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**HYDROMAGNETIC WAVE PROPAGATION NEAR  $1c/s$   
IN THE UPPER ATMOSPHERE AND THE PROPERTIES  
AND INTERPRETATION OF Pc 1 MICROPULSATIONS**

JOHN A. DAWSON



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U. S. DEPARTMENT OF COMMERCE  
NATIONAL BUREAU OF STANDARDS

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# NATIONAL BUREAU OF STANDARDS

## *Technical Note 342*

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CONTENTS

PART I

Page

Hydromagnetic Wave Propagation near 1 c/s in the Upper Magnetosphere

1. Introduction . . . . .	1
2. Hydromagnetic Propagation in a Uniform Cold Plasma . . . . .	3
3. The Effect of Ion-Electron Collisions . . . . .	12
4. Justification of Assumptions . . . . .	15
Figure 1 . . . . .	17
Figure 2 . . . . .	19
Table 1 . . . . .	20
Table 2 . . . . .	22
Table 3 . . . . .	25
5. Propagation in a Multicomponent Plasma . . . . .	26
Figure 3 . . . . .	33
Table 4 . . . . .	35
Figure 4 . . . . .	36
Table 5 . . . . .	38
Figure 5 . . . . .	39

PART II

The Properties and Interpretation of Pc 1 Micropulsations

1. Introduction . . . . .	41
Figure 6 . . . . .	42
Figure 7 . . . . .	43
2. Properties of Pc 1's . . . . .	46
3. Theoretical Interpretation of Pc 1's . . . . .	50



Table 6 . . . . .	51
Figure 8 . . . . .	53
Figure 9 . . . . .	56
Acknowledgments . . . . .	62
References . . . . .	63





HYDROMAGNETIC WAVE PROPAGATION NEAR  $1 \text{ c/s}$  IN THE UPPER ATMOSPHERE  
AND THE PROPERTIES AND INTERPRETATION OF Pc 1 MICROPULSATIONS

by

John A. Dawson

PART I

Hydromagnetic Wave Propagation near  $1 \text{ c/s}$  in the Upper Magnetosphere

Dispersion and polarization relations are developed from basic considerations for a hydromagnetic wave at about  $1 \text{ c/s}$  propagating in a cold uniform plasma. The treatment is then extended to include the effects of ion-electron collisions. It is shown that the approximations used are valid in the earth's magnetosphere for heights between 10,000 and 50,000 km (2.6 to 8.8 earth radii). At lower elevations the plasma must be considered as nonuniform. Above 50,000 km the small amplitude assumption is invalid. Finally the theory is extended to include the effects of a multicomponent plasma.

1. Introduction

The subject of hydromagnetic wave propagation in all its complexity has been treated extensively in the literature. The purpose of this paper is to develop those aspects of the subject which are particularly apropos to the propagation of hydromagnetic waves near  $1 \text{ c/s}$  in the upper magnetosphere. It was considered worthwhile to develop the theory from elementary considerations, to keep in mind the various assumptions made in order to keep the problem tractable, and to justify their application to the magnetosphere.

The assumption of a uniform plasma is commonly made, so that the solutions will have the form of plane harmonic waves. However, as will be shown later, such an assumption is not really valid at heights below 10,000 km. In this region the plasma density changes significantly within one wavelength.

Another assumption often made is that of a cold plasma, i. e., all collisions and hence, all pressure terms and momentum exchange be-

tween classes of particles are neglected. This is valid as long as the collision frequencies are less than the wave frequency, which assumption can safely be made for the frequency range of interest for heights above 3000 km, but lower than this, ion-electron collisions become important. For the magnetosphere this assumption is less restrictive than that of a uniform plasma.

The small amplitude assumption is usually made, which allows the equations to be linearized. This condition is invalid for large pulsations as the limits of the magnetosphere are approached. However, micropulsations in the 1 c/s range seldom exceed a few gamma, and the assumption can safely be justified for heights lower than 50,000 km.

The plasma will be considered to consist only of electrons and ions. Effects of neutral particles may be ignored as long as the wave frequency is greater than the ion-neutral collision frequency. This condition is met by staying above 300 km. Since the atmosphere consists largely of hydrogen above 3000 km, only one type of ion need be considered. The plasma will be assumed to be neutral and the ions to be singly ionized, i. e.,  $n_e = n_i$ .

Thus, analysis of small amplitude plane waves in a cold plasma is sufficient to explain hydromagnetic wave propagation in those regions with heights between 10,000 and 50,000 km, but more elaborate treatments will be required elsewhere. It is this region which will be considered in this paper.

Mks rationalized units will be used in the following derivations.

Useful references in the field are Alfvén and Fälthammar [1963], Spitzer [1962], Stix [1962], and Denise and Delcroix [1963].

## 2. Hydromagnetic Propagation in a Uniform Cold Plasma

The purpose of this section will be to derive a dispersion relation, suitable for all directions of propagation, for hydromagnetic waves at frequencies around 1 c/s. The polarization properties of these waves will also be discussed.

We shall start with Maxwell's equations,

$$\nabla \times \vec{B} = \mu \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t} \quad (1)$$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad (2)$$

and define the conductivity tensor by

$$\vec{J} = \vec{\sigma} \cdot \vec{E} \quad (3)$$

The important distinction between a hydromagnetic wave and an electromagnetic wave is that the first considers the material current important while the latter emphasizes the displacement current. As will be shown in section four, the displacement current can be neglected in the magnetosphere for ultra-low frequencies. However, under the ionosphere the displacement current is important and propagation becomes electromagnetic.

Let  $\vec{B}$  be composed of a steady component,  $\vec{B}_0$ , which is derivable from a multipole potential expansion, and a time varying component,  $\vec{b}$ , which is small, i. e.,  $|\vec{b}| \ll |\vec{B}_0|$ . Neglect the displacement current. Let  $\vec{b}$ ,  $\vec{J}$ , and  $\vec{E}$  vary as  $e^{-i\omega t}$ , which states that the various plasma parameters do not change significantly during one period. Since  $\nabla \times \vec{B}_0 = \frac{\partial \vec{B}_0}{\partial t} = 0$ , Maxwell's equations become

$$\nabla \times \vec{b} = \mu \vec{J} \quad (4)$$

and

$$\nabla \times \vec{E} = i\omega \vec{b} \quad (5)$$

Take the curl of (5), and substitute from (4) and (3) to obtain the wave equation

$$\nabla \times (\nabla \times \vec{E}) = i\omega \mu (\vec{\sigma} \cdot \vec{E}) \quad (6)$$

So far nothing has been stated concerning the structure of the plasma, nor the form of the conductivity tensor, other than that it is of second order.

The conductivity tensor for a cold plasma can be obtained by considering the equations of motion for electrons and ions, and the definition of current density.

$$m_e \frac{d\vec{v}_e}{dt} = -i\omega m_e \vec{v}_e = -e(\vec{E} + \vec{v}_e \times \vec{B}_0) \quad (7)$$

$$m_i \frac{d\vec{v}_i}{dt} = -i\omega m_i \vec{v}_i = e(\vec{E} + \vec{v}_i \times \vec{B}_0) \quad (8)$$

$$\vec{J} = n_e e(\vec{v}_i - \vec{v}_e) \quad (9)$$

Let  $\vec{B}_0$  lie along the z-axis, so that (7) becomes in rectangular coordinates

$$-i\omega m_e v_{ex} = -e(E_x + v_{ey} B_0) \quad (10)$$

$$-i\omega m_e v_{ey} = -e(E_y - v_{ex} B_0) \quad (11)$$

$$-i\omega m_e v_{ez} = -eE_z \quad (12)$$

Solving for  $v_{ex}$ ,  $v_{ey}$ , and  $v_{ez}$  gives

$$v_{ex} = \frac{i\omega m_e e E_x + e^2 B_0 E_y}{e^2 B_0^2 - \omega^2 m_e^2} \quad (13)$$

$$v_{ey} = \frac{-e^2 B_0 E_x + i\omega m_e e E_y}{e^2 B_0^2 - \omega^2 m_e^2} \quad (14)$$

$$v_{ez} = -\frac{ie E_z}{\omega m_e} \quad (15)$$

Similarly,

$$v_{ix} = \frac{-i\omega m_i e E_x + e^2 B_0 E_y}{e^2 B_0^2 - \omega^2 m_i^2} \quad (16)$$

$$v_{iy} = \frac{-e^2 B_0 E_x - i\omega m_i e E_y}{e^2 B_0^2 - \omega^2 m_i^2} \quad (17)$$

$$v_{iz} = \frac{ie E_z}{\omega m_i} \quad (18)$$

Make the following approximations

$$\omega \ll \omega_e, \quad \omega_i \ll \omega_e, \quad \omega^2 \ll \omega_e \omega_i$$

where  $\omega_e = \frac{eB_0}{m_e}$  and  $\omega_i = \frac{eB_0}{m_i}$  are the gyrofrequencies. The second inequality is equivalent to  $m_e \ll m_i$ . Then from (9)

$$J_x \approx -\frac{in m_i e^2 \omega E_x}{e^2 B_0^2 - m_i^2 \omega^2} + \frac{n m_i^2 e \omega^2 E_y}{B_0 (e^2 B_0^2 - m_i^2 \omega^2)} \quad (19)$$

$$J_y \approx -\frac{n m_i^2 e \omega^2 E_x}{B_0 (e^2 B_0^2 - m_i^2 \omega^2)} - \frac{in m_i e^2 \omega E_y}{e^2 B_0^2 - m_i^2 \omega^2} \quad (20)$$

$$J_z \approx \frac{in_e e^2 E_z}{m_e \omega} \quad (21)$$

Thus, it can be seen that the conductivity is given by

$$\bar{\sigma} = \begin{bmatrix} \sigma_1 & \sigma_2 & 0 \\ -\sigma_2 & \sigma_1 & 0 \\ 0 & 0 & \sigma_o \end{bmatrix} \quad (22)$$

where

$$\sigma_o \approx \frac{in_e e^2}{m_e \omega} \quad (23)$$

$$\sigma_1 \approx -\frac{in_e m_i e^2 \omega}{e^2 B_o^2 - m_i^2 \omega^2} \quad (24)$$

$$\sigma_2 \approx \frac{n_e m_i^2 e \omega^2}{B_o (e^2 B_o^2 - m_i^2 \omega^2)} \quad (25)$$

$\sigma_1$  is often called the Pedersen conductivity, while  $\sigma_2$  is the Hall conductivity.

To obtain the dispersion relation for a plane wave, allow  $\vec{E}$  to vary spatially as  $e^{i\vec{k} \cdot \vec{r}}$ . Thus,  $\nabla X$  becomes  $i\vec{k} X$  and the wave equation

(6) becomes

$$-\vec{k} \times (\vec{k} \times \vec{E}) = i\omega\mu(\bar{\sigma} \cdot \vec{E}) \quad (26)$$

Without loss of generality, let  $\vec{k}$  lie in the z-x plane at an angle  $\theta$  with respect to  $\vec{B}_o$ , so that

$$\left. \begin{aligned} k_x &= k \sin \theta \\ k_y &= 0 \\ k_z &= k \cos \theta \end{aligned} \right\} (27)$$

Writing (26) out in rectangular coordinates using (22) and (27) gives

$$\begin{aligned}
 k^2 E_x \cos^2 \theta - k^2 E_z \sin \theta \cos \theta &= i\omega\mu(\sigma_1 E_x + \sigma_2 E_y) \\
 k^2 E_y &= i\omega\mu(-\sigma_2 E_x + \sigma_1 E_y) \\
 -k^2 E_x \sin \theta \cos \theta + k^2 E_z \sin^2 \theta &= i\omega\mu\sigma_0 E_z
 \end{aligned}
 \tag{28}$$

In order for (28) to have nontrivial solutions, the following determinant must be zero.

$$\begin{vmatrix}
 k^2 \cos^2 \theta - i\omega\mu\sigma_1 & -i\omega\mu\sigma_2 & -k^2 \sin \theta \cos \theta \\
 i\omega\mu\sigma_2 & k^2 - i\omega\mu\sigma_1 & 0 \\
 -k^2 \sin \theta \cos \theta & 0 & k^2 \sin^2 \theta - i\omega\mu\sigma_0
 \end{vmatrix} = 0
 \tag{29}$$

This yields a quadratic in  $k^2$

$$\begin{aligned}
 (\sigma_0 \cos^2 \theta + \sigma_1 \sin^2 \theta)k^4 - i\omega\mu[\sigma_0 \sigma_1 (1 + \cos^2 \theta) + (\sigma_1^2 + \sigma_2^2) \sin^2 \theta]k^2 \\
 - \omega^2 \mu^2 \sigma_0 (\sigma_1^2 + \sigma_2^2) = 0
 \end{aligned}
 \tag{30}$$

which produces the following dispersion relation

$$\frac{k^2}{\omega^2} = \frac{i\mu \left[ \frac{\sigma_0 \sigma_1 (1 + \cos^2 \theta)}{\sigma_1^2 + \sigma_2^2} + \sin^2 \theta \right] \pm \mu \sqrt{4\sigma_0^2 \sigma_2^2 \cos^2 \theta - \left( \frac{\sigma_0 \sigma_1}{\sigma_1^2 + \sigma_2^2} - 1 \right)^2 \sin^4 \theta}}{2(\sigma_0 \cos^2 \theta + \sigma_1 \sin^2 \theta)\omega}
 \tag{31}$$

From (23), (24), and (25), we have

$$\frac{\sigma_0}{\sigma_1^2 + \sigma_2^2} = - \frac{iB_0^2 (e^2 B_0^2 - m_i^2 \omega^2)}{n_e m_e m_i \omega^3} = - \frac{iB_0^2 (\omega_i^2 - \omega^2)}{n_e m_e \omega^3}
 \tag{32}$$

$$\frac{\sigma_1}{\sigma_1^2 + \sigma_2^2} = \frac{iB_o^2}{n_e m_i \omega} \quad (33)$$

$$\frac{\sigma_2}{\sigma_1^2 + \sigma_2^2} = -\frac{B_o}{n_e e} \quad (34)$$

Eliminating the conductivities, (31) becomes

$$\frac{k^2}{\omega^2} = \frac{i\mu \left[ -\frac{e^2 B_o^2 (1 + \cos^2 \theta)}{m_e m_i \omega^2} + \sin^2 \theta \right] + \mu \sqrt{-\frac{4e^2 B_o^2 \cos^2 \theta}{m_e^2 \omega^2} - \left( -\frac{e^2 B_o^2}{m_e m_i \omega^2} - 1 \right)^2 \sin^4 \theta}}{2 \left[ -\frac{iB_o^2 (\omega_i^2 - \omega^2) \cos^2 \theta}{n_e m_e \omega^3} + \frac{iB_o^2 \sin^2 \theta}{n_e m_i \omega} \right]} \quad (35)$$

Divide numerator and denominator by  $i\mu$ , introduce the plasma frequency,

$$\omega_p^2 = \frac{n_e e^2}{\epsilon m_e}, \text{ and the Alfvén velocity, } V_A = \frac{B_o}{\sqrt{\mu n_e m_i}}, \text{ and use } c = \frac{1}{\sqrt{\mu \epsilon}} \text{ to}$$

obtain

$$\frac{k^2}{\omega^2} = \frac{1 + \cos^2 \theta - \frac{\omega^2 c^2 \sin^2 \theta}{\omega_p^2 V_A^2} + \sqrt{4\omega^2 \cos^2 \theta + \left(1 + \frac{\omega^2 c^2}{\omega_p^2 V_A^2}\right)^2 \sin^4 \theta}}{2V_A^2 \left[ \left(1 - \frac{\omega^2}{\omega_i^2}\right) \cos^2 \theta - \frac{\omega^2 c^2 \sin^2 \theta}{\omega_p^2 V_A^2} \right]} \quad (36)$$

Though  $\frac{\omega^2 c^2}{\omega_p^2 V_A^2} \ll 1$  in the problem under consideration, these terms are

retained to allow propagation across the field to be considered. If  $\theta$  is not near  $\frac{\pi}{2}$ , these terms may safely be neglected. This is a general dispersion relation which will treat hydromagnetic wave propagation in those regions of the magnetosphere with heights between 10,000 and 50,000 km, for frequencies around 1 c/s. The approximations used will be justified in the next section. The approximation  $\omega \ll \omega_i$  was



not made, as this does not hold above 20,000 km.

