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Technical Note 259

A FORTRAN PROGRAM FOR DETERMINING AN EMPIRICAL EXPRESSION FOR A QUANTITY MEASURED AT COMBINATIONS OF SEVERAL LEVELS OF EACH OF TWO VARIABLES

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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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Contents

A FORTRAN Program for Determining an Empirical Expression for a Quantity Measured at Combinations of Several Levels of Each of Two Variables

Mary Nan Steel, Frank L. McCrackin, and John Mandel

This note relates to a general procedure, reported elsewhere, for finding an empirical expression for the functional dependence of a measured quantity on two variables. The procedure is applicable when measurements of the dependent variable are available at m.n combinations of m levels of one variable and n levels of the other; it leads to an algebraic expression relating the three variables.

A Fortran program to perform the various steps of the general procedure is presented along with detailed instructions for its use. The program also includes the calculation of an analysis of variance, of components of variance, of significance tests, tables of residuals, and a number of other quantities.

1. Introduction

In many scientific studies, a quantity is measured at several levels for each of two variables. The data are often presented in a table with the rows of the table representing one variable and the columns representing the other variable.

When both variables are continuous, the data can be considered as representing either a surface in three-dimensional space or a family of plane curves [1][1]. In the latter representation each curve represents the relation between the measured quantity and one of the variables, for a selected fixed value of the second variable. For different fixed values of this second variable, different curves are obtained. In this fashion a family of curves is generated. An example of this type is the relationship between the volume, the pressure and the temperature of a gas. The objective is to find a mathematical representation of such a relationship.

One or both of the variables may represent categories rather than continuous variables. For example, in interlaboratory studies of test methods [2], the data are often classified in accordance with laboratories (rows) and materials (columns). In such cases, the main objective is to

[1]Figures in brackets indicate the literature references at the end of this paper.

determine the effect of the variables (laboratories and materials) on the variability of the data. Methods to determine the relationship between the measured quantity and the variables have been developed by Mandel, et al [1,2,3,4,5].

These methods, which have been referred to as the linear model and the quadratic model, constitute essentially a general procedure for the empirical fitting of the data by means of algebraic expressions. The fitting procedure comprises several steps:

1) the selection of a particular algebraic expression from among a class of such expressions for the most appropriate representation of the data;
2) the calculations of numerical values for the parameters and coefficients occurring in the chosen expression;
3) the estimation of the precision of the calculated parameters and coefficients, tests of significance related to them, and the calculation of confidence intervals for their true values;
4) the derivation of components of variance related to the variability of some of the parameters and coefficients;
5) further statistical interpretations of the parameters, coefficients, and components of variance in terms of the general objectives of the study.

A computer program was developed to perform these analyses and this note presents the program. Section 2 describes how the data are organized to be read into the computer and the instructions required to perform the calculations. Possible modifications to be made to the data by the computer are also described. Section 3 presents two examples with interpretations of the computer output. The tables of computer output examined in Section 3 are given in Appendix A. Appendixes B through D give the FORTRAN program and information required for understanding the program. Appendix E consists of Figures 1 through 9, the flow diagrams for the computer program and the principal subprograms.

2. Computer Input

Two examples are presented as a guide for the practical use of the program. Section 2.1 describes a setup in which each cell contains a single measurement. Section 2.2 presents the case in which replicate measurements in each cell are to be analyzed. It will be noted that the data are set up in a different way in each case (Examples 1 and 2). Each line of data in the lower portion of both examples represents a single card of computer input and the first card is the title card in both cases. The nominal number of replicates of a particular test is put in columns 1 and 2 of this card, followed by any descriptive title.

Example 1. A Single Value In Each Cell

The Data
Round Robin on Dielectric Strength

	Material							
Lab	1	2	3	4	5	6	7	8
LOH	.80	1.13	.34	.47	.22	.34	.17	.20
LMG	.96	1.26	.32	.61	.23	.32	.17	.23
LST	.98	1.24	.34	.52	.31	.45	.20	.24
LUM	.88	1.33	.32	.51	.22	.31	.15	.18

Setup for Computer Input

```
Column Number
1          7        15       25       35       45       55       65
01         ROUND ROBIN ON DIELECTRIC STRENGTH

           MAT1     MAT2     MAT3     MAT4     MAT5     MAT6     MAT7
LOH        .80      1.13     .34      .47      .22      .34      .17
LMG        .96      1.26     .32      .61      .23      .32      .17
LST        .98      1.24     .34      .52      .31      .45      .20
LUM        .88      1.33     .32      .51      .22      .31      .15
MOREDA
           MAT8
LOH        .20
LMG        .23
LST        .24
LUM        .18
ENDATA
LMWCOLCAL
STOP
```

2.1 An Example With A Single Value In Each Cell

Example 1 gives experimental data with a single value in each cell. The top table gives the original data. The lower table gives the setup for computer input. In the latter, the first card has 01 in columns 1 and 2 and the title that identifies the data follows. The second card contains the column headings from the table and may have as many as seven per card, beginning in card columns 7, 15, 25, 35, 45, 55, 65 as shown in the example. Next follow the data cards, each of which has a row heading in columns 1-6 followed by data corresponding in number and order to the column headings, i.e. the data are placed in card columns 7 through 14, 15 through 24, 25 through 34, 35 through 44, 45 through 54, 55 through 64

Example 2. Replicate Values In Each Cell

The Data

Adhesion of Plastic Patches, Method A

	Condition		
Material	1385	1415	2385
AA	18.2	20.0	22.1
	18.6	19.7	22.0
	17.9	20.0	20.8
AB	16.8	24.5	17.3
	15.2	25.1	17.7
	15.6	23.4	17.7
AC	14.6	14.7	14.6
	13.1	14.7	14.2
	14.2	14.7	14.9
AD	19.5	15.9	18.0
	18.7	16.5	20.1
	16.5	17.7	20.3
		16.7	

Setup for Computer Input

```
Column Number
1     7         15        25        35        45
03  ADHESION OF PLASTIC PATCHES, METHOD A
AA    1385      18.2      18.6      17.9
AB    1385      16.8      15.2      15.6
AC    1385      14.6      13.1      14.2
AD    1385      19.5      18.7      16.5
AA    1415      20.0      19.7      20.0
AB    1415      24.5      25.1      23.4
AC    1415      14.7      14.7      14.7
AD    1415    0415.9      16.5      17.7      16.7
AA    2385      22.1      22.0      20.8
AB    2385      17.3      17.7      17.7
AC    2385      14.6      14.2      14.9
AD    2385      18.0      20.1      20.3
ENDATA
QMODEL
REVERS
LMODEL
STOP
```

and 65 through 72. The column and row headings must all be different and each heading may consist of from 1 to 6 characters. All data must contain decimal points. If more than 7 columns of data are to be read, a card with MOREDA in columns 1-6 is placed after the last row of data of the first group of columns. That is, all rows for each set of columns are read before additional column headings with their respective rows of data are introduced. After the MOREDA card, the next set of column headings, again beginning in column 7 are read, followed by rows of data corresponding to the new column headings. At this point it is important that the rows are in the exact order and the same in number as before, for the row headings are ignored after the first reading, the identity of the order is assumed, and no provision is made for a difference in the number of rows per column. The MOREDA card occurs only once, regardless of the number of columns over 7 that are to be read.

The reading of the data is concluded by placing a card with ENDATA in columns 1-6 after the last row of data.

2.2 An Example With Replicate Values In Each Cell

In Example 2, the case in which replicate values occur in each cell, we note that 03 is placed in columns 1 and 2, followed by the title on the first card. On the second card, a row and column heading appear, for in the case of replicate measurements row and column headings and replicates for the cell thus designated, all appear on the same data card. The row heading is put in columns 1-6 and the column heading in card columns 7-12. Columns 13-14 are blank if the number of replicates in the cell is the same as the number appearing on the title card - in our example 03. If the number of replicates for any one cell differs from that on the title card, that number is placed in columns 13-14 (note 04 in line AD 1415 in Example 2). The data then follow on the same line beginning in columns 15, 25, 35, 45, 55, 65. If more than six replicates occur per cell, the additional data are placed in groups of six on successive cards, making certain that the number of replicates is either the same number indicated on the title card or that placed in columns 13-14 of the initial card of the cell listing. There may be as many as 99 replicate values per cell.

It is important that there be the same number of rows with the same row headings per column and the same number of columns with the same column headings per row, otherwise, the program will not run. The data cards may be placed in any order. The program will handle a table of data containing as many as 120 rows and 50 columns. A card with ENDATA in columns 1-6 is always placed after the final data card.

2.3 Analysis of the Data

After the data are read into the computer some basic calculations are made. In the case of replicate measurements per cell, the cell averages, standard deviations, and the coefficients of variation are computed.

Row and column averages are calculated for either the cell averages (in the case of replicate measurements per cell) or for the cell entries (in the case of single measurements in the cells). Then the data of the table are rearranged in such a way that the column-averages are in ascending order from left to right and the row-averages are in ascending order from top to bottom. A table of averages and, in the case of replicates, a table of standard deviations with the coefficients of variation are printed. Analysis of the data is accomplished by placing instruction cards for the desired analysis at the end of the data deck after the card ENDATA. The instruction cards are the same for data consisting of either single or replicate measurements per cell and the choice of instructions determines the analysis that will be performed on the data.

Examples 1 and 2 show the way in which the instruction cards appear in an input setup. Details for making these cards and notes on what the instruction accomplishes are given below.

2.31 The Linear Model - LMODEL

By placing LMODEL in columns 1-6 of a card following the data, the equation $y_{ij}=A_i+B_iC_j$, the linear model, is fitted to the cell averages y_{ij}. The residuals and their sums of squares are calculated and printed and the analysis of variance is computed. The analysis is then printed along with the values calculated for A, B and C. Interpretations of the computer output for this and other calculations are given in Section 3.

2.32 The Weighted Linear Model - LMWCOL

If one wishes to weight the columns before analyzing the data, LMWCOL is placed in columns 1-6 on a card. A linear model analysis is then performed, in which the columns are weighted. Weights may be given or calculated. When the weights are given, they are placed on cards immediately following the LMWCOL card. One weight is given for each column and six weights may be read from a single card beginning in card columns 15, 25, 35, 45, 55, and 65, as in Example 3. If a column is not to be weighted, 1. is given as its weight and of course the weights are given in the same order as the columns of data being weighted, i.e., in the ascending order of the column averages, from left to right, as ordered by the computer.

Example 3

Column Number

1	7	15	25	35	45	55	65
ENDATA							
LMWCOL							
		1.35	1.	.5	1.5	.75	1.45
STOP							

If the user wishes the weights to be calculated he places CAL in columns 7-9 following the LMWCOL, thus LMWCOLCAL appears on the instruction card. The calculation of weights is an iterative process discussed in reference [5]. When the weights are calculated, the analysis is not valid unless the data consist of a minimum of eight columns. The weights are derived from the original data themselves. It is well to note that if the user has sufficient information to provide his own weights, it is at this point in the program that they should be introduced.

After the weights have been read or computed, the square root of the weights, referred to as the weighting factor, is determined and both weights and weighting factors are printed. In addition to these, the weighted values for A, B, and C, the point of concurrence* and the analysis of variance are calculated and the results printed. The residuals with the sums of squares of both rows and columns are computed and the Table of Deviations, preceded by the number of rows and columns, is printed.

2.33 The Quadratic Model - QMODEL

The quadratic model fits the data to the equation

$$y_{ij} = A_i + B_i C_j + D_i E_j = A_i' + B_i' C_j + D_i C_j^2,$$

where E_j is a quadratic function of C_j. The instruction QMODEL is placed on a card in columns 1-6 when this model is desired. All the variables for the equation are calculated and printed as in the analysis of variance. In addition to the Table of Deviations with the row and column sums of squares for the quadratic model, this instruction also calculates and prints the linear model Table of Deviations with sums of squares.

* See Section 3.1

2.4 Modification of the Data

It is frequently desirable to analyze a set of data, then modify it in some fashion and reanalyze the new set. Instructions are included to accomplish a number of these modifications.

2.41 Omit Data - OMIT

The OMIT instruction placed in columns 1-6 of a card causes the designated rows and/or columns to be omitted in the analyses following it, or until another OMIT card is placed in the sequence of instructions.

On the same card with OMIT, in columns 13-14, appears the number of rows and columns to be omitted followed by their labels beginning in columns 15, 25, 35, 45, 55, 65. These may be given in any order. If there are more than 6, the additional labels in groups of six appear on cards immediately following the first OMIT instruction.

An OMIT card cancels all previous omit cards. In other words a single set of data may be analyzed using different rows and columns within the same run by first placing the OMIT card followed by the card designating the analysis desired, followed by another OMIT card and another analysis card, etc.

Example 4

```
Column Number
1     7     13           25
ENDATA
QMODEL
OMIT        02A1         B2
QMODEL
OMIT        02DUPONT     A1
LMODEL
STOP
```

In Example 4, the first QMODEL analysis will include all laboratories and materials, then materials A1 and B2 will be omitted in the second QMODEL analysis. LMODEL will be computed omitting laboratory DUPONT and material A1 but will include material B2 previously omitted.

2.42 Transform Data - TRAFOR

This instruction may be on a card in columns 1-6 placed before or after the data. Placed before the data, after the title card, the function TRAFOR causes each measurement to be transformed before any other calculations are made. Any transformation may be employed. This program does not include a transformation, in order to use the instruction a special subprogram called TRAFOR must be written and added by the user.

If one wishes to transform the averages after all of the cell averages have been computed, TRAFOR is placed on a card in columns 1-6 after the ENDATA card. The subroutine TRAFOR will then transform the averages. As in the case of the function TRAFOR, any transformation may be used, and again no subroutine TRAFOR is included and it is left to the user to write his own.

It is important to note that the user must choose either one or the other of these two transformations and include only the one desired in using the program. Of course neither is necessary to the running of the program; the decision depends entirely on the nature of the data and the wishes of the user.

2.43 Correct Data - CORECT

The averages and standard deviations may be replaced by values corrected in the form AX + B and Aσ respectively, where $\underline{A}$ is a multiplicative correction, $\underline{B}$ is an additive correction, $\bar{X}$ is the original average and $\underline{\sigma}$ is the original standard deviation.

The data may be corrected before the initial printing of the Table of Averages and Standard Deviations, in which case CORECT should appear in columns 7-12 on the same card with ENDATA. (Example 5a) If one wishes the original data to be printed before correction is made, the CORECT in columns 1-6 should appear on a separate card after ENDATA. (Example 5b)

In either case the row and column designations of those values to be corrected are placed on a card immediately following the CORECT card in columns 1-6 and 7-12, respectively, along with the multiplicative term (columns 15-24) and additive term (columns 25-34). These designations may be for single values or, by placing ALL in place of a single row or column heading, may include all row values for a single column or all column values for a single row, as shown in Example 5. After these instructions an END card (columns 1-3) is placed.

