NBS TECHNICAL NOTE

356

Comparison of Incompressible Flow and Isothermal Compressible Flow Formulae

U.S. DEPARTMENT OF COMMERCE
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COMPARISON OF INCOMPRESSIBLE FLOW AND ISOTHERMAL COMPRESSIBLE FLOW FORMULAE

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COMPARISON OF INCOMPRESSIBLE FLOW AND ISOTHERMAL COMPRESSIBLE FLOW FORMULAE

by

Jesse Hord

Mass flow formulae for incompressible and "modified-incompressible" flow are compared with the isothermal compressible flow relation under the following conditions: The gas flow is steady, isothermal, and fully developed in a horizontal pipe of constant cross section with a prescribed static pressure drop $(P_1 - P_2)$. The comparative data are limited to static pressure ratios $(P_2/P_1) > \frac{1}{2}$, and subsonic isothermal flow. Laminar and turbulent flows are treated. Under the limitations of the comparison, modified-incompressible flow and isothermal gas flow relations are identical when $fL/2D \gg \ell n(P_1/P_2)$. Graphical plots indicate the degree of approximation or error involved in using incompressible relations to solve compressible flow problems. Pressure losses due to end effects are briefly discussed.

Key Words: Compressible flow, flow comparison, fluid flow, incompressible flow, mass flow, pressure drop.

1. Introduction

Engineers frequently [1] use incompressible flow theory to solve compressible flow problems; a large class of fluid problems may be solved within the framework of isothermal compressible flow, and many textbook authors justifiably devote considerable attention to this subject. While compressible flow computations are not difficult to perform--working charts are available [2]--they are frequently replaced by the simpler incompressible flow calculations; this is particularly true in analyses where the flow expression is part of an integrand and closed form solutions are impossible using the compressible formula. Examples are the computations of 1) rate of gas transfer between two vessels and 2) the response of a pressure gage with an interconnecting tube. Limits of applicability of the incompressible flow relations must then be established. These limits are obviously dependent upon the desired computational precision. The textbook author will usually advise use of the compressible flow relations if any question of application exists. However, intelligent use of a significant portion of the literature requires knowledge of the limits of application of the incompressible theory. The applicability of incompressible flow theory to evaluate compressible flow problems has not been clearly established, and the purpose of this paper is to provide this information.

Comparisons [1, 3] between incompressible formulae and isentropic and adiabatic compressible relations have appeared in the literature. The approximations involved in computing pressure drop for isothermal gas flow (at fixed inlet velocities) via incompressible flow relations have also been treated in the literature [2, 4, 5]. This paper treats the parallel problem of determining the error involved in using incompressible flow theory to predict mass flow for fixed pressure drop ratios and constant gas temperature in horizontal pipes.

Some examples of the limits of applicability of the incompressible relations given in several textbooks will emphasize the need for this information. Binder [6] states that "sometimes the rule is given that the incompressible flow relation can be applied to isothermal compressible flow problems if the pressure drop ($P_1 - P_2$) is less than 10 percent of the initial pressure P_1." He then compares the two theories for flow in "long," horizontal pipes. The assumption of long pipes permits the compressible relation to be simplified for comparison. In a later edition, Binder [5] does not impose the "long pipe" restriction and illustrates the difference between the two flow relations in terms of pipe inlet Mach number. The error involved in using the incompressible relation to compute pressure drop at a fixed inlet Mach number can be deduced from these illustrations. King, Wisler, and Woodburn [7] also cite the 10 percent rule for pressure drop ratio, i.e., $[(P_1 - P_2) / P_1] < 0.10$ is the criterion used to determine the applicability of incompressible relations to compressible flow problems. Schlichting [8] points out that gaseous flow may be considered incompressible when the pipe line Mach number is less than 0.3. Rouse and Howe [9] neglect the momentum term in the compressible flow relation (which is equivalent to assuming a long pipe) and derive a simple expression for estimating pressure drop in isothermal compressible flow. This expression is then mathematically compared with the incompressible relation to indicate the error involved in pressure drop calculations for flow of gases at constant temperature in long pipes. Shapiro [2] provides a mathematical relationship for the comparison of incompressible and compressible formulae at "low" pressure drop ratios and inlet Mach numbers. Hall [4] presents a unique comparison of the theories by developing a "compressibility correction factor" in terms of the kinetic energy of the flowing fluid. He carefully defines the Mach number limitations of the approximate comparative formulae which he gives for computing the differences between static and stagnation temperatures and pressures.

It is the purpose of this paper to indicate the error involved in using incompressible and simplified compressible flow relations to compute the isothermal mass flow of gas through horizontal tubes with prescribed static pressure ratios. The results may be used to 1) serve as a guide to determine whether a compressible calculation is needed, 2) obtain a "correction factor" which converts the incompressible flow to compressible flow and 3) estimate the applicable error or appropriate "correction factor" when using incompressible formulae in integrands where compressible relations cannot be integrated. With respect to item 2), it is to be emphasized that there is no net advantage in using the comparative data given here as a calculation method for compressible flow. This topic is discussed in detail in section 5. The purpose of the paper is to assess the error involved in using the incompressible flow relations and express the results in terms of dimensionless flow parameters.

2. Review of Theoretical Relations

In arriving at the limits of application of the simplified flow formulae, it is necessary to make some assumptions and note pertinent restrictions. It is assumed that the flow in a horizontal pipe is isothermal, steady, and fully developed. Binder [5] gives the critical (limiting) pressure ratio for isothermal flow of gas in a horizontal pipe as $(P/P_1)_{cr} = M_1 \sqrt{k}$, i.e., isothermal sonic velocity is attained at the point in the pipe where this relation is satisfied. He points out that isothermal flow involves heat transfer and friction and exists only at Mach numbers below $1/\sqrt{k}$. From the pressure drop comparisons given by Binder [5], it is apparent that static pressure ratios (P_2/P_1) smaller than one-half involve rapidly increasing differences between compressible and incompressible theories; static pressure ratios smaller than one-half indicate large errors (up to 2:1) in computing the pressure drop via incompressible formulae. Thus, it becomes clear why many authors limit the use of simplified flow expressions to static pressure ratios greater than one-half.

The assumptions and restrictions which pertain to the following comparison of compressible and incompressible theories are tabulated below:

a) Steady, isothermal, fully developed gas flow in a horizontal pipe of constant cross section. See figure 2.1.

b) The limiting (critical) pressure ratio is given by $(P/P_1)_{cr} = M_1 \sqrt{k}$.

c) Pipe line exit Mach number, $M_2 < 1\sqrt{k}$.

d) The comparative data will be limited to static pressure ratios, $(P_2/P_1) > \frac{1}{2}$.

e) The gas viscosity is a function of temperature only.