To obtain the classical Alfvén modes, we now make the approximation  $\omega \ll \omega_i$ . Then (36) becomes, provided  $\theta$  is not near  $\frac{\pi}{2}$

$$\frac{k^2}{\omega^2} = \frac{1 + \cos^2\theta \mp \sin^2\theta}{2V_A^2 \cos^2\theta} = \begin{cases} \frac{1}{V_A^2 \cos^2\theta} & \text{Alfvén or slow mode} \\ \frac{1}{V_A^2} & \text{modified Alfvén or fast mode} \end{cases} \quad (37)$$

For low frequency waves the Alfvén mode is guided along the field lines, while the modified Alfvén mode has a isotropic phase velocity. As the approximation  $\omega \ll \omega_i$  is not a particularly good one, it will be necessary, in general, to use the more complete dispersion relation (36).

For longitudinal propagation,  $\theta = 0$ , (36) yields for the phase velocity

$$V_p^2 = \frac{\omega^2}{k^2} = V_A^2 \left( 1 \mp \frac{\omega}{\omega_i} \right) \quad (38)$$

where the upper sign corresponds to the Alfvén mode and the lower to the modified Alfvén mode. It is evident that the Alfvén mode will cease to propagate when  $\omega \geq \omega_i$ . The group velocity will be given by

$$V_g = \frac{d\omega}{dk} = \frac{V_A \left( 1 \mp \frac{\omega}{\omega_i} \right)^{3/2}}{1 \mp \frac{\omega}{2\omega_i}} \approx V_A \left( 1 \mp \frac{\omega}{\omega_i} \right) \quad (39)$$

Both the phase and group velocities of the Alfvén mode decrease with increasing frequency, while the modified wave does the opposite.

For transverse propagation, set  $\theta = \frac{\pi}{2}$  in (36), and obtain

$$\frac{\omega}{k} = \begin{cases} \frac{i\omega c}{\omega_p} & \text{Alfvén} \\ V_A & \text{modified Alfvén} \end{cases} \quad (40)$$

An interesting result is that the Alfvén mode degenerates into an evanescent plasma oscillation for this direction of propagation. The resonance angle for which this mode ceases to propagate is given by

$$\tan^2 \theta_r = \frac{\omega_p^2 V_A^2}{\omega^2 c^2} \left( 1 - \frac{\omega^2}{\omega_i^2} \right) \quad (41)$$

No resonance exists at this angle or any other angle for the modified Alfvén mode, since the numerator in (36) goes to zero at this angle when the minus sign is chosen. By applying L'Hospital's rule to (36), it can be seen that  $\lim_{\theta \rightarrow \theta_r} \frac{k^2}{\omega^2}$  remains finite and non-zero for the modified Alfvén mode.

The polarization of these waves when  $\theta = 0$  can be obtained from the y-component of (28)

$$i\omega\mu\sigma_2 E_x + (k^2 - i\omega\mu\sigma_1) E_y = 0 \quad (42)$$

Substituting for the conductivities from (24) and (25), we have for the electric polarization in the x-y plane

$$\frac{iE_x}{E_y} = \frac{eB_0}{m_i \omega} - \frac{(e^2 B_0^2 - m_i^2 \omega^2) B_0 k^2}{n_e m_i^2 e \omega^3} = \frac{\omega_i}{\omega} - \frac{(\omega_i^2 - \omega^2) V_A^2 k^2}{\omega_i \omega^3} \quad (43)$$

To determine the polarization for longitudinal propagation, substitute (38) into (43) and obtain

$$\frac{iE_x}{E_y} = \frac{\omega_i}{\omega} - \frac{\omega_i^2 - \omega^2}{\omega(\omega_i \mp \omega)} = \frac{\omega_i}{\omega} - \frac{\omega_i \mp \omega}{\omega} = \mp 1 \quad (44)$$

Again the upper sign refers to the Alfvén mode, and we can see that it is left-hand circularly polarized. The modified Alfvén wave is a right-hand circular wave.

In general these waves will be elliptically polarized. This can be seen by calculating  $\frac{i(E_x \cos \theta - E_z \sin \theta)}{E_y}$ , which will give the elliptical polarization in the plane normal to the propagation direction. From the z-component of (28) obtain

$$-k^2 E_x \sin \theta \cos \theta + (k^2 \sin^2 \theta - i\omega\mu\sigma_0)E_z = 0 \quad (45)$$

Combining this with (42), (23), (24), and (25) we have

$$\frac{i(E_x \cos \theta - E_z \sin \theta)}{E_y} = \frac{\omega_i \omega_p^2 \cos \theta \left[ 1 - \left( 1 - \frac{\omega^2}{\omega_i^2} \right) \frac{V_A^2 k^2}{\omega^2} \right]}{\omega^3 c^2 \left( \frac{k^2 \sin^2 \theta}{\omega^2} + \frac{\omega_p^2}{\omega^2 c^2} \right)} \quad (46)$$

Introducing the general dispersion relation (36) and using  $\omega^2 c^2 \ll \omega_p^2 V_A^2$ , wherever it can be made without losing the ability to treat transverse propagation, the right side of (46) becomes

$$\frac{\omega_i \omega_p^2 V_A^2 \cos \theta \left( 1 - \frac{\omega^2}{\omega_i^2} \right) \left[ \sin^2 \theta + \sqrt{\frac{4\omega^2 \cos^2 \theta}{\omega_i^2} + \left( 1 + \frac{\omega^2 c^2}{\omega_p^2 V_A^2} \right)^2 \sin^4 \theta} \right]}{\omega^3 c^2 \left\{ \left[ \sin^2 \theta + \sqrt{\frac{4\omega^2 \cos^2 \theta}{\omega_i^2} + \left( 1 + \frac{\omega^2 c^2}{\omega_p^2 V_A^2} \right)^2 \sin^4 \theta} \right] \sin^2 \theta + \frac{2V_A^2 \omega_p^2}{\omega^2 c^2} \left( 1 - \frac{\omega^2}{\omega_i^2} \right) \cos^2 \theta \right\}} \quad (47)$$

If one stays away from  $\theta = \frac{\pi}{2}$ , this rather complicated expression reduces to

$$\frac{i(E_x \cos \theta - E_z \sin \theta)}{E_y} = -\frac{\omega_i}{2\omega \cos \theta} \left( \sin^2 \theta + \sqrt{\frac{4\omega^2 \cos^2 \theta}{\omega_i^2} + \sin^4 \theta} \right) \quad (48)$$

Both (47) and (48) reduce to (44) when  $\theta = 0$ . When  $\omega \ll \omega_i$ , (48) shows that, as long as  $\theta$  is not near  $\theta = 0$ , the Alfvén mode is linearly polarized in the z-x plane, while the modified Alfvén or fast mode is linearly polarized in the y-direction.

For transverse propagation it can be seen from (47) that both modes are linearly polarized in the y-direction, though it should be remembered that the Alfvén mode does not propagate transversely.

Since one often works with magnetic records it would be desirable to express the polarization in terms of the components of the magnetic perturbation vector instead of the electric vector. The transformation can easily be made by means of  $\vec{k} \times \vec{E} = \omega \vec{b}$ , from which it follows that

$$E_x \cos \theta - E_z \sin \theta = \frac{\omega b_y}{k}$$

and

$$E_y = - \frac{\omega(b_y \cos \theta - b_z \sin \theta)}{k}$$

Thus, the electric vector in the z-x plane is associated with the magnetic y-component, and the electric y-component is associated with the magnetic z-x component with the vector aimed in the negative x-direction. With these changes all the preceding statements on polarization apply to the magnetic perturbation vector. Though  $\vec{E}$  has a component parallel to  $\vec{k}$ ,  $\vec{b}$  does not.

### 3. The Effect of Ion-Electron Collisions

It is possible that under 20,000 km the ion-electron collision frequency,  $\nu_{ie}$ , may approach the wave frequency,  $\omega$ . Thus, it is desirable to develop a technique for handling ion-electron collisions, when

their frequency is comparable to the wave frequency, but still considering  $\omega \gg \frac{m_e \nu_{ie}}{m_i}$  and  $\nu_{ie} \ll \omega_e$ .

The effects of collisions will appear as momentum exchange terms in the equations of motion, (7) and (8). If we assume that  $m_e \ll m_i$ , the rate of momentum exchange between electrons and ions is given by  $m_e \nu_{ie} (\vec{v}_e - \vec{v}_i)$  and the equations of motion become

$$-i\omega m_e \vec{v}_e = -e(\vec{E} + \vec{v}_e \times \vec{B}_0) - m_e \nu_{ie} (\vec{v}_e - \vec{v}_i) \quad (49)$$

$$-i\omega m_i \vec{v}_i = e(\vec{E} + \vec{v}_i \times \vec{B}_0) + m_e \nu_{ie} (\vec{v}_e - \vec{v}_i) \quad (50)$$

or approximately,

$$m_e (\nu_{ie} - i\omega) \vec{v}_e = -e(\vec{E} + \vec{v}_e \times \vec{B}_0) + m_e \nu_{ie} \vec{v}_i \quad (51)$$

$$-i m_i \omega \vec{v}_i = e(\vec{E} + \vec{v}_i \times \vec{B}_0) + m_e \nu_{ie} \vec{v}_e \quad (52)$$

The z-components of the above equations are

$$m_e (\nu_{ie} - i\omega) v_{ez} - m_e \nu_{ie} v_{iz} = -eE_z \quad (53)$$

$$-m_e \nu_{ie} v_{ez} - i m_i \omega v_{iz} = eE_z \quad (54)$$

Solving for the  $v_z$ 's,

$$v_{ez} = -\frac{ieE_z}{m_e(\omega + i\nu_{ie})} \quad (55)$$

$$v_{iz} = \frac{ieE_z}{m_i(\omega + i\nu_{ie})} \quad (56)$$

Then

$$J_z = n_e e (v_{iz} - v_{ez}) = \frac{in_e e^2 E_z}{\omega + i\nu_{ie}} \left( \frac{1}{m_i} + \frac{1}{m_e} \right) \approx \frac{in_e e^2 E_z}{m_e(\omega + i\nu_{ie})} \quad (57)$$

Thus,

$$\sigma_o = \frac{i n_e e^2}{m_e (\omega + i \nu_{ie})} \quad (58)$$

Similarly, the x- and y-components of (51) and (52) can be solved for  $v_{ex}$ ,  $v_{ey}$ ,  $v_{ix}$ , and  $v_{iy}$ . From these  $J_x$  and  $J_y$ , and hence  $\sigma_1$  and  $\sigma_2$ , can be found. However, if the above approximations are used, it will be found that  $\sigma_1$  and  $\sigma_2$  are not changed from (24) and (25). Hence, the main effect of introducing ion-electron collisions is to replace  $m_e$  by  $\frac{m_e (\omega + i \nu_{ie})}{\omega}$ .

The basic dispersion relation (36) now becomes

$$\frac{k^2}{\omega^2} = \frac{1 + \cos^2 \theta - \frac{\omega(\omega + i \nu_{ie})c^2 \sin^2 \theta}{\omega_p^2 V_A^2} + \sqrt{4\omega^2 \cos^2 \theta + \left[1 + \frac{\omega(\omega + i \nu_{ie})c^2}{\omega_p^2 V_A^2}\right]^2 \sin^4 \theta}}{2V_A^2 \left[ \left(1 - \frac{\omega^2}{\omega_i^2}\right) \cos^2 \theta - \frac{\omega(\omega + i \nu_{ie})c^2 \sin^2 \theta}{\omega_p^2 V_A^2} \right]} \quad (59)$$

The actual effect of ion-electron collisions on the dispersion relation is very small. Provided  $\theta$  is not near  $\frac{\pi}{2}$ , we can use the ap-

proximation  $\left| \frac{\omega(\omega + i \nu_{ie})c^2}{\omega_p^2 V_A^2} \right| \ll 1$ , and obtain the same dispersion relation as approximating  $\frac{\omega c^2}{\omega_p^2 V_A^2} \ll 1$  in (36) would give. In the case of

transverse propagation the modified Alfvén wave is unaffected, but the Alfvén wave is given by

$$\frac{k^2}{\omega^2} \approx \frac{i\omega_p}{c\omega} \left(1 - \frac{i\nu_{ie}}{2\omega}\right) = \frac{\omega_p \nu_{ie}}{2c\omega^2} + \frac{i\omega_p}{c\omega}$$

The propagation constant now has a real part and propagation will be

possible where  $\frac{\nu_{ie}}{\omega}$  becomes large enough, though attenuation will be significant. Thus, the only place where ion-electron collisions play a significant role is in the transverse propagation of the Alfvén mode.

The dispersion relation (59) bears some resemblance to the Appleton-Hartree formula for radio frequencies, especially as to the role played by collisions. However, the approximations used in deriving the respective equations are different, as the Appleton-Hartree relation is for much higher frequencies than (59).

#### 4. Justification of Assumptions

It will now be worthwhile to tabulate the various plasma parameters and justify the assumptions used in the preceding sections. Four models of the magnetosphere will be considered, corresponding to day and night, and maximum and minimum of the sunspot cycle. The basic data has been taken from Prince and Bostick [1964].