If a purely multiplicative correction is desired, it is not necessary to put 0. in column 25 but if a purely additive correction is desired, a 1. must be placed in column 15.

Example 5

(a)

Column Number

1	7	15	25
ENDATACORECT			
ALL	MAT02	1.	2.
LAB03	MAT11	2.	
LAB07	ALL	2.	5.
END			

(b)

Column Number

1	7	15	25
ENDATA			
CORECT			
ROW3	COL7	1.	3.
ALL	COL2	2.	1.5
ROW6	ALL	1.	2.
END			

2.44 Reverse Data - REVERS

This instruction card with REVERS in columns 1-6 causes the rows and columns to be interchanged in the analyses that follow.

2.45 Combine Data - COMBIN

Sometimes data previously treated as two or more sets are found not to be significantly different and the experimenter wishes to treat them as one set. The instruction COMBIN serves the purpose of combining such sets into new averages. After the COMBIN in columns 1-6, the designation "ROWS" or "COLS" is placed on the card in columns 7-10 and the labels of the sets to be combined are placed in columns beginning at 15, 25, 35, 45, 55. This causes the cell averages in the rows and/or columns thus designated to be averaged together into a single row or column. The label for the newly formed row or column is placed on the card in the position immediately following the listing of the rows and columns to be combined. Thus it is possible to combine as many as 5, their new label being placed on the card beginning in position 65.

If more than one combination is to be computed, additional lines are added as shown in Example 6. COMBIN is placed on the first card only, its place being left blank on the following lines. An instruction card with END in columns 7-9 completes the setup for this subroutine.

It should also be noted that the new table of averages is treated as though it were a table of single values and no standard deviations are calculated.

Example 6

```
Column Number
1     7       15       25      35      45        55        65
________________________________________________________________
ENDATA
COMBINROWS    A1       A2      A3      A10
      COLS    C1       C3      C6
      ROWS    L7       L8      L9      L10       L14       L15
      END
```

These instructions combine rows A1, A2 and A3 in a single row labeled A10; combine columns C1 and C3 in a single column called C6; combine rows L7, L8, L9, L10, L14 in a single row called L15.

2.46 New Data - NEWDAT

If the user wishes to analyze more than one set of data in a single run, a card with NEWDAT in columns 1-6 is placed after the last analysis card of the preceding set of data. This is followed by the title card for the new set of data. The rest of the setup for the set follows the same rules as for the initial set. Again, the number of such sets to be run at one time is the user's choice.

2.47 Stop - STOP

The very last card in any analysis is the card with STOP in columns 1-4. If several sets of data are to be analyzed the STOP card is placed only at the end of the last set of data.

3. Interpretation of Computer Output

3.1 The Table With A Single Value In Each Cell

The output from the computer is illustrated by two examples. The data shown in table 1 will give the output shown in table 2. Each line of table 1 represents one card of computer input. The first line is the title preceded by 01 designating a table with a single value in each cell. The second line gives the column headings which, in this particular example, are numbers. The data follow and the instructions come after the data.

The first page of output, table 2a, gives the original data with row and column averages. The data have been rearranged so that the row and column averages are in ascending order of magnitude, from top to bottom and from left to right, respectively. The data were then fitted by the linear model with the results shown in table 2b. The instruction LMODEL is printed on the second line to identify the source of the print out; the -0 following LMODEL indicates that no further information was on the instruction card. The parameters A, B, and C fit the original data by the model equations

$$\left.\begin{aligned} y_{ij} &= \hat{y}_{ij} + \text{residual error} \\ \hat{y}_{ij} &= A_i + B_i C_j \end{aligned}\right\} \qquad (1)$$

where $\hat{y}_{ij}$ is the fitted value for the observation y_{ij}. A and B values are given for each row; C is given for each column.

The analysis of variance for the fitting is given next. The total sum of square, SS, and degrees of freedom, DF, are partitioned into the effect of the mean (MEAN), rows (ROWS), columns (COLS) and interaction (RxC). The mean squares (MS) for the row, column and interaction effects are also given. This part of the analysis of variance corresponds to the analysis of the data by the additive model given by

$$(\hat{y}_{ij})_a = A_i + C_j \qquad (2)$$

where $(\hat{y}_{ij})_a$ is the fitted value for the observation y_{ij}.

The interaction (RxC) is next partitioned into two components according to the following equation (which is just a different way of writing equation (1)):

$$\begin{aligned} y_{ij} &= \hat{y}_{ij} + \text{residual error} \\ &= A_i + C_j + \underbrace{(B_i - 1)C_j + \text{residual error}}_{\text{interaction (RxC)}} \end{aligned} \qquad (3)$$

The component denoted linear (LIN) corresponds to the term $(B_i-1)C_j$, while the component denoted residual (RESID) corresponds to the residual error term in equation (3).

Mandel and McCrackin [1] have shown that if B_i is a linear function of A_i, the linear model, equation (1), may be reduced to the simpler concurrent model given by

$$(\hat{y}_{ij})_c = y_o + P_i Q_j \qquad (4)$$

In particular, if

$$B_i = 1 + \alpha(A_i - \bar{A}) \qquad (5)$$

then $y_o = \bar{A} - 1/\alpha$ (6)

$$P_i = A_i - y_o \qquad (7)$$

and $Q_j = 1 + \alpha C_j$ (8)

To aid in determining the applicability of the concurrent model, the sum of squares for the linear part has been further divided into a concurrent (C) and non-concurrent (NC) part. The concurrent model, equation (4), applies when the non-concurrent part is sufficiently small.

The ratios, F, of mean squares are given to be compared with tabulated values of the F-statistic to determine the significance level of the various effects. The ratios given are

	F
ROWS	MS(ROWS)/MS(RESID)
COLS	MS(COLS)/MS(RESID)
RXC	MS(RXC)/MS(RESID)
LIN	MS(LIN)/MS(RESID)
C	MS(C)/MS(NC)
NC	MS(NC)/MS(RESID)

For the example in table 2b, the F value for rows (10000) is larger than $F_{10,30}$ for the 0.005 significance level (3.344), so the effect of rows is significant at better than the 0.005 level. Likewise, the columns, interaction, and linear parts are significant. Since the interaction and linear parts are significant, the data cannot be adequately represented by the additive model.

The F values for both the concurrent and non-concurrent parts are also highly significant. Therefore, the data can be fitted approximately by equation (4), the concurrent model, but the fit by equation (4) is not as good as that by equation (1), the linear model.

The next page, table 2c, shows the deviations of the data from the linear model, i.e., $y_{ij} - \hat{y}_{ij}$. The sums of squares of the deviations over both the rows and columns are also given. This table should be carefully scrutinized for trends, either in the rows or in the columns, and for excessive individual residuals. Trends can be detected by systematic patterns in the + and - signs. If a row or column contains excessive residuals, the analysis should be run omitting the suspect row or column, or else a weighted analysis should be run. Other systematic patterns or residuals may indicate that the linear model is inadequate to represent the data to within their precision. In this case, the data should be fitted by a more general model, such as the quadratic model. In order that conclusions drawn from the analysis of variance be valid, it is essential that the residuals be free of marked systematic effects or serious outliers.

The linear model may also be applied with rows and columns interchanged. For this purpose the data were reversed (i.e., the rows and columns were interchanged) by the instruction REVERS, and the data were reanalyzed by the linear model with the results shown in table 2d. The residual mean square (0.233×10^{-5}) is seen to be larger than the residual mean square (0.879×10^{-6}) in table 2b, before the data were reversed, so for this case the first analysis gave the better fit and is to be preferred.

3.2 The Table With Replicate Values In Each Cell

Table 3 is an example of a set of data with 2 replicate values in each cell. The computer output for these data are given in table 4. The title for the data appears as the first line of table 3 and the 02 preceding the title indicates that there are two values in each cell to be used in calculating the cell averages and standard deviations. The second line gives a row heading, followed by a column heading and the two replicate values of the designated cell.

The data are from an inter-laboratory test on natural rubber. The rows labeled LAB12 through LAB59 in table 4a represent different laboratories that performed the test and the columns labeled MAT345 through MAT245 represent the materials on which the test of stress at 100 percent elongation was made. In table 4a the average of the two replicate measurements is given for each cell. The row and column averages are also given. The data have been arranged so that their row and column averages appear in ascending order of magnitude. Table 4b gives the standard deviations per cell and the percent coefficient of variation for each column (material).

Table 4c is a print out of the COMBIN instructions which appear after the data cards in table 3. It will be noted that in each line the instruction is given to combine four rows into a single row with a new label. (See Section 2.45 - COMBIN) The four rows in each case are really four tests within a single laboratory but with different labels. In a previous computer run of these data in which each of the four tests was analyzed as a separate laboratory, it was found that there was no significant difference within the set of four. Therefore in this run the computer has been instructed to combine each set of four - averaging the cell averages into a new "table of averages", table 4d. The result is that the cell averages from table 4a for the rows labeled LAB09, LAB 10, LAB11, LAB12 are averaged in a single row of averages labeled LAB01 in table 4d, etc. This new table is treated as a set of data with single measurements per cell and therefore no new standard deviations are computed. In table 4e the first line after the heading, LMWCOLCAL, indicates that the computer was instructed to analyze the data using the linear model weighted by columns, with the weights to be calculated. Printed in this table are the computed weights (WEIGHT) and their square roots (SQRTW) for the various columns of combined data. A, B, and C are the values obtained for the parameters in the model equation (1) discussed on page 12. B*(B STAR) is the estimate of B based on the assumption that B_i is a linear function of A_i. If $B_i^* = B_i$, equation (5) holds and the model is concurrent. To what extent equation (5) holds (i.e., to what extent the model is concurrent) can be judged by comparing B and B STAR: a close agreement indicates a high degree of concurrence. In table 4e they do not agree, so these data do not fit the concurrent model.

The calculated value of α is also given in table 4e. The last line of table 4e gives the "Point of Concurrence" which is discussed in reference [4].

Table 4f gives the analysis of variance of the weighted data. The usual row and column SS are given and the row x column interaction. The latter is partitioned into "Fit" and "Slope". The value of the ratio

$$F = \frac{MS_{slope}}{MS_{fit}} \qquad (9)$$

is compared with the tabulated value of the F-statistic with the degrees of freedom given in the column DF (in this case 5 and 30). A significant value for this F indicates that the model is not additive; it may then be either concurrent or non-concurrent. To decide on this point, the sum of squares for "slope" is partitioned into "concurrence" and "non-concurrence". If the mean square for "non-concurrence" is not appreciably larger than that for "fit" and the mean square for "concurrence" is significantly larger than that for "non-concurrence", the model is probably a concurrent one. This must be verified by:
(a) comparing BSTAR with B in table 4e
(b) plotting B versus A: a straight line indicates the

concurrent model
(c) examining R SQUARED which is the square of the correlation coefficient between A and B. A value of R squared close to unity indicates a tendency for concurrence. Conclusions drawn from the analysis of variance are always tentative subject to an examination of the residuals, as described below.

The Table of Deviations for this example is given in table 4g. These are the residuals in the weighted scale, i.e.

$$\text{deviation}_{ij} = \sqrt{w_j}\,(y_{ij}-\hat{y}_{ij}) \qquad (10)$$

As noted in the preceding example, the table should be carefully scrutinized for trends, either across the rows or down the columns, and for excessive individual residuals. Trend can be detected by systematic patterns in the + and - signs. The total sum of squares in this table is identical with the sum of squares for "fit" in the analysis of variance. Again it should be emphasized that in order that conclusions drawn from the analysis of variance be valid, it is essential that the residuals be free of marked systematic effects or serious outliers.

In table 4h the line below the title "QMODEL" indicates that the data were analyzed by the quadratic model as requested in the instructions at the end of Table 3.

Analysis by the quadratic model is in accordance with the relation

$$y_{ij} = A_i + B_i C_j + D_i E_j \qquad (11)$$

or the equivalent form:

$$y_{ij} = A_i' + B_i' C_j + D_i C_j^2 \qquad (12)$$

In equation (11), A_i, B_i, and C_j are the estimates corresponding to a fit by the linear model. The set E_j is orthogonal to C_j, i.e.: $\sum_j C_j E_j = 0$.

E_j is a quadratic function of C_j. If terms of the same order are collected in equation (11), the expression given by equation (12) is obtained, where C_j and D_i are the same as in equation (11).

The analysis of variance for the quadratic model is given in table 4i. The total sum of squares, SS, and the total degrees of freedom, DF, are partitioned into the effect of the mean (MEAN), rows (ROWS), columns (COLS) and interaction (RxC). The row-column interaction is further partitioned into three parts: linear (LIN), quadratic (QUAD), and residual (RES). The mean squares (MS) = (SS)/(DF) are also computed.

By means of the F-test, both the mean squares of the "linear" and "quadratic" can be tested against "residual". Comparing the resulting quotients with the values in the F table, a conclusion may be reached as to whether the model is essentially additive (both "linear" and "quadratic" non-significant against "residual"), linear ("linear" significant, "quadratic" non-significant), or quadratic ("quadratic" significant against "residual"). As pointed out in the previous examples of analysis of variance, the conclusions drawn here are not valid unless substantiated by a table of residuals free of large systematic effects.

The lower part of table 4i contains the table of residuals from the linear fit. On careful examination for systematic patterns or excessive residuals in this example, it is obvious that material 745 gives rise to residuals that are substantially larger than those corresponding to all other materials. Therefore the analysis of variance is not usable for the drawing of valid inferences from the data.

Table 4j contains the quadratic model residuals. Again a careful scrutiny is necessary and for the present example, the residuals for material 745 are abnormally large as compared to all others. The analysis of variance can therefore not be trusted and the data are reanalyzed omitting the abnormal material 745.

Table 5a shows the instruction OMIT with 1 before the label MAT745 on the second line. Thus the computer is instructed to omit 1 column. The Table of Averages is the same as that in table 4d except that material 745 is omitted and the row averages are therefore different.

Table 5b gives the values determined by the QMODEL analysis on the data without material 745, and table 5c gives the analysis of variance from the quadratic model and the table of residuals from the linear model both excluding material 745. Note the marked decrease in error variance resulting from the omission of the abnormal material. There also appears evidence in the new analysis of a quadratic effect.

The sums of squares of residuals from the linear model for the materials are now all of the same order of magnitude. And in table 5d the sums of squares of the residuals from the quadratic model are also all of the same order of magnitude. Therefore the quadratic effect observed in the analysis of variance appears to be real.

This program was written as a research tool. The first program "multiple curve fitting" was written by Alfred E. Beam. Necessary changes were made as the research progressed and the present FORTRAN program was written as a result of the experiences gained in the use of Mr. Beam's program.

