed in a manner similar to that of subroutine ELEMENT in amplitude calculations. In addition, three other quantities are calculated. In the foregoing iteration the velocity of sound, C(I), was determined for each element. In line 55, the parameter, Z(I), is calculated. This is simply the product of the sonic velocity and the density of the gas in the element, RHO(I). Also computed is its reciprocal, A(I). These two parameters are needed in pressure and velocity calculations which determine frequency. The remainder of ELEMENT is for print out purposes.

11. Detail of Program FREQENCY

The program computes the frequency of thermal oscillations from the known temperature gradient in the pipe, using equations derived in Appendix A. In the theory, a sudden overpressure is assumed in the dewar which travels towards the closed end according to the velocity of sound and the derived equations. In the program of this report the equivalent assumption is made that there is a sudden underpressure in the pipe. Frequencies calculated by the two methods are identical; the usage is one of convenience. In either case the initial gas velocities are zero.

The time required for sound to traverse one element, DXXOVC, is one computation time. At the start of the program, with everything at rest, the time, J, is considered to be one. After the passage of one time interval, DXXOVC, J becomes two. At the time [J = 2] a pulse has traversed from the dewar across element N. At the time [J = 1] the program inserts initial values of overpressure, P, and velocity, U. At the time [J = 2] the only change has occurred in element N, and the program determines the proper values. After these first two preliminary calculations,

the program computes, at each time J, the overpressure and velocity in each element. Because of computer storage limitations, the maximum value of J is 102. Then, if desired, the pressures and velocities in the N elements are printed for the values of J from 1 to 100. The values of pressure and velocity in each element at times [J = 1] and [J = 2] are now set equal to the values at [J=101] and [J = 102], and the calculations proceed for another hundred time intervals. On each occasion that J is thus shifted, the quantity KOUNT is increased by 100 and the quantity J2 is always equal to J+KOUNT, with the result that J2 is the number of the calculation time. Because of the nature of the equations used, a different route through the program is used for times when J is even or odd.

When first devised, the frequency was determined by plotting the closed end pressure versus time. Then the number of cycles per second was determined by examination of the plot. In the program presented here, the frequency is determined in the program itself. Because of the somewhat erratic nature of the overpressure, the routine used to calculate frequency is quite complicated.

In principle the method is as follows. During each calculation time for each element the quantity, PMIN(I, JK), is determined. If the overpressure, P(I, J), at the time is less than the previously determined PMIN(I, JK), then the value of PMIN(I, JK) is adjusted downward to this new value. Simultaneously the quantity, TMIN(I, JK), is set equal to the calculation time, J2, at which the overpressure went to a new minimum. The subscript, I, refers to the element number, and the subscript, JK, to the cycle number that is in progress. Initially the cycle number and JK are unity and the closed end overpressure is -1000. The closed end pressure then increases to a maximum overpressure and decreases. Eventually it becomes negative. At this time a new low is being approached for a

new cycle and JK is increased by one. When a new minimum pressure, PMIN(I, JK), occurs at a new time, TMIN(I, JK), the process is repeated.

Without some safety provision, the cycle number, JK, would increase prematurely. This results from the fact that during a pressure trend that is generally up or down, there may well be a number of temporary reversals. This reversal occurring just as the overpressure crossed zero would erroneously increase JK. The safety provision introduced is the quantity, MAXCOUNT. At each change of JK, MAXCOUNT is set to zero. During each calculation, if the overpressure is slightly over zero, one is added to MAXCOUNT. Until MAXCOUNT is greater than N, the quantity JK cannot change. By this time the danger of inadvertently having the overpressure cross zero is past.

TMIN of the first element is used as a criterion of the interval of one cycle. This quantity and others are computed for all the elements. These are,

TMAX(I, JK), the calculation time of the maximum overpressure
PMAX(I, JK), the maximum overpressure of cycle JK
UMAX(I, JK), the maximum velocity in cycle JK
TMIN(I, JK), the calculation time of the minimum overpressure
PMIN(I, JK), the minimum overpressure of cycle JK
UMIN(I, JK), the minimum velocity in cycle JK

The interval, ZINTERVL, of one cycle is obtained by subtracting a given value of TMIN from the next value of TMIN. The period, PERIOD, is this quantity multiplied by the calculation time, DXXOVC. The reciprocal of any one period is the frequency, FREQ, and the average frequency, AVG FREQ, is the average of the first nine frequencies calculated. The average frequency is then used in the subroutine AMPLITUD. Program FREQENCY is given in Appendix K.

Lines 5 and 10 call the subroutines READ DATA and ELEMENT.

Line 20 sets the initial value of KOUNT at - 100 so that when the calculation starts and 100 is added, the value of KOUNT during the first 100 calculations will be zero. The quantity SAMBDA is the value of the initial assumed under pressure in the pipe. GAMBDA is a small quantity used as a criterion to determine MAXCOUNT.

Line 25 sets the values of several quantities which are used later.

Lines 30 and 40000 sets the pressure in all elements at times [J = 1] and [J = 2] to - SAMBDA and the velocities to zero.

Lines 40000 and 35 set the initial values of quantities which may change later.

The first cycle starts at its minimum; therefore, line 40 sets PMIN equal to - SAMBDA. For other cycles it is set to + SAMBDA to insure the low value calculated later is the proper one.

Line 50 sets the velocity and pressure in the N'th element and second calculation time equal to initial values.

In line 60 the value of KOUNT is increased to zero before the first series of a hundred calculations, and increased by 100 after each series.

Lines 65 through 141 are the main DO LOOP that calculates the pressures and velocities. In line 65, if the calculation time is the 3rd, or any other odd time, the program goes to line 101. If even, it goes to line 121.

Lines 101 through 115 are the DO LOOP that calculates odd time overpressure, velocity, and parameters associated with PMAX and PMIN. The overpressures and velocities are calculated in lines 105 through 113 by formulas derived in Appendix A. Lines 2114 through 2129 determine if

the overpressures and velocities calculated are a maximum, a minimum or neither for the cycle and set up a new cycle when MAXCOUNT is greater than N and the closed end overpressure, P(1, J) is negative.

Lines 121 through 5141 perform the corresponding calculations for even values of J.

At line 150 the overpressures and velocities have just been calculated by the DO LOOP ending in line 141 for a hundred values of J. In line 150, if KOUNT equals zero or is greater than 1000, detailed printing of pressures and velocities is desired and carried out by lines 160 through 225.

In line 226, if not enough cycles have been completed, the program jumps to line 229 where the overpressures and velocities of the 101st and 102nd calculations times are transferred to the 1st and 2nd time, and control returns to line 60 for computation of another hundred values of J.

In line 226, if enough cycles have been completed, the program continues to line 6227 for final calculations.

Line 6228 finds the number of calculation times, ZINTERVL(J), for each cycle.

Line 6229 converts ZINTERVL(J) to cycle period, PERIOD(J), by multiplying by the calculation time, DXXOVC.

Line 6230 takes the reciprocal of PERIOD(J) to find the frequency of each cycle, FREQ(J).

Line 6231 adds all nine frequencies together.

Line 6232 determines the final answer needed for AMPLITUD, which is the average frequency, AVGFREQ.

The remainder of the program is for print-out.

12. Results and Conclusions

Typical results from the frequency program are shown in figure 3. To obtain this plot a version of the program is used which does not compute the frequency. Instead, the closed end overpressures are printed and then plotted. The plot shows the almost sinusoidal overpressure obtained. It also shows the tendency for the calculated overpressure to be erratic even though the general trend is clearly recognizable. Since the temperature profile for the calculations is from figure 4 which is from the work of Bannister (1965), it is possible to compare measured with calculated frequencies.

The curve of figure 3 shows three oscillation cycles over a range of 321 calculation points which followed 3984 previous calculation points. Over the whole calculation, the period averaged 107.2 calculation intervals of 0.000302 seconds each. From this the calculated frequency is 30.9 cps. From figure 3, the period is about 105 calculation intervals, or 0.0317 seconds, and the frequency is 31.5 cps. From the program of this report which averaged the first 10 cycles, the frequency is 31.25 cps.

It should be noted that figure 3 is from a slightly different program than the one described in detail in this report. In figure 3 an initial overpressure in the dewar of 10130 dynes/cm^2 is assumed. In the described program an initial underpressure of 1000 dynes/cm^2 is assumed throughout the pipe. This difference has no effect on the calculated frequency.

The calculated frequency is needed in the determination of pressure amplitude. In all the amplitude determinations of this report the temperature profile of figure 4 is used, so the frequency of each calculated point is taken as 31.25 cps. The program is designed however to accommodate any predescribed temperature profile by modifying input data.