In order to derive a dispersion relation for the Alfvén modes, it was necessary to assume that  $\vec{b}$ ,  $\vec{E}$ ,  $\vec{J}$ , and  $\vec{v}$  vary spatially as  $e^{i\vec{k} \cdot \vec{r}}$ , which implies that the plasma is uniform. A reasonable criterion for uniformity is that  $|\vec{k}|$  must not change by more than ten per cent within one wavelength

$$\frac{1}{|\vec{k}|} \frac{d|\vec{k}|}{ds} \leq \frac{0.1}{\lambda} \quad \text{or} \quad \left| \frac{dV}{ds} \right| \leq 0.1 f$$

where  $V$  is the phase velocity,  $s$ , the path length along the direction of propagation, and  $f$ , the wave frequency.

The first case to be considered is that of wave propagating radially in the equatorial plane at the Alfvén velocity. Here the

requirement is stated simply as

$$\left| \frac{dV_{Ae}}{dr} \right| \leq 0.1 f$$

where  $V_{Ae}$  is the equatorial Alfvén velocity and  $r$  the geocentric height.

Figure 1 shows the magnetosphere divided into uniform and nonuniform regions. As the frequency becomes higher more of the magnetosphere can be considered uniform. For frequencies greater than 45 c/s, all of the magnetosphere above 500 km is uniform. If the wave period is greater than 100 sec, none of the magnetosphere can be considered uniform. Off the equatorial plane the Alfvén velocity will be increased by the factor  $\sqrt{1 + 3 \sin^2 \lambda}$ , where  $\lambda$  is the geomagnetic latitude, and the severity of the restriction will be increased. The restriction will be relaxed if the propagation direction is different from that of the gradient of the Alfvén velocity.

A more realistic type of propagation is that of an Alfvén wave traveling along a field line. If the field is assumed to be a dipole field, the equation of a field line is

$$r = r_e \cos^2 \lambda$$

Then, if  $s$  is the arc length along the field line

$$\frac{ds}{dr} = \sqrt{1 + r^2 \frac{d\lambda}{dr}^2} = \frac{\sqrt{1 + 3 \sin^2 \lambda}}{2 \sin \lambda}$$

and

$$\frac{dV_A}{ds} = \frac{dr}{ds} \frac{dV_A}{dr} = \frac{dr}{ds} \sqrt{1 + 3 \sin^2 \lambda} \frac{dV_{Ae}}{dr} + \frac{3V_{Ae} \sin \lambda \cos \lambda}{\sqrt{1 + 3 \sin^2 \lambda}} \frac{d\lambda}{dr}$$



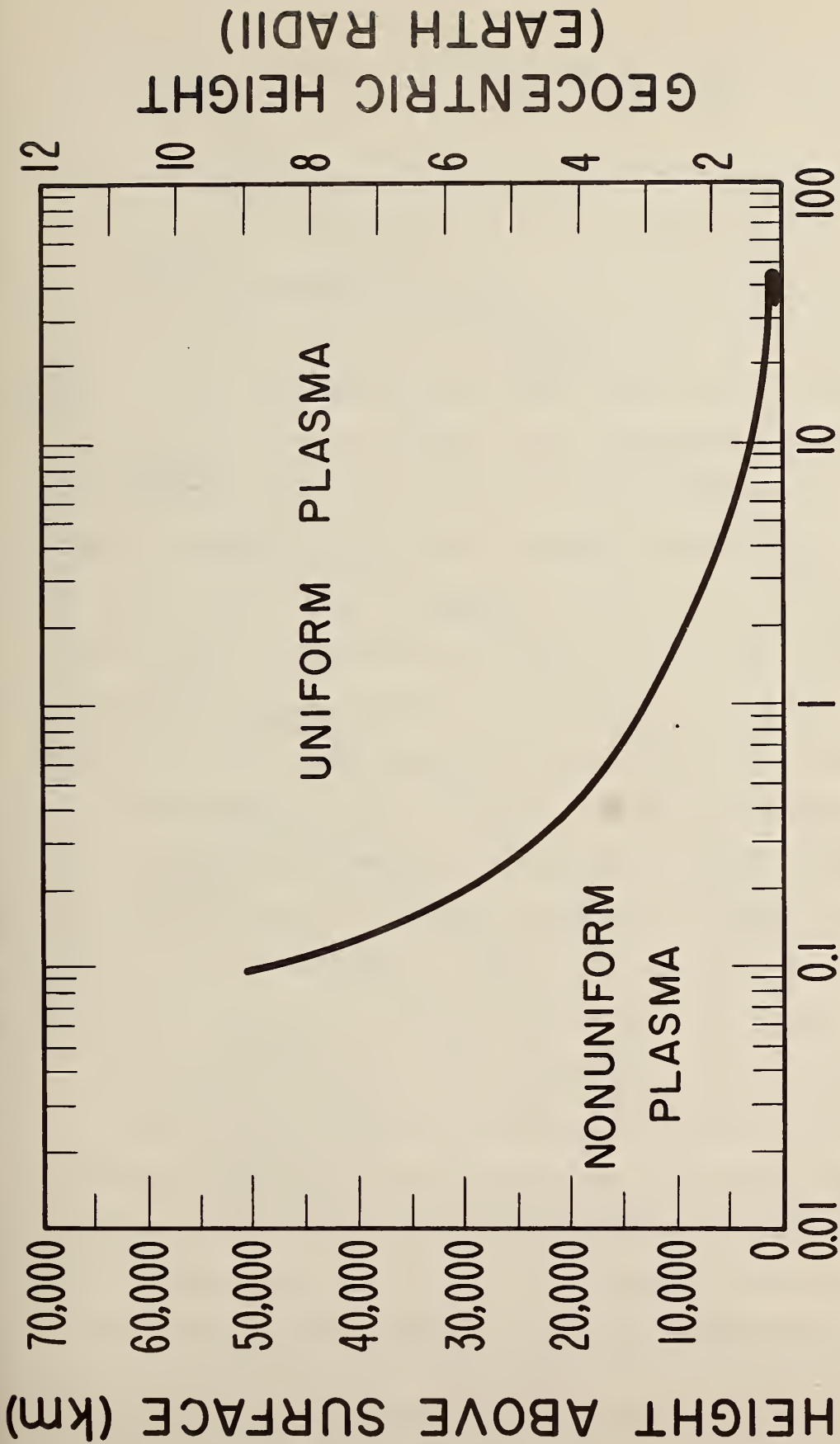


Figure 1 - The magnetosphere divided into uniform and non-uniform regions for a hydromagnetic wave propagating radially in the equatorial plane. The criterion for uniformity is that  $|\vec{k}|$  shall change by less than 10 per cent within one wavelength.

$$\frac{dV_A}{ds} = 2 \sin \lambda \frac{dV_{Ae}}{dr} - \frac{3V_{Ae} \sin \lambda \cos^2 \lambda}{r(1 + 3 \sin^2 \lambda)} \leq 0.1 f$$

The boundary of this restriction is shown on a polar plot (fig. 2) for several frequencies. Near the equator the boundary is closer to the earth, since the field lines there are almost perpendicular to the velocity gradient. But toward the poles the field lines are close to being parallel to the velocity gradient and the gradient itself is greater, which pushes the boundary away from the earth. The plot is to be interpreted as representing the inner boundary of the uniform plasma for a wave with the specified frequency. The values used in figures 1 and 2 are given in table 1. They are an average of day, night, sunspot maximum, and minimum conditions, and are merely typical values. Though the Alfvén velocity at a given point may vary by a factor of ten, the change in the plots will be relatively small. At any rate, it is clear that the assumption of a uniform plasma for waves with frequencies around 1 c/s is not really valid over much of the magnetosphere. It is necessary to make this assumption in order to derive a dispersion relation and the polarization properties for Alfvén waves, but it should not be surprising that these waves show observed properties which cannot be explained by, or are inconsistent with, the simple theory derived in the foregoing sections. In particular, reflection and refraction of energy would be expected if the plasma parameters have large gradients.

The assumption of a collisionless plasma depends mostly upon the ion-electron and electron-electron collision frequencies, which in turn depend on the electron temperature, a quantity which is largely spec-

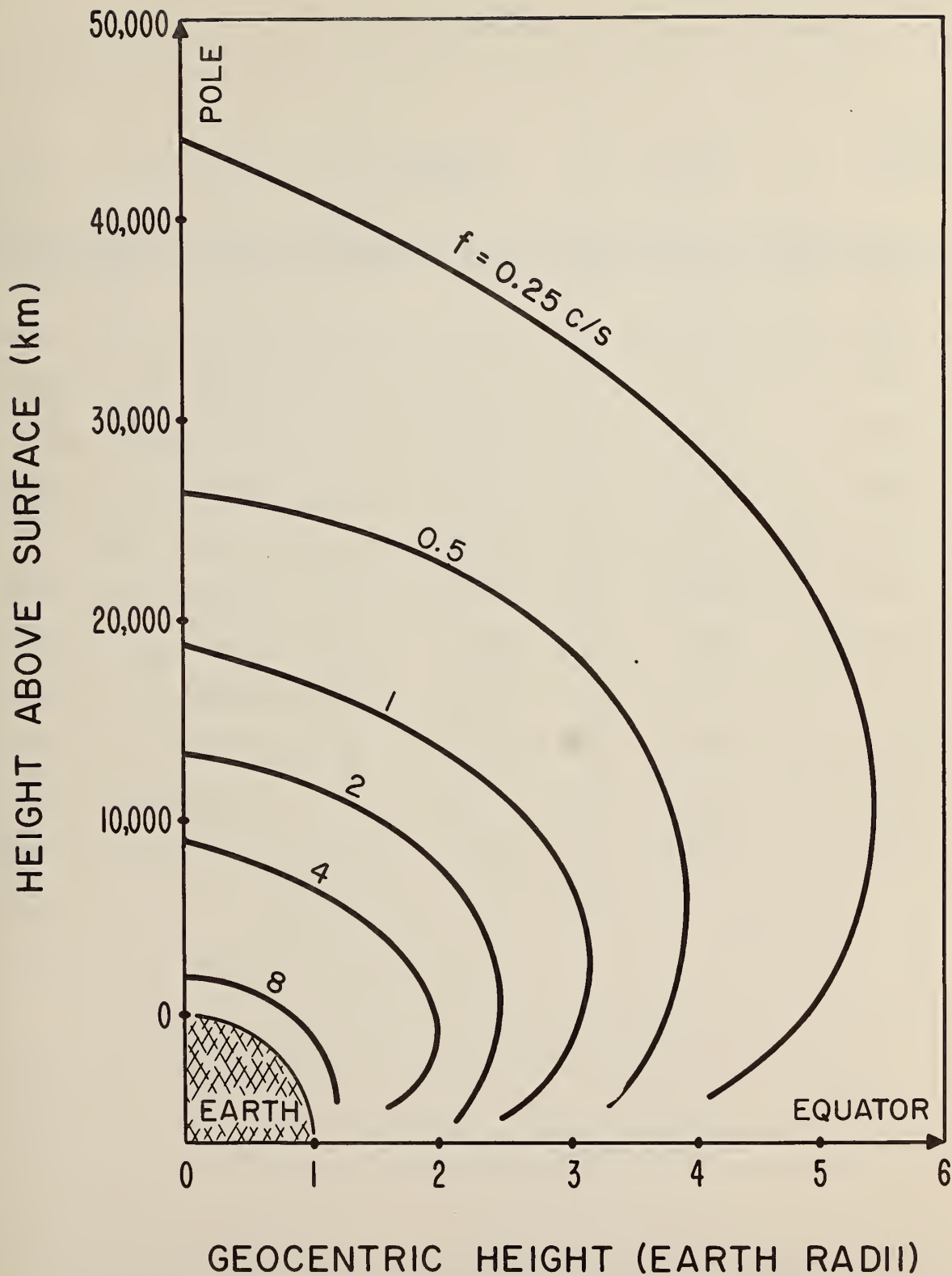


Figure 2 - Uniformity conditions for a hydromagnetic wave propagating along a field line. The plasma beyond the curve for a given frequency may be considered uniform for that frequency. A dipole field is assumed.

TABLE 1

Typical Values for the Gradient of the Equatorial Alfvén Velocity

Height (km)	Geocentric height (km)	$V_{Ae}$ (km/sec)	$\frac{dV_{Ae}}{dr}$ (sec <sup>-1</sup> )	$\frac{V_{Ae}}{r}$ (sec <sup>-1</sup> )
1000	7370	2020	+ 3.80	0.274
1500	7870	3570	+ 2.00	0.454
2000	8370	4020	+ 0.437	0.480
3000	9370	4200	- 0.156	0.448
4000	10,370	3720	- 0.469	0.359
5000	11,370	3270	- 0.418	0.288
7000	13,370	2490	- 0.330	0.186
10,000	16,370	1620	- 0.178	0.0990
15,000	21,370	1060	- 0.0822	0.0496
20,000	26,370	799	- 0.0357	0.0303
30,000	36,370	526	- 0.0217	0.0145
40,000	46,370	366	- 0.0127	0.00789

ulative. Nicolet [1963] gives the ion-electron collision frequency as

$$\nu_{ie} = \left( 59.1 + 4.18 \log \frac{T_e^3}{n_e} \right) \frac{n_e \times 10^{-6}}{T_e^{1.5}}$$

when  $\frac{\epsilon \kappa T_e}{2 n_e v_s} \gg 1$ . The great problem here is deciding what  $T_e$  should be.

If we assume that it is the same as the ion temperature, and that this is  $1500^\circ$  K at sunspot maximum and  $1000^\circ$  at minimum, we obtain the results displayed in table 2. But there are reasons to believe that the actual electron temperatures are much higher than what are assumed here.

Chapman [1960] believes that the solar corona plays an important role in determining the temperature of the upper atmosphere. Best estimates of the coronal temperature at the distance of the earth from the sun are between  $50,000^\circ$  and  $100,000^\circ$  K. With conduction of heat downward, a smooth transition from coronal to ionospheric temperatures would be expected. It is not unreasonable to expect temperatures in excess of  $10,000^\circ$  K above 10,000 km. This would reduce the values in table 2 by a factor of at least 20 and certainly make the collision frequency negligible above 4000 km.

According to Liemohn and Scarf [1962] evidence from cutoff frequencies observed in whistler data indicate temperatures on the order of  $100,000^\circ$  K in the outer magnetosphere. Their calculations are based on the assumption that the cutoff is controlled by thermal broadening of the cyclotron resonance.

