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Technical Note No. 311

COMPUTATION OF THE PERMEABILITY AND PERMITTIVITY OF A RELATIVELY SMALL RING SAMPLE IN A TOROIDAL COIL

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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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ISSUED JANUARY 25, 1966

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CONTENTS

Computation of the Permeability and Permittivity of a Relatively Small Ring Sample in a Toroidal Coil

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The derivation and the FORTRAN machine program are presented for a formula which gives the impedance of a partially filled toroid coil. This formula shows the relationship of the complex permeability and permittivity of the ring material, the dimensions of the coil and sample, and the frequency of the applied electromagnetic field. Limitations of the formula due to proximity of the ring and coil, range of permeability and permittivity of the ring, and uniformity of the coil winding are considered. The cost of the program in terms of computer time is also considered. The principal value of the formula is to accurately determine the permeability and permittivity of ferrites.

Key Words: Complex permeability and permittivity, FORTRAN, impedance, toroid coil.

1. Introduction

In this technical note we develop the theory and the associated computer program for the impedance, Z, of a toroid sample with rectangular cross section imbedded in a toroid shaped frame of similar cross section around which a wire is wound. In order that the problem be amenable to theoretical solution it is necessary to require that the wire coil and the ferrite sample both have a rectangular cross section. The expected range of validity of this theory can be estimated by the requirement that $2\pi \frac{Df}{c} \mu \epsilon < 1$. Here D is the largest length in meters of the toroid cross section, the f is the frequency in hertz, and the ϵ, μ are the relative permittivity and permeability, respectively, of the sample. The c is the velocity of light which is in M.K.S. units 3×10^8 m/sec.

2. Procedure

Assume a ferrite sample with the following boundary conditions:

1. The experimental circumstances are such that the surface of a rectangular cross-sectioned toroid can be assumed to be a spacially uniform current sheet.

2. A second rectangular cross-sectioned toroid consisting of the ferrite material inside the current sheet toroid is to be constructed so that its size is electrically small compared with the current sheet toroid. This means that the ferrite sample is small enough that non-uniformity of a uniform current sheet will produce no essential difference in the determination of the permeability and permittivity of the ferrite sample.

3. Two complex frequency dependent parameters--permeability μ and permittivity ϵ are used to characterize the ferrite.

4. The excitation mechanism of the toroid system is such that no angular field dependence exists. This means that the basic electromagnetic field equations will be reduced to a single equation. The basic equations that are used are

$$\underset{\sim}{\nabla} \times \underset{\sim}{E} = -\hat{i}\, \omega\, \mu\, \underset{\sim}{H}, \tag{1}$$

and

$$\underset{\sim}{\nabla} \times \underset{\sim}{H} = \hat{i}\, \omega\, \epsilon\, \underset{\sim}{E}\, .$$

(natural units are used, namely c = 1)

5. The coordinates of interest are given in Figure 1.

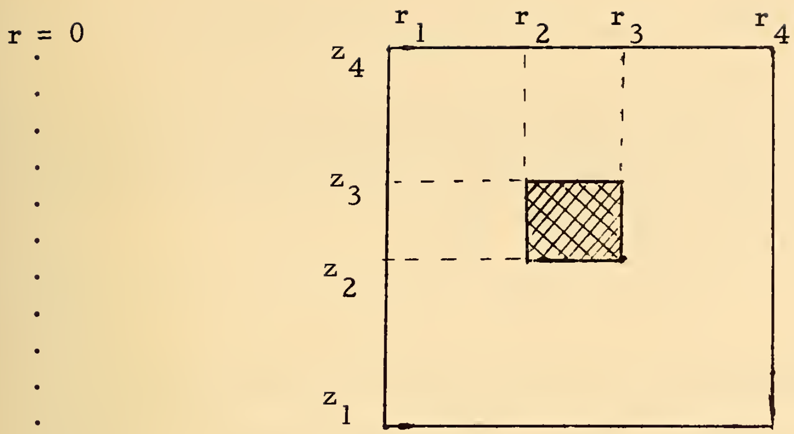

Figure 1

The r_1, r_2, r_3, and r_4 are the radial coordinates. The z_1, z_2, z_3, and z_4 are the "z" axis coordinates. The crosshatched area characterizes the ferrite sample.

By defining the function

$$P(r, z) = \left[\theta(r - r_2) - \theta(r - r_3)\right] \left[\theta(z - z_2) - \theta(z - z_3)\right],$$

$$\text{with } \theta(x) = 1 \qquad \text{if } x \geq 0$$

$$\text{and } \theta(x) = 0 \qquad \text{if } x < 0 \quad ,$$

the permeability and permittivity may be written as $\mu = 1 + (\mu_M - 1)P(r, z)$, and $\epsilon = 1 + (\epsilon_M - 1)P(r, z)$. The μ_o, ϵ_o which are the vacuum permeability and permittivity, respectively, are set equal to one. The μ_M, ϵ_M are the ferrite's permeability and permittivity, respectively. The μ and ϵ give the permeability and permittivity for the entire cross section of Figure 1.

Upon reducing (1) with assumptions 1 to 5 and letting $f = rH_\varphi$, the following differential equation results:

$$\frac{r}{\mu} \partial_r \left(\frac{1}{r\epsilon} \partial_r f \right) + \frac{1}{\mu} \partial_z \frac{1}{\epsilon} \partial_z f + \omega^2 f = 0 \, . \qquad (2)$$

Here $\left(\partial_r \equiv \frac{\partial}{\partial_r}\right)$, $\left(\partial_z \equiv \frac{\partial}{\partial_z}\right)$.

In order to get rapidly convergent results in the machine calculation, we extract the quasi-static part from this equation, and to make the function continuous to its first derivative, we write

$$f = I \left(1 + \epsilon\, g(r, z)\right) . \tag{3}$$

This transformation assumes that the discontinuity of the material surface tends under the condition of the experimental system to form a surface current,

$$V = \hat{i}\omega \int \frac{\mu f}{r}\, dr\, dz = \hat{i}\,\omega I \left[\int \frac{\mu}{r} (1 + \epsilon g)\, dr\, dz \right] , \tag{4}$$

and that the total current is given by $I_{total} = 2\pi H_\varphi r$; where $r = r_1$ and H_φ is the magnetic field intensity along the azimuthal direction. Since $I_{total} = 2\pi f(r_1, z) = Nj$, where j is the current in each wire loop, we deduce that $I = \frac{Nj}{2\pi}$. Therefore, the measured impedance may be written as

$$Z = \frac{NV}{j} = \hat{i}\frac{\omega N^2}{2\pi} \left[\ln\left(\frac{r_4}{r_1}\right)\left(z_4 - z_1\right) + \left(\mu_M - 1\right) \ln\left(\frac{r_3}{r_2}\right)\left(z_3 - z_2\right) + \int dr\, dz\, \frac{\mu\epsilon}{r}\, g(r, z) \right] . \tag{5}$$

Because natural units are used, Z is dimensionless. The first and second terms represent the quasi-static approximation. The last term gives the corrections. To evaluate Z we need g. The differential equation for g is obtained by substituting (3) into (2). The following equation is obtained:

$$r\, \partial_r \frac{1}{r} \partial_r g + \partial_z^{\,2} g + \omega^2 \mu \epsilon g = -\mu\omega^2 . \tag{6}$$

Assume the following form for the solution:

$$g = \Sigma R_n(r)\, Z_n(z) \tag{7}$$

in which g = 0 on the surface of the current toroid.

3. Analysis

Once the above steps have been taken, the rest of the analysis uses in the usual manner normal modes and perturbation theory.[1] Because of the cylindrical symmetry, the best orthogonal set to use is Bessel functions for the radial dependence. This gives integrable results.

The Bessel functions used are implied by the equation

$$\left(r\, \partial_r \frac{1}{r}\, \partial_r + k_n^{\,2} \right) R_n = 0\,, \tag{8a}$$

which can be obtained from the homogeneous version of (6), with (7) substituted for g and with the boundary conditions $R_n(r_1) = R_n(r_4) = 0$. The dual function to R_n is given by $\overline{R}_n = \frac{1}{r} R_n$, where $\overline{R}_n$ satisfies

$$\overline{R}_n \left[r\, \overleftarrow{\partial}_r \frac{1}{r}\, \overleftarrow{\partial}_r + k_n^{\,2} \right] = 0\,, \tag{8b}$$

where the $\overleftarrow{\partial}_r$ denotes differentiation to the left. The orthogonality condition is

$$\int_{r_1}^{r_4} \overline{R}_n\, R_{n'}\, dr = \delta_{nn'}\,, \tag{8c}$$

where
$$\delta_{nn'} = 1 \qquad n = n'$$
$$\quad\;\; = 0 \qquad n \neq n'\,.$$

$Z_n(z)$ obeys the following differential equation:

$$\left(\partial_z^{\,2} + \omega^2 - k_n^{\,2} \right) Z_n + \Sigma\, A_{nn'}\, P(z)\, Z_{n'} = -B_n - D_n\, P(z). \tag{9}$$

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[1] P. M. Morse and H. Feshback, Methods of Theoretical Physics (McGraw Hill, New York, 1953), Vol. II, Chapter 9, pp. 1001-1025.

This is derived by taking (6), multiplying by $\overline{R}_n$, and integrating from r_1 to r_4. In (9),

$$A_{nn'} \equiv \int_{r_2}^{r_3} dr\, \overline{R}_n(r)\, R_{n'}(r)\, \omega^2 (\mu_M \epsilon_M - 1),$$

$$B_n \equiv \omega^2 \int_{r_1}^{r_4} \overline{R}_n(r)\, dr\,,$$

$$D_n \equiv \omega^2 (\mu_M - 1) \int_{r_2}^{r_3} \overline{R}_n(r)\, dr\,,$$

and

$$P(z) \equiv \theta(z - z_2) - \theta(z - z_3).$$

Define the variables $\lambda_n^{\;2} = -\omega^2 + k_n^{\;2}$, and $\overline{\lambda}_n^{\;2} = \lambda_n^{\;2} - A_{nn}$, or $\lambda_n^{\;2}(z) = \lambda_n^{\;2} + (\overline{\lambda}_n^{\;2} - \lambda_n^{\;2})P(z)$, and use the Green's function defined by

$$G_n\left[\overleftarrow{\partial}_z^{\,2} - \lambda_n^{\;2}(z)\right] = -\delta(z - \overline{z})$$

$\left(\delta(z - \overline{z})\right.$ is the Dirac delta function$\left.\right)$

to solve the differential (9), the resulting integral equation is

$$Z_n(\overline{z}) = \sum_{n' \neq n} \int_{z_2}^{z_3} A_{nn'} G_n(\overline{z}, z)\, Z_{n'}(z) dz + \int_{z_1}^{z_4} G_n(\overline{z}, z) B_n dz$$
$$+ D_n \int_{z_2}^{z_3} G_n(\overline{z}, z)\, dz. \qquad (10)$$

By considering only first order mode correlation, $Z_n(\bar{z})$ may be approximately written as

$$Z_n(\bar{z}) \cong \left(\int_{z_1}^{z_4} G_n\,(\bar{z},z)\,dz B_n + D_n \int_{z_2}^{z_3} G_n(\bar{z},z)\,dz \right) \tag{11}$$

$$+ \sum_{n' \neq n} \int_{z_2}^{z_3} A_{nn'}\, G_n(\bar{z},\bar{\bar{z}})d\bar{\bar{z}} \left[\int_{z_1}^{z_4} G_{n'}(\bar{\bar{z}},z)B_{n'}dz + \int_{z_2}^{z_3} D_{n'}G_{n'}(\bar{\bar{z}},z)dz \right] .$$

This approximation implies that the sample is electrically small. G_n is $G_n(z,\bar{z}) = A_n C_n(z_>) S_n(z_<)$, where $z_>$ is the larger of z, $\bar{z}$ pair and $z_<$ is the smaller of z, $\bar{z}$ pair. A_n is the normalization. It is such that (12) is true:

$$A_n \quad C_{n'} S_n - C_n S_{n'} \quad = -1 \;, \tag{12}$$

where $z = \bar{z}$.