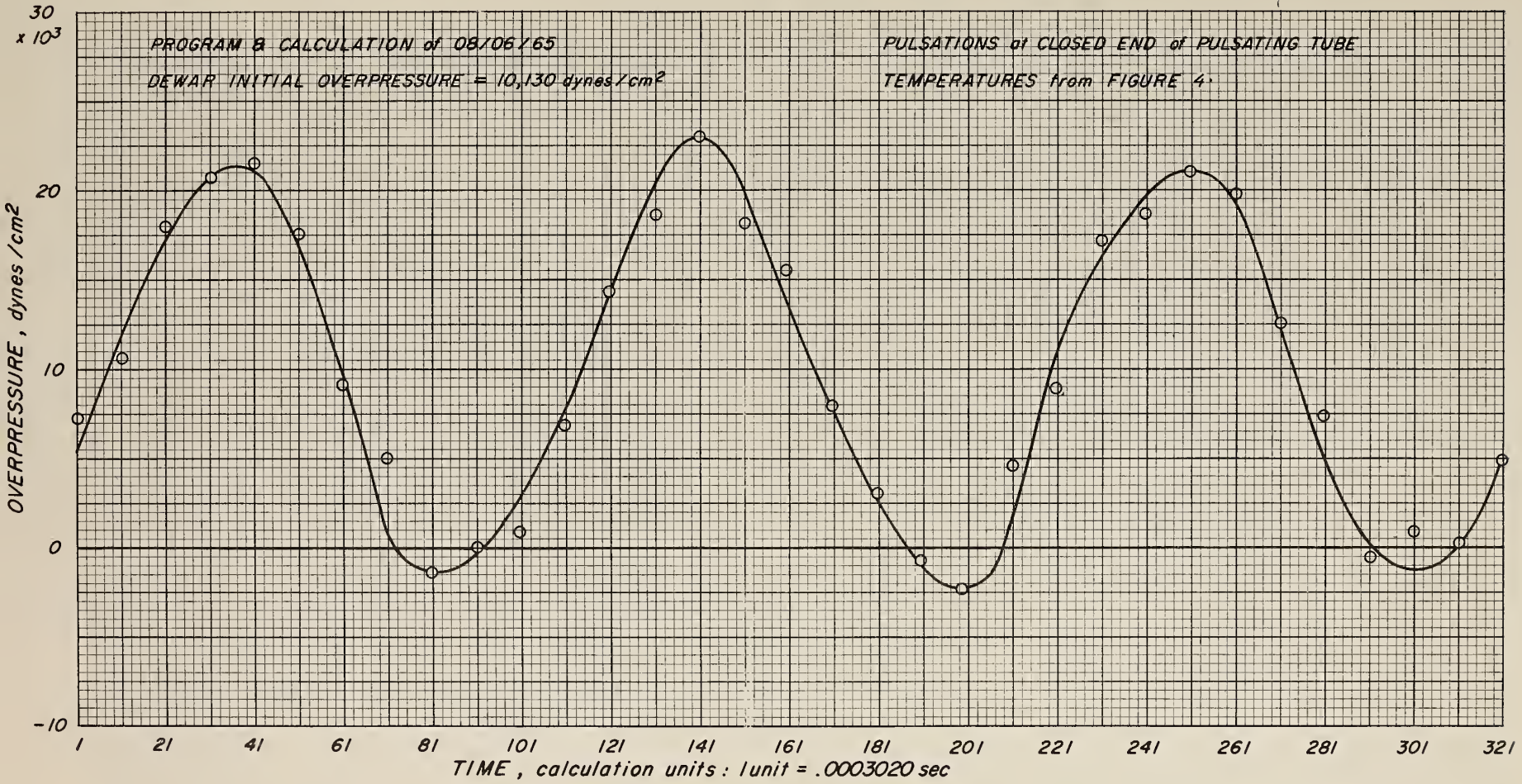

Figure 3. Typical Calculated Oscillation Frequency

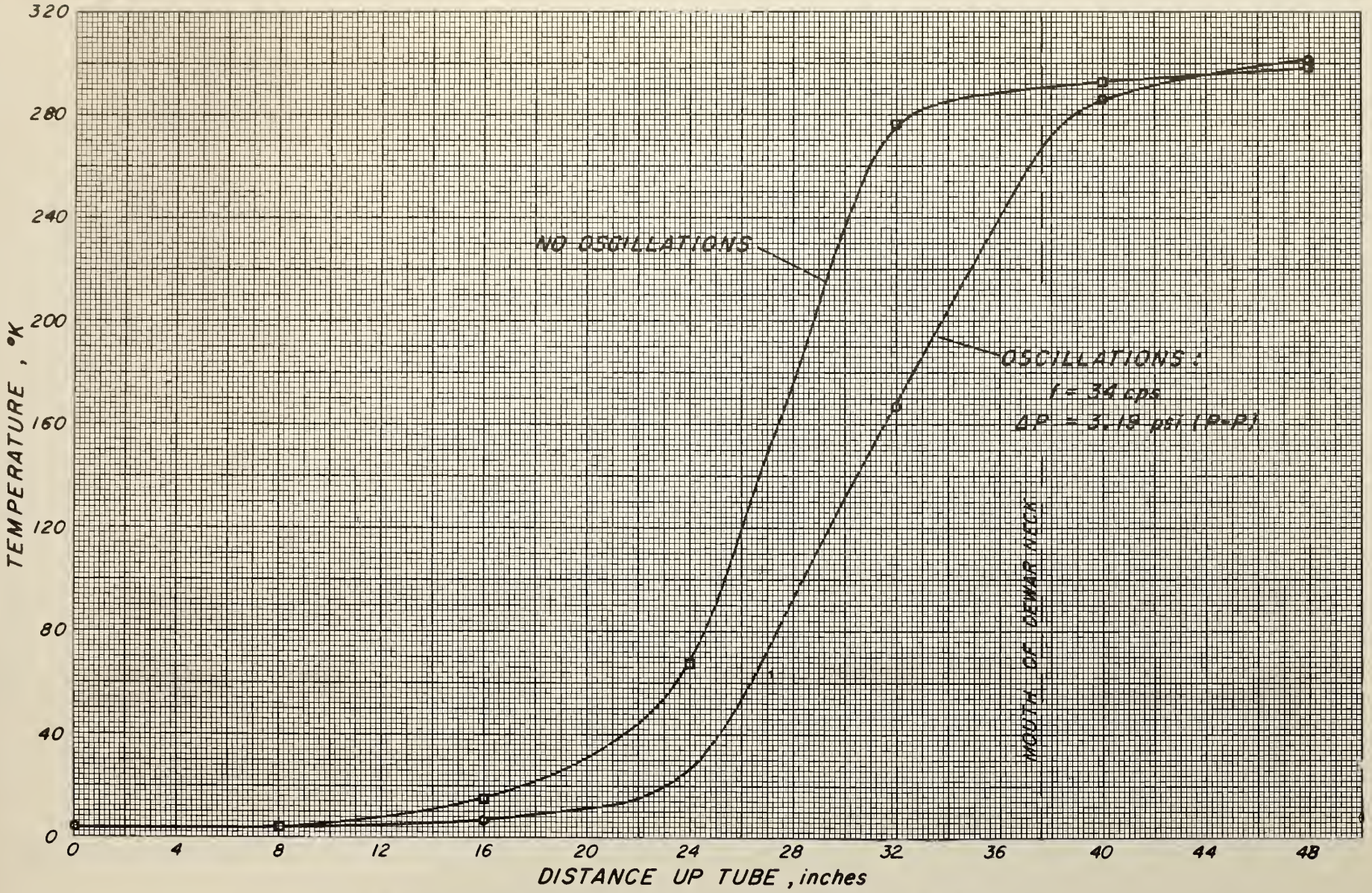

Figure 4. Test Data Courtesy of John Bannister, Air Reduction Co.

The amplitude calculations depend on heat transfer coefficients and friction factors. Neither of these quantities is precisely known for the type of fluid movement that exists in thermal oscillations. In McAdams (1954) is information for fully developed flow. Unfortunately the flow in this case is far from fully developed. Furthermore, it is intended that a high value of friction factor is to be used, to allow for inlet turbulence.

Accordingly, the results are presented for two types of heat transfer coefficients. With the one type, heat transfer coefficients are obtained directly from the text book. With the other, they are multiplied by 0.9.

Two methods are used for presenting friction factor. With one method, friction factors are assumed to be constant during a run regardless of the momentary Reynold's number. Then a series of runs are presented, each with a given constant friction factor. With the other method, friction factors are calculated from a text book formula for each momentary Reynold's number, but the calculated value is increased by a constant multiplier for each run. Then a series of runs is presented, each with a constant friction factor multiplier.

Three figures are presented, using a variety of friction factors and two coefficient multipliers, showing the effect of pipe diameter on the amplitude of oscillations. Figure 5 uses unmodified text book heat transfer coefficients and a series of four constant friction factors. Figure 6 uses unmodified text book heat transfer coefficients and a series of five friction factor multipliers. Figure 7 uses text book heat transfer coefficients times a multiplier of 0.9 and the same series of five friction factor multipliers.

The general shape of all the curves is the same. At the small diameter end, the oscillations tend towards zero amplitude. As the diameter increases, the calculated amplitude increases to a maximum,

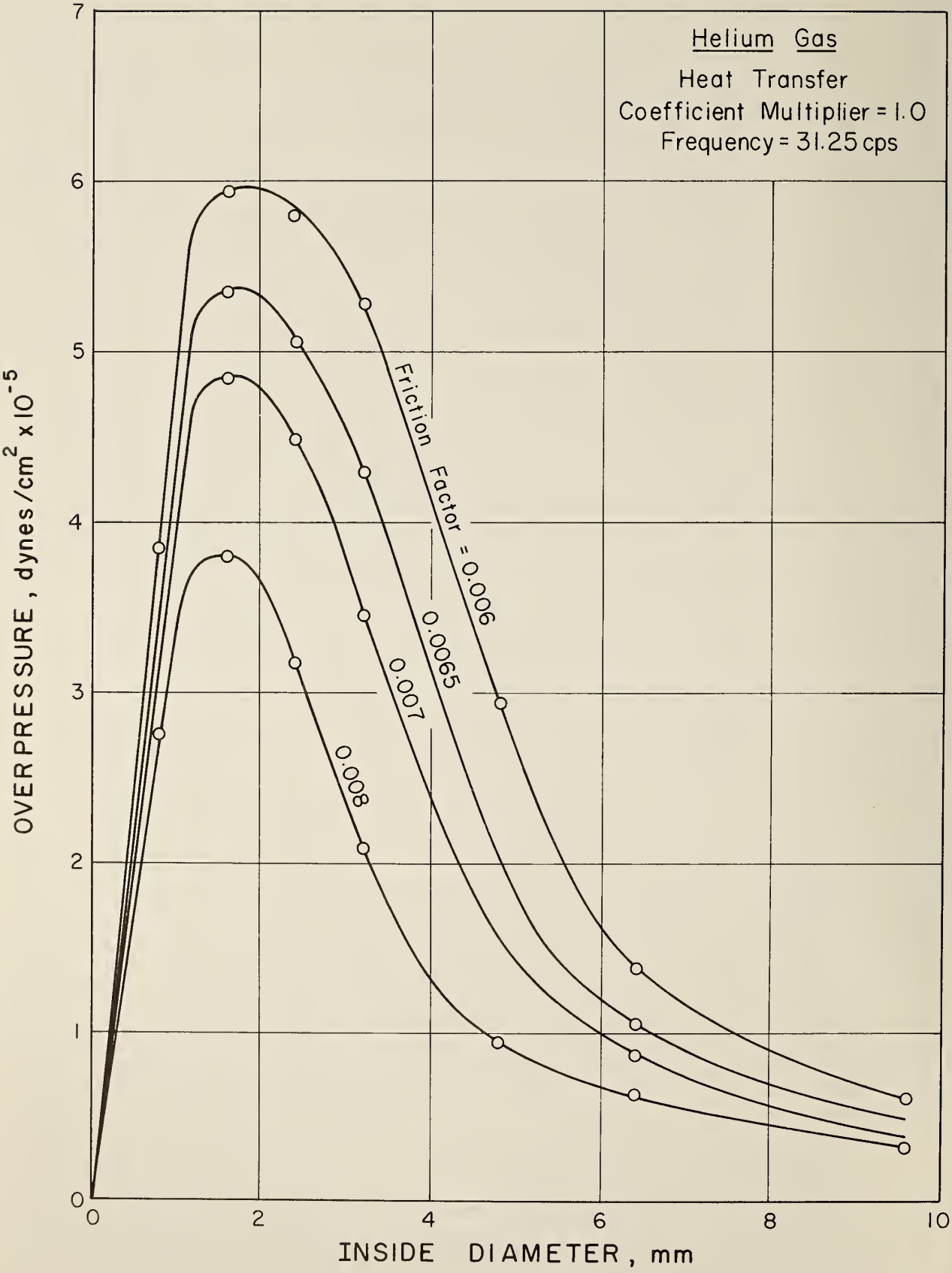

Figure 5. Amplitude Versus Pipe Diameter Using Constant Friction Factors and Text Book Heat Transfer Coefficients

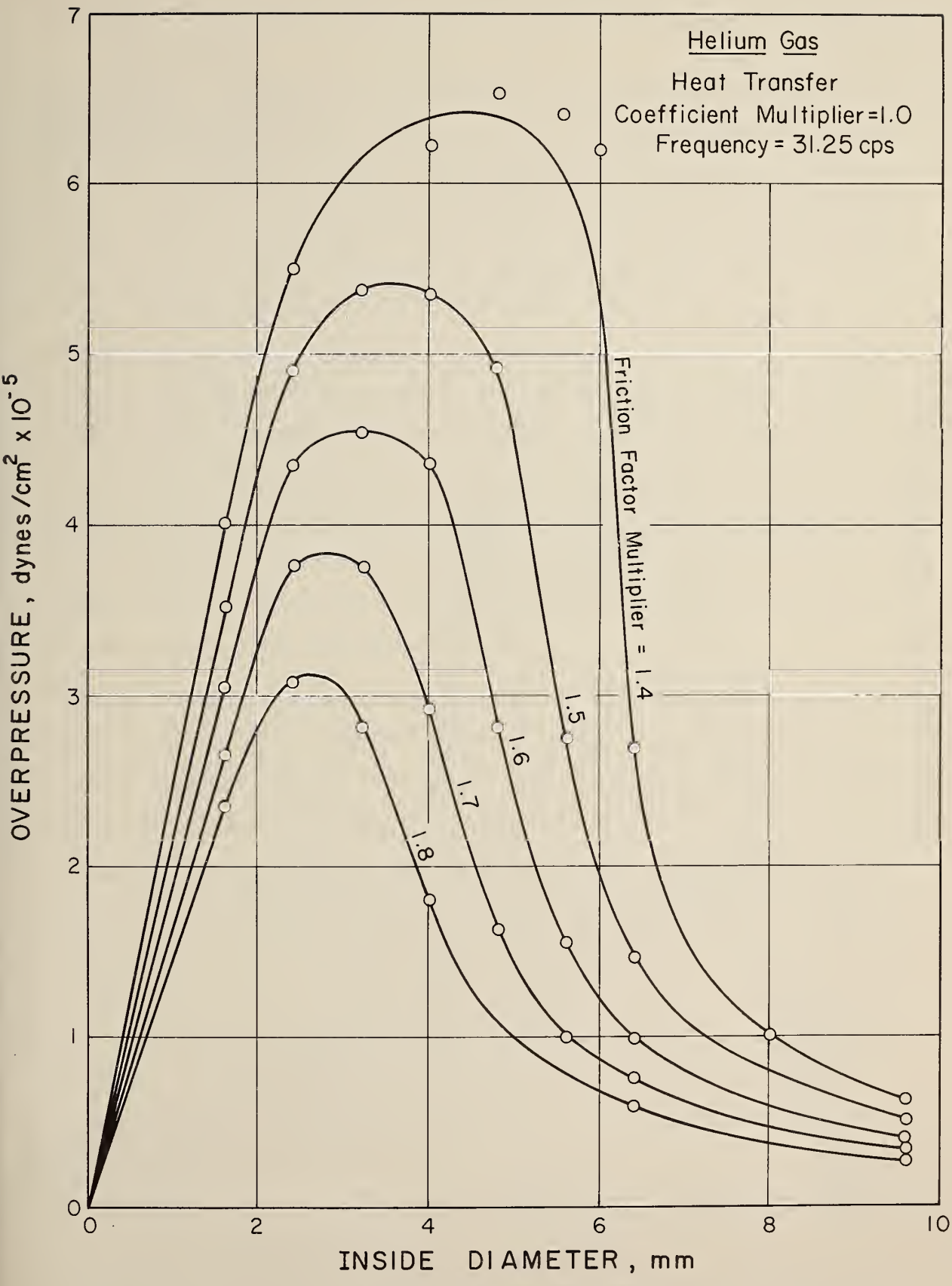

Figure 6. Amplitude Versus Pipe Diameter Using Modified Calculated Friction Factors and Text Book Heat Transfer Coefficients

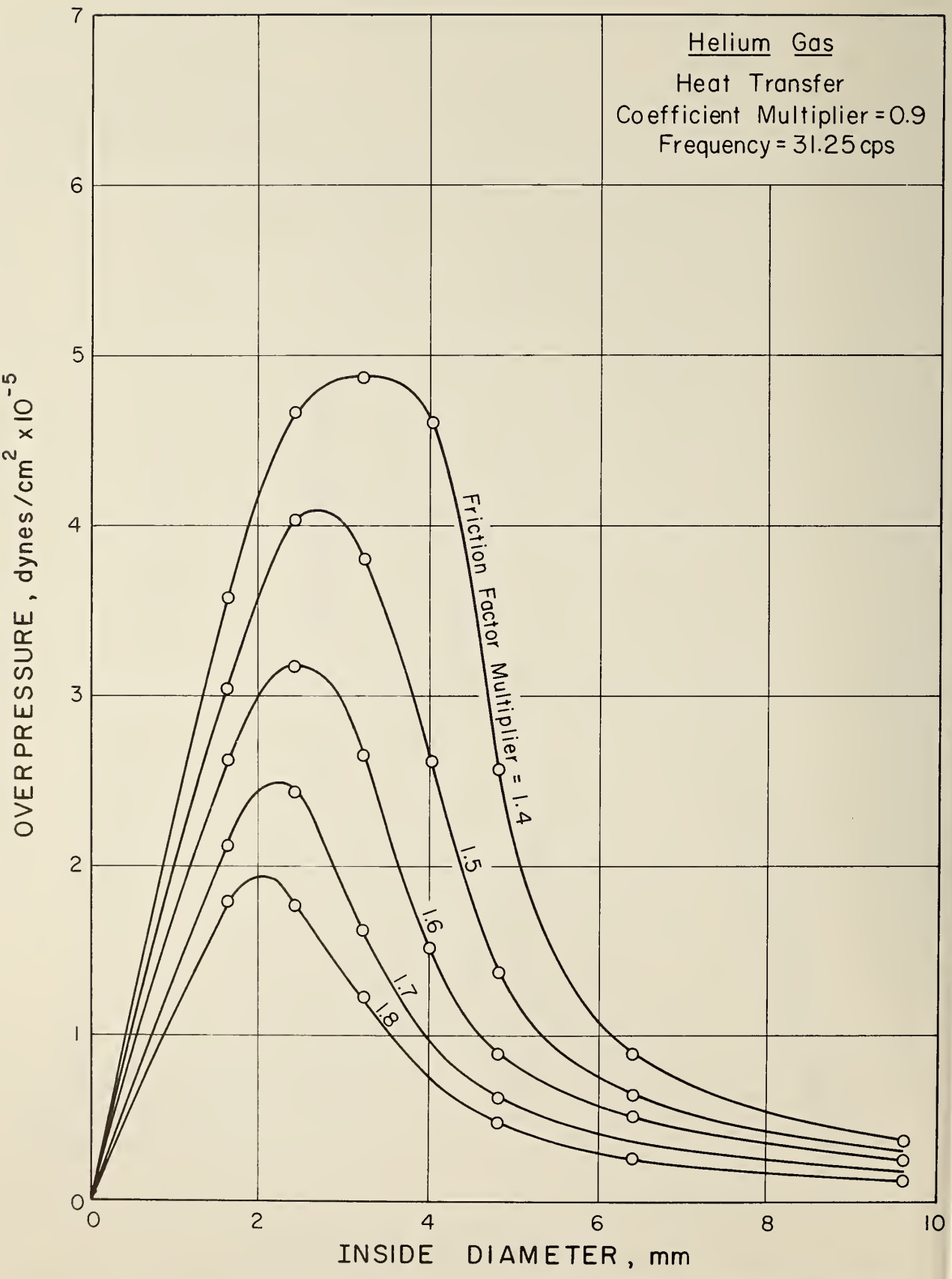

Figure 7. Amplitude Versus Pipe Diameter Using Modified Calculated Friction Factors and 0.9 times Text Book Heat Transfer Coefficients.

