NBS TECHNICAL NOTE

354

The Single-Engine Claude Cycle As A 4.2°K Refrigerator

U.S. DEPARTMENT OF COMMERCE
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THE SINGLE-ENGINE CLAUDE CYCLE AS A 4.2°K REFRIGERATOR

R. C. Muhlenhaupt and T. R. Strobridge

The performance of the 4.2°K Claude-cycle refrigerator has been computed taking into account the efficiencies of the various components. The results are presented in graphical form. These charts give the input power requirements, mass flow rates for both the compressor and expander, pertinent temperatures, and allow selection of the optimum high pressure for a given set of component characteristics.

Key Words: Claude, cryogenics, refrigeration

1. Introduction

The Claude cycle, as it has become known after the Frenchman who constructed one of the first really successful low temperature expansion engines, has been used for many years in air liquefaction and separation plants. This thermodynamic cycle has also found use in other applications at lower temperatures and has been incorporated in several different liquid helium temperature refrigerators. Virtually all liquid helium temperature refrigerators have a heat exchanger and a Joule-Thomson expansion valve as the lowest temperature stage of the cycle.[1] Before the Joule-Thomson process becomes effective, the helium gas entering the exchanger must be cooled below the inversion temperature, about 55°K. The difference between the thermodynamic cycles that are used for continuous cooling at about 4°K is the manner in which the precooling is provided. Variations of the Cascade Joule-Thomson, the Gifford-Mc Mahon and the Stirling cycles have all been incorporated in liquid helium temperature refrigerators. In the Claude

1. A complete analysis of the Joule-Thomson process is given by Dean and Mann (1965).

cycle, the precooling is accomplished by a work producing expansion machine. High pressure gas is cooled upon expanding, converting its potential energy to work. This cold fluid, which must be brought below the inversion temperature, is then used to refrigerate the gas which circulates through the lowest temperature stage of the cycle.

The performance of the Claude cycle has been computed taking into account the efficiencies of the various components. The results are presented in graphical form. These charts give the input power requirements, mass flow rates for both the compressor and expander, pertinent temperatures, and allow selection of the optimum high pressure for a given set of component characteristics.

2. Nomenclature List

m_1	total mass flow	(g/s)
m_2	expander flow	(g/s)
F_t	compressor flow rate per unit of refrigeration	(scfm/w)
h	enthalpy	(J/g)
k	specific heat ratio	(unitless)
K	conversion factor	(3.5314455×10^{-5} ft^3/cm^3)
M	molecular weight	(4.0028 g/g mole for helium)
P	pressure	(atm)
R	universal gas constant	(8.3143 J/g mole °K)
T	temperature	(°K)
V_{std}	specific volume at 1.0 atm and 273.15°K	(5603.24 cm^3/g for helium)
W_c/Q	compressor power per unit of refrigeration available at 4.2°K	(watts/watt)

W_e/Q	expander power per unit of refrigeration available at 4.2°K	(watts/watt)
η_c	isothermal compressor efficiency	(unitless)
η_e	isentropic expander efficiency	(unitless)
scfm	standard cubic feet per minute measured at 1 atm and 273.15°K	(ft^3/min)

Numerical subscripts refer to the state of the refrigerant as shown on the schematic flow diagram in figure 1.

3. Discussion of Process

Basically the Claude refrigerator is a combination of the modified Brayton and the Joule-Thomson cycles where the modified Brayton system precools the primary refrigerant stream for the Joule-Thomson process. The streams pass through the same compressor and heat exchangers Depending upon the particular application, any number of expansion engines or expansion valves (J-T) may be included in the system.

A schematic flow diagram of a double-valve Claude cycle is shown in figure 1, and the corresponding process paths are shown on temperature-entropy coordinates in figure 2. A single-valve Claude cycle would not have the lower heat exchanger, HX IV, or the first expansion valve between stations 4 and 5. The assumed processes for the single-valve cycle are (primary refrigerant stream) isobaric cooling between stations 1 and 6, isenthalpic expansion between 6 and 7, isothermal heat addition constituting the refrigeration of the load between 7 and 8, isobaric heating between 8 and 12, isothermal compression between the pressure levels at T_1 (heating from T_{12} to T_1 is considered to take place in connecting

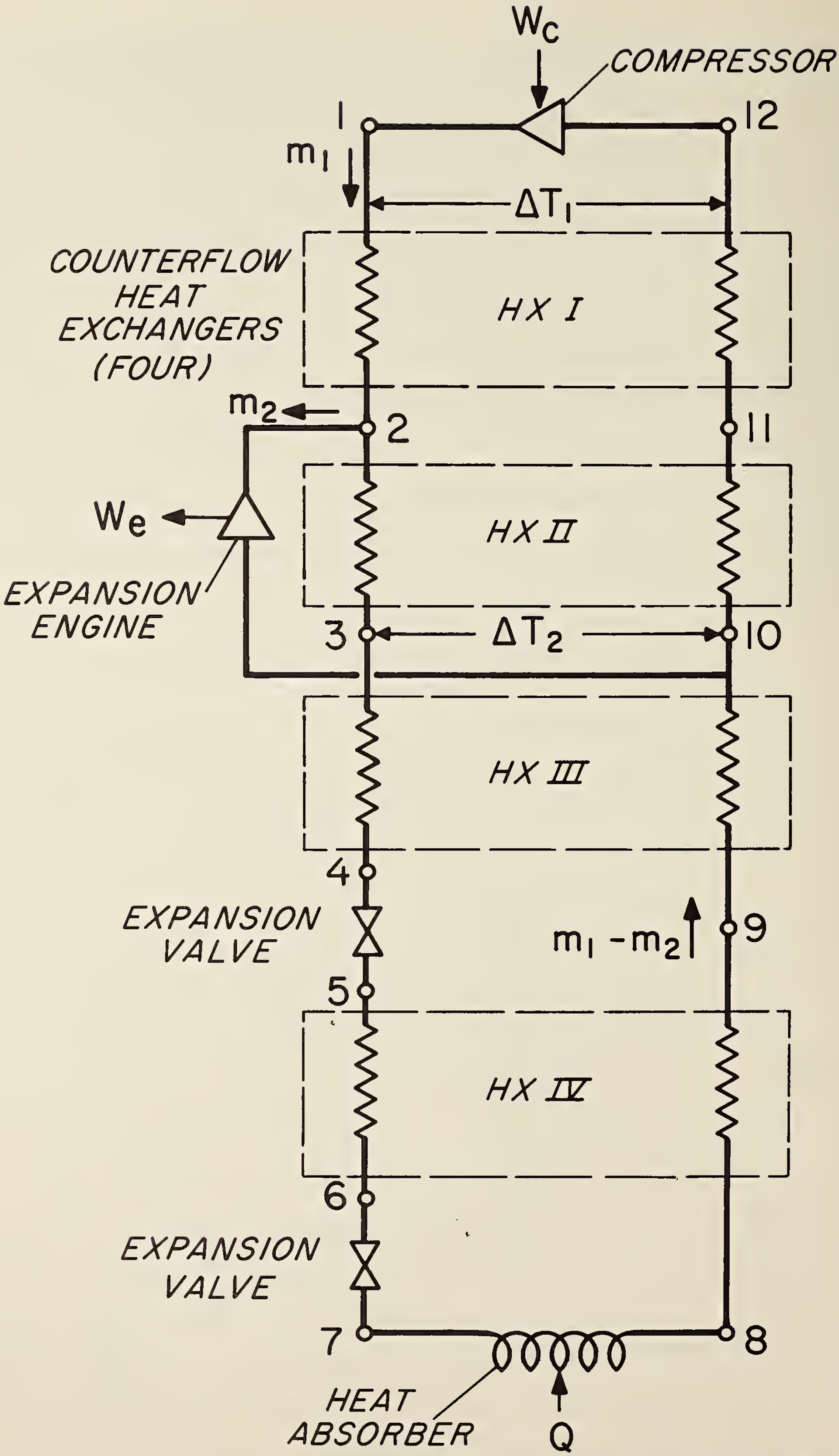

Figure 1. Schematic Flow Diagram of the Single-Engine Claude Refrigerator

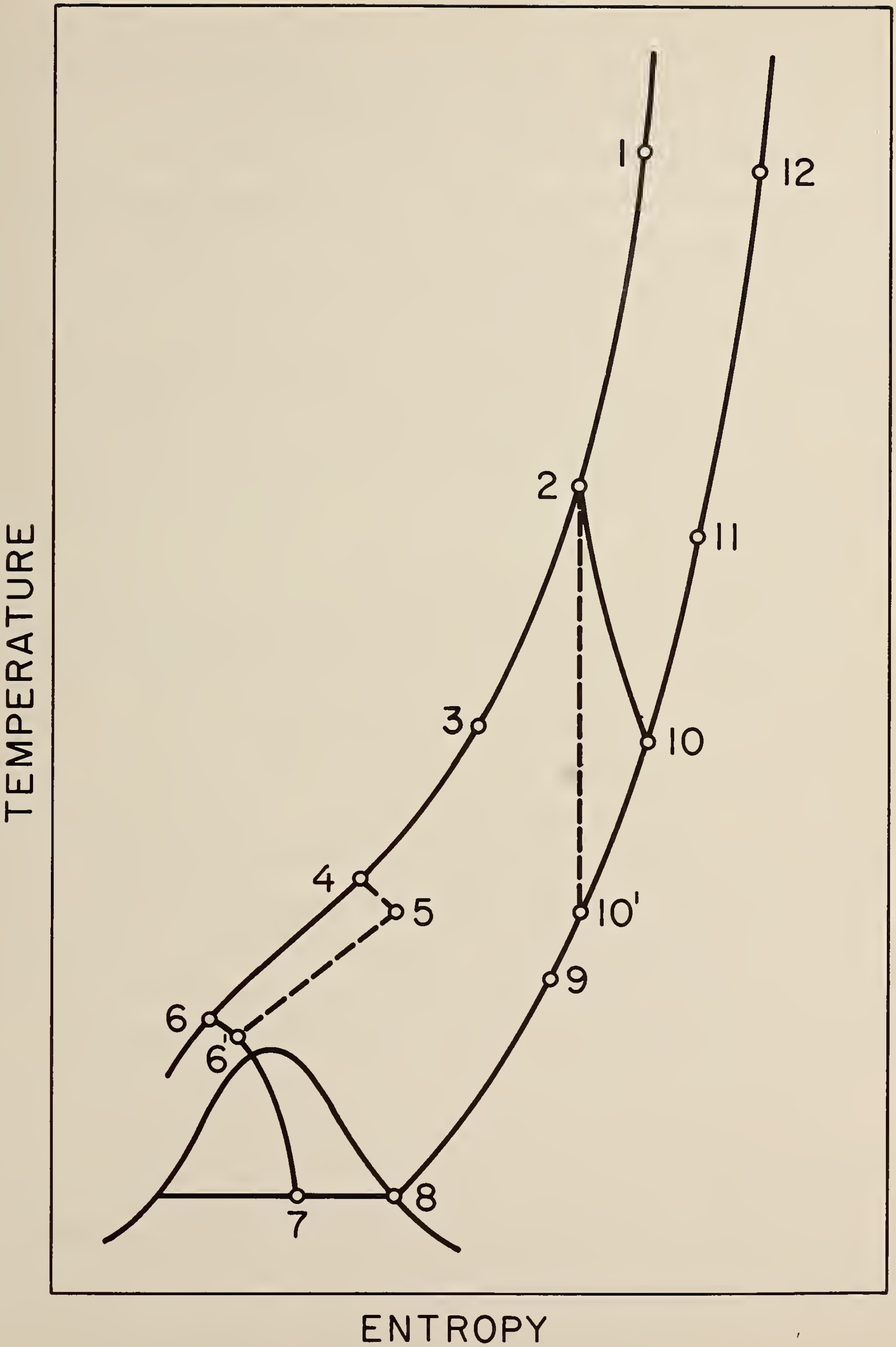

Figure 2. Temperature-Entropy Diagram of the Single-Engine Claude Refrigerator

lines by heat exchange with the ambient), and adiabatic expansion between 2 and 10 of the precoolant stream. The inclusion of a second valve produces essentially the same set of conditions with the addition of isenthalpic expansion between 4 and 5 and isobaric cooling between 5 and 6 . Deviations from these assumed processes will occur in practice. Specifically, pressure drop in the piping and heat exchangers precludes isobaric heat transfer and the compression will not be isothermal. A means of estimating the effect of pressure drop is presented in the Appendix. Further, it is possible to calculate the actual work of compression by applying an efficiency factor to the energy required to isothermally compress an ideal gas.

The compressor circulates the helium at mass flow rate m_1. As the high pressure stream passes through the warmest of the counterflow heat exchangers, HX I, it is cooled by the low pressure stream to temperature T_2. At that point, some of the fluid is diverted through the expansion engine at rate m_2. The remainder of the flow, $m_1 - m_2$, is further cooled in HX II by both the low pressure stream returning from the load and by the cold exhaust of the expansion engine, and then passes through the final heat exchanger before entering the J-T valve. A portion of the stream is liquefied during the expansion process, and this liquid is evaporated as the stream passes through the heat absorber. The vapor then returns through the low pressure side of the heat exchangers.

Steady state conditions are assumed throughout for the analysis. Elevation and kinetic energy contributions to the fluid energy are negligible. With these considerations, application of the first law of thermodynamics to the various components of the system yields the following equations. For the upper heat exchanger HX I

$$h_1 - h_2 = h_{12} - h_{11} \quad . \tag{1}$$

For heat exchanger HX II

$$(m_1 - m_2)(h_2 - h_3) = m_1 (h_{11} - h_{10}) \quad . \tag{2}$$

For heat exchanger HX III

$$h_3 - h_4 = h_{10} - h_9 \quad . \tag{3}$$

For the lower heat exchangers HX III and HX IV

$$h_3 - h_6 = h_{10} - h_8 \quad . \tag{4}$$

For the expansion valve

$$h_7 = h_6 \quad . \tag{5}$$

For the heat absorber evaporator

$$Q = (m_1 - m_2)(h_8 - h_7) \quad . \tag{6}$$

4. Calculations

The performance of the Claude-cycle refrigerator is a function of the efficiencies of the heat exchangers, the expander, and the compressor. The amount of refrigeration that is lost in the counterflow heat exchangers is reflected by the temperature difference, ΔT, between the streams at the warm ends of the exchangers. All real expanders produce less work than an ideal isentropic machine, reducing potential refrigeration.

The power required to produce a unit of refrigeration is a complicated function of the characteristics of the heat exchangers and the expansion machine, but is simply inversely proportional to the efficiency of the compressor and drive motor. All the calculations were made assuming 100 percent efficient, isothermal compression of an ideal gas since the results can be related to any real compressor through its actual efficiency.

Calculations were made for perfect heat exchangers and expander to provide a reference for the degradation of performance caused by non-ideal components. Further computations were performed for realistic component efficiencies and for nitrogen precooling. A low pressure of one atmosphere was used for all cases. The parameter values are given in Table I.

Table I

Independent Parameters

ΔT_1 (°K)	ΔT_2 (°K)	η_e (percent)	T_1 (°K)	P_1 (atm)
0	0	50	300	2 to 60
1	0.25	70	77.36	
2	0.50	80		
5	0.75	100		
10	1.0			
	1.5			

The results give the compressor power requirement per unit of refrigeration, the expander power output per unit of refrigeration, the fraction of total mass flow through the expander, the total flow per unit of refrigeration, the expander inlet temperature, and the expander outlet temperature.

The thermodynamic properties of helium used for the computations are from Mann (1962) and can be calculated directly from the equation of state. However, this would be a time consuming effort since data at over five million states were required for the present work. Four point interpolation in the tabular data was found to be sufficiently accurate and a much more rapid way of obtaining the values. Comparisons of the power required per unit of refrigeration, calculated from data obtained by either method, show differences of less than 0.01 percent. The differences in the other cycle characteristics are less than 0.5 percent.

The power required to isothermally compress an ideal gas from P_{12} to P_1 at temperature T_1 is

$$W_c = \frac{m_1 R T_1 \ln (P_{12}/P_1)}{M} , \tag{7}$$

or when an efficiency factor for a real machine compressing a real gas is applied

$$W_c = \frac{m_1 R T_1 \ln (P_{12}/P_1)}{\eta_c M} . \tag{8}$$

Dividing (8) by (6) gives the compressor power requirement per unit of refrigeration or the figure of merit

$$W_c/Q = \frac{R T_1 \ln (P_{12}/P_1)}{\eta_c M (m_1 - m_2)(h_8 - h_7)} . \tag{9}$$

The work of compression is negative; however, in the charts that follow, the absolute values of W_c/Q are plotted.

An isentropic expansion machine would discharge the fluid at state 10', figure 2. The losses incurred in a real machine result in an entropy increase and a discharge at a higher temperature at state 10. Since the process is adiabatic, the work produced by the expander is equal to the mass flow rate times the enthalpy change of the fluid from inlet to outlet.

The efficiency of such a device is defined as the ratio of the actual work to the isentropic work or

$$\eta_e = \frac{h_2 - h_{10}}{h_2 - h_{10'}} \tag{10}$$

and the power produced per unit of refrigeration at 4.2°K is

$$\frac{W_e}{Q} = \frac{m_2 (h_2 - h_{10})}{(m_1 - m_2)(h_8 - h_7)} \quad . \tag{11}$$

Rearranging (2) and substituting equalities gives the expression for percent of total mass flow through the expander

$$\frac{m_2}{m_1} = \left(\frac{h_1 - h_{12} + h_{10} - h_3}{h_2 - h_3}\right) x\ 100 \ . \tag{12}$$

The volumetric flow per unit of refrigeration is

$$F_t = \frac{W_c}{Q} \frac{KMV_{std}}{RT_1 \ \ell n\, (P_{12}/P_1)} \quad . \tag{13}$$

For a given set of heat exchanger and expander efficiencies and a high pressure, there is, in terms of the amount of power required per unit of refrigeration, an optimum engine inlet temperature. Equation (12) shows that the flow distribution is a function of the engine inlet temperature. The Joule-Thomson portion of the cycle becomes more efficient as the inlet temperature of the exchanger (governed by the inlet temperature of the expansion machine and its efficiency) is lowered. However, at lower temperatures the precooling refrigeration from the expander decreases so the expander flow rate must increase and more compressor power for precooling is required. The optimum engine inlet temperature occurs at a balance between these two opposing effects. For each set of conditions, the best value of T_2 was found to within 0.05°K and the curves in figures 4 through 21 are all for optimum T_2.

The data for the constant T_2 and T_{11} curves were found by iteration to within 0.05°K, and the data for the constant expander flow curves were also found by iteration to within 0.1 percent.

It is possible to select numerous sets of independent parameters from Table I which are invalid for a single expansion-valve refrigerator due to implied violations of the second law of thermodynamics in the Joule-Thomson heat exchanger. The fluid entering the low pressure side of that heat exchanger is assumed to be saturated vapor at one atmosphere, station 8, figure 1 and state 8, figure 2. Optimization of T_2 determines state 10, selection of ΔT_2 gives state 3, and solution of (4) determines state 6. All these manipulations are straightforward applications of the first law of thermodynamics. However, because the specific heats of the high and low pressure streams are different, it may be that under the assumed conditions, the calculated temperature at state 6 is lower than at state 8. This impossible situation is illustrated schematically in figure 3 where the temperatures of the high and low pressure streams are plotted as a function of the heat transferred between them. This cooling curve shows, with the dashed high pressure line, the invalid temperature profile in the heat exchanger calculated from the first law of thermodynamics for a single-valve arrangement. The effect of a secondary valve between 4 and 5 is to lower the pressure to some intermediate level where the specific heat of the fluid will give a positive temperature difference throughout the exchanger. The heat balance below the expansion machine is not affected by the second valve. During the computations, if a negative temperature difference was found between 6 and 8, a second valve was included in an attempt to correct the situation. The region where such valves are required is indicated on the performance charts. Under

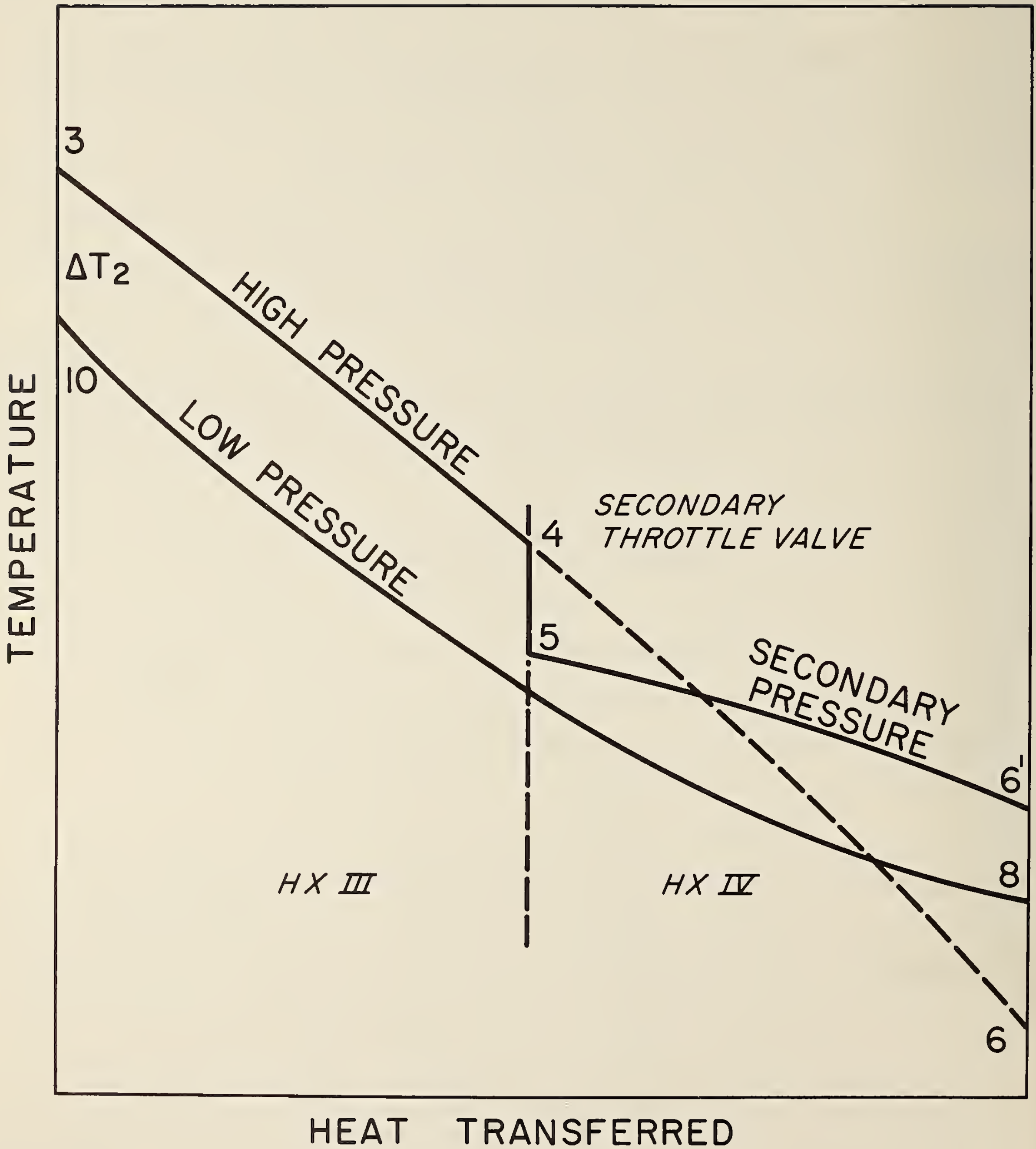

Figure 3. Cooling Curve for Heat Exchanger with Secondary Throttle Valve

certain circumstances, the thermal properties of the fluid made it impossible to have a positive temperature difference even with two valves, and the curves for $\Delta T_2 = 0$ are discontinued at higher pressures for this reason. In practice, additional valves could be used.

5. Results

The calculated performance characteristics of the single-engine Claude-cycle refrigerator at 4.2°K are shown in figures 4 through 31. Figures 4a through 15a show the compression power per unit of refrigeration W_c/Q as a function of high pressure P_1 for various values of expander inlet temperature T_2 and upper heat exchanger inlet temperature (low pressure side) T_{11}. The values on the charts represent the conditions for which the expander inlet temperature T_2 is at an optimum level and the cycle inlet temperature T_1 is at a temperature of 300°K. The solid line marked "two valves" indicates the limit to the range of possible operating pressures for the single-valve refrigerator. Hence, an additional valve must be included in the system before higher pressures are valid. In general, W_c/Q reaches a minimum value; thus, the optimum high pressure P_1 (i.e., the pressure at which the minimum occurs) can be determined. Therefore, the cycle can be optimized for high pressure as well as for expander inlet temperature. Figures 4b through 15b show expander power per unit of refrigeration at 4.2°K W_e/Q as a function of high pressure P_1 for various values of expander flow m_2.

The same information (W_c/Q versus P_1 and W_e/Q versus P_1) is presented for nitrogen precooling (i.e., T_1 equals 77.36°K) in figures 16 through 21.

Figures 22 through 29 show the expansion engine outlet temperature T_{out} as a function of the expander inlet temperature T_{in} for several values of helium inlet pressure P_{in}. For an isentropic expansion of an ideal gas, the relationship of T_{out} to T_{in} is represented by

$$T_{out} = T_{in} \left(\frac{P_{out}}{P_{in}}\right)^{\frac{k-1}{k}} \tag{14}$$

where k is the specific heat ratio of the gas and P_{out} is some constant value. Therefore, a plot of T_{out} versus T_{in} would be a set of straight lines with slopes

$$\frac{dT_{out}}{dT_{in}} = \left(\frac{P_{out}}{P_{in}}\right)^{\frac{k-1}{k}} , \tag{15}$$

and the slope of a high pressure line will always be less than that of a low pressure line. A plot of T_{out} versus T_{in} for helium should be very similar, and this is true except for the few cases where the high pressure curves intersect. It can be shown that this behavior results from the non-ideal properties of the gas and the efficiency of the expander. The more general variable subscripting in (14) and (15) and figures 22 through 29 was used because the equations and data apply to expansion machines in any cycle.

Figures 30 and 31 show total volumetric flow F_t as a straight-line function of the compression work per unit of refrigeration W_c/Q for several values of high pressure P_1. The slope of each curve is represented by

$$\left[\frac{dF_t}{d\,(W_c/Q)}\right]_{P_1} = \frac{KMV_{std}}{RT_1 \,\ell n\,(P_{12}/P_1)} . \tag{16}$$

The logarithmic term in (16) is negative; consequently, the slope is also negative. For convenience, however, the curves are plotted with a positive slope. The values of W_c/Q used to find the flow rates from figures 30 and 31 must be for 100 percent compressor efficiency with no pressure drop in the system.

Since the calculations were performed assuming 100 percent isothermal compression, the plotted values of W_c/Q in figures 4a through 21a must be divided by the actual isothermal compressor efficiency to obtain the power requirement for a lower efficiency.

The charts do not contain any information regarding the effect of pressure drop on the performance characteristics. This is because many conceivable sets of conditions could be assumed, thereby making the work approach undesirable proportions. In addition, the effect of pressure drop can be readily estimated. As will be shown later, the power requirement increases, the amount depending upon the pressure loss in the system.