The functions S_n, C_n are defined as follows:

$$\left.\begin{aligned} S_n &= \exp\left[\lambda_n(z - z_1)\right] - \exp\left[-\lambda_n(z - z_1)\right] \\ C_n &= C^1_n \exp\left[\lambda_n(z - z_2)\right] + C^2_n \exp\left[-\lambda_n(z - z_2)\right] \end{aligned}\right\} z_2 \geq z \geq z_1$$

$$\left.\begin{aligned} S_n &= \bar{S}^1_n \exp\left[\bar{\lambda}_n(z - z_2)\right] + \bar{S}^2_n \exp\left[-\bar{\lambda}_n(z - z_2)\right] \\ C_n &= \bar{C}^1_n \exp\left[\bar{\lambda}_n(z - z_3)\right] + \bar{C}^2_n \exp\left[-\bar{\lambda}_n(z - z_3)\right] \end{aligned}\right\} z_3 \geq z \geq z_2$$

$$\begin{aligned} S_n &= S^1_n \exp\left[\lambda_n(z - z_3)\right] + S^2_n \exp\left[-\lambda_n(z - z_3)\right] \\ C_n &= \exp\left[\lambda_n (z - z_4)\right] - \exp\left[-\lambda_n(z - z_4)\right] . \end{aligned}$$

Here,

$$\lambda_n = + \left[- \omega^2 + k_n^{\ 2} \right]^{\frac{1}{2}}$$

and (13)

$$\bar{\lambda}_n = + \left[\lambda_n^{\ 2} - A_{nn} \right]^{\frac{1}{2}} .$$

The coefficients $C^1_{\ n}$, $C^2_{\ n}$, $\bar{C}^1_{\ n}$, $\bar{C}^2_{\ n}$ and $S^1_{\ n}$, $S^2_{\ n}$, $\bar{S}^1_{\ n}$, $\bar{S}^2_{\ n}$ are defined with the aid of the function f_1:

$$f_1(\lambda_n, \bar{\lambda}_n, 3, 4) \equiv \frac{1}{2}\left[\left(\frac{\lambda_n}{\bar{\lambda}_n} + 1 \right) a^n_{\ 34} + \left(\frac{\lambda_n}{\bar{\lambda}_n} - 1 \right) a^n_{\ 43} \right] .$$

Thus

$$\bar{C}^1_{\ n} \equiv f_1(\lambda_n, \bar{\lambda}_n, 3, 4)$$

$$\bar{C}^2_{\ n} \equiv f_1(\lambda_n, -\bar{\lambda}_n, 3, 4)$$

$$\bar{S}^1_{\ n} \equiv f_1(\lambda_n, \bar{\lambda}_n, 2, 1) \qquad (14)$$

and

$$\bar{S}^2_{\ n} \equiv f_1(\lambda_n, -\bar{\lambda}_n, 2, 1) .$$

By defining a second function

$$f_2(\lambda_n, \bar{\lambda}_n, 2, 3, \bar{C}^1_{\ n}, \bar{C}^2_{\ n}) \equiv \frac{1}{2}\left[\bar{C}^1_{\ n}\left(1 + \frac{\bar{\lambda}_n}{\lambda_n}\right) a^n_{\ 23} + \bar{C}^2_{\ n}\left(1 - \frac{\bar{\lambda}_n}{\lambda_n}\right) a^n_{\ 32} \right],$$

the $C^1_{\ n}$, etc., are:

$$C^1_{\ n} = f_2(\lambda_n, \bar{\lambda}_n, 2, 3, \bar{C}^1_{\ n}, \bar{C}^2_{\ n})$$

$$C^2_{\ n} = f_2(-\lambda_n, \bar{\lambda}_n, 2, 3, \bar{C}^1_{\ n}, \bar{C}^2_{\ n})$$

$$S^1_{\ n} = f_2(\lambda_n, \bar{\lambda}_n, 3, 2, \bar{S}^1_{\ n}, \bar{S}^2_{\ n}) \qquad (15)$$

$$S^2_{\ n} = f_2(-\lambda_n, \bar{\lambda}_n, 3, 2, \bar{S}^1_{\ n}, \bar{S}^2_{\ n}).$$

The a^n_{34}, etc. are defined as

$$a^n_{34} = \exp\left[\lambda_n(z_3 - z_4)\right] = (a^n_{43})^{-1}$$

$$a^n_{23} = \exp\left[\overline{\lambda}_n(z_2 - z_3)\right] = (a^n_{32})^{-1}$$

and

$$a^n_{12} = \exp\left[\lambda_n(z_1 - z_2)\right] = (a^n_{21})^{-1} .$$

Using the above definitions, the normalization A_n is

$$A_n = {}^1\!/_2 \lambda_n \left[C^1_n a^n_{12} + C^2_n (a^n_{12})^{-1}\right]$$

and the impedance reduces to

$$Z = \hat{i}\,\frac{\omega N^2}{2\pi}\left[\begin{array}{l}(z_4 - z_1)\ln(r_4/r_1) + (\mu_M - 1)(z_3 - z_2)\ln(r_3/r_2) \\ + \sum\limits_{n=1} \left\{\begin{array}{l}\left[T_n(r_4) - T_n(r_1)\right]\ (V_n + W_n) \\ + \quad (\mu_M - 1)\left[T_n(r_3) - T_n(r_2)\right] W_n\end{array}\right\} N_n\end{array}\right] . \qquad (16)$$

Here we have further defined

$$V_n \equiv \int_{z_1}^{z_4} Z_n dz ,$$

$$W_n \equiv \int_{z_2}^{z_3} Z_n dz ,$$

and have noted that the T_n is a solution to the equation

$\left[d_r^{\,2} + \frac{1}{r} d_r + k_n^{\,2}\right] T_n = 0$. The R_n is related to T_n by $R_n = r\frac{dT_n}{dr} N_n$. The k_n is fixed by the condition that at $r = r_1$ and r_4, $\frac{dT_n}{dr} = 0$. The N_n is the normalization factor needed to have (8c) true. We can write T_n in terms of Bessel functions of the zero and first order as

$$T_n = \left[J_o(k_n r) Y_1(k_n r_1) - J_1(k_n r_1) Y_o(k_n r) \right] . \qquad (17a)$$

The coefficients $A_{nn'}$, B_n, D_n, and N_n are given in terms of the Bessel functions of zero and first order as

$$B_n = N_n \omega^2 \left[T_n(r_4) - T_n(r_1) \right] ,$$

$$D_n = N_n \omega^2 (\mu_n - 1) \left[T_n(r_3) - T_n(r_2) \right] ,$$

and

$$A_{nn'} = \omega^2 \frac{(\mu_M - 1)}{(k_{n'}^2 - k_n^2)} N_n N_{n'} \left[-rk_n^2 T_n d_r T_{n'} + rk_{n'}^2 T_{n'} d_r T_n \right] \Bigg|_{r_2}^{r_3} \qquad (17b)$$

where $n \neq n'$; when

$n = n'$, we have

$$A_{nn} = \omega^2 (\mu_M - 1) N_n^2 \left[r \left\{ T_n \partial_r T_n + k_n^2 \left(\partial_{k^2}(T_n) \partial_r T_n - T_n \partial_r \partial_{k^2}(T_n) \right) \right\} \right] \Bigg|_{r_2}^{r_3} .$$

The N_n^2 is given by

$$N_n^2 = 1 \Big/ \left[k_n^2 \left(r_1 T_n(r_1) \partial_r \partial_{k^2} T_n(r_1) - r_4 T_n(r_4) \partial_r \partial_{k^2} T_n(r_4) \right) \right] .$$

If we use the recurrence relationships

$$d_r J_o(k_n r) = -k_n J_1(k_n r) ,$$

$$d_r Y_o(k_n r) = -k_n Y_1(k_n r) ,$$

$$d_r J_1(k_n r) = k_n J_o(k_n r) - \frac{1}{r} J_1(k_n r) ,$$

and

$$d_r Y_1(k_n r) = k_n Y_o(k_n r) - \frac{1}{r} Y_1(k_n r),$$

we find that $N_n^{\ 2}$ and A_{nn} become

$$N_n^{\ 2} = 1/\left[k_n^{\ 2}\left(T_n(r)\,r\right)\left(rT_n + r_1\left(Y_o(r_1)\,J_1(r) - Y_1(r)\,J_o(r_1)\right)\right)\Bigg|_{r_1}^{r_4}\right], \tag{18}$$

$$A_{nn} = \omega^2(\mu_M - 1)\quad N_n^{\ 2}\quad\Bigg[r\Bigg\{T_n d_r T_n + \frac{d_r T_n}{2}\Big[rd_r T_n k_n r_1\left(Y_o(k_n r_1)J_o(k_n r)\right.$$
$$\left.-Y_o(k_n r)J_o(k_n r_1)\right)\Big] + \frac{T_n}{2}k_n^{\ 2}\Big[rT_n + r_1\Big(Y_o(r_1)J_1(r)$$
$$- Y_1(r)J_o(r_1)\Big)\Big]\Bigg\}\Bigg|_{r_2}^{r_3}\ \Bigg] \tag{19}$$

In order to evaluate V_n and W_n, we note the definitions

$$B'_n \equiv T_n(r_4) - T_n(r_1),$$

and

$$D'_n \equiv T_n(r_3) - T_n(r_2), \tag{20}$$

and explicitly insert into the impedance, the G_n for V_n and W_n. The impedance finally becomes

$$Z = \frac{N^2\hat{i}\omega}{2\pi}\Bigg[\quad (z_4 - z_1)\ln(r_4/r_1) + (\mu_M - 1)(z_3 - z_2)\ln(r_3/r_2) + \tag{21}$$
$$+ \sum_{n=1}^{\infty} A_n N_n^{\ 2} B'_n\left\{B_n(R^1_n + R^3_n) + D_n(R^2_n + R^4_n)\right\} +$$
$$+ \sum_{n=1}^{\infty} A_n N_n^{\ 2} B'_n \sum_{n'>n} N_{n'}^{\ 2} A_{nn'} A_{n'}\Big\{B_{n'}(R^5_{nn'} + R^7_{nn'} + R^5_{n'n} + R^7_{n'n})$$
$$+ D_{n'}(R^6_{nn'} + R^8_{nn'} + R^6_{n'n} + R^8_{n'n})\Big\}$$

$$+(\mu_M - 1)\sum_{n=1}^{\infty} A_n D'_n N_n^{\,2} \left(D_n R^4_n + B_n R^3_n\right)$$

$$+(\mu_M - 1)\sum_{n=1}^{\infty} D'_n A_n N_n^{\,2} \sum_{n'>n} A_{nn'} A_{n'} N_{n'}^{\,2} \left\{ B_{n'}\left(R^7_{nn'} + R^7_{n'n}\right) + D_{n'}\left(R^8_{nn'} + R^8_{n'n}\right)\right\}$$

