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A Standard for Accurate Phase-Angle Measurements at Audio Frequencies

WINSTON W. SCOTT, JR.

U.S. DEPARTMENT OF COMMERCE
National Bureau of Standards

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Abstract

A method is described for the measurement of phase angles from 0° to 360° (except for a band ±10° about the 180° point) with an uncertainty at audio frequencies of ±0.05° to ±0.01°, depending on the phase-angle. The method utilizes readily available equipment which may be assembled and connected to serve as a standard. The principal advantage of this standard is in its potentially broad frequency-response; namely, 10 Hz to 10 kHz or higher. It makes use of inductive voltage dividers and thermal voltage converters to compare voltage magnitudes, which are related to phase-angle by the law of cosines. The thermal voltage converters are compared with each other before each phase-angle measurement. Their outputs are connected so that short-time power supply variations do not affect the measurements. The repeatability of measurements with this standard at a phase-angle of 60° is within a range of 0.001°.

A model has been built which shows good measurement precision, but it has not been thoroughly tested or studied at many frequencies and phase-angles for sources of error. This report serves as a record of the development work completed on this standard to date.

Key Words: Phase-Angle
Standard
Audio Frequency
Thermal Voltage Converter
Inductive Voltage Divider
Measurement

A Standard for Accurate Phase-Angle Measurements at Audio Frequencies

Winston W. Scott, Jr.

1. Introduction

The phase-angle standard is used for the accurate determination of phase-angle between two alternating voltages supplied externally and for the calibration of precise phase-angle standards. The standard has a broad frequency response, a feature lacking in a previously described standard of phase-angle [Park and Cones, 1960], by using thermal voltage converters and appropriate inductive voltage dividers, and is useful within the frequency range 10 Hz to 10 kHz. Tests were made only to 10 kHz, but there seems little reason, with an appropriate power supply and high frequency inductive voltage divider, why these circuits cannot be useful up to 100 kHz. Measurement uncertainty depends upon the phase-angle being measured and the uncertainty varies from less than ±0.01° to ±0.05° except for phase-angles between 170° and 190° where the sensitivity of the standard is low.

The basis for operation of the standard is the "three-voltmeter method" described by Kinnard [1956]. The three-voltmeter method is a scheme for determining the phase-angle between two periodic, steady-state, alternating voltages by measuring the magnitude of three voltages between terminals of the source. Kinnard's method utilizes three voltmeters to measure the three voltages. Figure 1 shows how the three voltmeters are connected. The three terminals are marked 1, 2, and 3. If an alternating voltage supply is connected between terminals 1 and 2, and if another supply, operating at the same frequency, is connected between terminals 3 and 2, then three voltages designated E_{12}, E_{23}, and E_{31} exist between terminals. From Kirchhoff's Voltage Law, the vector sum of the voltages must equal zero, i. e., $\overline{E}_{12} + \overline{E}_{23} + \overline{E}_{31} = 0$.

Figure 2 shows the phasor voltage relationship for ϕ_2 approximately 60°. From the trignometric identity for oblique triangles for any ϕ_2 from 0° to ±180°,

$$\phi_2 = \cos^{-1} \frac{|E_{12}|^2 + |E_{23}|^2 - |E_{31}|^2}{2|E_{12}|\,|E_{23}|} . \tag{1.1}$$

or expressing voltages in rms and in terms of voltage ratio,

$$\phi_2 = \cos^{-1} \left(\frac{E_{12}}{2E_{23}} + \frac{E_{23}}{2E_{12}} - \frac{E_{31}}{2E_{12}} \frac{E_{31}}{E_{23}} \right).$$

Equation (1. 1) gives the phase-angle between voltages E_{12} and E_{23} in terms of the voltage magnitudes between each pair of terminals. Since the equation is one of voltage ratio, the voltage level is not of primary interest. In other words, the observer is not really interested in measuring the voltage levels E_{12}, E_{23}, and E_{31} accurately, but rather is interested in determining the ratios between the voltages accurately. The measurement of voltage ratio is accomplished through the use of an inductive voltage divider and several thermal voltage converters.

The standard is used to determine the phase-angle between two voltages generated by a power supply. In order to calibrate a phase meter, connections can be made directly to the standard. The phase meter indication can then be compared with the indication from the standard. A phase-shift device can be calibrated by using impedance compensation [Park and Cones, 1960] and voltage E_{12} as the source voltage.

Appendix 1 describes shielding for the control of capacitance effects in connection with circuitry to be discussed in section 5. Appendix 2 gives information related to typical measurements for phase-angle while appendix 3 discusses phase-angle errors for periodic waveforms other than sinusoidal.

2. Plan

When voltages E_{12} and E_{23} are made equal in magnitude (=E) by a technique to be described later, then (1.1) is simplified as follows:

$$\phi_2 = \cos^{-1}\left[1 - \frac{1}{2}\left(\frac{E_{31}}{E}\right)^2\right] . \tag{2.1}$$

But voltage E can be expressed in terms of voltage E_{31} as the result of two different inductive voltage divider arrangements. The first arrangement is for phase angles from 0° to ±60°, where

$$E_{31} = nE \qquad 0 < n < 1$$

and therefore,

$$\phi_2 = \cos^{-1}\left[1 - \frac{n^2}{2}\right] . \tag{2.2}$$

The second arrangement is for phase-angles from ±60° to ±180°, where

$$E_{31} = \frac{1}{n} E . \qquad 1 \geqslant n \geqslant 0.5$$

and therefore,

$$\phi_2 = \cos^{-1}\left[1 - \frac{1}{2n^2}\right] . \tag{2.3}$$

However, for values of ϕ_2 near 180° the usefulness of (2. 3) is poor, because n is near 0. 5 where a change in n affects the value of ϕ_2 by a relatively large amount.

The particular inductive voltage divider arrangements are described in detail in sections 4 and 5.

3. Voltage Supply and Detector Systems

The supply and detector systems are an important part of the phase-angle standard. A supply fixture provides convenient physical terminals to which voltages can be supplied, voltage comparisons can be made, and phase-angle standards can be connected for calibration. Figure 3 indicates how the supply fixture is provided with voltages of variable phase. The voltage level for each phase is most conveniently controlled by a potentiometer (voltage divider) connected between the generator and the power amplifier. The variable-phase generator indicates the approximate phase difference between the reference and variable phases so that any ambiguity in angle may be resolved. In the experimental setup, transformers T_1 and T_2 had 100-volt primaries and tapped secondaries of 1, 2, 3, and 4 volts. Rated output power was 50 watts. The voltage supply system was designed to have low impedance outputs, so that different loads at the supply fixture did not significantly change the voltage levels.

To help establish definite relationships between the three voltages E_{12}, E_{23}, and E_{31}, thermal voltage converters are used. A thermal voltage converter (TVC) consists of a resistive multiplier in series with the heater circuit of a thermoelement. The thermocouple circuit of the thermoelement is the output of the thermal voltage converter. If the outputs of two thermal voltage converters are differentially connected and if the inputs are connected in parallel as shown in figure 4, then short-time amplitude effects tend to be eliminated. The outputs at rated input voltage are typically about 7 mV dc. The Lindeck potentiometer adds a few microvolts of the correct polarity as required for galvanometer null. A null indication then represents equal input voltages applied to the two thermal voltage converters.

A major source of short-time amplitude instabilities at the supply fixture is within the oscillator section of the phase generator. Both reference phase and variable phase voltages at the supply fixture fluctuate alike, because the oscillator is common to both voltages. So, instead of connecting the inputs of the thermal voltage converters in parallel (see fig. 4), assume the two inputs were connected to the proper supply fixture voltages. Then because of the differential connection in the thermal voltage converter outputs, the galvanometer indication will tend to remain steady in the presence of changes affecting both outputs, although sensitive to changes in either output alone.