The shapes of the curves in figures 4 through 21 reflect the characteristics of the Brayton cycle and the Joule-Thomson process, which in turn depend upon the thermodynamic properties of the working fluid. The charts are in pairs (a and b) with each pair giving data for constant ΔT_1 and η_e. For illustration, refer to figures 4a and 4b which are for a 10°K temperature difference at the warm end of HX I and a 100 percent efficient expander. In figure 4a, for a ΔT_2 of 1.0°K, the minimum W_c/Q occurs at about 25 atmospheres high pressure with the engine inlet temperature of about 39°K. The outlet temperature of a 100 percent efficient expander with an inlet at 39°K will be ~ 11°K (see fig. 22). Since ΔT_2 is 1.0°K, the inlet to the Joule-Thomson heat exchanger will be at 12°K. From Dean and Mann (1965), figure 12, the optimum high pressure for the Joule-Thomson process, with the inlet at 12°K and a 1°K temperature difference, is ~ 22 atmospheres with the performance decreasing as the high pressure is decreased. The optimum high pressure for a Brayton cycle with the discharge of the expander at 11°K is greater than 25 atmospheres for an ambient temperature difference in the heat exchanger of 10°K. Therefore, the best high pressure for the combination of the cycles falls somewhere in between - in this case ~ 25 atmospheres.

Consider now the effect of changing the value of ΔT_2 while holding the high pressure constant at ~ 25 atmospheres. The performance of the cycle increases as ΔT_2 becomes smaller because the Joule-Thomson process efficiency increases. The optimum engine inlet temperature rises slightly, since the most favorable inlet temperature for the Joule-Thomson process varies in the same way. Helium behaves almost as an ideal gas at moderate pressures and down to fairly low temperatures. This means that the temperature difference at the cold end of HX I will be about the same as at the warm end, and the curves for constant values of T_{11} will roughly parallel, or at least have the same general characteristics as, those for constant T_2.

At lower pressures, the performance of both cycles becomes progressively unfavorable and the power requirement per unit of refrigeration increases rapidly. There is still a general trend of increasing optimum engine inlet temperature with better Joule-Thomson heat exchanger performance at constant pressure (see the curve for $T_2 = 30°K$).

In figure 4b, the expansion engine power output, the amount of precooling performed by the expander, is given as a function of pressure for several values of ΔT_2. At low pressures, two effects are evident. The relative amount of flow through the expander increases as does the amount of energy removed from the cycle per unit of refrigeration. The total flow rate is proportional to W_c/Q, so the amount of precooling required is also proportional to W_c/Q as reflected by the shape of the curves in figure 4b. The relative flow through the expander decreases at higher pressures where a greater amount of work is produced per unit flow.

Figures 4 through 21 for the single-engine Claude cycle show that reasonable thermal efficiency can be expected for this type of refrigerator - much better than a cascade Joule-Thomson cycle, for example, but less efficient than a two-expander cycle. For discussion purposes, consider a set of realistic component efficiencies (i.e., realistic in the sense that no advance in technology would be required to design and fabricate such components today). Depending on the mass flow rate, expander efficiencies between 50 and 80 percent can be achieved. The temperature difference at the warm end of HX I could be between 5 and 10°K, while ΔT_2 across HX III could be less than 1°K, with 0.5°K considered to be a reasonable value. Choosing ΔT_1 as 5°K, ΔT_2 as 0.5°K, and η_e as 70 percent, figure 10a shows a power requirement of about 345 watts/watt for a 100 percent efficient compressor and no pressure drop. Typical isothermal compressor efficiencies are about 60 percent. Estimates can be made of the effect of fluid pressure losses. For example, if the pressure drop were 10 percent of total pressure in both the high and low pressure streams, the intake and discharge at the compressor would be 0.9 and 27.5 atmospheres, respectively, for optimum conditions. Ideally, this would raise W_c/Q to about 365. If the compressor efficiency is included, W_c/Q would be about 610. In addition, there are other losses, such as heat leak to the low temperature space, which becomes critical in smaller capacity refrigerators and which should be considered when translating the information given here to real situations.

Inspection of the performance charts shows the possible improvement in thermal efficiency if the components could be slightly upgraded. Consider making the following changes in efficiencies given in the example.

1. Decrease ΔT_2 from 0.5°K to 0.25°K,
2. Increase η_e from 70 to 80 percent, and
3. Increase η_c from 60 to 70 percent while holding ΔT_1 constant at 5°K.

From figure 9a, W_c/Q is about 245. The assumed pressure drop increases that value to about 260, and the compressor efficiency increases it further to the final value of 371 which is a considerable improvement.

About the same improvement in performance as in the previous example is shown in figure 17a which illustrates the effect of precooling the cycle with liquid nitrogen. Precooling involves depressing T_1 to the temperature of an evaporating liquid-nitrogen bath and providing an additional heat exchanger to communicate with the ambient temperature compressor. The liquid nitrogen consumption is determined by the efficiency of the warmest heat exchanger. The cooling provided by the liquid can be converted into an equivalent power requirement to be added to the compressor power. In most instances it is estimated that the increase in W_c/Q would not exceed ten percent and is considered to be negligible. Inclusion of the equivalent power required for precooling would involve additional parametric studies beyond the scope of this work. Typical conditions could be

T_1 = 77.36°K,

ΔT_1 = 2.0°K,

ΔT_2 = 1.0°K,

η_e = 70 percent,

η_c = 100 percent, and

W_c/Q = 243 at a high pressure of 19 atmospheres with no pressure drop.

Many existing Claude-cycle refrigerators are liquid-nitrogen precooled, but only the largest of these approach the power requirement of 468 watts per watt of refrigeration which comes from adjusting the above conditions for 10 percent pressure drop and 60 percent compressor efficiency (see Strobridge and Chelton, 1966).

6. Conclusions

The proper choice of a low temperature refrigeration cycle to meet a given set of duty requirements rests on a balance between economic and technical requirements. Present applications for such refrigerators are so varied that it is impossible to make definite statements about the "best" cycle for general use. It is, however, possible to discuss the advantages and disadvantages of a particular cycle.

The single-engine Claude-cycle refrigerator is capable of reaching potentially high thermal efficiencies with reasonable improvements in present heat exchangers, expanders, and compressors, and competitive performance can be obtained with present components. This cycle selected for analysis is an elementary configuration. The single expansion engine allows for simple control. Refrigerators operating in the liquid region provide refrigeration at constant temperatures which is imperative for some applications. The refrigeration temperatures are limited, but control of those temperatures is easily affected by regulating the pressure over the liquid in the evaporator.

The minima in the performance curves are reasonably flat; thus, for the sample set of conditions, where W_c/Q is a minimum of 345 at 25 atmospheres, the high pressure could vary between 17 and 36 atmospheres without increasing W_c/Q by more than 10 percent. This behavior permits good flexibility in component selection. The optimum pressures are generally between 20 and 35 atmospheres.

The Carnot efficiency, a useful tool of the refrigerator designer, indicates the extent to which the real refrigerator deviates from ideal performance. In general, the Carnot efficiencies of the single-engine Claude refrigerator, operating at a cycle inlet temperature of 300.0°K, are somewhat low. For example, the following reasonable conditions

$$\Delta T_1 = 5.0°K,$$

$$\Delta T_2 = 0.5°K,$$

$$\eta_e = 80 \text{ percent},$$

$$\eta_c = 60 \text{ percent, and}$$

$$\text{Pressure drop} = 10 \text{ percent},$$

yield a Carnot efficiency of only ~ 15 percent at the optimum high pressure of 24 atmospheres.

A simple hand calculation of the two-engine Claude-cycle refrigerator was performed for comparison purposes. In addition to the same set of conditions, the isentropic efficiency of the second engine was assumed to be 80 percent, and the temperature difference at the top of the additional heat exchanger was assumed to be 0.25°K. This configuration yielded a Carnot efficiency of approximately 23 percent. Liquid nitrogen precooling of the single-engine cycle would produce about the same effect without the added complexity of the second expander.

Any heat leak to the low temperature region of the refrigerator will decrease the amount of useful refrigeration, so the calculated performance is better than can be realized in practice. The magnitude of the heat leak depends upon the size and geometry of the low temperature enclosure and is especially critical in small capacity refrigerators.

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8. Appendix

The performance charts may be used in combination to determine the design parameters of a single-engine Claude refrigerator. An illustration will best describe their use. Consider the conditions

T_1	=	300°K,
$\eta_{e \text{ isentropic}}$	=	80 percent,
$\eta_{c \text{ isothermal}}$	=	60 percent,
ΔT_1	=	5.0°K,
ΔT_2	=	0.5°K,
Q	=	10 watts, and
P_8	=	1.0 atm,

which are practical and readily attainable in the current state-of-the-art. In addition, the heat exchangers experience some pressure losses with a 10 percent drop considered to be an average value. The greatest pressure losses occur in HX I, because the fluid passing through it is at a lower density than in the other exchangers and HX I is the largest (in terms of heat transferred) of the exchangers. Hence, it is reasonable to assume that the total drop occurs here, and the example that follows will be based on this assumption.

The appropriate performance chart (i.e., figure 9a) shows that the optimum operating high pressure is 23 atm for which T_2, T_{11}, and W_c/Q are 28.5°K, 21°K, and 265 watts/watt, respectively. In addition, it shows that a second expansion valve is required to prevent second law violations from occurring in the lower heat exchanger.

The W_c/Q value must be modified to account for pressure loss, and a close approximation may be obtained through the use of

$$\left(\frac{W_c}{Q}\right)_{PD} = \left(\frac{W_c}{Q}\right)_{NPD} \frac{\ln\left(\frac{P_1}{P_{12}}\right)_{PD}}{\ln\left(\frac{P_1}{P_{12}}\right)_{NPD}} \tag{17}$$

where the subscripts PD and NPD refer to pressure drop and no pressure drop, respectively. The pressure drop terms may be found from

$$P_{1\,PD} = \frac{P_{1\,NPD}}{0.9} \quad \text{and} \tag{18}$$

$$P_{12\,PD} = 0.9\, P_{12\,NPD}. \tag{19}$$

Thus,

$$\left(\frac{W_c}{Q}\right)_{PD} = 282 \text{ watts/watt.} \tag{20}$$

The isothermal compressor efficiency must also be considered, and this results in

$$\left(\frac{W_c}{Q}\right)_{isothermal} = \frac{\left(\frac{W_c}{Q}\right)_{PD}}{\eta_{c\ isothermal}} \tag{21}$$

$$= \frac{282}{0.6}$$

$$= 470 \text{ watts/watt.}$$

Thus, a 4.7 kw compressor is needed to provide the required amount of refrigeration.

The expander performance may be determined from figure 9b. Using a high pressure of 23 atm, slightly less than half the total flow (i.e., about 49.5%) will pass through the expander, and this device will deliver about 5.5 watts of power per watt of refrigeration. The expander outlet temperature is found in figure 24, and its value is about 12.0°K.

Figures 24 and 25 give flow rate information for an isothermal compressor efficiency of 100 percent. Thus, for a W_c/Q of 265 watts/watt and a high pressure of 23 atm, the flow will be about 1.7 scfm/watt or 17 scfm for the required amount of refrigeration.

The various refrigerator parameters will then be

T_1 = 300°K $\quad P_1$ = 25.5 atm,

T_2 = 28.5°K $\quad P_2 = P_3 = P_4$ = 23.0 atm,

T_3 = 12.5°K $\quad P_{11} = P_{10} = P_8$ = 1.0 atm,

T_8 = 4.2°K $\quad P_{12}$ = 0.9 atm,

T_{10} = 12°K $\quad F_t$ = 17 scfm,

T_{11} = 21.5°K $\quad M_2$ = 50 percent,

T_{12} = 295°K $\quad W_c$ = 4700 watts,

η_e = 80 percent $\quad W_e$ = 55 watts, and

η_c = 60 percent $\quad Q$ = 10 watts.

NOTE

Figures 4a through 31

follow.

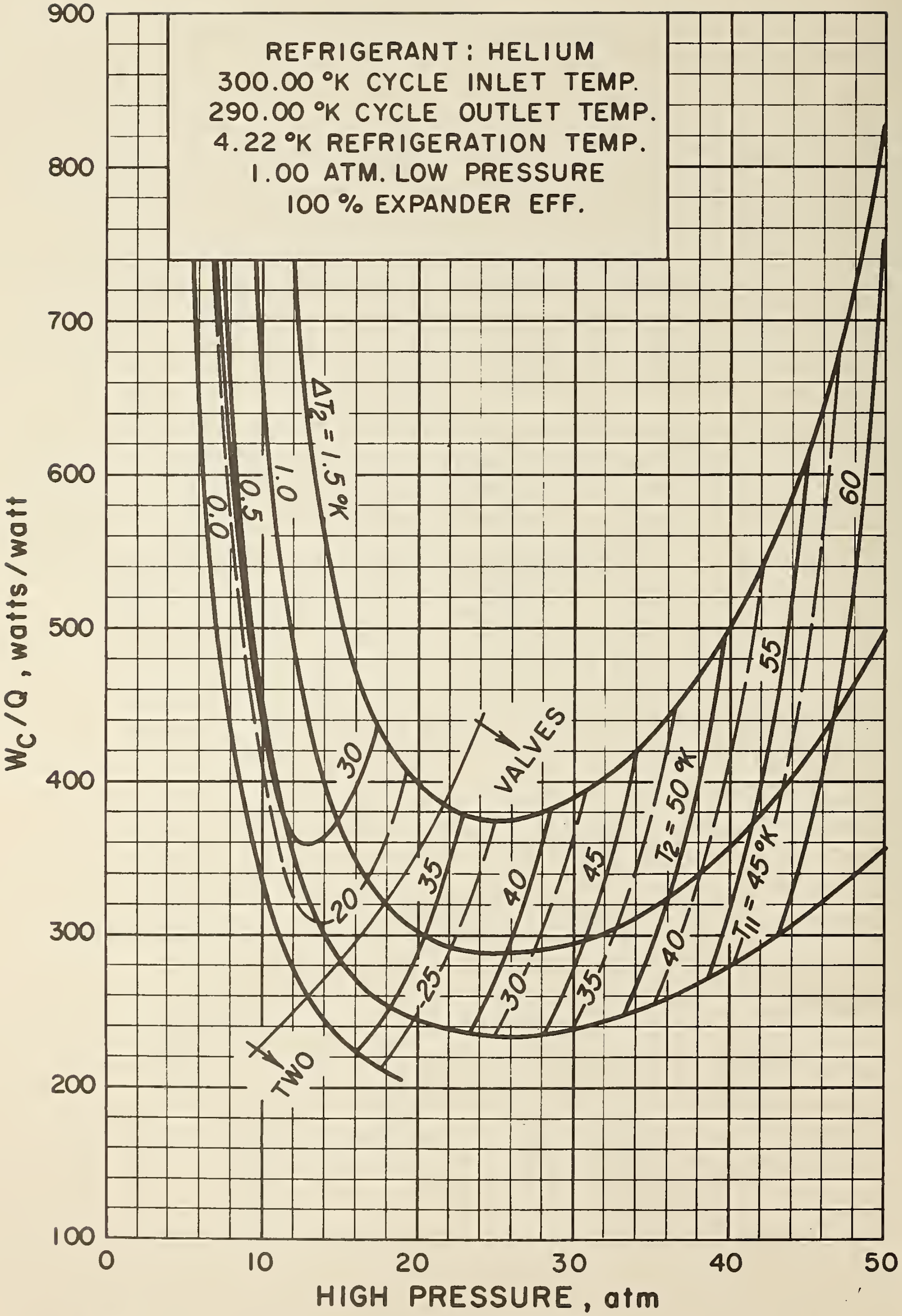

Figure 4a. Claude Refrigerator Performance - Compressor Power - 290°K Cycle Outlet Temperature - 100% Expander Efficiency

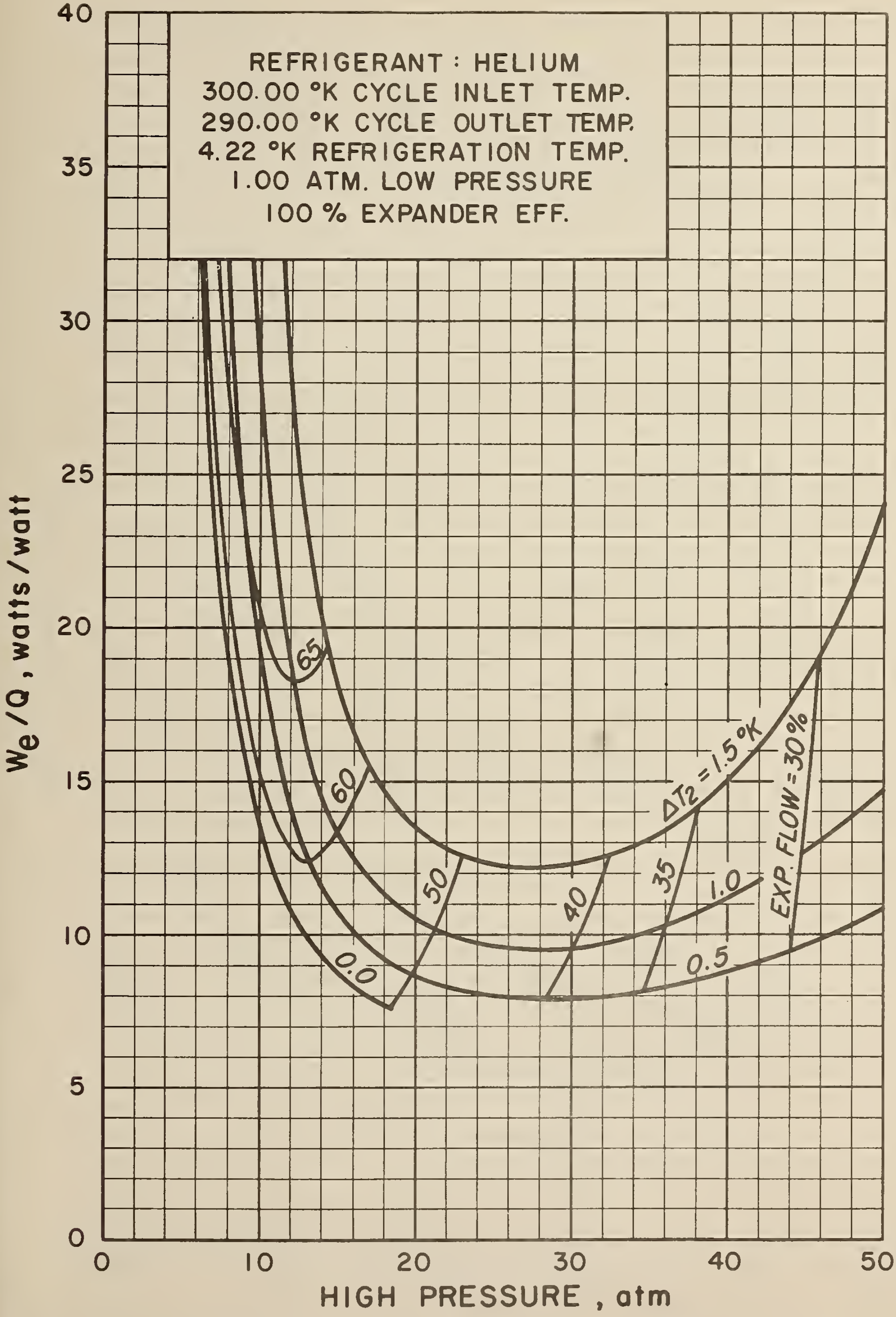

Figure 4b. Claude Refrigerator Performance - Expander Power - 290°K Cycle Outlet Temperature - 100% Expander Efficiency

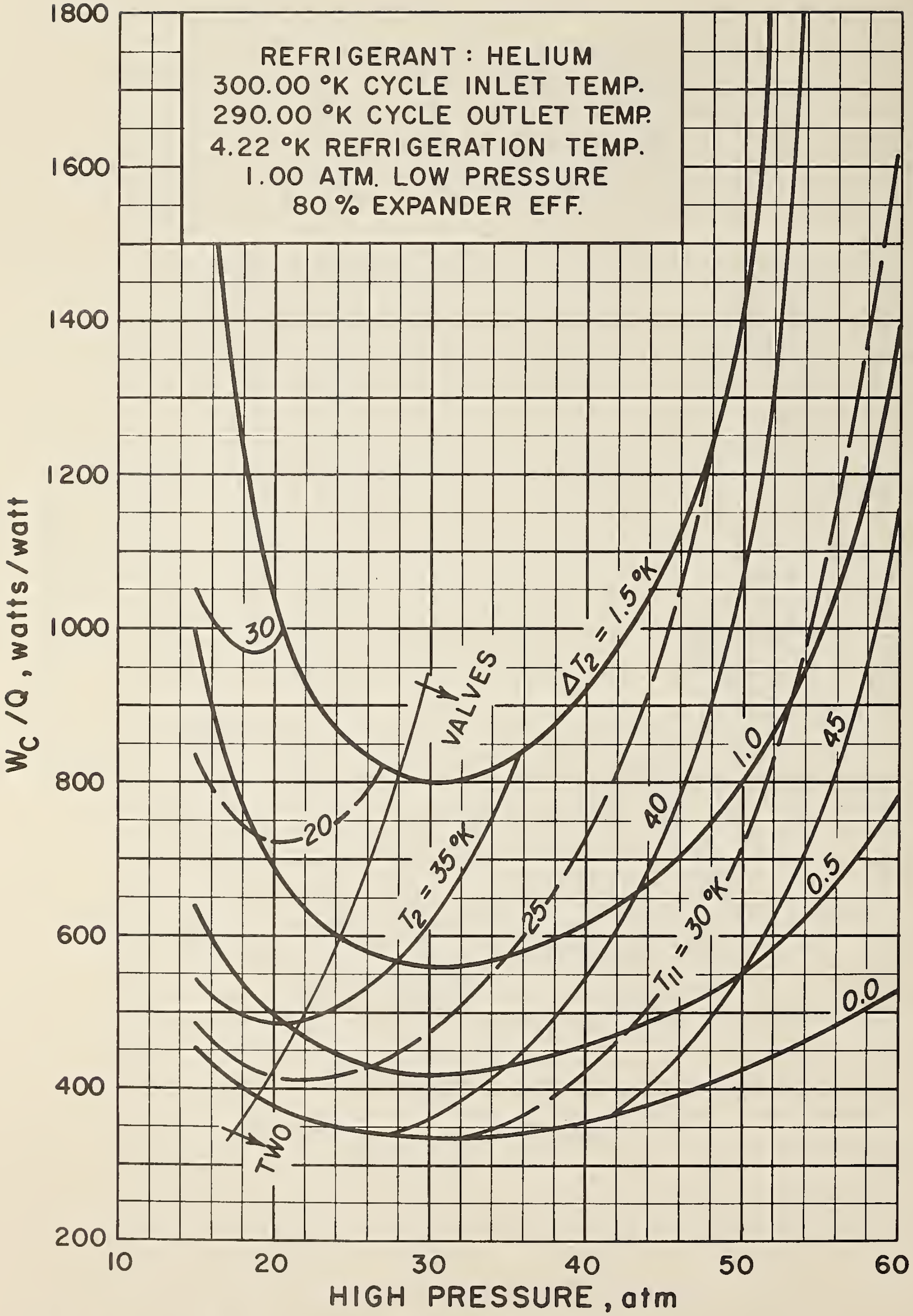

Figure 5a. Claude Refrigerator Performance - Compressor Power - 290°K Cycle Outlet Temperature - 80% Expander Efficiency

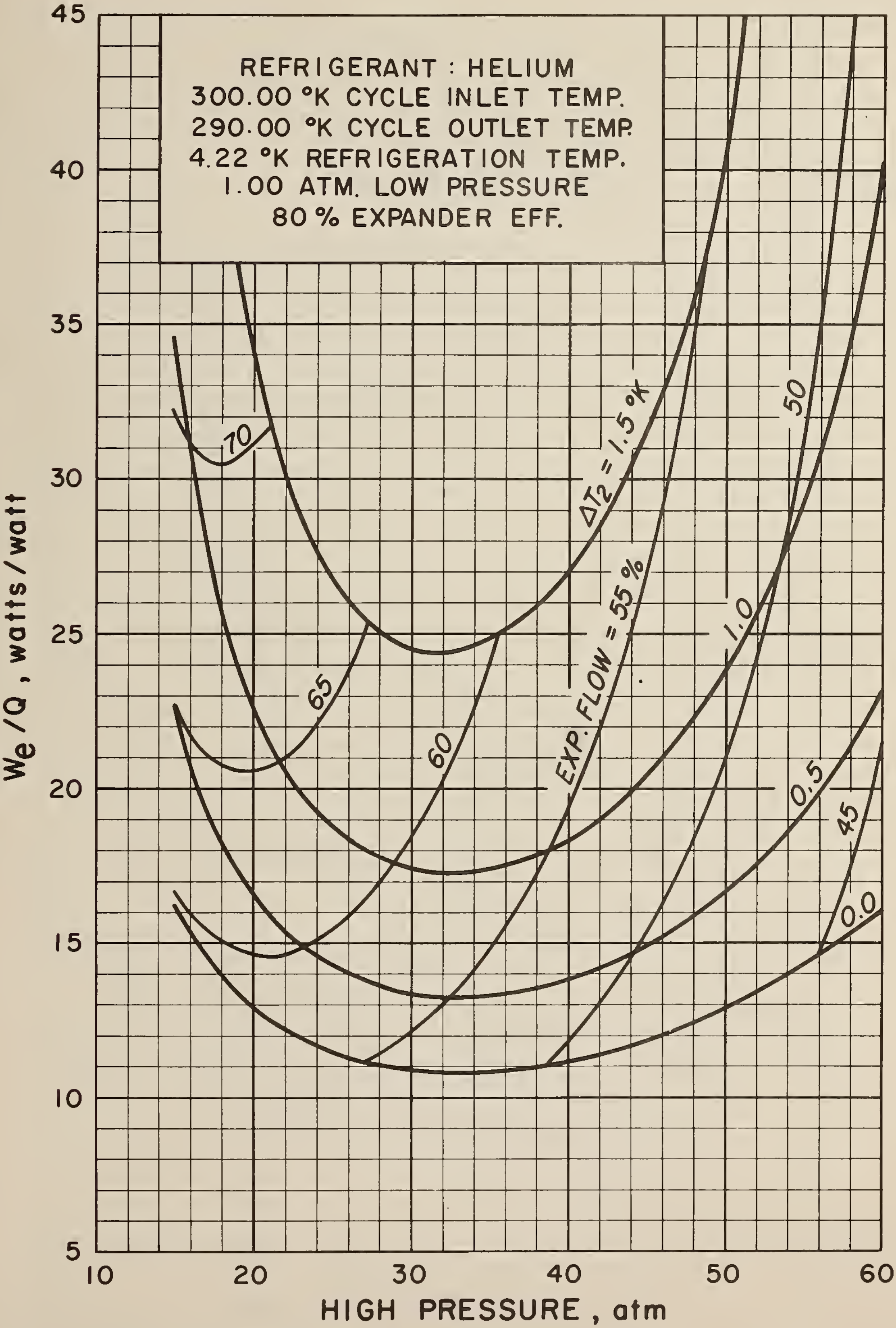

Figure 5b. Claude Refrigerator Performance - Expander Power - 290°K Cycle Outlet Temperature - 80% Expander Efficiency