4. References

[1] Mandel, John and McCrackin, F. L., Analysis of Families of Curves, J. Res. Natl. Bur. Standards 67A (Phys. and Chem.), No. 3, 259-267 (May-June 1963).

[2] Mandel, John and Lashof, T. W., The Interlaboratory Evaluation of Testing Methods, ASTM Bull. 239, 53-61 (July 1959).

[3] Mandel, John, The Measuring Process, Technometrics, 1, 251-267 (Aug. 1959).

[4] Mandel, John, Non-Additivity in Two-Way Analysis of Variance, J. Am. Stat. Assoc. 56, 878-888 (Dec. 1961).

[5] Mandel, John, Estimation of Weighting Factors in Linear Regression and Analysis of Variance, Technometrics 6, 1-25 (Feb. 1964).

Appendix A. Tables of Computer Input and Output

Table 1

COMPUTER INPUT WITH A SINGLE VALUE IN EACH CELL

```
01   SPECIFIC VOLUME OF RUBBER - WEIR
       81.5    64.0     50.2     38.5     21.0
0    .96677  .95639   .94826   .94143   .93397
1    .94077  .93344   .92673   .92204   .91678
2    .91360  .90953   .90464   .90189   .89941
3    .89436  .89231   .88880   .88645   .88447
4    .87937  .87864   .87572   .87336   .87214
5    .86680  .86697   .86424   .86188   .86056
6    .85636  .85654   .85410   .85134   .85038
7    .84708  .84717   .84480   .84201   .84113
8    .83828  .83828   .83608   .83345   .83263
9    .83074  .82996   .82859   .82563   .82647
10   .82360  .82249   .82129   .81829   .81834
ENDATA
LMODEL
REVERS
LMODEL
STOP
```

Table 2. Computer Output For Data In Table 1

11 ROWS 5 COLUMNS

SPECIFIC VOLUME OF RU88ER - WEIR

Table 2a

TABLE OF AVERAGES

	21.0	38.5	50.2	64.0	81.5	AVG
10	0.818339996	0.818289995	0.821289994	0.822489999	0.823599994	0.820801981
9	0.826469995	0.825629994	0.828589998	0.829959996	0.830739997	0.828277975
8	0.832629994	0.833449997	0.836080000	0.838280000	0.838280000	0.835743986
7	0.841129996	0.842009999	0.844799995	0.847169995	0.847080000	0.844437987
6	0.850379996	0.851339996	0.854099996	0.856539994	0.856359996	0.853743978
5	0.860560000	0.861879997	0.864239998	0.866969995	0.866799995	0.864089988
4	0.872139998	0.873359993	0.875719994	0.878639996	0.879369996	0.875845976
3	0.884469993	0.886449993	0.888799995	0.892309994	0.894359998	0.889277980
2	0.899409994	0.901839995	0.904639997	0.909529999	0.913599998	0.905813977
1	0.916779995	0.922039993	0.926729999	0.933440000	0.940769993	0.927951992
0	0.933969997	0.941429995	0.948259994	0.956389993	0.966769993	0.949363977
AVG	0.866934516	0.868888147	0.872113600	0.875610881	0.877975419	0.872304499

SPECIFIC VOLUME OF RU88ER - WEIR

Table 2b

LMOOEL -0

	A	8		A	8
10	0.820801981	0.51668030	4	0.875845976	0.68988606
9	0.828277975	0.45763269	3	0.889277980	0.89047448
8	0.835743986	0.56575614	2	0.905813977	1.24970494
7	0.844437987	0.59856942	1	0.927951992	2.04841876
6	0.853743978	0.60197268	0	0.949363977	2.77372935
5	0.864089988	0.61487932			

	C		C
21.0	-0.005369984	64.0	0.003306381
38.5	-0.003416352	81.5	0.005670920
50.2	-0.000190899		

ANALYSIS OF VARIANCE

	OF	SS	MS	F
TOTAL	55	41.9396214		
MEAN	1	41.8503318		
ROWS	10	0.087871805	0.008787177	10000.4635
COLS	4	0.000919990	0.000229992	261.756298
RXC	40	0.000494681	0.123672E-04	14.0747378
LIN	10	0.000468358	0.468365E-04	53.3034325
C	1	0.000391498	0.000391498	45.8394866
NC	9	0.768659E-04	0.854066E-05	9.71989989
RESIO	30	0.263603E-04	0.878677E-06	

11 ROWS 5 COLUMNS

SPECIFIC VOLUME OF RU88ER - WEIR

Table 2c

TABLE OF OEVIATIONS

	21.0	38.5	50.2	64.0	81.5	SUM SQ
10	0.312582E-03	-0.746824E-03	0.586651E-03	-0.203177E-04	-0.132039E-03	0.101746E-05
9	0.649504E-03	-0.108454E-02	0.399388E-03	0.168920E-03	-0.133172E-03	0.180387E-05
8	-0.758842E-04	-0.361167E-03	0.444017E-03	0.665411E-03	-0.672340E-03	0.122817E-05
7	-0.936761E-04	-0.383057E-03	0.476278E-03	0.752911E-03	-0.752419E-03	0.151536E-05
6	-0.131398E-03	-0.347428E-03	0.470936E-03	0.805669E-03	-0.797719E-03	0.164521E-05
5	-0.228092E-03	-0.109345E-03	0.267394E-03	0.846982E-03	-0.776924E-03	0.145647E-05
4	-0.129640E-05	-0.129089E-03	0.572205E-05	0.512995E-03	-0.388265E-03	0.430611E-06
3	-0.261515E-04	0.214189E-03	-0.307992E-03	0.877678E-04	0.322163E-04	0.150161E-06
2	0.306919E-03	0.345454E-03	-0.935413E-03	-0.415973E-03	0.699051E-03	0.175024E-05
1	-0.172019E-03	0.108612E-02	-0.830948E-03	-0.128485E-02	0.120158E-02	0.499436E-05
0	-0.499092E-03	0.154205E-02	-0.574477E-03	-0.214498E-02	0.167643E-02	0.103684E-04
SUM SQ	0.976957E-06	0.588329E-05	0.321133E-05	0.910076E-05	0.718798E-05	0.263603E-04

SPECIFIC VOLUME OF RUBBER - WEIR

REVERS -0

SPECIFIC VOLUME OF RUBBER - WEIR

Table 2d

LMODEL -0

	A	B
21.0	0.866934516	0.90821072
38.5	0.868888147	0.96198194
50.2	0.872113600	0.98464904
64.0	0.875610881	1.03570965
81.5	0.877975419	1.10952595

	C		C
10	-0.051502518	4	0.003541477
9	-0.044026524	3	0.016973481
8	-0.036560513	2	0.033509478
7	-0.027866513	1	0.055647492
6	-0.018560521	0	0.077059478
5	-0.008214511		

ANALYSIS OF VARIANCE

	DF	SS	MS	F
TOTAL	55	41.9396214		
MEAN	1	41.8503323		
ROWS	4	0.000919990	0.000229992	98.7544155
COLS	10	0.087871805	0.008787177	3772.93661
RXC	40	0.000494681	0.123672E-04	5.31006300
LIN	4	0.000410840	0.000102706	44.1007395
C	1	0.000391461	0.000391461	60.5937004
NC	3	0.193813E-04	0.646044E-05	2.77390614
RESID	36	0.838441E-04	0.232900E-05	

5 ROWS 11 COLUMNS

SPECIFIC VOLUME OF RUBBER - WEIR

Table 2e

TABLE OF DEVIATIONS

	10	9	8	7	6	5	4	3
21.0	-0.181938E-02	-0.479154E-03	-0.109987E-02	-0.495851E-03	0.302352E-03	0.108600E-02	0.198908E-02	0.211999E-02
38.5	-0.105365E-02	-0.905432E-03	-0.267595E-03	-0.710636E-04	0.306740E-03	0.894062E-03	0.106502E-02	0.123367E-02
50.2	-0.111699E-03	-0.172928E-03	-0.343248E-04	0.125133E-03	0.262000E-03	0.214815E-03	0.119284E-03	-0.265241E-04
64.0	0.220776E-03	-0.521839E-04	0.535198E-03	0.420734E-03	0.152431E-03	-0.133030E-03	-0.638820E-03	-0.880480E-03
81.5	0.276796E-02	0.161316E-02	0.869423E-03	0.232011E-04	-0.102204E-02	-0.206121E-02	-0.253478E-02	-0.244793E-02
SUM SQ	0.121431E-04	0.368429E-05	0.232483E-05	0.444132E-06	0.132195E-05	0.629117E-05	0.119381E-04	0.127846E-04

SPECIFIC VOLUME OF RUBBER - WEIR

TABLE OF DEVIATIONS

	2	1	0	SUM SQ
21.0	0.204182E-02	-0.694163E-03	-0.295076E-02	0.280748E-04
38.5	0.766337E-03	-0.380032E-03	-0.158797E-02	0.880963E-05
50.2	-0.468679E-03	-0.176847E-03	0.269860E-03	0.512698E-06
64.0	-0.786968E-03	0.194475E-03	0.967875E-03	0.333311E-05
81.5	-0.155506E-02	0.105224E-02	0.329509E-02	0.431139E-04
SUM SQ	0.801346E-05	0.180259E-05	0.230958E-04	0.838441E-04

SPECIFIC VOLUME OF RUBBER - WEIR

STOP -0

Table 3. Computer Input With Replicate Values In Each Cell

```
02  ISO NATURAL RUBBER TEST     STRESS AT 100  40
LAB09 MAT145         6.93       6.78
LAB09 MAT245         8.86       8.76
LAB09 MAT345         4.26       4.24
LAB09 MAT445         6.48       6.46
LAB09 MAT545         5.17       5.82
LAB09 MAT645         4.58       4.56
LAB09 MAT745         4.37       4.38
LAB09 MAT845         5.50       5.47
LAB10 MAT145         6.96       6.81
LAB10 MAT245         9.13       9.09
LAB10 MAT345         4.39       4.40
LAB10 MAT445         6.32       6.42
LAB10 MAT545         5.13       5.12
LAB10 MAT645         4.58       4.57
LAB10 MAT745         4.48       4.51
LAB10 MAT845         5.55       5.55
LAB11 MAT145         6.81       6.94
LAB11 MAT245         9.12       9.29
LAB11 MAT345         4.32       4.37
LAB11 MAT445         6.29       6.30
LAB11 MAT545         5.15       5.12
LAB11 MAT645         4.50       4.55
LAB11 MAT745         4.44       4.41
LAB11 MAT845         5.51       5.54
LAB12 MAT145         6.82       6.82
LAB12 MAT245         8.57       8.61
LAB12 MAT345         4.41       4.37
LAB12 MAT445         6.46       6.41
LAB12 MAT545         5.13       5.11
LAB12 MAT645         4.48       4.46
LAB12 MAT745         4.20       4.16
LAB12 MAT845         5.52       5.48
LAB21 MAT145         6.99       7.14
LAB21 MAT245         8.03       8.80
LAB21 MAT345         4.29       4.37
LAB21 MAT445         6.44       6.43
LAB21 MAT545         4.95       5.12
LAB21 MAT645         4.57       4.59
LAB21 MAT745         6.61       6.21
LAB21 MAT845         5.58       5.54
LAB22 MAT145         6.91       7.10
LAB22 MAT245         8.75       8.82
LAB22 MAT345         4.45       4.49
LAB22 MAT445         6.46       6.47
LAB22 MAT545         5.02       5.21
LAB22 MAT645         4.53       4.50
LAB22 MAT745         4.89       4.72
LAB22 MAT845         5.41       5.48
```

Table 3. Computer Input With Replicate Values in Each Cell (Cont.)

LAB23	MAT145	7.09	7.14
LAB23	MAT245	8.40	8.46
LAB23	MAT345	4.34	4.35
LAB23	MAT445	6.35	6.19
LAB23	MAT545	5.16	5.18
LAB23	MAT645	4.45	4.40
LAB23	MAT745	4.83	4.82
LAB23	MAT845	5.52	5.47
LAB24	MAT145	7.18	7.81
LAB24	MAT245	8.83	8.87
LAB24	MAT345	4.45	4.45
LAB24	MAT445	6.51	6.46
LAB24	MAT545	5.44	5.26
LAB24	MAT645	4.61	4.64
LAB24	MAT745	6.82	6.62
LAB24	MAT845	5.56	5.67
LAB57	MAT145	6.97	6.97
LAB57	MAT245	9.29	9.25
LAB57	MAT345	4.51	4.54
LAB57	MAT445	6.61	6.67
LAB57	MAT545	5.24	5.29
LAB57	MAT645	4.77	4.79
LAB57	MAT745	7.60	7.51
LAB57	MAT845	5.85	5.88
LAB58	MAT145	7.14	7.14
LAB58	MAT245	9.37	9.28
LAB58	MAT345	4.60	4.57
LAB58	MAT445	6.72	6.64
LAB58	MAT545	5.31	5.37
LAB58	MAT645	4.86	4.85
LAB58	MAT745	8.41	8.34
LAB58	MAT845	6.02	5.86
LAB59	MAT145	7.11	7.14
LAB59	MAT245	9.70	9.68
LAB59	MAT345	4.67	4.61
LAB59	MAT445	6.76	6.71
LAB59	MAT545	5.41	5.37
LAB59	MAT645	4.94	4.84
LAB59	MAT745	8.32	8.32
LAB59	MAT845	5.90	5.99
LAB60	MAT145	7.04	7.06
LAB60	MAT245	9.16	9.21
LAB60	MAT345	4.63	4.59
LAB60	MAT445	6.74	6.76
LAB60	MAT545	5.43	5.39
LAB60	MAT645	4.78	4.78
LAB60	MAT745	8.28	8.22
LAB60	MAT845	5.88	5.99
LAB61	MAT145	6.90	6.95
LAB61	MAT245	9.59	9.90
LAB61	MAT345	4.97	5.06
LAB61	MAT445	6.91	6.88

Table 3. Computer Input With Replicate Values in Each Cell (Cont.)