Scarf [1962] also used the cutoffs observed in micropulsation data

TABLE 2

Ion-electron Collision Frequencies Based on the Low Temperature Model

Height (km)	Geocentric height (earth radii)	Day Max (sec <sup>-1</sup> )	Day Min (sec <sup>-1</sup> )	Night Max (sec <sup>-1</sup> )	Night Min (sec <sup>-1</sup> )
1000	1.16	75.2	27.6	20.6	6.96
1500	1.24	22.0	11.4	8.96	3.84
2000	1.31	12.4	7.47	6.19	2.93
3000	1.47	7.27	4.38	4.15	2.10
4000	1.63	5.02	3.07	3.22	1.63
5000	1.78	3.86	2.35	2.60	1.32
7000	2.10	2.50	1.52	1.77	0.932
10,000	2.57	1.44	0.876	1.02	0.621
15,000	3.36	0.683	0.413	0.483	0.291
20,000	4.15	0.353	0.215	0.249	0.153
30,000	5.71	0.125	0.0759	0.0884	0.0536
40,000	7.28	0.0601	0.0365	0.0427	0.0264
50,000	8.85	0.0349	0.0212	0.0246	0.0150

to support the existence of high temperatures. He argues that the thermal motion of the ions will have a Larmor radius associated with it, which will determine the smallest wavelength which can propagate. Since few micropulsations are observed with frequencies higher than 3 c/s, temperatures of  $10,000^{\circ}$  to  $100,000^{\circ}$  K would be expected.

These conclusions support Chapman's hypothesis that heat is being conducted downward from an extended solar corona to the ionosphere. More commonly, the upper magnetosphere or exosphere is assumed to be isothermal. Above 500 km disassociation and recombination is assumed to be negligible and diffusion is the most important process at work. Evidence from various sources indicate the temperature at the 500 km level to lie between  $1000^{\circ}$  and  $2000^{\circ}$  K [U. S. Standard Atmosphere, 1962; Prince and Bostick, 1964]. Hence, under the foregoing assumptions these temperatures would be expected to pervade the exosphere. The crux of the argument lies in deciding the importance of conduction from a hot solar corona.

Even if the ion-electron collision frequency approaches the wave frequency, the results of the last section indicate that the theory is not greatly affected by its inclusion.

Some comment should be made about electron-electron collisions, or in other words, the electron gas pressure. In the first chapter of his book, Stix [1962] discusses briefly some of the effects. To use one his results, pressure terms will not be important if

$$\frac{n_e v_A^2}{\kappa T_e \omega_p^2} \ll 1$$

where  $\kappa$  is Boltzmann's constant. This condition is easily met.

Table 3 tabulates the various parameters appearing in equations (36) and (46) for the equatorial plane.  $B_0$ ,  $\omega_i$ ,  $\omega_e$ ,  $\sqrt{\omega_e \omega_i}$ , and  $V_A$  should be increased by the factor  $\sqrt{1 + 3 \sin^2 \lambda}$  for locations with latitude  $\lambda$ .  $B_0$  was calculated for a dipole field,  $B_0 = \frac{8.1 \times 10^{15}}{r^3}$ , the ion gyrofrequency from  $\omega_i = \frac{eB_0}{m_1 M} = 9.66 \times 10^7 \frac{B_0}{M}$ , where  $M$  is the mean molecular weight and  $m_1$  is the mass of a unit molecular weight. Above 2000 km  $M$  is always 1.

If  $\omega$  is on the order of a few rad/sec, it is clear that  $\omega \ll \omega_e$  and  $\omega \ll \sqrt{\omega_e \omega_i}$  is always satisfied. The latter inequality also guar-

antees that  $\frac{\omega^2 c^2}{\omega_p^2 V_A^2} \ll 1$ , since  $\omega_e \omega_i = \frac{V_A^2 \omega_p^2}{c^2}$ .

In section 3 we used the approximations  $\frac{m_e \nu_{ie}}{m_i} \ll \omega$ , and  $\nu_{ie} \ll \omega_e$ .

The largest value of  $\nu_{ie}$  appearing in table 2 is 75.2, which means that

$\frac{m_e \nu_{ie}}{m_i} \leq 0.04$  rad/sec for heights above 1000 km, which is much smaller

than the wave frequencies of interest. Comparison of tables 2 and 3 supports the second inequality.

To justify  $\frac{\epsilon \kappa T_e}{e n_e^{1/3}} \gg 1$  in the derivation of the ion-electron col-

lision frequency, note that the smallest possible value of  $T_e$  is  $1000^\circ$  K, and the largest value of  $n_e$  above 1000 km is  $8.2 \times 10^{10} \text{ m}^{-3}$ . Thus

$\frac{\epsilon \kappa T_e}{e n_e^{1/3}} \gg 10^3$  and the condition is met.



TABLE 3

## Some Magnetospheric Parameters

Height (km)	$B_0$ (Wb/m <sup>2</sup> )	$\omega_i$ (rad/s)		$\omega_e$ (rad/s)	$\sqrt{\omega \omega_i}$ (rad/s)		$V_A$ (m/s)				$\omega_p$ (rad/s)			
		Max	Min		Max	Min	Day		Night		Day		Night	
							Max	Min	Max	Min	Max	Min	Max	Min
1000	$2.02^{-5}$	124	848	$3.55^6$	$2.10^4$	$5.49^4$	$3.89^5$	$2.28^6$	$7.64^5$	$4.66^6$	$2.86^7$	$7.15^6$	$8.24^6$	$3.51^6$
1500	$1.66^{-5}$	150	1600	$2.92^6$	$2.10^4$	$6.85^4$	$7.32^5$	$4.50^6$	$1.16^6$	$7.88^6$	$1.52^7$	$4.53^6$	$5.35^6$	$2.59^6$
2000	$1.38^{-5}$	475	1330	$2.43^6$	$3.40^4$	$5.70^4$	$1.60^6$	$4.66^6$	$2.29^6$	$7.54^6$	$6.34^6$	$3.64^6$	$4.42^6$	$2.25^6$
3000	$9.84^{-6}$	951	951	$1.73^6$	$4.06^4$	$4.06^4$	$2.52^6$	$4.37^6$	$3.36^6$	$6.57^6$	$4.83^6$	$2.76^6$	$3.60^6$	$1.89^6$
4000	$7.23^{-6}$	698	698	$1.27^6$	$2.96^4$	$2.96^4$	$2.24^6$	$3.85^6$	$2.81^6$	$5.98^6$	$3.98^6$	$2.30^6$	$3.16^6$	$1.66^6$
5000	$5.47^{-6}$	528	528	$9.62^5$	$2.26^4$	$2.26^4$	$1.94^6$	$3.36^6$	$2.37^6$	$5.40^6$	$3.47^6$	$2.00^6$	$2.83^6$	$1.49^6$
7000	$3.36^{-6}$	325	325	$5.92^5$	$1.39^4$	$1.39^4$	$1.49^6$	$2.58^6$	$1.78^6$	$4.10^6$	$2.82^6$	$1.60^6$	$2.32^6$	$1.24^6$
10,000	$1.84^{-6}$	178	178	$3.24^5$	$7.59^3$	$7.59^3$	$1.08^6$	$1.87^6$	$1.29^6$	$2.24^6$	$2.09^6$	$1.20^6$	$1.75^6$	$1.01^6$
15,000	$8.27^{-7}$	79.9	79.9	$1.46^5$	$3.41^3$	$3.41^3$	$7.15^5$	$1.24^6$	$8.54^5$	$1.44^6$	$1.42^6$	$8.18^5$	$1.19^6$	$6.84^5$
20,000	$4.40^{-7}$	42.5	42.5	$7.74^4$	$1.82^3$	$1.82^3$	$5.34^5$	$9.24^5$	$6.38^5$	$1.10^6$	$1.01^6$	$5.84^5$	$8.45^5$	$4.90^5$
30,000	$1.69^{-7}$	16.3	16.3	$2.97^4$	$6.96^2$	$6.96^2$	$3.51^5$	$6.08^5$	$4.17^5$	$7.26^5$	$5.92^5$	$3.42^5$	$4.95^5$	$2.86^5$
40,000	$8.12^{-8}$	7.84	7.84	$1.43^4$	$3.35^2$	$3.35^2$	$2.45^5$	$4.25^5$	$2.92^5$	$5.02^5$	$4.07^5$	$2.35^5$	$3.41^5$	$1.98^5$
50,000	$4.53^{-8}$	4.38	4.38	$7.97^3$	$1.87^2$	$1.87^2$	$1.61^5$	$3.15^5$	$2.16^5$	$3.74^5$	$3.08^5$	$1.78^5$	$2.58^5$	$1.48^5$

Note.  $2.02^{-5}$  means  $2.02 \times 10^{-5}$

The assumption,  $\frac{n_e e^2 V_A^2}{T_e \omega_p^2} \ll 1$ , allowed electronic pressure terms to be neglected. Using the largest possible Alfvén velocity and the smallest temperature, we find that  $\frac{n_e e^2 V_A^2}{T_e \omega_p^2} < 4 \times 10^{-8}$ .

At the beginning of the derivation, the displacement current was neglected, which is equivalent to stating that  $\frac{V_A^2}{c^2} \ll 1$ . To see this, note that we want  $\frac{\epsilon \omega E}{J} \ll 1$ . From (26),  $|k^2 E| = |\omega \mu J|$ . Thus we need

$$\left| \frac{\mu \epsilon \omega^2}{k^2} \right| \ll 1 \text{ or } \frac{V_A^2}{c^2} \ll 1$$

## 5. Propagation in a Multicomponent Plasma

So far the treatment has only considered a plasma consisting of one kind of ion. When several species are present, it is valid to consider all the ions as having an average ionic mass,  $\bar{m}_i = \frac{\sum_i n_i m_i}{\sum_i n_i}$ , provided the approximation  $\omega \ll \omega_i$  can be made for each of the ion species. Fortunately for geomagnetic micropulsation studies, ions heavier than hydrogen exist only below 3000 km and in this region all the ion gyrofrequencies stay above 100 rad/sec, well above the normal range of micropulsations. Though micropulsation theory is not concerned with multicomponent propagation and the subject has been treated in depth by Smith and Brice [1964], some of the simpler aspects of the theory are included here because of the ease with which the foregoing treatment can be extended to include multicomponent effects.

Consider the plasma to consist of electrons and  $j$  kinds of singly charged positive ions. Negative ions and multiply charged ions are rare in the ionosphere and can be neglected. The same dispersion relation (31) will still be valid, but it will be necessary to calculate new values for the conductivities. With the realization that equation (8) now represents  $j$  equations, one for each of the ion species, the same equations of motion can be used. The definition of current density must now be written as  $\vec{J} = e(\sum n_i \vec{v}_i - n_e \vec{v}_e)$  and the neutrality condition becomes  $n_e = \sum n_i$ . Throughout this section all summations will be over  $i$  and will run from 1 to  $j$ . The approximations  $\omega \ll \omega_e$  and  $\omega_i \ll \omega_e$  will still be used.

Following a procedure similar to that used in section 2, we obtain

$$\sigma_0 = \frac{ie^2}{\omega} \left( \sum \frac{n_i}{m_i} + \frac{n_e}{m_e} \right) \approx \frac{in_e e^2}{m_e \omega} \quad (60)$$

$$\sigma_1 = -ie^2 \omega \left( \sum \frac{n_i m_i}{e^2 B_0^2 - \omega^2 m_i^2} + \frac{n_e m_e}{e^2 B_0^2 - \omega^2 m_e^2} \right) \approx -ie^2 \omega \sum \frac{n_i}{m_i (\omega_i^2 - \omega^2)} \quad (61)$$

and

$$\sigma_2 = e^3 B_0 \left( \sum \frac{n_i}{e^2 B_0^2 - \omega^2 m_i^2} - \frac{n_e}{e^2 B_0^2 - \omega^2 m_e^2} \right) \approx \frac{e\omega^2}{B_0} \sum \frac{n_i}{\omega_i^2 - \omega^2} \quad (62)$$

Making use of the identity

$$\left( \sum \frac{n_i \omega_i}{\omega_i^2 - \omega^2} \right)^2 - \left( \sum \frac{n_i \omega}{\omega_i^2 - \omega^2} \right)^2 = \sum \frac{n_i}{\omega_i - \omega} \sum \frac{n_i}{\omega_i + \omega} \quad (63)$$

we obtain

$$\sigma_1^2 + \sigma_2^2 = -\frac{e^2 \omega^2}{B_0^2} \sum \frac{n_i}{\omega_i - \omega} \sum \frac{n_i}{\omega_i + \omega} \quad (64)$$

If we substitute for the conductivities in (31) and divide the numerator and denominator by

$$- \frac{i\omega_p^2 B_o^2 \sum \frac{n_i}{m_i(\omega_i^2 - \omega^2)}}{c^2 \omega^2 \sum \frac{n_i}{\omega_i - \omega} \sum \frac{n_i}{\omega_i + \omega}}$$

we obtain a new dispersion relation, equation (65) on next page.