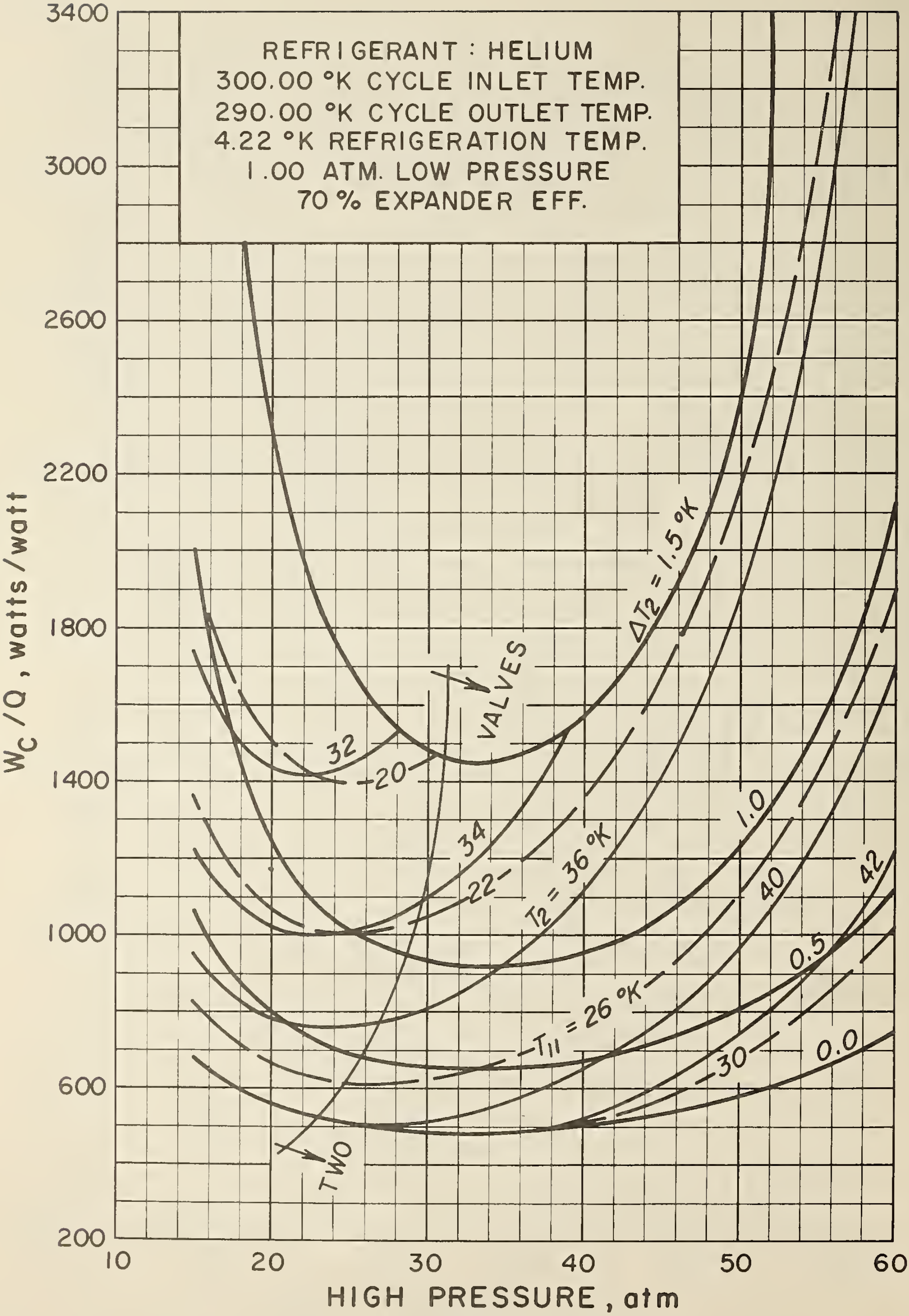

Figure 6a. Claude Refrigerator Performance - Compressor Power - 290°K Cycle Outlet Temperature - 70% Expander Efficiency

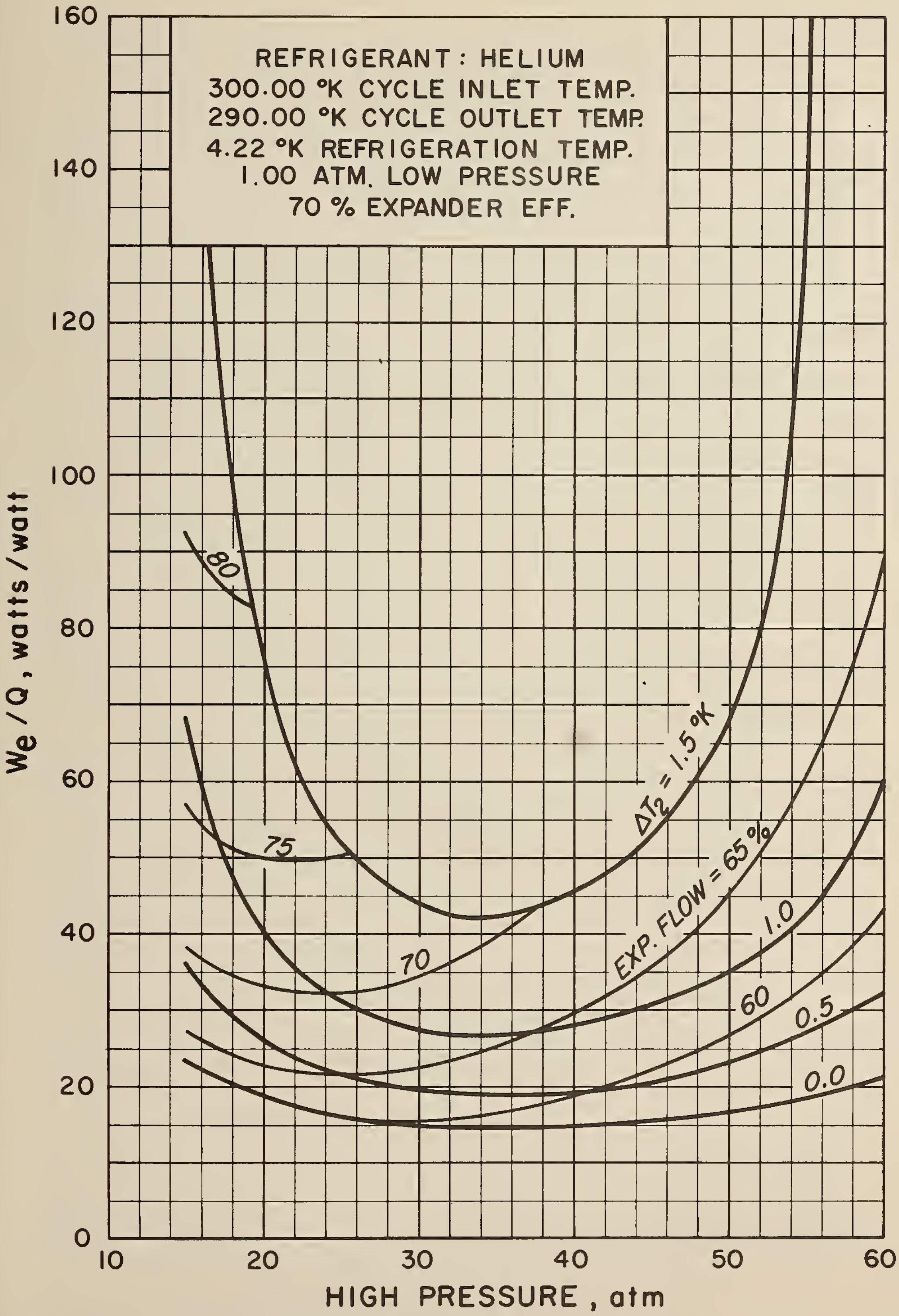

Figure 6b. Claude Refrigerator Performance - Expander Power - 290°K Cycle Outlet Temperature - 70% Expander Efficiency

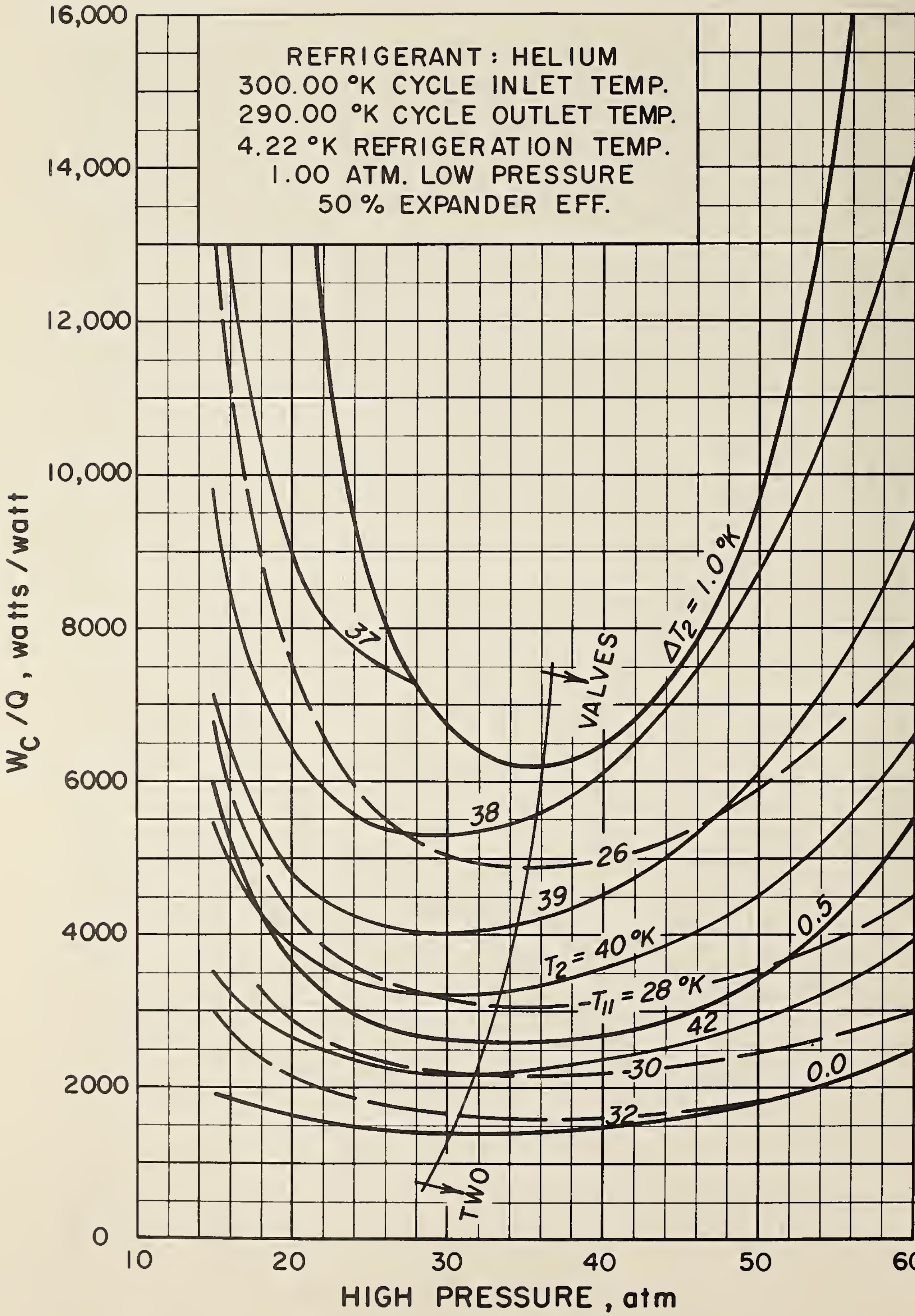

Figure 7a. Claude Refrigerator Performance - Compressor Power 290°K Cycle Outlet Temperature - 50% Expander Efficiency

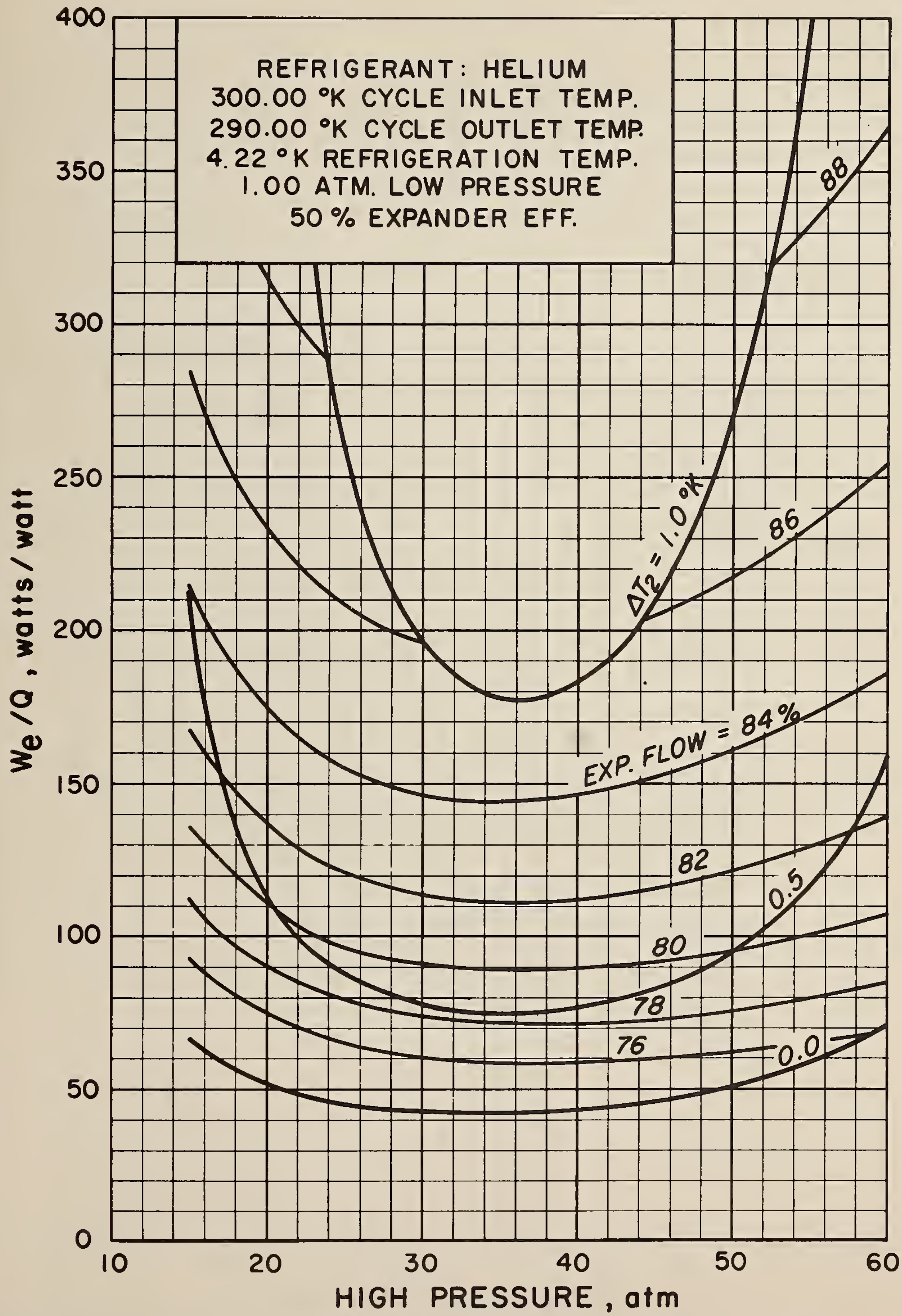

Figure 7b. Claude Refrigerator Performance - Expander Power - 290°K Cycle Outlet Temperature - 50% Expander Efficiency

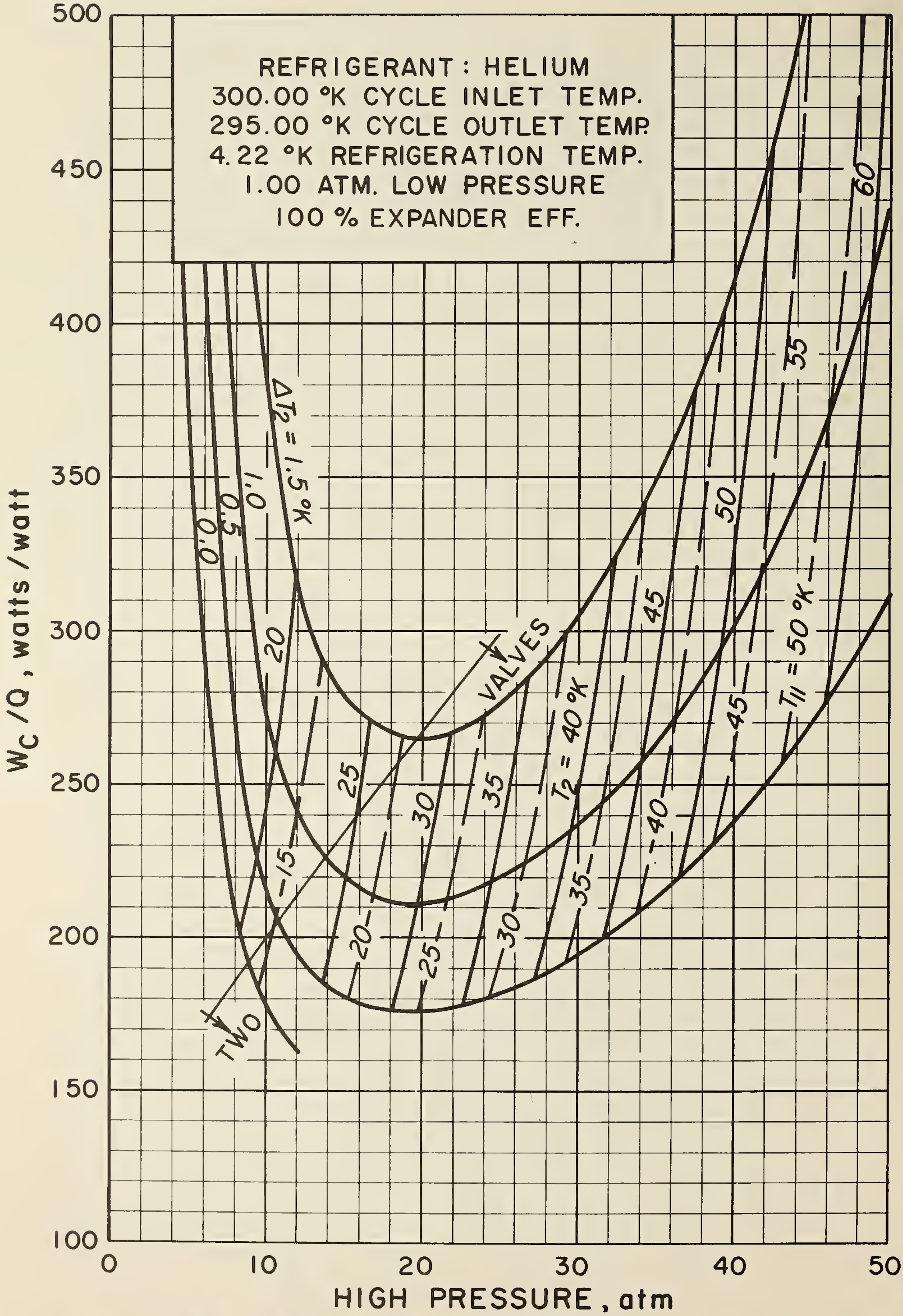

Figure 8a. Claude Refrigerator Performance - Compressor Power - 295°K Cycle Outlet Temperature - 100% Expander Efficiency

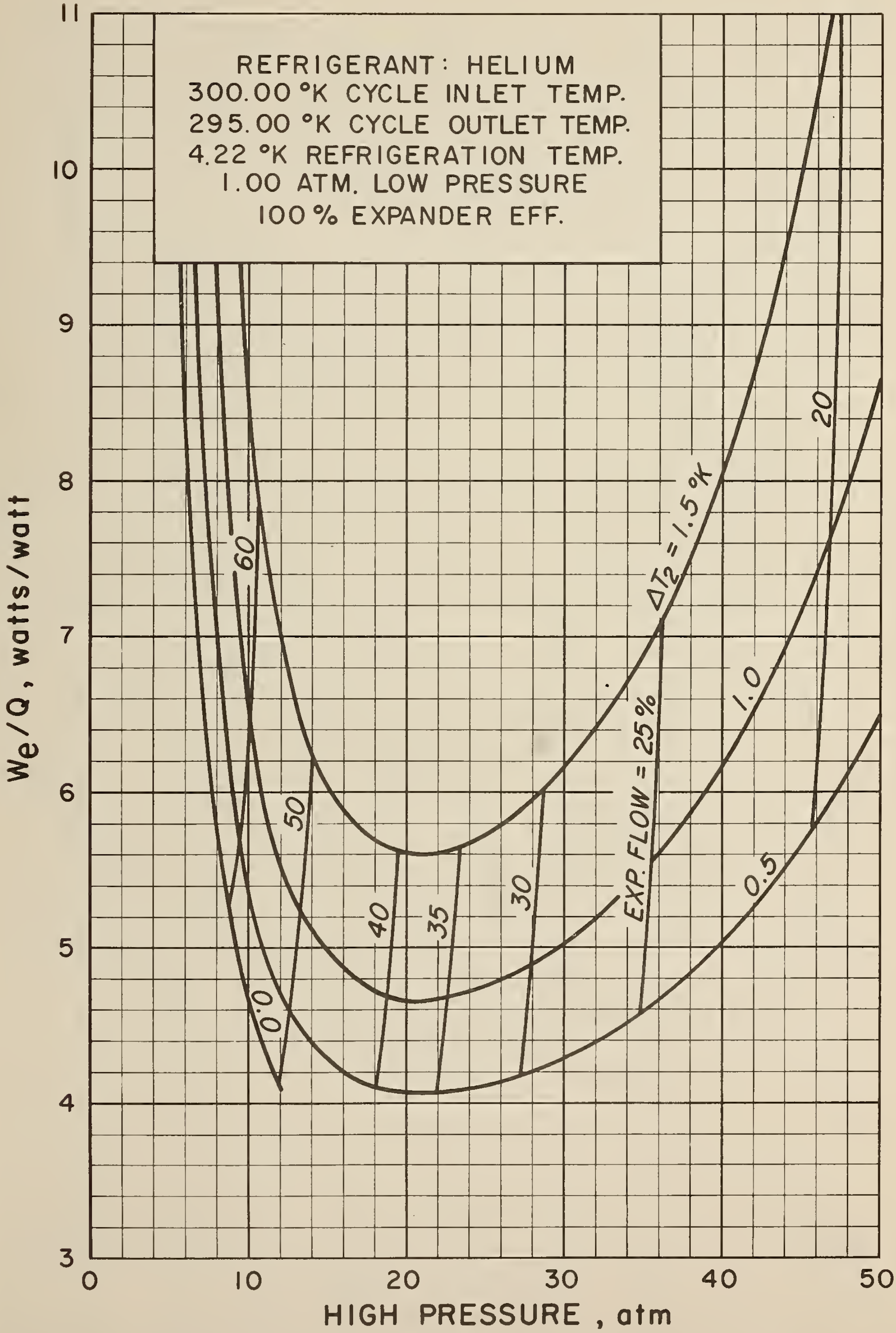

Figure 8b. Claude Refrigerator Performance - Expander Power - 295°K Cycle Outlet Temperature - 100% Expander Efficiency

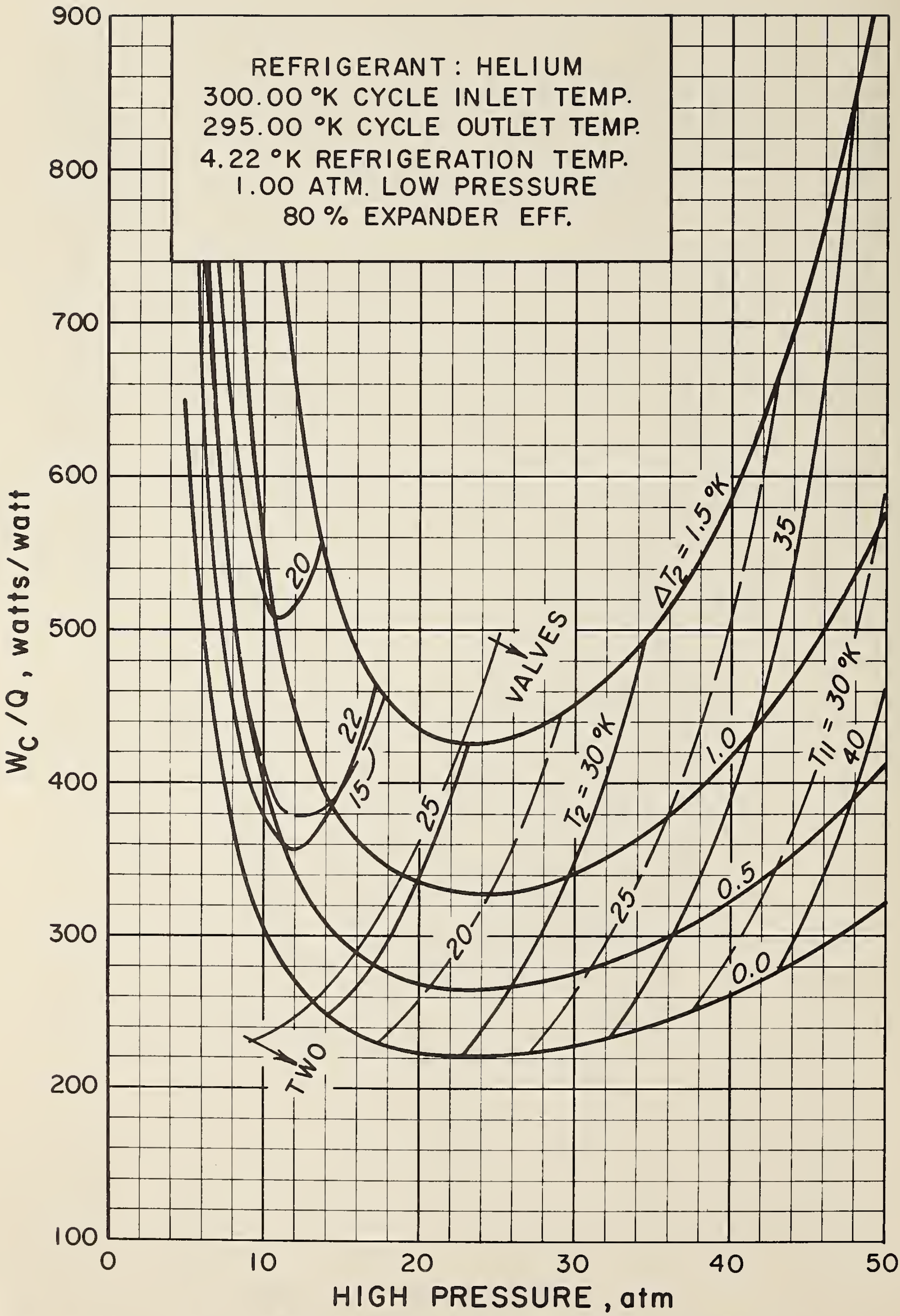

Figure 9a. Claude Refrigerator Performance - Compressor Power - 295°K Cycle Outlet Temperature - 80% Expander Efficiency