and then decrease asymptotically towards zero at large diameters. As might be expected, the higher friction factors result in lower peak overpressures in figures 5, 6, and 7. In figures 6 and 7 the effect of heat transfer coefficients may be seen. In all cases, reducing the heat transfer coefficient multiplier from 1.0 to 0.9 reduced the peak overpressure. As an approximation, the reduction is about 30 percent.

It is of interest to compare the calculated overpressures with the data from Bannister (1965). For a 0.635 cm (1/4 inch) diameter pipe the measured peak to peak pressure oscillation was 220,000 dynes/cm^2 (3.19 psi) which corresponds to a peak overpressure of 110,000 dynes/cm^2. On figure 5 the constant friction factor that produces 110,000 dynes/cm^2 is a value of 0.0065. On figure 6, by interpolation, the friction factor multiplier, which produces this overpressure, is about 1.57, and on figure 7, by extrapolation, the value is 1.37.

The constant friction factor method is not favored. At small diameters, the Reynold's number will be smaller and the friction factor larger than at large diameters. As a result, the maximum overpressure occurs at too low a diameter as is the case on figure 5.

It is not usually expected that heat transfer coefficients are known precisely. Therefore the effect of diameter on amplitude can be followed equally well with a heat transfer coefficient multiplier of 1.0 or 0.9. The value of 1.0 from figure 6 is therefore used. From the one datum point the friction factor multiplier of 1.57 has been selected. From the figure it is found that the maximum overpressure, for the given temperature gradient, occurs at 0.3175 cm (1/8 inch) and its value is about 480,000 dynes/cm^2 (7 psi), and double that value for peak to peak variation. At 0.635 cm (1/4 inch) the value coincides with test data and at large diameters tends towards zero.

Considering that no allowance exists in the program for entrance turbulence, and further considering that friction factor determination cannot be precise, it is quite reasonable to have to multiply by 1.37 or 1.57 to correlate calculated oscillation overpressures with test data.

It is concluded that the program in its present form calculates overpressure peak amplitudes that are approximately correct. Therefore, the qualitative explanation of the phenomenon, given in this report, which generally follows Rayleigh (1945), is correct. Further,the heating effect which evaporates helium is the result of the operation of the work cycles which expel kinetic energy and warmed gas into the liquid.

Thus the method of oscillation suppression, by inserting a knotted string in the pipe is in effect increasing the coefficient of friction. Similarly the insertion of glass wool or other packing does the same thing. Clement and Gaffney (1954) found that a vacuum insulated line does not oscillate. This case corresponds to a heat transfer coefficient multiplier of about zero. Since a 0.9 multiplier greatly reduces oscillations, it is expected that a multiplier close to the value of zero would essentially eliminate them. Furthermore, since the oscillations are a thermal cycle, a source of heat is needed to provide the cycle work. Complete insulation eliminates the source. In the same reference it is found that closure of the cold end suppresses oscillations. For this case there is no kinetic energy ejected into the dewar. Any chance oscillations that start come to equilibrium when the sum of all the work terms equal the kinetic energy. This equilibrium occurs when the work terms are all zero, or the oscillations are non-existent.

Although the program does simulate frequency and amplitude of thermal oscillations, it has a defect which should be improved upon. The assumption that the maximum overpressure varies as a cosine function from one end to the other of the pipe is not needed. The assumption that

the closed end pressure is sinusoidal with time is correct and should be retained. Then using the gas laws and Newton's laws of motion the boundaries of the elements can be calculated one by one from the closed to the open end. Simultaneously the instantaneous pressure variation at all of the boundaries can be obtained. Such a calculation would be an improvement over the present one and is recommended for future considerations. Additional work is needed, particularly in experimental phases such that more appropriate comparisons can be made with the mathematical model.

Although calculations have been performed and comparisons made for one particular case, the program is expressly designed to accommodate different fluids, tube lengths and temperature gradients along the tube.

13. References

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14. Appendix A

To analyze the behavior of pressure pulses traversing a non homogeneous medium it was necessary to make an assumption. The gas temperature in the tube varies continuously from one end to the other. If an attempt is made to analyze this situation, an infinite number of reflected pressure pulses would be obtained, which cannot be handled in a satisfactory manner. It therefore was assumed that the behavior of the continuous temperature gradient could be closely approximated by an equivalent gradient composed of a large number of constant temperature elements. As the number of these elements becomes sufficiently large, the behavior of the continuous gradient and the discontinuous gradient should become identical. Such an assumption is quite common in analyses of physical situations. The assumption makes no requirement of the length of each gas element other than the tacit one that each element length should be small. As a convenience in analysis, the element lengths were selected such that the time required for sound to traverse each gas element would be the same. As a further convenience in analysis an even number of elements was used.

In the derivations, all pressures were considered as absolute. However, since the derived expressions showed pressure differences, it was often possible in the digital computer program to ignore the absolute pressure level and to consider only over or underpressure. Of course, in determining gas densities, the absolute pressure must be used. The quantity, J, is used in analysis. When J is one the overpressure has just occurred. After the length of time it takes sound to traverse one element, J has increased to two. After each such time increment, J is increased by one.

At time [J = 1], with reference to figure 8, the process is just ready to start. In each element the overpressure and gas velocity are

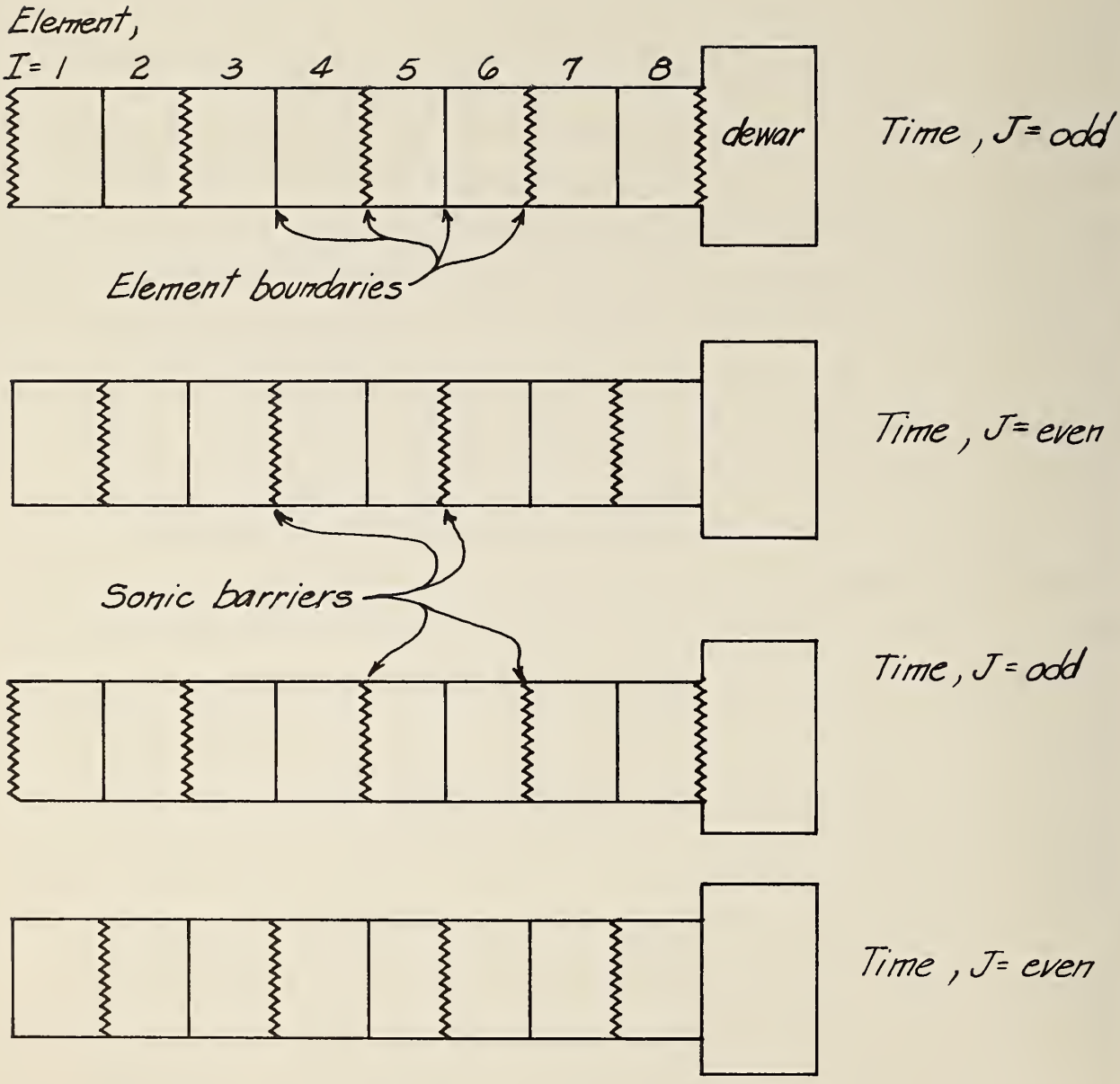

Figure 8. Sonic Front Patterns At Odd And Even Calculation Times.

zero. In the dewar there is a small overpressure λ. A moving sonic front exists at the tube end between the dewar overpressure and the tube.

At time [J = 2] the dewar overpressure has penetrated across the gas element nearest the dewar, element [I =N]. Because of the discontinuity in gas properties between element [I =N] and [I = N - 1], a portion of the overpressure is transmitted into element [I = N - 1] and a portion is reflected through element [I = N]. The values of the resulting gas velocities and pressures will be derived later. At time [J = 2] there is no longer a sonic front at the tube end but there now is one between elements [I = N] and [I = N - 1].

At time [J = 3] the transmitted wave has traversed element [I = N - 1] and is at the border of elements [I = N - 1] and [I = N - 2]. At the same time the reflected wave is at the tube end between the dewar and element [I = N]. At time [J = 3] there is a sonic front between the dewar and element [I = N], and between elements [I = N - 1] and [N -2].

At each succeeding time there will be a new pattern of sonic fronts. After the transmitted pulses have reached the closed end it may be seen that the pattern will be continuously repetitive. At each time [J = odd number] there will be sonic fronts at the two tube ends and at the dewar end of all even gas elements. (This was the reason for using an even number of elements.) At each time [J = even number], there will be no sonic fronts at the tube ends, but there will be some at the dewar end of all odd gas elements.

For a tube with N = 8 gas elements this situation is illustrated on figure 8.

From figure 8 it may be seen that at any time [J = odd] all gas elements are grouped in pairs, with each pair surrounded by sonic fronts.

For any time [J = even] most of the gas elements are grouped in such pairs, but elements, [I = 1] and [I = N], are arranged independently.

In the following derivations the gas velocities and pressures are derived in terms of gas velocities and pressures at previous times. The first derivation is for element pairs surrounded by sonic fronts. Separate derivations then follow for the cases, [Time J = even, Element I = 1 and I = N].

On figure 9 at time [J -2] there are two regions, [I] and [I - 1], surrounded by sonic fronts. Later at time [J] they are again in the same condition. At time [J - 2] two pressure pulses have just started towards the center. At time [J - 1] they have met at a density discontinuity, and at time [J] they are widely separated again. At the intermediate times [t_1] and [t_2] the pressures, velocities and other gas functions φ are known on each side of the sonic fronts in terms of functions φ at times [J -2, J - 1, and J].