Some variable-phase oscillators of older design do not have their outputs independent of phase or frequency setting. For convenience, the operator does not want to change to a new phase-angle and then drastically change the oscillator voltage outputs back to their former level. Good practice in the use of a thermoelement dictates keeping the applied heater voltage somewhat constant in magnitude not only to avoid possible burnout to its heater, but also to avoid undesired thermocouple voltage drift. The easiest solution is to use modern oscillators which have specifications limiting this kind of interaction.

4. Phase-Angle Measurements, 0° to ±60°

The circuit used to obtain accurate phase-angle measurements from 0° to ±6° will be described later. An inductive voltage divider is used in the circuit shown in figure 5 to obtain accurate phase-angle measurements between ±6° and ±60°.

To determine phase-angle ϕ_2, the following calibration procedure is used:

1. Connect TVC_2 in parallel with TVC_1, apply reference-phase voltage (E_{12}) and adjust Lindeck potentiometer #1 for galvanometer null (G_1). The null now indicates equality of input voltage (magnitudes only) applied to the thermal voltage converters.

2. Reconnect TVC_2 as shown in figure 5 and adjust the variable-phase voltage (E_{23}) for the same null. Now $E_{12} = E_{23} = E$.

3. Connect TE#1 in parallel with TE#4 and adjust Lindeck potentiometer #2 for a null in galvanometer G_2. This null now identifies the condition when the voltage across TE#1 equals E_{31}.

4. Reconnect TE#1 as shown in figure 5 and adjust the inductive voltage divider and the decade resistance box for a minimum in the a-c detector indication and a null in galvanometer G_2. The best measurement is realized when the two galvanometers are at null at the same time. The value n is read directly from the inductive voltage divider and used with formula (2. 2) to determine phase-angle ϕ_2:

$$\phi_2 = \cos^{-1}\left[1 - \frac{n^2}{2}\right].$$

The current for Thermoelement #1 is supplied through the decade resistance branch of the bridge circuit to avoid a loading error on the inductive voltage divider. The heater circuit of TE#4 is above ground, but a measurement error due to stray capacitance is not expected. At worst ($\phi_2 = 60°$), TE#4 will be above ground by only about one volt rms. This error will not be significant provided the thermoelement is constructed with an insulating bead between the heater and thermocouple circuits.

Providing they are reasonably small, a-c response errors in the thermal voltage converters do not affect the determination of phase-angles. The reason is that when the thermal voltage converters are connected in parallel and their outputs equated, any a-c error differences (which is a function of frequency) between them are, in effect, eliminated by the adjustment of potentiometer #1. Likewise, any differences in a-c error between TE#1 and TE#4 is accommodated in the adjustment of potentiometer #2. Ordinarily, the a-c response of the decade resistance box does not affect the measurements because it serves only to supply current to TE#1. If however, the decade resistance box is excessively poor at high frequencies, an a-c detector null will not be obtained resulting in an imprecise inductive voltage divider indication.

The resistance of the thermal voltage converters (including leads, multiplier resistor, and thermoelements) need not be known, but they must remain constant over the period of calibration, or at least change in the same proportion and direction. Multiplier resistors constructed at NBS have shown excellent short-time resistance stability (typically less than 0.001% in ten minutes) and have a very flat frequency response from dc to 100 kHz. Thermoelements have a high temperature coefficient but if TE#2 and TE#3 as well as TE#1 and TE#4 are placed in close physical proximity in a thermal lag enclosure (such as a massive copper block or a thermistatically controlled oven), then temperature changes, if they do occur, will tend to affect the thermoelements equally and therefore create no galvanometer difference indications.

Unfortunately, a limitation is reached with thermoelements for values of n between 0 and 1/10 or in terms of phase angle, between 0° and ±6°. This limitation is reached for small phase angles because voltage E_{31} is not large enough to energize TE#4 (and TE#1) with resulting good sensitivity. Fortunately, as table 1 indicates, the "maximum phase-angle error" is nearly constant with "resultant allowable voltage-ratio error" (expressed in microvolts) over the phase-angle ϕ_2 range of 60° to 0°. This condition results from a study of (1.1). The uncertainty in setting voltage $E_{12} = E_{23} = E$ is easily done to within ±0.001% (which is negligible) and is independent of phase-angle. The major source of uncertainty affecting phase-angle occurs within the determination of the ratio E_{31}/E as identified by a study of (2.1). As the phase-angle ϕ_2 decreases from 60°, voltage E_{31} decreases (E remains constant). As E_{31} decreases, a larger uncertainty in E_{31}/E is allowable for the same phase-angle uncertainty because of the action of the cosine term near 0°. For example, at a phase-angle of about 6°, E_{31}/E need be only within ±0.3% for a maximum phase-angle error of ±0.017° (see table 1). The ratio uncertainty restriction of ±0.3% can easily be met by using an a-c differential voltmeter to give the same indication at the output of the inductive voltage divider as was obtained across terminals 1 - 3. Since the a-c differential voltmeter is a high impedance device and takes very little current to operate, the loading on the inductive voltage divider is not significant and the decade resistance box and a-c detector can be removed. An ac-dc differential voltmeter with a null sensitivity better than 1 mV ac is recommended.

When the phase angle ϕ_2 equals 60°, the inductive voltage divider, decade resistance box, and a-c detector are no longer needed and can be removed from the circuit shown in figure 5. For this phase-angle, TE#1 and TE#4 will be operating at 1.0 volt, which is 67% over the 0.6 volt recommended operating voltage for the thermoelements. A small possibility of heater burnout exists, especially for transient overloads above this voltage level, but burnouts do not ordinarily occur except for maintained overloads in excess of 100% of 0.6 volt. If a thermoelement must be replaced, it can be easily done with no preliminary calibration, such as d-c reversal, insulating bead efficiency, ac-dc difference, or sensitivity curve determination, such as is necessary for ac-dc transfer standard calibration. In other words, the thermoelements are referenced against each other just before each determination of phase-angle.

5. Phase-Angle Measurements, ±60° to ±170°

The circuit used to obtain accurate phase-angle measurements between ±60° and ±170° is shown in figure 6.

In order to obtain phase-angle measurements, the following calibration procedure is used:

1. Connect TVC_2 in parallel with TVC_1, apply reference-phase voltage and adjust Lindeck potentiometer #1 for galvanometer null. The null now indicates equality in input voltage applied to the thermal voltage converters, TVC_1 and TVC_2.
2. Reconnect TVC_2 as shown in figure 6 and adjust the variable-phase voltage for the same null. Now $E_{12} = E_{23} = E$.
3. Connect the inductive voltage divider and associated circuitry input in parallel with TVC_1 and adjust Lindeck potentiometer #2 for galvanometer null. The decade resistance box must be set to zero and the inductive voltage divider set to unity. This null now identifies the condition when the voltage across the inductive voltage divider equals E_{12} and corresponds with $n = 1$.
4. Reconnect the inductive voltage divider as shown in figure 6 and adjust its dials for galvanometer null and the decade resistance box for an a-c detector minimum indication. The value n is read directly from the inductive voltage divider and used with formula (2.3) to determine phase-angle ϕ_2:

$$\phi_2 = \cos^{-1}\left[1 - \frac{1}{2n^2}\right].$$

In this setup, a-c response errors in the thermal voltage converters do not create a measurement error because when the Lindeck potentiometers are adjusted, any differences in a-c error with frequency are accommodated. The best measurement precision would be realized if a fourth thermal voltage converter were connected permanently in parallel with TVC_1 and the resulting thermocouple output compared with TE#4. This change would allow E_{12} to equal E_{23} at the same time E_{12} equals nE_{31} as a consequence of nulling both galvanometers at the same time. Shielding is used around the circuitry connected to E_{31} and is described in appendix 1.