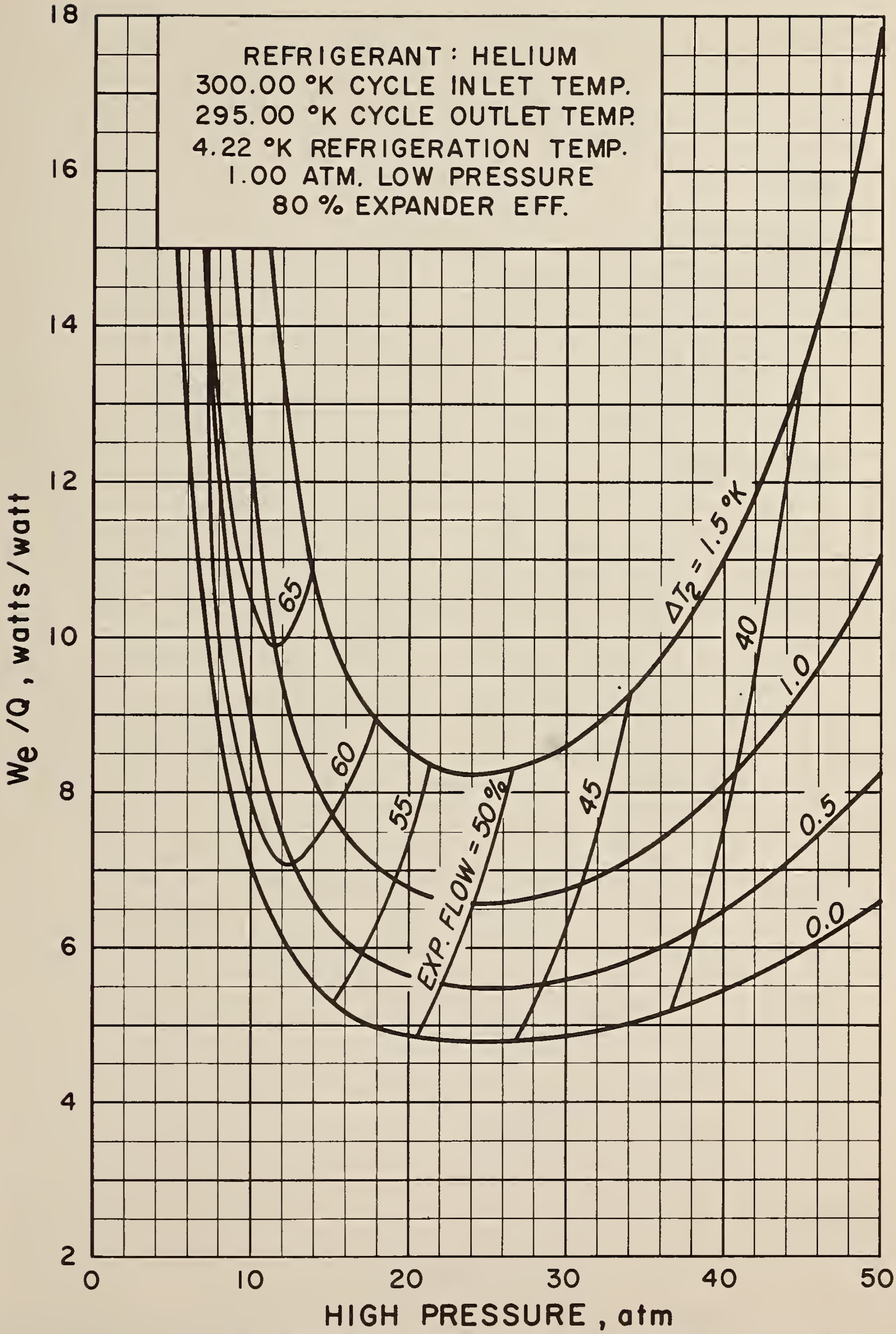

Figure 9b. Claude Refrigerator Performance - Expander Power - 295°K Cycle Outlet Temperature - 80% Expander Efficiency

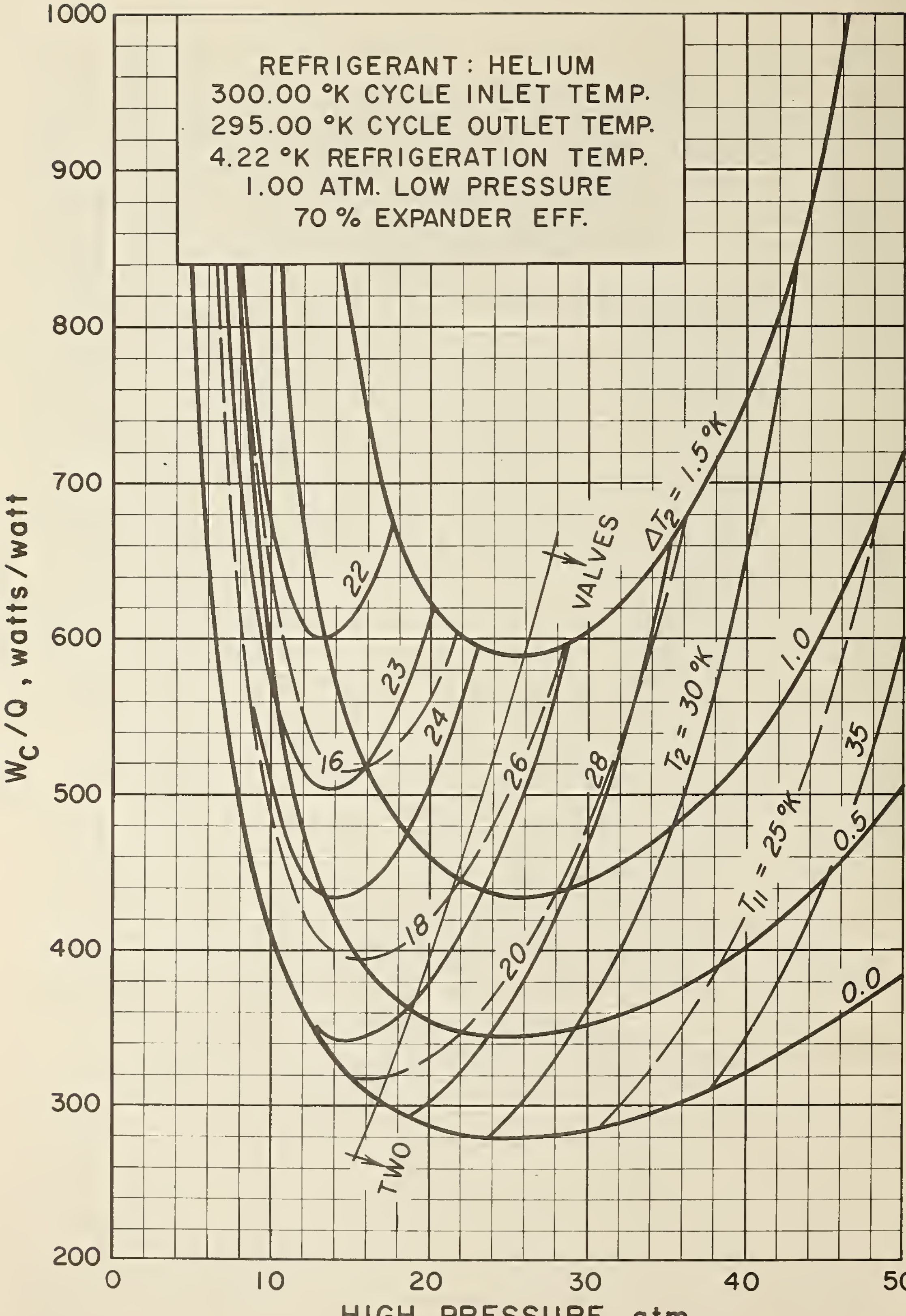

Figure 10a. Claude Refrigerator Performance - Compressor Power - 295°K Cycle Outlet Temperature - 70% Expander Efficiency

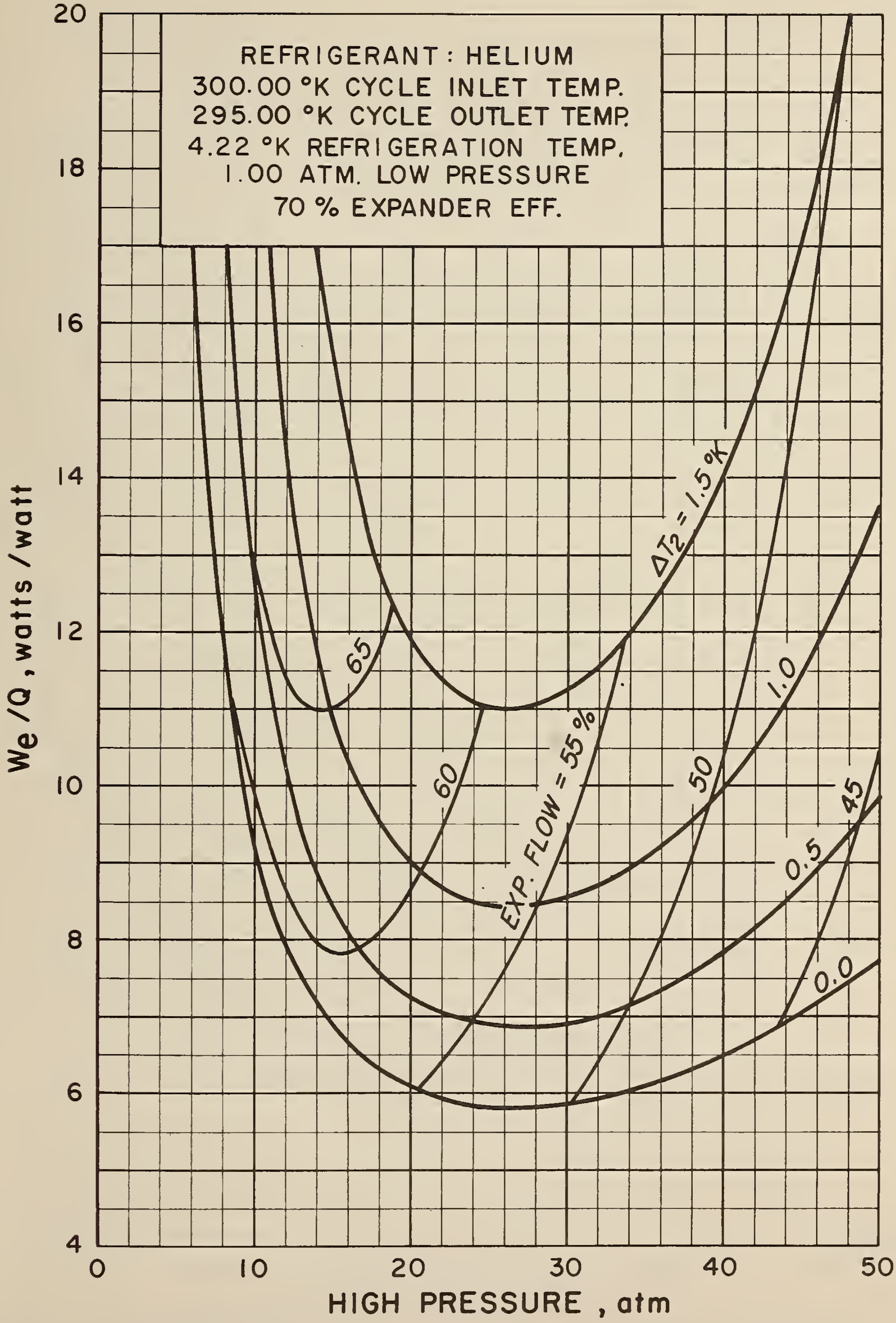

Figure 10b. Claude Refrigerator Performance - Expander Power - 295°K Cycle Outlet Temperature - 70% Expander Efficiency

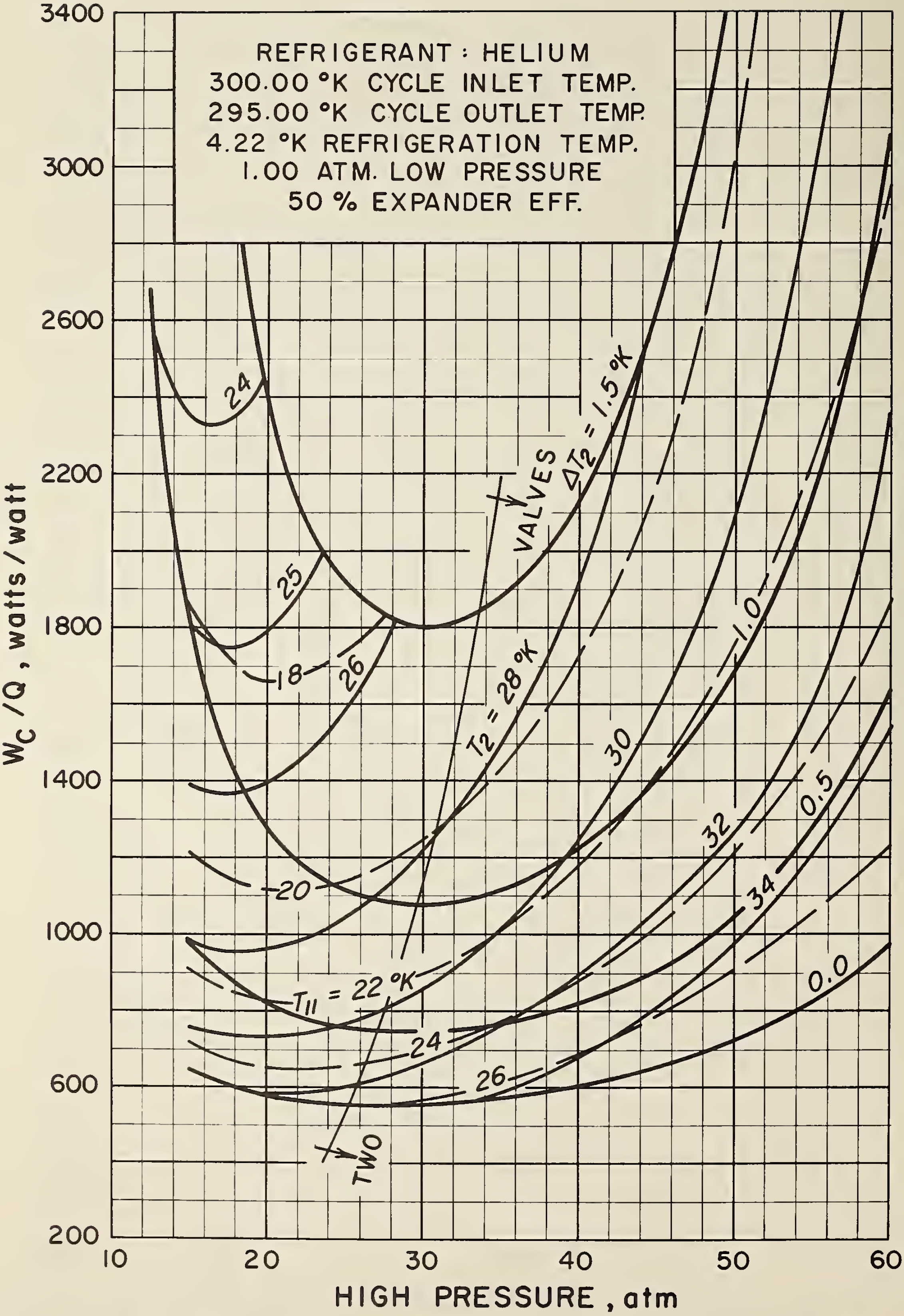

Figure 11a. Claude Refrigerator Performance - Compressor Power - 295°K Cycle Outlet Temperature - 50% Expander Efficiency

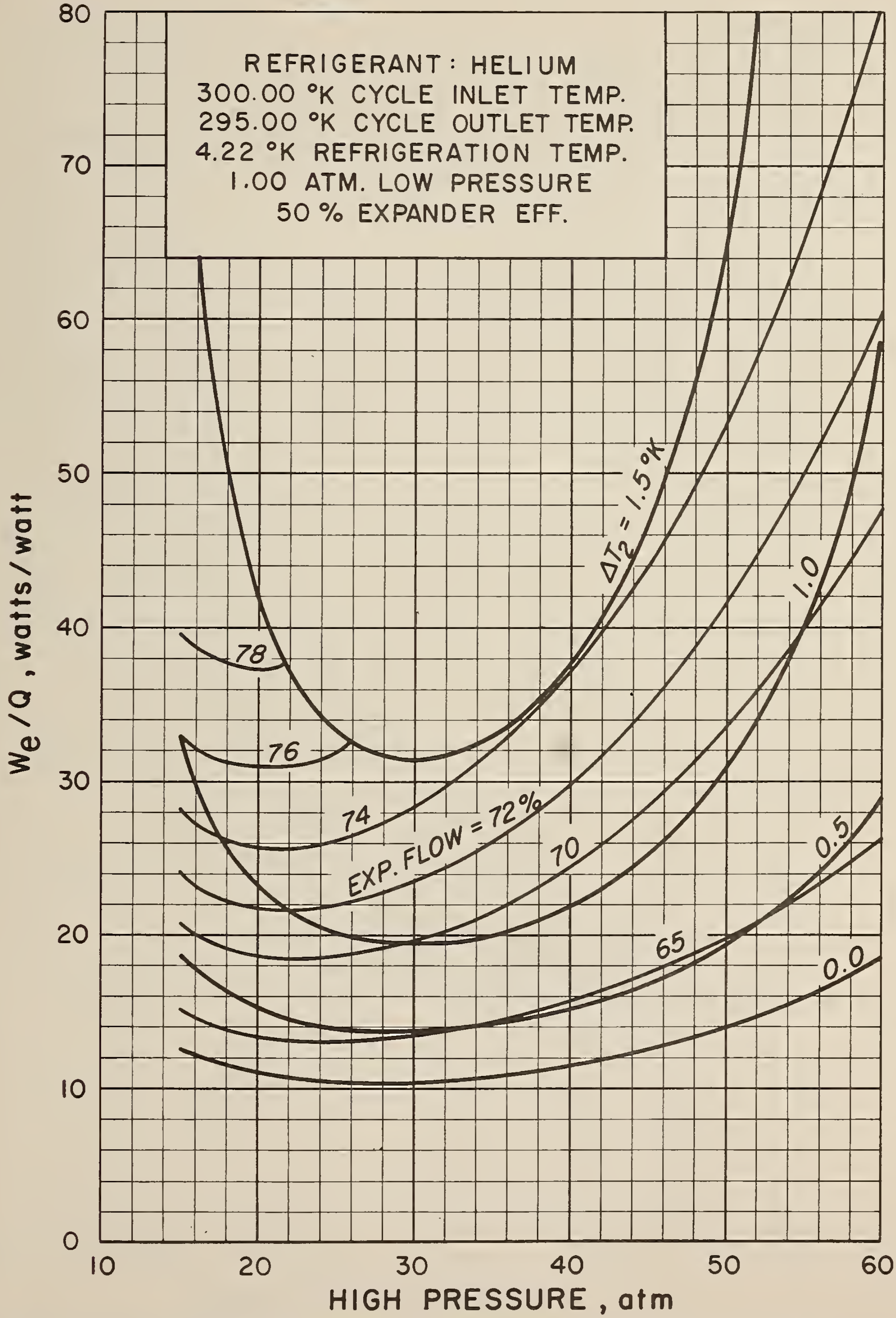

Figure 11b. Claude Refrigerator Performance - Expander Power - 295°K Cycle Outlet Temperature - 50% Expander Efficiency

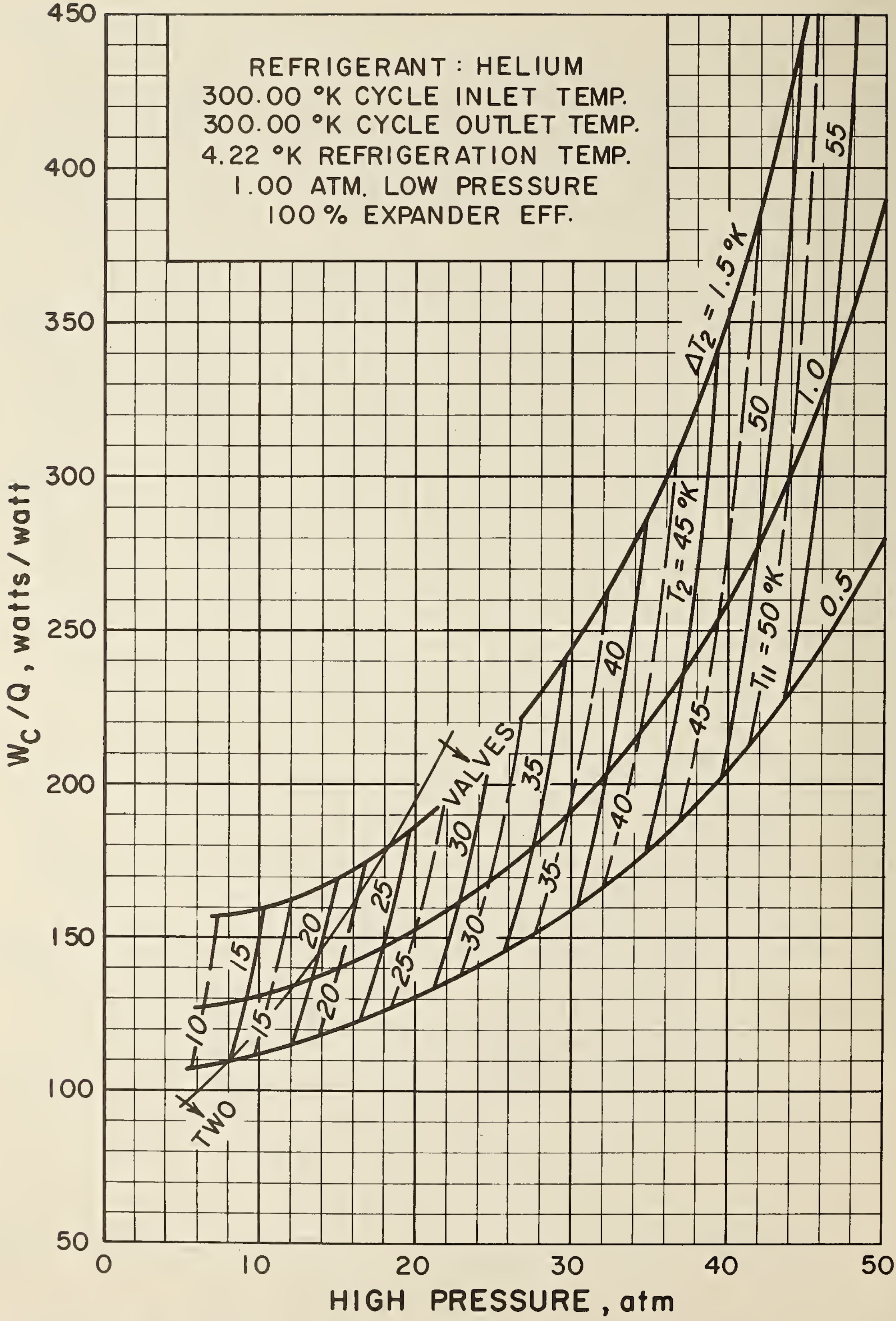

Figure 12a. Claude Refrigerator Performance - Compressor Power - 300°K Cycle Outlet Temperature - 100% Expander Efficiency

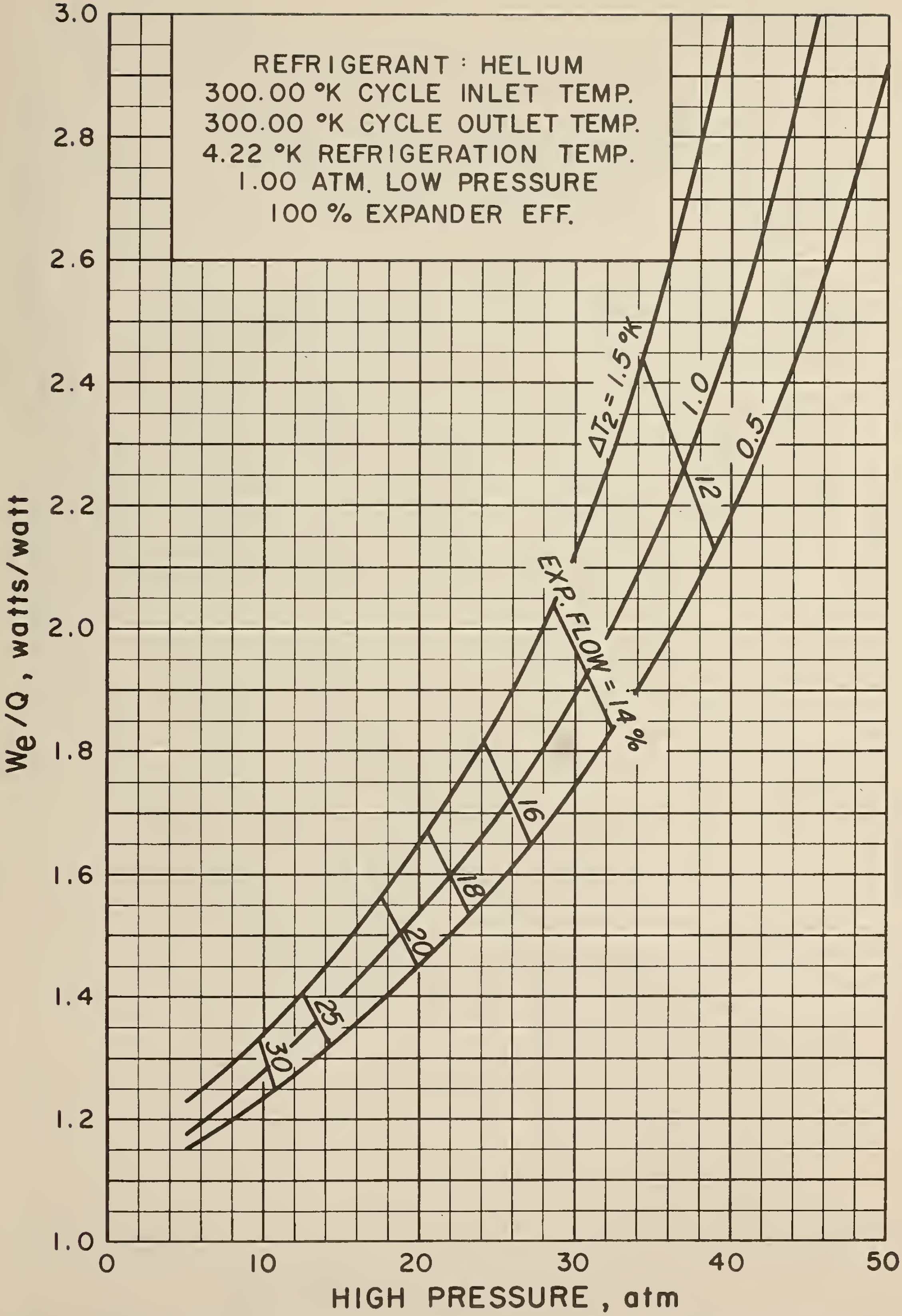

Figure 12b. Claude Refrigerator Performance - Expander Power - 300°K Cycle Outlet Temperature - 100% Expander Efficiency