2.1 Incompressible Flow

The mass flow of an incompressible fluid may be written as

$$\mathring{m}_{ic} = A\left\{\rho_1 g_c (P_1 - P_2) / (fL/2D)\right\}^{\frac{1}{2}} . \tag{2.1}$$

2.2 "Modified-incompressible" Flow

The mass flow of gas given by the product of incompressible volumetric flow and arithmetic <u>mean</u> density is

$$\mathring{m}_{mic} = A\left\{\bar{\rho} g_c (P_1 - P_2) / (fL/2D)\right\}^{\frac{1}{2}} . \tag{2.2}$$

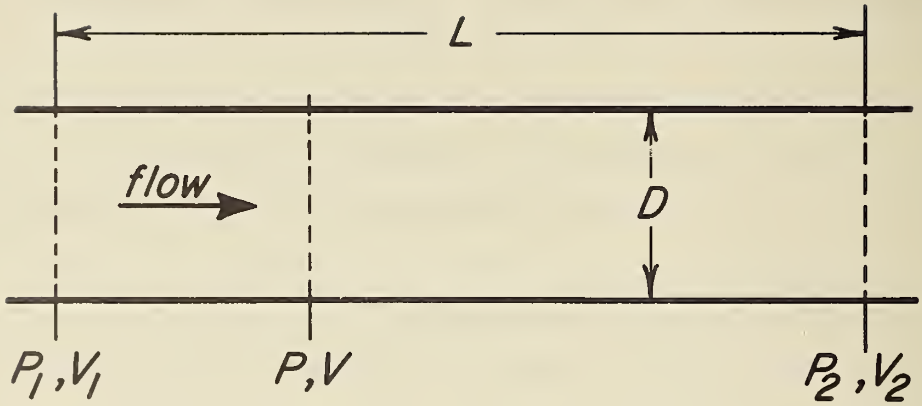

Figure 2.1 Notation for fully developed isothermal flow of ideal gas in horizontal pipes.

2.3 Isothermal Compressible Flow

The mass flow of an ideal gas is given by

$$\mathring{m}_c = A\left\{g_c(P_1^2 - P_2^2) \,/\, 2R_G T\left[\ell n(P_1/P_2) + f_c L/2D\right]\right\}^{\frac{1}{2}}. \qquad (2.3)$$

3. Comparison of Incompressible and Isothermal Compressible Flows

Dividing (2.3) by (2.1) and utilizing the perfect gas law we obtain the mass flow ratio,

$$\mathring{m}_c/\mathring{m}_{ic} = N = \left\{\left[(fL/2D)\,(1+P_2/P_1)/2\right] / \left[f_c L/2D + \ell n\,(P_1/P_2)\right]\right\}^{\frac{1}{2}}. (3.1)$$

To account for laminar and turbulent flow the friction factor may be expressed in the Blasius form

$$f = \alpha/Re^{\beta}. \qquad (3.2)$$

For laminar flow $\alpha = 64$, and $\beta = 1$; in turbulent flow typical values of α and β are $\alpha = 0.01$ to 0.2 and $\beta = 0.0$ to 0.2.

Equation (3.1) may be evaluated for a fixed pressure ratio by assuming as a first try $f = f_c$. Since the friction factor is related to the mass flow through the Reynolds number, a correction in f_c is required for the second and succeeding iterations. Where $N > \frac{1}{2}$, the compressible and incompressible friction factors may be evaluated from (3.2) using identical values of α and β, i.e., a 2:1 variation in Re is permissible for a given α and β. Then, (3.2) may be used to obtain the following relation

$$f/f_c = N^{\beta}. \qquad (3.3)$$

Combining (3.1) and (3.3) we obtain

$$N = \{ \sigma_1 / (\sigma_2 + N^{-\beta}) \}^{\frac{1}{2}}, \tag{3.4}$$

where $\sigma_1 = (1 + P_2/P_1)/2$ and $\sigma_2 = \left[\ell n(P_1/P_2) \right] / (fL/2D)$.

We may now examine the effect of assuming a <u>long</u> pipe, i.e. $fL/2D >> \ell n\,(P_1/P_2)$. Since $N^{-\beta}$ is near unity the first term in the denominator of (3.4) is negligible and (3.4) becomes

$$N = \sigma_1^{1/(2-\beta)}. \tag{3.5}$$

Figure 3.1 illustrates the results given by (3.5) for various values of P_2/P_1 and β. For arbitrary values of fL/2D, a trial and error solution of (3.4) is required. A graphical solution of (3.4) is given on figures 3.2, 3.3, and 3.4. Comparison of figure 3.1 and figures 3.2 - 3.4 at identical values of P_2/P_1 and β indicates that a <u>long</u> pipe requires fL/D values of 10^2 to 10^3 (the exact value of fL/D is dependent upon the computational precision desired). In laminar flow (3.4) becomes a simple quadratic equation with the solution

$$(N)_{\beta=1} = \left[-1 + (1 + 4\sigma_1\sigma_2)^{\frac{1}{2}} \right] / 2\sigma_2. \tag{3.6}$$

4. Comparison of "Modified-incompressible" and Compressible Flows

In a similar manner, we obtain the mass flow ratio for modified-incompressible flow; dividing (2.3) by (2.2),

$$\mathring{m}_c / \mathring{m}_{mic} = \overline{N} = \{1/(\sigma_2 + \overline{N}^{-\beta})\}^{\frac{1}{2}}. \tag{4.1}$$