At time [t_1] on the left side, from continuity

$$dM = (C_{I-1,\ t_1} - U_{I-1,\ J-2})\ \rho_{I-1,\ J-2}\ Adt \tag{1}$$

$$dM = (C_{I-1,\ t_1} - U_{I-1,\ J-1})\ \rho_{I-1,\ J-1}\ Adt\ . \tag{2}$$

From momentum it follows that

$$(P_{I-1,\ J-1} - P_{I-1,\ J-2}) = dM\ (U_{I-1,\ J-1} - U_{I-1,\ J-2})/(Adt)\ . \tag{3}$$

Combining (1) and (3) results in

$$(P_{I-1,\ J-1} - P_{I-1,\ J-2}) = (C_{I-1,\ t_1} - U_{I-1,\ J-2})\ \rho_{I-1,\ J-2}\ (U_{I-1,\ J-1} - U_{I-1,\ J-2}). \tag{4}$$

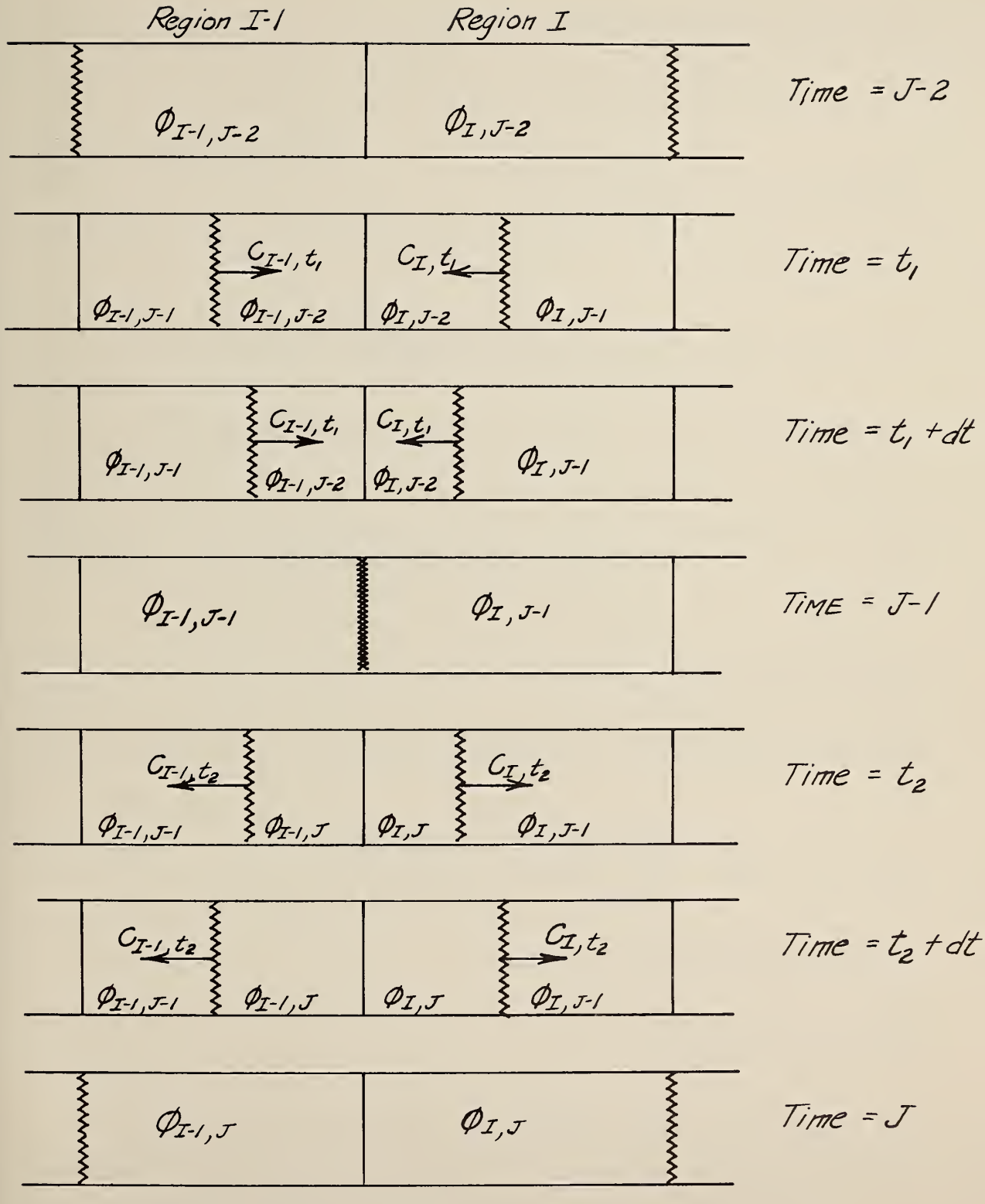

Figure 9. Element Pair Surrounded by Sonic Fronts.

Combining (1) and (2) leads to

$$(C_{I-1,t_1} - U_{I-1,J-2})\,\rho_{I-1,J-2} = (C_{I-1,t_1} - U_{I-1,J-1})\,\rho_{I-1,J-1}\,. \quad (5)$$

By algebraic manipulation of (5)

$$(C_{I-1,\,t_1} - U_{I-1,\,J-2}) =$$

$$(U_{I-1,\,J-1} - U_{I-1,\,J-2})\,\rho_{I-1,\,J-1}/(\rho_{I-1,\,J-1} - \rho_{I-1,\,J-2})\,. \quad (6)$$

Solving (4) for $(C_{I-1,\,t_1} - U_{I-1,\,J-2})$ and multiplying by (6) produces

$$(C_{I-1,\,t_1} - U_{I-1,\,J-2})^2 = \frac{P_{I-1,\,J-1} - P_{I-1,\,J-2}}{\rho_{I-1,\,J-1} - \rho_{I-1,\,J-2}}\,\frac{\rho_{I-1,\,J-1}}{\rho_{I-1,\,J-2}} \quad (7)$$

At time t_1 on the right side, by the same operations

$$(P_{I,J-2} - P_{I,\,J-1}) = (C_{I,\,t_1} + U_{I,\,J-2})\,\rho_{I,\,J-2}\,(U_{I,\,J-1} - U_{I,\,J-2}) \quad (4)'$$

$$(C_{I,\,t_1} + U_{I,\,J-2})^2 = \frac{(P_{I,\,J-2} - P_{I,\,J-1})}{(\rho_{I,\,J-2} - \rho_{I,\,J-1})}\,\frac{\rho_{I,\,J-1}}{\rho_{I,\,J-2}} \quad (7)'$$

At time t_2 on the left side

$$(P_{I-1,\,J-1} - P_{I-1,\,J}) =$$

$$(C_{I-1,\,t_2} + U_{I-1,\,J-1})\,\rho_{I-1,\,J-1}\,(U_{I-1,\,J} - U_{I-1,\,J-1}) \quad (4)''$$

.

$$(C_{I-1,\ t_2} + U_{I-1,\ J-1})^2 = \frac{P_{I-1,\ J-1} - P_{I-1,\ J}}{\rho_{I-1,\ J-1} - \rho_{I-1,\ J}} \frac{\rho_{I-1,\ J}}{\rho_{I-1,J-1}} \quad . \qquad (7)''$$

At time t_2 on the right side

$$(P_{I,\ J} - P_{I,\ J-1}) = (C_{I,\ t_2} - U_{I,\ J-1})\ \rho_{I,\ J-1}\ (U_{I,\ J} - U_{I,\ J-1}) \qquad (4)'''$$

$$(C_{I,\ t_2} - U_{I,\ J-1})^2 = \frac{P_{I,\ J} - P_{I,\ J-1}}{\rho_{I,\ J} - \rho_{I,\ J-1}} \frac{\rho_{I,\ J}}{\rho_{I,\ J-1}} \quad . \qquad (7)'''$$

It may be noted that where the velocity of the fluid is small compared with sonic velocity and where the density in a given element changes only slightly with the passage of a pulse, the velocity of sound relative to the wall is the usual expression for sonic velocity.

Since each sonic front during any time period is in a medium of constant properties, each front is at a constant velocity. From this fact and from continuity it follows that pressure and gas velocity between two adjacent sonic fronts is the same on each side of the density discontinuity.

$$P_{I-1,\ J-2} = P_{I,\ J-2} \qquad (8)$$

$$U_{I-1,\ J-2} = U_{I,\ J-2} \qquad (9)$$

$$P_{I-1,\ J} = P_{I,\ J} \qquad (10)$$

$$U_{I-1,\ J} = U_{I,\ J} \qquad (11)$$

For small pressure pulses the equations may be simplified. The density in a given region will not change appreciably, and the gas velocities will be small compared with sonic velocities. After this

simplification and using (9) through (12), (4) becomes

$$P_{I-1,\ J-1} - P_{I,\ J-2} = C_{I-1}\,\rho_{I-1}\,(U_{I-1,\ J-1} - U_{I,\ J-2}) \tag{12}$$

$$P_{I,\ J-2} - P_{I,\ J-1} = C_I\,\rho_I\,(U_{I,\ J-1} - U_{I,\ J-2}) \tag{12}'$$

$$P_{I-1,\ J-1} - P_{I,\ J} = C_{I-1}\,\rho_{I-1}\,(U_{I,\ J} - U_{I-1,\ J-1}) \tag{12}''$$

$$P_{I,\ J} - P_{I,\ J-1} = C_I\,\rho_I\,(U_{I,\ J} - U_{I,\ J-1})\ . \tag{12}'''$$

Adding (12) to (12)' produces the intermediate equation

$$P_{I-1,\ J-1} - P_{I,\ J-1} = C_{I-1,}\,\rho_{I-1}\,(U_{I-1,J-1} - U_{I,\ J-2})$$

$$+ C_I\,\rho_I\,(U_{I,\ J-1} - U_{I,\ J-2})\ .$$

Adding (12)'' to (12)''' gives

$$P_{I-1,\ J-1} - P_{I,\ J-1} = C_{I-1}\,\rho_{I-1}\,(U_{I,\ J} - U_{I-1,\ J-1})$$

$$+ C_I\,\rho_I\,(U_{I,\ J} - U_{I,\ J-1})\ .$$

Upon adding the intermediate equations

$$(U_{I,\ J} - U_{I,\ J-2}) = 2\,\frac{P_{I-1,\ J-1} - P_{I,\ J-1}}{C_I\,\rho_I + C_{I-1}\,\rho_{I-1}}\ . \tag{13}$$

Upon subtracting the intermediate equations

$$(U_{I,\ J} + U_{I,\ J-2}) = 2\,\frac{U_{I,\ J-1}\left(\frac{1}{C_{I-1}\,\rho_{I-1}}\right) + U_{I-1,J-1}\left(\frac{1}{C_I\,\rho_I}\right)}{\left(\frac{1}{C_I\,\rho_I}\right) + \left(\frac{1}{C_{I-1}\,\rho_{I-1}}\right)}\ . \tag{13}'$$

Similarly

$$(P_{I,\ J} - P_{I,\ J-2}) = 2\frac{U_{I-1,\ J-1} - U_{I,\ J-1}}{\left(\frac{1}{C_I\ \rho_I}\right) + \left(\frac{1}{C_{I-1}\ \rho_{I-1}}\right)}\ , \tag{14}$$

$$(P_{I,\ J} + P_{I,\ J-2}) = 2\frac{P_{I,\ J-1}\ (C_{I-1}\ \rho_{I-1}) + P_{I-1,\ J-1}\ (C_I\ \rho_I)}{C_I\ \rho_I + C_{I-1}\ \rho_{I-1}}\ . \tag{14}'$$

Equations (13) and (14) determine the gas velocity and pressure at any time [J = odd] for any gas element [I = even] in terms of previously determined velocities and pressures. In each element [(I - 1) = odd], the velocities and pressures are identical to those in the elements [I = even] because of (8), (9), (10), and (11).

Equations (13) and (14) also determine the gas velocity and pressure at any time [J = even] for all gas elements [I = odd] except for the first element [I = 1]. In each element [(I - 1) = even], the velocities and pressures are identical to those in the elements [I = odd].

For all times [J = even] the pressure in the element [I = N] is dewar pressure. The gas velocities are obtained with the aid of figure 10. Using the principles of momentum and continuity as before

$$U_{N,\ J} - U_{N,\ J-1} = \frac{(P_{N,\ J-1} - P_{N,\ J})}{\rho_{N,\ J-1}\ (C_{N,\ t_1} + U_{N,\ J-1})}\ . \tag{15}$$

For small pulses the density does not change appreciably from one time to another. Also the velocity of the gas is small compared with sonic velocity. So for small pulses for all times [J = even] the gas velocity in the element [I = N] may be found from,

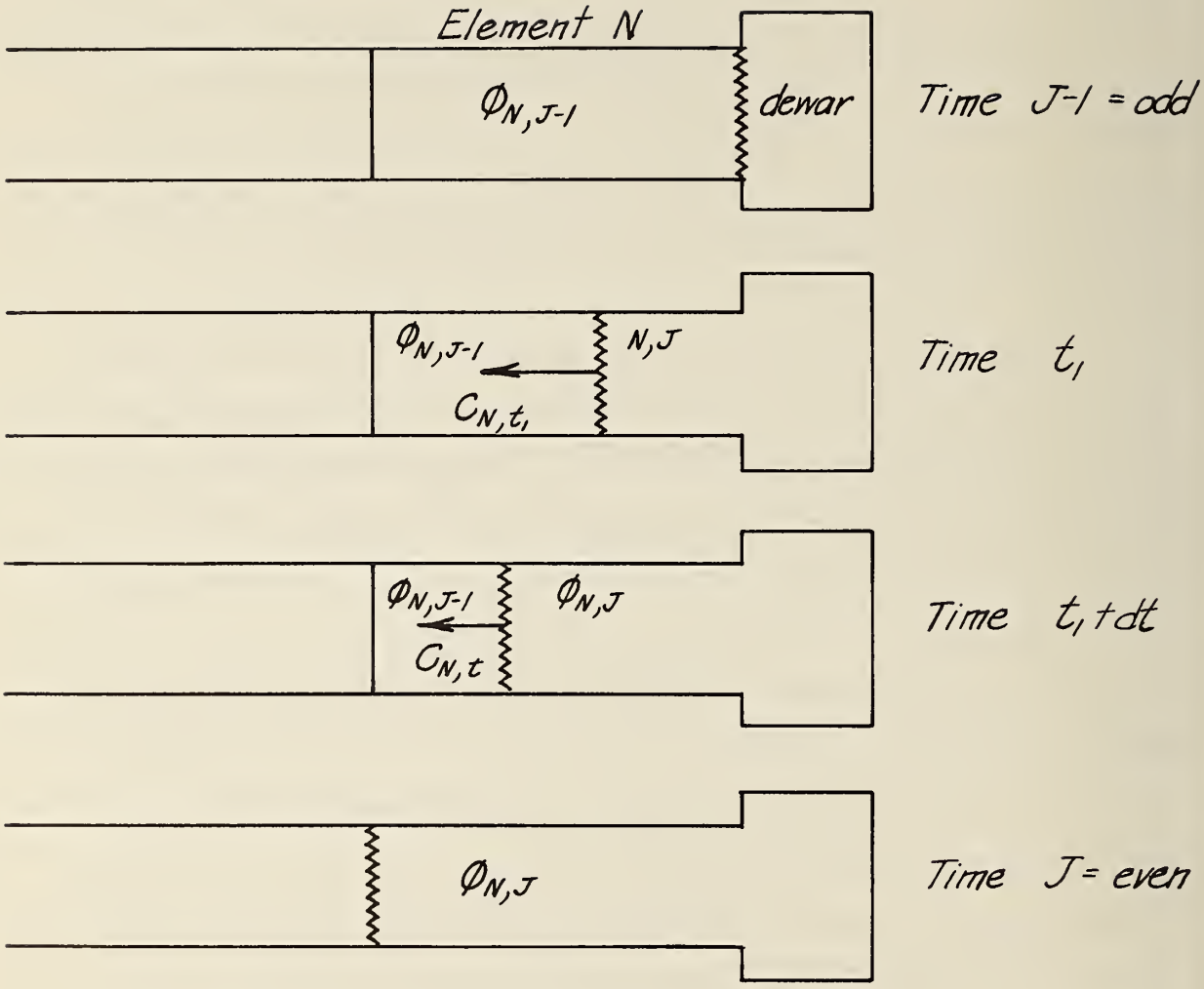

Figure 10. Sonic Front in Element N.