LAB61	MAT545	5.42	5.37
LAB61	MAT645	4.92	5.12
LAB61	MAT745	4.47	4.42
LAB61	MAT845	5.73	5.65
LAB62	MAT145	7.14	7.07
LAB62	MAT245	9.33	9.65
LAB62	MAT345	4.92	5.13
LAB62	MAT445	6.51	6.50
LAB62	MAT545	5.44	5.37
LAB62	MAT645	5.07	5.12
LAB62	MAT745	7.34	7.25
LAB62	MAT845	6.01	6.13
LAB63	MAT145	7.20	7.16
LAB63	MAT245	9.77	10.07
LAB63	MAT345	4.59	4.57
LAB63	MAT445	6.02	6.20
LAB63	MAT545	5.27	5.20
LAB63	MAT645	4.99	5.17
LAB63	MAT745	6.53	6.46
LAB63	MAT845	5.80	5.90
LAB64	MAT145	7.09	7.01
LAB64	MAT245	9.20	9.26
LAB64	MAT345	4.16	4.35
LAB64	MAT445	6.28	6.29
LAB64	MAT545	5.26	5.16
LAB64	MAT645	4.84	4.88
LAB64	MAT745	7.54	7.72
LAB64	MAT845	5.91	5.94
LAB69	MAT145	7.16	7.14
LAB69	MAT245	8.85	8.89
LAB69	MAT345	4.5	4.42
LAB69	MAT445	6.56	6.64
LAB69	MAT545	5.34	5.39
LAB69	MAT645	4.77	4.73
LAB69	MAT745	7.39	7.43
LAB69	MAT845	5.72	5.83
LAB70	MAT145	7.13	7.15
LAB70	MAT245	9.16	9.81
LAB70	MAT345	4.73	4.68
LAB70	MAT445	6.82	6.87
LAB70	MAT545	5.58	5.48
LAB70	MAT645	5.05	5.05
LAB70	MAT745	7.64	7.59
LAB70	MAT845	5.81	5.77
LAB71	MAT145	7.03	7.03
LAB71	MAT245	9.34	9.29
LAB71	MAT345	4.46	4.47
LAB71	MAT445	6.61	6.66
LAB71	MAT545	5.28	5.23
LAB71	MAT645	4.83	4.83
LAB71	MAT745	7.51	7.52
LAB71	MAT845	5.84	5.79

Table 3. Computer Input With Replicate Values in Each Cell (Cont.)

```
LAB72 MAT145        7.17       7.10
LAB72 MAT245        9.22       9.14
LAB72 MAT345        4.47       4.47
LAB72 MAT445        6.62       6.67
LAB72 MAT545        5.48        5.5
LAB72 MAT645        4.76       4.86
LAB72 MAT745        6.94       7.05
LAB72 MAT845        5.86       5.72
LAB85 MAT145         7.0        7.0
LAB85 MAT245         8.8        8.7
LAB85 MAT345         4.5        4.5
LAB85 MAT445         6.6        6.7
LAB85 MAT545         4.9        5.3
LAB85 MAT645         4.5        4.5
LAB85 MAT745         4.8        5.0
LAB85 MAT845         5.7        5.7
LAB86 MAT145         7.0        7.0
LAB86 MAT245         8.7        8.9
LAB86 MAT345         4.4        4.5
LAB86 MAT445         6.6        6.6
LAB86 MAT545         5.3        5.3
LAB86 MAT645         4.6        4.7
LAB86 MAT745         4.5        4.7
LAB86 MAT845         5.9        5.9
LAB87 MAT145         7.0        7.1
LAB87 MAT245         9.0        9.1
LAB87 MAT345         4.4        4.5
LAB87 MAT445         6.5        6.6
LAB87 MAT545         4.9        5.0
LAB87 MAT645         4.5        4.6
LAB87 MAT745         5.0        5.2
LAB87 MAT845         5.6        5.7
LAB88 MAT145         7.0        7.0
LAB88 MAT245         9.2        9.2
LAB88 MAT345         4.5        4.5
LAB88 MAT445         6.7        6.8
LAB88 MAT545         5.1        5.2
LAB88 MAT645         4.8        4.8
LAB88 MAT745         5.1        5.3
LAB88 MAT845         5.7        5.8
ENDATA
COMBINROWS    LAB09    LAB10    LAB11    LAB12    LAB01
COMBINROWS    LAB21    LAB22    LAB23    LAB24    LAB02
COMBINROWS    LAB57    LAB58    LAB59    LAB60    LAB03
COMBINROWS    LAB61    LAB62    LAB63    LAB64    LAB04
COMBINROWS    LAB69    LAB70    LAB71    LAB72    LAB05
COMBINROWS    LAB85    LAB86    LAB87    LAB88    LAB06
      END
LMWCOLCAL
QMODEL
OMIT        01MAT745
QMODEL
STOP
```

Table 4. Computer Output For Data in Table 3

24 ROWS   8 COLUMNS

ISO NATURAL RUBBER TEST   STRESS AT 100 40

TABLE OF AVERAGES

	MAT345	MAT645	MAT545	MAT845	MAT745	MAT445	MAT145	MAT245
LAB12	4.38999993	4.46999997	5.11999995	5.49999994	4.17999995	6.43499994	6.81999999	8.58999991
LAB23	4.34499997	4.42499995	5.16999996	5.49499995	4.82499999	6.26999998	7.11499995	8.42999995
LAB09	4.24999994	4.56999993	5.49499995	5.48499995	4.37499994	6.46999997	6.85499996	8.80999994
LAB11	4.34499997	4.52499998	5.13499993	5.52499998	4.42499995	6.29499996	6.87499994	9.20499992
LAB10	4.39499998	4.57499999	5.12499994	5.54999995	4.49499995	6.36999995	6.88499993	9.10999990
LAB22	4.46999997	4.51499999	5.11499995	5.44499993	4.80499995	6.46499997	7.00499994	8.78499997
LAB85	4.50000000	4.50000000	5.09999996	5.69999999	4.89999998	6.64999998	7.00000000	8.74999988
LAB86	4.44999999	4.64999998	5.29999995	5.89999998	4.59999996	6.59999996	7.00000000	8.79999995
LAB87	4.44999999	4.54999995	4.94999999	5.64999998	5.09999996	6.54999995	7.04999995	9.04999995
LAB21	4.32999992	4.57999998	5.03499997	5.55999994	6.40999997	6.43499994	7.06499994	8.41499996
LAB88	4.50000000	4.79999995	5.14999998	5.74999994	5.19999993	6.74999994	7.00000000	9.19999993
LAB61	5.01499993	5.01999992	5.39499992	5.68999994	4.44499993	6.89499998	6.92499995	9.74499989
LAB24	4.44999999	4.62499994	5.34999996	5.61499995	6.71999997	6.48499995	7.49499995	8.84999990
LAB69	4.45999998	4.74999994	5.36499995	5.77499998	7.40999997	6.59999996	7.14999998	8.86999989
LAB64	4.25499994	4.85999995	5.20999998	5.92499995	7.62999994	6.28499997	7.04999995	9.22999990
LAB63	4.57999998	5.07999992	5.23499995	5.84999996	6.49499995	6.10999995	7.17999995	9.91999996
LAB72	4.46999997	4.80999994	5.48999995	5.78999996	6.99499995	6.64499992	7.13499993	9.17999995
LAB71	4.46499997	4.82999998	5.25499994	5.81499994	7.51499999	6.63499993	7.02999997	9.31499994
LAB57	4.52499998	4.77999997	5.26499993	5.86499995	7.55499995	6.63999993	6.96999997	9.26999998
LAB60	4.60999995	4.77999997	5.40999997	5.93499994	8.24999988	6.74999994	7.04999995	9.18499994
LAB62	5.02499998	5.09499997	5.40499997	6.06999999	7.29499996	6.50500000	7.10499996	9.48999989
LAB70	4.70499992	5.04999995	5.52999997	5.78999996	7.61499995	6.84499997	7.13999999	9.48499990
LAB58	4.58499998	4.85499996	5.33999991	5.93999994	8.37499988	6.67999995	7.13999999	9.32499993
LAB59	4.63999993	4.88999999	5.38999993	5.94499993	8.31999993	6.73499995	7.12499994	9.68999994
AVG	4.50874972	4.73270798	5.26395798	5.73187459	6.16395783	6.54583275	7.04854119	9.11249936

ISO NATURAL RUBBER TEST   STRESS AT 100 40

TABLE OF AVERAGES

	AVG
LAB12	5.68812484
LAB23	5.75937486
LAB09	5.78874981
LAB11	5.79124975
LAB10	5.81312484
LAB22	5.82562482
LAB85	5.88749987
LAB86	5.91249985
LAB87	5.91874981
LAB21	5.97874987
LAB88	6.04374981
LAB61	6.14124978
LAB24	6.19874978
LAB69	6.29749978
LAB64	6.30562478
LAB63	6.30624992
LAB72	6.31437486
LAB71	6.35749984
LAB57	6.35874981
LAB60	6.49624979
LAB62	6.49874979
LAB70	6.51999986
LAB58	6.52999973
LAB59	6.59187484
AVG	6.13851511

24 ROWS 8 COLUMNS

ISO NATURAL RUBBER TEST STRESS AT 100 40

TABLE OF STANDARD DEVIATIONS

	MAT345	MAT645	MAT545	MAT845	MAT745	MAT445	MAT145	MAT245
LA812	0.028284281	0.014142118	0.014142163	0.028284281	0.028284281	0.035355344	0.	0.028284244
LA823	0.007071055	0.035355344	0.014142118	0.035355344	0.007071055	0.113137059	0.035355344	0.042426363
LA809	0.014142163	0.014142163	0.459619425	0.021213219	0.007071100	0.014142118	0.106065996	0.070710607
LA811	0.035355300	0.035355300	0.021213219	0.021213181	0.021213219	0.007071055	0.091923915	0.120208204
LA810	0.007071055	0.007071055	0.007071100	0.	0.021213219	0.070710652	0.106066041	0.028284326
LA822	0.028284244	0.021213181	0.134350285	0.049497463	0.120208159	0.007071055	0.134350285	0.049497426
LA885	0.	0.	0.282842688	0.	0.141421385	0.070710689	0.	0.070710689
LA886	0.070710689	0.070710689	0.	0.	0.141421340	0.	0.	0.141421385
LA887	0.070710689	0.070710652	0.070710689	0.070710689	0.141421340	0.070710652	0.070710652	0.070710607
LA821	0.056568526	0.014142118	0.120208122	0.028284281	0.282842688	0.007071100	0.106066041	0.544472203
LA888	0.	0.	0.070710689	0.070710652	0.141421340	0.070710652	0.	0.
LA861	0.063639589	0.141421340	0.035355344	0.056568526	0.035355344	0.021213181	0.035355344	0.219203144
LA824	0.	0.021213219	0.127279222	0.077781752	0.141421385	0.035355344	0.445477262	0.028284244
LA869	0.056568570	0.028284281	0.035355344	0.077781752	0.028284244	0.056568570	0.014142118	0.028284326
LA864	0.134350285	0.028284281	0.070710689	0.021213219	0.127279222	0.007071055	0.056568526	0.042426445
LA863	0.014142118	0.127279222	0.049497463	0.070710689	0.049497463	0.127279222	0.028284281	0.212131999
LA872	0.	0.070710652	0.014142163	0.098994933	0.077781707	0.035355344	0.049497463	0.056568488
LA871	0.007071055	0.	0.035355344	0.035355344	0.007071055	0.035355344	0.	0.035355300
LA857	0.021213181	0.014142118	0.035355344	0.021213219	0.063639589	0.042426407	0.	0.028284244
LA860	0.028284281	0.	0.028284244	0.077781707	0.042426445	0.014142163	0.014142118	0.035355300
LA862	0.148492448	0.035355300	0.049497508	0.084852815	0.063639589	0.007071055	0.049497463	0.226274200
LA870	0.035355344	0.	0.070710689	0.028284244	0.035355344	0.035355300	0.014142118	0.459619388
LA858	0.021213181	0.007071055	0.042426407	0.113137096	0.049497508	0.056568526	0.	0.063639544
LA859	0.042426407	0.070710689	0.028284281	0.063639589	0.	0.035355344	0.021213219	0.014142118
AVG	0.037123099	0.034471445	0.075719349	0.048024334	0.073951580	0.040658630	0.057452418	0.109012276

ISO NATURAL RUBBER TEST STRESS AT 100 40

TABLE OF STANDARD DEVIATIONS

	AVG
LA812	0.022097088
LA823	0.036239207
LA809	0.088388346
LA811	0.044194177
LA810	0.030935928
LA822	0.068059012
LA885	0.070710681
LA886	0.053033009
LA887	0.079549491
LA821	0.144956879
LA888	0.044194162
LA861	0.076013975
LA824	0.109601550
LA869	0.040658653
LA864	0.060987964
LA863	0.084852807
LA872	0.050381340
LA871	0.019445427
LA857	0.028284259
LA860	0.030052029
LA862	0.083085045
LA870	0.084852800
LA858	0.044194162
LA859	0.034471452
AVG	0.059551641

	PERCENT CV		PERCENT CV
MAT345	0.81497547	MAT745	1.27107066
MAT645	0.71768180	MAT445	0.62585288
MAT545	1.43588310	MAT145	0.80329586
MAT845	0.83216653	MAT245	1.19633342

Table 4c

ISO NATURAL RUBBER TEST   STRESS AT 100   40

```
COMBINROWS  -0LAB09   LAB10   LAB11   LAB12   LAB01
      ROWS    LAB21   LAB22   LAB23   LAB24   LAB02
      ROWS    LAB57   LAB58   LAB59   LAB60   LAB03
      ROWS    LAB61   LAB62   LAB63   LAB64   LAB04
      ROWS    LAB69   LAB70   LAB71   LAB72   LAB05
      ROWS    LAB85   LAB86   LAB87   LAB88   LAB06
      END
```

Table 4d

6 ROWS   8 COLUMNS

ISO NATURAL RUBBER TEST   STRESS AT 100   40

TABLE OF AVERAGES

	MAT345	MAT645	MAT545	MAT845	MAT745	MAT445	MAT145	MAT245
LAB01	4.34499991	4.53499991	5.21874988	5.51499987	4.36874992	6.39249992	6.85874993	8.92874992
LAB02	4.39874989	4.53624994	5.16749990	5.52874988	5.68999988	6.41374993	7.16999990	8.61999989
LAB06	4.47499996	4.62499994	5.12499994	5.74999988	4.94999993	6.63749987	7.01249999	8.94999981
LAB04	4.71874988	5.01374990	5.31124985	5.88374990	6.46624988	6.44874990	7.06499994	9.59624982
LAB05	4.52499992	4.85999990	5.40999985	5.79249996	7.38374990	6.68124986	7.11374992	9.21249986
LAB03	4.58999991	4.82624996	5.35124987	5.92124993	8.12499988	6.70124984	7.07124990	9.36749983
AVG	4.50874984	4.73270822	5.26395816	5.73187476	6.16395813	6.54583317	7.04854143	9.11249971

ISO NATURAL RUBBER TEST   STRESS AT 100   40

TABLE OF AVERAGES

	AVG
LAB01	5.77031231
LAB02	5.94062483
LAB06	5.94062483
LAB04	6.31296849
LAB05	6.37234348
LAB03	6.49421853
AVG	6.13851535

Table 4e

ISO NATURAL RUBBER TEST   STRESS AT 100   40

LMWCOLCAL   -0

	WEIGHT	SQRT W		WEIGHT	SQRT W
MAT345	0.130694E 04	36.1516733	MAT745	0.575277E 00	0.7584704
MAT645	0.178694E 03	13.3676336	MAT445	0.904790E 02	9.5120463
MAT545	0.845907E 02	9.1973194	MAT145	0.294502E 02	5.4268070
MAT845	0.439850E 03	20.9726107	MAT245	0.885176E 01	2.9751900