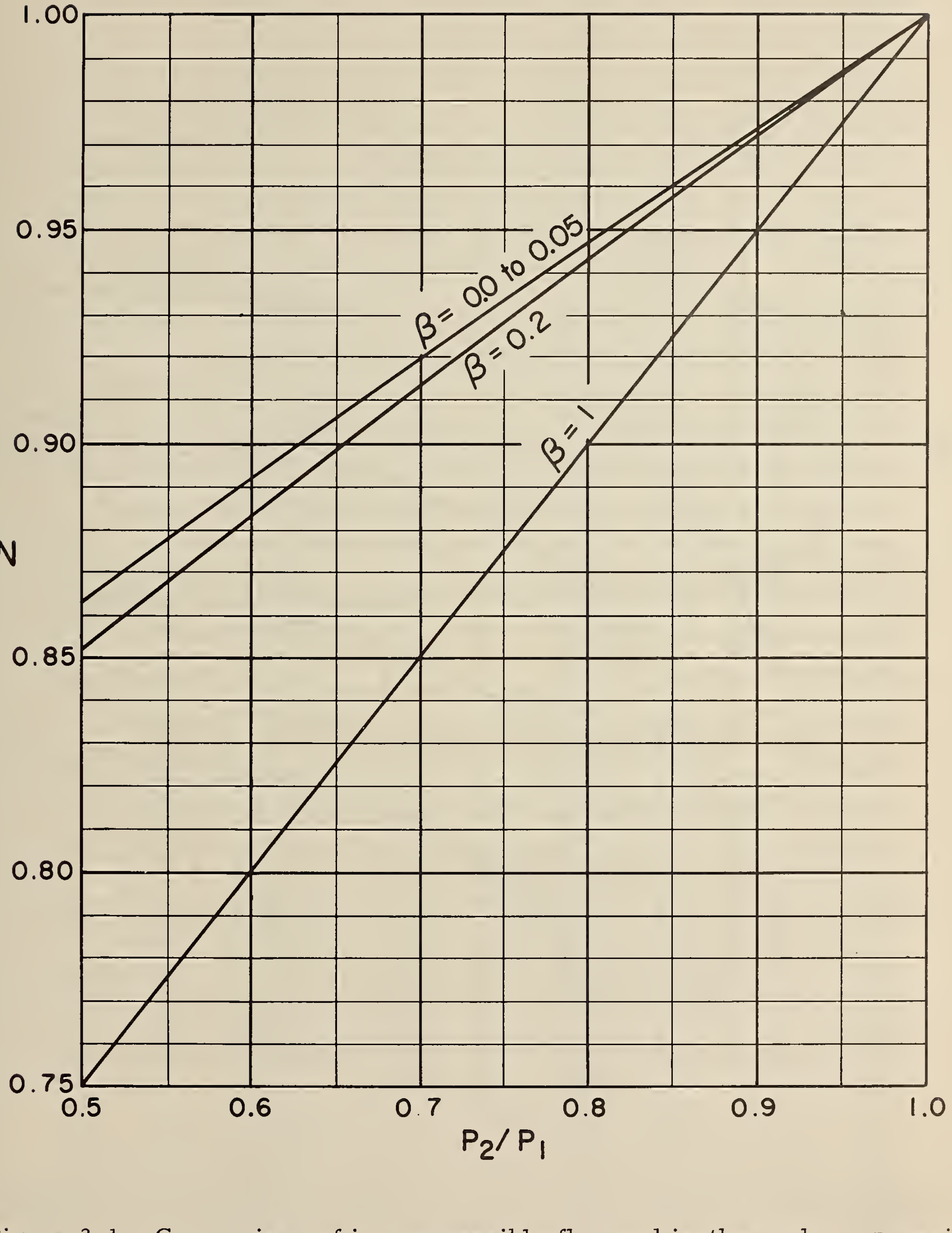

Figure 3.1 Comparison of incompressible flow and isothermal compressible flow at various static pressure ratios in long pipes, i.e., $fL/2D >> \ell n\,(P_1/P_2)$. Laminar flow ($\beta$=1); Turbulent flow ($\beta$=0.0 to 0.2).

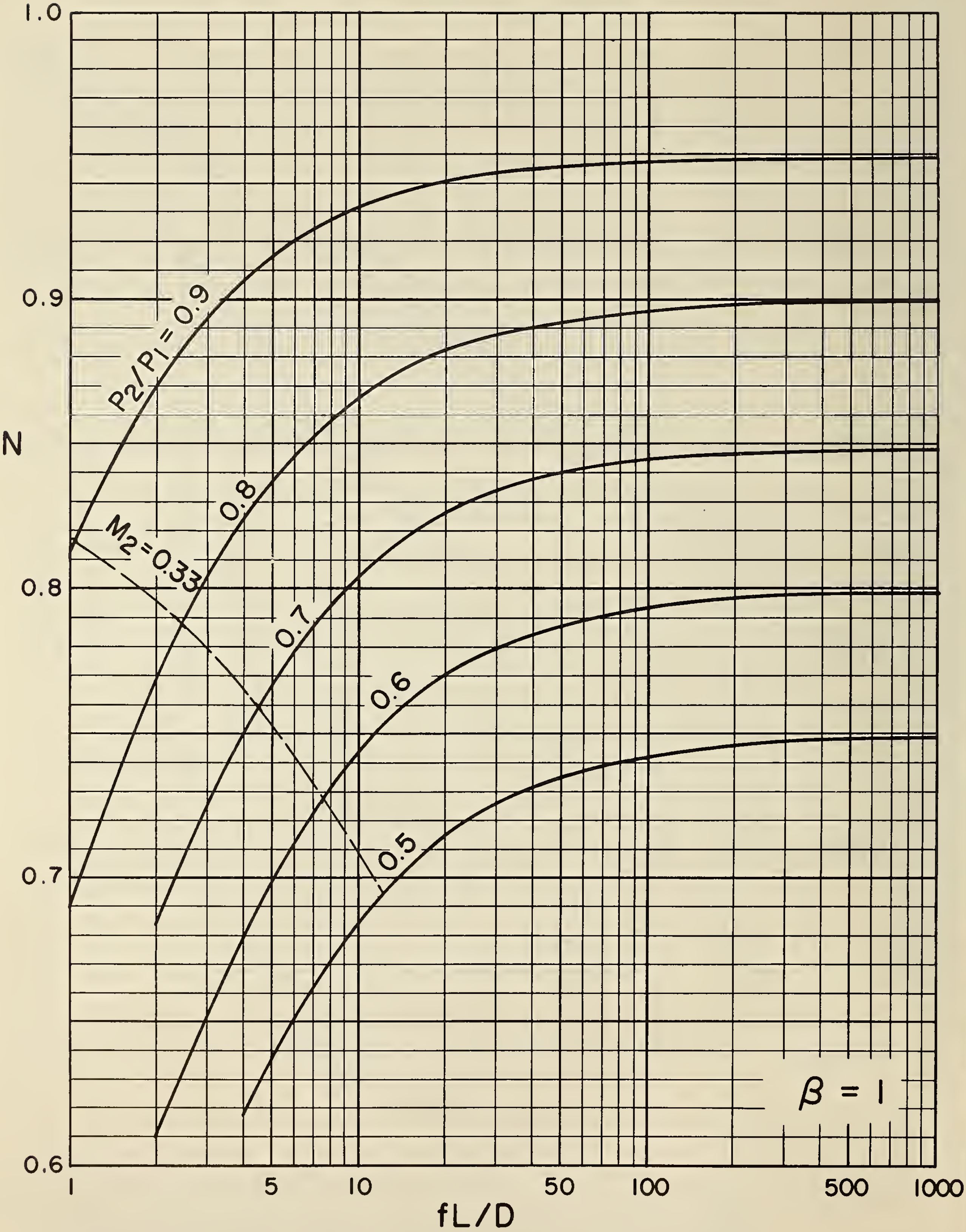

Figure 3.2 Comparison of incompressible flow and isothermal compressible flow as a function of fL/D and P_2/P_1 for laminar flow (β=1). M_2= 1/3 is evaluated at k = 1.4.