$$U_{N,\ J} - U_{N,\ J-1} = \frac{P_{N,\ J-1} - P_{N,\ J}}{\rho_N C_N} \quad . \tag{16}$$

For all times [J = even], from continuity, the velocity in the element [I = 1] is zero. The gas pressures are obtained with the aid of figure 11. Using the momentum and continuity principles again and assuming small pressure pulses yields

$$P_{1,\ J} - P_{1,\ J-2} = -2\, C_1\, \rho_1\, U_{1,\ J-1} = -2\, C_1\, \rho_1\, U_{2,\ J-1} \quad . \tag{17}$$

Equations have now been derived which show gas velocity and pressure in each of N gas elements at any calculation time J provided the velocities and pressures are known for the previous two calculation times. For the first calculation time [J = 1], all velocities and pressures are zero. For the second calculation time [J = 2] all velocities and pressures are zero except for the element [I = N]. In that element the pressure is λ above original and the velocity is obtained from equation (16).

Nomenclature for Appendix A

Symbols

P	Pressure	dynes/cm^2
C	Sonic velocity relative to tube	cm/sec
ρ	Density	g/cm^3
U	Gas velocity (positive toward dewar)	cm/sec
λ	Initial overpressure	dynes/cm^2
M	Mass	g
φ	Sonic function, pressure, density, velocity as determined by context.	
A	Area	cm^2

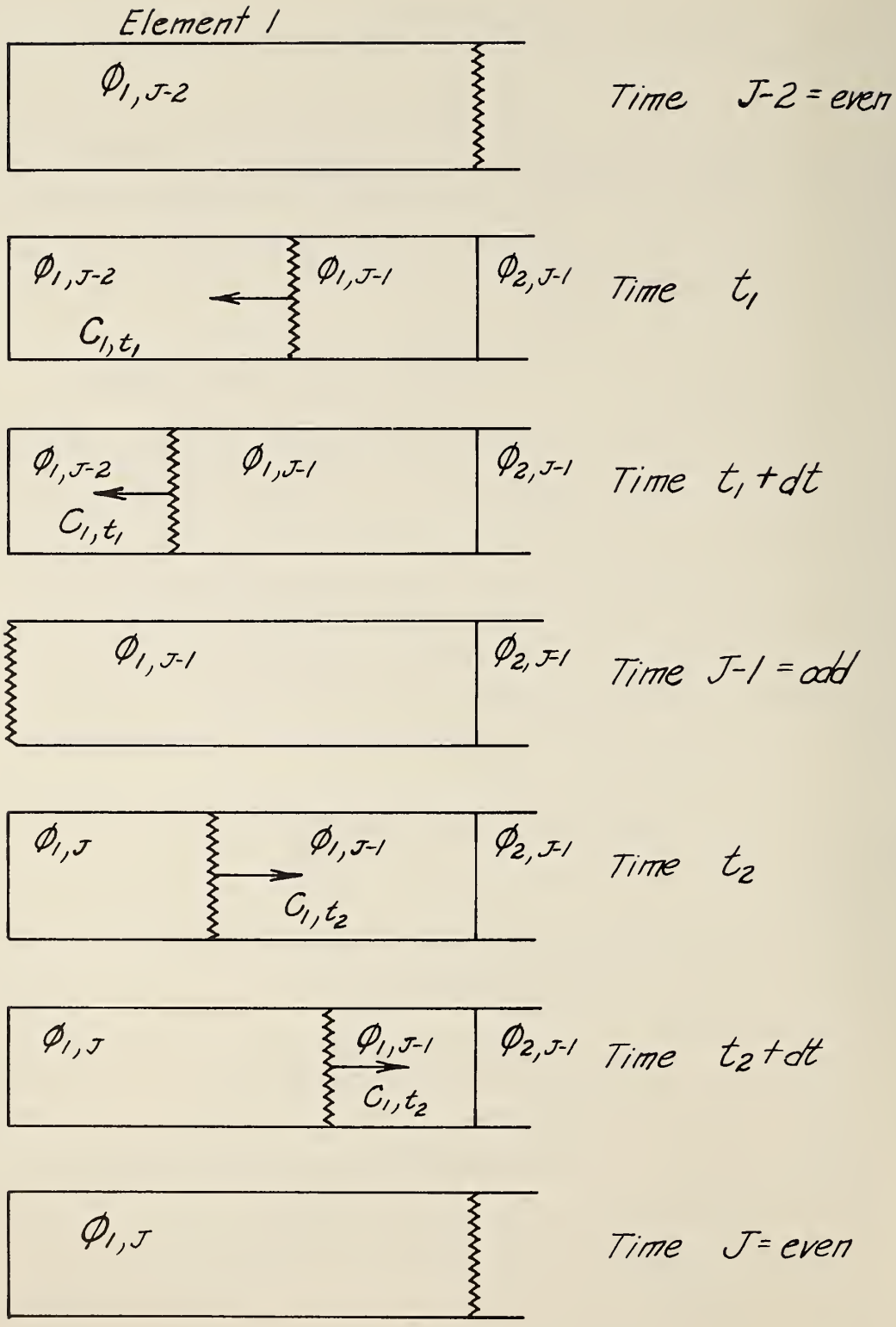

Figure 11. Sonic Front in Element 1.

Nomenclature for Appendix A (continued)

Subscripts

I	Gas element, I = 1 at closed end, N at open end
N	Gas element closest to dewar
J	Calculation time. At J = 1 process is just ready to start. Time between J and J + 1 is time required for sound to traverse one gas element.

Where double subscripts are used, the first refers to a gas element, the second to the calculation time interval.

15. Appendix B

The infinitesimal net work done by a gas element in a short time interval is,

$$dW = P\,dV - dF$$

where

dW = work done --- ergs

P = pressure --- dynes/cm^2

dV = volume change --- cm^3

dF = friction work --- ergs .

From the gas laws we have,

$$PV = MRT$$

$$\frac{dV}{V} = \frac{dT}{T} - \frac{dP}{P}$$

$$PdV = MR\,dT - VdP$$

where

R = gas constant --- ergs/g-°K

T = gas temperature --- °K

M = gas mass --- g .

The temperature rise may be expressed as,

$$dT = \frac{dQ - dW}{M\,C_v}$$

where

dQ = heat received --- ergs

C_v = specific heat --- ergs/g-°K.

Substituting,

$$PdV = \frac{R}{C_v}(dQ - dW) - VdP$$

$$PdV = \frac{R}{C_v} dQ - \frac{R}{C_v} (PdV - dF) - VdP$$

$$PdV = \frac{\frac{R}{C_v} dQ + \frac{R}{C_v} dF - VdP}{\left(1 + \frac{R}{C_v}\right)}$$

$$PdV = \frac{(\gamma-1)}{\gamma} dQ + \frac{(\gamma-1)}{\gamma} dF - \frac{VdP}{\gamma}$$

$$dW = PdV - dF = \left(\frac{\gamma-1}{\gamma}\right) dQ - \frac{VdP}{\gamma} - \frac{dF}{\gamma}$$

where,

γ = specific heat ratio.

In Fortran notation this becomes,

```
DWK  = C9*DQ-VOLM/XK*(P(2)-P(1))-DFR/XK
```

where,

DWK	=	net work	- - -	ergs
XK	=	specific heat ratio		
C9	=	(XK-1)/XK		
DQ	=	heat received	- - -	ergs
VOLM	=	average volume	- - -	cm^3
P(2)	=	overpressure at end of interval	- - -	$dynes/cm^2$
P(1)	=	overpressure at start of interval	- - -	$dynes/cm^2$
DFR	=	friction work	- - -	ergs .

16. Appendix C

When a fluid is moving in a pipe there will be a frictional force between any gas element and the wall. This is expressed in equation (6-2c) of McAdams (1954) as

$$\tau = f\left(\frac{\rho V^2}{2}\right)$$

where

τ = force per unit wall area - - - dynes/cm^2

f = Fanning friction factor

ρ = fluid density - - - g/cm^3

V = fluid velocity - - - cm/sec .

From this, the frictional work overcome by an element with a given area is,

$$F\ dX = f\left(\frac{\rho V^2}{2}\right) A\ V\ dt$$

where

F = wall force - - - dynes

dX = distance moved - - - cm

A = wall contact area - - - cm^2

V = fluid velocity - - - cm/sec

dt = time increment - - - sec .

In Fortran notation

```
DFR = ABSF(FRF/2.*AREA*ROE*VEL**3*DT)
```

where

DFR = friction work - - - ergs

AREA = element wall contact area - - - cm^2

ROE = average density - - - g/cm^3

VEL = fluid velocity - - - cm/sec .

DT = time increment - - - sec .

The absolute value is used because friction work is always a positive number to be subtracted from the work done.

17. Appendix D. Program THM LOS

```
      PROGRAM THMLOS
      CALL READ DATA
      CALL ELEMENT
      CALL AMPLITUD
      END THMLOS
C
```

18. Appendix E. Subroutine READ DATA for Amplitude

```
      SUBROUTINE READ DATA
C FRICTION FACTOR (FRF) IS READ, BUT IT MAY BE CHANGED IN (HEAT TR)
C IF THE INPUT FRICTION FACTOR (FRF) IS ZERO,THE QUANTITY (IFRF) IS SET EQUAL TO
C ZERO IN SUBROUTINE (AMPLITUD), AND THE SUBROUTINE (HEAT TR) COMPUTES THE HEAT
C TRANSFER COEFFICIENT ACCORDING TO A FORMULA IN MC ADAMS, MODIFIED BY THE
C MULTIPLIER (NFR).
C
C IF THE INPUT FRICTION FACTOR(FRF)IS NON ZERO, THE QUANTITY (IFRF) IS SET EQUAL
C TO UNITY IN SUBROUTINE (AMPLITUD) AND THE INPUT VALUE OF THE FRICTION FACTOR
C IS USED REGARDLESS OF REYNOLDS NUMBER (REYNO).
      COMMON/1/PSTART,DIAM,FRF,FACTOR,AVGFREQ,PM,N,NJJ,PI,AF,XK,CPG,M,
     1 TMP(32),S(32),TMRVIS(16),VIS(16),TCONGAS(12),CONGAS(12),ZNU(16),
     1 GRPR(16),CVG,RHODEW,TDEW,R,JD
      COMMON/4/NFR
      TYPE REAL NFR
    5 READ8001,PSTART,DIAM,FRF,FACTOR,AVGFREQ,PM,N,NJJ,NFR,JD
 8001 FORMAT(6F10.0,2I5,F5.0,I5)
   10 READ 8000,(TMP(I),I=1,32) $ READ 8000,(S(I),I=1,32)
   15 READ 8000,(TMR VIS(IN),IN=1,16) $READ 8000,(VIS    (IN),IN=1,16)
   20 READ8000,(T CON GAS(IN),IN=1,12)$READ8000,(CON GAS(IN),IN=1,12)
   25 READ 8000,(Z NU   (IN),IN=1,16) $READ 8000,(GR PR  (IN),IN=1,16)
 8000 FORMAT(16F5.0)
CONVERSION OF INPUT DATA TO UNITS USED IN CALCULATIONS
   30 DO 40012 K=1,32 $S(K)=S(K)*2.54
40012 CONTINUE  $DIAM=DIAM*2.54
   35  DO 40011 K=1,16 $VIS(K)=VIS(K)/1 000 000.
40011 CONTINUE
   40  DO 40010 K=1,12 $CONGAS(K)=CONGAS(K)*10 000.
40010 CONTINUE
C VISCOCITY OF GAS,VIS,IS READ IN MICROPOISE AND CONVERTED TO POISE
C CONDUCTIVITY,CON GAS, IS READ IN MILLIWATTS/(CM DEG K)AND CONVERTED TO
C ERG/(S CM DEG K). TMR VIS,DEG K,GOES WITH VIS
C T CON GAS, DEG K, GOES WITH CON GAS.   FOR THE GRASHOF
C PRANDTL PRODUCT,GR PR,VERSUS THE NUSSELT NUMBER,Z NU, THE LOG 10 OF THE
C QUANTITIES ARE READ IN WITH NO CONVERSION.  THE LENGTH S IS READ IN
C INCHES AND CONVERTED TO CENTIMETERS. THE SAME IS TRUE FOR DIAM
C
   45 WT=4.003$RU=8.317E07$R=RU/WT$PI=3.1415926536$AF=PI/4.*DIAM**2
   50 XK=5./3.$ CPG=R*XK/(XK-1.)$CVG=R/(XK-1.)
   55 T DEW=4.3 $M=N+N/2 $RHODEW=PSTART/R/TDEW
COMPUTATIONAL VALUES ARE PRINTED EXCEPT VISCOCITY IS PRINTED IN MICROPOISE
   60 PRINT 9310
 9310 FORMAT(//)
   65 PRINT 9000,PSTART,DIAM,FACTOR,AVGFREQ,PM,N,NJJ,NFR,JD
   70 PRINT9001,(TMR VIS(K),K=1,16) $PRINT 9003,(VIS(K),K=1,16)
   75 PRINT 9002,(T CON GAS(K),K=1,12) $PRINT 9004,(CON GAS(K),K=1,12)
   80 PRINT 9005,(Z NU(K),K=1,16) $ PRINT 9006,(GR PR  (K),K=1,16)
   85 PRINT 9009,(TMP(K),K=1,32) $PRINT 9010,(S(K),K=1,32)
 9000 FORMAT(5X6HPSTART,6X4HDIAM,4X6HFACTOR,3X7HAVGFREQ,
     1 8X2HPM,9X1HN,7X3HNJJ,7X3HNFR,8X2HJD/
     1 1XF10.2,F10.4,F10.6,F10.4,F10.0,2I10,F10.2,I10)
 9001 FORMAT(//1X7HTMR VIS,2F7.2,14F8.2)
 9002 FORMAT(1X9HT CON GAS,F5.2,F7.2,14F8.2)
 9003 FORMAT(1X9HVIS (E06),1(6PF5.2),1(6PF7.2),14(6PF8.2))
 9004 FORMAT(1X6HCONGAS,F8.0,F7.0,14F8.0)
 9005 FORMAT(1X6HZ NU  ,F7.3,15F8.3)
```

```
 9006   FORMAT(1X6HGR PR ,F7.3,15F8.3)
 9009   FORMAT(1X6HTMP   ,F7.3,15F8.3)
 9010   FORMAT(1X6HS     ,F7.3,15F8.3)
99999   END READ DATA
C
```