By introducing a composite ion gyrofrequency defined as

$$\Omega = \frac{eB_o \sum \frac{n_i}{m_i(\omega_i^2 - \omega^2)}}{\sum \frac{n_i}{\omega_i^2 - \omega^2}} = \frac{\sum \frac{n_i \omega_i}{\omega_i^2 - \omega^2}}{\sum \frac{n_i}{\omega_i^2 - \omega^2}} \quad (66)$$

and a new Alfvén velocity defined as

$$V_A^2 = \frac{B_o^2 \sum \frac{n_i}{m_i(\omega_i^2 - \omega^2)}}{\mu \sum \frac{n_i}{\omega_i - \omega} \sum \frac{n_i}{\omega_i + \omega}} \quad (67)$$

the dispersion relation (65) can be put into the same form as (36)

$$\frac{k^2}{\omega^2} = \frac{1 + \cos^2 \theta - \frac{\omega^2 c^2 \sin^2 \theta}{\omega_{pA}^2} + \sqrt{4\omega^2 \cos^2 \theta + \left(1 + \frac{\omega^2 c^2}{\omega_{pA}^2}\right)^2 \sin^4 \theta}}{2V_A^2 \left[ \left(1 - \frac{\omega^2}{\Omega^2}\right) \cos^2 \theta - \frac{\omega^2 c^2 \sin^2 \theta}{\omega_{pA}^2} \right]} \quad (68)$$

With the above substitutions and some algebraic manipulation the polarization equation

$$\frac{i(E_x \cos \theta - E_z \sin \theta)}{E_y} = \frac{i\sigma_o \cos \theta (k^2 - i\omega\mu\sigma_1)}{\sigma_2 (k^2 \sin^2 \theta - i\omega\mu\sigma_o)} \quad (69)$$

$$\begin{aligned}
& \frac{k^2}{\omega^2} = \frac{1 + \cos \theta - \frac{\omega^2 c^2 \mu \sum \frac{n_i}{\omega_i - \omega} \sum \frac{n_i}{\omega_i + \omega} \sin^2 \theta}{\omega_{p0}^2} - \frac{2 \omega_{p0}^2}{m_i (\omega_i^2 - \omega^2)} + \sqrt{\frac{4 \omega^2 \left( \sum \frac{n_i}{\omega_i^2 - \omega} \right)^2 \cos^2 \theta}{e^{2B_c} \left[ \sum \frac{n_i}{m_i (\omega_i^2 - \omega^2)} \right]^2} + \frac{1 + \frac{\omega^2 c^2 \mu \sum \frac{n_i}{\omega_i - \omega} \sum \frac{n_i}{\omega_i + \omega} \sin^2 \theta}{\omega_{p0}^2} - \frac{2 \omega_{p0}^2}{m_i (\omega_i^2 - \omega^2)}}{\omega_{p0}^2} \sin^4 \theta}}{\frac{2 \omega_{p0}^2}{\sum \frac{n_i}{\omega_i - \omega} \sum \frac{n_i}{\omega_i + \omega}} \left[ \frac{1 - \frac{\omega^2 \left( \sum \frac{n_i}{\omega_i^2 - \omega} \right)^2}{e^{2B_0} \left[ \sum \frac{n_i}{m_i (\omega_i^2 - \omega^2)} \right]^2} \cos^2 \theta - \frac{\omega^2 c^2 \mu \sum \frac{n_i}{\omega_i - \omega} \sum \frac{n_i}{\omega_i + \omega} \sin^2 \theta}{\omega_{p0}^2}}{\omega_{p0}^2} \sum \frac{n_i}{m_i (\omega_i^2 - \omega^2)} \right]} \right]} \quad (65)
\end{aligned}$$

can be put in the same form as (46)

$$\frac{i(E_x \cos \theta - E_z \sin \theta)}{E_y} = \frac{\Omega \omega_p^2 \cos \theta \left[ 1 - \left( 1 - \frac{\omega^2}{2} \right) \frac{V^2 k^2}{\omega^2} \right]}{\omega^3 c^2 \left( \frac{k^2 \sin^2 \theta}{\omega^2} + \frac{\omega_p^2}{\omega^2 c^2} \right)} \quad (70)$$

Thus, all the dispersion and polarization properties for propagation in a multicomponent plasma can be obtained from the equivalent equations for a single component plasma simply by making the substitutions (66) and (67) for  $\omega_i$  and  $V_A$  respectively. However, formulas which involve taking a derivative with respect to frequency, such as the formula for group velocity, will not carry over, as  $V_A$  and  $\Omega$  now are functions of  $\omega$ . It can be seen that for  $j = 1$ , (66) and (67) reduce to  $\omega_i$  and the classical form of  $V_A$ .

For low frequencies,  $\omega \ll \omega_i$ ,

$$V_A^2 = \frac{B_o^2}{\Sigma n_i m_i} = \frac{B_o^2}{\mu N m_i}$$

and

$$\Omega = \frac{e B_o \Sigma n_i m_i}{\Sigma n_i m_i^2} = \frac{e B_o m_i}{m_i^2}$$

where  $N = \Sigma n_i$ . Since at low frequencies,  $\omega \ll \Omega$ , it is sufficient to account for multiple ion species by simply using the average ionic mass in the single ion equations. As  $\omega$  approaches any of the ion gyrofrequencies,  $\Omega$  becomes equal to that gyrofrequency.

Since  $\Omega$  may be negative the upper sign does not necessarily go with the Alfvén (left-hand) mode as before. This point may be clari-

fied by examining the polarization at  $\Omega = \infty$ , a point known as the crossover frequency. When (48) is translated into multicomponent form, it becomes

$$\frac{i(E_x \cos \theta - E_z \sin \theta)}{E_y} = - \frac{\Omega}{2\omega \cos \theta} \left( \sin^2 \theta \mp \sqrt{\frac{4\omega^2 \cos^2 \theta}{2} + \sin^4 \theta} \right) \quad (71)$$

which is indeterminate as both  $\Omega \rightarrow \infty$  and  $\theta \rightarrow 0$ . (Letting  $\omega \rightarrow 0$  would produce the same problem, except that waves of zero frequency are uninteresting.) What is happening is that the angle at which polarization changes from circular to linear goes to zero as  $\frac{\omega}{\Omega} \rightarrow 0$ . If  $\sin^4 \theta \ll \frac{4\omega^2 \cos^2 \theta}{\Omega^2}$ , the polarization is essentially circular and the propaga-

tion is considered quasi-longitudinal. But if the reverse approximation holds, the polarization is linear and propagation is quasi-transverse [Booker and Dyce, 1965]. Paying close attention to signs, the polarization for  $\theta = 0$  becomes

$$\frac{iE_x}{E_y} = \mp \frac{\Omega}{|\Omega|}, \quad \Omega \neq \pm \infty$$

Thus when  $\Omega > 0$ , the upper sign will give left-hand polarization, but when  $\Omega < 0$ , the lower sign must be taken to obtain the same mode. A consequence of the sign interchange as  $\omega$  goes through the crossover frequency is that the two modes will interchange their wave-normal surfaces. The faster mode is always the isotropic mode. If, because of changing plasma parameters, a wave propagates through a crossover point, it will interchange modes rather than exchange wave-normal surfaces. This point will be made clearer in the ensuing discussion.

For longitudinal propagation,  $\theta = 0$ , the phase velocity is given by

$$V_p = \frac{\omega}{k} = V \sqrt{1 + \frac{3\epsilon}{\Omega}} = \frac{B_0}{\sqrt{\mu \sum \frac{n_i m_i}{1 + \frac{\omega}{\omega_i}}}} \quad (72)$$

From this point on the upper sign will refer to the Alfvén (left-hand) mode, regardless of the sign of  $\Omega$ .

Resonances will occur wherever  $\sum \frac{n_i}{\omega_i \mp \omega} = \infty$ . Since  $\omega > 0$  in this treatment, only the Alfvén mode produces resonances, and these will occur at  $\omega = \omega_i$ . (The other mode has a resonance at  $\omega = \omega_e$ , but this result cannot be obtained from (68) as the approximation,  $\omega \ll \omega_e$ , was used in the derivation.) A cutoff will occur where  $\sum \frac{n_i}{\omega_i \mp \omega} = 0$ ,

and again only the Alfvén mode will have cutoffs in the frequency range

of interest. Since  $\sum \frac{n_i}{\omega_i - \omega} = 0$  is an equation of order  $j - 1$ , there

will be  $j - 1$  cutoffs. By noting that  $V_p$  is real as  $\omega$  approaches  $\omega_i$  from below, and imaginary when  $\omega$  is just above  $\omega_i$ , it is clear that a cutoff must occur between each pair of ion gyrofrequencies. At the crossover point, determined by  $\Omega = \infty$ , both modes will have the same phase velocity when propagating longitudinally. This will happen wher-

ever  $\sum \frac{n_i}{\omega_i^2 - \omega^2} = 0$ , and one such point will occur above each cutoff.

Figure 3 shows these critical frequencies for a plasma with three types of ions. Propagation can occur wherever the phase velocity is



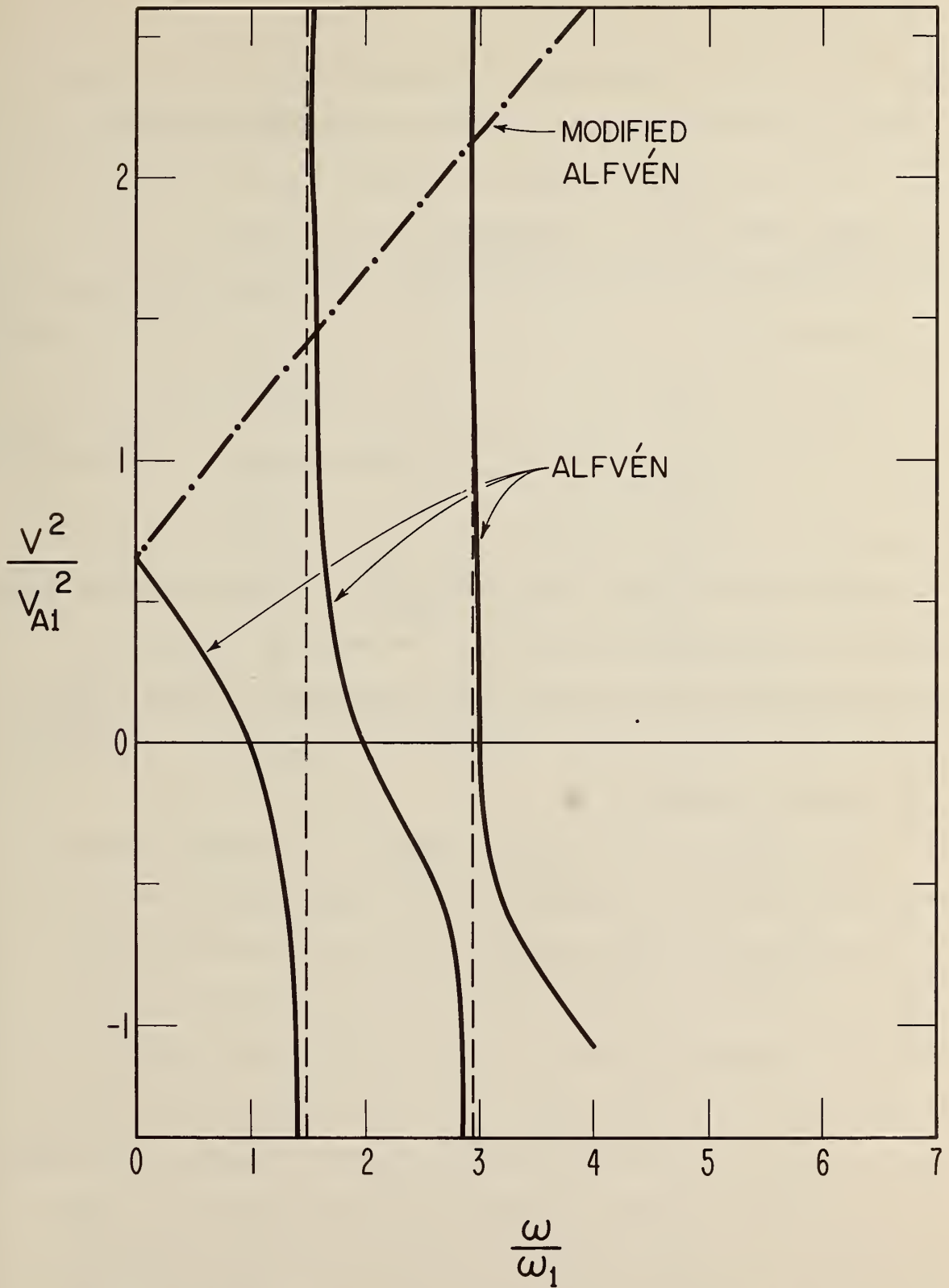


Figure 3 - Phase velocity for longitudinal propagation in a plasma with three ion components. ( $\omega_2 = 2\omega_1$ ,  $\omega_3 = 3\omega_1$ ,  $n_2 = n_1$ ,  $n_3 = 0.1n_1$ ,  $v_{A1} = \frac{B_0}{\sqrt{\mu n_1 m_1}}$ .)

real, which for the Alfvén mode is for frequencies up to the first gyrofrequency, then from the first cutoff to the second gyrofrequency, etc., until the last gyrofrequency is reached. The effect of letting the number density for one species go to zero is to move all the cutoffs toward that ion's gyrofrequency, which narrows the passband controlled by that ion.

Another significant set of frequencies is generated by the condition,  $\Omega = 0$ , i. e.,  $\sum \frac{n_i \omega_i}{\omega_i^2 - \omega^2} = 0$ . At these frequencies the Pedersen conductivity is zero and no current will flow in the direction of the transverse electric field. There will be  $j - 1$  of these zero current points and one will occur below each of the cutoffs. Here only the modified Alfvén wave will propagate and it undergoes a reshaping transition at this point, the wave-normal surface changing from a spheroid to a dumbbell lemniscoid.

Table 4 summarizes the types of significant frequencies arranged in order of increasing frequency. The last column indicates the type of transition in the notation used by Stix [1962]. The entire cycle will be repeated  $j - 1$  times. The situation can perhaps be best displayed in a diagram which shows the effect of the various transitions on the wave-normal surfaces (fig. 4). At low frequencies the surfaces are as shown in the upper right corner, with the Alfvén wave field guided. As the frequency increases, we proceed around the diagram in a clockwise direction. At the resonance,  $L = \infty$ , the Alfvén mode disappears. At the zero current point,  $S = 0$ , the modified Alfvén wave is transformed

TABLE 4

The Significant Frequencies for Propagation in a Multicomponent Plasma

	Determining Condition	Type of Transition
resonance	$\omega = \omega_i$	$L = \infty$
zero current	$\sum \frac{n_i \omega_i}{\omega_i^2 - \omega^2} = 0$	$S = 0$
cutoff	$\sum \frac{n_i}{\omega_i - \omega} = 0$	$L = 0$
crossover	$\sum \frac{n_i}{\omega_i^2 - \omega^2}$	$D = 0$