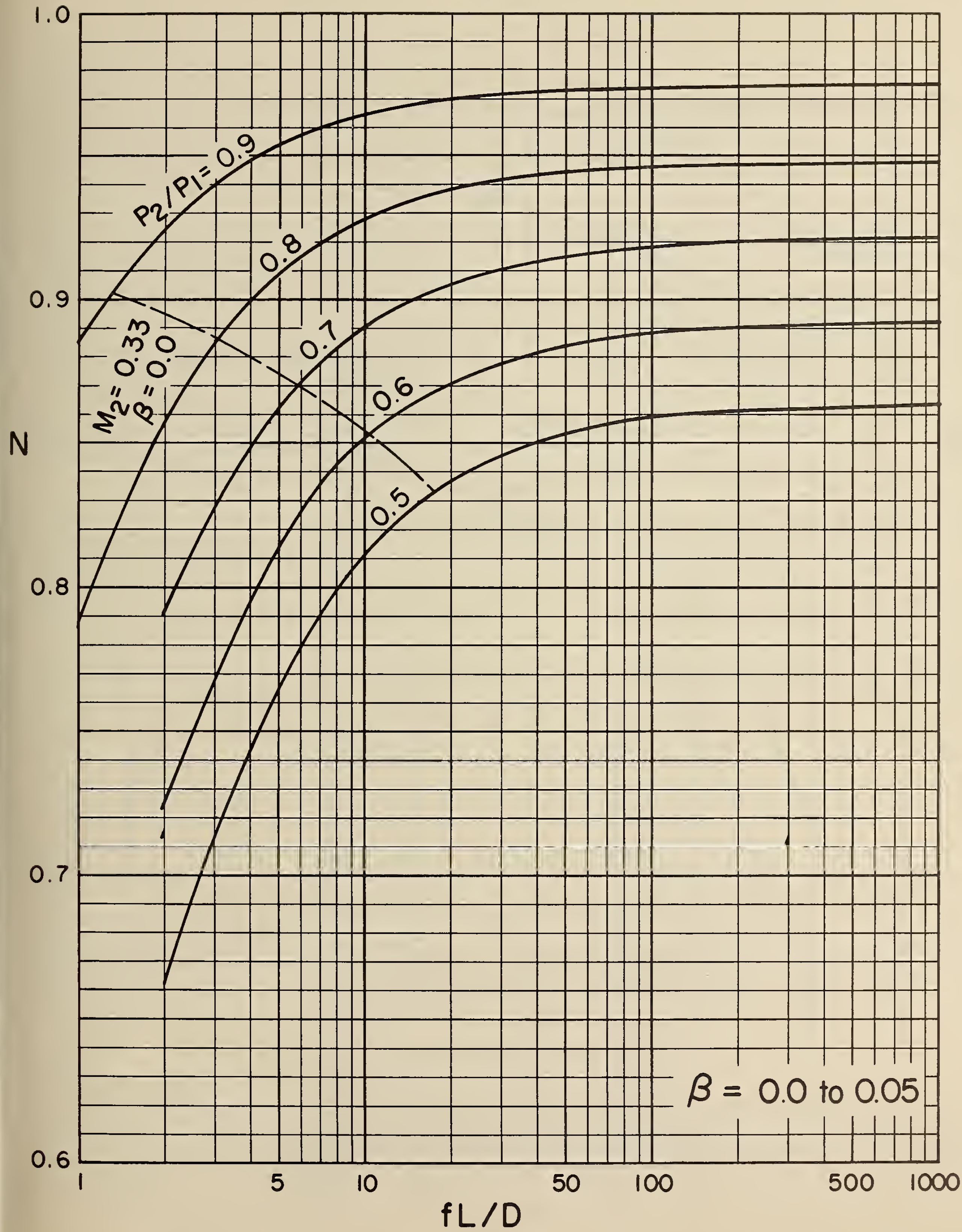

Figure 3.3 Comparison of incompressible flow and isothermal compressible flow as a function of fL/D and P_2/P_1 for turbulent flow ($\beta = 0.0$ to 0.05). $M_2 = 1/3$ is evaluated at $k = 1.4$ and $\beta = 0.0$.