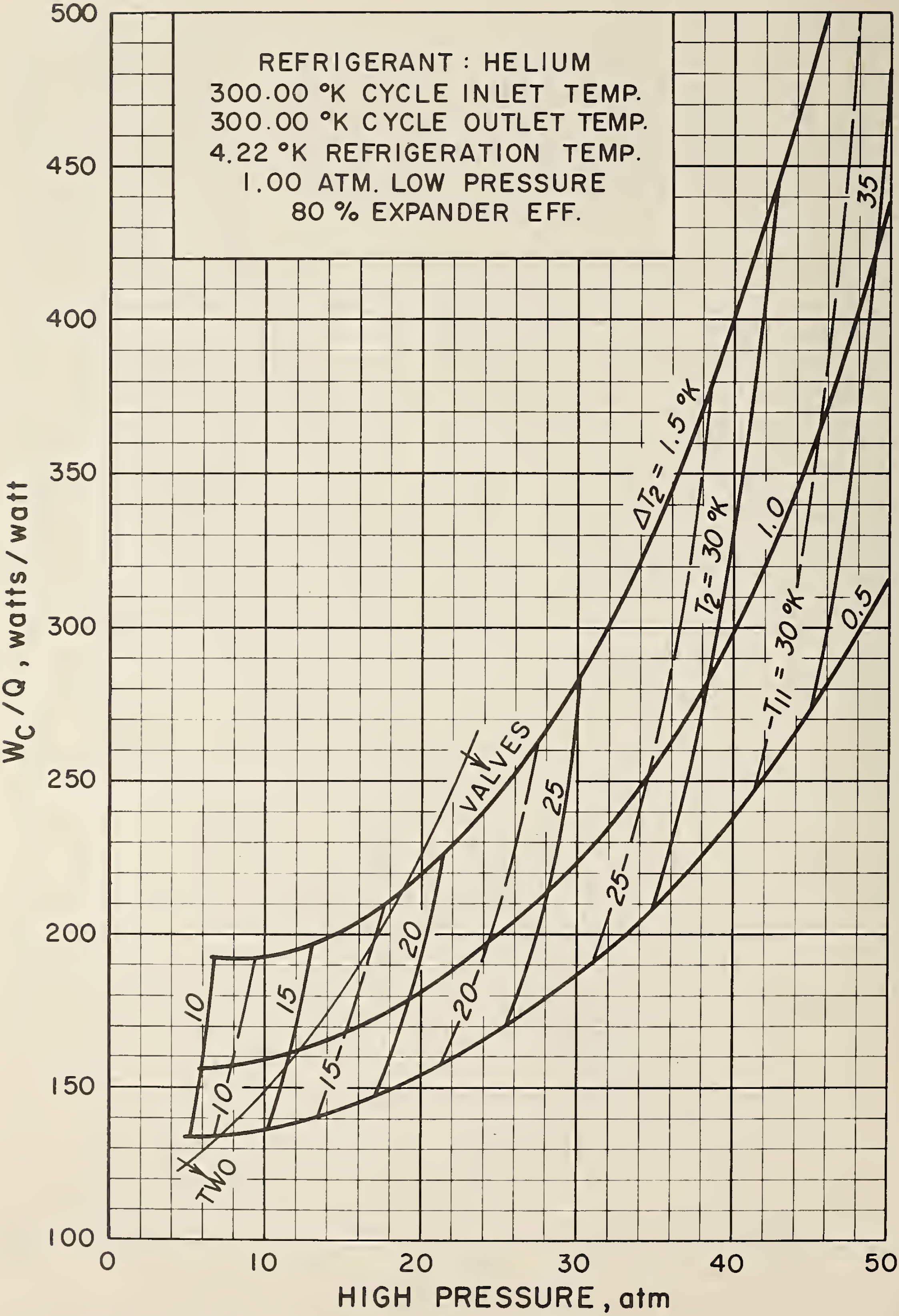

Figure 13a. Claude Refrigerator Performance - Compressor Power - 300°K Cycle Outlet Temperature - 80% Expander Efficiency

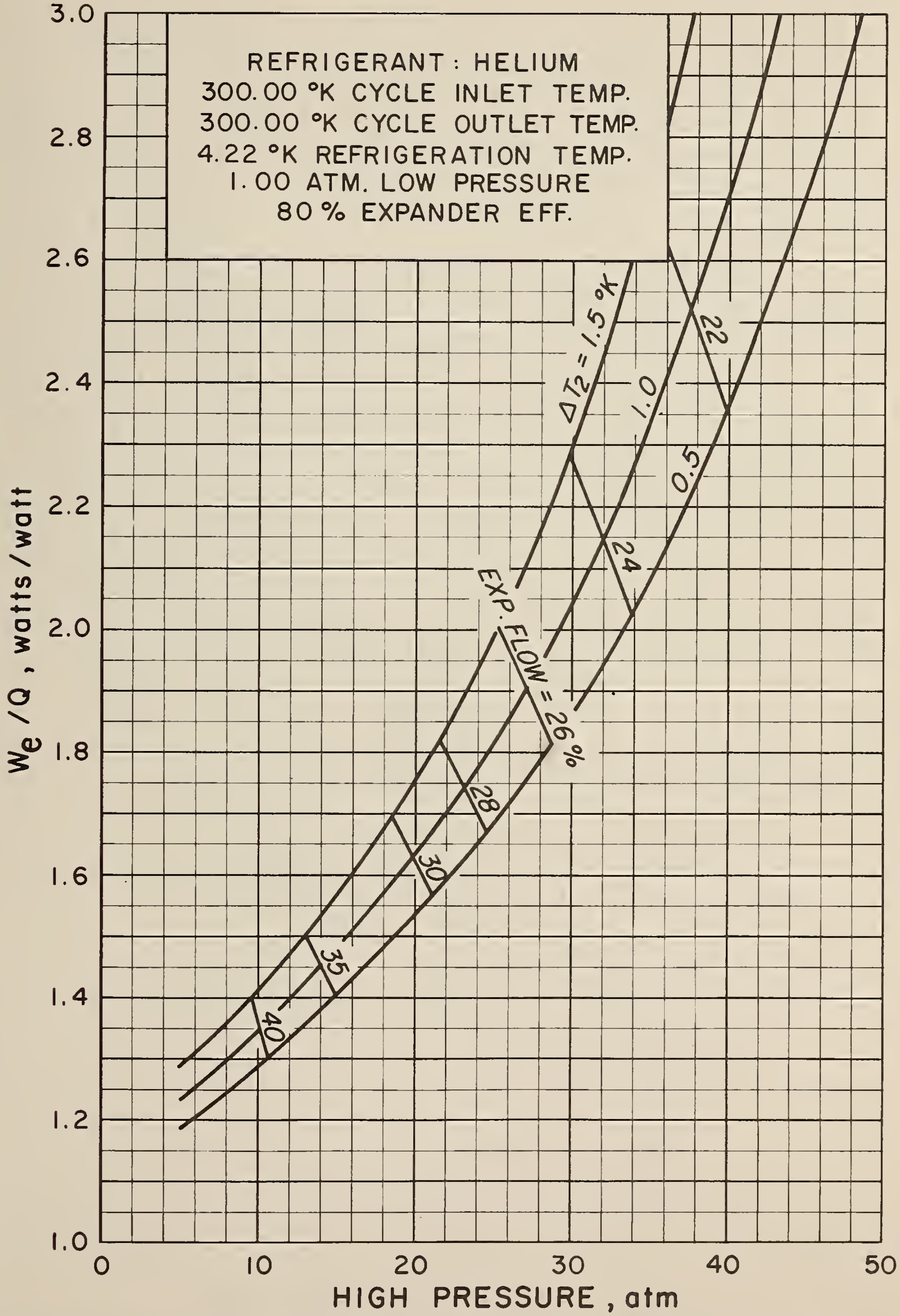

Figure 13b. Claude Refrigerator Performance - Expander Power - 300°K Cycle Outlet Temperature - 80% Expander Efficiency

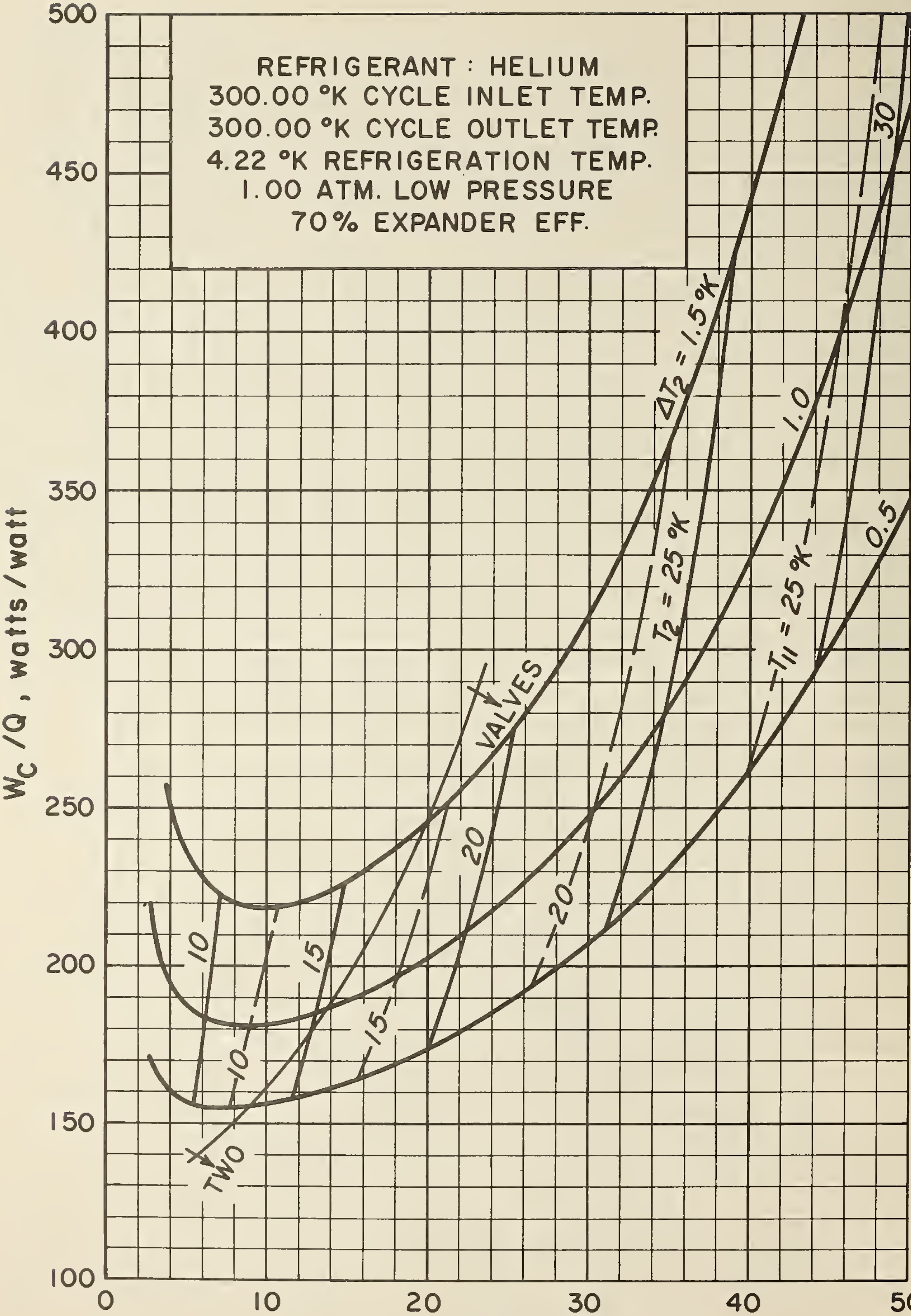

Figure 14a. Claude Refrigerator Performance - Compressor Power
300°K Cycle Outlet Temperature - 70% Expander Efficiency

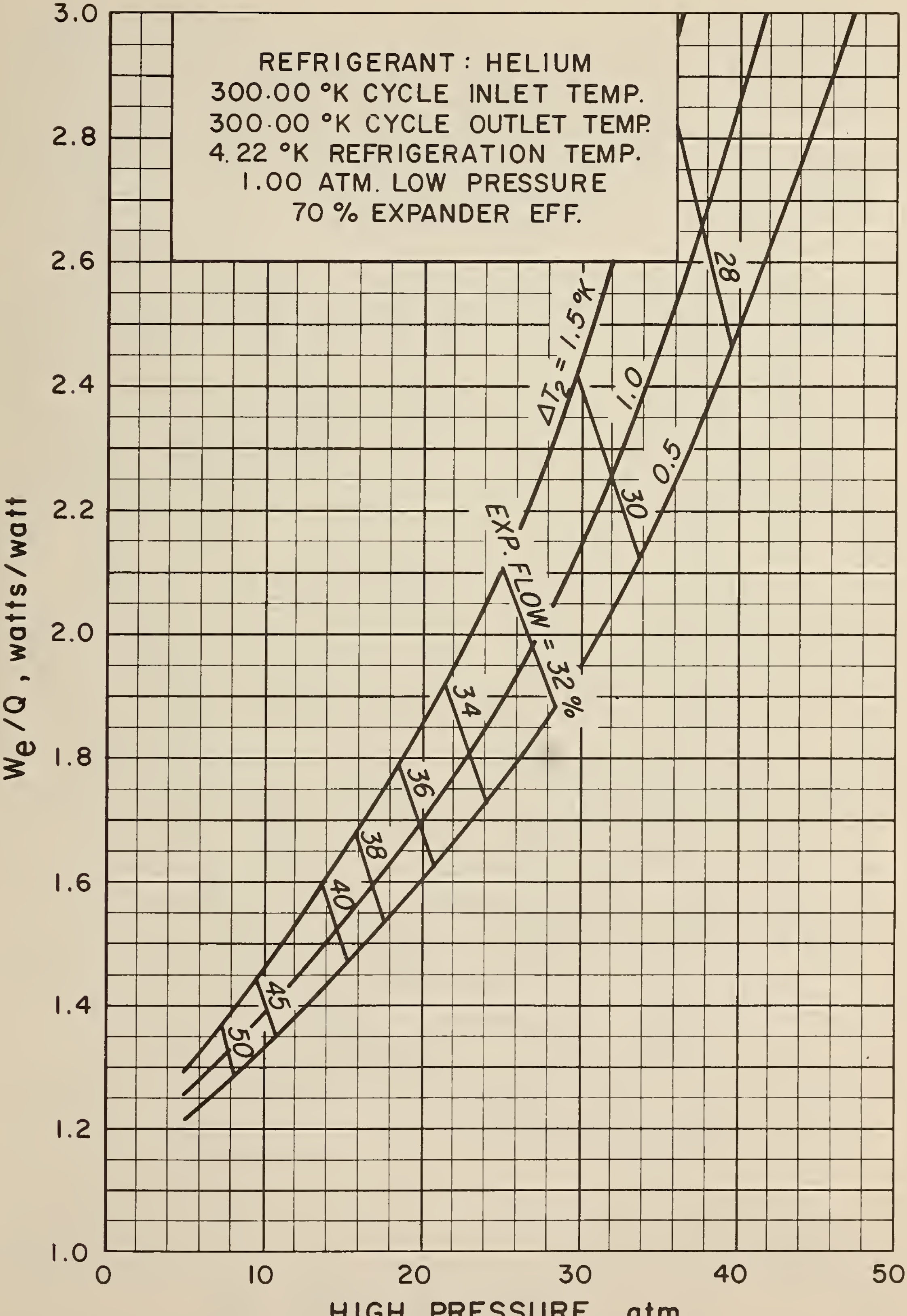

Figure 14b. Claude Refrigerator Performance - Expander Power - 300°K Cycle Outlet Temperature - 70% Expander Efficiency

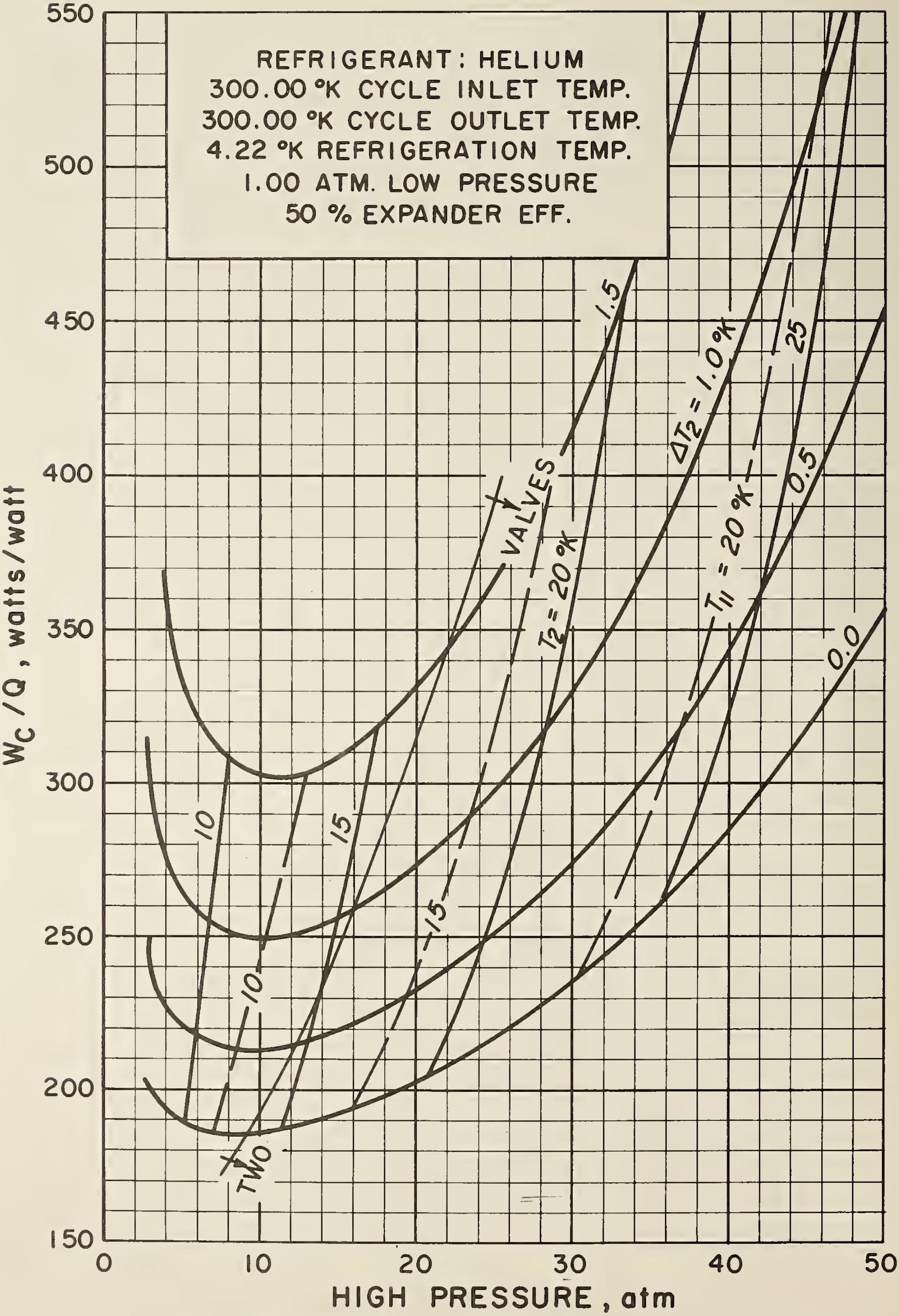

Figure 15a. Claude Refrigerator Performance - Compressor Power - 300°K Cycle Outlet Temperature - 50% Expander Efficiency

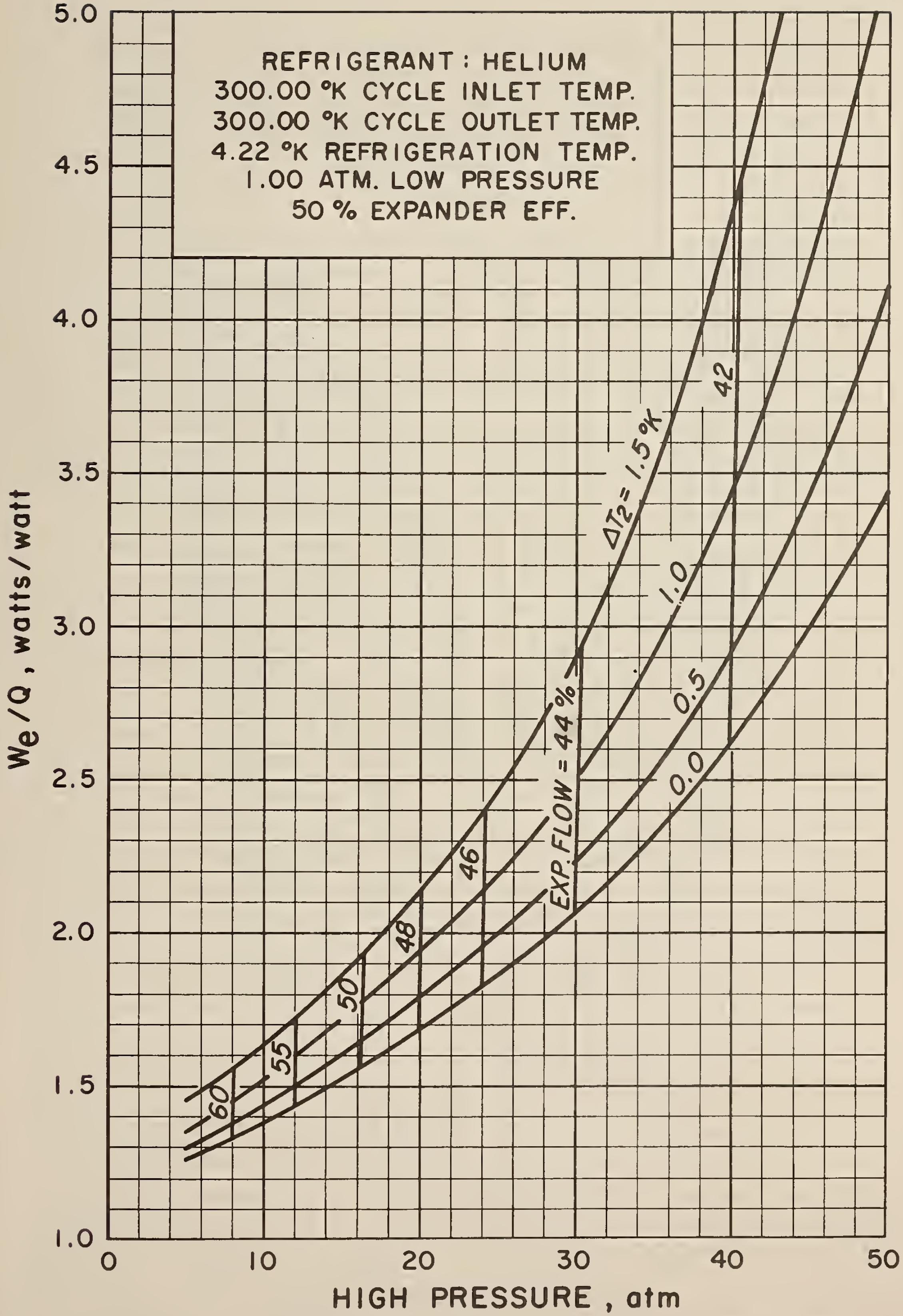

Figure 15b. Claude Refrigerator Performance - Expander Power - 300°K Cycle Outlet Temperature - 50% Expander Efficiency

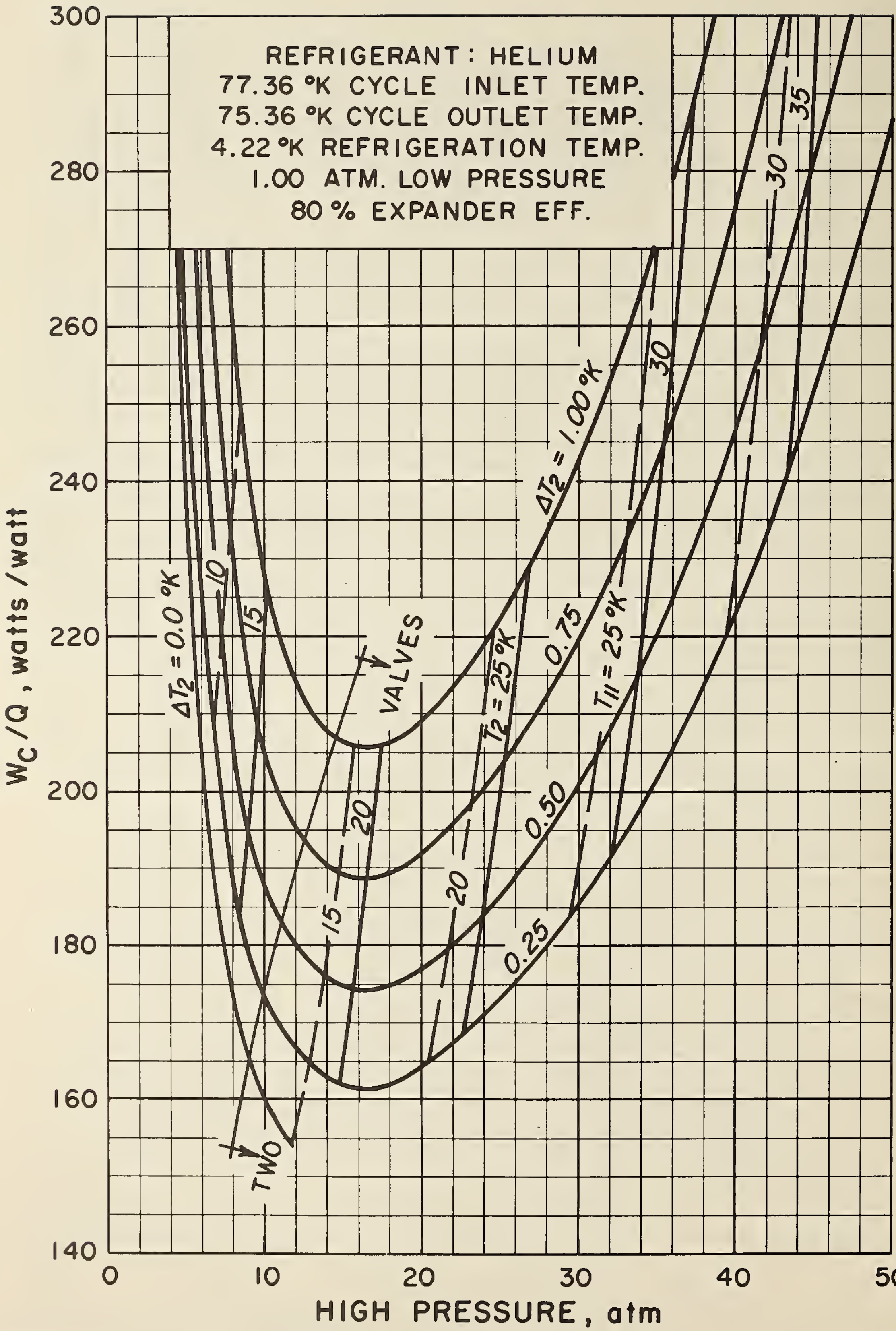

Figure 16a. Claude Refrigerator Performance - Compressor Power - 75.36°K Cycle Outlet Temperature - 80% Expander Efficiency