As the phase-angle ϕ_2 approaches 170°, the requirement for precision voltage comparisons relative to a specific phase-angle uncertainty rapidly becomes more difficult to achieve. In fact, for a phase-angle error of ±0.017° near 170°, the total voltage comparison uncertainty must be within ±0.0013%. Proportionally, for a phase-angle error of ±0.05°, the total voltage uncertainty cannot be larger than ±0.0038%. For phase-angles 170° to 180°, some other method is required for good precision. Figure 7 shows the maximum phase-angle error as a function of phase-angle for different allowed voltage-ratio errors.

Appendix 2 summarizes data obtained for a phase-angle of approximately 70°. The data show good measurement repeatability as well as high sensitivity.

A disadvantage with the standard, apparent at this stage of its development, is its "blind spot" near 180°. The balancing procedure could be simplified by the use of automatic balancing equipment to keep $E_{12} = E_{23}$ and by careful planning of plug-in components for use when changing setups.

The standard is used with sinusoidal or slightly distorted sinusoidal waveforms. Appendix 3 shows that the phase-angle ϕ_2, determined by the standard, is independent of the phases of the harmonics. It also shows that if the maximum magnitudes of the harmonics remain constant during the measurement, the standard can be used to calibrate a narrow-band phase-angle device. If the maximum magnitudes of the harmonics in the two power supplies are equal, measurement error due to waveshape will not occur. If the harmonics are not equal and the difference is small compared with the fundamental, a corrected phase-angle equation is used as derived in the appendix. If the phase-angle device has the same bandwidth as the standard, or all significant harmonics are responded to by both the standard and the device, then the restrictions of equal harmonic magnitudes are eliminated.

6. Conclusion

The principal advantages of this phase-angle standard are its broad frequency range--potentially 10 Hz to 100 kHz, its self-alignment features, its high accuracy, and its ability to be used with sinusoidal or slightly harmonically-distorted waveforms. If the measurement errors that have been identified are accounted for, there seems little reason why more sensitive galvanometers cannot be used and even higher accuracies realized in the measurements of phase-angle.

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9. Appendix 1

Shielding for Control of Capacitance Leakage Effects

Proper shielding is a requirement for accurate phase-angle measurements from ±60° to ±170° (see fig. 8). Shielding is required in order to make certain that capacitance leakage paths do not change when terminal "a" is grounded, then ungrounded at the supply fixture. For configuration 1, terminal "a" is grounded and distributed capacitances C_2 and C_3 are across voltage source E_{12}. Distributed capacitance C_1 does not exist. For configuration 2, terminal "a" is ungrounded and distributed capacitance C_2 is across voltage source E_{23}. Likewise, C_3 is across E_{31} and C_1 is across E_{12}. These capacitances are all across the low impedance power supply, where the leakage current does not affect the measurement. With efficient shielding, capacitances within the shields are fixed and independent of whether terminal "a" is grounded or ungrounded. This independence obviates a source of systematic error when the circuitry is referenced in configuration 1 and used for phase-angle measurements in configuration 2.

10. Appendix 2

Experimental Setup, Calibration Procedure, and Summary of typical measurements for phase-angle ϕ_2 approximately 70° and frequency 400 Hz.

The calibration procedure used was as follows:

1. Turn on the two voltage supplies and adjust each to about 0.5 volt with a vacuum tube voltmeter. Connect leads "A" to "Z" terminals (refer to fig. 9 and fig. 10) and with the switch in #1 position, adjust Lindeck potentiometer A for a galvanometer null indication.

2. Reconnect leads "A" to "X" terminals and adjust the E_{23} supply for an exact balance in the galvanometer. Voltage E_{12} is now equal to voltage E_{23}.

3. Connect leads "B" to "Z" terminals and with the switch in #2 position adjust Lindeck potentiometer B for a null indication on the galvanometer. The inductive voltage divider must be at setting 0.9999X and the decade resistor at setting 0.0 ohms.

4. Set phase-angle for about 70°, reconnect leads "B" to "Y" terminals and adjust the decade resistor for a null indication on the galvanometer. Then adjust the inductive voltage divider for a minimum indication on the a-c detector and readjust the decade resistor. The formula used is (2.3):

$$\phi_2 = \cos^{-1}\left[1 - \frac{1}{2n^2}\right].$$

where n is the inductive voltage divider indication.

A direct indication of reading precision was obtained when the switch was placed in the #2 position and the phase was changed from 60° to 59°. The galvanometer, which indicates the difference between E_{23} and E_{12}, changed from a reading of 43 cm to about 3 cm. Therefore:

$\Delta 1°$ = 400 divisions on the galvanometer scale (40 cm)
$\Delta 0.1°$ = 40 divisions (4 cm)
$\Delta 0.01°$ = 4 divisions (4 mm)

No fluctuations in the galvanometer indications was observed. The interconnection of the thermoelements is effective in eliminating the effects of power supply variations. A slight drift of galvanometer indication with time was observed, however, with all measurements. It is believed it could be largely eliminated if all thermoelements are placed in a massive thermal conductor such that changes in ambient temperature tends to increase each thermoelement output in the same direction and magnitude.

The calibration procedure was then repeated three additional times. The data indicated that variations in voltage ratio are less than ±0.00001 in the region where $n = 0.84222$. This is equivalent to a variation in phase-angle of ±0.001° in the region of 73°. This excellent repeatability would not be expected at phase-angles near 180°. The data indicate stability in phase-angle for both the variable-phase generator and the standard. It was considered highly unlikely that the generator was changing phase-angle (drifting) at the same rate and direction as the standard. However, a source of systematic error exists, in the use of an unshielded isolation transformer; its effective voltage-ratio changes depending upon whether its input is grounded or not. The isolation transformer was used to avoid shorting E_{12} because of the ground in the inductive voltage divider circuit. This source of error is avoided by using a shielded isolation transformer, but the problem is better solved by the proper use of circuit shielding as shown in appendix 1.

11. Appendix 3

Phase-Angle Errors for Periodic Waveforms other than Sinusoidal

Harmonic voltages may be expressed algebraically in terms of the scale of angles for the harmonics, as follows:

$$e = E_{m1} \sin(\omega t+\theta_1) + E_{m2} \sin 2(\omega t+\theta_2) + E_{m3} \sin 3(\omega t+\theta_3) + \text{---} + E_{mn} \sin n(\omega t+\theta_n) \; . \qquad \text{(A-1)}$$

To determine the effective or root-mean-square value as responded to by the thermal voltage converters, (A-2) must be used:

$$E = \left[\frac{1}{T} \int_0^T e^2 \, dt \right]^{1/2} , \qquad \text{(A-2)}$$

where T represents the waveform period. Reference [Lawrence, 1922] proves by derivation that the effective value of a non-sinusoid is the following:

$$E = \left(\frac{E_{m1}^2 + E_{m2}^2 + E_{m3}^3 + \text{---} + E_{mn}^2}{2} \right)^{1/2} . \qquad \text{(A-3)}$$

This equation for rms voltage is a function of the relative magnitudes of the fundamental and harmonics, and is independent of the phases of the harmonics relative to the fundamental.