Here we have explicitly extracted the normalization factors N_n, $N_{n'}$, from B_n, D_n, and $A_{nn'}$. The R^1_n, R^2_n, etc., are defined as follows:

$$R^1_n = \int_{z_1}^{z_2} C_n(\bar{z})d\bar{z} \int_{z_1}^{\bar{z}} S_n(z)dz + \int_{z_1}^{z_2} S_n(\bar{z})d\bar{z} \int_{\bar{z}}^{z_2} C_n dz$$

$$+\int_{z_3}^{z_4} C_n d\bar{z} \int_{z_3}^{\bar{z}} S_n dz + \int_{z_3}^{z_4} S_n d\bar{z} \int_{\bar{z}}^{z_4} C_n dz + \left(\int_{z_1}^{z_2} S_n d\bar{z}\right)$$

$$\left(\int_{z_3}^{z_4} C_n dz + \int_{z_2}^{z_3} C_n dz\right) + \left(\int_{z_3}^{z_4} C_n d\bar{z}\right)\left(\int_{z_1}^{z_2} S_n dz + \int_{z_2}^{z_3} S_n dz\right)$$

$$R^2_n = \left(\int_{z_1}^{z_2} S_n(\bar{z})d\bar{z}\right)\left(\int_{z_2}^{z_3} C_n dz\right) + \left(\int_{z_3}^{z_4} C_n d\bar{z}\right)\left(\int_{z_2}^{z_3} S_n dz\right)$$

$$R^3_n = \int_{z_2}^{z_3} C_n d\bar{z} \int_{z_2}^{\bar{z}} S_n dz + \int_{z_2}^{z_3} S_n d\bar{z} \int_{\bar{z}}^{z_3} C_n dz + \left(\int_{z_2}^{z_3} C_n d\bar{z}\right)\left(\int_{z_1}^{z_2} S_n dz\right)$$

$$+\left(\int_{z_2}^{z_3} S_n d\bar{z}\right)\left(\int_{z_3}^{z_4} C_n dz\right)$$

$$R^4_n = \int_{z_2}^{z_3} C_n d\bar{z} \int_{z_2}^{\bar{z}} S_n dz + \int_{z_2}^{z_3} S_n d\bar{z} \int_{\bar{z}}^{z_3} C_n dz$$

$$R^5_{nn'} = \left(\int_{z_1}^{z_2} S_n dz\right)\left[\int_{z_2}^{z_3} C_n(\bar{z})d\bar{z} \left\{C_{n'} \int_{z_2}^{z} S_{n'}dz + S_{n'}\int_{z}^{z_3} C_{n'}dz + C_{n'}\left(\int_{z_1}^{z_2} S_{n'}dz\right)\right.\right.$$

$$\left.\left. + S_{n'}\left(\int_{z_3}^{z_4} C_{n'}dz\right)\right\}\right] + \left(\int_{z_3}^{z_4} C_n dz\right)\left[\int_{z_2}^{z_3} S_n(\bar{z})d\bar{z} \left\{C_{n'} \int_{z_2}^{\bar{z}} S_{n'}dz\right.\right.$$

$$\left.\left. + S_{n'}\int_{\bar{z}}^{z_3} C_{n'}dz + C_{n'}\left(\int_{z_1}^{z_2} S_{n'}dz\right) + S_{n'}\left(\int_{z_3}^{z_4} C_{n'}dz\right)\right\}\right]$$

$$R^6_{nn'} = \left(\int_{z_1}^{z_2} S_n dz\right)\left[\int_{z_2}^{z_3} C_n d\bar{z} \left(C_{n'} \int_{z_2}^{\bar{z}} S_{n'}dz + S_{n'} \int_{\bar{z}}^{z_3} C_{n'}dz\right)\right]$$

$$+ \left(\int_{z_3}^{z_4} C_n dz\right)\left[\int_{z_2}^{z_3} S_n d\bar{z} \left(C_{n'} \int_{z_2}^{\bar{z}} S_{n'}dz + \int_{\bar{z}}^{z_3} C_{n'}dz\right)\right]$$

$$R^7_{nn'} = \int_{z_2}^{z_3} C_n(\bar{z})d\bar{z} \int_{z_2}^{\bar{z}} S_n(z)dz \left[C_{n'} \int_{z_2}^{z} S_{n'}d\bar{\bar{z}} + S_{n'} \int_{z}^{z_3} C_{n'}d\bar{\bar{z}} + \right.$$

$$\left. + C_{n'}\left(\int_{z_1}^{z_2} S_{n'}d\bar{\bar{z}}\right) + S_{n'}\left(\int_{z_3}^{z_4} C_{n'}d\bar{\bar{z}}\right)\right] + \int_{z_2}^{z_3} S_n(\bar{z})dz \int_{\bar{z}}^{z_3} C_n(z)dz$$

$$\cdot \left[C_{n'} \int_{z_2}^{z} S_{n'} d\bar{\bar{z}} + S_{n'} \int_{z}^{z_3} C_{n'} d\bar{\bar{z}} + C_{n'} \left(\int_{z_1}^{z_2} S_{n'} d\bar{\bar{z}} \right) + S_{n'} \left(\int_{z_3}^{z_4} C_{n'} d\bar{\bar{z}} \right) \right]$$

$$R^8_{nn'} = \int_{z_2}^{z_3} C_n(\bar{z}) d\bar{z} \int_{z_2}^{\bar{z}} S_n(z) dz \left[C_{n'} \int_{z_2}^{z} S_{n'} d\bar{\bar{z}} + S_{n'} \int_{z}^{z_3} C_{n'} d\bar{\bar{z}} \right]$$

$$+ \int_{z_2}^{z_3} S_n(\bar{z}) d\bar{z} \int_{\bar{z}}^{z_3} C_n(z) dz \left[C_{n'} \int_{z_2}^{z} S_{n'} d\bar{\bar{z}} + S_{n'} \int_{z}^{z_3} C_{n'} d\bar{\bar{z}} \right] .$$

By making use of common factors in R^1_n, etc., the above may be reduced to

$$R^1_n = R^2_n + 2F_{n12} G_{n34} + L^1_n \tag{22}$$

$$R^2_n = F_{n12} G_{n23} + F_{n23} G_{n34} \tag{23}$$

$$R^3_n = R^4_n + R^2_n \tag{24}$$

$$R^5_{nn'} = R^6_{nn'} + H^1_{nn'} F_{n12} F_{n'12} + H^2_{nn'} G_{n34} G_{n'34} + F_{n12} G_{n'34} H^3_{nn'} + G_{n34} F_{n'12} H^4_{nn'} \tag{25}$$

$$R^7_{nn'} = R^8_{nn'} + F_{n'12} L^4_{nn'} + G_{n'34} L^5_{nn'} \tag{26}$$

$$R^6_{nn'} = L^2_{nn'} F_{n12} + L^3_{nn'} G_{n34} \tag{27}$$

The remaining definitions are

$$L^1_n = \int_{z_1}^{z_2} C_n d\bar{z} \int_{z_1}^{\bar{z}} S_n dz + \int_{z_1}^{z_2} S_n d\bar{z} \int_{\bar{z}}^{z_2} C_n dz + \int_{z_3}^{z_4} C_n d\bar{z} \int_{z_3}^{\bar{z}} S_n d\bar{z}$$

$$+ \int_{z_3}^{z_4} S_n d\bar{z} \int_{\bar{z}}^{z_4} C_n dz$$

$$H^{1}_{nn'} = \int_{z}^{z_3} C_n C_{n'} dz$$

$$H^{2}_{nn'} = \int_{z_2}^{z_3} S_n S_{n'} dz$$

$$H^{3}_{nn'} = \int_{z_2}^{z_3} C_n S_{n'} dz$$

$$H^{4}_{nn'} = \int_{z_2}^{z_3} S_n C_{n'} dz$$

$$L^{2}_{nn'} = \int_{z_2}^{z_3} \left[C_n d\bar{z} \left(C_{n'} \int_{z_2}^{\bar{z}} S_{n'} dz + S_{n'} \int_{\bar{z}}^{z_3} C_{n'} dz \right) \right]$$

$$L^{3}_{nn'} = \int_{z_2}^{z_3} \left[S_n d\bar{z} \left(C_{n'} \int_{z_2}^{\bar{z}} S_{n'} dz + S_{n'} \int_{\bar{z}}^{z_3} C_{n'} dz \right) \right]$$

$$L^{4}_{nn'} = \int_{z_2}^{z_3} C_n d\bar{z} \int_{z_2}^{\bar{z}} S_n C_{n'} dz + \int_{z_2}^{z_3} S_n d\bar{z} \int_{\bar{z}}^{z_3} C_n C_{n'} dz$$

$$L^{5}_{nn'} = \int_{z_2}^{z_3} \left[C_n \int_{z_2}^{\bar{z}} S_n S_{n'} dz + S_n \int_{\bar{z}}^{z_3} C_n S_{n'} dz \right] d\bar{z}$$

$$F_{n12} = \int_{z_1}^{z_2} S_n dz$$

$$F_{n23} = \int_{z_2}^{z_3} S_n dz$$

$$G_{n23} = \int_{z_2}^{z_3} C_n dz \quad ,$$

and

$$G_{n34} = \int_{z_3}^{z_4} C_n dz \; .$$

The results of integration of the above definitions are listed as follows. The single integral expressions become

$$F_{n12} = \frac{1}{\lambda_n} \left[a_{n12} + (a_{n12})^{-1} - 2 \right] \tag{28}$$

$$F_{n23} = \frac{1}{\bar{\lambda}_n} \sum_i \bar{S}_n^{\,i} (-1)^{i+1} \left[(a_{n23})^{(-1)^i} - 1 \right] \tag{29}$$

$$G_{n23} = \frac{1}{\bar{\lambda}_n} \sum_i \bar{C}_n^{\,i} (-1)^{i+1} \left[1 - (a_{n23})^{(-1)^{i+1}} \right] \tag{30}$$

$$G_{n34} = \frac{1}{\lambda_n} \left[2 - a_{n34} - (a_{n34})^{-1} \right] . \tag{31}$$