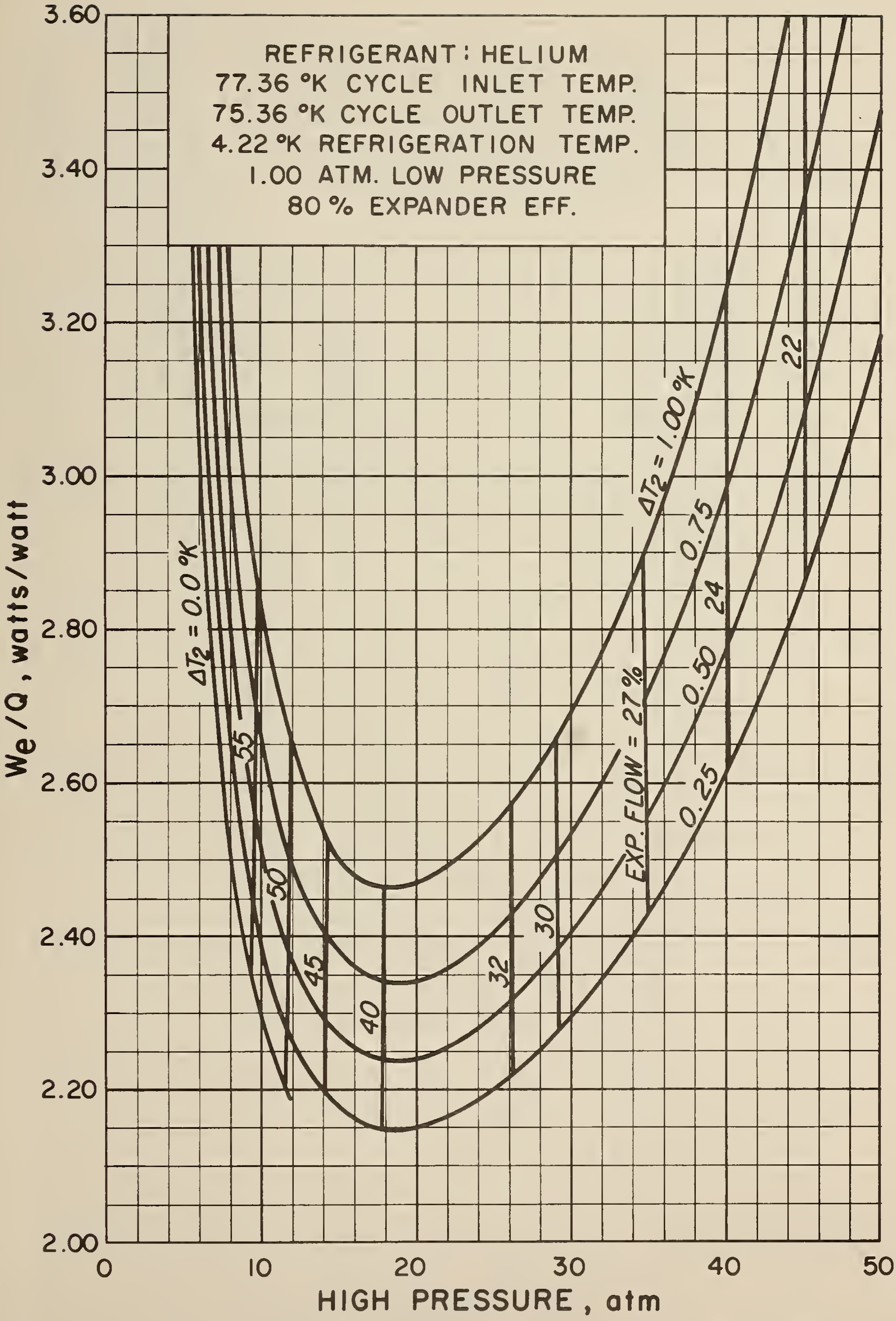

Figure 16b. Claude Refrigerator Performance - Expander Power - 75.36°K Cycle Outlet Temperature - 80% Expander Efficiency

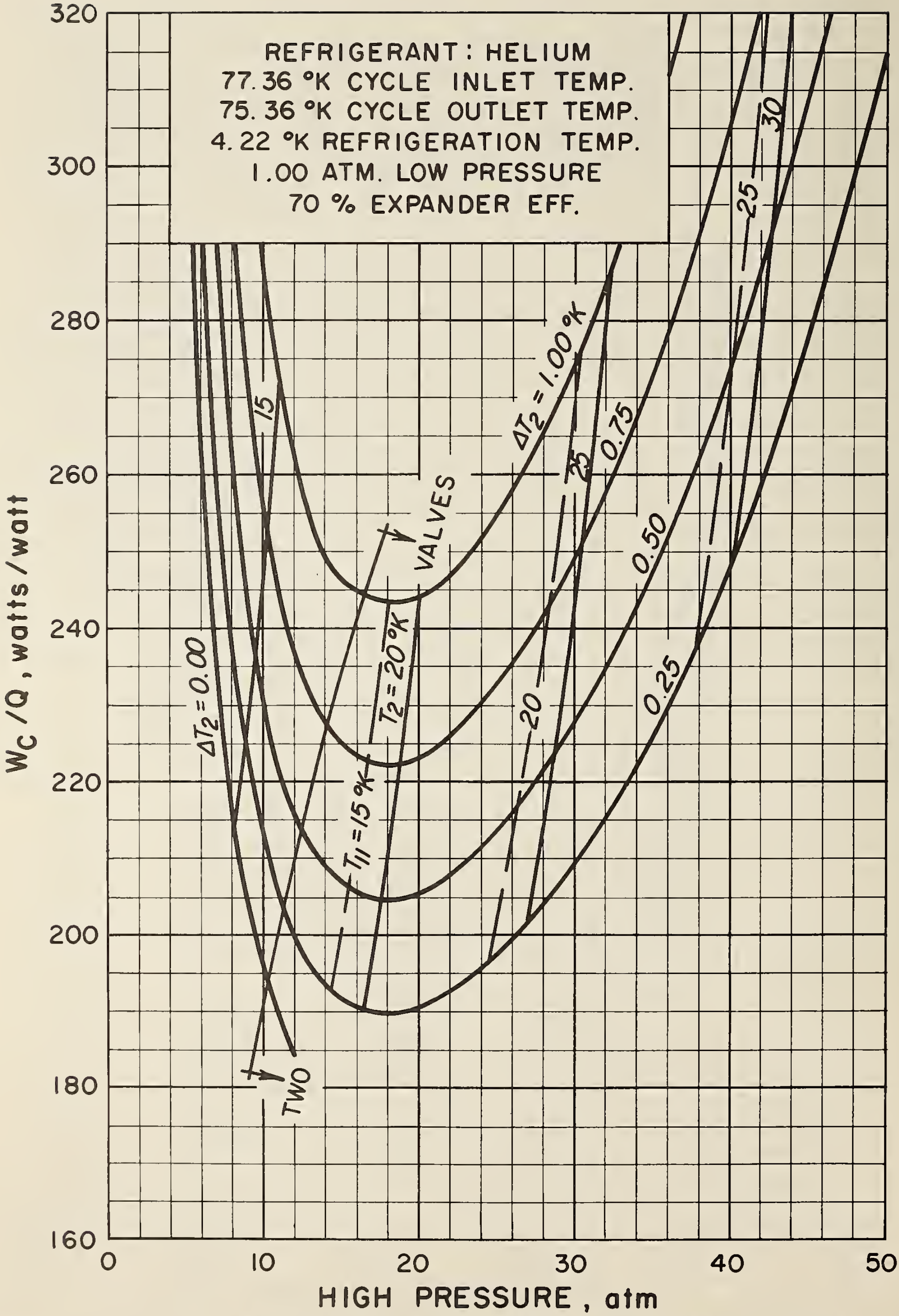

Figure 17a. Claude Refrigerator Performance - Compressor Power - 75.36°K Cycle Outlet Temperature - 70% Expander Efficiency

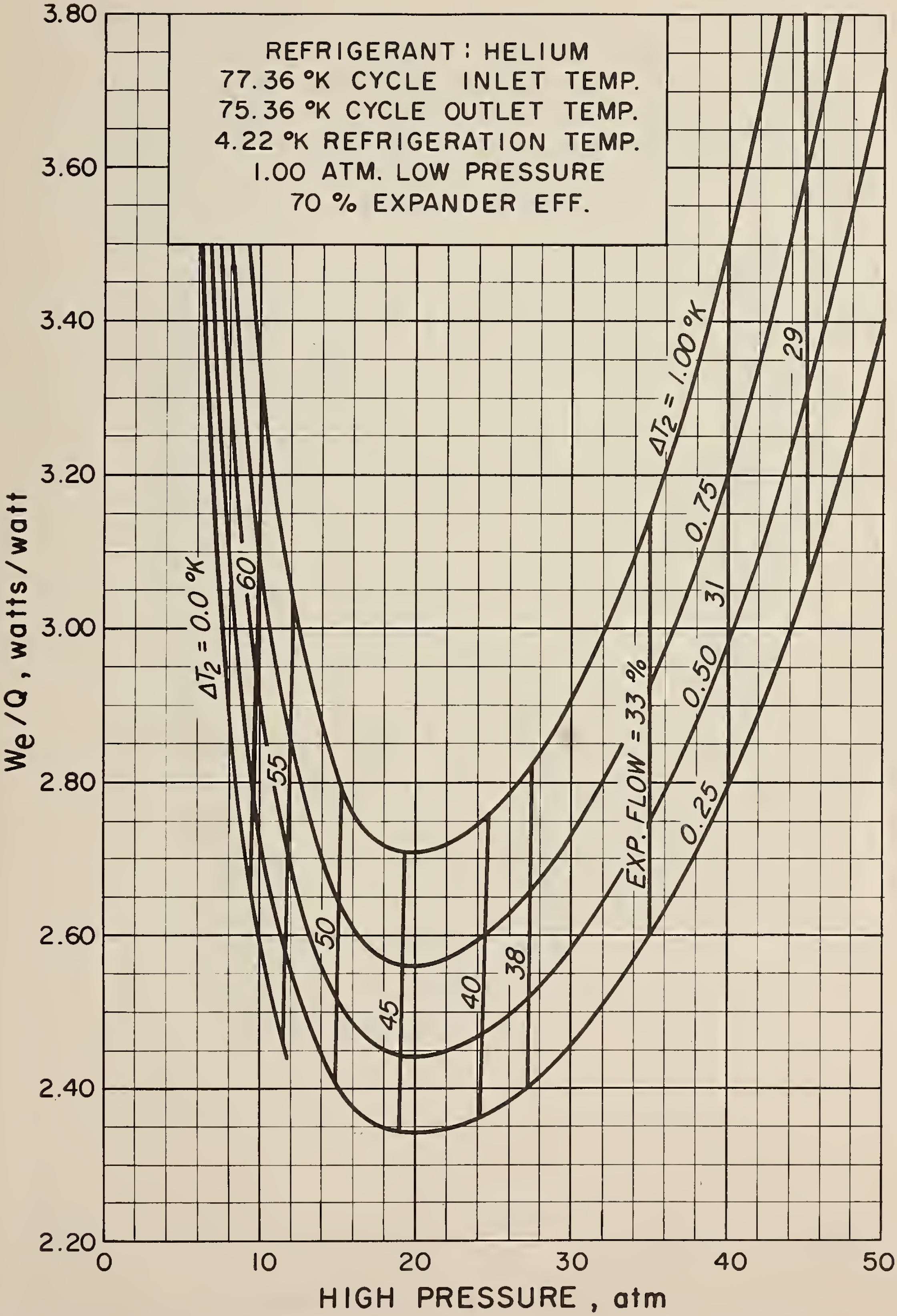

Figure 17b. Claude Refrigerator Performance - Expander Power - 75.36°K Cycle Outlet Temperature - 70% Expander Efficiency

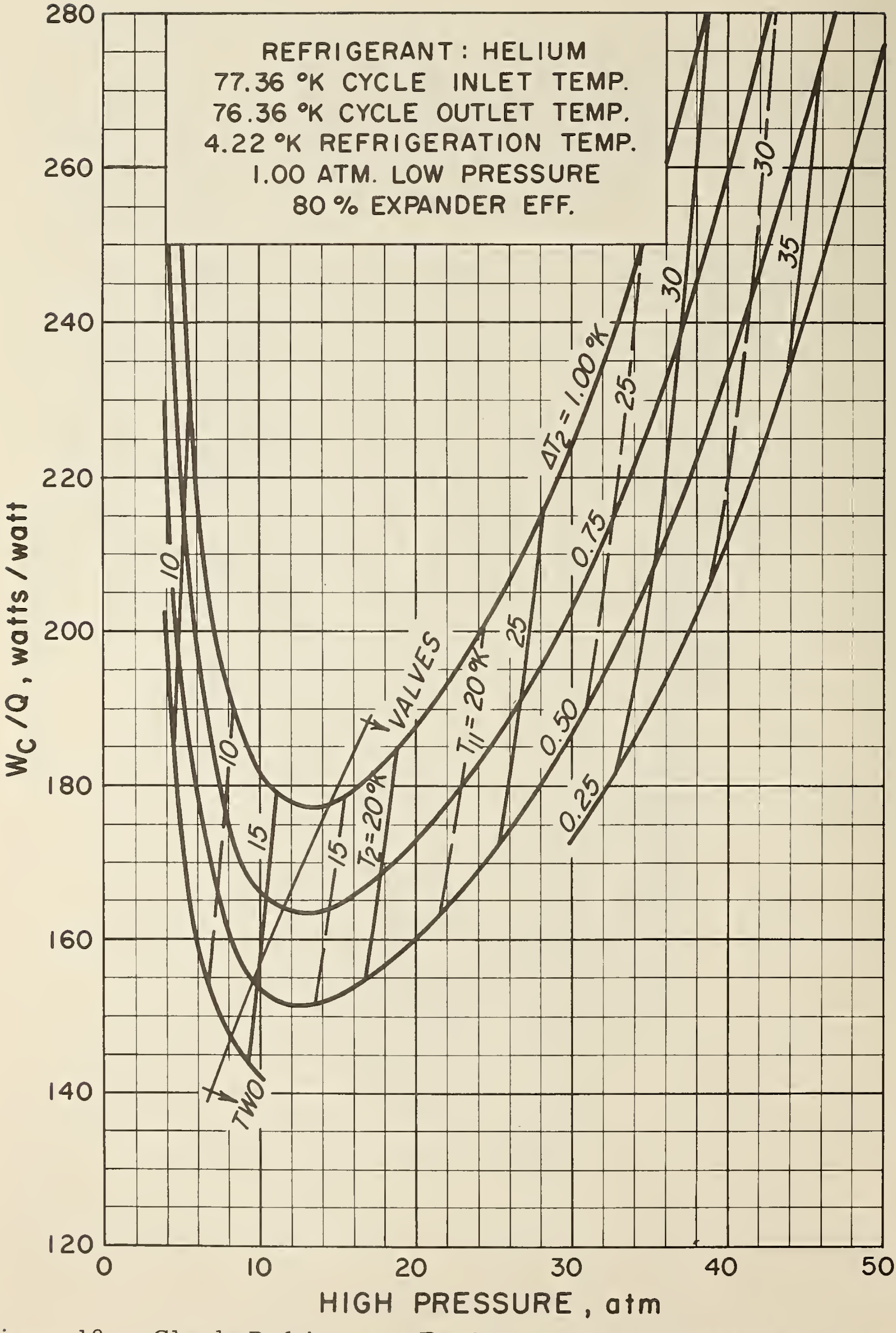

Figure 18a. Claude Refrigerator Performance - Compressor Power - 76.36°K Cycle Outlet Temperature - 80% Expander Efficiency

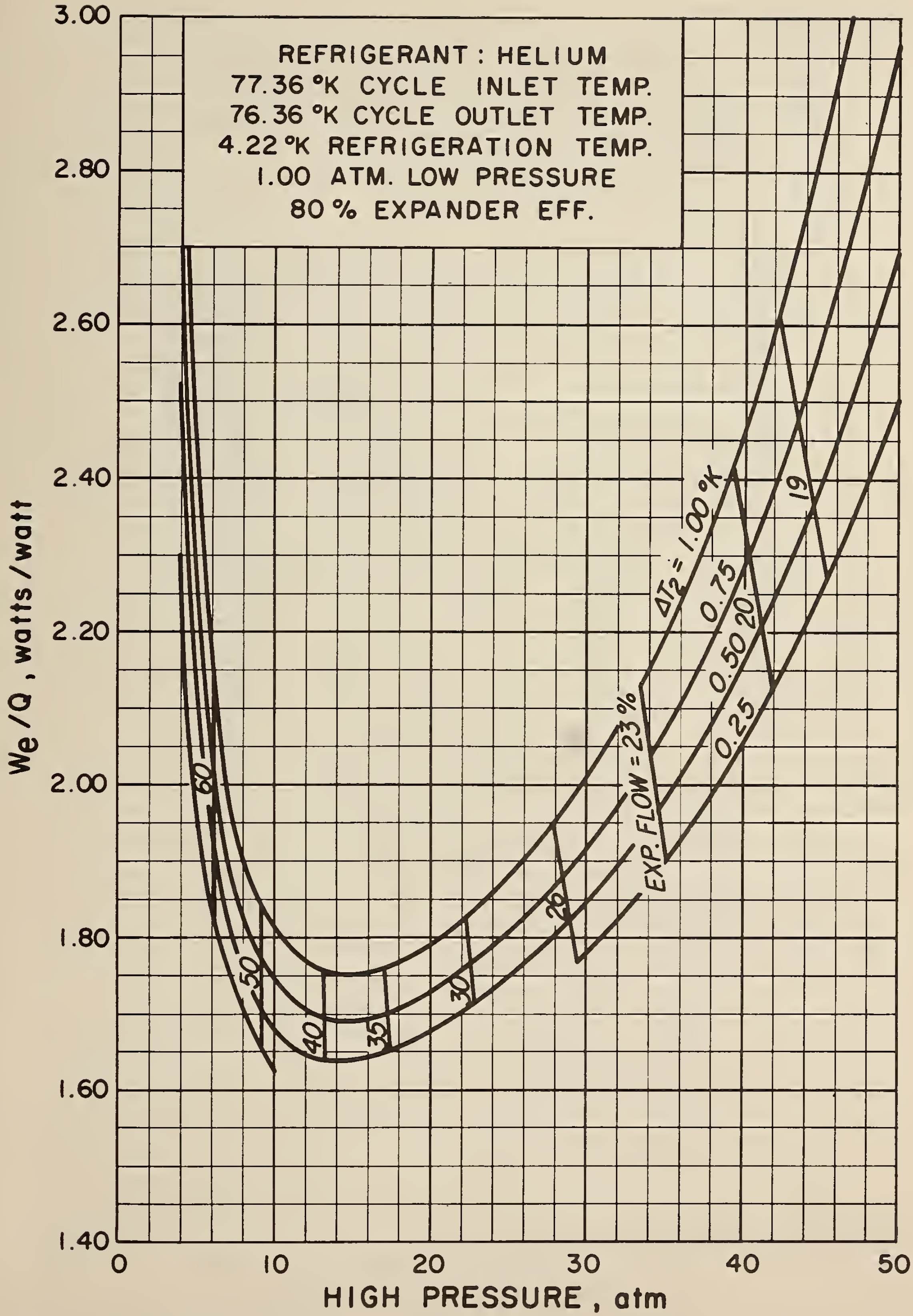

Figure 18b. Claude Refrigerator Performance - Expander Power - 76.36°K Cycle Outlet Temperature - 80% Expander Efficiency

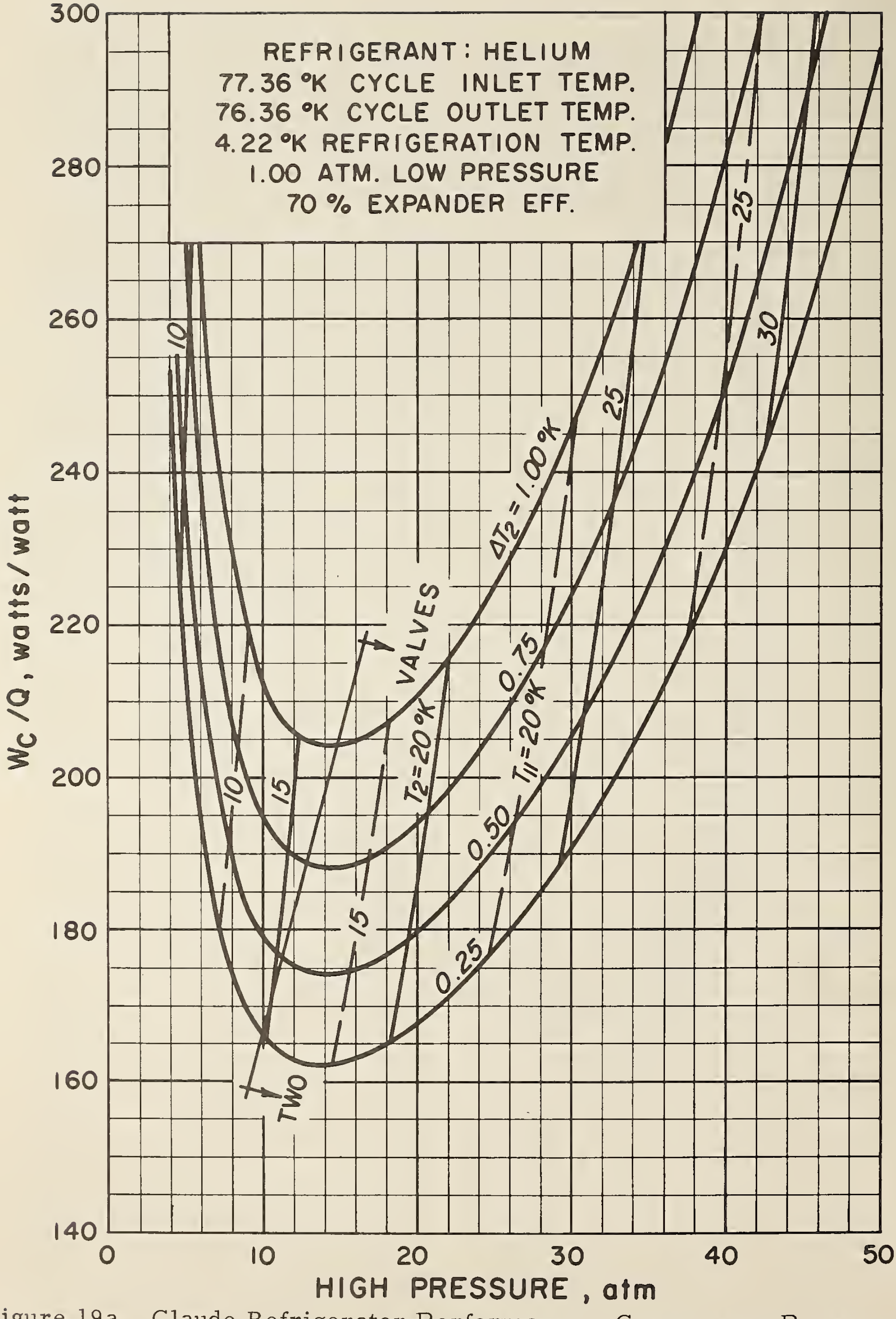

Figure 19a. Claude Refrigerator Performance - Compressor Power - 76.36°K Cycle Outlet Temperature - 70% Expander Efficiency

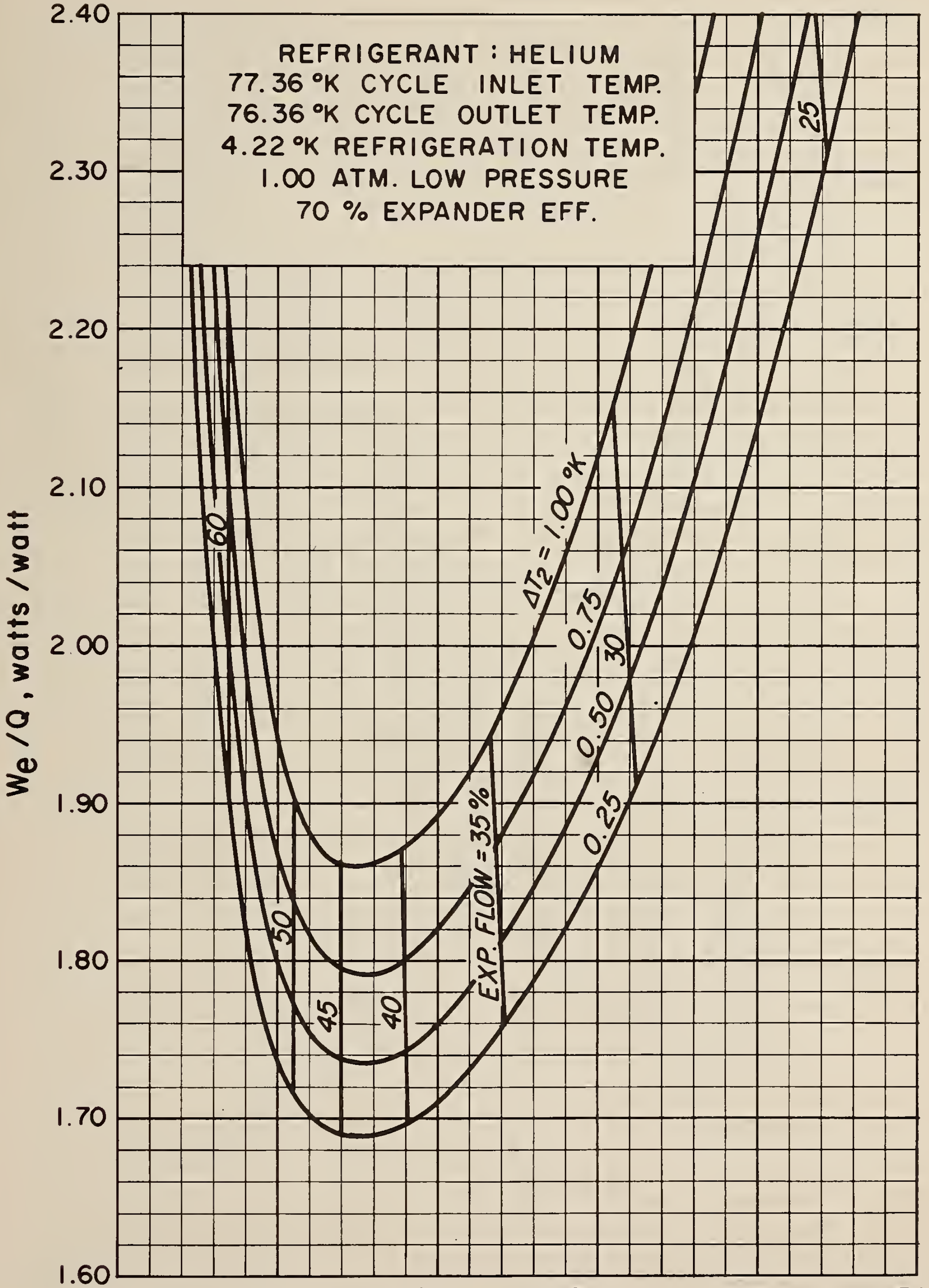

Figure 19b. Claude Refrigerator Performance - Expander Power - 76.36°K Cycle Outlet Temperature - 70% Expander Efficiency