	A	B	B STAR		A	B	B STAR
LAB01	4.77612406	0.98501815	1.00000301	LAB04	5.13244158	0.93052941	1.00000323
LAB02	4.81412500	0.97005575	1.00000304	LAB05	4.99554825	1.03660026	1.00000314
LAB06	4.92038530	1.02875741	1.00000310	LAB03	5.05768508	1.04903851	1.00000319

	C		C
MAT345	-0.44063497	MAT745	1.21457332
MAT645	-0.21667659	MAT445	1.59644830
MAT545	0.31457335	MAT145	2.09915662
MAT845	0.78248996	MAT245	4.16311491

ALPHA= 0.632179E-06

POINT OF CONCURRENCE= -1581831.50

Table 4f

ANALYSIS OF WEIGHTED DATA

SOURCE	DF	SS	MS
ROWS	5	206.509474	41.3018947
COLS	7	6326.78949	903.827065
RXC	35	40.9357047	1.16959156
FIT	30	29.8441887	0.994806290
SLOPE	5	11.0915161	2.21830320
CONC	1	0.939660E-04	0.939660E-04
NONC	4	11.0914221	2.77285552

R SQUARED= 0.847188E-05

R= 0.002910644

Table 4g

6 ROWS 8 COLUMNS

ISO NATURAL RUBBER TEST STRESS AT 100 40

TABLE OF DEVIATIONS

	MAT345	MAT645	MAT545	MAT845	MAT745	MAT445	MAT145	MAT245
LA801	0.105176158	-0.37019988	1.22108589	-0.66883728	-1.21639760	0.41705905	0.08095985	0.15435924
LA802	0.436184779	-0.90481069	0.44350605	-0.93189596	-0.22930755	0.48493462	1.73427707	-0.69196562
LA806	0.286363162	-0.96885531	-1.09452666	0.51639310	-0.92524794	0.71108788	-0.36581131	-0.75337961
LA804	-0.132600397	1.10860850	-1.04767998	0.48611278	0.15443379	-1.60975917	-0.11270709	1.75508659
LA805	-0.498385459	1.19050403	0.81271965	-0.29734046	0.85644557	0.29318916	-0.31358596	-0.29315839
LA803	-0.196740396	-0.05524824	-0.33510166	0.89558908	1.36007446	-0.29649965	-1.02312286	-0.17093369
SUM SQ	0.588000752	4.54377592	4.75617659	2.70923194	4.99544007	3.67994228	4.30590874	4.26571304

ISO NATURAL RUBBER TEST STRESS AT 100 40

TABLE OF DEVIATIONS

	SUM SQ
LA801	3.7704466
LA802	5.8483440
LA806	4.5484633
LA804	8.2887406
LA805	3.4183493
LA803	3.9698453
SUM SQ	29.8441887

Table 4h

ISO NATURAL RUBBER TEST STRESS AT 100 40

QMODEL -0

	A	B	D	AP	BP
LA801	5.77031231	0.98995132	0.086043470	5.60465795	0.88223404
LA802	5.94062483	0.94821443	-0.027748495	5.99404734	0.98295260
LA806	5.94062483	0.98772637	0.038834378	5.86585927	0.93910985
LA804	6.31296849	1.03774434	0.055445835	6.20622188	0.96833204
LA805	6.37234348	1.00552733	-0.063444003	6.49448842	1.08495247
LA803	6.49421853	1.03083800	-0.089132071	6.66581917	1.14242187

	C	E		C	E
MAT345	-1.62976551	2.77118859	MAT745	0.02544278	-1.95644450
MAT645	-1.40580714	1.81097460	MAT445	0.40731782	-2.26925099
MAT545	-0.87455720	-0.06553726	MAT145	0.91002607	-2.23634863
MAT845	-0.40664059	-1.25081283	MAT245	2.97398436	3.19623044

Table 4i

ANALYSIS OF VARIANCE

	DF	SS	MS
TOTAL	48	1912.73088	
MEAN	1	1808.70580	
ROWS	5	3.40423223	0.680846445
COLS	7	92.4115286	13.2016469
RXC	35	8.20929074	0.234551162
LIN	5	0.082239047	0.016447805
QUAD	5	0.907975405	0.181595080
RES	25	7.21907640	0.288763054

ISO NATURAL RUBBER TEST   STRESS AT 100 40

TABLE OF DEVIATIONS LINEAR MODEL

	MAT345	MAT645	MAT545	MAT845	MAT745	MAT445	MAT145	MAT245
LA801	0.188076138	0.156368256	0.314206660	0.147241950	-1.42674947	0.218962848	0.187556148	0.214337945
LA802	0.003492236	-0.071368277	0.056142867	-0.026292443	-0.27475011	0.086900473	0.366475224	-0.140599728
LA806	0.144137502	0.072927892	0.048198342	0.211024702	-1.01575536	0.294556499	0.173018456	0.071892261
LA804	0.097061336	0.159649849	-0.094151855	-0.007229567	0.12687832	-0.286910295	-0.192342937	0.197045922
LA805	-0.208569765	-0.098766088	-0.082952440	-0.170955241	0.98582304	-0.100662768	-0.173649609	-0.150266170
LA803	-0.224194407	-0.218809128	-0.241441846	-0.153788030	1.60455400	-0.212847352	-0.361058056	-0.192414761
SUM SQ	0.159345828	0.117983185	0.178240769	0.119831622	6.70539916	0.280014448	0.396929681	0.169307984

ISO NATURAL RUBBER TEST   STRESS AT 100 40

TABLE OF DEVIATIONS LINEAR MODEL

	SUM SQ
LA801	2.34490645
LA802	0.24606062
LA806	1.22657490
LA804	0.21806434
LA805	1.12407701
LA803	2.96736929
SUM SQ	8.12705255

Table 4j

6 ROWS   8 COLUMNS

ISO NATURAL RUBBER TEST   STRESS AT 100 40

TABLE OF DEVIATIONS

	MAT345	MAT645	MAT545	MAT845	MAT745	MAT445	MAT145	MAT245
LA801	-0.050366543	0.000545710	0.319845706	0.254866228	-1.25841019	0.414217077	0.379979342	-0.060676813
LA802	0.080388561	-0.021116443	0.054324299	-0.061000615	-0.32903851	0.023932159	0.304419905	-0.051909126
LA806	0.036520094	0.002599806	0.050743438	0.259599246	-0.93977804	0.382681459	0.259865671	-0.052231379
LA804	-0.056589536	0.059238844	-0.090518080	0.062122792	0.23535502	-0.161089770	-0.068346709	0.019828245
LA805	-0.032754466	0.016129389	-0.087110385	-0.250311807	0.86169836	-0.244633131	-0.315532513	0.052515484
LA803	0.022807382	-0.057393201	-0.247283317	-0.265275568	1.43017203	-0.415110402	-0.560388446	0.092471890
SUM SQ	0.015128233	0.007516332	0.184758082	0.272956006	5.41835326	0.576705560	0.722853191	0.020806432

ISO NATURAL RUBBER TEST   STRESS AT 100 40

TABLE OF DEVIATIONS

	SUM SQ
LA801	2.07303306
LA802	0.21778555
LA806	1.17119324
LA804	0.10517071
LA805	0.97626529
LA803	2.67562920
SUM SQ	7.21907705

Table 5. Computer Output For Data In Table 3 Omitting Column MAT745

ISO NATURAL RUBBER TEST STRESS AT 100 40

OMIT 1MAT745

6 ROWS AND 7 COLUMNS

ISO NATURAL RUBBER TEST STRESS AT 100 40

Table 5a

TABLE OF AVERAGES

	MAT345	MAT645	MAT545	MAT845	MAT445	MAT145	MAT245	AVG
LAB01	4.34499991	4.53499991	5.21874988	5.51499987	6.39249992	6.85874993	8.92874992	5.97053552
LAB02	4.39874989	4.53624994	5.16749990	5.52874988	6.41374993	7.16999990	8.61999989	5.97642833
LAB06	4.47499996	4.62499994	5.12499994	5.74999988	6.63749987	7.01249999	8.94999981	6.08214265
LAB05	4.52499992	4.85999990	5.40999985	5.79249996	6.68124986	7.11374992	9.21249986	6.22785687
LAB03	4.58999991	4.82624996	5.35124987	5.92124993	6.70124984	7.07124990	9.36749983	6.26124978
LAB04	4.71874988	5.01374990	5.31124985	5.88374990	6.4487499C	7.06499994	9.59624982	6.29107118
AVG	4.50874984	4.73270822	5.26395816	5.73187476	6.54583311	7.04854143	9.11249971	6.13488066

ISO NATURAL RUBBER TEST STRESS AT 100 40

Table 5b

QMODEL -0

	A	B	D	AP	BP
LAB01	5.97053552	0.99264501	-0.000944868	5.97261435	0.99383824
LAB02	5.97642833	0.94873317	-0.050493434	6.08752239	1.01249915
LAB06	6.08214265	0.98964411	-0.026128277	6.13962924	1.02264038
LAB05	6.22785687	1.00366606	-0.003878571	6.23639035	1.00856414
LAB03	6.26124978	1.02780862	0.009729356	6.23984349	1.01552182
LAB04	6.29107118	1.03750479	0.071714893	6.13328630	0.94693914

	C	E
MAT345	-1.62613082	2.49770308
MAT645	-1.40217245	1.53666186
MAT545	-0.87092251	-0.34181234
MAT845	-0.40300590	-1.52881627
MAT445	0.41095245	-2.55026C96
MAT145	0.91366076	-2.51921552
MAT245	2.97761905	2.90573967

ANALYSIS OF VARIANCE

Table 5c

	DF	SS	MS
TOTAL	42	1674.41855	
MEAN	1	1580.74394	
ROWS	5	0.727348790	0.145469755
COLS	6	92.4070892	15.4011815
RXC	30	0.540115476	0.018003844
LIN	5	0.076744035	0.015348807
QUAD	5	0.274498112	0.054899618
RES	20	0.188873298	0.009443663

ISO NATURAL RUBBER TEST STRESS AT 100 40

TABLE OF DEVIATIONS LINEAR MODEL

	MAT345	MAT645	MAT545	MAT845	MAT445	MAT145	MAT245	SUM SQ
LAB01	-0.011364937	-0.043676138	0.112731278	-0.055493832	0.01403451C	-0.018726349	0.002495766	0.018378541
LAB02	-0.034914196	-0.109890878	0.017344654	-0.065333366	0.047437429	0.326751292	-0.181394339	0.159784913
LAB06	0.002148092	-0.069490969	-0.095239341	0.066689670	0.14866060C	0.026158392	-0.078925967	0.047365196
LAB05	-0.070764661	0.039455950	0.056258380	-0.030873537	0.040933967	-0.031117201	-0.003892183	0.013341606
LAB03	0.000101447	0.006165147	-0.014858246	0.074213088	0.01761961C	-0.129068255	0.045827627	0.024835601
LAB04	0.114797235	0.177439392	-0.076235056	0.010799289	-0.268686354	-0.173998654	0.215884686	0.199665628
SUM SQ	0.019538827	0.051892117	0.031277262	0.018372908	0.098725647	0.155703768	0.087860957	0.463371493

6 ROWS 7 COLUMNS

Table 5d,

ISO NATURAL RUBBER TEST STRESS AT 100 40

TABLE OF DEVIATIONS

	MAT345	MAT645	MAT545	MAT845	MAT445	MAT145	MAT245	SUM SQ
LAB01	-0.009004921	-0.042224184	0.112408E-00	-0.056938365	0.011624835	-C.021106683	0.005241312	0.018349670
LAB02	0.091203421	-0.032299533	0.853722E-04	-0.142528556	-0.081334017	0.199547432	-0.034673542	0.077312373
LAB06	0.067408770	-0.029340632	-0.104170E-00	0.026744328	0.082026668	-0.039664373	-0.003003985	0.025282182
LAB05	-0.061077125	0.045416005	0.549326E-01	-0.036803164	0.031042576	-0.040888168	0.007377952	0.012855016
LAB03	-0.024199605	-0.008785583	-0.115326E-01	0.089087486	0.04243201C	-C.104557894	0.017556638	0.021773450
LAB04	-0.064325280	0.067237847	-0.517220E-01	0.120438181	-0.085794650	0.006666623	0.007499866	0.033300601
SUM SQ	0.021396868	0.010347739	0.293129E-01	0.048068032	0.023603573	0.054486565	0.001657665	0.188873313