If we define

$$\lambda_{ml} (i, j) \equiv \left[(-1)^{i+1} \bar{\lambda}_n + (-1)^{j+1} \bar{\lambda}_{n'} \right] \tag{32a}$$

$$P_1(i, j) \equiv \left[1 - (a_{n23})^{(-1)^{i+1}} (a_{n'23})^{(-1)^{j+1}} \right] / \lambda_{ml}(i, j) \quad ,$$

and

$$P_2(i, j) \equiv \left[(a_{n'23})^{(-1)^j} - (a_{n23})^{(-1)^{i+1}} \right] / \lambda_{ml}(i, j) \quad , \tag{32b}$$

then the remaining single integral becomes

$$H^1_{nn'} = \sum_{ij} \bar{C}^i_n \bar{C}^j_{n'} P_1(i,j)$$

$$H^2_{nn'} = \sum_{ij} \bar{S}^i_n \bar{S}^j_{n'} P_1(i-1, j-1)$$

$$H^3_{nn'} = \sum_{ij} \bar{C}^i_n \bar{S}^j_{n'} P_2(i,j) = H^4_{n'n}$$

$$H^4_{nn'} = \sum_{ij} \bar{S}^i_n \bar{C}^j_{n'} P_2(i-1, j-1) = H^3_{n'n} \; . \tag{33}$$

The double integrations give

$$R^4_n = \sum_{ij} \frac{\bar{C}^i_n \bar{S}^j_n}{\bar{\lambda}_n} \left[\frac{(-1)^{j+i}}{\bar{\lambda}_n} \left[-2 + (a_{n23})^{(-1)^{i+1}} + (a_{n23})^{(-1)^j}\right] \right.$$
$$\left. + (z_2 - z_3)(a_{n23})^{(-1)^j} \left[(-1)^j - (-1)^i\right] \right] \tag{34}$$

$$L^1_n = \frac{1}{\lambda_n} \sum_{ij} \left[C^i_n (-1)^{j+1} \left\{ (-1)^{j+i} \left[-2 + (a_{n12})^{(-1)^{i+1}} + (a_{n12})^{(-1)^j}\right] / \lambda_n \right. \right.$$
$$\left. + (z_1 - z_2)(a_{n12})^{(-1)^j} \left[(-1)^j - (-1)^i\right] \right\}$$
$$+ S^j_n (-1)^{i+1} \left\{ (-1)^{j+i} \left[-2 + (a_{n34})^{(-1)^{i+1}} + (a_{n34})^{(-1)^j}\right] / \lambda_n \right.$$
$$\left. \left. + (z_3 - z_4)(C_{n34})^{(-1)^j} \left[(-1)^j - (-1)^i\right] \right\} \right] \tag{35}$$

Upon further definition

$$P_4(i) \equiv \left[1.0 - (a_{n23})^{(-1)^{i+1}}\right] / \bar{\lambda}_n (-1)^{i+1} \tag{36}$$

and

$$P_7(i,j) \equiv P_4(i)\ (a_{n'23})^{(-1)^j} \quad ; \tag{37}$$

the two-fold integrations give

$$L^2_{nn'} = \sum_{i\,j\,\ell} \frac{\overline{C}^{\ell}_{n}\,\overline{C}^{i}_{n'}\,\overline{S}^{j}_{n'}}{\overline{\lambda}_{n'}} \Big[-\Big\{(-1)^j - (-1)^i\Big\} P_7(\ell,j) + (-1)^j P_1(\ell,i)$$

$$+ (-1)^{i+1} P_2(\ell,j) \Big] \tag{38}$$

$$L^3_{nn'} \equiv \sum_{i\,j\,\ell} \frac{\overline{S}^{\ell}_{n}\,\overline{C}^{i}_{n'}\,\overline{S}^{j}_{n'}}{\overline{\lambda}_{n'}} \Big[-\Big\{(-1)^j - (-1)^i\Big\} P_7(\ell-1,j) + (-1)^j$$

$$\cdot\ P_2(\ell-1, i+1) + (-1)^{i+1} P_1(\ell-1, j-1) \Big].$$

(39)

If we define

$$\lambda_{m2}(i,j,\ell) \equiv \overline{\lambda}_n \left[(-1)^{j+1} + (-1)^{i+1} \right] + \overline{\lambda}_{n'} (-1)^{\ell+1} \quad , \tag{40}$$

$$\lambda_{m3}(\ell,i,j) \equiv \overline{\lambda}_{n'} \left[(-1)^{i+1} + (-1)^{j+1} \right] + \overline{\lambda}_n (-1)^{\ell+1} \quad , \tag{41}$$

and

$$R_3(i,j,\ell) \equiv \left[(a_{n23})^{(-1)^j} - (a_{n23})^{(-1)^{i+1}} (a_{n'23})^{(-1)^{\ell+1}} \right] / \lambda_{m2}(i,j,\ell), \tag{42}$$

the remaining two-fold integrations are

$$L^4_{nn'} = \sum_{i\,j\,\ell} \overline{C}^{i}_{n}\,\overline{S}^{j}_{n}\,\overline{C}^{\ell}_{n'} \Big[\Big(P_3(i,j,\ell) - P_7(i,\ell+1) \Big) / \lambda_{m1}(j,\ell)$$

$$+ \Big(-P_3(i,j,\ell) + P_4(j-1) \Big) / \lambda_{m1}(i,\ell) \Big] \tag{43}$$

$$L^5_{nn'} = \sum_{i\,j\,\ell} \overline{C}^i_n \overline{S}^i_n \overline{S}^\ell_{n'} \left[\Big(P_3(j-1, i+1, \ell-1) - P_4(i) \Big) / \lambda_{m1}(j,\ell) + \Big(-P_3(j-1, i+1, \ell-1) + P_7(j-1, \ell) \Big) / \lambda_{m1}(i,\ell) \right] \tag{44}$$

The triple integration gives

$$R^8_{nn'} = -\sum_{i\,j\,\ell\,m} \frac{\overline{C}^i_n \overline{C}^j_{n'} \overline{S}^\ell_n \overline{S}^m_{n'}}{\overline{\lambda}_{n'}}$$

$$\cdot \Bigg[(-1)^m \Big[\Big(P_5(i,j,\ell,m) - P_7(i,j+1) \Big) / \lambda_{m3}(\ell,j,m)$$

$$\Big(-P_3(i,\ell,j) + P_7(i,j+1) \Big) / \lambda_{m1}(\ell,j) \Big]$$

$$(-1)^j \Big[\Big(-P_5(i,j,\ell,m) + P_7(i,j+1) \Big) / \lambda_{m3}(\ell,j,m)$$

$$\Big(P_3(\ell-1, i+1, m-1) - P_4(i) \Big) / \lambda_{m1}(\ell,m) \Big]$$

$$(-1)^m \Big[\Big(-P_5(i,j,\ell,m) + P_7(\ell-1,m) \Big) / \lambda_{m3}(i,j,m)$$

$$\Big(P_3(i,\ell,j) - P_4(\ell-1) \Big) / \lambda_{m1}(i,j) \Big]$$

$$(-1)^j \Big[\Big(P_5(i,j,\ell,m) - P_7(\ell-1,m) \Big) / \lambda_{m3}(i,j,m)$$

$$\Big(-P_3(\ell-1, i+1) + P_7(\ell-1,m) \Big) / \lambda_{m1}(i,m) \Big] \Bigg]. \tag{45}$$

Here the definition

$$P_5(i,j,\ell,m) \equiv \frac{\left[(a_{n23})^{(-1)^\ell} (a_{n'23})^{(-1)^m} - (a_{n23})^{(-1)^{i+1}} (a_{n'23})^{(-1)^{j+1}} \right]}{\overline{\lambda}_n \left[(-1)^{i+1} + (-1)^{\ell+1} \right] + \left[(-1)^{j+1} + (-1)^{m+1} \right] \overline{\lambda}_{n'}} \tag{46}$$

is used.

4. Conclusion

The derivation of the formula for the impedance of a toroid sample in a toroid has been completed. In (21), one can note that the Z is a dimensionless quantity. To make it have the dimensions of ohms, we must multiply Z by Z_o, where $Z_o = \sqrt{\frac{\mu_o}{\epsilon_o}} = 376.7\Omega$ is the free space impedance. In addition, if one examines (9) and (10), one can see that under the circumstance that the ferrite sample has $r_1 = r_2$ and $r_3 = r_4$, that $A_{nn'} = 0$ when $n' \neq n$. This means that the formula (21) is now mathematically exactly correct. However, the physical effects of the closeness of the winding on the outside toroid becomes a major source of error. This means that the condition under which the large disk toroid may be used must be such that the effects of the winding can be neglected. Such condition can be accurately determined only if one used a theory for which the non-uniformity of the windings has been explicitly accounted for. Rather than to have to develop such a theory, we avoid such needs by using the present formula in a way that will make them sufficiently accurate. This may be done by mathematically varying the sample size relative to the outside current sheet toroid. When the double sum correction in the impedance formula (21) is less than some figure of merit, say 2%, of the total correct impedance, one can be reasonably sure that the resulting impedance values are known to 0.5%.*

---

*Note - The criteria used to establish the 0.5% figure is developed in the FORTRAN subroutine IMPED. Here the double sum series is truncated by the requirement that the last correction to the double sum correction be less than 10%. The single sum series is truncated by the requirement that the last correction to the single sum part be less than 0.5%. Because the two series have alternating signs and apparently converge like $1/n^2$ for the double sum and like $1/n^3$ for the single sum, the error made in the evaluation of the impedance is determined by the value of the largest finally accepted correction - namely 0.5%.

In the theory and experiment the effects of the reactive coupling of the wire coil to the sample can be minimized by placing the sample as far as possible from the wire coil--namely the exact center of the coil. To make explicit comparison of the theory with the experiment, one must have the following operational procedure: first, the experiment is done with and without the sample. The two resulting impedances are then subtracted to produce an effective impedance Z_{eff}. The reason for this subtraction is to attempt the removal of the effects of external fringing in the impedance results. These effects are due to the coil arrangement which allows magnetic flux to exist outside the toroidal cross section. For this operation to be meaningful the assumption is made that the fringing field is the same for the system , with or without the sample. This condition is the same as the theoretical assumption that one can use a uniform current sheet to characterize the internal impedance of the toroid. However, to have a more precise relationship to the experimental procedure, one must produce a Z_{eff} (for the theory) in exactly the same manner as was the Z_{eff} produced for the experiment. This means that we compute the case where the ferrite core has $\mu = \epsilon = 1$ to obtain Z_{vac} and then calculate for the case of μ, ϵ equal the desired values to get Z_t.* The effective impedance is given by $Z_{eff} = Z_t - Z_{vac}$. This result may now be compared with the experimental effective impedance. To get a reliable comparison

---

*Note - The experiment produces two numbers, while we need four numbers to fix the theoretical impedance. It should be obvious that to fix the four numbers it is necessary to do a second measurement under known conditions, so that we have sufficient experimental data to get the μ and ϵ. The second measurement is to use a parallel plate capacitor.

one should have several sample sizes for the same material. By assuming that the material is sufficiently homogenous, this comparison will give a check on the accuracy of the above correspondence.

The appendix I gives the FORTRAN and FLOW chart needed to calculate the correct impedance. The main program, TOROID, gives the definitions and indicates the needed input. The number appearing on the far right of the listing corresponds roughly to the number of the same equation in the text. This program takes about 0.1 minute per point. If one is to do very many points, say 6000 or more, it would be desirable to make the program more efficient. The dimensions of the system must be such that $\left(\frac{Z_4 - Z_1}{r_4 - r_1}\right)$ $3n \leq \ln$ [Largest number allowed in the computing machine.] The n is the largest number of modes needed to satisfy the figure of merit. (Usually $n \leq 7$.)