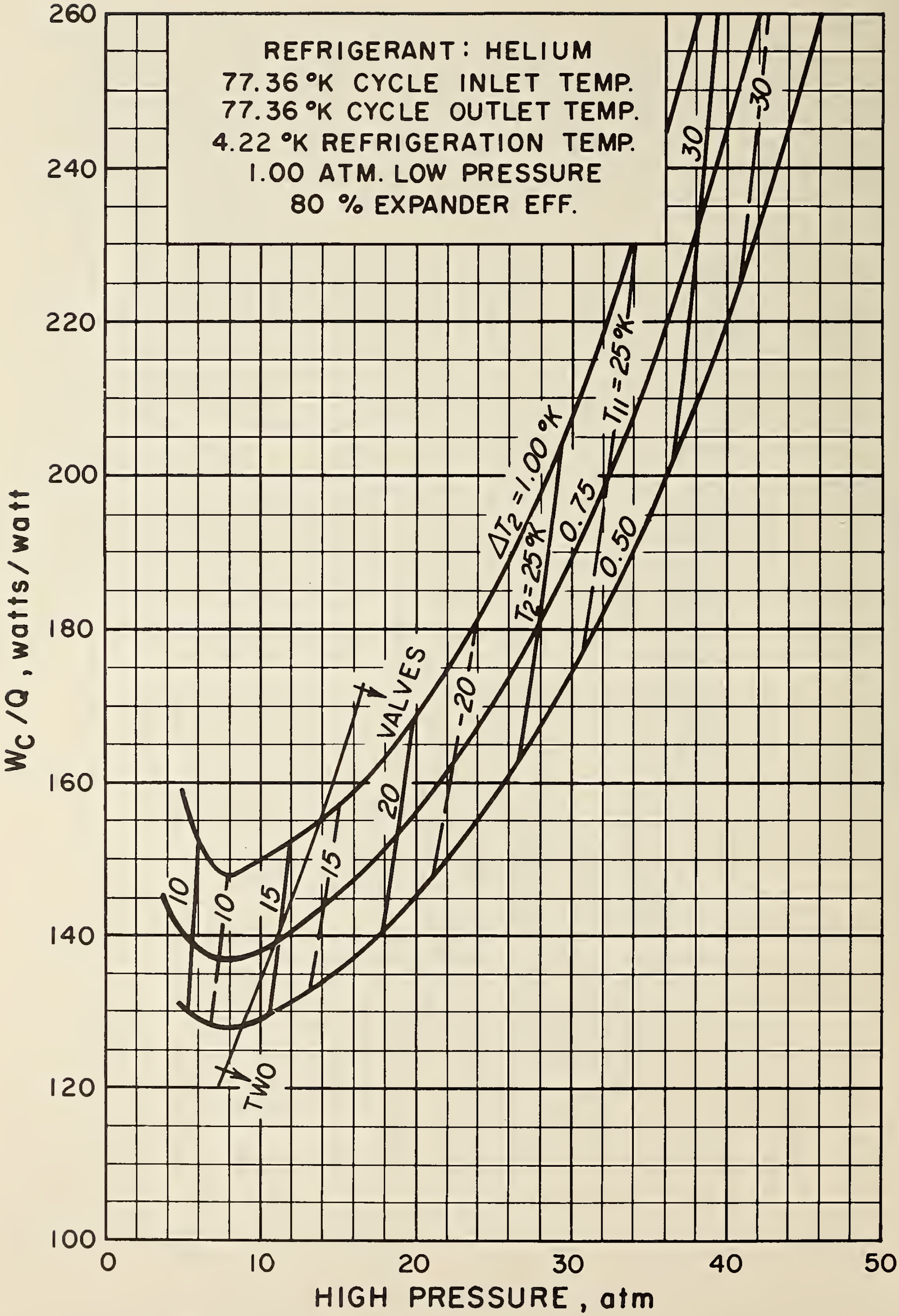

Figure 20a. Claude Refrigerator Performance - Compressor Power - 77.36°K Cycle Outlet Temperature - 80% Expander Efficiency

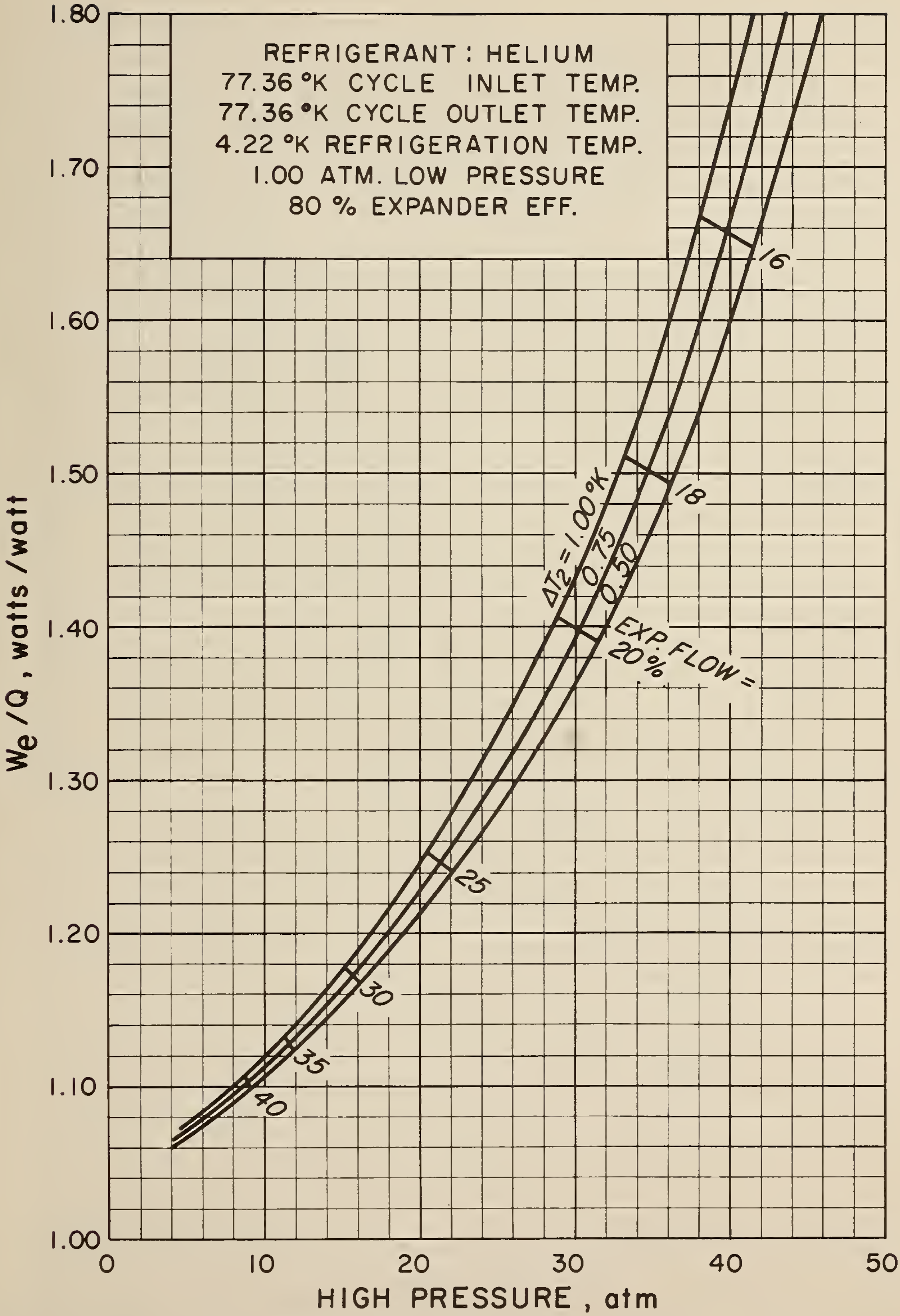

Figure 20b. Claude Refrigerator Performance - Expander Power - 77.36°K Cycle Outlet Temperature - 80% Expander Efficiency

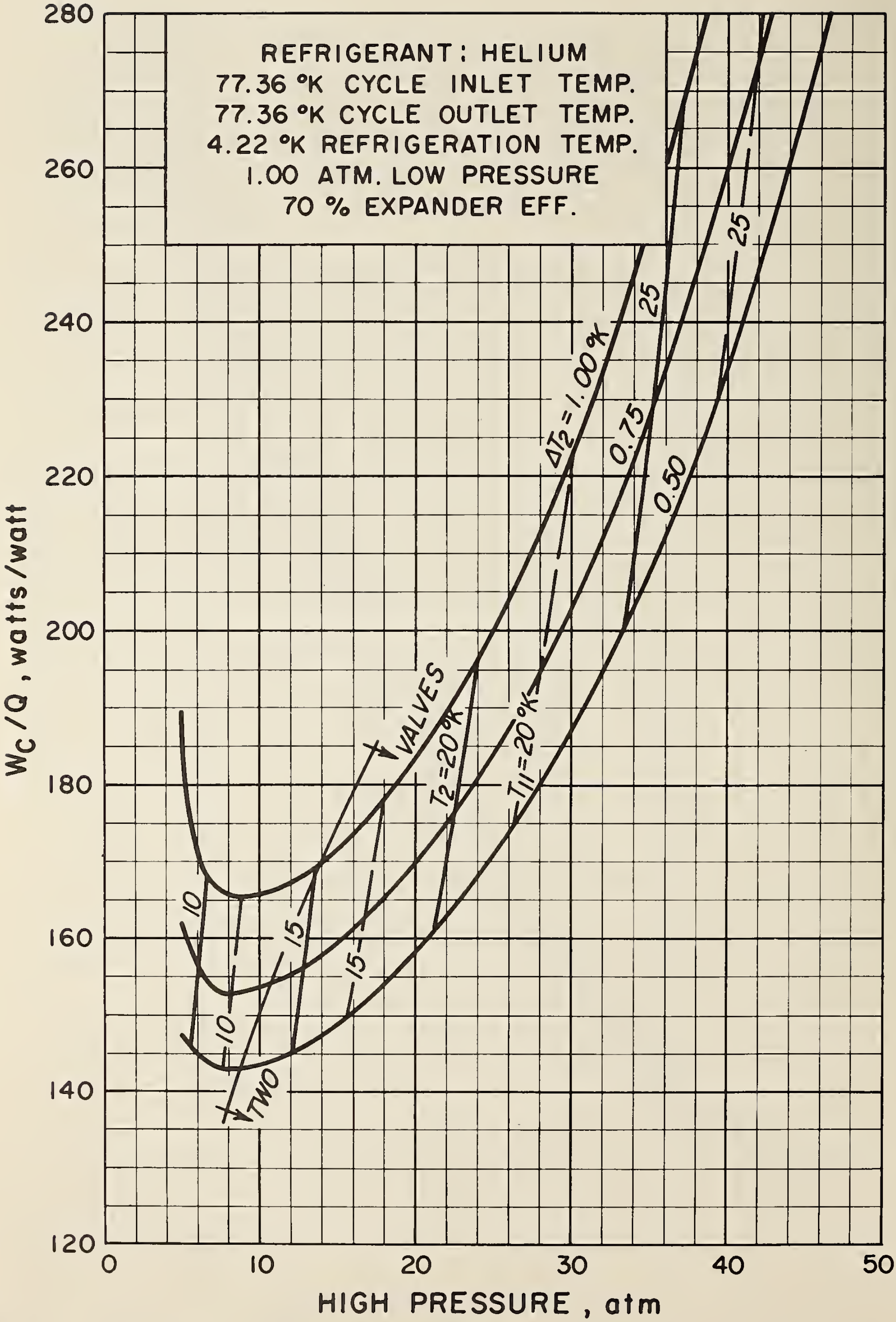

Figure 21a. Claude Refrigerator Performance - Compressor Power - 77.36°K Cycle Outlet Temperature - 70 Expander Efficiency

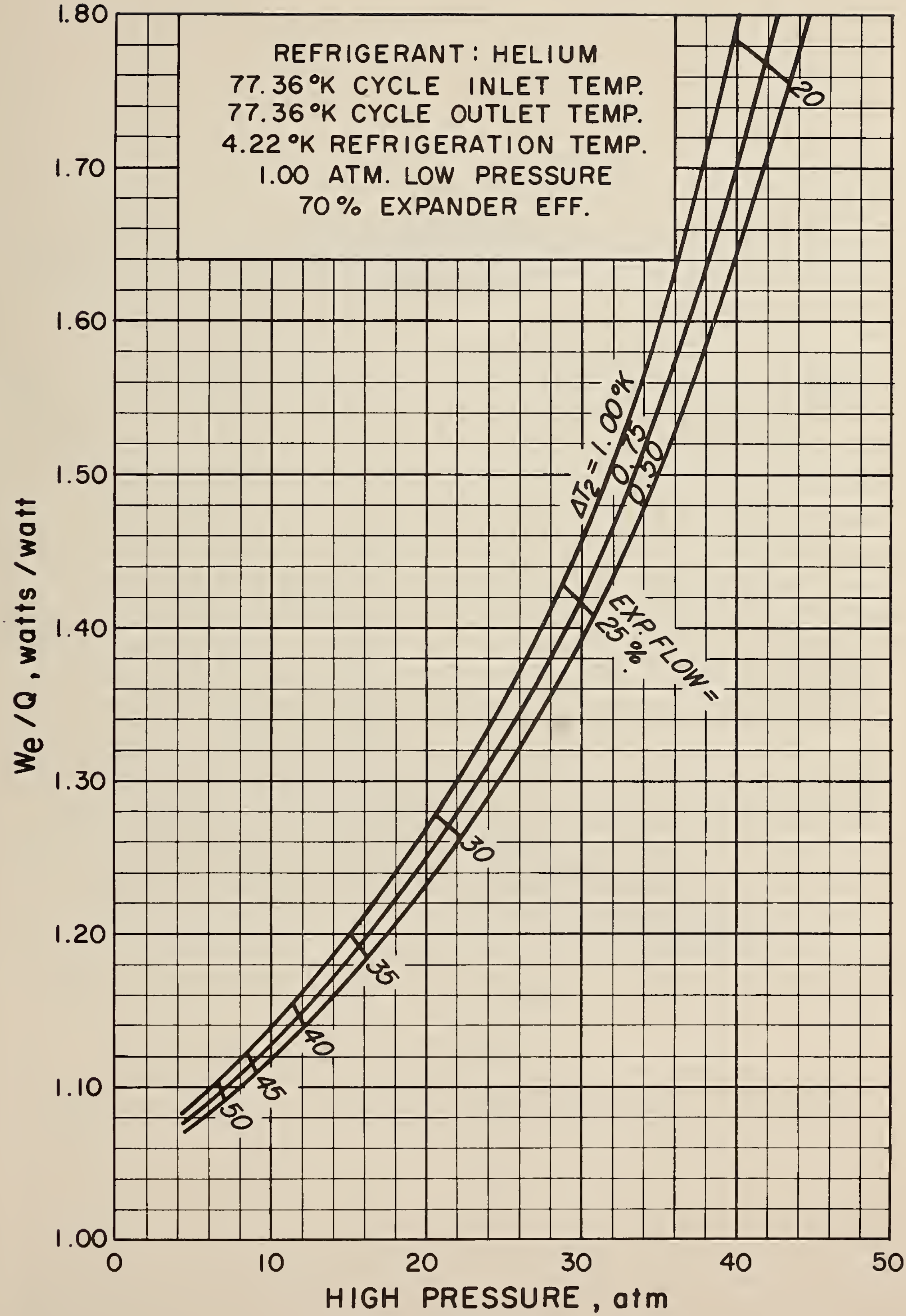

Figure 21b. Claude Refrigerator Performance - Expander Power - 77.36°K Cycle Outlet Temperature - 70% Expander Efficiency