For purpose of illustration, assume a power supply generates two voltages described by

$$e_{12} = E_{m1} \sin(\omega t+\theta_1) + E_{mj} \sin j(\omega t+\theta_3)$$

where j represents the j harmonic and,

$$e_{23} = E'_{m1} \sin(\omega t+\theta_1') + E'_{mj} \sin j(\omega t+\theta_3')$$

which as a result of (A-3) reduce to

$$E_{12} = \frac{\left(E_{m1}^2 + E_{mj}^2\right)^{1/2}}{\sqrt{2}} \qquad \text{(A-4)}$$

and

$$E_{23} = \frac{\left(E'^2_{m1} + E'^2_{mj}\right)^{1/2}}{\sqrt{2}} . \qquad \text{(A-5)}$$

The standard described in this paper uses the law of cosines to determine the phase-angle ϕ_2 between voltages E_{12} and E_{23}. In the procedure we force E_{23} to equal E_{12} and E_{31} to equal either nE_{12} or $\frac{1}{n}E_{12}$, depending on the inductive voltage divider configuration. In this case, because of the fact that the law of cosines is an equation of ratio, the components of E_{12} and E_{23} need not be known. It is necessary however, that the components remain constant in magnitude during the measurement.

In order to use this standard to calibrate another phase-angle device, no measurement error due to waveform will result providing the device is also broad-band because it responds to the same harmonics of significance. However, if the device is narrow-band, it will not respond to some harmonics and a measurement error due to waveform could result.

Consider the narrow-band device as equivalent to another standard which uses internal filters to block all harmonics above the fundamental. When the adjustment of voltages $E_{12} = E_{23}$ is made with the broad-band standard, the narrow-band device may see an error voltage e (assuming e represents the error voltage due to harmonic voltages) defined by:

$$E_{12}^{*} = E_{23}^{*} + \frac{e}{\sqrt{2}} ,$$

where

$$E_{12}^{*} = \left(\frac{E_{m1}^{2} + 0}{2} \right)^{1/2}$$

and

$$E_{23}^{*} = \left(\frac{E'^{2}_{m1} + 0}{2} \right)^{1/2}$$

or

$$e = E_{m1} - E_{m1}' \ . \qquad \text{(A-6)}$$

Referring to the balance with the broad-band standard and using (A-4) and (A-5),

$$E_{12} = E_{23}$$

or

$$\left(E_{m1}^{2} + E_{mj}^{2} \right)^{1/2} = \left(E'^{2}_{m1} + E'^{2}_{mj} \right)^{1/2} ,$$

or

$$E_{ml}^{2} - E'^{2}_{ml} = E'^{2}_{mj} - E_{mj}^{2} .$$

If the j ·harmonics in both E_{12} and E_{23} supplies have the same magnitude, then $E_{mj} = E_{mj}'$ and

$$E_{ml}^{2} - E'^{2}_{ml} = 0$$

$$(E_{ml} - E_{ml}')(E_{ml} + E_{ml}') = 0 .$$

Since $E_{ml} + E_{ml}' = 0$ is trivial, then the error voltage $e = E_{ml} - E_{ml}'$ must equal zero. In other words, if the magnitudes of the harmonics are equal in the power supplies, then the device may be calibrated without error due to waveshape, even though the device is narrow-band. On the other hand, if the harmonics are unequal, then a corrected phase-angle equation must be used. The equation is developed as follows:

$$\phi_2' = \cos^{-1}\left[\frac{E_{12}^{*2} + E_{23}^{*2} - E_{31}^{*2}}{2E_{12}^{*} E_{23}^{*}}\right] ,$$

where $E_{12}^{*} = E_{ml}/\sqrt{2}$, $E_{23}^{*} = E_{ml}'/\sqrt{2}$, $E_{31}^{*} = kE_{ml}'/\sqrt{2}$ where k may be n or $1/n$ depending upon the inductive voltage divider connection:

$$\phi_2' = \cos^{-1}\left[\frac{E_{ml}^{2} + E'^{2}_{ml} - k^{2}E'^{2}_{ml}}{2E_{ml}E_{ml}'}\right] ,$$

but from (A-6), $E_{ml}' = E_{ml} - e$:

$$\phi_2' = \cos^{-1}\left[\frac{E_{ml}^2 + (E_{ml}-e)^2 - k^2(E_{ml}-e)^2}{2E_{ml}(E_{ml}-e)}\right]$$

$$= \cos^{-1}\left[\frac{E_{ml}}{2(E_{ml}-e)} + \frac{(E_{ml}-e)(1-k^2)}{2E_{ml}}\right].$$

If e is sufficiently small compared with E_{ml} such that the e^2 term may be neglected, then the term $E_{ml}/2(E_{ml}-e)$ can be multiplied by $E_{ml} + e/E_{ml} + e$ to obtain $E_{ml} + e/2E_{ml}$, so

$$\phi_2' = \cos^{-1}\left[\frac{E_{ml} + e + E_{ml} - k^2E_{ml} - e + k^2e}{2E_{ml}}\right]$$

$$= \cos^{-1}\left[1 - \frac{k^2}{2}\left(1 - \frac{e}{E_{ml}}\right)\right].$$

Substituting $k = n$ or $1/n$ gives us formulas similar in form to (2.2) and (2.3):

$$\phi_2' = \cos^{-1}\left[1 - \frac{n^2}{2}\left(1 - \frac{e}{E_{ml}}\right)\right] \text{ for } 0° \text{ to } \pm 60° \qquad \text{(A-7)}$$

and

$$\phi_2' = \cos^{-1}\left[1 - \frac{1}{2n^2}\left(1 - \frac{e}{E_{ml}}\right)\right] \text{ for } \pm 60° \text{ to } \pm 170°. \qquad \text{(A-8)}$$

Voltage E_{ml} can be obtained closely enough by measuring and using the E_{12} value while voltage e can be obtained using a harmonic wave analyzer

Equations (A-7) and (A-8) are mathematically valid and phase-angle is meaningful only if e is small compared with E_{m1}. In other words, the distorting harmonics must be small compared with the fundamental. For example, if $\phi = 60°$ $(k = 1)$, a 0.1% distortion <u>difference</u> between the two power supplies would be equivalent to a 0.01% voltage error which is, using figure 7, about 0.005°. Note that these equations also predict the effect on phase-angle that an offset from galvanometer null would create. ($E_{12} = E_{23} + e$, where e is the difference voltage in the supplies which creates an offset galvanometer deflection.)