Because of the transcendental nature of the Equation (21), we cannot solve for the μ, ϵ, explicitly. This means that the best one can do is to prepare a table of impedance in which the values of μ and ϵ are varied over ranges of interest. Such a table will be forthcoming when the needs of the various personnel have been determined.

To aid in the use of this program, sample output is included in appendix II. The reader should note that the 1.5×10^6 Hz case strongly violates the region of validity of the theory.

APPENDIX I

```
'EQUIP,5=60
'EQUIP,6=61
'FTN,X,*
C     MAIN PROGRAM FOR TOROID CALCULATION                                 TOROID
CTOROID
      TYPE COMPLEX CZII,CFIX,CZ,CZI,CZIII,ZC,CR,CV,DUM,DELV,W
      COMMON R,Z,V,FIX,   FR  ,ZI,ZII,IT,IC,ITT ,FR2,ICYLE ,DUM,ZC        TOROID
      DIMENSION IC(6),FR2( 5004)
      DIMENSION ZII(2),FIX(2),Z(8),CZ(4),ZI(2),ZIII(2),ZC(58),CR(5),
     1            R(10),CV(2),V(4),DUM(10),DELV(2),W(2),UM(20)
      EQUIVALENCE (CZII,ZII),(CFIX,FIX),(CZ,Z),(CZI,ZI),(CZIII,ZIII),
     1            (CR,R),(CV,V),(DUM,UM)
      DO 980 I=1,5796
  980 R(I)=0.
    1 READ(5,2)(R(I),I=1,8,2),(Z(I),I=1,8,2),TURN,IC(5),IC(6)
 2    FORMAT( 4E18.5/4E18.5/E18.5,4I4)
      IF(EOF,5)100,999
999   CONTINUE
      IT=-1                                                               TOROID
 101  READ INPUT TAPE 5,102,ER,IC(4),IS,ITES
      IF(EOF,5)100,200
  200 CONTINUE
      FR=ER*(2.0944E-10)
 102  FORMAT(E18.5,3I4)
C    1THE INPUT IS THE RADIUS VARIABLES,THE Z AXIS VARIABLES,THE MAGNETITOROID
C    2C PERMIBILITY,AND DIELECTRIC PERMITIVITY,TURN COUNT,AND FREQUENCY TOROID
C    3ALL DIMENSIONS ARE ASSUMED TO BE IN CM. THE FREQUENCY IS HERTZ.
C         FREE SPACE MU AND EPSILON IS ONE. THE INPUT FORMAT IS FLOATINGTOROID
C     V(1)+IV(3) IS THE DIELECTRIC PERMITIVITY AND V(2)+IV(4) IS THE
C       COMPLEX PERMIBILITY
      DO 4 I=2,8,2
    4 Z(I) = R(I) = 0.
      FIX(1)=0.                                                           TOROID
    6 WRITE(6,7)IS,ER,TURN,(R(I),I=1,8,2),(Z(I),I=1,8,2)
 7    FORMAT( 44H1IMPEDANCE RESULTS FOR  TOROID GEOMETRY PAGE I4/        1TOROID
     18H THE FREQUENCY IS E18.4,25HHZ.   THE TURNS COUNT IS  F4.0,/      2TOROID
     27H THE RADIAL DIMENSIONS ARE     4E18.4    /                       2TOROID
     37H THE Z AXIS DIMENSIONS ARE  4E18.4  //                           9TOROID
     41H THE LENGTH MEASUREMENTS ARE IN CM.  IMPEDANCES ARE IN OHMS.  THE
     5E REST ARE DIMENSIONLESS.     //115H            MU
     6   EPSILON              QUASI STATIC IMP          CORRECT IMP
     7       DBL SUM IMP  /125H U1           +I U2            E1          +I E2
     8           Z1          +I Z2         ZC1       +I ZC2         DZ1
     9 +I DZ2         CNT              )
      ITT=1
 9    IS=IS+1                                                             TOROID
      IF(IT) 71,71,13
 71   IV=IC(4)                                                            TOROID
 72   FIX(2)=  FR*(59.95     )*(TURN**2)
      DO 13 I1=1,IV                                                       TOROID
 211  FORMAT(4E18.5)
      READ(5,211)V(1),V(3),V(2),V(4)
      CALL  IMPED                                                         TOROID
      CZII=CFIX*CZII
```

```
      CZI=CZII+CFIX*CZI
      CZIII=ZC(1)*CFIX
      WRITE(6,8)V(3),V(4),V(1),V(2),ZIII,ZI,ZII,ICYLE
      ITT=ITT+1                                                         TOROID
 8    FORMAT( E13.4,9E12.4,I4)
      IF(ITT-46)13,6,6
 13    CONTINUE
      IF(ITES)1,1,103                                                   TOROID
 103  IT=0                                                              TOROID
      GO TO 101                                                         TOROID
  100 END
      SUBROUTINE IMPED                                                  IMPED
      TYPE COMPLEX CFR2,R,Z,VE,Z12,Z23,Z34,ZA1,ZA,SV,FR3,ZC,AD,AE,DUM,
     1             AN,XI,XJ,A4,B1,D1,A5,A12,A34,A23,CB,ZB,V,FIX,ZI,ZII,
     2             F12,G34,ALAM
      DIMENSION IC(6),TD(100,2),WK(100),T(100,2),ADI(100) ,TW(8)        IMPED
      DIMENSION FR2(2),R(5),Z(4),ZC(58),DUM(10),AN(100,5),A4RL(2),B1(100
     1            ),D1(100),A5RL(2),A12(100,2),A34(100,2),A23(100,2),
     2            UM(20),
     2             CB(100,8),V(2),F12(100),ALAM(100,2),G34(100)
      EQUIVALENCE (CFR2,FR2),(A4,A4RL),(XI,XIRL),(XJ,XJRL),(A5,A5RL),
     1(DUM,UM)
      COMMON R,Z,V,FIX,FR,ZI,ZII,IT,IC,ITT,FR2,FR3,B1,D1,AN,  F12,G34,  IMPED
     1ALAM,CB,A23,A34,A12 ,ICYLE,DUM,ZC,TD,WK,T,ADI ,Z12,Z23,Z34        IMPED
     2,TW,SV,VE
      ICYLE=0                                                           IMPED
      IF OVERFLOW FAULT 10,10
 10   IF(IT)1,101,2                                                     IMPED
 1    Z34=Z(3)-Z(4)                                                     IMPED
      Z12=Z(1)-Z(2)                                                     IMPED
      Z23= Z(2)-Z(3)                                                    IMPED
      ZA1=Z23*CLOG(R(3)/R(2))                                           IMPED
      ZA=  (Z(4)-Z(1))*CLOG(R(4)/R(1))                                  IMPED
      FR2(2)=0.0                                                        IMPED
      IA=IC(5)                                                          IMPED
 101  FR2(1)=(FR)**2                                                    IMPED
      CALL TXX(1,1)                                                     IMPED
      IT=1                                                              IMPED
 2    SV=V(2)-(1.0,0.0)                                                 IMPED
      VE= V(1)*V(2)-(1.0,0.0)                                           IMPED
      FR3=CFR2*VE
      N2=1                                                              IMPED
      CALL SCX(N2,1)                                                    IMPED
 102  IF(N2)42,42,201                                                   IMPED
 201  DO 3 I=2,58                                                       IMPED
 3    ZC(I)=(0.0,0.0)                                                   IMPED
      ZC(1)=ZA-ZA1*SV                                                   IMPED
      DO 40 N=1,IA                                                      IMPED
      N1=N+1                                                            IMPED
      N2=N                                                              IMPED
      IB=IC(6) +N                                                       IMPED
      IF ( IB .GT. 100 ) IB=100
 301  DO 303 L=25,28                                                    IMPED
 303  ZC(L)=(0.0,0.0)                                                   IMPED
```

```
      DO 30 M=N1,IB                                                     IMPED
      ICYLE=ICYLE+1                                                     IMPED
      M2=M                                                              IMPED
      CALL TXX(N2,M2)                                                   IMPED
      CALL SCX(N2,M2)                                                   IMPED
  103 IF(N2)42,42,304                                                   IMPED
  304 AD= AN(M,1)*DUM(8)                                                IMPED
      AE= AN(M,2)*DUM(8)                                                IMPED
      ZC(35)=       DUM(4)*AD*AN(N,1)                                   IMPED
      ZC(36)=       DUM(5)*AE*AN(N,1)                                   IMPED
      ZC(37)=DUM(6)*AE*AN(N,2)                                          IMPED
      ZC(38)=        DUM(7)*AE*AN(N,2)                                  IMPED
      ZC(26)=ZC(35)+ZC(36)*SV+(ZC(37)+ZC(38)*SV)*VE                     IMPED
      ZC(25)=ZC(25)+ZC(26)                                              IMPED
 1000 PRINT 1001,M ,AD,AE,AN(M,1),AN(M,2),SV,ZC(25),ZC(26),(ZC(I),I=35,38
     X8), (DUM(I),I=4,8)
 1001 FORMAT(6H 1001 I2,5C(E12.4,E12.4)/(8X,5C(E12.4,E12.4)  )  )
      XI=CABS(ZC(26))
      XJ=CABS(ZC(25))
      XK=XJRL*(0.1)-XIRL
      IF(XK)30,31,31
   30  CONTINUE                                                         IMPED
      ITT=ITT+2                                                         PTEST
      A4=FR3*CFR2*FIX*ZC(26)
       WRITE OUTPUT TAPE 6,11,N2,A4RL
   11  FORMAT(54H WARNING CONVERGENCE IS SUCH THAT ON  DBLE SUM CYCLE   PTEST
     1   I4/25H THE LAST IMPEC COR. WAS  E14.4,  4H +I  E14.4  )        PTEST
   31 ZC(5)=ZC(5)+ZC(25)
      ZC(12)=       AN(N,1)*B1(N)*DUM(1)                                IMPED
      ZC(13)=       AN(N,1)*D1(N)*DUM(2)                                IMPED
      ZC(14)=       AN(N,2)*D1(N)*DUM(3)                                IMPED
      ZC(4)=ZC(12)+ZC(13)*SV+(ZC(13)+ZC(14)*SV)*VE
      ZC(2)=ZC(2)+ZC(4)
      XI=CABS(ZC(4))
      XJ=CABS(ZC(2))
      XK=XJRL*(0.005)-XIRL
      IF(XK)40,41,41
   40 CONTINUE                                                          IMPED
      ITT=ITT+2
      A5=CFR2*FIX*(ZC(4)+FR3*ZC(25))
       WRITE OUTPUT TAPE 6,22,IA,A5RL
   22 FORMAT(57H WARNING TOTAL CONVERGENCE IMPLIES THE SINGLE SUM COUNT PTEST
     1  I4/25H THE LAST IMPED COR. WAS  E14.4,  4H +I   E14.4)          PTEST
   41 ZI=ZC(1)+CFR2*ZC(2)
      ?II=FR3*CFR2*ZC(5)
      IF OVERFLOW FAULT 43,42
   43 WRITE OUTPUT TAPE 6,44
   44 FORMAT(47H ACCUMULATOR OVERFLOW NEXT LINE HAS NO MEANING    )
   42 RETURN
      END                                                               IMPED
      SUBROUTINE TXX(N,M)                                               TXX
      TYPE COMPLEX TWO,ONE,AVE,ZRO,CX22,CR,B1,D1,AN,ATEM,A12,A34,G34,F12
     1          ,ALAM
      DIMENSION X22(2),CR(5),R(10),B1(100),B1RL(200),D1(100),D1RL(200),
```

```
     1          AN(100,5),A12(100,2),A34(100,2),F12(100),G34(100),
     2          ALAM(100,2),ANRL(1)
      DIMENSION IC(6),TD(100,2),WK(100),T(100,2),ADI(100)              IMPED
     1,TB(2)     ,T11(4),T12(2)
      EQUIVALENCE (CX22,X22),(CR,R),(B1,B1RL),(D1,D1RL),(AN,ANRL)
      COMMON R,Z,V,FIX,FR,ZI,ZII,IT,IC,ITT,FR2,FR3,B1,D1,AN,  F12,G34, IMPED
     1ALAM,CB,A23,A34,A12 ,ICYLE,DUM,ZC,TD,WK,T,ADI ,Z12,Z23,Z34       IMPED
     2,ONE,TWO,ZRO,AVE
      COMPLEX Z,V,FIX,ZI,ZII,FR2,FR3,CB,A23,DUM,ZC,Z12,Z23,Z34
      DIMENSION Z(4),V(2),CB(100,8),A23(100,2),DUM(10),ZC(58)
      IF(IT  )1,101,2
 1    N1=1                                                              TXX
      TWO=(2.0,0.0)                                                     TXX
      ONE=(1.0,0.0)                                                     TXX
      AVE=(0.5,0.0)                                                     SCX
      ZRO=(0.0,0.0)                                                     SCX
      X22(2)=0.0                                                        TXX
      R(9)=1.5/(R(7)-R(1))
      R(10)=0.1 *R(9)
 101  N2=1
 2    IF(M-N1)41,3,3
 3    N1=N1+1                                                           TXX
      B1RL(2*M)=D1RL(2*M) =ANRL( 2*M+400 ) = 0.
      WK(M)= BESSEL(5,1)                                                TXX
      WK1=WK(M)                                                         TXX
      X2=BESSEL(1,4)                                                    TXX
      X3=BESSEL(1,2)                                                    TXX
      X4= BESSEL(1,1)                                                   TXX
      X1= BESSEL(1,3)                                                   TXX
      DO 33 I=2,3                                                       TXX
       K=I-(-1)**I                                                      TXX
       J=I-1                                                            TXX
      TD(M,J)=(BESSEL(I,4)*X1-BESSEL(I,3)*X2)*WK1                       TXX
      TB(J)=BESSEL(K,1)*X2-BESSEL(K,2)*X1
      T(M,J)=BESSEL(I,1)*X2-BESSEL(I,2)*X1
      T12(J)=BESSEL(I,1)*X3-BESSEL(I,2)*X4
      T11(I)=BESSEL(I,3)*X3-BESSEL(I,4)*X4
 33   T11(K)=BESSEL(K,3)*X3-BESSEL(K,4)*X4
      AN2=0.0                                                           TXX
      B1(M)=TB(2)-TB(1)                                                 TXX
      D1(M)=T(M,2)-T(M,1)                                               TXX
      WK2=WK(M)**2                                                      TXX
      ADI(M)=0.0                                                        TXX
      DO 35 I=2,3                                                       TXX
      IF(I.EQ.2)340,341
  340 K=KK=I       $ II=3 $ GO TO 342
  341 K=4 $ KK=7 $ II=5
  342 J=I-1
      AN2=AN2-(-1.0)**I*(TB(J)*R(KK))*(TB(J)*R(KK)+R(1)*T11 (   K ))    TXX
 35   ADI(M)=ADI(M)-(-1.0)**I*R(II)*(T(M,J)*WK2*(R(II)*T(M,J)+R(1)*T11 (  TXX
     1I  ))+TD(M,J)*(2.0*T(M,J)+R(II)*TD(M,J)+ WK(M)*R(1)*T12 (J)))    TXX
      ADI(M)=0.5*ADI(M)                                                 TXX
      AN(M,3)= 1.0/( WK2*AN2)                                           TXX  18
      ADI(M)=ADI(M)*AN(M,3)                                             TXX
```

```
      X22(1)= WK2-FR2                                                   TXX
      GO TO 39
41    IF(M-N2)4,38,38
38    X22(1)=WK(M)**2-FR2
39    ALAM(M,1)= CSQRT(CX22)
      N2=N2+1
      A12(M,1)= CEXP(ALAM(M,1)*Z12)
      A34(M,1)= CEXP(ALAM(M,1)*Z34)
      A34(M,2)=ONE/A34(M,1)                                             TXX
      A12(M,2)= ONE/A12(M,1)                                            TXX
      G34(M)=(TWO-A34(M,1)-A34(M,2))/ALAM(M,1)                          TXX
      F12(M)=(A12(M,1)+A12(M,2)-TWO)/ALAM(M,1)                          TXX
      ATEM   =AN(M,3)/ALAM(M,1)
      AN(M,1)=B1(M)*ATEM
      AN(M,2)=D1(M)*ATEM
      AN(M,4)=ONE
4      RETURN                                                           TXX
       END                                                              TXX
      FUNCTION BESSEL(I,K)
      DIMENSION R(10),B(12,4)
      DIMENSION S(19)
      COMMON R,S,IT                                                     BESSEL
      IF(IT)100,1010,1010                                               BESSEL
100   A5= R(9)
      R(10)=5.0*R(10)
      IT=1
      ISV=2
      IM=1
      A4= 1.0E+04                                                       BESSEL
1010  GO TO (10,10,10,10,1),I                                           BESSEL
1     DEL=R(10)                                                         BESSEL
14    R(9)=R(9)+DEL
141   X=R(9)*R(1)
      Y= X/3.0                                                          BESSEL
      IF(Y-1.0) 101,110,110                                             BESSEL
101   I1=5                                                              BESSEL
      GO TO 3                                                           BESSEL
15    X=R(9)*R(7)
      Y= X/3.0                                                          BESSEL
      IF(Y-1.0) 102,103,103                                             BESSEL
102   I1=6                                                              BESSEL
      GO TO 3                                                           BESSEL
103   I1=5                                                              BESSEL