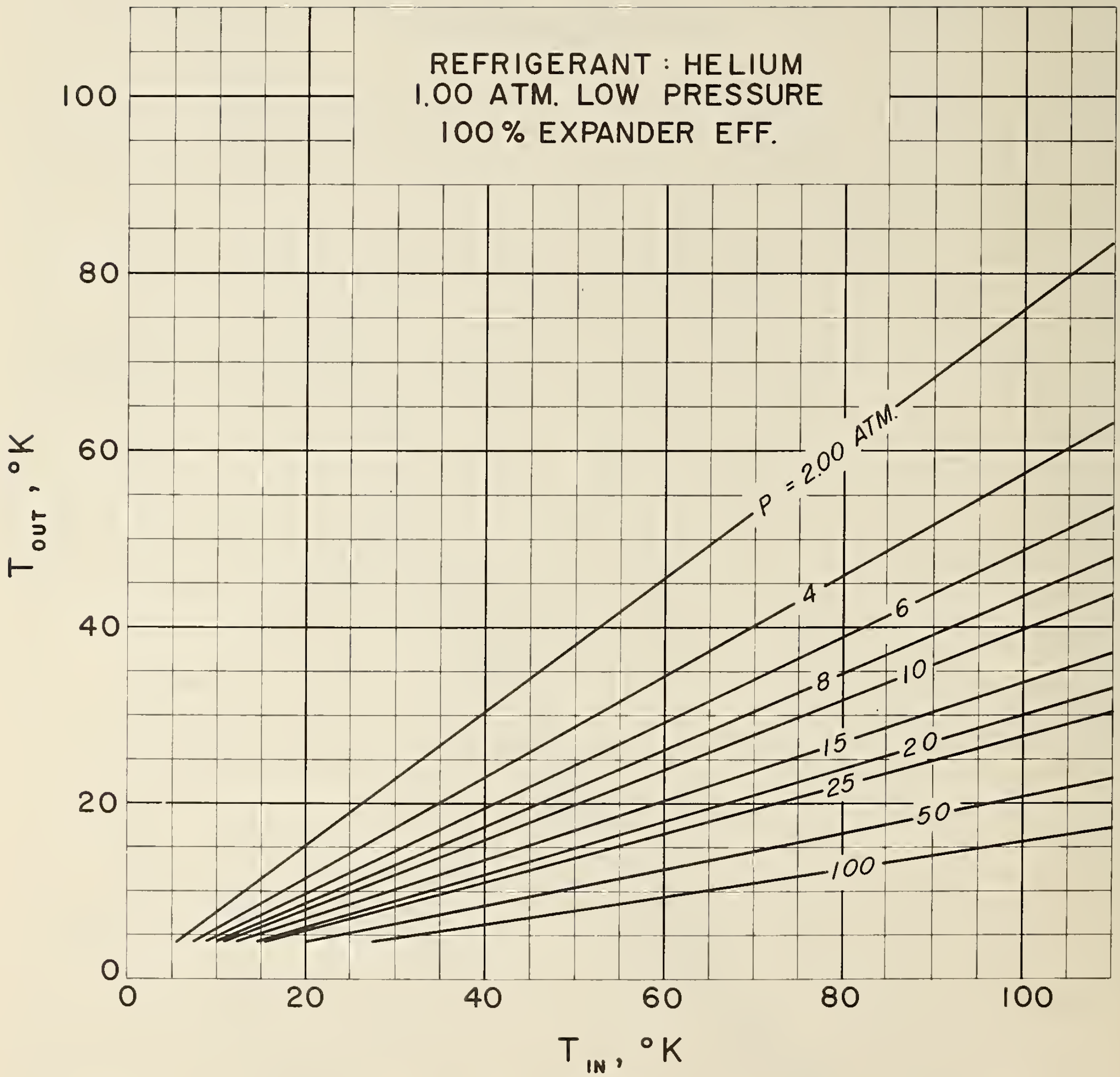

Figure 22. Expansion Engine Performance - 100% Efficiency

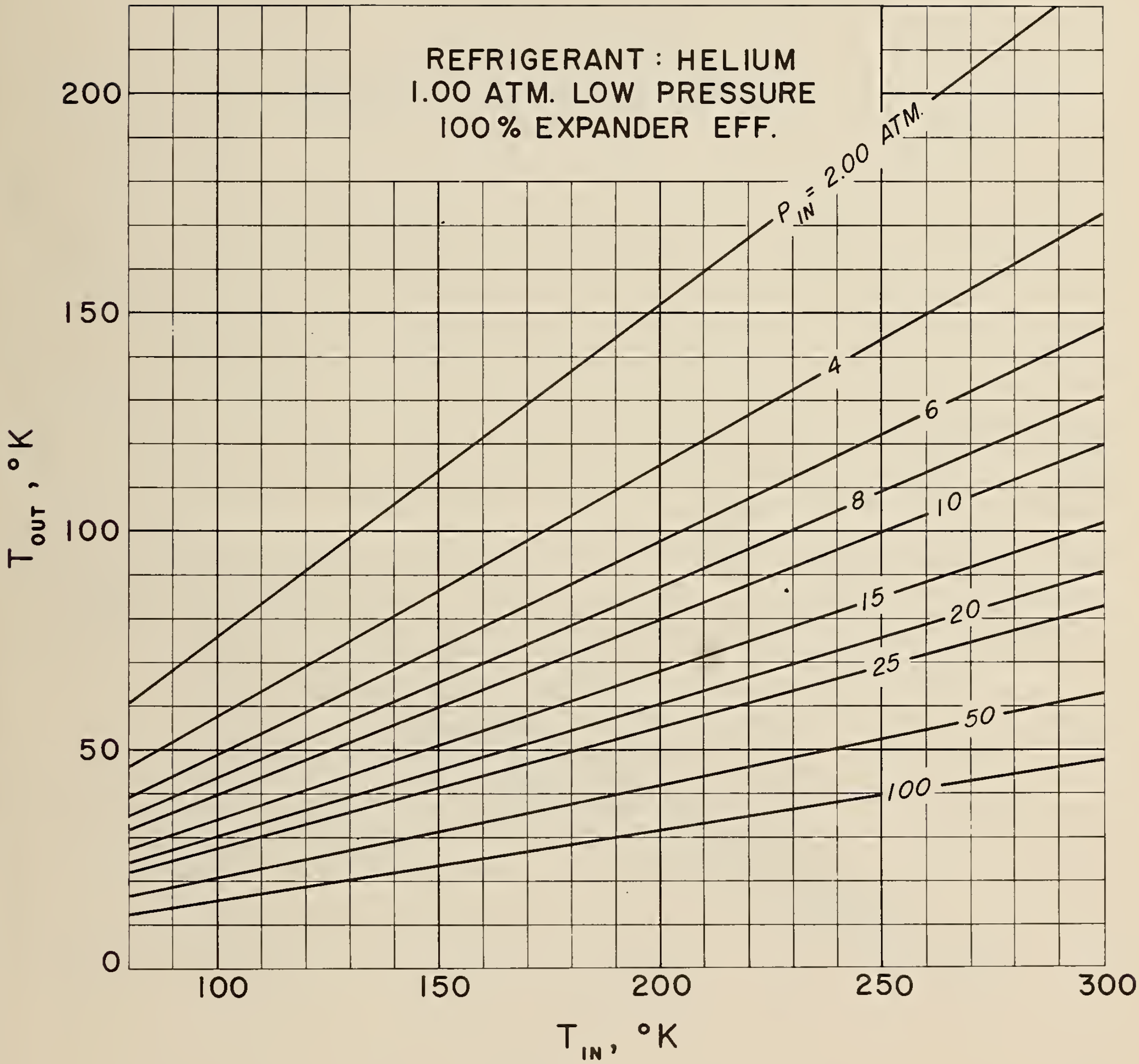

Figure 23. Expansion Engine Performance - 100% Efficiency

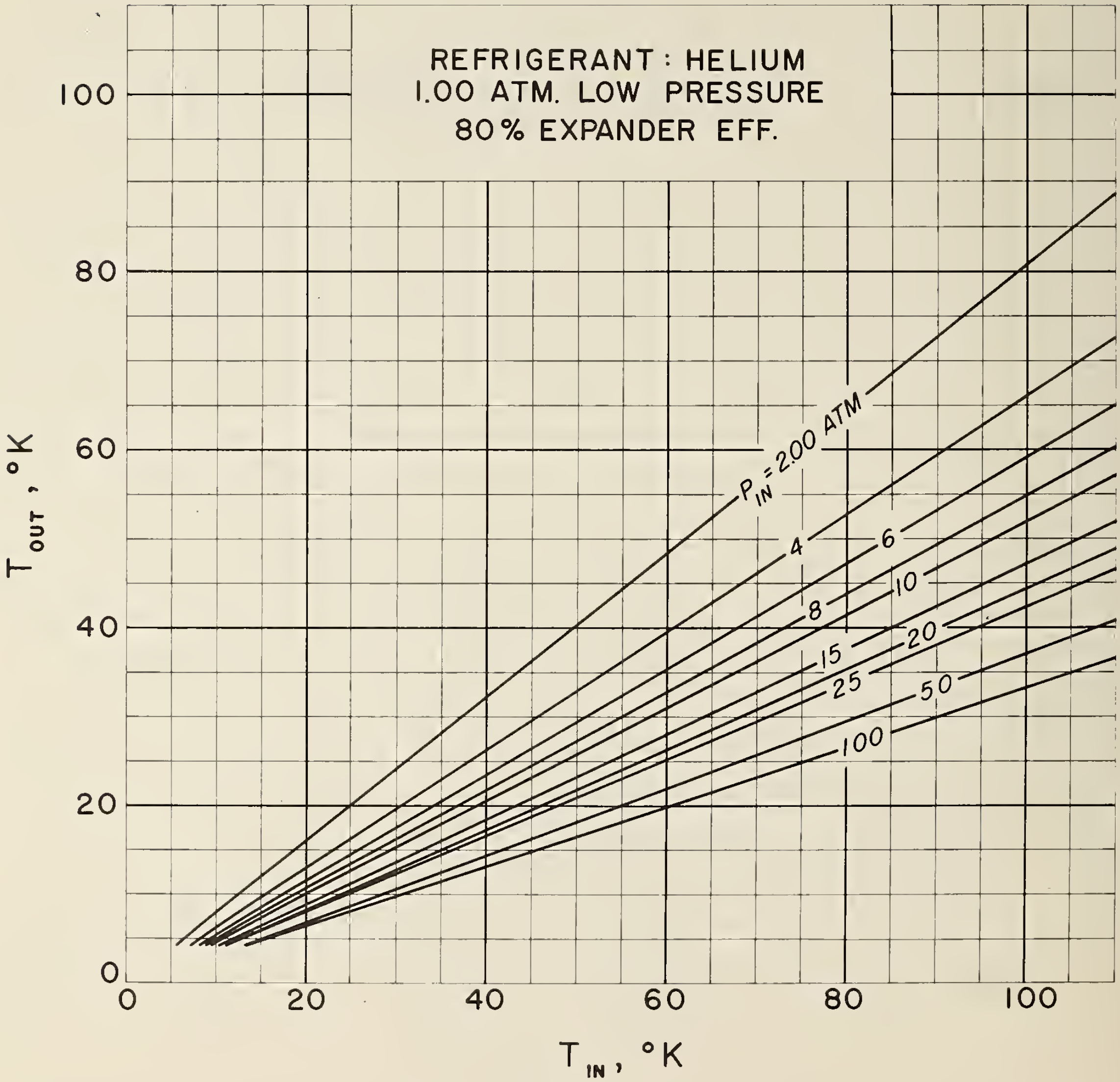

Figure 24. Expansion Engine Performance - 80% Efficiency

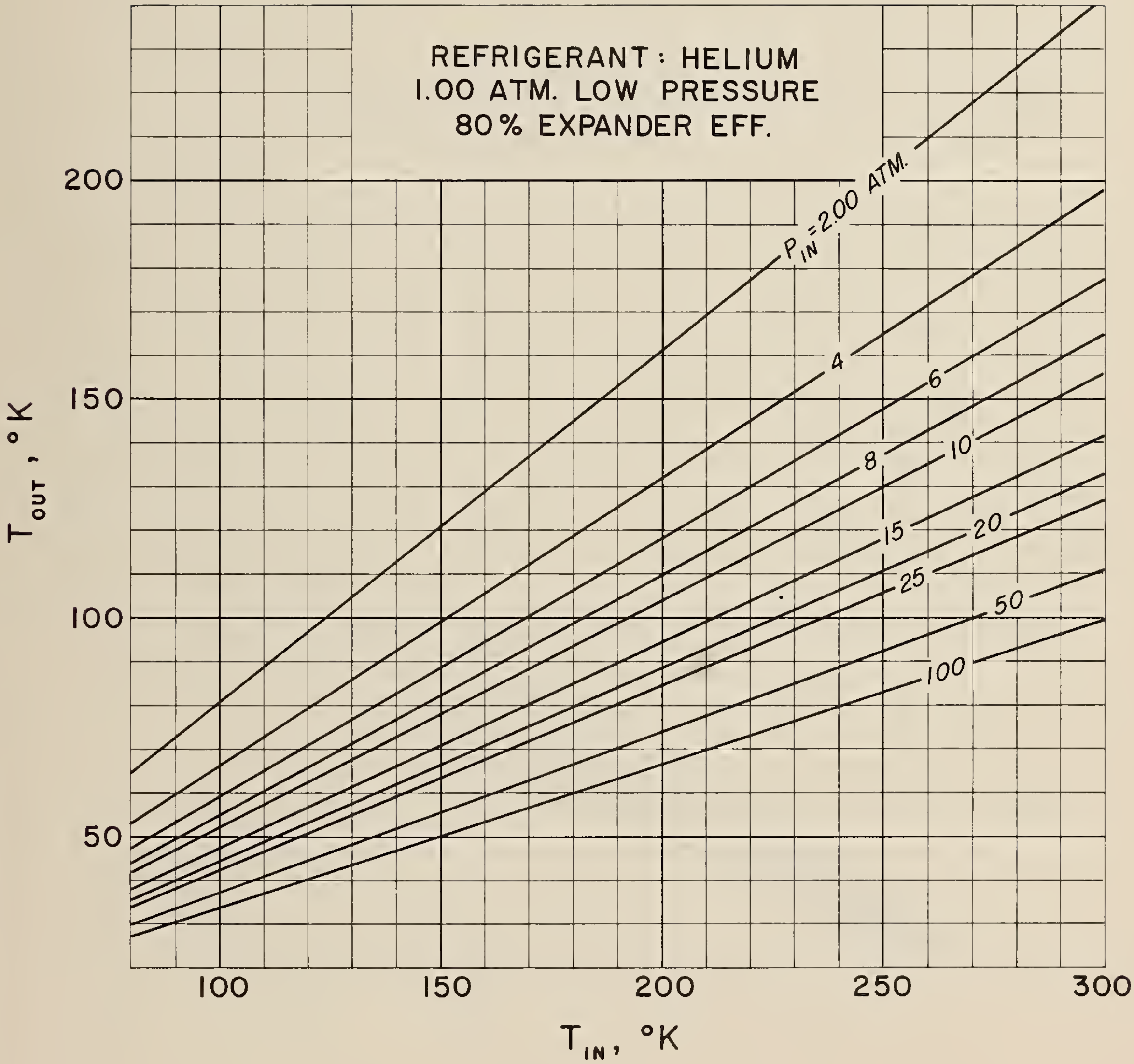

Figure 25. Expansion Engine Performance - 80% Efficiency

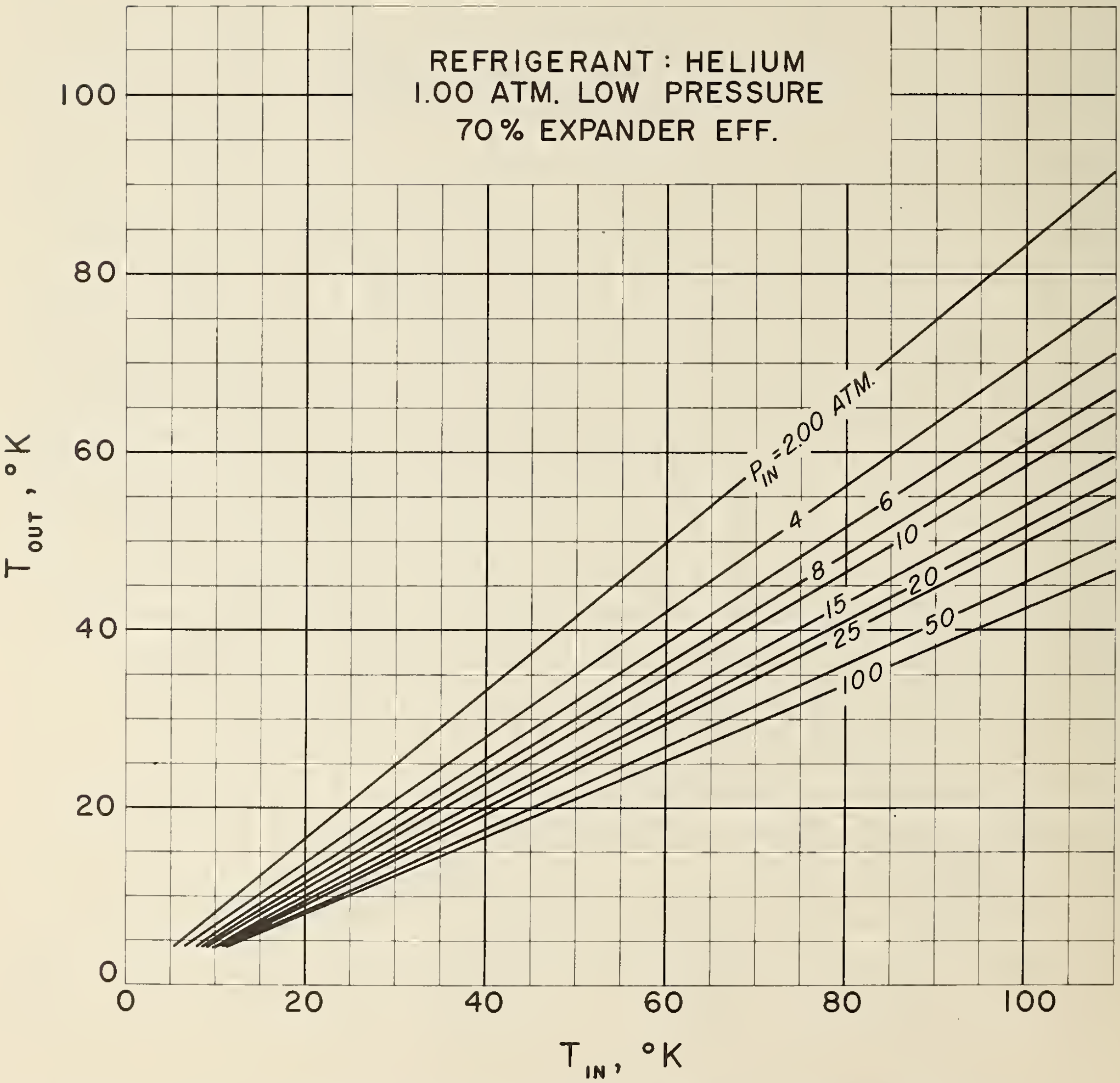

Figure 26. Expansion Engine Performance - 70% Efficiency

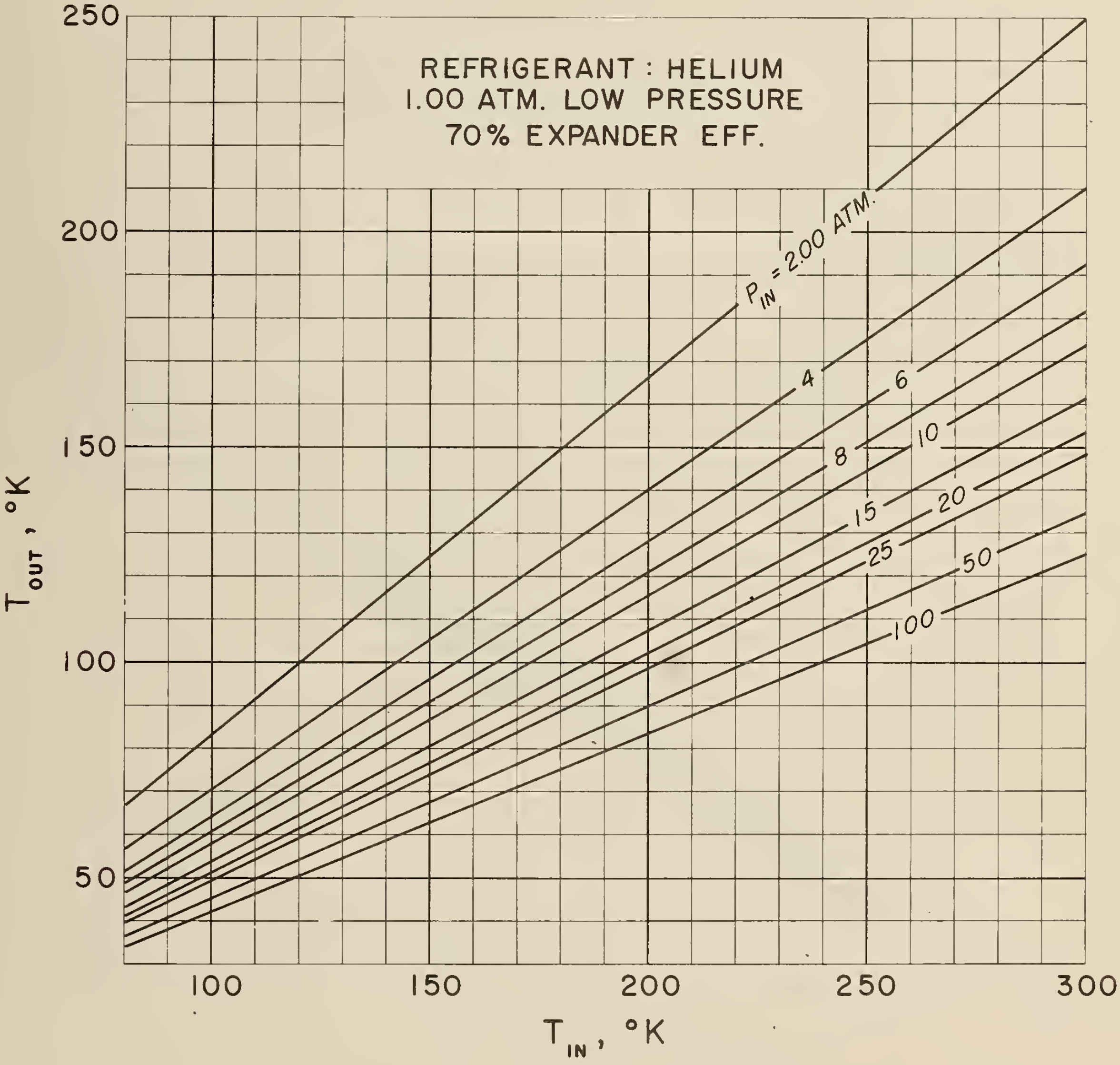

Figure 27. Expansion Engine Performance - 70% Efficiency

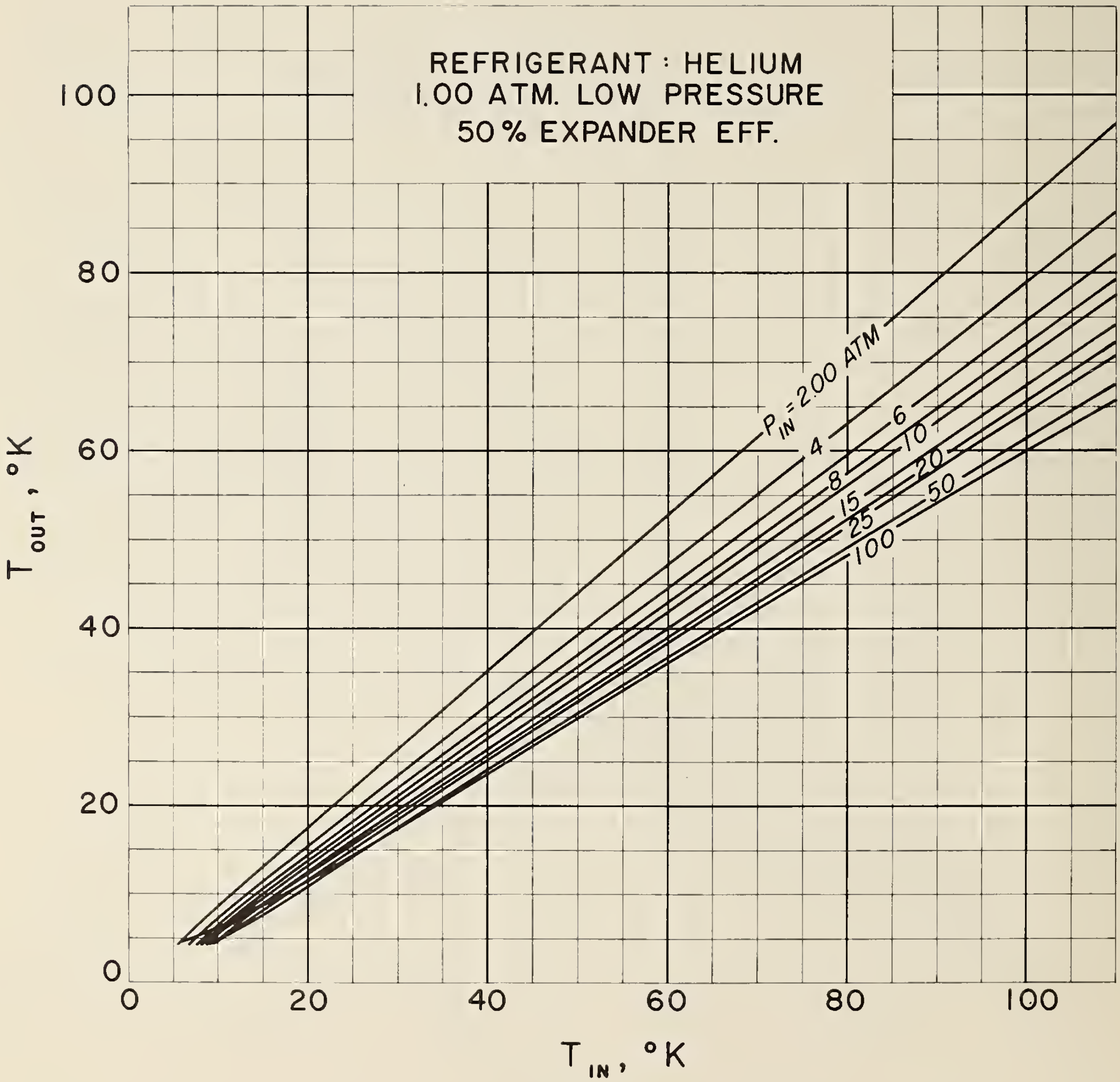

Figure 28. Expansion Engine Performance - 50% Efficiency

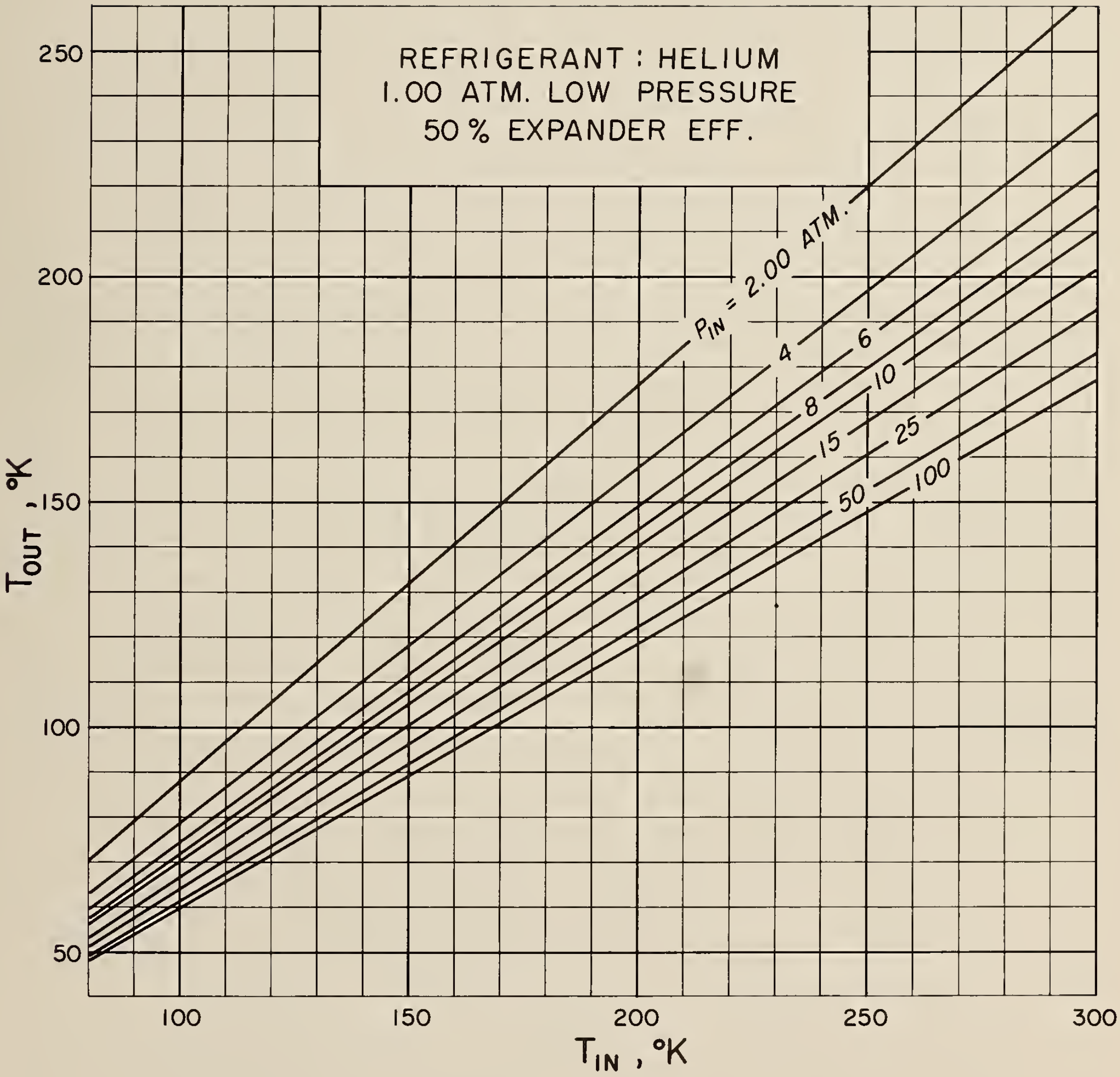

Figure 29. Expansion Engine Performance - 50% Efficiency

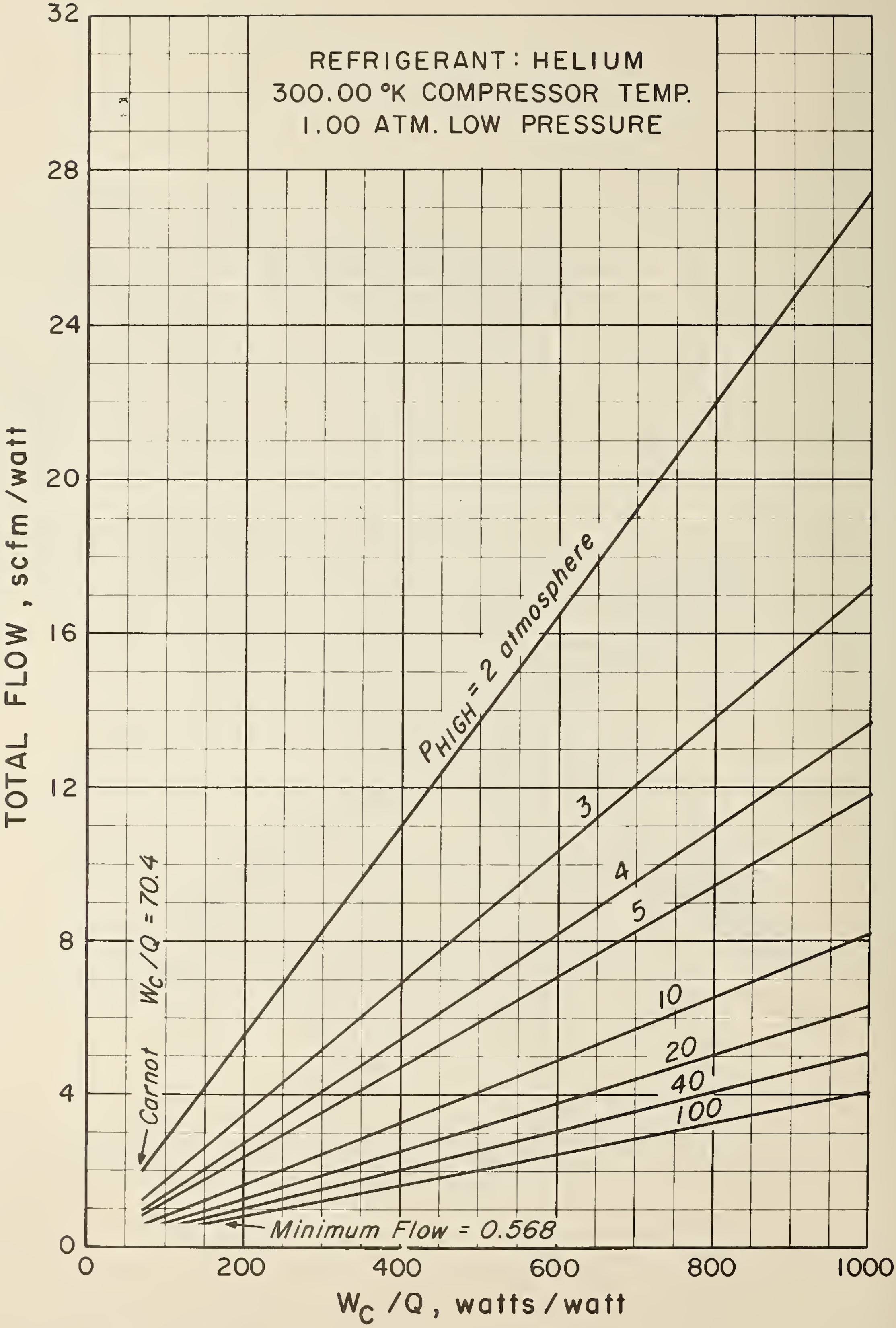

Figure 30. Helium Flow Rate

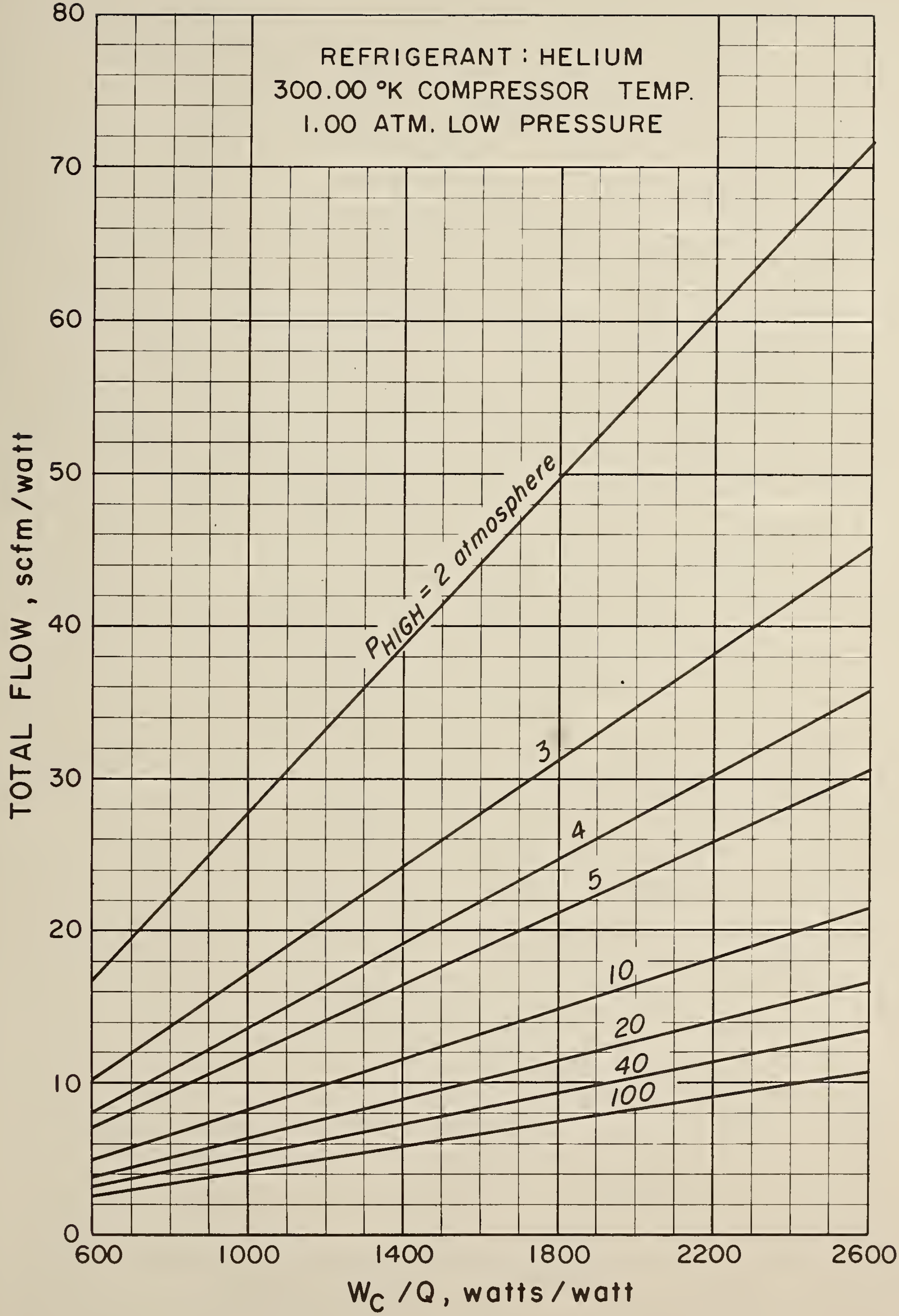

Figure 31. Helium Flow Rate

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