TABLE I: HOW PHASE ANGLE UNCERTAINTY RESTRICTIONS ON $\phi_2 \leq 60°$ AFFECT ALLOWABLE VOLTAGE-RATIO ERRORS

MAXIMUM PHASE ANGLE ERROR FOR $\phi 2$	VOLTAGE LEVEL E (VOLTS)	VOLTAGE LEVEL E_{31} (VOLTS)	PHASE ANGLE $\phi 2$	RESULTANT ALLOWABLE VOLTAGE-RATIO ERROR
± 0.017°	1	1	60.000°	± 0.026% (±260 μv)
± 0.017°	1	0.1	5.732°	± 0.30% (± 300 μv)
± 0.017°	1	0.01	0.573°	± 3.0 % (± 300 μv)

THREE-VOLTMETER METHOD

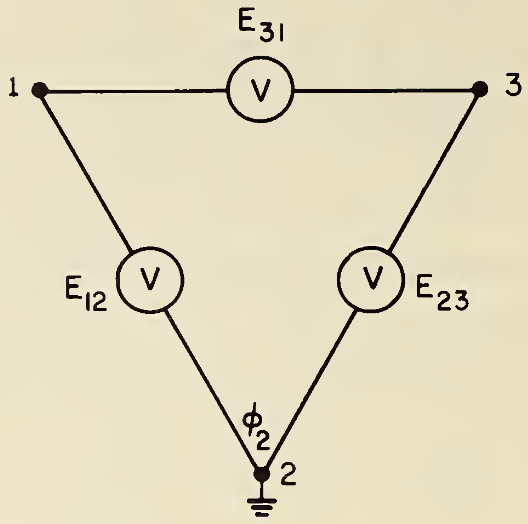


Figure 1

VOLTAGE PHASOR DIAGRAM

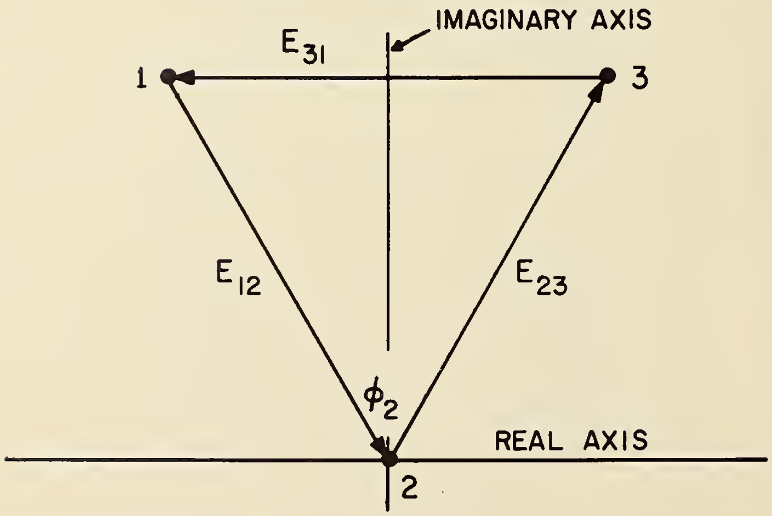
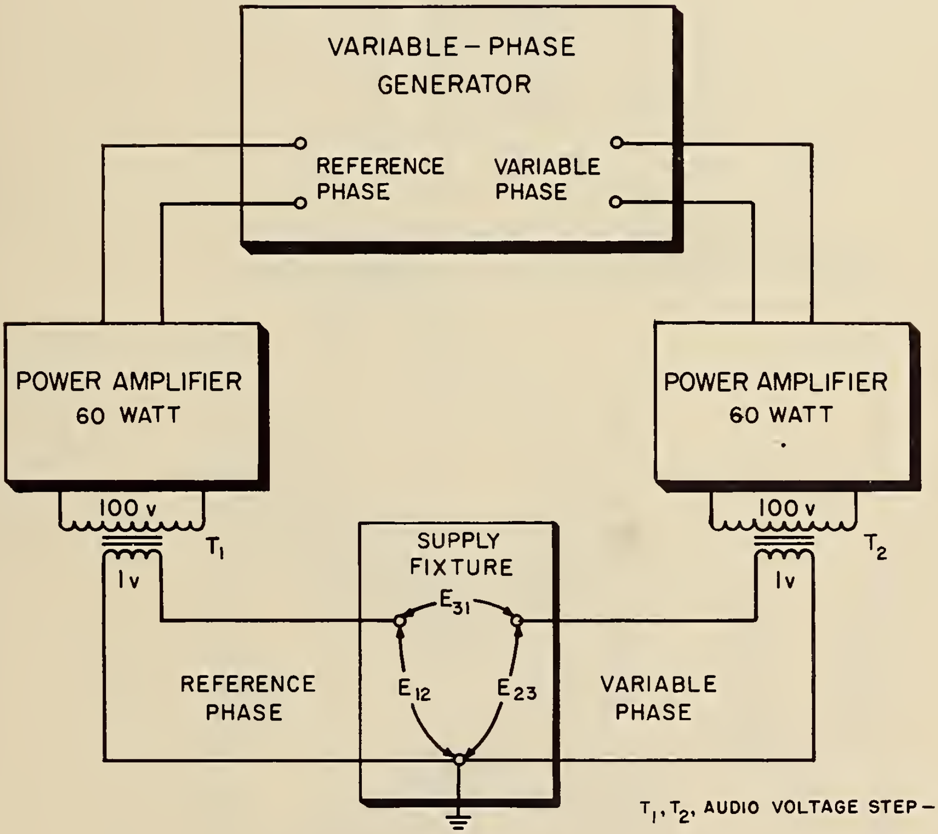


Figure 2

VOLTAGE SUPPLY SYSTEM



T_1, T_2, AUDIO VOLTAGE STEP – DOWN TRANSFORMER

Figure 3

DETECTION SYSTEM

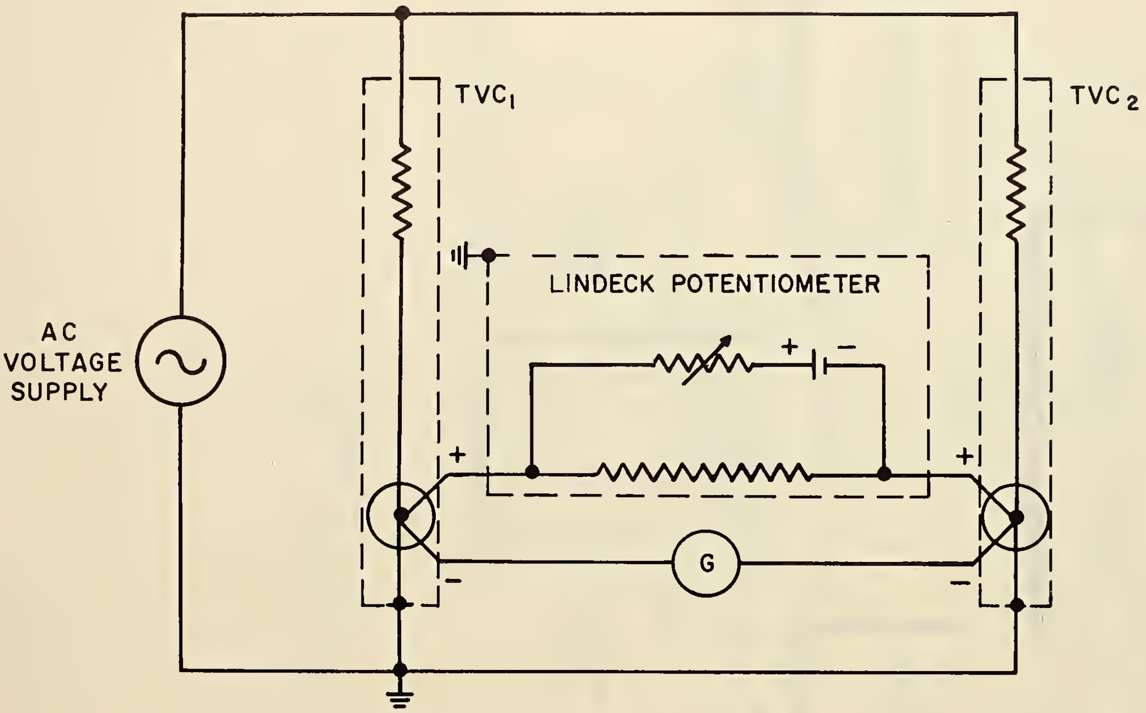
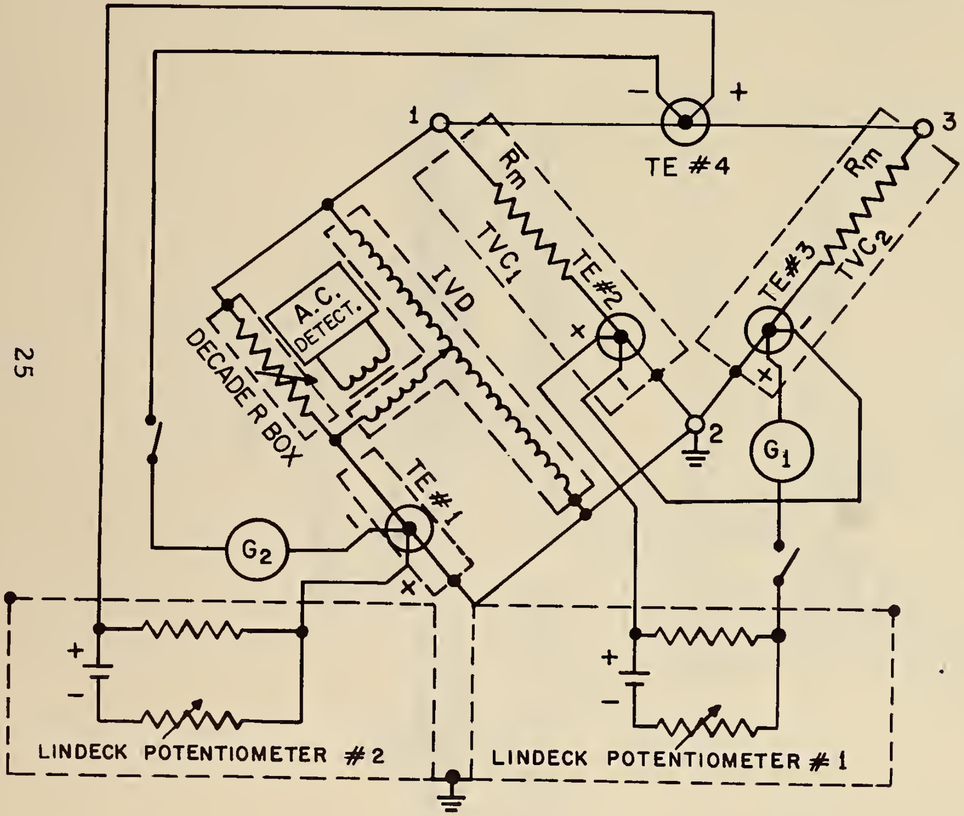