      GO TO 4                                                           BESSEL
16    ANULL=B( 5,3)*B(6,4)-B(6,3)*B(5,4)                                BESSEL
      GO TO 71                                                          BESSEL
20    ANULL=(B(5,3)*COSF(PH1)+B(5,4)*SINF(PH1))*(-F1)                   BESSEL
      GO TO 71                                                          BESSEL
110   I1=6                                                              BESSEL
       Y=1.0/Y                                                          BESSEL
      GO TO (41,400),  ISV
21     PH2=PH1                                                          BESSEL
      F2=F1
      I1=7                                                              BESSEL
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```
      X=R(9)*R(7)
      Y= 3.0/X                                                          BESSEL
     GO TO (41,400),  ISV
22   ANULL= PH1-PH2                                                     BESSEL
     ANULL=F2*F1*SINF(ANULL)
     GO TO 71
2     Y= X/3.0                                                          BESSEL
      IF(Y-1.0)3,3,4                                                    BESSEL
3     Y= Y**2                                                           BESSEL
     IF ( X ) 910,910,912
910  X4=0.   $ GO TO 914
912  X4=0.63661977*LOGF(X/2.0)                                          BESSEL
914   B(I1,3)= X*(0.5-Y*(0.56249985-Y*(0.21093573-Y*(0.03954289-Y*(0.00BESSEL
    1443319-Y*(0.00031761-Y*(0.00001109)))))))                          BESSEL
      B(I1,4)=                      X4*B(I1,3) -((0.6366198-Y*(0.2212091+YBESSEL
    1*(2.1682709-Y*(1.3164827-Y*(0.3123951-Y*(0.0400976-Y*(0.0027873)))BESSEL
    2)))))/ X                                                           BESSEL
     GO TO (6,6,6,6,15,16),I1                                           BESSEL
4     Y=1.0/Y                                                           BESSEL
400  X1= 1.0/SQRTF(X)                                                   BESSEL
     F1=         X1 *(0.79788456+Y*(0.00000156+Y*(0.01659667+Y*(0.00017105BESSEL
    1-Y*(0.00249511-Y*(0.00113653-Y*(0.00020033)))))))                  BESSEL
41   PH1= X-(0.78539816-Y*(0.12499612+Y*(0.00005650-Y*(0.00637879-Y*(   BESSEL
    10.00074348+Y*(0.00079824-Y*(0.00029166)))))))                      BESSEL
     GO TO(42,42,42,42,20,21,22),I1                                     BESSEL
42    B(I1,3)= F1*SINF(PH1)                                             BESSEL
      B(I1,4)=-F1*COSF(PH1)                                             BESSEL
     GO TO 7                                                            BESSEL
6    B(I1,1)=1.0-Y*(2.2499997-Y*(1.2656208-Y*(0.3163866-Y*(0.0444479-Y*BESSEL
    1(0.0039444-0.0002100*Y)))))                                        BESSEL
     B(I1,2)= X4*B(I1,1)+0.36746691+Y*(0.60559366-Y*(0.74350384-Y*(     BESSEL
    10.25300117-Y*(0.04261214-Y*(0.00427916-Y*(0.00024846))))))         BESSEL
     GO TO 8                                                            BESSEL
7    F0= X1*(0.79788456-Y*(0.00000077+Y*(0.00552740+Y*(0.00009512-Y*(   BESSEL
    10.00137237-Y*(0.00072805-Y*(0.00014476)))))))                      BESSEL
     PH0=X-(0.78539816+Y*(0.04166397+Y*(0.00003954-Y*(0.00262573-Y*(    BESSEL
    10.00054125+Y*(0.00029333-Y*(0.00013558)))))))                      BESSEL
     B(I1,1)=F0*COSF(PH0)                                               BESSEL
     B(I1,2)=F0*SINF(PH0)                                               BESSEL
     GO TO 8                                                            BESSEL
71    IF(SIGNF(ANULL,A4)-ANULL)741,74,741
74   IM=1                                                               BESSEL
740  A4=ANULL                                                           BESSEL
     GO TO 14                                                           BESSEL
741  GO TO (742,740),IM                                                 BESSEL
742  R(9)=R(9)+(DEL*ANULL)/(A4-ANULL)
     DEL=(-0.5)*DEL                                                     BESSEL
     A4=ANULL                                                           BESSEL
     ANULL=ABSF(ANULL)
     IF(ANULL-1.0E-05)75,75,141                                         BESSEL
 75  B(5,1)=R(9)
     GO TO (77,751),ISV
751  IF(R(9)*R(1)/3.0-1.0)77,76,76
76   ISV=1
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```
      F1=1.0
77    DO 8 J=1,4                                                        BESSEL
      I1=J                                                              BESSEL
      JJ=J+J-1
      X=R(JJ)*R(9)
      GO TO 2                                                           BESSEL
8     CONTINUE                                                          BESSEL
      IM=2                                                              BESSEL
      R(9)=R(9)+ A5
10    BESSEL      =B(I,K)
      RETURN                                                            BESSEL
      END
      SUBROUTINE SCX(N,M)                                               SCX
      TYPE COMPLEX A12,A23,A34,ADD,AH,AH1,AH2,AH3,AH4,AK1,AK2,AK4,AK5,
     1            AK6,AK7,AK8,AL2,AL3,AL4,AL5,ALAM,ALM,ALM2,ALM3,AN,AVE
     2           ,B1, CB, D1,DIV,DUM, F12,F23,FIX,FR2,FR3, G23,G34,ONEC
     3           ,R1,R2,RD1,RD2,RS1,RS2, TWO, V, X1,X2,XA,XAL,XP1,XP2,
     4            XP3,XP4,XP5,XP7,XPP,XT,XY, Y2, Z,Z23,ZB,ZC,ZI,ZII,ZRO
     5           ,Z12,Z34
      DIMENSION DUMRL(20),R(10),DIVRL(2),
     1          A12(100,2),A23(100,2),A34(100,2),ALAM(100,2),ALM(2,2),
     2          ALM2(2,2,2),ALM3(2,2,2),AN(100,5),B1(100),CB(100,8),
     3          D1(100),DUM(10),F12(100),G34(100),XP1(2,2),XP2(2,2),
     3          XP3(2,2,2),XP4(2),XP5(4,4),XP7(2,2),Z(4),ZC(58),V(2)
      DIMENSION IC(6),TD(100,2),WK(100),T(100,2),ADI(100)               IMPED
      EQUIVALENCE(DUM,DUMRL),(DIV,DIVRL)
     1          ,(ONEC,ONE)
      COMMON R,Z,V,FIX,FR,ZI,ZII,IT,IC,ITT,FR2,FR3,B1,D1,AN,  F12,G34,  IMPED
     1ALAM,CB,A23,A34,A12 ,ICYLE,DUM,ZC,TD,WK,T,ADI ,Z12,Z23,Z34        IMPED
     2,ONEC,TWO,ZRO,AVE
7001  IF( ICYLE ) 1,1,2
1     N3=1
      IT=IT+1
2     IF(M-N3)4,3,3
3     IF DIVIDE CHECK 31,31                                             SCX
31    N3=N3+1
      IF(IT-2)12,11,12
11    AN(M,5)=ONE
      GO TO 311
12    AN(M,5)=ONE/AN(M,4)
      IF DIVIDE CHECK 310,311
 310  AN(M,5)=(1.0,0.0)
 311  ADD=ADI(M)
      ALAM(M,2)= CSQRT(ALAM(M,1)**2-ADD*FR3)
      A23(M,1)= CEXP( ALAM(M,2)*Z23)
      A23(M,2)=  ONE/ A23(M,1)                                          SCX
      IF DIVIDE CHECK  5,32                                             SCX
32     R1=  ALAM(M,1)/ALAM(M,2)                                         SCX
       R2=  ONE/R1                                                      SCX
       RS1=  R1+ONE                                                     SCX
       RD1=  R1-ONE                                                     SCX
       RS2=  R2+ONE                                                     SCX
       RD2=  R2-ONE                                                     SCX
      CB(M,1)= AVE*( RS1*A34(M,1)+RD1*A34(M,2))                         SCX 14
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      CB(M,2)=(-AVE)*(RD1*A34(M,1)+RS1*A34(M,2))                        SCX
      CB(M,3)=(AVE)*( RS1*A12(M,2)+RD1*A12(M,1))                        SCX
      CB(M,4)=(-AVE)*(RD1*A12(M,2)+RS1*A12(M,1))                        SCX
      CB(M,5)=(AVE)*(CB(M,1)*RS2*A23(M,1)-CB(M,2)*RD2*A23(M,2))         SCX 15
      CB(M,6)=(AVE)*(CB(M,2)*RS2*A23(M,2)-CB(M,1)*RD2*A23(M,1))         SCX
      CB(M,7)=(AVE)*(CB(M,3)*RS2*A23(M,2)-CB(M,4)*RD2*A23(M,1))         SCX
      CB(M,8)=(AVE)*(CB(M,4)*RS2*A23(M,1)-CB(M,3)*RD2*A23(M,2))         SCX
      XA=  AVE/ ( CB(M,5)*A12(M,1)+CB(M,6)*A12(M,2))
      IF DIVIDE CHECK 5,34                                              SCX
34    AN(M,4)=CSQRT(XA)
      AN(M,5)=AN(M,5)*AN(M,4)
      DO 35 I=1,8
35    CB(M,I)=CB(M,I)*AN(M,4)
      F12(M)=F12(M)*AN(M,5)
      G34(M)=G34(M)*AN(M,5)
4     IF(M-N-1)500,41,430                                               SCX
41    G23=(CB(N,2)*( A23(N,2)-ONE)-CB(N,1)*(A23(N,1)-ONE))/ALAM(N,2)    SCX
      F23=(CB(N,3)*( A23(N,2)-ONE)-CB(N,4)*(A23(N,1)-ONE))/ALAM(N,2)    SCX
      AK1= ZRO                                                          SCX
      AK4=ZRO                                                           SCX
      AK2=F12 (N)*G23+F23*G34(N)                                        SCX 23
      DO 43  I=1,2                                                      SCX
      I1=I+4                                                            SCX
      DO 43 J=1,2                                                       SCX
      J2=J+6                                                            SCX
      J1=J+2                                                            SCX
      K= 3-J                                                            SCX
      XY=((-ONE)**J-(-ONE)**I)                                          SCX
      AK1=AK1-(((-ONE)**J)*CB(N,I1)*(A12(N,K)*Z12*XY+                   SCX 35
     1((-ONE)**(I+J))*(A12(N,I)+A12(N,K)-TWO)/ALAM(N,1)                )SCX
     2+((-ONE)**I)*CB(N,J2)*(A34(N,K)*Z34*XY+                           SCX
     3((-ONE)**(I+J))*(A34(N,I)+A34(N,K)-TWO)/ALAM(N,1)               ))SCX
43    AK4=AK4+CB(N,I)*CB(N,J1)*(A23(N,K)*Z23*XY+                        SCX 34
     1((-ONE)**(I+J))*(A23(N,I)+A23(N,K)-TWO)/ALAM(N,2))                SCX
      DUM(3)=AK4/ALAM(N,2)                                              SCX
      DUM(2)=DUM(3)+AK2                                                 SCX  24
      AK1=AK1*AN(N,4)
      DUM(1)=DUM(2)+AK2+TWO*F12(N)*G34(N)+ AK1/ALAM(N,1)                SCX  22
      WK2=WK(N)**2                                                      SCX
430   WK2P=WK(M)**2                                                     SCX
      DUMRL(15)=((R(5)*T(N,2)*TD(M,2)-R(3)*T(N,1)*TD(M,1))*WK2
     1 -(R(5)*T(M,2)*TD(N,2)-R(3)*T(M,1)*TD(N,1))*WK2P)/(WK2-WK2P)      SCX
4308  DUMRL(16)=0.0
      DO 431 I=4,7                                                      SCX
431   DUM(I)=ZRO                                                        SCX
      DO 499 I1=1,2                                                     SCX
      GO TO(432,433),I1                                                 SCX
432   N1=N                                                              SCX
      N2=M                                                              SCX
      DO 4321 I2=1,2                                                    SCX
      DO 4321 I3=1,2                                                    SCX
      ALM(I2,I3)=((-ONE)**(I2+1))*ALAM(N1,2)+((-ONE)**(I3+1))*ALAM(N2,2)SCX 32A
      XP1(I2,I3)=(ONE - A23(N1,I2)*A23(N2,I3))/ALM(I2,I3)               SCX32A
      IP=3-I3                                                           SCX
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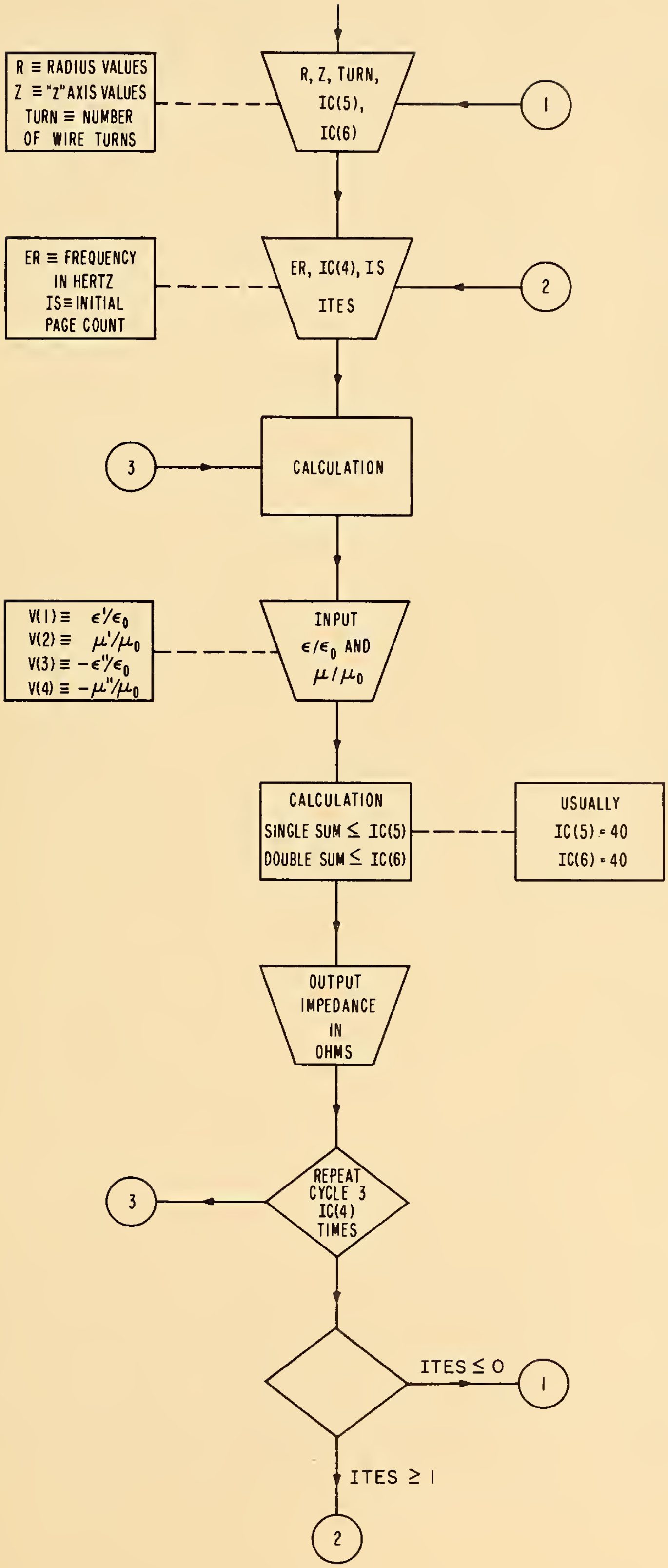
```
4321 XP2(I2,I3)=(A23(N2,IP)-A23(N1,I2) )/ALM(I2,I3)                    SCX
     AH4=ZRO                                                           SCX
     AH1=ZRO                                                           SCX
     AH2=ZRO                                                           SCX
     AH3=ZRO                                                           SCX
     DO 4322 I2=1,2                                                    SCX
     I6=I2+2                                                           SCX
     I4=3-I2                                                           SCX
     DO 4322 I3=1,2                                                    SCX
     I5=3-I3                                                           SCX
     I7=I3+2                                                           SCX
     AH2=AH2+(CB(N1,I6  )*CB(N2,I7   )*XP1(I4,I5))                     SCX  33
     AH3=AH3+(CB(N1,I2)*CB(N2,I7   )*XP2(I2,I3))                       SCX
     AH4=AH4+(CB(N1,I6  )*CB(N2,I3)*XP2(I4,I5))                        SCX
4322 AH1=AH1+(CB(N1,I2)*CB(N2,I3)*XP1(I2,I3))                          SCX
     IF         ( ALAM(N2,2)-2.*ALAM(N1,2)  )    438,4341,438
4341 IF((0.,1.)*( ALAM(N2,2)-2.*ALAM(N1,2)  )  )  438,4342,438
4342 ISET=1                                                            SCX
     GO TO 4381                                                        SCX
438  ISET=2                                                            SCX
4381 DO 45 I=1,2                                                       SCX
     IP=3-I                                                            SCX
     DO 45 J=1,2                                                       SCX
     JP=3-J                                                            SCX
     DO 45 L=1,2                                                       SCX
     LP=3-L                                                            SCX
     L2=2*(L-1)+I                                                      SCX
     Y2=(-ONE)**I+(-ONE)**J                                            SCX
     ALM2(I,J,L)=-( ALAM(N1,2)*Y2+ALAM(N2,2)*(-ONE)**L)                SCX 40
     ALM3(L,I,J)=-( ALAM(N2,2)*Y2+ALAM(N1,2)*(-ONE)**L)                SCX 41
     DO 45  K=1,2                                                      SCX
     K2=2*(K-1)+J                                                      SCX
     KP=3-K                                                            SCX
     DIV=   ( ALM(I,J)+ALM(L,K))                                       SCX
     IF( CABS(DIV) .GT. 1.E-38   )  GO TO 449 $  DIV=0.
448  XP5(L2,K2)=A23(N1,LP)*A23(N2,KP)*(-Z23)                           SCX  46
     GO TO 45                                                          SCX
449  XP5(L2,K2)=(A23(N1,LP)*A23(N2,KP)-A23(N1,I)*A23(N2,J))/DIV        SCX
45   CONTINUE                                                          SCX
     GO TO 65                                                          SCX
433  N2=N                                                              SCX
     ALM(1,2)=-ALM(1,2)                                                SCX
     ALM(2,1)=-ALM(2,1)                                                SCX
     N1=M                                                              SCX
     XA=XP1(1,2)                                                       SCX
     XP1(1,2)=XP1(2,1)                                                 SCX
     XP1(2,1)=XA                                                       SCX
     XA=XP2(1,1)                                                       SCX
     XP2(1,1)= XP2(2,2)                                                SCX
     XP2(2,2)= XA
     XA=XP2(1,2)                                                       SCX
     XP2(1,2)= XP2(2,1)                                                SCX
     XP2(2,1)= XA                                                      SCX
     AH=AH3                                                            SCX
```