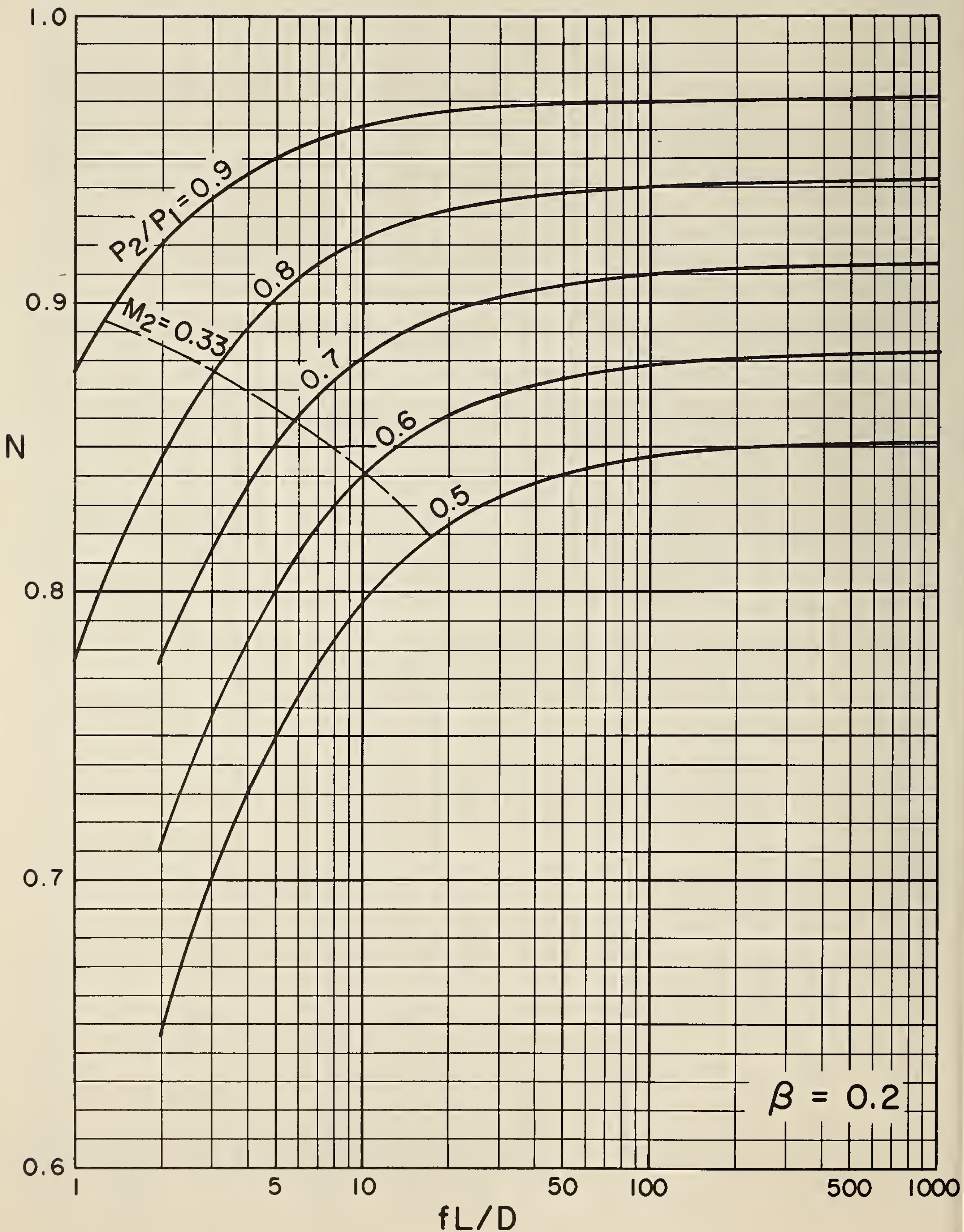

Figure 3.4 Comparison of incompressible flow and isothermal compressible flow as a function of fL/D and P_2/P_1 for turbulent flow ($\beta = 0.2$). $M_2 = 1/3$ is evaluated at $k = 1.4$.

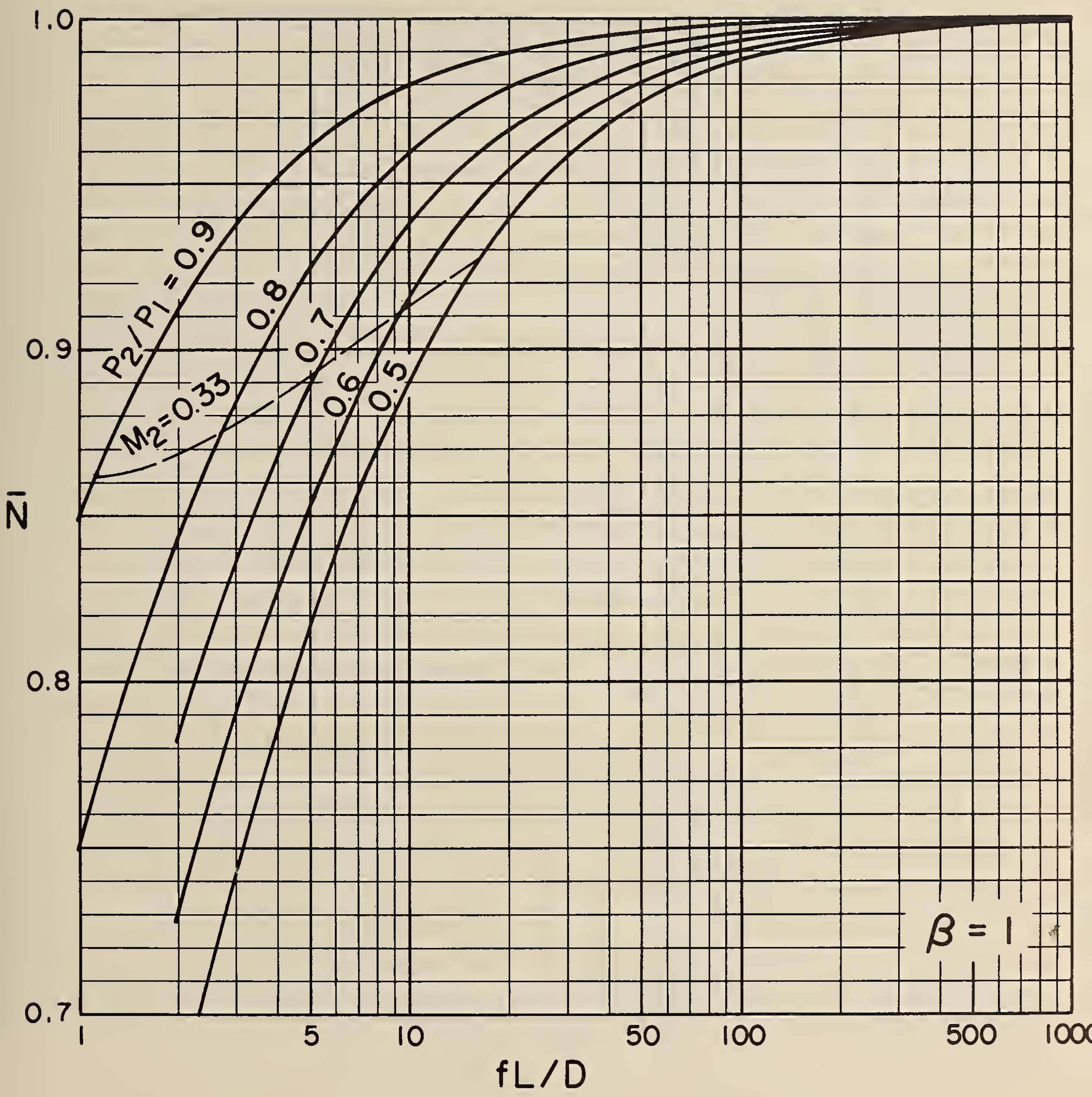

Figure 4.1 Comparison of "modified-incompressible" flow and isotherma compressible flow as a function of fL/D and P_2/P_1 for lamina flow (β=1). M_2 = 1/3 is evaluated at k = 1.4.