Figure 4

SCHEMATIC FOR PHASE MEASUREMENTS
± 6° TO ± 60°



TVC$_1$,TVC$_2$, 1 VOLT THERMAL VOLTAGE CONVERTERS

TE #1, TE #4, 0.6 VOLT THERMOELEMENTS

TE #2, TE #3, 0.45 VOLT THERMOELEMENTS

AC DETECTOR, TUNED AMPLIFIER AND NULL DETECTOR

DECADE R BOX, SIX DIAL LOW RESISTANCE BOX

IVD, INDUCTIVE VOLTAGE DIVIDER

R$_m$, RESISTANCE MULTIPLIER

Figure 5

SCHEMATIC FOR PHASE MEASUREMENTS
± 60° TO ± 170°

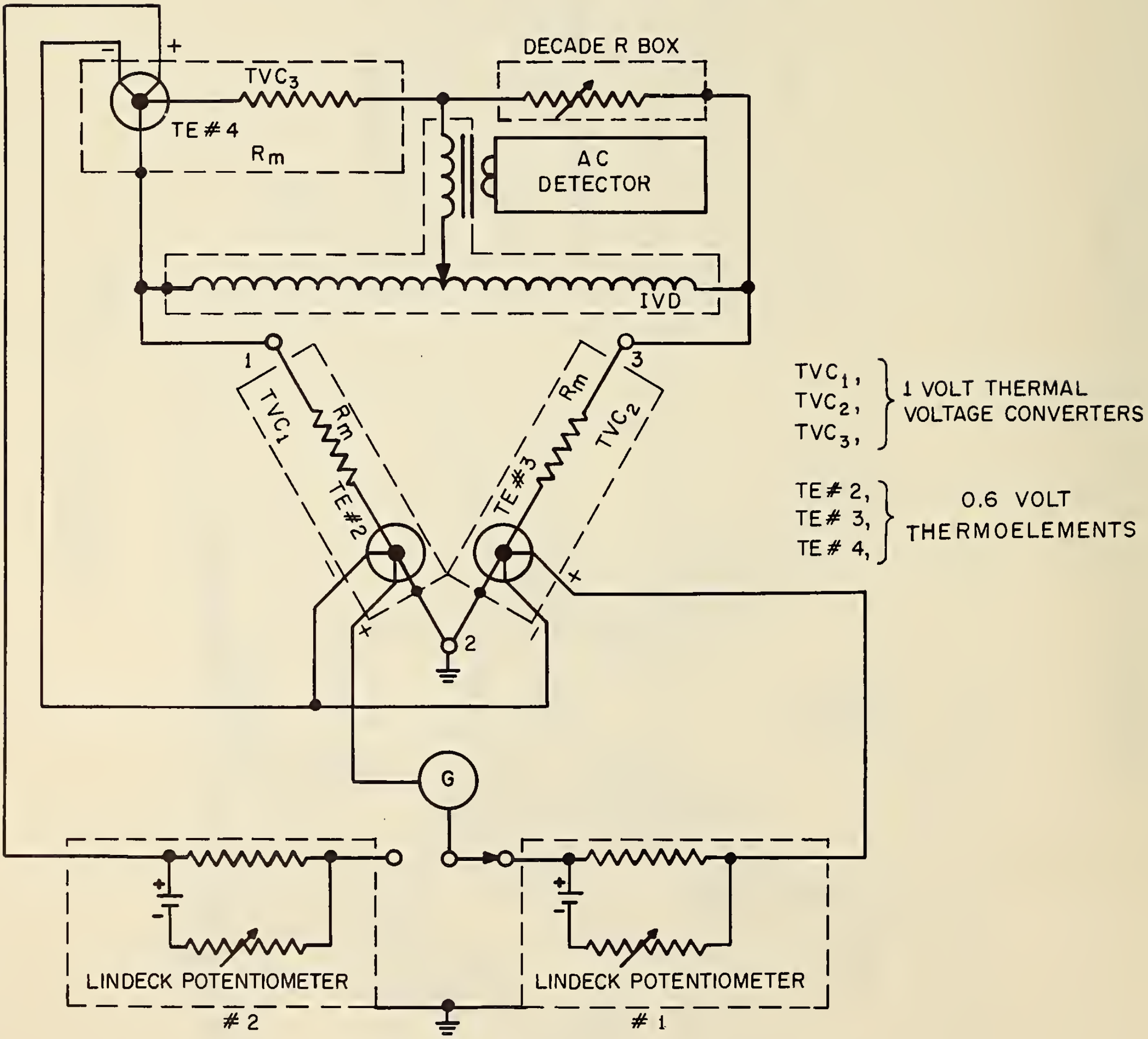
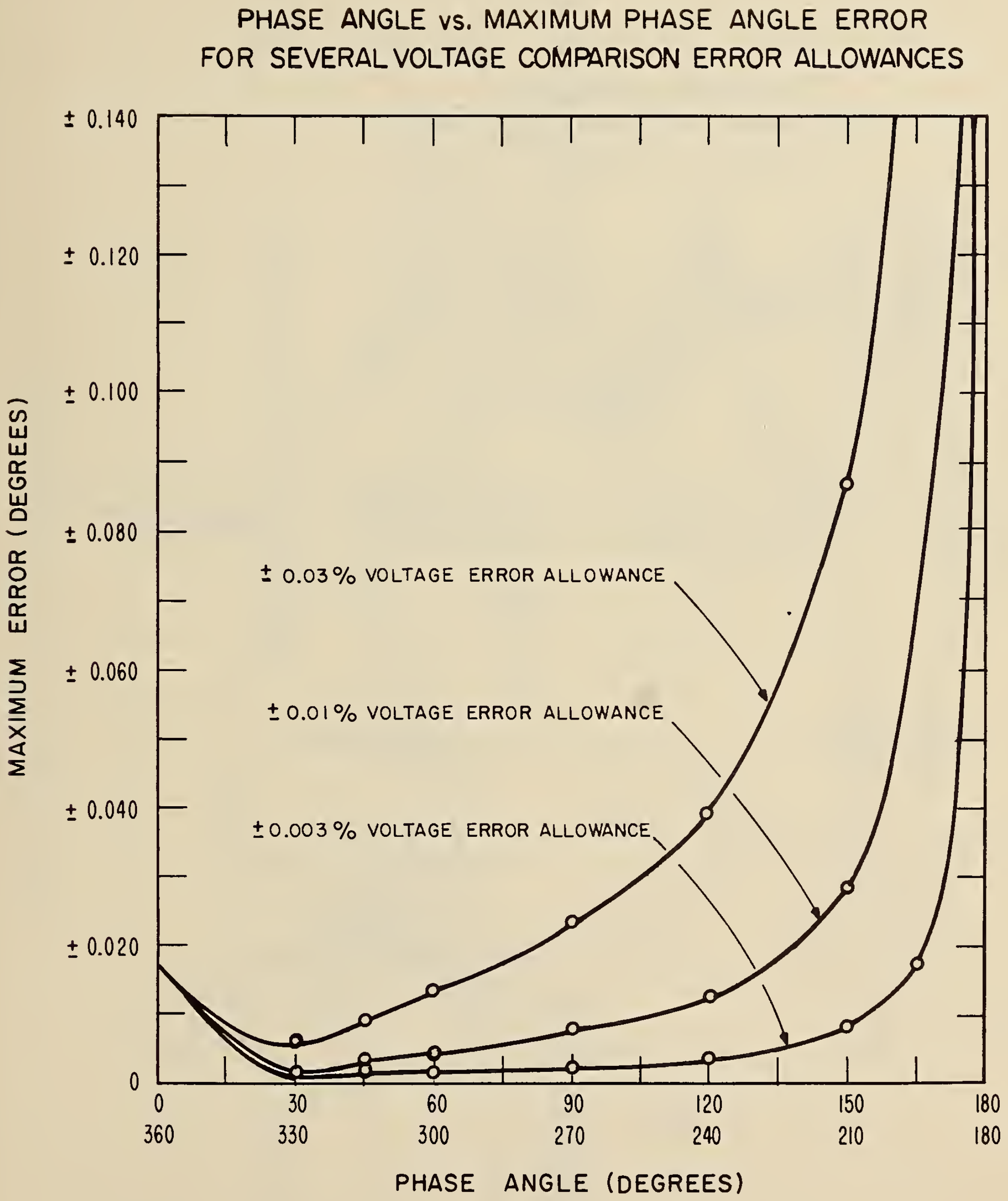