```
      AH3=AH4                                                           SCX
      AH4=AH                                                            SCX
      DO 62 I=1,3                                                       SCX
      IA=4-I                                                            SCX
      DO 62 L=1,IA                                                      SCX
      J=I+L                                                             SCX
      XPP=XP5(I,J)                                                      SCX
      XP5(I,J)=XP5(J,I)                                                 SCX
      XP5(J,I)=XPP                                                      SCX
   62 CONTINUE                                                          SCX
      DO 64 J=1,2                                                       SCX
      DO 64 I=1,2                                                       SCX
      DO 64 K=1,2                                                       SCX
      XAL=ALM2(I,J,K)                                                   SCX
      ALM2(I,J,K)=ALM3(K,I,J)                                           SCX
   64 ALM3(K,I,J)=XAL                                                   SCX
      ISET=2                                                            SCX
   65 DO 70 I=1,2                                                       SCX
      IP=3-I                                                            SCX
      XP4(I)=(ONE-A23(N1,I  ))/(ALAM(N1,2)*(-ONE)**(I+1))               SCX
      DO 70 J=1,2                                                       SCX
      JP=3-J                                                            SCX
      XP7(I,J)=A23(N2,JP)*XP4(I)                                        SCX   37
      DO 70 K=1,2                                                       SCX
      GO TO (69,692),ISET                                               SCX
   69 IF((-1.0)**I+(-1.0)**J+2.0*(-1.0)**K) 692,691,692                 SCX
  691 XP3(I,J,K)=-A23(N1,JP)*Z23                                        SCX   42
      GO TO 70                                                          SCX
  692 XP3(I,J,K)=(A23(N1,JP)-A23(N1,I)*A23(N2,K))/ALM2(I,J,K)           SCX
   70 CONTINUE                                                          SCX
      AL2=ZRO                                                           SCX
      AL3=ZRO                                                           SCX
      AL4=ZRO                                                           SCX
      AL5=ZRO                                                           SCX
      AK8=ZRO
      DO 48 I=1,2                                                       SCX
      IP=3-I                                                            SCX
      DO 48 J=1,2                                                       SCX
      J1=J+2                                                            SCX
      JP=3-J                                                            SCX
      XT =((-ONE)**I-(-ONE)**J)                                         SCX
      DO 48 L=1,2                                                       SCX
      LP=3-L                                                            SCX
      L1=L+2                                                            SCX
      L2=2*(L-1)+I                                                      SCX
      AL2=AL2+(CB(N2,I)          ) *CB(N1,L)*(XT*XP7(L,J)+((-ONE)**J*XP1(L,SCX
     1I))            - (-ONE)**I*XP2(L,J  )) *CB(N2,J1)                 SCX
      AL3=AL3+(CB(N2,I)          ) *CB(N1,L1)*(XT*XP7(LP,J)+((-ONE)**J*XP2(SCX
     1LP,IP))        -((-ONE)**I*XP1(LP,JP))   ) *CB(N2,J1)             SCX
      AL4=AL4+CB(N1,I)*CB(N1,J1)*CB(N2,L)*((XP3(I,J,L)-XP7(I  ,LP))     SCX
     1/ALM(J,L)+(XP4(JP)-XP3(I,J,L))/ALM(I,L))
      AL5=AL5+CB(N1,I)*CB(N1,J1)*CB(N2,L1)*((XP3(JP,IP,LP)-XP4(I  ))/ALM(SCX
     1J,L)+(XP7(JP,L)           -XP3(JP,IP,LP))/ALM(I,L))               SCX
      DO 48  K=1,2                                                      SCX
```