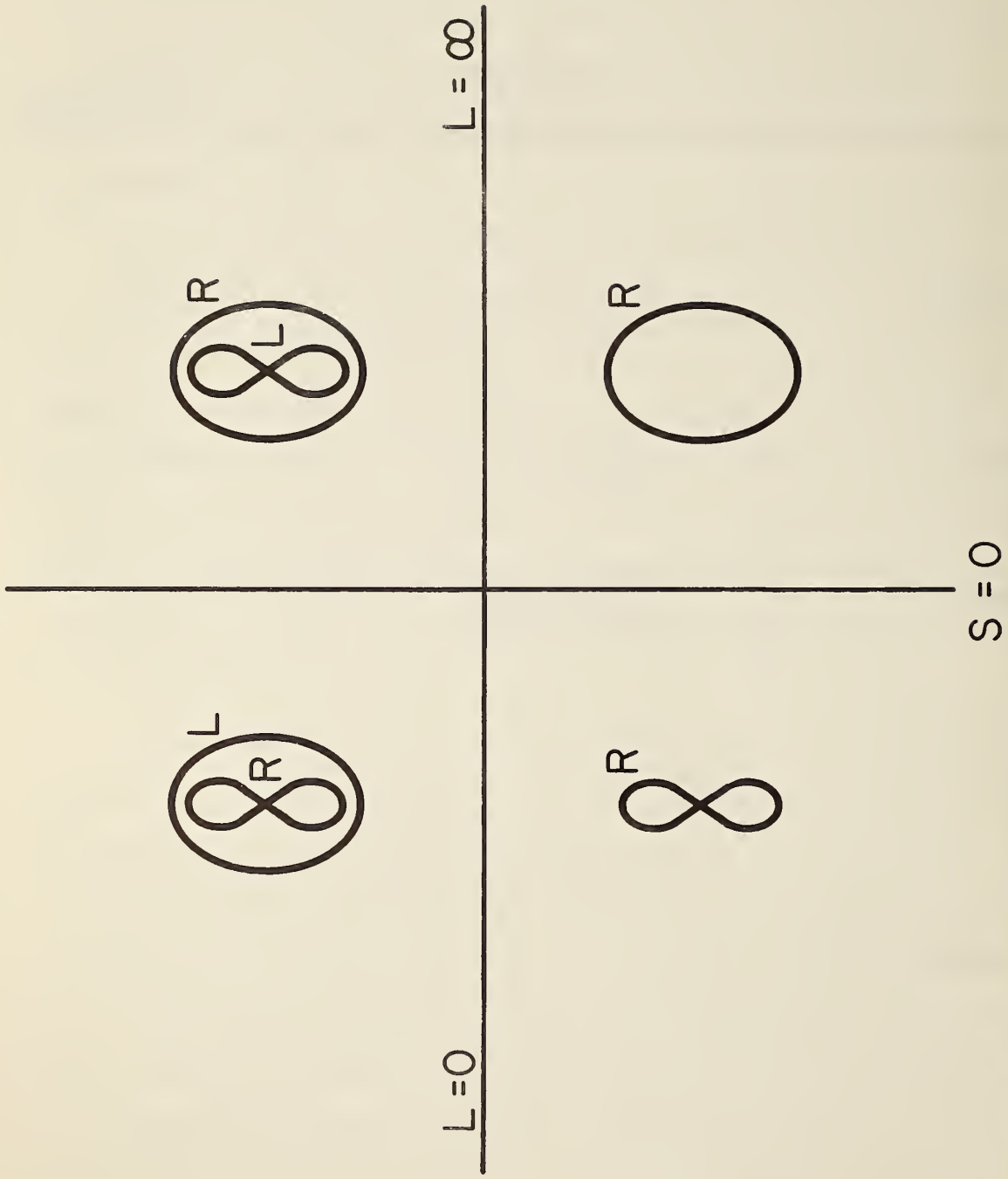


Figure 4 - Wave normal surfaces for a multicomponent plasma. At low frequencies surfaces are as shown at upper right. As frequency increases, surfaces deform continuously in clockwise progression. Magnetic field is vertical.

into a field guided wave. The Alfvén mode reappears at the cutoff,  $L = 0$ , as an isotropic wave. Finally, at the crossover or  $D = 0$  transition, the surfaces become tangent and they interchange their polarizations. After completing the circuit  $j - 1$  times, the final resonance is passed and only the modified Alfvén mode is present, as shown in the lower right corner.

Table 5 specifies a six component ( $H^+$ ,  $He^+$ ,  $N^+$ ,  $O^+$ ,  $NO^+$ , and  $O_2^+$ ) model ionosphere. The number densities are averages obtained from the papers cited at the bottom of the table. The resonances, cutoffs, and crossovers for the Alfvén mode were calculated as a function of height and are displayed in figure 5. The shaded areas delineate the regions in which the Alfvén mode can propagate. The resonances, located along the right edge of the passbands, depend only on the field strength and the ionic mass. The cutoffs, in contrast, depend greatly on the relative number densities of the ions. For example, it is seen that  $N^+$ , which is relatively scarce, controls a very narrow passband. The crossover frequencies, which are shown by a dashed line, are located in the shaded regions, but close to the left edge or the cutoff frequencies.

The existence of a crossover point at any height acts as a block to the transmission of a modified Alfvén wave through that height. Such a wave propagating downward through a crossover point will be transformed to an Alfvén wave and then reflected at the next lower cutoff. Similarly, a modified Alfvén wave attempting to propagate upward through a crossover point will be stopped by a resonance. Thus, a major effect produced by a multicomponent plasma is that a modified Alfvén wave with

TABLE 5  
A Model Multicomponent Ionosphere

height (km)	H <sup>+</sup> n <sub>1</sub>	He <sup>+</sup> n <sub>2</sub>	N <sup>+</sup> n <sub>3</sub>	O <sup>+</sup> n <sub>4</sub>	NO <sup>+</sup> n <sub>5</sub>	O <sub>2</sub> <sup>+</sup> n <sub>6</sub> (m <sup>-3</sup> )
150				2.4 x 10 <sup>10</sup>	2.2 x 10 <sup>11</sup>	1.5 x 10 <sup>11</sup>
200				3.0 x 10 <sup>11</sup>	2.0	1.2
300			6.0 x 10 <sup>10</sup>	1.1 x 10 <sup>12</sup>		
400	1.8 x 10 <sup>8</sup>	1.2 x 10 <sup>9</sup>	5.5	7.0 x 10 <sup>11</sup>		
500	8.6	2.9	4.6	4.4		
600	1.2 x 10 <sup>9</sup>	3.6	3.5	2.7		
700	1.6	3.9	1.8	1.6		
800	1.8	4.0	3.2 x 10 <sup>9</sup>	1.0		
900	2.1	4.1	7.8 x 10 <sup>8</sup>	6.2 x 10 <sup>10</sup>		
1000	2.4	4.1	2.7	3.8		
1100	2.6	4.0	1.2	2.4		
1200	2.7	3.9	5.5 x 10 <sup>7</sup>	1.4		
1300	2.8	3.7	2.5	8.8 x 10 <sup>9</sup>		
1400	2.8	3.4	1.4	5.4		
1500	2.7	3.0		3.4		
1600	2.6	2.7		2.1		
1700	2.5	2.2		1.3		
1800	2.3	1.7		8.0 x 10 <sup>8</sup>		
1900	2.1	1.0		4.9		
2000	1.8	5.0 x 10 <sup>8</sup>		3.0		

The following references were used in the above compilation: [Johnson, 1960; Hanson, 1965; Nicolet, 1962; Taylor et al., 1963; Nawrocki and Papa, 1961; Taylor and Brinton, 1961]. Exponents are assumed to carry down a column until the next is reached.

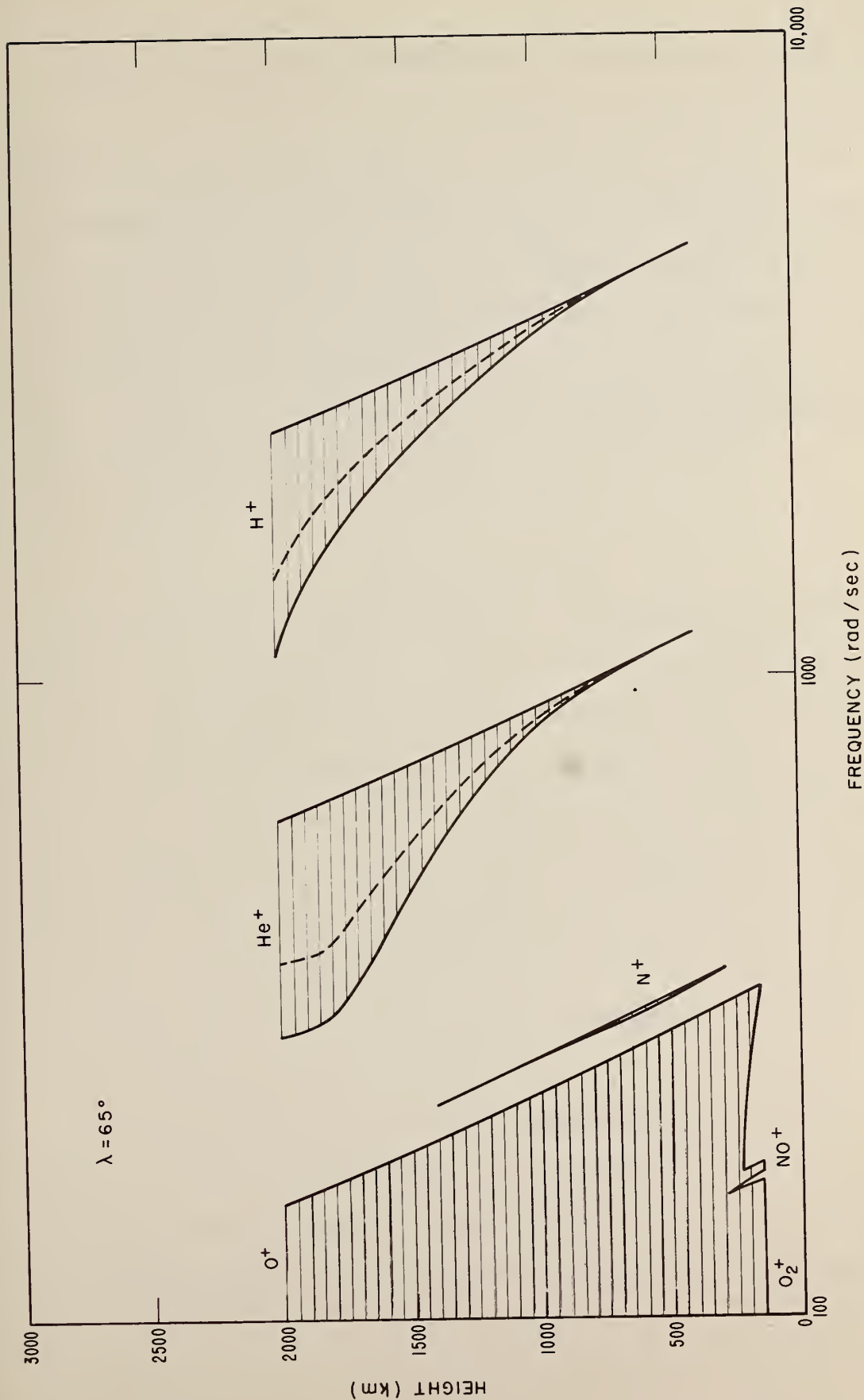


Figure 5 - The resonances, cutoffs, and crossovers (dashed lines) for the six component model ionosphere tabulated in table 5. Hatched areas represent the regions in which the left-hand mode can propagate.

a frequency in the vicinity of the ion gyrofrequencies will be unable to propagate through the ionosphere.

An Alfvén wave, propagating in one of the shaded regions but above a crossover point, is blocked from traveling upward by the resonance, but is free to propagate downward. At the crossover point it is converted to a modified Alfvén wave and it can then propagate through the cutoff. Similarly, an Alfvén wave generated below a crossover point can travel upwards, but not downward.



## PART II

### The Properties and Interpretation of Pc 1 Micropulsations

A reasonable hydromagnetic model for the propagation of pc 1 micropulsations is one in which Alfvén (left-hand) waves travel along field lines to reach the earth's surface at auroral latitudes and then propagate equatorward as modified Alfvén (right-hand) waves just above the ionosphere. The isotropic right-hand wave is constrained to a duct between 400 and 2000 km above the earth by refractive processes. It is shown how many of the observed properties of pc 1's can be related to such a model. Mechanisms for the generation of pc 1's are also discussed and it is concluded that particle instabilities are the most likely source.

#### 1. Introduction

One of the more interesting classes of geomagnetic micropulsations are the pc 1 or pearl pulsations, which have periods ranging from 0.2 to 4 sec. On an amplitude chart of sufficient resolution they appear as a smooth sinusoidal carrier signal with irregular amplitude modulation (fig. 6). The envelope has a tendency to repeat its general shape over an interval of several minutes. Often the period remains constant, ignoring minor fluctuations, for the entire event, which can last several hours. When displayed on a frequency-time plot, such as a Sonagram, pc 1's reveal a regular fine-structure consisting of a series of tones, usually rising rapidly (fig. 7).

For the most part the early work in geomagnetic pulsations was confined to those of longer period. But often there was reference to faster oscillations, e. g., Van Bemmelen's [1908] "spasms" and Harang's [1936] "vibrations". Of particular interest here is a paper by Sucks-

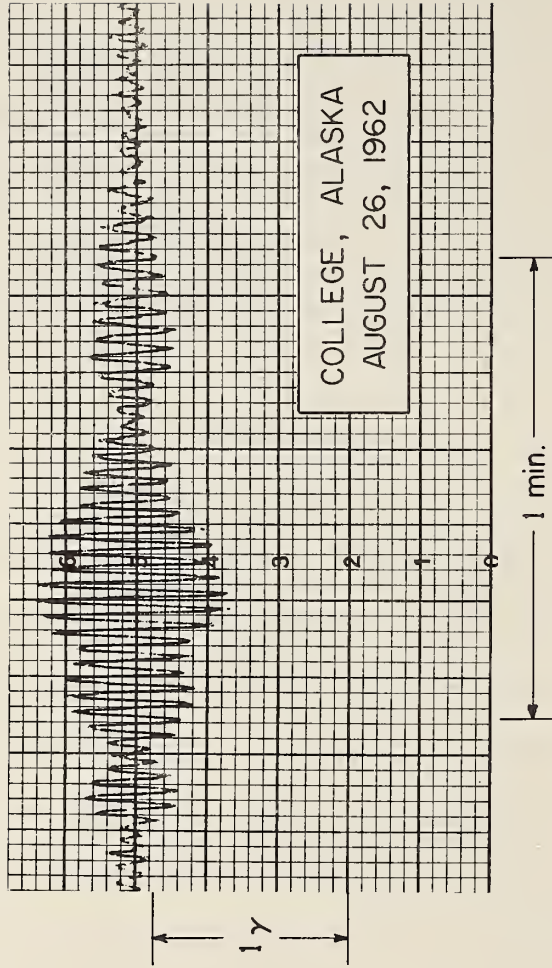


Figure 6 - A typical pc 1 event at College, Alaska as recorded on a strip chart at 3 in/minute.