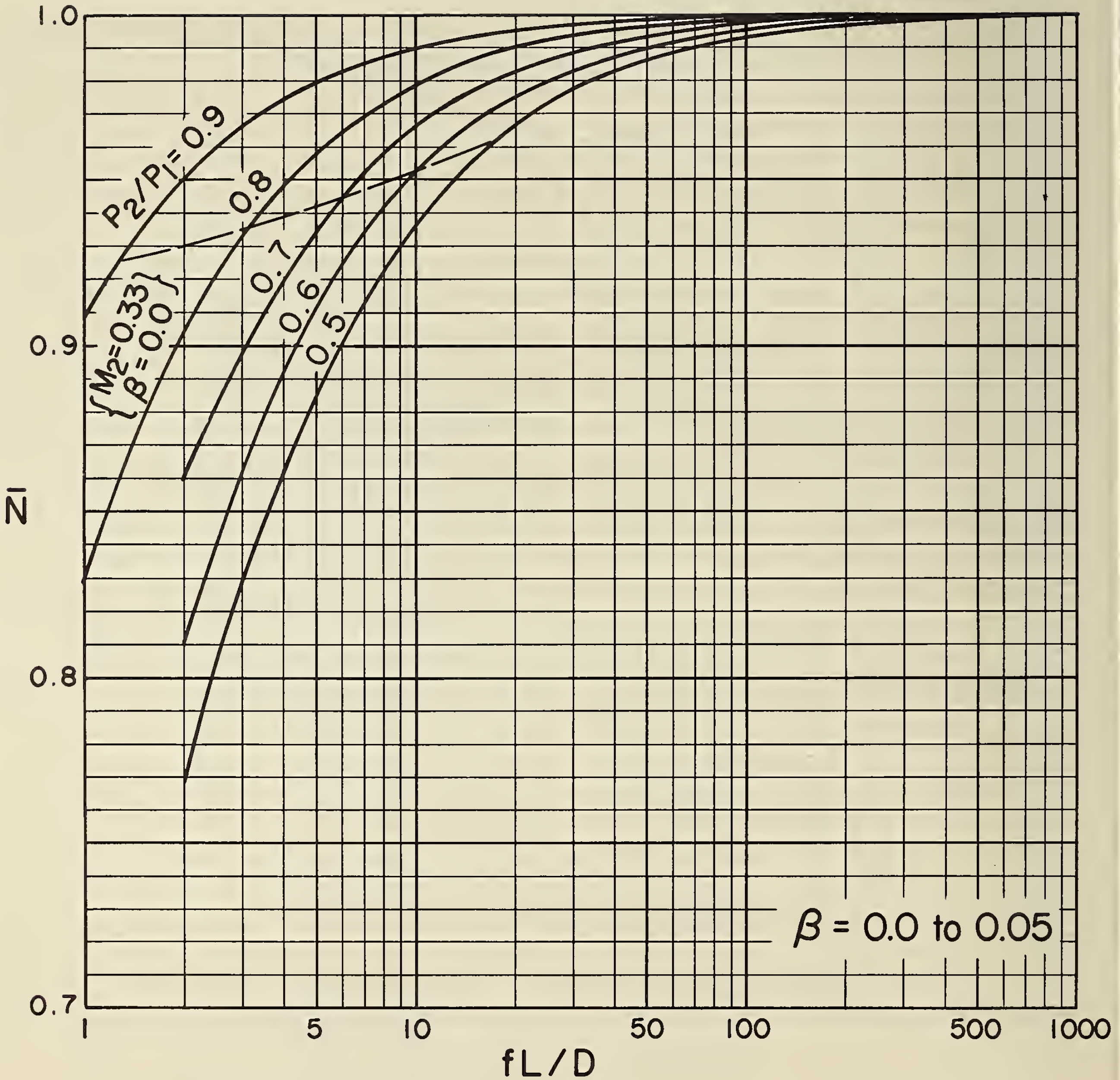

Figure 4.2 Comparison of "modified-incompressible" flow and isothermal compressible flow as a function of fL/D and P_2/P_1 for turbulent flow (β = 0.0 to 0.05). $M_2 = 1/3$ is evaluated at k = 1.4 and β = 0.0.