```
          KP=3-K                                                          SCX
          K1=K+2                                                          SCX
          K2=2*(K-1)+J                                                    SCX
          AK8=AK8-(CB(N1,I)*CB(N2,J)        ) *CB(N2,K1)*((-ONE)**K*(((XP5(L2SCX  45
         1,K2)-XP7(I,JP))/ALM3(L,J,K))+(-XP3(I,L,J)+XP7(I,JP))/ALM(L,J)-((XPSCX
         25(L2,K2)-XP7(LP,K ))/ALM3(I,J,K))+(XP3(I,L,J)-XP4(LP))/ALM(I,J))  SCX
         3+((-ONE)**J*(-((XP5(L2,K2)-XP7(I,JP))/ALM3(L,J,K))+(XP3(LP,IP,KP) SCX
         4-XP4(I))/ALM(L,K)+((XP5(L2,K2)-XP7(LP,K ))/ALM3(I,J,K))+((-XP3(   SCX
         5LP,IP,KP)+XP7(LP,K ))/ALM(I,K))))       )  *CB(N1,L1)             SCX
    48    CONTINUE                                                        SCX
          AK8=AK8/ALAM(N2,2)                                              SCX
          AL2=AL2/ALAM(N2,2)                                              SCX
          AL3=AL3/ALAM(N2,2)                                              SCX
          AK6= F12(N1)*AL2+G34(N1)*AL3                                    SCX 27
          AK5= AK6+F12(N1)*(AH1*F12(N2)+AH3*G34(N2))+G34(N1)*(AH2*G34(N2)+  SCX 25
         1 AH4*F12(N2))                                                   SCX
          AK7=AK8+F12(N2)*AL4+G34(N2)*AL5                                 SCX 26
          DUM(4)=DUM(4)+AK5                                               SCX
          DUM(5)=DUM(5)+AK6                                               SCX
          DUM(6)=DUM(6)+AK7                                               SCX
    499   DUM(7)=DUM(7)+AK8                                               SCX
          DUM(5)=DUM(5)+DUM(7)                                            SCX
          DUM(4)=DUM(4)+DUM(6)                                            SCX
    500   RETURN                                                          SCX
    5     N=-N
          WRITE OUTPUT TAPE 6,51                                          SCX
    51    FORMAT(54H OMEGA IS SUCH THAT STRUCTURE IS RESONANT-NOT ALLOWED  )SCX
          RETURN                                                          SCX
          END
             SCOPE
```

INPUT DATA DEFINED TOROID



APPENDIX II

IMPEDANCE RESULTS FOR TOROID GEOMETRY PAGE 1
THE FREQUENCY IS 0.1000E 07HZ. THE TURNS COUNT IS 1.
THE RADIAL DIMENSIONS ARE 0.4130E-00 0.6450E 00 0.1275E 01 0.2540E 01
THE Z AXIS DIMENSIONS ARE -0.1590E 01 -0.4470E-00 0.4930E-00 0.1590E 01

THE LENGTH MEASUREMENTS ARE IN CM. IMPEDANCES ARE IN OHMS. THE REST ARE DIMENSIONLESS.

MU		EPSILON		QUASI STATIC IMP		CORRECT IMP		DBL SUM IMP		
U1	+I U2	E1	+I E2	Z1	+I Z2	ZC1	+I ZC2	DZ1	+I DZ2	CNT
0.2024E 04	-0.1030E 04	0.7540E 05	-0.2950E 05	0.8290E 01	0.1635E 02	0.1595E 02	0.1608E 02	0.9130E 00	-0.1048E 01	11

```
IMPEDANCE RESULTS FOR   TOROID GEOMETRY PAGE    1
THE FREQUENCY IS          0.1500E 07HZ.    THE TURNS COUNT IS    1.
THE RADIAL DIMENSIONS ARE          0.4130E-00          0.6450E 00          0.1275E 01          0.2540E 01
THE Z AXIS DIMENSIONS ARE         -0.1590E 01         -0.4470E-00          0.4930E-00          0.1590E 01

THE LENGTH MEASUREMENTS ARE IN CM.  IMPEDANCES ARE IN OHMS.  THE REST ARE DIMENSIONLESS.

             MU                    EPSILON                QUASI STATIC IMP            CORRECT IMP             DBL SUM IMP
U1          +I U2          E1         +I E2          Z1          +I Z2          ZC1          +I ZC2          DZ1          +I DZ2          CNT
  0.1670E 04 -0.1060E 04  0.7390E 05 -0.2880E 05  0.1280E 02  0.2026E 02  0.3090E 02  0.8581E 01  0.7944E 00 -0.7310E 01  11
```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