Figure 6



PHASE ANGLE vs. MAXIMUM PHASE ANGLE ERROR FOR SEVERAL VOLTAGE COMPARISON ERROR ALLOWANCES

Figure 7

SHIELDING DETAILS
FOR PHASE MEASUREMENTS
± 60° TO ± 170°

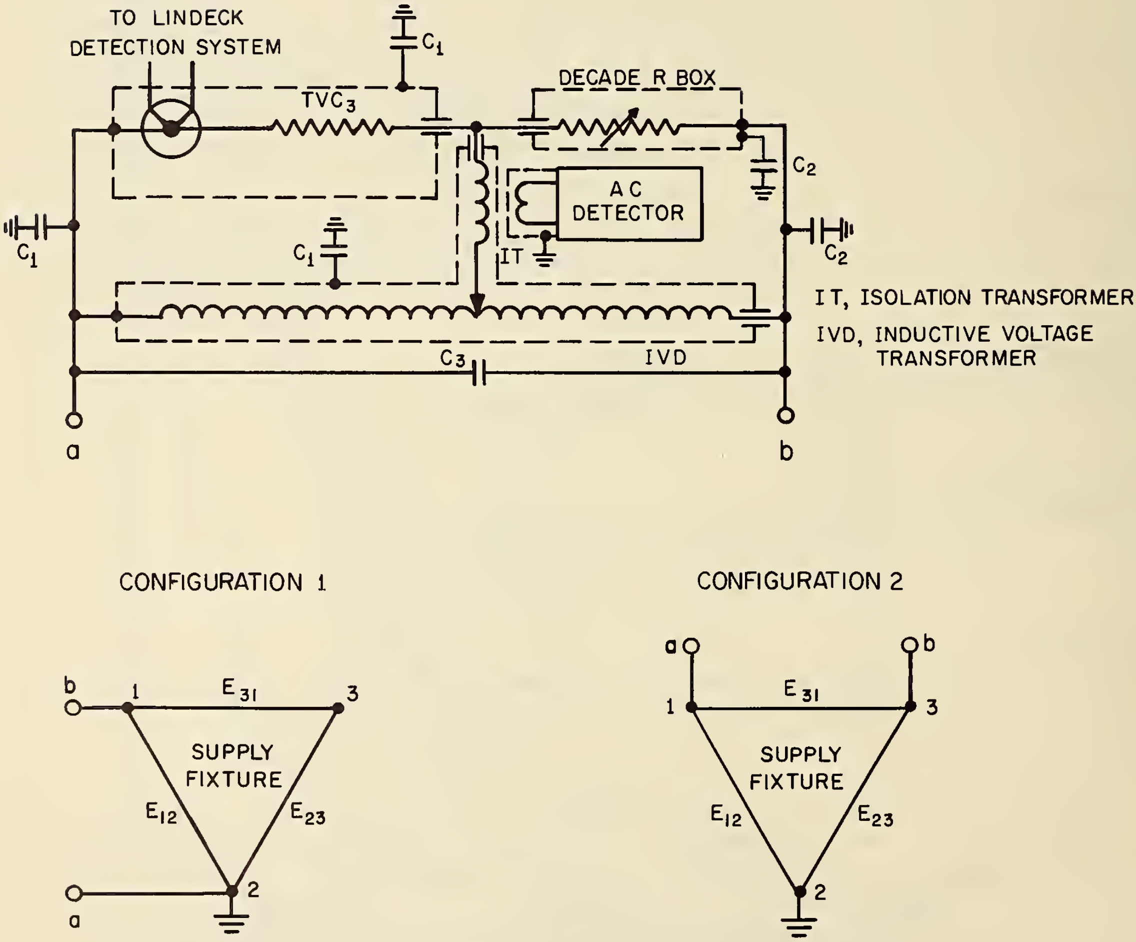
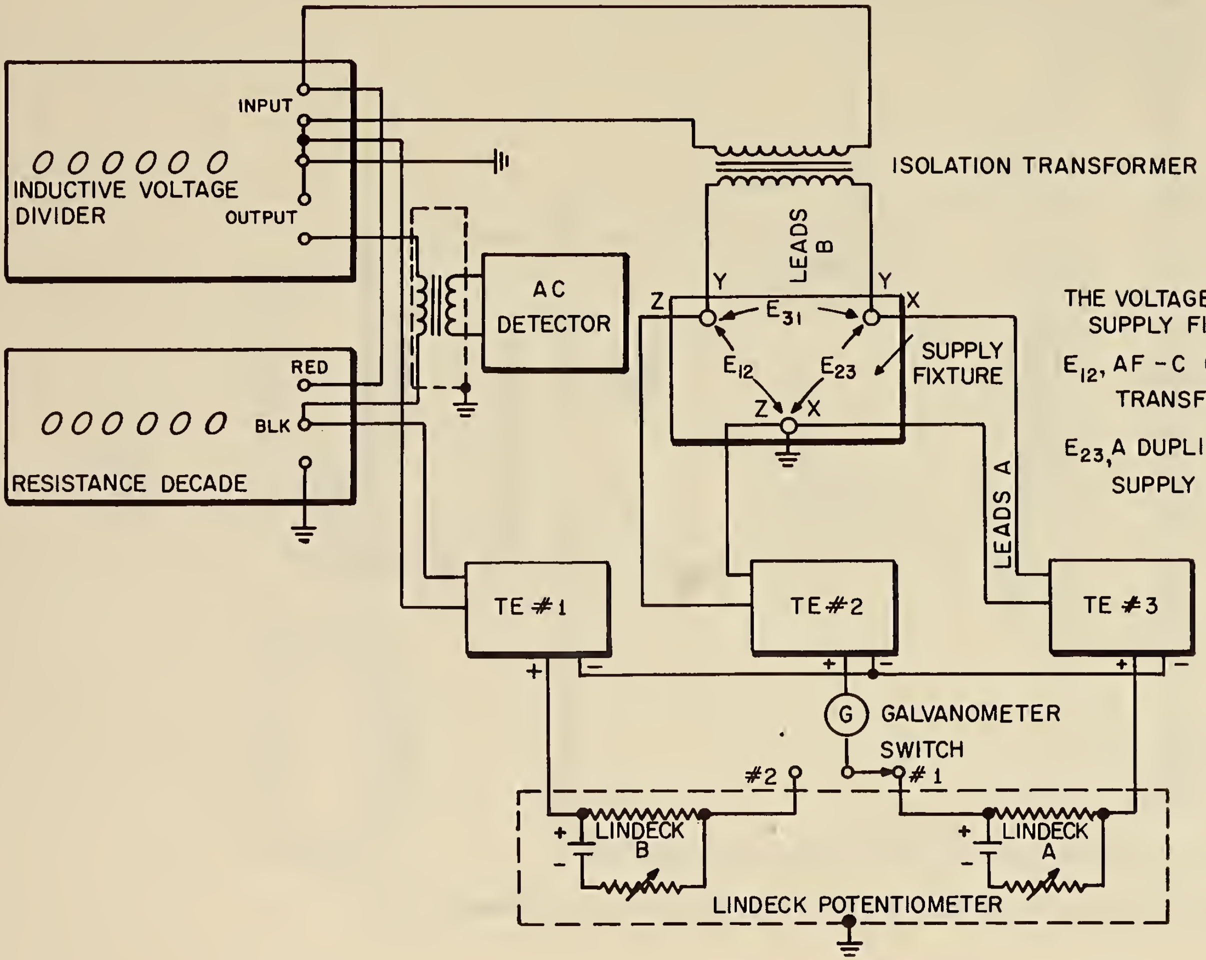