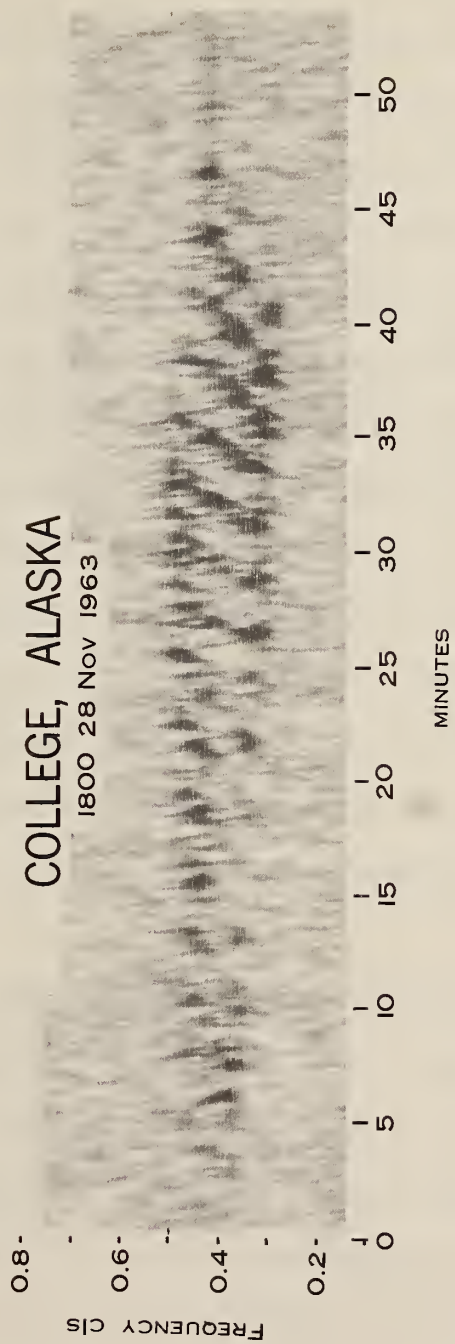


Figure 7 - A Sonagram of a p c l event taken at  
College, Alaska.

dorff [1936], in which pulsations of 2 to 3 sec period and with the general appearance of a "shuttle" are described. "It is seldom that such an oscillation occurs singly; usually there is a number of oscillations, interrupted by short intervals, for one hour, for several hours, sometimes even for more than 24 hours continuously, when the curve often resembles a pearl necklace consisting of oval pearls of different sizes." To the best of the author's knowledge, this is the earliest reference to pearls by name. It is interesting that Sucksdorff reported pearls to be an essentially daytime phenomenon with a maximum about 1400 local time. Much later pearls were considered to be nocturnal [Benioff, 1960; Tepley and Wentworth, 1962] on the basis of studies made in middle latitudes. Heacock and Hessler [1962] restated that in auroral latitudes pearls occur primarily during the day. To return to Sucksdorff's work, he concluded that September was the most favorable month, that no 27 day recurrence pattern was evident, and that though the pulsations were local in detail, general characteristics might be spread over several hundred kilometers.

Attempts to relate pc 1's to other geophysical phenomena have at best met with limited success. Troitskaya [1961] indicated a tendency for them to precede magnetic storms and to coincide with increases in cosmic ray intensity. Heacock [1963] showed several examples of pc 1's having rather sudden onsets accompanied by increased cosmic noise absorption. However, it should be emphasized that, in general, no correlation between pc 1's and ionospheric data has been shown. Some authors have stated that pc 1's occur during magnetically quiet periods,

but other evidence indicates pc 1's to be independent of general magnetic activity. Possibly, the relative ease of detecting pc 1's at quiet times led to the former conclusion. It has been suggested that pc 1's are less common at sunspot maximum [Troitskaya, 1961]. There is, however, evidence that the occurrence of pc 1's is enhanced during the week following a magnetic storm [Wentworth, 1964a].

Along with other types of micropulsations, pc 1's have been plagued with a multiplicity of nomenclature [Matsushita, 1963]. The term pearls originated by Sucksdorff [1936] was used by Troitskaya [1961], who also introduced the abbreviation PP. Benioff [1960] classified micropulsations into four period ranges, with Type A corresponding to pearls. Later Tepley [1961] used the term hm-emission to emphasize their hydromagnetic character. The International Association of Geomagnetism and Aeronomy (IAGA) at their Berkeley meeting in 1963 agreed on a system of classification which divided micropulsations into regular and irregular categories, and subdivided these by period. The classification is as follows:

Regular:	pc 1	0.2 to 5 sec period
	pc 2	5 to 10 sec
	pc 3	10 to 45 sec
	pc 4	45 to 150 sec
	pc 5	150 to 600 sec
Irregular:	pi 1	1 to 40 sec
	pi 2	40 to 150 sec

Still other terms have been used such as hydromagnetic whistlers [Oba-

yashi, 1965] and CPsp [Yanagihara, 1963]. The author prefers pc 1, since it uniquely specifies the phenomenon and it has been accepted by the IAGA. However, the term pearl is still frequently used in the literature and it should be considered interchangeable with pc 1.

## 2. Properties of Pc 1's

Many of the most significant properties of pc 1's are best displayed by a frequency-time plot. The distinguishing feature of a pc 1 is a fine-structure consisting of tones repeating at regular intervals usually on the order of one or two minutes. Most often the tones are rising, but sometimes they are vertical and occasionally falling tones are observed. Sometimes a fan-shaped structure is observed, i. e., successive tones show a slower rise rate, and this has been considered evidence of dispersion during the propagation of the wave. It should be noted, however, that the majority of pc 1's do not show such a fan-shaped structure.

The complexity of the amplitude charts is readily explained by the frequency spectrum. Generally the tones overlap to a considerable degree. Thus, at any instant the chart is recording the beating effects of several tones. It is also easy to see how two stations may have similar frequency structures and yet have dissimilar amplitude records.

When simultaneous Sonagrams for conjugate points were compared, it was found that between hemispheres the fine-structure was displaced by half the repetition rate [Tepley, 1964]. This result has since been verified over a wide range of latitudes [Yanagihara, 1963; Campbell

and Stiltner, 1965] and appears to be a consistent property of pc 1's. The probable explanation is that pc 1's are produced by waves bouncing between hemispheres. In contrast simultaneous Sonagrams from two stations in the same hemisphere show no such shift in the fine-structure. Tepley reports that the equatorial station of Canton Island sometimes records a doubling of the fine-structure, i. e., a repetition time half that of stations away from the equator. This could mean that Canton occasionally sees signals generated in both hemispheres. Careful measurements indicate that the signals do not occur quite simultaneously in the same hemisphere. Tepley, Landshoff, and Wentworth [1965] found that waves reach College, Alaska six seconds sooner than Kauai, Hawaii. This would indicate an equatorward propagation velocity of about  $8 \times 10^5$  m/sec.

As mentioned in the introduction, pc 1's are essentially a nighttime phenomenon at low and middle latitudes, but occur during the day at high latitudes. Wentworth [1964b] suggests that this may be caused by differences in ionospheric attenuation. Wentworth argues that though the maximum production of pc 1's is probably in the afternoon, higher ion densities at that time greatly attenuate the waves. Thus, most pc 1's are seen at night, when the ionosphere is more or less transparent. At auroral latitudes ionospheric attenuation will be much less and the waves can get through at the time of their maximum production. In addition auroral absorption may provide partial blocking at night.

Attempts to relate observationally pc 1's and ionospheric activity at College have shown no correlation between the two phenomena [Dawson, 1965]. There seems to be no significant agreement between the occur-

rence of pc l's or their amplitude and various ionospheric parameters, such as  $f_oF_2$ ,  $f_{min}$ , or riometer absorption. Also, some long events have been followed through sunrise or sunset with no apparent ionospheric effects [Campbell and Stiltner, 1965]. Pc l's at conjugate points often show similarity even though ionospheric conditions at the two stations may differ drastically. It would be incorrect to state that attenuation effects are never observed, but it appears to be a minor factor in determining their occurrence.

All this does not necessarily contradict Wentworth, as he predicted strong attenuation only for the lower latitudes. It would be worthwhile to investigate the question of pc l ionosphere correlation for these latitudes.

Though it has been suggested that large auroral zone pc l's with sudden onsets are accompanied by an increase in riometer absorption [Heacock, 1963], the author found little correlation between the two phenomena at College [Dawson, 1965]. Pc l's occurred during times of no, moderate, and heavy absorption with the distribution one would expect from chance occurrence. In general no consistent relationship between pc l's and particle events has been shown [Campbell and Stiltner, 1965; Tepley, 1965].

In the auroral zone the polarization of the magnetic perturbation vector has been observed to be confined to a plane perpendicular to the magnetic field line [Dawson, 1965]. When a similar measurement was attempted at Boulder, Colorado, the result showed a random orientation of the polarization plane [Pope, 1965]. Apparently the situ-



ation at middle latitudes is more complex than at high latitudes. The property of polarization perpendicular to the field line would explain why it is difficult to detect pc 1's with a total field detector, such as a rubidium or helium magnetometer.

Attempts to measure the sense of polarization have produced confused results. Both left- and right-hand elliptical, as well as linear, polarizations have been observed. Indeed, the sense will often reverse itself in the middle of a beat. One problem arises from the overlapping of the structural elements. Under some conditions two left-hand elliptical waves can combine to produce an apparent right-hand wave [Pope, 1964]. Ideally, one wishes to observe the polarization of a single element. Two possible solutions are to either select an event in which the elements are well separated, so that there is no overlapping, or to remove the overlapping artificially with a suitable filter. Though the author found a predominance of left-hand waves at College, Alaska, McPherron and Ward [1965] observed that right-hand waves were favored two to one at Flin Flon, Manitoba, also in the auroral zone. Hessler and Heacock [1966] also observed more right-hand waves, especially when the rising tones were well separated.

Studies at low latitudes have indicated that pc 1 events sometimes occur simultaneously at stations separated by several thousand kilometers [Tepley, 1964]. In contrast, Baie St. Paul and Great Whale River, Quebec, at geomagnetic latitudes of  $58.7^{\circ}$  and  $66.6^{\circ}$  N respectively, seldom see the same event, even though they are only separated by 700 km [Campbell and Stiltner, 1965]. Even Fort Yukon and College, Alaska,

only 200 km apart, show a low correlation. The implication here is that pc l's are fairly well localized in the auroral zone, but become widespread at lower latitudes. Pc l's are usually seen simultaneously at pairs of stations which are magnetically conjugate to each other [Campbell and Stiltner, 1965; Dawson, 1965].

### 3. Theoretical Interpretation of Pc l's

Most of the available evidence indicates that pc l's are the result of a magnetic disturbance at about five or six earth radii in the vicinity of the equatorial plane. Left-hand waves would be guided along field lines reaching the surface at auroral latitudes. Bouncing between hemispheres would explain the regular repetition observed on Sonagrams and also the  $180^\circ$  phase shift between hemispheres. The waves could propagate to lower latitudes as a right-hand traveling in a duct between the ionosphere and the height of maximum Alfvén velocity at about 2000 km.

The high correlation between conjugate stations at high latitudes establishes pc l's as a field line phenomenon. The degree of correlation for conjugate stations at middle latitudes is not known. At low latitudes travel along field lines becomes indistinguishable from propagation in the ionosphere.

The repetition times can vary from about 45 to 300 sec, though usually it is around 180 sec [Tepley, 1965]. It can be seen from table 6 that only L-values between 3.5 and 7 are suitable, if the repetition time represents the time for an Alfvén wave to make a round trip between

TABLE 6

## Travel Times for Alfvén Waves

Latitude	L-value	Bounce time (sec)
35°	1.5	12
40°	1.7	15
45°	2.0	17
50°	2.4	23
55°	3.0	36
60°	4.0	69
65°	5.6	140
70°	8.5	390

hemispheres. A travel time of 180 sec would correspond to an L-value of about 6.

As only the left-hand waves are guided along the field lines, the bouncing waves would have to be the left-hand waves. Since these waves cannot propagate at frequencies above the ion gyrofrequency, the equatorial value of the gyrofrequency represents the highest frequency which can bounce on that field line. This resonance could explain why pc 1's are never observed with frequencies higher than 5 c/s. At  $L = 4.5$ , the equatorial gyrofrequency is 5 c/s (see fig. 8). Thus, if the bouncing never occurs at lower L-values, higher frequencies could not propagate.

The most common type of fine-structure observed is a repeating series of rising tones. Several workers have suggested that the rising tones are the result of dispersion [Obayashi, 1965]. Since for a left-hand wave, higher frequencies travel slower, this could explain the rising tones. But in order to obtain significant dispersion,  $\frac{\omega}{\omega_i}$  must be comparable to unity, though always smaller, which would be further evidence that high latitude field lines play the primary role. Some pc 1 events show a fan-shaped fine-structure, i. e., successive elements have an increasingly slow rise time. This could be interpreted as evidence of cumulative dispersion by each successive bounce. However, only a few events show the fan-shaped structure; most events have elements with a constant rise rate. Then there is the problem of those events which have falling tones. It seems difficult to explain all these effects by dispersion alone. More likely the observed fine-structure is a result of generation plus dispersion.

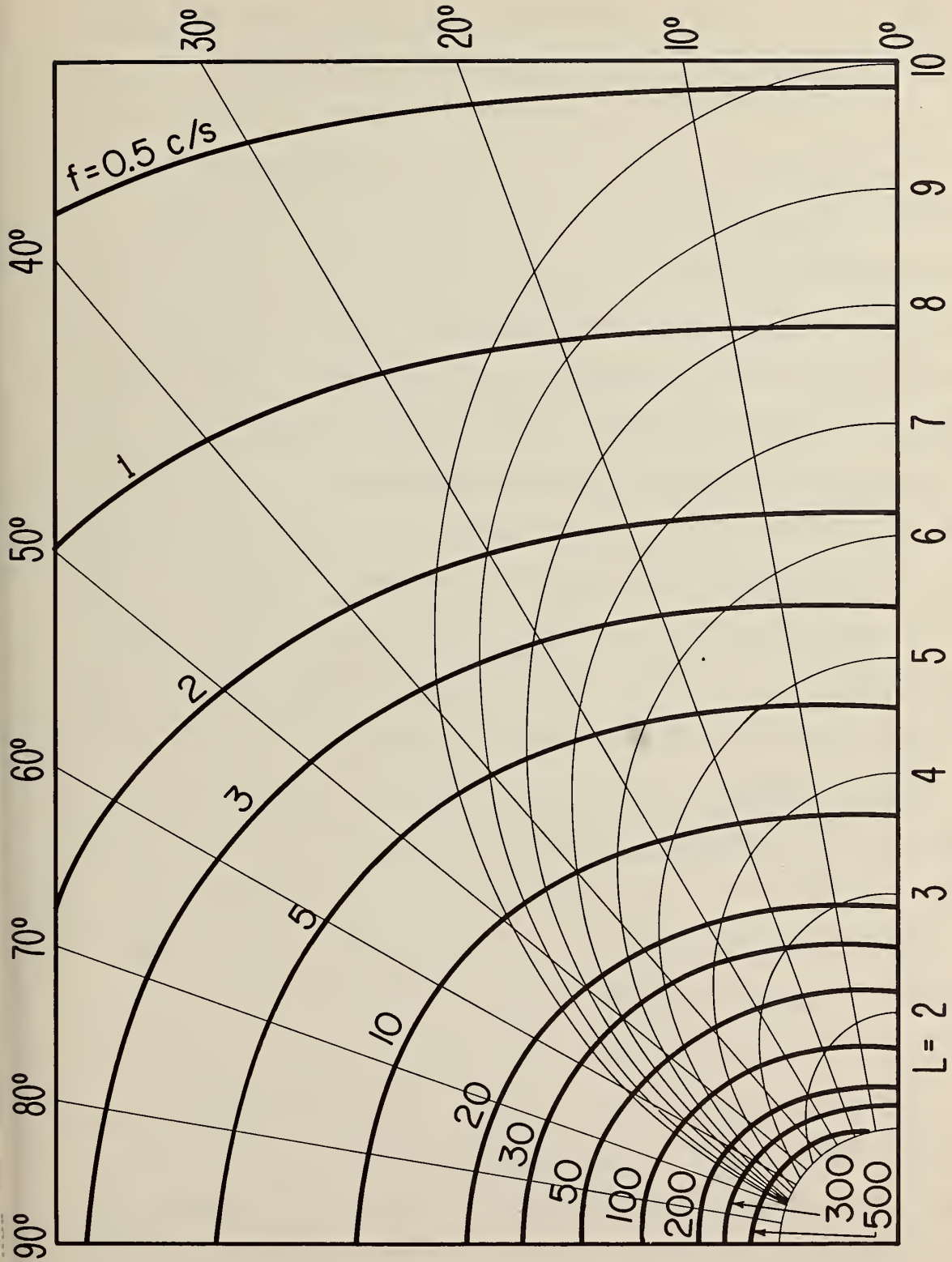


Figure 8 - Proton gyrofrequencies in the earth's magnetosphere. A dipole field is assumed.