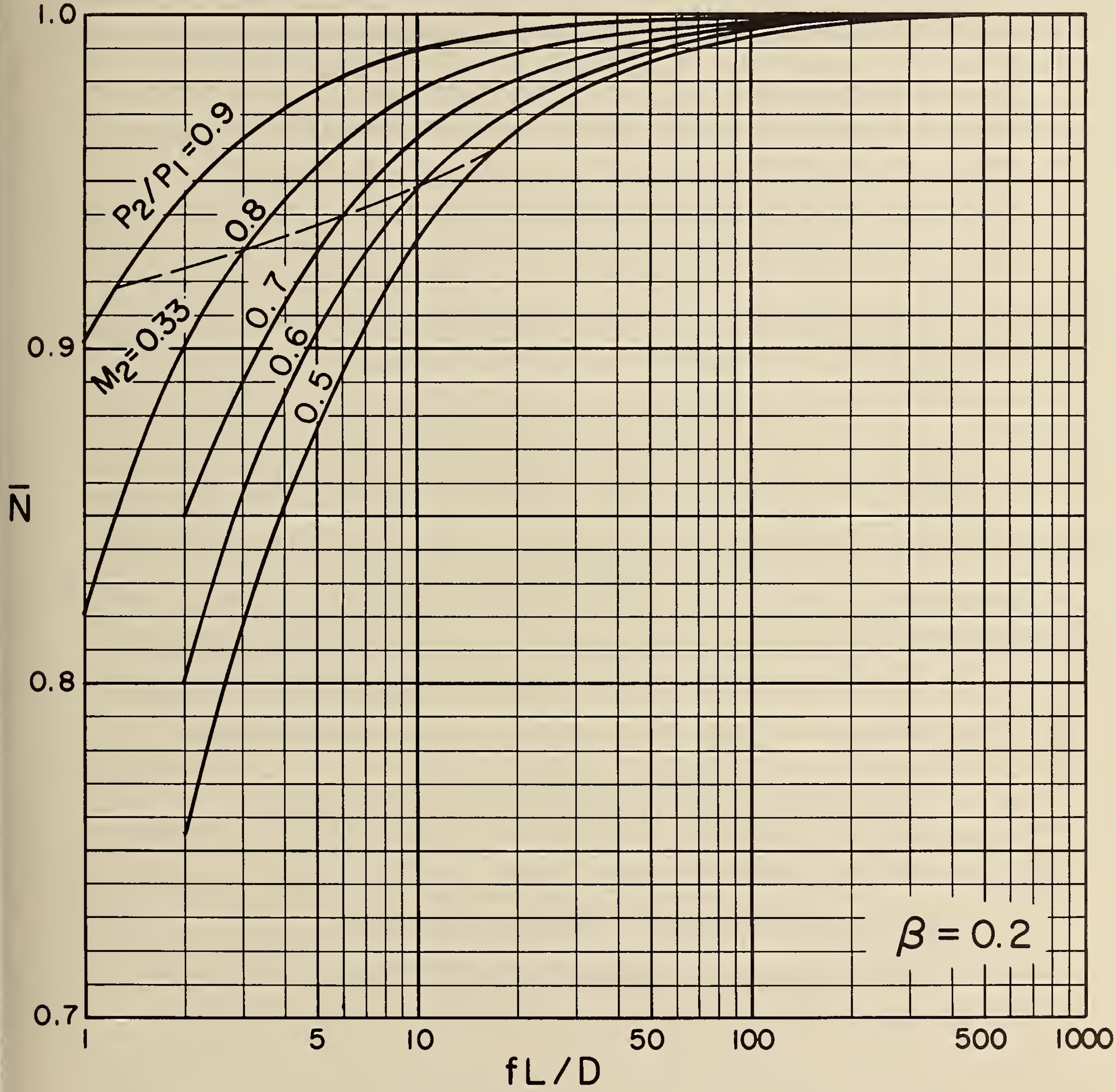

Figure 4.3 Comparison of "modified-incompressible" flow and isothermal compressible flow as a function of fL/D and P_2/P_1 for turbulent flow ($\beta = 0.2$). $M_2 = 1/3$ is evaluated at $k = 1.4$.

When the pipe is long, $fL/2D >> \ell n\,(P_1/P_2)$, $\overline{N} = 1$, and identical results are obtained from the compressible and modified incompressible theories. A trial and error solution of (4.1) is required for arbitrary values of $fL/2D$, and a simple quadratic solution arises in laminar flow ($\beta = 1$),

$$(\overline{N})_{\beta=1} = [-1+(1+4\sigma_2)^{\frac{1}{2}}]/2\sigma_2 \quad . \tag{4.2}$$

The comparative results given by (4.1) and (4.2) are shown on figures 4.1, 4.2, and 4.3.

5. Discussion of Figures

Figure 3.1 depicts the mass flow ratio, N, as a function of the pressure ratio P_2/P_1 and β for long pipes. For the range of turbulent flow, $\beta = 0.0$ to 0.2, there is little variation in the flow ratio for a given pressure ratio; however, there is an appreciable difference between laminar ($\beta = 1$) and turbulent flows. A large portion of isothermal flow problems fall into the long pipe category. By comparing figure 3.1 and figures 3.2 - 3.4 at identical values of β and P_2/P_1, the implications of a long pipe become clear. Figures 3.2 - 3.4 are not restricted to long pipes and the various pressure ratio curves asymptotically approach the flow ratio given by figure 3.1. Depending upon the degree of agreement required the long pipe requires fL/D of 10^2 to 10^3 for the range of pressure ratios plotted.

The counterpart of figure 3.1 for modified-incompressible flow is not given since the mass flow ratio, $\overline{N}$, is unity for long pipes. Accordingly, all of the curves shown in figures 4.1 - 4.3 approach $\overline{N} = 1$ asymptotically.

A few words concerning the construction of figures 3.2 - 3.4 and figures 4.1 - 4.3 will clarify their use. Figures 3 and 4 are plotted for values of β which are representative of the entire flow range: i. e., $\beta = 0.0$ to 0.05 is appropriate for constant friction at large values of Re and for turbulent flow in very rough pipes at lower Re; $\beta = 0.2$ applies to turbulent flow in smooth pipes at lower values of Re (10^4 to 10^5), and $\beta = 1$ is applicable to laminar flow. Comparison of figure 3.3 with figure 3.4 and figure 4.2 with figure 4.3 indicates very small differences in flow ratio occur for a particular P_2/P_1 due to change in β. Hence we have essentially two sets of figures; one is for laminar flow (figures 3.2 and 4.1) and the other is for turbulent flow (figures 3.3 and 4.2 or figures 3.4 and 4.3 may be used where Re $> 2 \times 10^3$). The pressure ratio curves are limited to values of fL/D greater than those at which sonic flow would occur at the pipe outlet, i. e., $M_2 < 1/\sqrt{k}$. To indicate the implications of the rule that $M_2 < 0.3$ permits neglect of compressibility, the locus of $M_2 = 1/3$ has been plotted on figures 3.2 - 3.4 and figures 4.1 - 4.3 This locus was constructed from the following relation which is derived by combining (2.3) and (3.3);

$$(fL/D) = N^{\beta} \left[(1/kM_2^2) \left\{ (P_1/P_2)^2 - 1 \right\} - 2\ln(P_1/P_2) \right], \qquad (5.1)$$

where k is evaluated at 1.4 and $M_2 = 1/3$. $\overline{N}$ was used in (5.1) for figures 4.1 - 4.3. In these figures, $M_2 < 1/3$ exists only for values of fL/D larger than those intersected by the dotted line ($M_2 = 1/3$) for a prescribed pressure ratio. For $M_2 = 1/3$, the error is observed to increase with decreasing pressure ratios for incompressible flow (figures 3.2 - 3.4), and to decrease with decreasing pressure ratios for modified-incompressible flow (figures 4.1 - 4.3). In both cases, N or $\overline{N}$ increases with decreasing values of M_2 at a fixed pressure ratio.

The improvement in computational precision offered by the modified-incompressible flow relation is elucidated by comparing figure 3.2

with figure 4.1, figure 3.3 with 4.2, etc. It should be mentioned that equation (2.2) represents the familiar Poiseuille flow relation when laminar flow ($\beta = 1$) is considered. Consequently, figure 4.1 illustrates the agreement between Poiseuille and compressible flows. Poiseuille flow is frequently utilized by vacuum textbook authors in analyzing the flow of gas in systems at low pressures.