19. Appendix F. Subroutine ELEMENT for Amplitude

```
      SUBROUTINE ELEMENT
CONSTANT LENGTH OF EACH ELEMENT TYPE
CALCULATION OF ELEMENT LENGTHS AND TEMPERATURES
      COMMON/1/PSTART,DIAM,FRF,FACTOR,AVGFREQ,PM,N,NJJ,PI,AF,XK,CPG,M,
     1 TMP(32),S(32),TMRVIS(16),VIS(16),TCONGAS(12),CONGAS(12),ZNU(16),
     1 GRPR(16),CVG,RHODEW,TDEW,R,JD
      COMMON/2/TEMP(97),XX(97),RHO(96),XMID(96),GM(96),TMG(96)
      TYPE REAL NFR
    5 TEMP(1)=TMP(1) $ XX(1)=S(1) $NP1=N+1 $MP1=M+1
   10 DXX=(S(32)-S(1))/N
   15  DO 40000 I=1,N  $XX(I+1)=XX(I)+D XX
   20  DO 40001 K=2,32 $IF(S(K).GE.XX(I+1))GO TO 33
40001 CONTINUE $K=32
   33 TEMP(I+1)=TMP(K)+(TMP(K)-TMP(K-1))/(S(K)-S(K-1))*(XX(I+1)-S(K))
   35 TMG(I)=(TEMP(I)+TEMP(I+1))*0.5 $RHO(I)=PSTART/R/TMG(I)
   40 X MID(I)=(XX(I)+XX(I+1))*0.5 $GM(I)=RHO(I)*AF*DXX
40000 CONTINUE
   45  DO 40002 I=NP1,M$TEMP(I+1)=TMG(I)=T DEW
   50 XX(I+1)=XX(I)+D XX $XMID(I)=(XX(I)+XX(I+1))*0.5 $RHO(I)=RHO DEW
   55 GM(I)=RHO(I)*AF*D XX
40002 CONTINUE
   60 PRINT 9310
 9310 FORMAT(/)
   65 PRINT 8038,(TEMP(K),K=1,NP1)
 8038 FORMAT(1X 4HTEMP,10F8.2,7F7.2)
   70 PRINT 8039,(XX(K),K=1,NP1)
 8039 FORMAT(3X 2HXX,10F8.2,7F7.2)
   75 PRINT 9206,(X MID(K),K=1,N)
 9206 FORMAT(4X 5HX MID,10F8.2,6F7.2)
   80 PRINT 9205,(TMG(K),K=1,N)
 9205 FORMAT(6X 3HTMG,10F8.2,6F7.2)
   85 PRINT 9000,(GM(K),K=1,N)
 9000 FORMAT(7X2HGM,8E12.4)
   90 PRINT 9207,(RHO(K),K=1,N)
 9207 FORMAT(6X3HRHO,8E12.4)
 9208 FORMAT(8X1HC,8E12.4)
   95 PRINT 9310 $PRINT8038,(TEMP(K),K=NP1,MP1)
  100 PRINT 8039,(XX(K),K=NP1,MP1) $ PRINT 9206,(X MID(K),K=NP1,M)
  105 PRINT 9205,(TMG(K),K=NP1,M) $ PRINT9000,(GM(K),K=NP1,M)
  110 PRINT 9207,(RHO(K),K=NP1,M)
99999 END ELEMENT
C
```

```
      SUBROUTINE HEAT TR(ITESTFRF)
      COMMON/1/PSTART,DIAM,FRF,FACTOR,AVGFREQ,PM,N,NJJ,PI,AF,XK,CPG,M,
     1 TMP(32),S(32),TMRVIS(16),VIS(16),TCONGAS(12),CONGAS(12),ZNU(16),
     1 GRPR(16),CVG,RHODEW,TDEW,R,JD
      COMMON/2/TEMP(97),XX(97),RHO(96),XMID(96),GM(96),TMG(96)
      COMMON/3/G(2),P(2),T(  360),PRES(2),X(2,360),XAVG(2),EKE(360),
     1 VOL(   2),RO(   2),FR(96),ZKE(96),Q(96),WORK(96),IFRF,
     1 VEL,ROE,THETA,AREA,DQ,DT,TWI,TAVG,REYNO,HHT1,HHT2,HHT
      COMMON/4/NFR
      TYPE REAL NFR
    1 IF(ITESTFRF.EQ.1)GO TO 386
    2  DO 40014 K=2,12 $IF(T CON GAS(K).GE.T AVG )GO TO 3
40014  CONTINUE $K=12
    3  CON G=CON GAS(K)+(CON GAS(K)-CON GAS(K-1 ))/(T CON GAS(K)-
     1 T CON GAS(K-1 ))*(T AVG -T CON GAS(K))
    4   DO 40013 K=2,16 $IF(TMR VIS(K).GE.T AVG )GO TO 5
40013  CONTINUE $K=16
    5  VISC=VIS(K)+(VIS(K)-VIS(K-1 ))/(TMR VIS(K)-TMR VIS(K-1 ))*
     1 (T AVG -TMR VIS(K))
    6  IF(VISC.EQ.0.)GO TO 400
    8  REYNO=ABSF(DIAM*VEL*ROE/VISC)
   10 IF(REYNO.LT.0.001)REYNO=0.001
   20  CONTINUE
   22 IF(CONG)400,400,25
   25  PRANDTL=CPG    *VISC/CON G
   26 IF(PRANDTL) 400,400,30
   30 IF(TAVG)400,400,31
   31  ZNUSSELT=0.023*REYNO**0.8*PRANDTL**0.4
   32  HHT1=  ZNUSSELT*CON G/DIAM*FACTOR
   33  THETA=TWI-T AVG  $IF(THETA) 35,385,35
   35  GP=CPG*ROE**2*980.66/T AVG*DIAM**3/VISC/CONG*ABSF(THETA)
   37 IF(GP) 410,40,40
   40 IF(GP.EQ.0.)ZNUS=0.
   41 IF(GP.EQ.0.)GO TO 380
   42  GP L=LOGF(GP)*0.43429
   43   DO 40016 K=2,16 $IF(GR PR(K).GE.GP L)GO TO 370
40016  CONTINUE $K=16
  370  ZNUL=ZNU(K)+(ZNU(K)-ZNU(K-1 ))/
     1 (GR PR(K)-GR PR(K-1 ))*(GP L-GR PR(K))
  375  ZNUS=10.**ZNUL
  380  HHT2=ZNUS*CON G/DIAM*FACTOR $HHT   =HHT1   +HHT2
  385  DQ=HHT*AREA*THETA*DT
  386  CONTINUE $IF(     REYNO .LT.0.001)REYNO=0.001
 2386 IF(IFRF.EQ.1)GO TO 99999
  387 IF(REYNO.LT.1000)FRF=16./REYNO *NFR
  388 IF(REYNO.LT.1000)GO TO 99999
  389  FRF=0.01*NFR
  390  FRF1=FRF
  392  FRF=(1./(3.2*0.43429*LOGF(REYNO*SQRTF(FRF1))+1.2))**2*NFR
  393  IF(ABSF((FRF1-FRF)/FRF).GT.0.0001)GO TO 390
  395  GO TO 99999
  400  PRINT 9000,CON G,VISC,PRANDTL,TWI,T AVG
 9000  FORMAT(//16X5HCON G,16X4HVISC,13X7HPRANDTL,17X3HTWI,15X5HT AVG/
     1 1X5E20.5)
  410  PRINT 9001,CON G,VISC,GP,TWI,T AVG
 9001  FORMAT(//16X5HCON G,16X4HVISC,18X2HGP     ,17X3HTWI,15X5HT AVG/
```

```
     1 1X5E20.5)
  420  DT=0.
99999  END HEAT TR
C
```