There appears to be a qualitative difference in the pc l's observed at auroral latitudes and those observed closer to the equator. At College the magnetic perturbation vector was observed to be perpendicular to the field line, indicating that the incoming wave was traveling down the field line. At Boulder the plane of polarization was randomly oriented. Apparently Boulder sees waves arriving from a variety of directions. Effects of ground conductivity will introduce a measure of confusion, but at least the limited polarization data available does support the concept of left-hand waves traveling down field lines to the auroral regions and then propagating equatorward via some other mode. If this model is correct, waves reaching the top of the ionosphere at high latitudes should have left-hand polarization. However, in the transmission through the ionosphere, a right-hand wave will be generated, and because of differential attenuation and reflection, the polarization observed on the ground could conceivably have either sense.

Further evidence of the different nature of pc l's at high and middle latitude is obtained from studies of the areal coherence of the events. Pc l's appear to be a local phenomenon in the auroral zone but widespread at lower latitudes. Again this could support the idea of primary impingement being in the auroral zone.

There are three conceivable ways in which pc l's could arrive at low latitudes. If it is assumed that pc l's reach the auroral zone as Alfvén waves, then they could propagate equatorward as an electromagnetic wave under the ionosphere. The extremely long wavelength would put all stations in the near field of the auroral zone and pc l's would

be expected simultaneously at all latitudes. It would also be expected that a pc 1 event would be very widespread, when in actuality they are not.

A more promising way to propagate pc 1's to lower latitudes is as a right-hand wave in the duct at the top of the ionosphere. Alfvén velocity profiles for four model ionospheres are shown in figure 9. The profiles show a rather sharp minimum at about 400 km. Thus, refractive processes could guide a right-hand wave in a duct at this altitude. Recent measurements [Tepley, Landshoff, and Wentworth, 1965] indicate that College sees a pc 1 six seconds earlier than Kauai, implying a velocity of 800 km/sec, which is the right magnitude for a wave traveling at about 900 km. Presumably energy would leak through the ionosphere to be received at the ground as the right-hand wave is propagated overhead toward the equator. Apparently the wave is mostly attenuated by the time the equator is reached, as equatorial stations see a minimum number of events. Reception of the signals from both hemispheres could explain the double fine-structure occasionally observed at Canton Island.

One problem is that the simple hydromagnetic theory postulated here is not really adequate to explain this mode of propagation. About all it can do is to hint at what might occur. In general the available wave guide will have a height less than one wavelength, which makes the uniform plasma assumption somewhat suspect. Collisions also become important and can no longer be neglected. The little data so far available indicate a tendency for pc 1's to propagate along geomagnetic me-

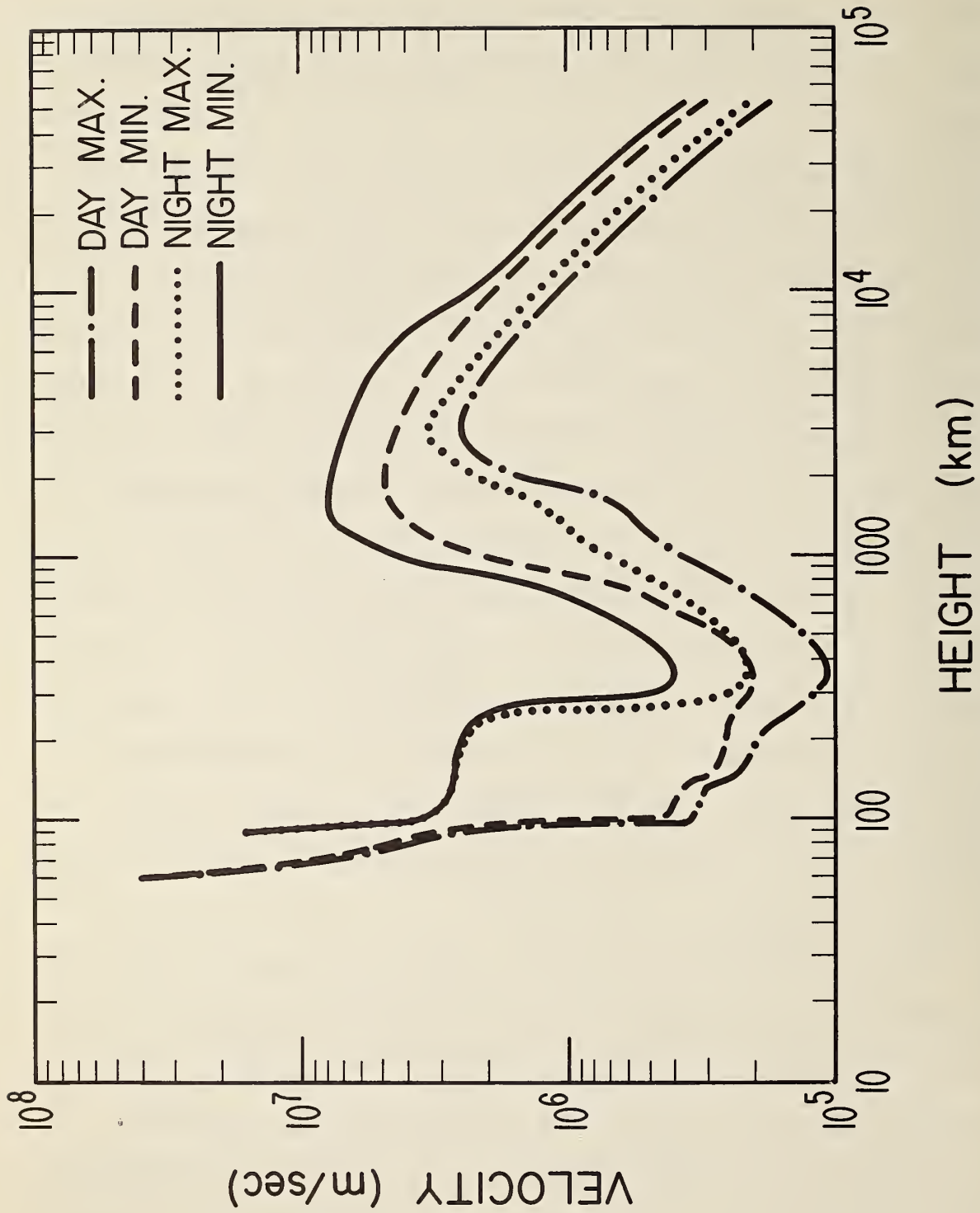


Figure 9 - Alfvén velocity profiles for four model magnetospheres. Calculated from data given by Prince and Bostick [1964].



ridians. If this is so, it will be necessary to explain why the right-hand wave cannot propagate as easily east-west as it does north-south. The answer could lie in the nature of the coupling of the two modes in the ionosphere.

A third way in which a hydromagnetic disturbance could propagate to low latitudes is as a right-hand wave propagating directly in along the equatorial plane from the original source. Unfortunately, there are many problems associated with this explanation. If the frequency fine-structure is the result of interhemispheric bouncing, it would not be present at low latitudes. However, it is observed that both low and high latitude events show similar fine-structure. Also, it would be expected that the magnetic vector would be linearly polarized in the north-south direction. To the best of the author's knowledge, this has not been observed. The same refractive processes that would allow a right-hand wave to propagate in a duct under 2000 km would refract a right-hand wave away from the earth. Only a wave coming straight in along a radial path could penetrate the high speed region.

One possible source for pc 1's is a fast beam of protons with an instability occurring at the Doppler-shifted ion gyrofrequency. Such a beam could radiate hydromagnetic energy backwards at a frequency of

$$\omega = \frac{\omega_i V_A}{V + V_A}$$

where  $V$  is the beam velocity along the field line. Cornwall [1965] showed that it was necessary for the proton velocity distribution to be anisotropic in order for the instability to have a positive growth rate. Fortunately, the existence of a loss cone usually provides suf-

ficient anisotropy.

Such a generative model would tend to be self-destructive. In successive passages of the beam through the wave packet, the wave would react on the protons so as to scatter them into the loss cone. After a period of time the source particles would be removed and the wave would decay, mostly because of collisional and coupling losses at the lower elevations, bringing the whole process to a natural conclusion.

A possible objection to the foregoing is that it violates the Sturrock [1958] criterion for a convective instability. The criterion says that only a convective instability, i. e., one that can propagate away from its origin, is suitable for the generation of hydromagnetic waves. In general, if the particle beam radiates in the forward direction the instability is convective, otherwise it is nonconvective. If this criterion is applicable, a proton beam can only produce a right-hand wave and an electron beam a left-hand wave.

Stefant [1965] and Gendrin [1965] showed that conditions for the transfer of energy from a proton beam to a hydromagnetic wave were particularly favorable when the frequency was either near half the electron gyrofrequency or twice the ion gyrofrequency. At these frequencies the group velocity of the wave is equal to the beam velocity and the transfer of energy may be made over a large distance. Gendrin also showed that the frequency range over which the instability can take place is greatly enlarged under these conditions.

As mentioned earlier the proton beam would feed energy into a right-hand wave, but it is desirable to have a left-hand wave which will be

field guided and thus can penetrate the ionosphere. Gendrin suggested the possibility that a beam could interact simultaneously with both hydromagnetic modes and could serve as a means to transfer energy from one mode to the other, but he did not outline explicitly the conditions under which this could happen.

Stefant considered the case when the beam velocity is slightly different from the group velocity. This produces two right-hand waves at about twice the ion gyrofrequency, which could beat together to produce a left-hand wave at a much lower frequency. If a nonlinearity exists, an independent left-hand pc 1 wave could be generated, which could penetrate the ionosphere, where the higher frequency carriers would be greatly attenuated. The beating also serves the purpose of producing a frequency low enough to explain pc 1 occurrence.

So far the discussion has centered about a model which relies on a particle instability mechanism to generate hydromagnetic waves which are observed on the ground as geomagnetic micropulsations. Other speculations have involved the idea of hydromagnetic waves generated by fluctuations in the magnetopause.

Typical of such models is the one suggested by Jacobs and Watanabe [1962]. The interaction of the solar wind with the magnetopause is postulated to generate noisy fluctuations which propagate as left-hand waves along the geomagnetic field lines down to the ionosphere at high latitudes. As the original fluctuations are noisy, some sort of filtering mechanism must operate so that only regular pulsations are transmitted to the surface. Jacobs and Watanabe suggested that standing waves

are established between the top of the ionosphere and the height of maximum Alfvén velocity. (Others have suggested that the entire field exhibits resonances, which are observed as regular pulsations. Except for field lines at very low latitudes, the fundamental resonance is much too long in period to account for pc l's and there is no reason to expect to see only the higher harmonics.) The effective width of the transmitted frequency determines the envelope or the beating of the pc l's. Secondary waves are then generated which propagate to lower latitudes via the mechanism discussed earlier.

This model can explain the characteristic frequency, the beating, and the polarization of pc l's, but it fails to explain the opposite phases observed between conjugate points and the series of rising tones seen on Sonagrams. In a second paper Jacobs and Watanabe [1963] changed their model to one of bouncing particles in order to remove these deficiencies.

In the revised model bouncing protons, instead of stochastic fluctuations, generate standing waves in the aforementioned duct. To account for the observed fine structure, protons are believed to be bouncing at several latitudes simultaneously. Since the Alfvén velocity and the angle of field line inclination is greater at higher latitudes, emissions stimulated at high latitudes will have a higher frequency than those generated at middle latitudes. At the same time, protons bouncing at the higher latitudes will have a longer bounce period, and therefore after each successive bounce, the higher frequencies will be further retarded with respect to the lower frequencies, producing the fan-shaped structure.

The difficulty with Jacobs and Watanabe's revised theory is that the predicted resonant frequency is greatly dependent on local ionospheric conditions, which will often differ drastically between conjugate points. In contrast, simultaneous Sonagrams taken at conjugate points show a similar frequency structure. Also, the fan-shaped structure is not the most common type, and the theory cannot explain the other forms observed.

Wentworth and Tepley [1962] postulated another type of bouncing particle model, one using electrons whose bounce period is equal to the wave period. The diamagnetic effect of the spiraling electrons would generate a hydromagnetic wave which is detected on the ground. Two fundamental difficulties are that beats are predicted to occur simultaneously in both hemispheres and the magnetic polarization should be aligned along the field lines. The reason for the latter point is that spiraling electrons would produce a perturbation magnetic field aligned against the main field. Since  $\vec{k} \cdot \vec{b} = 0$ , this could only produce a right-hand wave propagating across the field lines. Wentworth later rejected the bouncing electron model in favor of a proton model, whose bounce period was equal to the repetition time. Now the  $180^\circ$  phase shift was accounted for. A summary of all the particles was given by Tepley [1964]. As pc 1's can last for several hours, all these models share the formidable problem of explaining the stability of the postulated particle bunches. Finally, no good correlation between pc 1 events and particle events has been shown.

Models for the generation of pc 1's fall into three general categories, hydromagnetic, particle, and particle-wave interactions. All

of them have serious faults, but overall, the particle-wave models seem the most satisfactory. The question of whether an instability must be convective in order to serve as a satisfactory model needs to be settled. An acceptable theory must explain the conjugacy, the observed fine-structure, its hemispheric phase shift, the long duration of the pulsations, and the lack of correlation with other geophysical phenomena.

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