Use of the various figures is straightforward and some applications were listed in the introduction to this paper. For the class of problems of interest, P_1/P_2 and the pipe dimensions are known. The flow may be classified as laminar or turbulent by conventional means using the Darcy-Weisbach friction formula, and the Hagen-Poiseuille relation for laminar friction factor. For laminar flow the friction factor is computed and then the appropriate chart (figure 3.2 or 4.1) is entered with known values of fL/D and P_2/P_1 and the mass flow ratio read directly. When turbulent flow is indicated the friction factor can usually be estimated within a factor of about two. Again the appropriate figure is used--e.g., figure 4.2--and the mass flow ratio read directly. If the degree of approximation of the incompressible calculation is inadequate, as indicated by the mass flow ratio obtained, a compressible calculation is required. If $fL/D >> \ell n(P_1/P_2)$ exact results are obtained by using equation (2.2), or figure 3.1 may be used to assess the error involved in a true incompressible flow calculation. Although there is no advantage in using the various figures to compute the compressible mass flow, they may be used this way. The incompressible flow is computed in the usual manner, i.e., by assuming a value for the friction factor and performing an iterative calculation to obtain f and $\mathring{m}_{ic}$, or by using one of the direct solution nomographs [10] available in the literature. A mass flow ratio is obtained from the pertinent figure and used as a "correction factor" to obtain the compressible mass flow. Where incompressible relations are substituted for compressible formulae in integrands, the applicable error or appropriate "correction factor" may be estimated as follows: the "correction factor" associated with the initial and final pressure differences in the gas transfer process are obtained as outlined above; an average "correction factor" may then be obtained and applied over the appropriate time interval.

6. End Effects

In most practical applications involving isothermal gas flow the effect of end losses is not significant[11]; however, in some short pipe installations, end losses may warrant consideration. Entrance and exit losses are dependent upon the area expansion or contraction ratio and the geometry (i. e., abrupt, contoured, etc.) of the transition. The pressure loss, due to abrupt[11, 12] enlargements and contractions, has been shown to be a function of dimensionless flow parameters, e. g., Reynolds Number, pressure ratios and area ratios.

It has also been shown[11-13] that incompressible loss coefficients may be used for compressible fluids at low Mach numbers (<0. 15 according to Kays[11]). At higher Mach numbers the compressible and incompressible loss coefficients differ [12, 13] appreciably. The loss coefficients also depend on whether the flow is laminar or turbulent[11].

In addition to the pressure losses due to sudden changes in flow cross section there are additional friction losses associated with the establishment of "fully developed" flow in the entrance region of a tube. The establishment of a fully developed boundary layer and velocity profile in a pipe requires a certain entrance or "entry" length. The "entry length" which is sometimes referred to as the critical length is given [14] for laminar flow by $L_e/D = 0.0575\ Re$. The entry length for turbulent flow is given by Schlichting[8] (attributed to Nikuradse) as 25 to 40 pipe diameters. Latzko[15] has developed an expression for turbulent flow in a bellmouthed tube, $L_e/D = 0.693\ Re^{0.25}$. Foust, Wenzel, Clump, Maus, and Anderson[16] state that "the pressure drop through the entry length is considerably greater than the pressure drop after fully developed flow is established." They suggest the pressure drop through the entry length be taken as two to three times the value for fully developed flow in an equivalent length downstream of the entrance region. Most analyses treating entrance region losses include the momentum and friction losses incurred, while a simple contraction loss coefficient neglects the additional friction loss required to establish flow. Lundgren, Sparrow, and Starr[17] have shown that the friction component accounts for 25 to 40 percent of the total loss in the entrance region of ducts of arbitrary cross section in laminar incompressible flow. The bulk of the literature concerning entrance region losses has treated laminar incompressible flow[17-27]. A summary of the literature is given in reference[27]. Few[28] articles concerning turbulent entry regions have appeared.

From the foregoing discussion it is apparent that correction of the mass flow calculations for "end losses" will vary with each individual problem; therefore, it is not possible to account for end effects in the comparison figures presented herein. Further discussion of end effects is beyond the scope and objective of this paper and the reader is referred to the literature cited in this section for detailed information.

7. Summary

1. Incompressible and isothermal compressible mass flow relations have been graphically compared for prescribed static pressure ratios. The comparative data are dependent upon the level of turbulence. "Modified-incompressible" flow is shown to agree more closely with the compressible case than true incompressible flow; the compressible and

modified-incompressible formulae are identical when $fL/D >> \ell n(P_1/P_2)$. The results of the study are given in figures 3.1 through 4.3 and some applications have been outlined.

2. The error involved in neglecting compressibility when $M_2 < 1/3$, or $(P_1 - P_2)/P_1 < 0.10$, is indicated on figures 3.2 - 3.4 and figures 4.1 - 4.3. It has been demonstrated that the assumption of a <u>long</u> pipe requires $fL/D > 10^2$.

8. Nomenclature

A	=	cross sectional area of pipe $[\equiv \pi D^2/4]$
D	=	inside diameter of pipe
f	=	friction factor (Darcy-Weisbach) in incompressible flow $[\equiv \alpha/Re^{\beta}]$, dimensionless
g_c	=	conversion factor in Newton's law of motion [given in engineering units as g_c = 32.2 (ft) (pounds mass)/ (sec^2) (pounds force)]
k	=	ratio of specific heats $[\equiv C_p/C_v]$, dimensionless
L	=	length of pipe
L_e	=	entry length [defined in text]
M	=	Mach Number [≡ gas velocity/adiabatic acoustic velocity], dimensionless
$\mathring{m}$	=	mass flow rate
N	=	ratio of isothermal gaseous mass flow to incompressible mass flow $[\equiv \mathring{m}_c/\mathring{m}_{ic}]$, dimensionless
$\overline{N}$	=	ratio of isothermal gaseous mass flow to "modified-incompressible" mass flow $[\equiv \mathring{m}_c/\mathring{m}_{mic}]$, dimensionless
P	=	absolute pressure of gas in pipe
$\overline{P}$	=	arithmetic mean pressure in the pipe $[\equiv (P_1 + P_2)/2]$

Re = Reynolds Number (based on pipe diameter), dimensionless

R_G = gas constant in mechanical units, equals $P/\rho T$, equals universal gas constant divided by the molecular mass [typical engineering units are (ft)(pounds force)/(pounds mass((degree Rankine)]

T = absolute temperature of gas in pipe

V = bulk velocity of gas in pipe

Greek

α = constant in friction factor formula

β = constant in friction factor formula

ρ = gas mass density

$\bar{\rho}$ = arithmetic mean gas mass density $[\equiv(\rho_1 + \rho_2)/2]$

σ_1 = variable in comparative mass flow ratio $[\equiv(1 + P_2/P_1)/2]$

σ_2 = variable in comparative mass flow ratio $[\equiv\{\ell n(P_1/P_2)\}/(fL/2D)]$

Subscripts

c = denotes isothermal compressible flow

ic = denotes isothermal incompressible flow

mic = denotes "modified-incompressible" flow

1 = denotes pipe inlet condition

2 = denotes pipe exit condition

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