ISO NATURAL RUBBER TEST STRESS AT 100 40

STOP -0

```
C     A PROGRAM FOR FITTING FUNCTIONS OF TOO VARIABLES
      DIMENSION AV(120,50),Z(120,50),XR(120),XC(50),A(120),B(120),C(50),
     1 TITLE(12),XNS(26),R(99),D(120),WT(120),WFAC(120),XL(120)
      COMMON AV,Z,XR,XC,A,B,C,M,N,WT,WFAC,MT,NT,NREP,NR,NX,XNS,TITLE,XY,
     1XL,D,R
C     BOOLEAN STATEMENTS FOR INSTRUCTIONS AND GETTING TO SUBROUTINES
B     XNS(1)=626346476060                                           STOP
B     XNS(2)=452566242163                                           NEWDAT
B     XNS(3)=434446242543                                           LMODEL
B     XNS(4)=504446242543                                           QMODEL
B     XNS(5)=234446242543                                           CMODEL
B     XNS(6)=444446242543                                           MMODEL
B     XNS(7)=434466514666                                           LMWROW
B     XNS(8)=434466234643                                           LMWCOL
B     XNS(9)=626422016060                                           SUB1
B     XNS(10)=234651252363                                          CORECT
B     XNS(11)=464431636060                                          OMIT
B     XNS(12)=314523436424                                          INCLUD
B     XNS(13)=635121264651                                          TRAFOR
B     XNS(14)=474666255160                                          POWER
B     XNS(15)=234644223145                                          COMBIN
B     XNS(16)=626422026060                                          SUB2
B     XNS(17)=512565255162                                          REVERS
B     XNS(18)=314563432122                                          INTLAB
B     XNS(19)=254524216321                                          ENDATA
B     XNS(20)=232143606060                                          CAL
B     XNS(21)=606060606060                                          BLANK
B     XNS(22)=626444606250                                          SUM SQ
B     XNS(23)=444651252421                                          MOREDA
B     XNS(24)=606060606060                                          BLANK
B     XNS(25)=216527606060                                          AVG
  100 READ 110,NR,TITLE
C     PROGRAM RETURNS TO THIS STATEMENT FOR EACH NEW SET OF DATA
C     NR - NUMBER OF REPLICATES  TITLE - ANY SUITABLE IDENTIFICATION
  110 FORMAT(I2,11A6,A4)
      PRINT 120,NR,TITLE
  120 FORMAT(1H1I2,11A6,A4)
      M=0
      N=0
      TRANS=0.
      DO 130 I=1,120
  130 XR(I)=0.
      DO 140 J=1,50
  140 XC(J)=0.
C     IF NR IS GREATER THAN 1, REPLICATE VALUES FOR EACH CELL ARE READ
C     AT STATEMENT 250
C     IF NR EQUALS 1 PROGRAM CONTINUES AND READS IN SINGLE VALUES / CELL
      IF(NR-1)200,200,250
  200 READ  210,XRX,(XC(J),J=1,7)
C     READ COLUMN HEADINGS XC
  210 FORMAT(2A6,2X,5(A6,4X),A6)
      DO  220 J=2,7
      IF(XC(J)-XNS(21))220,221,220                                  BLANK
  220 CONTINUE
      N=7
      GO TO 222
  221 N=J-1
  222 DO 227 I=1,120
C     READ ROW HEADINGS XR AND DATA AV
      READ  225,XR(I),(AV(I,J),J=1,N)
  225 FORMAT(A6,F8.0,5F10.0,F8.0)
      IF(XR(I)-XNS(19))226,228,226                                  ENDATA
C     ENDATA - END OF DATA
  226 IF(XR(I)-XNS(23))227,230,227                                  MOREDA
C     MOREDA - MORE DATA, WHEN MORE THAN 7 COLUMNS
```

```
  227 CONTINUE
      CALL ENDJOB
  228 M=I-1
  229 MT=M
      NT=N
      MT1=MT+1
      NT1=NT+1
      GO TO 425
C     JUMPS TO 425 WHERE ROW AND COLUMN AVERAGES ARE COMPUTED
  230 M=I-1
      DO  244 JX=8,50,7
      JXX=JX+6
      READ  210,XRX,(XC(J),J=JX,JXX)
C     AT THIS POINT MORE DATA ARE READ
      IF(XRX-XNS(19))240,229,240                                       ENDATA
  240 DO  241 J=JX,JXX
      IF(XC(J)-XNS(21))241,242,241                                     BLANK
  241 CONTINUE
      N=JXX
      GO TO 243
  242 N=J-1
  243 DO  244 I=1,M
  244 READ  245,(AV(I,J),J=JX,N)
  245 FORMAT(6X,F8.0,5F10.0,F8.0)
      CALL ENDJOB
  250 DO 300 I=1,120
      DO 300 J=1,50
  300 AV(I,J)=-1.0E37
C     USED NEAR STATEMENT 402 TO CHECK FOR MISSING VALUES
  301 READ 305,XRX,XCX,NRX,(R(K),K=1,6)
C     DATACONSISTING OF REPLICATE MEASUREMENTS ARE READ
  305 FORMAT(2A6,I2,5F10.0,F8.0)
      IF(XRX-XNS(19))310,400,310                                       ENDATA
  310 IF(XRX-XNS(13))312,311,312                                       TRAFOR
  311 TRANS=1.
      AX=R(1)
      BX=R(2)
      CX=R(3)
C     FOR TRANSFORMING DATA BEFORE AVERAGING OR PRINTING
      GO TO 301
  312 DO 313 I=1,M
      IF(XRX-XR(I))313,314,313
  313 CONTINUE
      M=M+1
      XR(M)=XRX
      I=M
  314 DO 315 J=1,N
      IF(XCX-XC(J))315,320,315
  315 CONTINUE
      N=N+1
      XC(N)=XCX
      J=N
  320 IF(NRX)331,330,331
  330 NRX=NR
  331 IF(NRX-6)336,336,332
  332 READ 335,(R(K),K=7,NRX)
C     AS MANY AS 99 REPLICATES PER CELL MAY BE READ
  335 FORMAT(14X,5F10.0,F8.0)
  336 IF(TRANS)337,339,337
  337 DO 338 K=1,NRX
  338 R(K)=TRAFOR(R(K),AX,BX,CX)
C     CALLS FUNCTION TRANSFOR **NOTE** USERWRITES OWN TRANSFORMATION
  339 FNRX=NRX
      AV(I,J)=SUM1(R,NRX)/FNRX
      IF(NRX-1)350,301,350
  350 Z(I,J)=0.
      DO 351 K=1,NRX
  351 Z(I,J)=(R(K)-AV(I,J))**2+Z(I,J)
      Z(I,J)=SQRTF(Z(I,J)/(FNRX-1.))
```

```
      GO TO 301
  400 MT=M
      NT=N
      MT1=MT+1
      NT1=NT+1
      MEMORY=0
  401 DO 408 I=1,M
      DO 408 J=1,N
      IF(AV(I,J)+1.0E37)402,402,408
C     CHECK FOR MISSING VALUES
  402 PRINT 405,XR(I),XC(J)
  405 FORMAT(16HOMISSING VALUE  A6,2X,A6)
      MEMORY=1
  408 CONTINUE
  409 IF(MEMORY)420,421,420
  420 CALL ENDJOB
C     CUTS OFF BAD PROGRAM WITH MISSING VALUES
  421 IF(XCX-XNS(10))425,422,425                                        CORECT
C      CORECT IN COLUMNS 7-12 AFTER ENDATA - CORRECTS DATA BEFORE
C          PRINTING TABLES OF AVERAGES AND STANDARD DEVIATIONS
  422 CALL CORECT
  425 MP1=M+1
      NP1=N+1
      FM=M
      FN=N
C     CALCULATES ROW AND COLUMN AVERAGES FOR TABLE OF AVERAGES AND IF
C     REPLICATE MEASUREMENTS, FOR TABLE OF STANDARD DEVIATIONS
  450 DO 451 J=1,N
  451 AV(M+1,J)=SUM1(AV(1,J),M)/FM
  452 DO 453 I=1,MP1
  453 AV(I,N+1)=SUM2(AV,I,N)/FN
  454 IF(NR-1)500,500,455
  455 DO 456 J=1,N
  456 Z(M+1,J)=SUM1(Z(1,J),M)/FM
  457 DO 458 I=1,MP1
  458 Z(I,N+1)=SUM2(Z,I,N)/FN
C     ORDER AV,Z,XR,XC IN ASCENDING ORDER OF AVERAGES OF AVERAGE
  500 DO 505 I=1,M
      IS=I
      DO 502 IX=I,M
      IF(AV(IX,N+1)-AV(IS,N+1))501,502,502
  501 IS=IX
  502 CONTINUE
      HOLD=XR(I)
      XR(I)=XR(IS)
      XR(IS)=HOLD
      DO 505 J=1,NT1
      HOLD=AV(I,J)
      AV(I,J)=AV(IS,J)
      AV(IS,J)=HOLD
      IF(NR-1)505,505,503
  503 HOLD=Z(I,J)
      Z(I,J)=Z(IS,J)
  504 Z(IS,J)=HOLD
  505 CONTINUE
      DO 520 J=1,N
      JS=J
      DO 507 JX=J,N
      IF(AV(M+1,JX)-AV(M+1,JS))506,507,507
  506 JS=JX
  507 CONTINUE
      HOLD=XC(J)
      XC(J)=XC(JS)
      XC(JS)=HOLD
      DO 520 I=1,MT1
      HOLD=AV(I,J)
      AV(I,J)=AV(I,JS)
      AV(I,JS)=HOLD
      IF(NR-1)520,520,508
```

```
  508 HOLD=Z(I,J)
      Z(I,J)=Z(I,JS)
  509 Z(I,JS)=HOLD
  520 CONTINUE
      XR(M+1)=XNS(25)                                                   AVG
      XC(N+1)=XNS(25)                                                   AVG
C     PRINTING OF TABLE OF AVERAGES AND STANDARD DEVIATIONS
C     CALCULATE AND PRINT PER CENT COEFFICIENT OF VARIATION
      PRINT 521,M,N
  521 FORMAT (1H1I3,5H ROWSI9,8H COLUMNS)
      CALL PRMXLT (AV,MP1,NP1,XR,XC,TITLE,17HTABLE OF AVERAGES)
      IF(NR-1)522,540,522
  522 PRINT 521,M,N
      CALL PRMXLT(Z,MP1,NP1,XR,XC,TITLE,28HTABLE OF STANDARD DEVIATIONS)
      DO 531 J=1,N
      C(J)=0.
      DO 530 I=1,M
  530 C(J)=C(J)+Z(I,J)/AV(I,J)*100.
  531 C(J)=C(J)/FM
      CALL PRL1(XC,C,N,10HPERCENT CV)
C     INSTRUCTIONS TO GET TO VARIOUS SUBROUTINES AND FOR CALCULATING SS
C      OF DEVIATIONS
  540 READ 541,XX,XY,NX,(XL(K),K=1,6)
  541 FORMAT(2A6,I2,5(A6,4X),A6)
      PRINT 542,TITLE,XX,XY,NX,(XL(K),K=1,6)
  542 FORMAT(1H112A6/1H02A6,I2,5(A6,4X),A6)
C     CALCULATED GO TO STATEMENT
      DO 550 K=1,18
      IF(XX-XNS(K))550,552,550
  550 CONTINUE
  551 CALL ENDJOB
C     THESE CAN BE READILY IDENTIFIED FROM LIST OF BOOLEAN STATEMENTS
C     AT BEGINNING OF PROGRAM.  EXAMPLE - XNS(1) = STOP CAUSES PROGRAM
C     TO GO TO STATEMENT 551 WHERE ENDJOB IS CALLED.
  552 GO TO (551,100,1100,1200,1300,1400,1500,1600,1700,422,1900,2000,
     1  2100,2200,2300,2400,2500,2600),K
 1100 CALL LMODEL
      GO TO 3000
 1200 CALL QMODEL
      GO TO 3000
 1300 CALL CMODEL(PTCONC)
      GO TO 3000
 1400 CALL MMODEL
      GO TO 3000
C     CALCULATES ROW WEIGHTS
 1500 IF(XY-XNS(20))1501,1515,1501                                      CAL
 1501 READ 1502,(WT(I),I=1,M)
 1502 FORMAT(14X,5F10.0,F8.0)
 1503 DO 1504 I=1,M
 1504 WFAC(I)=SQRTF(WT(I))
      CALL PRL2(XR,WT,WFAC,N,6HWEIGHT,6HSQRT W)
 1505 CALL LMWROW
      GO TO 3000
 1515 CALL CALWR
      GO TO 1503
C     CALCULATES COLUMN WEIGHTS
 1600 IF(XY-XNS(20))1601,1615,1601                                      CAL
 1601 READ 1502,(WT(J),J=1,N)
 1602 DO 1603 J=1,N
 1603 WFAC(J)=SQRTF(WT(J))
      CALL PRL2(XC,WT,WFAC,N,6HWEIGHT,6HSQRT W)
 1605 CALL LMWCOL
      GO TO 3000
 1615 CALL CALWC
      GO TO 1602
 1700 CALL SUB1
      GO TO 3000
 1900 CALL OMIT
      GO TO 425
 2000 CALL INCLUD
```

```
      GO TO 425
 2100 CALL TRAFOR
      GO TO 425
 2200 CALL POWER
      GO TO 425
 2300 CALL COMBIN
      GO TO 425
 2400 CALL SUB2
      GO TO 540