21. Appendix H. Subroutine AMPLITUD

```
      SUBROUTINE AMPLITUD
COMPUTATION ASSUMES PRESSURE VARIATION IS A COSINE FUNCTION
      DIMENSION PX(96),KOUNTMAX(96),JJMAX(96)
      COMMON/1/PSTART,DIAM,FRF,FACTOR,AVGFREQ,PM,N,NJJ,PI,AF,XK,CPG,M,
     1 TMP(32),S(32),TMRVIS(16),VIS(16),TCONGAS(12),CONGAS(12),ZNU(16),
     1 GRPR(16),CVG,RHODEW,TDEW,R,JD
      COMMON/2/TEMP(97),XX(97),RHO(96),XMID(96),GM(96),TMG(96)
      COMMON/3/G(2),P(2),T(  360),PRES(2),X(2,360),XAVG(2),EKE(360),
     1 VOL(   2),RO(   2),FR(96),ZKE(96),Q(96),WORK(96),IFRF,
     1 VEL,ROE,THETA,AREA,DQ,DT,TWI,TAVG,REYNO,HHT1,HHT2,HHT
      TYPE REAL NFR
     IF(FRF.NE.0.)IFRF=1 $IF(FRF.EQ.0.)IFRF=0
    1  PRINT 9004
    2  DO 40009 K=1,7 $REYNO=10.**K $ITESTFRF=1 $CALL HEAT TR(ITESTFRF)
    3  PRINT 9005,REYNO,FRF
40009  CONTINUE
 9004  FORMAT(//10X,5X5HREYNO,7X3HFRF)
 9005  FORMAT(10X,2E10.3)
      PRINT 9006
    4  PRINT 9006
    5   DO 40004 J=1,360 $X(2,J)=XX(1) $EKE(J)=0.0
40004  CONTINUE
    6  NJJL2=NJJ-2
   10  DT=JD/360./AVGFREQ  $NL1=N-1 $ NP1=N+1     $ SUM WORK=SUM Q=0.
   15  SUMFR=GID2=SUMEKE=0. $ROID2=RHODEW
   25  MP1=M+1 $XF=XX(N+1) $C8=PI*DIAM $C9=(XK-1.)/XK  $XL=XF-XX(1)
   30   DO 40000 I=1,M
   31  JJMAX(I)=0
   32  G(2)=GM(I)
   35   DO 40008 K=1,360 $X(1,K)=X(2,K)  $T(K)=TMG(I)
40008  CONTINUE
   40  IQ=0
   45  VOL(2)=GM(I)/RHO(I)
   50  X(2,360)=X(1,360)+VOL(2)/AF
   55  RO(2)=RHO(I) $XAVG(2)=(X(1,360)+X(2,360))*0.5$Q(I)=0.
   60   DO 40002 JJ=1,NJJ $IF(IQ.GT.2)GO TO 40002
   61 IF(JJ.GT.JJMAX(I))JJMAX(I)=JJ
   65  QI=Q(I) $KOUNTMAX(I)=0 $WORKI=WORK(I)
   70  FR(I)=   ZKE(I)=Q(I)=WORK(I)=0.
   80  DO 40003 J=JD,360,JD $R2=J*PI/180.$SIN2=SINF(R2)
   85  IF(J.EQ.JD)JLJD=360 $IF(J.GT.JD)JLJD=J-JD
   90  RO(  1)=RO(  2) $VOL(  1)=VOL(  2)
   95  XAVG(  1)=XAVG(  2)  $ROID1=ROID2
  100  PRES(  1)=PRES(  2) $G(  1)=G(  2) $ P  (  1)=P  (  2)
  105  KOUNT=0      $GID1=GID2
  110 IF(X(1,J).GE.XF.AND.X(1,JLJD).GE.XF)GO TO 690
  120  DQ=AQ       =0.
  240  CONTINUE $KOUNT=KOUNT+1 $DQ1=AQ
  241 IF(KOUNT.GT.KOUNTMAX(I))KOUNTMAX(I)=KOUNT
  245  X(  2,J )=X(1,J )+VOL(  2)/AF
  250  XAVG(  2)=(X(  2,J )+X(1,J ))*0.5
  255  VEL=(XAVG(  2)-XAVG(  1))/DT
  260  TAVG=(T(JLJD)+T(J))*0.5
  265 IF(XAVG(2).LT.XF)PX(I)=PM*COSF(PI/2.*(XAVG(2)-XX(1))/XL)
  270 IF(XAVG(  2).GE.XF)PX(I)=0.
  275  P(  2)=PX(I)*SIN2 $PRES(  2)=P(  2)+PSTART
```

```
  280  PRESS=(PRES(  1)+PRES(  2))*0.5 $ROE=(RO(  2)+RO(  1))*0.5
  300  IF(X(2,JLJD).GT.XF.OR.X(2,J).GT.XF)GO TO 500
  305  VOLM=GM(I)/ROE
  400  AREA=C8*(X(2,J)-X(1,J)+X(2,JLJD)-X(1,JLJD))*0.5
  404  G(2)=GM(I)
  405  AVGX=(XAVG(  1)+ XAVG(  2)) *0.5
  410   DO 40005  K=2,NP1 $IF(XX(K).GE.AVGX)GO TO 420
40005  CONTINUE $K=NP1
  420  TWI=TEMP(K)+(TEMP(K)-TEMP(K-1))/(XX(K)-XX(K-1))*(AVGX-XX(K))
  425      ITESTFRF=0 $CALL HEAT TR(ITEST FRF)
  430 IF(DT.EQ.0.)GO TO 99999
  435  D FR=ABSF(FR F/2. *AREA*ROE*VEL**3*DT)
  440  DWK    =C9*DQ-VOLM/XK*(P(  2)-P(  1))-DFR/XK
  455  QW=DQ-DWK        $AQ=DQ
  460  T(J)=T(JLJD)+QW/GM(I)/CVG
  465  RO(  2)=PRES(  2)/R/T(  J)
  470  VOL(  2)=GM(I)/RO(  2)
  475 IF(KOUNT.LT.2)GO TO 240
  480 IF(KOUNT.GT.10.)GO TO 700
  485 IF(AQ.EQ.0.)AQ=0.000001 $ IF(ABSF((AQ-DQ1)/AQ).GT.0.0001 )GO TO240
  490  GO TO 700
  500 IF(X(  2,J ).GE.XF)C2=XF $IF(X(  2,J ).LT.XF)C2=X(  2,J )
  510 IF(X(1  ,J ).GE.XF)W2=XF $IF(X(1  ,J ).LT.XF)W2=X(1  ,J )
  520  IF(X(2,JLJD).GE.XF)C1=XF $IF(X(2,JLJD).LT.XF)C1=X(2,JLJD)
  530 IF(X(1,JLJD).GE.XF)W1=XF $IF(X(1,JLJD).LT.XF)W1=X(1,JLJD)
  550 IF(VEL.GT.0.0)EKE(J)=0.5*(VEL*AF*ROE*DT)*VEL*VEL
  551 IF(VEL.LE.0.0)EKE(J)=0.0
  600  AREA1=(C1-W1)*C8 $ AREA2=(C2-W2)*C8 $AREA=(AREA1+AREA2)*0.5
  605  AVGX=0.25*(W2+W1+C2+C1)
  610   DO 40006 KR=1,NL1 $K=N -KR $IF(XX(K).LE.AVGX)GO TO 620
40006  CONTINUE
  620  TWI=TEMP(K)+(TEMP(K)-TEMP(K+1))/(XX(K)-XX(K+1))*(AVGX-XX(K))
  625      ITEST FRF=0 $CALL HEAT TR(ITESTFRF)
  630 IF(DT.EQ.0.0)GO TO 99999
  635  D FR=ABSF(FR F/2. *AREA*ROE*VEL**3*DT)
  639  VOLM=(G(1)+G(2))*0.5/ROE
  640  DWK    =C9*DQ-VOLM/XK*(P(  2)-P(  1))-DFR/XK
  655  QW=DQ-DWK        $AQ=DQ
  670  VOLIP2=AREA2*DIAM/4. $G(2)=RO(2)*VOLIP2 $GID2=GM(I)-G(2)
  674 IF(GID2.GT.GID1.AND.GID2.GT.0.)
     1 ROID2=(ROID1*GID1+RO(1)*(GID2-GID1))/GID2
  675 IF(GID2.LE.GID1)ROID2=ROID1
  676  VOLID2=GID2/ROID2   $VOL(2)=VOLIP2+VOLID2
  680 IF(G(  2).LE.0.)T(  J)=T DEW $ IF(G(  2).LE.0.)GO TO 687
  681  IF(G(2)-G(1))685,685,683
  683  T(J)=(T(JLJD)*G(1)+TDEW*(G(2)-G(1)))/G(2)+QW/CVG/G(2)
  684  GO TO 687
  685  T(J)=T(JLJD)+QW/CVG/G(2)
  687  CONTINUE
  688  RO(  2)=PRES(  2)/R/T(  J) $IF(KOUNT.LT.2 )GO TO 240
 2688 IF(KOUNT.GT.10)GO TO 700
  689 IF(AQ.EQ.0.)AQ=0.000001 $ IF(ABSF((AQ-DQ1)/AQ).GT.0.0001 )GO TO240
 2689    GO TO 700
  690  DQ=DQ1=AQ=DWK=QW=D FR=0. $REYNO=.001 $HHT1=HHT2=HHT=0.
  691  ITEST FRF=1 $CALLHEAT TR(ITEST FRF)
  692  G(  2)=0. $T(  J)=T DEW $RO(  2)=ROID2   $VOL  (2)=GM(I)/RO(  2)
```

```
  693  X(  2,J )=X(1,J )+VOL(  2)/AF $XAVG(  2)=(X(2  ,J )+X(1,J ))*0.5
  694  VEL=(XAVG(  2)-XAVG(  1))/DT
  695  PRES(  2)=PSTART
  696  TWI=0. $KOUNT=KOUNT+1
  697 IF(KOUNT.GT.KOUNTMAX(I))KOUNTMAX(I)=KOUNT
  700  CONTINUE
  701  WORK(I)=WORK(I)+DWK $ Q(I)=Q(I)+DQ $FR(I)=FR(I)+D FR
  703 IF(VEL.GT.0.)DKE=0.5*(VEL*AF*ROE*DT)*VEL*VEL
  704 IF(VEL.LE.0.)DKE=0. $ZKE(I)=ZKE(I)+DKE
  705 IF(IQ.LT.1)GO TO 40003
  711 IF(J.EQ.90)PRINT 9001
 9001  FORMAT(///3X,1HI,3X,1HJ,1X,2HJJ,1X,5HKOUNT,  5X,6H  P(2),4X,7H  RO(2),
     1 5X,6H  T(J),5X,3HTWI,8X,3HVEL,9X,2HDQ,8X,3HDWK,8X,3HDFR,3X,8H  X(2,J),
     1 1X,8H  VOL(2),5X,6H  G(2)/63X,6X,5HREYNO,7X,4HHHT1,
     1 7X,4HHHT2,8X,3HHHT,8X,3HFRF)
  712 IF(J.EQ.90.OR.J.EQ.180.OR.J.EQ.270.OR.J.EQ.360)
     1 PRINT 9000,I,J,JJ,KOUNT,P  (  2),RO(  2),T(  J),TWI,VEL,DQ,DWK,
     1 DFR,X(2,J),VOL(2),G(2),                          REYNO,HHT1,
     1 HHT2,HHT,FRF
 9000  FORMAT(1X,I3,I4,I3,I6,F11.1,E11.4,F11.5,F8.3,4E11.3,F11.5,F9.4,
     1 E11.4/63X,6E11.4)
  720 IF(J.EQ.360)PRINT 9003,Q(I),WORK(I),ZKE(I) ,FR(I)
     1 ,QLWOQ,DELQIOQI,DELWKOWK
 9003  FORMAT(/12X,4HQ(I), 8X,7HWORK(I),  9X,6HZKE(I),10X,5HFR(I)
     1 ,10X,5HQLWOQ,7X,8HDELQIOQI,7X,8HDELWKOWK
     1 /1X,4F15.3,3F15.5/)
40003  CONTINUE
  723 IF(JJ.LT.2)GO TO 40002
  724  QLWOQ=DELQIOQI=DELWKOWK=0.0
  725 IF(Q(I).EQ.0.0.AND.WORK(I).EQ.0.0)GO TO 750
  726 IF(Q(I).EQ.0.0)Q(I)=0.000001 $IF(WORK(I).EQ.0.0)WORK(I)=0.000001
  730  QLWOQ=ABSF((Q(I)-WORK(I))/Q(I)) $DELQIOQI=ABSF((QI-Q(I))/Q(I))
  735  DELWKOWK=ABSF((WORKI-WORK(I))/WORK(I))
  738 IF(QLWOQ.LT.0.0001.AND.DELQIOQI.LT.0.0001)GO TO 750
  739 IF(QLWOQ.LT.0.0001.AND.DELWKOWK.LT.0.0001)GO TO 750
  740 IF(DELQIOQI.LT.0.0001.AND.DELWKOWK.LT.0.0001)GO TO 750
  742 IF(JJ.GE.NJJL2)GO TO 750
  745  GO TO 40002
  750  IQ=IQ+1
40002  CONTINUE
40000  CONTINUE
  756  PRINT 9006
 9006  FORMAT(1H1)
  757  PRINT 9501
 9501  FORMAT(1X,4X,1HI,1X,8HKOUNTMAX,1X,5HJJMAX,
     1                  5X,5HPX(I),3X,7HWORK(I),3X,7HSUMWORK,6X,4HQ(I),
     1 6X,4HSUMQ,5X,5HFR(I),5X,5HSUMFR,4X,6HZKE(I))
  760   DO 40001 I=1,M $SUMQ=SUMQ+Q(I) $SUM WORK=SUMWORK+WORK(I)
  761 IF(SUMWORK.EQ.0.)SUMWORK=0.000001
  765  SUMFR=SUMFR+FR(I)
  775  PRINT 9500,I,KOUNTMAX(I),JJMAX(I),PX(I)
     1 ,WORK(I),SUMWORK,Q(I),SUMQ,FR(I),SUMFR,ZKE(I)
 9500  FORMAT(1X,I5,I9,I6,8F10.0)
40001  CONTINUE
  780   DO 40007 J=1,360 $SUMEKE=SUMEKE+EKE(J)
 9310  FORMAT(//)
```

```
40007  CONTINUE
  785  EKEOSW=SUMEKE/SUMWORK $ZKENOSW=ZKE(N)/SUMWORK
  787  ZKEMOSW=ZKE(M)/SUMWORK
       PRINT 9008
 9008  FORMAT(///1X,9X6HSUMEKE,9X6HEKEOSW,8X7HZKENOSW,8X7HZKEMOSW)
       PRINT 9007,SUMEKE,EKEOSW,ZKENOSW,ZKEMOSW
 9007  FORMAT(1X,4F15.4)
       PRINT 9006
        DO40010 J=JD,360,JD
       PRINT 9009,J,EKE(J)
 9009  FORMAT(1X,6HEKE(J),I3,E11.4)
40010  CONTINUE
99999  END AMPLITUD
C
```

22. Appendix I. Subroutine READ DATA for Frequency

```
      SUBROUTINE READ DATA
COMBINATION OF STATEMENTS FOR FREQUENCY TYPE READ DATA
      COMMON/1/N,TMP(32),S(32),RHODEW,TDEW,M,XK,R,TEMP(97),XX(97),PI,
     1 C(96),TMG(96),XMID(96),RHO(96),GM(96),A(96),Z(96),AF,PSTART,
     1 DXXOVC
      READ 8001,PSTART,DIAM,N
 8001 FORMAT(2F10.0,1I10)
      READ 8000,(TMP(I),I=1,32) $READ 8000,(S(I),I=1,32)
 8000 FORMAT(16F5.0)
CONVERSION OF INPUT DATA TO UNITS USED IN CALCULATIONS
COMPUTATION USES CENTIMETERS BUT INPUT IS IN INCHES
       DO 40012 K=1,32 $S(K)=S(K)*2.54
40012 CONTINUE $DIAM=DIAM*2.54
      WT=4.003$RU=8.317E07$R=RU/WT$PI=3.1415926536$AF=PI/4.*DIAM**2
      XK=5./3. $TDEW=4.3 $M=N+N/2 $RHODEW=PSTART/R/TDEW
COMPUTATIONAL VALUES ARE PRINTED
      PRINT 9000,PSTART,DIAM,N
 9000 FORMAT(1X13HPSTART,DIAM,N,F8.0,F8.4,I8)
      PRINT 9009,(TMP(K),K=1,32) $PRINT 9010,(S(K),K=1,32)
 9009 FORMAT(1X5HTMP  16F8.2)
 9010 FORMAT(1X5H  S  16F8.2)
99999 END READ DATA
C
```