Figure 8

WIRING DIAGRAM OF EXPERIMENTAL SETUP



THE VOLTAGE SOURCES AT THE SUPPLY FIXTURE ARE :

E_{12}, AF - C ON AC - DC TRANSFER PANEL 0.5 VOLT LEVEL

E_{23}, A DUPLICATE AF - C SUPPLY TRANSFORMER 0.5 VOLT LEVEL

Figure 9

SCHEMATIC DIAGRAM OF EXPERIMENTAL SETUP

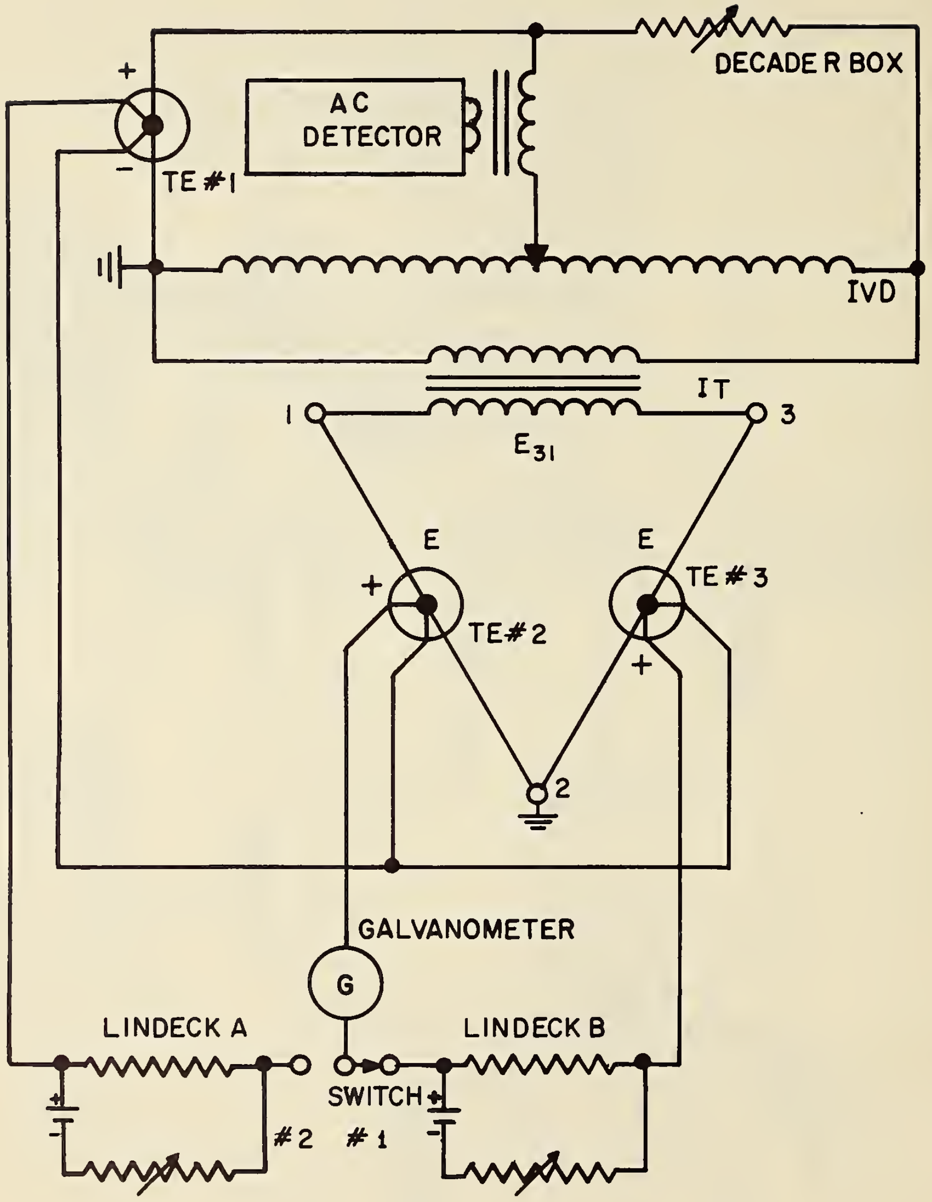


Figure 10

GPO 831-025

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