 2500 CALL REVERS
      GO TO 540
 2600 CALL INTLAB
      GO TO 540
C     CALCULATES ROW AND COLUMN SUM OF SQUARES FOR TABLE OF DEVIATIONS
 3000 DO 3001 I=1,M
 3001 Z(I,N+1)=SUM5(Z,I,N)
      DO 3002 J=1,N
 3002 Z(M+1,J)=SUM3(Z(1,J),Z(1,J),M)
      Z(M+1,N+1)=SUM2(Z,M+1,N)
      XR(M+1)=XNS(22)                                                   SUM SQ
      XC(N+1)=XNS(22)                                                   SUM SQ
      PRINT 3003,M,N
 3003 FORMAT (1H1I3,5H ROWSI9,8H COLUMNS)
      CALL PRMXLT(Z,M+1,N+1,XR,XC,TITLE,19HTABLE OF DEVIATIONS)
      GO TO 540
       END

       SUBROUTINE LMODEL
C      FITS AV(I,J)=A(I)+B(I)*C(J)    Z(I,J)=AV(I,J)-A(I)-B(I)*C(J)
C      ASSUMES AVERAGES FOR AV ARE IN AV
      DIMENSION AV(120,50),Z(120,50),XR(120),XC(50),A(120),B(120),C(50),
     1 TITLE(12),XNS(26),R(99),D(120),WT(120),WFAC(120),XL(120)
      COMMON AV,Z,XR,XC,A,B,C,M,N,WT,WFAC,MT,NT,NREP,NR,NX,XNS,TITLE,XY,
     1XL,D,R
       FM=M
       FN=N
      CALL LINM
      CALL PRL2(XR,A,B,M,1HA,1HB)
      CALL PRL1(XC,C,N,1HC)
      PRINT 101
  101 FORMAT(1H010X,20HANALYSIS OF VARIANCE/1H08X,2HDF8X,2HSS12X,2HMS12X
     1 ,1HF/1H )
      S2=SUM3(C,C,N)
      SSR=0.
      SST=0.
      SSRXC=0.
      RSQ=0.
      SSL=0.
      SSRES=0.
      AVDD=AV(M+1,N+1)
      DO 10 I=1,M
      SST=SST+SUM5(AV,I,N)
      SSL=SSL+(B(I)-1.)**2
      RSQ=RSQ+(A(I)-AVDD)*(B(I)-1.)
   10 SSR=SSR+(A(I)-AVDD)**2
      RSQ=RSQ**2/SSR/SSL
      SSM=FM*FN*AVDD**2
      SSR=FN*SSR
      SSC=FM*S2
      SSL=SSL*S2
      DO 12 I=1,M
      DO 12 J=1,N
      SSRXC=SSRXC+(AV(I,J)-A(I)-C(J))**2
      Z(I,J)=AV(I,J)-A(I)-B(I)*C(J)
   12 SSRES=SSRES+Z(I,J)**2
       SRES=SSRES/(FM-1.)/(FN-2.)
      CALL PRLI1(5HTOTAL,M*N,SST)
      CALL PRLI1(4HMEAN,1,SSM)
       SR=SSR/(FM-1.)
```

```
      CALL PRLI3(4HROWS,M-1,SSR, SR, SR/ SRES)
       SC=SSC/(FN-1.)
      CALL PRLI3(4HCOLS,N-1,SSC, SC, SC/ SRES)
       SRXC=SSRXC/(FM-1.)/(FN-1.)
      CALL PRLI3(3HRXC,(M-1)*(N-1),SSRXC, SRXC,SRXC/SRES)
       SL=SSL/(FM-1.)
      CALL PRLI3(5H  LIN,M-1,SSL,SL,SL/SRES)
      SSCN=SSL*RSQ
      SSNC=SSL-SSCN
      SNC=SSNC/(FM-2.)
      CALL PRLI3(5H    C,1,SSCN,SSCN,SSCN/SNC)
      CALL PRLI3(6H    NC,M-2,SSNC,SNC,SNC/SRES)
      CALL PRLI2(5HRESID,(M-1)*(N-2),SSRES, SRES)
      RETURN
      END

      SUBROUTINE LINM
C         TO DO BASIC CALCULATIONS FOR LINEAR AND QUADRATIC MODELS
      DIMENSION AV(120,50),Z(120,50),XR(120),XC(50),A(120),B(120),C(50),
     1 TITLE(12),XNS(26),R(99),D(120),WT(120),WFAC(120),XL(120)
      COMMON AV,Z,XR,XC,A,B,C,M,N,WT,WFAC,MT,NT,NREP,NR,NX,XNS,TITLE,XY,
     1XL,D,R
      DO 1 J=1,N
    1 C(J)=AV(M+1,J)-AV(M+1,N+1)
      S2=SUM3(C,C,N)
      DO 2 I=1,M
      A(I)=AV(I,N+1)
    2 B(I)=SUM4(AV,C,I,N)/S2
      RETURN
       END

      SUBROUTINE LMWCOL
C     WEIGHTED WITH WFAC. Z(I,J)=WFAC(J)*(Y(I,J)-A(I)-B(I)*C(J))
      DIMENSION AV(120,50),Z(120,50),XR(120),XC(50),A(120),B(120),C(50),
     1 TITLE(12),XNS(26),R(99),D(120),WT(120),WFAC(120),XL(120),U(50),BE
     2STAR(120)
      COMMON AV,Z,XR,XC,A,B,C,M,N,WT,WFAC,MT,NT,NREP,NR,NX,XNS,TITLE,XY,
     1XL,D,R
      CALL LMWCL
      FM=M
      SBIM1=0.
      SAIMAV=0.
      DO 106 I=1,M
  105 SBIM1=SBIM1+(B(I)-1.)*(A(I)-AVA)
  106 SAIMAV=SAIMAV+(A(I)-AVA)**2
      ALPHA=SBIM1/SAIMAV
      PTCONC=AVA-1./ALPHA
      DO 107 I=1,M
  107 BESTAR(I)=1.+ALPHA*(A(I)-AVA)
      DO 109 J=1,N
  109 Z(M+1,J)=SUM3(Z(1,J),Z(1,J),M)
      Z(M+1,N+1)=SUM2(Z,M+1,N)
      CALL PRL3 (XR,A,B,BESTAR,M,1HA,1HB,6HB STAR)
      CALL PRL1 (XC,C,N,1HC)
      CALL PLINE1(6HALPHA=,ALPHA)
      CALL PLINE1(21HPOINT OF CONCURRENCE=,PTCONC)
      AS=SUM1(A,M)
      W=SUM3(WFAC,WFAC,N)
      ABAR=AS/FM
      AZA=0.
      DO 1 I=1,M
    1 AZA=AZA+(A(I)-ABAR)**2
      SSR=W*AZA
      SWC=0.
      DO 2 J=1,N
    2 SWC=SWC+WFAC(J)**2*C(J)**2
      SSC=FM*SWC
      SSF = Z(M+1,N+1)
      BZB=0.
```

```
      DO 3 I=1,M
    3 BZB=BZB+(B(I)-1.)**2
      SSS = SWC*BZB
      RSQ = (SUM3(A,B,M)-AS)**2/(AZA*BZB)
      SSCO= SSS*RSQ
      SSNC= SSS*(1.-RSQ)
      SSRC= SSF+SSS
      PRINT 110
  110 FORMAT (26H1ANALYSIS OF WEIGHTED DATA/7H0SOURCE4X,2HDF 8X,2HSS12X,
     12HMS)
      CALL PRLI2 (4HROWS,M-1,SSR,SSR/FLOATF(M-1))
      CALL PRLI2 (4HCOLS,N-1,SSC,SSC/FLOATF(N-1))
      CALL PRLI2 (3HRXC,(M-1)*(N-1),SSRC,SSRC/FLOATF((M-1)*(N-1)))
      CALL PRLI2 (5H  FIT, (M-1)*(N-2),SSF,SSF/FLOATF((M-1)*(N-2)))
      CALL PRLI2 (5HSLOPE,  M-1 ,SSS,SSS/FLOATF(M-1))
      CALL PRLI2 (6H  CONC,1,SSCO,SSCO)
      CALL PRLI2 (6H  NONC,M-2,SSNC,SSNC/FLOATF(M-2))
      CALL PLINE1(10HR SQUARED=,RSQ)
      CALL PLINE1(2HR=,SQRTF(RSQ))
      RETURN
      END

      SUBROUTINE LMWCL
C     DOES BASIC CALCULATIONS FOR SUBROUTINES LMWCOL AND CALWC
      DIMENSION AV(120,50),Z(120,50),XR(120),XC(50),A(120),B(120),C(50),
     1 TITLE(12),XNS(26),R(99),D(120),WT(120),WFAC(120),XL(120)
      COMMON AV,Z,XR,XC,A,B,C,M,N,WT,WFAC,MT,NT,NREP,NR,NX,XNS,TITLE,XY,
     1XL,D,R
      FM=M
      DO 100 I=1,M
      DO 100 J=1,N
  100  Z(I,J)=AV(I,J)*WFAC(J)
      DO 101 J=1,N
  101 D(J)=SUM1(Z(1,J),M)/FM
      SW=SUM3(WFAC,WFAC,N)
      DO 102 I=1,M
  102  A(I)=SUM4(Z,WFAC,I,N)/SW
      AVA=SUM1(A,M)/FM
      DO 103 I=1,M
  103 B(I)=(SUM4(Z,D,I,N)-SW*A(I)*AVA)/(SUM3(D,D,N)-SW*AVA**2)
      DO 104 J=1,N
  104 C(J)=SUM1(AV(1,J),M)/FM-AVA
      DO 108 I=1,M
      DO 108 J=1,N
  108 Z(I,J)=(AV(I,J)-A(I)-B(I)*C(J))*WFAC(J)
      RETURN
      END


      SUBROUTINE CALWC
C       ITERATIVE F-PROCEDURE WITH ADJUSTMENT FOR HIGH WEIGHTS
      DIMENSION AV(120,50),Z(120,50),XR(120),XC(50),A(120),B(120),C(50),
     1 TITLE(12),XNS(26),R(99),D(120),WT(120),WFAC(120),XL(120),XM(50),L
     2Y(50)
      COMMON AV,Z,XR,XC,A,B,C,M,N,WT,WFAC,MT,NT,NREP,NR,NX,XNS,TITLE,XY,
     1XL,D,R
      FM=M
      NUM=25
      NUMX=20
      DO 1 J=1,N
    1 WT(J)=1.
      DO 50 LX=1,NUM
      DO 17 J=1,N
   17 WFAC(J)=SQRTF(WT(J))
      CALL LMWCL
      DO2 J=1,N
    2 XM(J)=SUM3(Z(1,J),Z(1,J),M)/(FM-1.)
      W=SQRTF(SUM1(WT,N))
      SUM=0.
```

```
      DO 3 J=1,N
    3 SUM=SUM+WT(J)*C(J)**2
      SUM=SQRTF(SUM)
      DO 4 J=1,N
      A(J)=WFAC(J)/W
    4 B(J)=WFAC(J)*C(J)/SUM
      DO 5 J=1,N
      A(J)=1.-A(J)**2-B(J)**2
    5 WT(J)=WT(J)*A(J)/XM(J)
      IF(LX-NUMX)50,50,18
   18 DO 19 J=1,N
   19 LY(J)=J
      N1=N-1
      DO 8 J=1,N1
      SMALL=WT(J)
      JX=J
      DO 7 K=J,N
      IF(SMALL-WT(K))7,7,6
    6 JX=K
      SMALL=WT(K)
    7 CONTINUE
      WT(JX)=WT(J)
      WT(J)=SMALL
      IHOLD=LY(JX)
      LY(JX)=LY(J)
    8 LY(J)=IHOLD
      FD=N-5
      R=LOGF(WT(N-2)/WT(3))
      NX=(N+1)/2
      NY=(N+2)/2
      S=0.5*LOGF(WT(NX)*WT(NY))
      SF1=EXPF(R*(.5501+1.0553/FD)+S)
      SF2=EXPF(R*(.6529+2.5405/FD)+S)
      IF(WT(N-1)-SF1)11,11,9
    9 PRINT 10,LY(N-1),WT(N-1),SF1
   10 FORMAT(14HOREPLACED  WT I2,E15.6,5H   BYE15.6)
      WT(N-1)=SF1
   11 IF(WT(N)-SF2)13,13,12
   12 PRINT 10,LY(N),WT(N),SF2
      WT(N)=SF2
   13 DO 16 J=1,N1
      DO 16 K=J,N
      IF(J-LY(K))16,15,14
   14 CALL ENDJOB
   15 LY(K)=LY(J)
      HOLD=WT(K)
      WT(K)=WT(J)
      WT(J)=HOLD
   16 CONTINUE
   50 CALL PRN1(WT,N,6HWEIGHT)
      RETURN
      END

      SUBROUTINE QMODEL
C     SUBROUTINE TO FIT AV(I,J)=A(I)+B(I)*C(J)+D(I)*E(J).
C                       AV(I,J)=AP(I)+BP(I)*C(J)+D(I)*C(J)**2
C                       Z(I,J)=AV(I,J)-A(I)-B(I)*C(J)-D(I)*E(J)
      DIMENSION AV(120,50),Z(120,50),XR(120),XC(50),A(120),B(120),C(50),
     1 TITLE(12),XNS(26),R(99),D(120),WT(120),WFAC(120),XL(120),E(50),AP
     2(120),BP(120)
      COMMON AV,Z,XR,XC,A,B,C,M,N,WT,WFAC,MT,NT,NREP,NR,NX,XNS,TITLE,XY,
     1XL,D,R
      FM=M
      FN=N
      CALL LINM
      S2=SUM3(C,C,N)
      S3=0.
      DO1 J=1,N
```

```
    1 S3=C(J)**3+S3
      DO 2 J=1,N
    2 E(J)=C(J)**2-S3/S2*C(J)-S2/FN
      Q=SUM3(E,E,N)
      DO 4 I=1,M
      R=0.
      DO 3 J=1,N
    3 R=AV(I,J)*C(J)**2+R
    4 D(I)=(R-B(I)*S3-A(I)*S2)/Q
      DO 5 I=1,M
      AP(I)=A(I)-D(I)*S2/FN
    5 BP(I)=B(I)-D(I)*S3/S2
      AVDD=AV(M+1,N+1)
      CALL PRL5(XR,A,B,D,AP,BP,M,1HA,1HB,1HD,2HAP,2HBP)
      CALL PRL2(XC,C,E,N,1HC,1HE)
      PRINT 101
  101 FORMAT(1H110X,21H ANALYSIS OF VARIANCE/1H08X,2HDF8X,2HSS12X,
     1       2HMS/1H )
      SST=0.
      DO 10 I=1,M
   10 SST=SST+SUM5(AV,I,N)
      SSM=FM*FN*AVDD**2
      SSR=0.
      DO 11 I=1,M
   11 SSR=SSR+(A(I)-AVDD)**2
      SSR=FN*SSR
      SSC=FM*S2
      SSRXC=0.
      DO 12 I=1,M
      DO 12 J=1,N
   12 SSRXC=SSRXC+(AV(I,J)-A(I)-C(J))**2
      SSL=0.
      DO 13 I=1,M
   13 SSL=SSL+(B(I)-1.)**2
      SSL=S2*SSL
      SSQ=Q*SUM3(D,D,M)
      SSRES=0.
      DO 14 I=1,M
      DO 14 J=1,N
   14 SSRES=SSRES+(AV(I,J)-A(I)-B(I)*C(J)-D(I)*E(J))**2
      CALL PRLI1 (5HTOTAL,M*N,SST)
      CALL PRLI1 (4HMEAN,1,SSM)
      CALL PRLI2 (4HROWS,M-1,SSR,SSR/FLOATF(M-1))
      CALL PRLI2 (4HCOLS,N-1,SSC,SSC/FLOATF(N-1))
      CALL PRLI2 (3HRXC,(M-1)*(N-1),SSRXC,SSRXC/FLOATF((M-1)*(N-1)))
      CALL PRLI2 (6H   LIN,M-1,SSL,SSL/FLOATF(M-1))
      CALL PRLI2 (6H  QUAD,M-1,SSQ,SSQ/FLOATF(M-1))
      CALL PRLI2 (6H   RES,(M-1)*(N-3),SSRES,SSRES/FLOATF((M-1)*(N-3)))
      DO 20 I=1,M
      DO 20 J=1,N
   20 Z(I,J)=AV(I,J)-A(I)-B(I)*C(J)
      DO 21 I=1,M
   21 Z(I,N+1)=SUM5(Z,I,N)
      DO 22 J=1,N
   22 Z(M+1,J)=SUM3(Z(1,J),Z(1,J),M)
      Z(M+1,N+1)=SUM2(Z,M+1,N)
      XR(M+1)=XNS(22)
      XC(N+1)=XNS(22)
      CALL PRMXLT(Z,M+1,N+1,XR,XC,TITLE,33H TABLE OF DEVIATIONS LINEAR M
     1ODEL)
      DO 23 I=1,M
      DO 23 J=1,N
   23 Z(I,J)=Z(I,J)-D(I)*E(J)
      RETURN
      END

      SUBROUTINE OMIT