23. Appendix J. Subroutine ELEMENT for Frequency

```
      SUBROUTINE ELEMENT
      COMMON/1/N,TMP(32),S(32),RHODEW,TDEW,M,XK,R,TEMP(97),XX(97),PI,
     1 C(96),TMG(96),XMID(96),RHO(96),GM(96),A(96),Z(96),AF,PSTART,
     1 DXXOVC
CALCULATION OF ELEMENT LENGTHS AND TEMPERATURES
   10  TEMP(1)=TMP(1) $ XX(1)=S(1) $NP1=N+1
   20  D XX OV C=(S(32)-S(1))/N/SQRTF(XK*R*TMP(16))
   24   DO 40002 I=1,N
   25  C(I)=SQRTF(XK*R*TEMP(I))
   27  CONTINUE
   28  XX(I+1)=XX(I)+D XX OV C*C(I)
   29   DO 40000 K=2,32 $ IF(S(K).GE.XX(I+1))GO TO 33
40000  CONTINUE $ K=32
   33  TEMP(I+1)=TMP(K)+(TMP(K)-TMP(K- 1))/(S(K)-S(K- 1))*(XX(I+1)-S(K))
   37  TMG(I)=(TEMP(I)+TEMP(I+1))/2. $ CCC=SQRTF(XK*R*TMG(I))
   38 IF(ABSF(CCC-C(I))/C(I)-. 001)43,43,39
   39  C(I)=CCC $ GO TO 27
   43  C(I)=CCC
40002  CONTINUE
   47 IF(ABSF(XX(N+1)-S(32))/S(32)-.0001)50,50,49
   49  D XX OV C=D XX OV C*(1.+1.*((S(32)-XX(N+1))/XX(N+1))) $GOTO 24
   50  DO 40017 I=1,N $X MID(I)=(XX(I)+XX(I+1))/2.
   55  RHO(I)=P START/R/TMG(I) $Z(I)=C(I)*RHO(I) $A(I)=1./Z(I)
   60  GM(I)=RHO(I)*AF    *(XX(I+1)-XX(I))
40017  CONTINUE
   85  PRINT 9310
 9310  FORMAT(/)
   90  PRINT 8038,(TEMP(K),K=1,NP1)
 8038  FORMAT(1X 4HTEMP,10F8.2,7F7.2)
   95  PRINT 8039,(XX(K),K=1,NP1)
 8039  FORMAT(3X 2HXX,10F8.2,7F7.2)
  100  PRINT 9206,(X MID(K),K=1,N)
 9206  FORMAT(4X 5HX MID,10F8.2,6F7.2)
  105  PRINT 9205,(TMG(K),K=1,N)
 9205  FORMAT(6X 3HTMG,10F8.2,6F7.2)
  110  PRINT 9000,(GM(K),K=1,N)
 9000  FORMAT(7X2HGM,8E12.4)
  120  PRINT 9207,(RHO(K),K=1,N) $ PRINT 9208,(C(K),K=1,N)
 9207  FORMAT(6X3HRHO,8E12.4)
 9208  FORMAT(8X1HC,8E12.4)
       END ELEMENT
```

24. Appendix K. Program FREQENCY

```
      PROGRAM FREQENCY
C PROGRAM OF JULY 1967  WITH OMISSIONS AND PRINTOUT DELETIONS OF OCT 6 1967
      COMMON/1/N,TMP(32),S(32),RHODEW,TDEW,M,XK,R,TEMP(97),XX(97),PI,
     1 C(96),TMG(96),XMID(96),RHO(96),GM(96),A(96),Z(96),AF,PSTART,
     1 DXXOVC
      COMMON/2/P(96,102),U(96,102),PMAX(96,12),UMAX(96,12),UMIN(96,12),
     1 TMIN(96,12),TMAX(96,12),PMIN(96,12)
     1,ZINTERVL(12),PERIOD(12),FREQ(12)
    5 CALL READ DATA
   10 CALL ELEMENT
   15 PRINT 9310
   20 KOUNT=-100 $ SAMBDA=1000. $GAMBDA=0.1*SAMBDA
   25 NJ=102 $NJ LESS 1=NJ-1 $ NJ LESS 2=NJ-2$NL1=N-1$N PLUS 1=N+1
   30 DO 40000J=1,2 $DO 40000 I=1,N $ P(I,J)=-SAMBDA $ U(I,J)=0.
40000 CONTINUE $ DO 40001 JK=1,10 $ DO 40001 I=1,N
   35 P MAX(I,JK)=U MAX(I,JK)=U MIN(I,JK)=0.$T MIN(I,JK)=T MAX(I,JK)=1.
   40 IF(JK.EQ.1)P MIN(I,JK)=-SAMBDA $IF(JK.GT.1)P MIN(I,JK)=SAMBDA
40001 CONTINUE $ JK=1 $ MAX COUNT=0
CALCULATION STARTS
   50 U(N,2)=(-1)*SAMBDA*A(N) $ P(N,2)=0.
   60 KOUNT=KOUNT+100 $ SUM FREQ=0.
   65 DO 141 J=3,NJ $J2=J+KOUNT $IF((J/2)*2-J)101,121,101
COMPUTATION OF  ODD TIME PRESSURE AND VELOCITY
  101 DO 115 ,I=2,N,2
  105 P(I,J)=P(I,J-2)+2.*(U(I-1,J-1)-U(I,J-1))/(A(I-1)+A(I))
  107 P(I-1,J)=P(I,J)
  111 U(I,J)=-U(I,J-2)+2.*(U(I-1 ,J-1)*A(I)+U(I,J-1)*A(I-1))
     1 /(A(I-1)+A(I))
  113 U(I-1,J)=U(I,J)
 2114 IF(P(I,J).GT.P MAX(I,JK))T MAX(I,JK)=J2
 2115 IF(P(I,J).GT.P MAX(I,JK))P MAX(I,JK)=P(I,J)
 2116 IF(U(I,J).GT.U MAX(I,JK))U MAX(I,JK)=U(I,J)
 2117 IF(P(I,J).LT.P MIN(I,JK))T MIN(I,JK)=J2
 2118 IF(P(I,J).LT.P MIN(I,JK))P MIN(I,JK)=P(I,J)
 2119 IF(U(I,J).LT.U MIN(I,JK))U MIN(I,JK)=U(I,J)
 2120 IF(P(I-1,J).GT.P MAX( I-1,JK))T MAX(I-1,JK)=J2
 2121 IF(P(I-1,J).GT.P MAX(I  -1,JK))P MAX( I-1,JK)=P(I-1,J)
 2122 IF(U(I-1 ,J).GT.U MAX(I-1,JK))U MAX(I-1,JK)=U(I-1,J)
 2123 IF(P(I-1,J).LT.P MIN(I-1,JK))T MIN(I-1,JK)=J2
 2124 IF(P(I-1,J).LT.P MIN(I-1,JK))P MIN(I-1,JK)=P(I-1,J)
 2125 IF(U(I-1,J).LT.U MIN(I-1,JK))U MIN(I-1,JK)=U(I-1,J)
 2126 IF(I.GT.2)GO TO 115
 2127 IF(P(1,J).GT.GAMBDA)MAX COUNT=MAX COUNT+1
 2128 IF(MAX COUNT.GT.N.AND.P(1,J).LT.0.     )JK=JK+1
 2129 IF(MAX COUNT.GT.N.AND.P(1,J).LT.0.     ) MAX COUNT=0
  115 CONTINUE
  117 GO TO 141
COMPUTATION OF EVEN TIME PRESSURE AND VELOCITY
  121 U(1,J)=0. $P(1,J)=P(1,J-2)-2.*U(2,J-1)/A(1)
 3122 IF(P(1,J).GT.P MAX(1,JK))T MAX(1,JK)=J2
 3123 IF(P(1,J).GT.P MAX(1,JK))P MAX(1,JK)=P(1,J)
 3124 IF(U(1,J).GT.U MAX(1,JK))U MAX(1,JK)=U(1,J)
 3125 IF(P(1,J).LT.P MIN(1,JK))T MIN(1,JK)=J2
 3126 IF(P(1,J).LT.P MIN(1,JK))P MIN(1,JK)=P(1,J)
 3127 IF(U(1,J).LT.U MIN(1,JK))U MIN(1,JK)=U(1,J)
 3128 IF(P(1,J).GT.GAMBDA)MAX COUNT=MAX COUNT+1
```

```
 3129  IF(MAX COUNT.GT.N.AND.P(1,J).LT.0.     )JK=JK+1
 3130  IF(MAX COUNT.GT.N.AND.P(1,J).LT.0.     ) MAX COUNT=0
  125   DO 134 I=3,NL1,2
  127  P(I,J)=P(I,J-2)+2.*( U(I-1,J-1)-U(I,J-1))/(A(I-1)+A(I))
  129  P(I-1,J)=P(I,J)
  131 U(I,J)=-U(I,J-2)+2.*(U(I-1 ,J-1)*A(I)+U(I,J-1)*A(I-1))
     1 /(A(I-1)+A(I))
  132  U(I-1,J)=U(I,J)
 4133  IF(P(I,J).GT.P MAX(I,JK))T MAX(I,JK)=J2
 4134  IF(P(I,J).GT.P MAX(I,JK))P MAX(I,JK)=P(I,J)
 4135  IF(U(I,J).GT.U MAX(I,JK))U MAX(I,JK)=U(I,J)
 4136  IF(P(I,J).LT.P MIN(I,JK))T MIN(I,JK)=J2
 4137  IF(P(I,J).LT.P MIN(I,JK))P MIN(I,JK)=P(I,J)
 4138  IF(U(I,J).LT.U MIN(I,JK))U MIN(I,JK)=U(I,J)
 4139  IF(P(I-1,J).GT.P MAX( I-1,JK))T MAX(I-1,JK)=J2
 4140  IF(P(I-1,J).GT.P MAX(I  -1,JK))P MAX( I-1,JK)=P(I-1,J)
 4141  IF(U(I-1 ,J).GT.U MAX(I-1,JK))U MAX(I-1,JK)=U(I-1,J)
 4142  IF(P(I-1,J).LT.P MIN(I-1,JK))T MIN(I-1,JK)=J2
 4143  IF(P(I-1,J).LT.P MIN(I-1,JK))P MIN(I-1,JK)=P(I-1,J)
 4144  IF(U(I-1,J).LT.U MIN(I-1,JK))U MIN(I-1,JK)=U(I-1,J)
  134  CONTINUE
  135  P(N,1)=0.      $U(N,J)=U(N,J-1)+(P(N,J-1)-P(N,J))*A(N)
 5136  IF(P(N,J).GT.P MAX(N,JK))T MAX(N,JK)=J2
 5137  IF(P(N,J).GT.P MAX(N,JK))P MAX(N,JK)=P(N,J)
 5138  IF(U(N,J).GT.U MAX(N,JK))U MAX(N,JK)=U(N,J)
 5139  IF(P(N,J).LT.P MIN(N,JK))T MIN(N,JK)=J2
 5140  IF(P(N,J).LT.P MIN(N,JK))P MIN(N,JK)=P(N,J)
 5141  IF(U(N,J).LT.U MIN(N,JK))U MIN(N,JK)=U(N,J)
  141  CONTINUE
  150  IF(KOUNT.GT. 0 .AND.KOUNT.LT.1000)GO TO 226
  160  DO 225 J=1,NJ LESS 2 $ J2=J+KOUNT
  220  IF(N.LE.16)PRINT 9220,J2,(P(I,J),I=1,N)
  221  IF(N.LE.16)PRINT 9221,(U(I,J),I=1,N)
  222  IF(N.GT.16)PRINT 9220,J2,(P(I,J),I=1,16)
  224  IF(N.GT.16)PRINT 9222,(P(I,J),I=17,N)
  223  IF(N.GT.16)PRINT 9221,(U(I,J),I=1,16)
 5223  IF(N.GT.16)PRINT 9221,(U(I,J),I=17,N)
  225  CONTINUE
  226  IF(JK.LT.11)GO TO 229
 6227  DO 40003 J=1,9
 6228  ZINTERVL(J)=T MIN(1,J+1)-T MIN(1,J)
 6229  PERIOD(J)=ZINTERVL(J)*0 XX OV C
 6230  FREQ(J)=1./PERIOD(J)
 6231  SUM FREQ=SUM FREQ+FREQ(J)
40003  CONTINUE
 6232  AVG FREQ=SUM FREQ/9.
 6233  OMEGA=2.*PI*AVG FREQ
 6234  PRINT 9310
 9310  FORMAT(//)
 6235  DO 40005 J=1,10
 6236  PRINT 9300,J ,(T MIN(I,J),I=1,16  ),(P MIN(I,J),I=1,16  ),
     1 (U MIN(I,J),I=1,16  ),(T MAX(I,J),I=1,16  ),
     1 (P MAX(I,J),I=1,16  ),(U MAX(I,J),I=1,16  )
 9300 FORMAT(107H0MAXIMUM AND MINIMUM PRESSURES AND VELOCITIES AND CORRE
     1SPONDING CALCULATION TIMES IN ELEMENTS 1 THROUGH 16,
     11X12HTHIS IS THE I2,9H TH CYCLE
```

```
     11X5HT MIN16F8.0,/1X5HP MIN16F8.0,/1X5HU MIN16F8.0,/
     1 1X5HT MAX16F8.0,/1X5HP MAX16F8.0,/1X5HU MAX16F8.0)
40005  CONTINUE
 6239  PRINT 9302,(J,J=1,9),(ZINTERVL(J),J=1,9),(PERIOD(J),J=1,9),
     1 (FREQ(J),J=1,9),AVG FREQ,OMEGA    ,D XX OV C
 9302 FORMAT(131H0ZINTERVL IS THE NUMBER OF CALCULATION UNITS IN ONE CYC
     1LE.  PERIOD IS THE TIME LENGTH OF ONE CYCLE AND THE RECIPROCAL OF
     1THE PERIOD /126H IS THE FREQUENCY. THE TOP LINE GIVES THE CYCLE NU
     1MBER FOLLOWING THE INITIAL PULSE. AVG FREQ IS THE AVERAGE OF ALL F
     1REQUENCIES /
     11X8HCYCLE     9I12,/1X8HZINTERVL 9F12.0,/1X8HPERIOD   9F12.5,/
     1 1X8HFREQ      9F12.5,/9H0AVG FREQ 1F12.5 ,5X5HOMEGA F12.5,
     1                3X9HD XX OV C,E12.3)
 6240  IF(JK.GE.11)GO TO 99999
  229  DO 231 I=1,N
 7230  P(I,1)=P(I,NJ LESS 1)$U(I,1)=U(I,NJ LESS 1)
 7231  P(I,2)=P(I,NJ) $U(I,2)=U(I,NJ)
  231  CONTINUE
  232  GO TO 60
 9220  FORMAT(1X,I4,1HP ,16F8.0)
 9221  FORMAT(5X,1HU ,16F8.1)
 9222  FORMAT(5X,1HP ,16F8.0)
99999  END  FREQENCY
C
```


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