C     TO OMIT COLS OR ROWS OR BOTH - READ NX AND XL(K)
C     EXAMPLE
```

```
C                COLUMN NUMBER
C     1              15          25          35          45
C     OMIT           NXROW1      ROW5        COL3        COL7
C         ANY NUMBER OF ROWS AND COLUMNS IN ANY ORDER MAY BE OMITTED
      DIMENSION AV(120,50),Z(120,50),XR(120),XC(50),A(120),B(120),C(50),
     1 TITLE(12),XNS(26),R(99),D(120),WT(120),WFAC(120),XL(120)
      COMMON AV,Z,XR,XC,A,B,C,M,N,WT,WFAC,MT,NT,NREP,NR,NX,XNS,TITLE,XY,
     1XL,D,R
      MT1=MT+1
      NT1=NT+1
      NR=1
      IF(XL(1)-XNS(21))10,180,10
   10 XR(M+1)=XR(MT1)
      XC(N+1)=XC(NT1)
      DO 20 I=1,MT
      AV(I,N+1)=AV(I,NT1)
   20 Z(I,N+1)=Z(I,NT1)
      DO 30 J=1,NT
      AV(M+1,J)=AV(MT1,J)
   30 Z(M+1,J)=Z(MT1,J)
      M=MT
      N=NT
      DO 40 K=1,6
   40 A(K)=XL(K)
      IF(NX-6)65,65,50
   50 READ 60,(A(K),K=7,NX)
   60 FORMAT(14X,A6,4X,A6,4X,A6,4X,A6,4X,A6,4X,A6)
      PRINT 61,(A(K),K=7,NX)
   61 FORMAT(1H014X,A6,4X,A6,4X,A6,4X,A6,4X,A6,4X,A6)
   65 DO 150 K=1,NX
      DO 70 I=1,M
      IF(A(K)-XR(I)) 70,80,70
   70 CONTINUE
      GO TO 100
   80 DO 90 J=1,NT1
      AV(M+1,J)=AV(I,J)
      AV(I,J)=AV(M,J)
      Z(M+1,J)=Z(I,J)
   90 Z(I,J)=Z(M,J)
      XR(M+1)=XR(I)
      XR(I)=XR(M)
      M=M-1
      GO TO 150
  100 DO 110 J=1,N
      IF(A(K)-XC(J)) 110,130,110
  110 CONTINUE
      PRINT 120,A(K)
  120 FORMAT(1H0A6,13H IS INCORRECT)
      CALL ENDJOB
  130 DO 140 I=1,MT1
      AV(I,N+1)=AV(I,J)
      AV(I,J)=AV(I,N)
      Z(I,N+1)=Z(I,J)
  140 Z(I,J)=Z(I,N)
      XC(N+1)=XC(J)
      XC(J)=XC(N)
      N=N-1
  150 CONTINUE
  160 PRINT 170,M,N
  170 FORMAT(1H0I6,9H ROWS ANDI6,8H COLUMNS)
  180 RETURN
      END

      SUBROUTINE CORECT
C     CORRECTS DATA(AV) AND STANDARD ERRORS(Z),AV=AX*AV+BX  Z=Z*AV
C     EXAMPLE
C       COLUMN NUMBER
C      1     7       15          25
C      CORECT
```

```
C       ALL   COL02   1.        2.
C       ROW02 COL11   2.
C       COL03 ALL     2.        5.
C       END
      DIMENSION AV(120,50),Z(120,50),XR(120),XC(50),A(120),B(120),C(50),
     1 TITLE(12),XNS(26),R(99),D(120),WT(120),WFAC(120),XL(120)
      COMMON AV,Z,XR,XC,A,B,C,M,N,WT,WFAC,MT,NT,NREP,NR,NX,XNS,TITLE,XY,
     1XL,D,R
B     ALL=214343606060
    1 READ 100,XX,XY,AX,BX
  100 FORMAT(2A6,2X,2F10.0)
      PRINT101, XX,XY,AX,BX
  101 FORMAT(1H02A6,2X,7HFACTOR=E15.8,5X,9HCONSTANT=E15.8)
      IF(XX-ALL)20,2,20
    2 IF(XY-ALL)7,3,7
    3 DO 4 I=1,M
      DO 4 J=1,N
      AV(I,J)=AX*AV(I,J)+BX
    4 Z(I,J)=AX*Z(I,J)
      GO TO 1
    7 DO 8 J=1,N
      IF(XY-XC(J))8,10,8
    8 CONTINUE
    9 RETURN
   10 DO 11 I=1,M
      AV(I,J)=AX*AV(I,J)+BX
   11 Z(I,J)=AX*Z(I,J)
      GO TO 1
   20 DO 21 I=1,M
      IF(XX-XR(I))21,23,21
   21 CONTINUE
      GO TO 9
   23 IF(XY-ALL)27,24,27
   24 DO 25 J=1,N
      AV(I,J)=AX* AV(I,J)+BX
   25 Z(I,J)=AX*Z(I,J)
      GO TO 1
   27 DO 28 J=1,N
      IF(XY-XC(J))28,29,28
   28 CONTINUE
      GO TO 9
   29 AV(I,J)=AX*AV(I,J)+BX
      Z(I,J)=AX*Z(I,J)
      GO TO 1
      END

      SUBROUTINE COMBIN
C     COMBINES ROWS OR COLUMNS OF AV
C       EXAMPLE
C          COLUMN NUMBER
C       1     7           15          25          35          45
C       COMBINROWS        LAB01       LAB02       LAB10
C             ROWS        LAB03       LAB34       LAB11
C       END
C         LAB01 AND LAB02 WILL BE COMBINED AND LABELED LAB10
C         LAB03 AND LAB04 WILL BE COMBINED AND LABELED LAB11
      DIMENSION AV(120,50),Z(120,50),XR(120),XC(50),A(120),B(120),C(50),
     1 TITLE(12),XNS(26),R(99),D(120),WT(120),WFAC(120),XL(120)
      COMMON AV,Z,XR,XC,A,B,C,M,N,WT,WFAC,MT,NT,NREP,NR,NX,XNS,TITLE,XY,
     1XL,D,R
B     ROWS=514666626060
B     COLS=234643626060
    1 IF(XY-ROWS)2,3,2
    2 IF(XY-COLS)24,14,24
    3 DO4 I=3,6
      IF(XL(I)-XNS(21))4,5,4
    4 CONTINUE
      NR=5
      GO TO 6
```

```
    5 NR=I-2
    6 DO 30 J=1,N
   30 A(J)=0.
      DO 11 K=1,NR
      DO 7 I=1,M
      IF(XL(K)-XR(I))7,8,7
    7 CONTINUE
      CALL ENDJOB
    8 DO 10 J=1,N
    9 A(J)=A(J)+AV(I,J)
   10 AV(I,J)=AV(M,J)
      XR(I)=XR(M)
   11 M=M-1
      M=M+1
      FNR=NR
      DO 12 J=1,N
   12 AV(M,J)=A(J)/FNR
      XR(M)=XL(NR+1)
   13 READ 100,XY,(XL(K),K=1,6)
  100 FORMAT(6X,A6,2X,5(A6,4X),A6)
      PRINT 101,XY,(XL(K),K=1,6)
  101 FORMAT(1H 6X,A6,2X,5(A6,4X),A6)
      GO TO 1
   14 DO 15 I=3,6
         IF(XL(I)-XNS(21))15,16,15
   15 CONTINUE
      NR=5
      GO TO 17
   16 NR=I-2
   17 DO 31 I=1,M
   31 A(I)=0.
      DO 22 K=1,NR
      DO 18 J=1,N
      IF(XL(K)-XC(J))18,19,18
   18 CONTINUE
      CALL ENDJOB
   19 DO 21 I=1,M
   20 A(I)=A(I)+AV(I,J)
   21 AV(I,J)=AV(I,N)
      XC(J)=XC(N)
   22 N=N-1
      N=N+1
      DO 23 I=1,M
   23 AV(I,N)=A(I)/FNR
      XC(N)=XL(NR+1)
      GO TO 13
   24 MT=M
      NT=N
      NR=1
      RETURN
      END

      SUBROUTINE REVERS
C     REVERSES ROWS AND COLUMNS OF DATA AV FOR MATRIX CURVE FITTING
      DIMENSION AV(120,50),Z(120,50),XR(120),XC(50),A(120),B(120),C(50),
     1 TITLE(12),XNS(26),R(99),D(120),WT(120),WFAC(120),XL(120)
      COMMON AV,Z,XR,XC,A,B,C,M,N,WT,WFAC,MT,NT,NREP,NR,NX,XNS,TITLE,XY,
     1XL,D,R
      MX=XMAXOF(M,N)
      MXP1=MX+1
      DO 9 I=1,MX
      HOLD=XR(I)
      XR(I)=XC(I)
      XC(I)=HOLD
      M2=I+1
      DO 9 J=M2,MXP1
      HOLD=AV(I,J)
      AV(I,J)=AV(J,I)
```

```
9     AV(J,I)=HOLD
      IOLD=M
      M=N
      N=IOLD
      RETURN
      END

      FUNCTION SUM1(A,M)
C     SUMS OVER ROWS.  MAY BE IN THE FORM SUM1(A,M) OR SUM1(A(I,J),M)
      DIMENSION A(101)
      SUM1=0.
      DO 1 I=1,M
    1 SUM1=A(I)+SUM1
      RETURN
      END

      FUNCTION SUM2(A,I,N)
C      SUMS ROW I OF A TWO DIMENSIONAL VARIABLE A OVER COLUMNS
      DIMENSION A(120,50)
      SUM2=0.
      DO 1 J=1,N
    1 SUM2=A(I,J)+SUM2
      RETURN
      END

      FUNCTION SUM3(A,B,M)
C      SUM OF A PRODUCT OVER ROWS.  MAY BE IN THE FORM SUM3(A,B,M) OR
C        SUM3(A(I,J),B,M) OR SUM3(A(I,J),B(I,J),M)
      DIMENSION A(101),B(101)
      SUM3 =0.
      DO1 I=1,M
    1 SUM3 =A(I)*B(I)+SUM3
      RETURN
      END

      FUNCTION SUM4(A,B,I,N)
C      SUM OF A PRODUCT CONSISTING OF A VECTOR TIMES THE VALUES IN A ROW
C          OF A MATRIX - SUMMED OVER COLUMNS
      DIMENSION A(120,50),B(120)
      SUM4 =0.
      DO 1 J=1,N
    1 SUM4 =A(I,J)*B(J)+SUM4
      RETURN
      END

      FUNCTION SUM5(A,I,N)
C      SUM OF SQUARES OVER COLUMNS
      DIMENSION A(120,50)
      SUM5 =0.
      DO 1 J=1,N
    1 SUM5 =A(I,J)**2+SUM5
      RETURN
      END


      SUBROUTINE PLINE1(L,X)
C     PRINTS 1 LINE OF 1 TO 48 CHARACTERS (1 TO 8 CELLS) AND 1 NUMBER
      DIMENSION NFMT(4),L(8),LX(8),FMT(4)
      EQUIVALENCE (NFMT,FMT)
C     CALCULATE FORMAT (NFMT OR FMT)
B      FMT(1)=740130006010
B      FMT(2)=210673606060
      NFMT(3)=NFRMT(X,1)
B      FMT(4)=016734606060
      CALL ORDHOL(L,LX,8)
      PRINT FMT  ,(LX(J),J=1,8),X
      RETURN
      END
```

```
      SUBROUTINE PRMXLT(A,N,M,XR,XC,TITLE,NT)
C       PRINT MATRIX A WITH N ROWS AND M COLUMNS
C       MATRIX LABELED WITH TITLE
C       DIMENSION STATEMENT MAY BE CHANGED
C       ROWS LABELED WITH XR AND COLS LABELED WITH XC
C        TITLE(12) IS PRINTED BEFORE NT, THE SUBTITLE
      DIMENSION FMT(11),NT(10),NU(10),A(120,50),XR(120),XC(50),TITLE(12)
     1,NFMT(11)
      EQUIVALENCE (FMT,NFMT),(XND,ND)
B     FMT(1)=740130606060
B     FMT(2)=210673606060
B     FMT(11)=016734606060
B     XL4=   606060606060
B     XND=   000000000100
B     XNE=   250104330673
B     XNF9=  260104331173
      CALLORDHOL (NT,NU,10)
      PRINT 102
  102 FORMAT(1H )
      DO 500 I=1,M,8
      PRINT 104,TITLE,(NU(J),J=1,10)
  104 FORMAT(1H012A6/1H010A6)
      J=XMINOF(I+7,M)
      PRINT 106,(XC(K),K=I,J)
  106 FORMAT(1H06X,8(6X,A6,2X))
      DO 120 K=3,10
  120 FMT(K)=XL4
      DO 190 K=I,J
      KK=K-I+3
      FMT(KK)=XNF9
      A1=.0001
      A2=1.
      DO 170 JX=1,10
      DO 150 IX=1,N
      IF (A(IX,K)) 130,150,130
  130 IF (ABSF(A(IX,K))-A1) 180,140,140
  140 IF (ABSF(A(IX,K))-A2) 150,150,160
  150 CONTINUE
      GO TO 190
  160 A1=A1*10.
      A2=A2*10.
  170 NFMT(KK)=NFMT(KK)-ND
  180 FMT(KK)=XNE
  190 CONTINUE
      DO 500 L=1,N,5
      LL=XMINOF(N,L+4)
      PRINT 200
  200 FORMAT(1H )
  500 PRINT FMT,(XR(KK),(A(KK,K),K=I,J),KK=L,LL)
      RETURN
       END

      SUBROUTINE PRL1(LABEL,A,N,X)
C      PRINTS 1 COLUMN HEADED BY X CONSISTING OF N VALUES A WITH
C        RESPECTIVE LABELS
      DIMENSION LABEL(100),A(100),NFMT(7),FMT(7),X(2)
      EQUIVALENCE (NFMT,FMT)
C     FMT(1)=(1H    FMT(2)=A6,    NFMT(4)=10X,   NFMT(5)=A6,   NFMT(7)=1X)
B     FMT(1)=740130606060
B     FMT(2)=210673606060
B     FMT(4)=010067736060
B     FMT(5)=210673606060
B     FMT(7)=016734606060
B     IF(-X(0)) 1,2,1
    1 Y=X(0)
      GO TO 3
B   2 Y=606060606060
    3 PRINT 4, X(1),Y,X(1),Y
    4 FORMAT (1H09X,2A6,18X,2A6)
```

```
      NFMT(3)=NFRMT(A,N)
      NFMT(6)=NFMT(3)
      ND2=N/2
      J=N-ND2
      DO 5 K=1,ND2
      KX=K+J
    5 PRINT FMT,LABEL(K),A(K),LABEL(KX),A(KX)
      IF(J-ND2) 6,7,6
    6 PRINT FMT,LABEL(J),A(J)
    7 RETURN
      END

      SUBROUTINE  PRL2(LABEL,A,B,N,XA,XB)
C      PRINTS 2 COLUMNS HEADED BY XA AND XB CONSISTING OF N VALUES EACH
C          OF A AND B WITH THEIR RESPECTIVE LABELS
      DIMENSION LABEL(100),A(100),B(100),NFMT(9),FMT(9),XA(2),XB(2)
      EQUIVALENCE (NFMT,FMT)
B     IF(-XA(0)) 1,2,1
    1 X=XA(0)
      GO TO 3
B   2 X=606060606060
B   3 IF(-XB(0)) 4,5,4
    4 Y=XB(0)
      GO TO 6
B   5 Y=606060606060
    6 PRINT 7, XA(1),X,XB(1),Y,XA(1),X,XB(1),Y
    7 FORMAT(1H09X,2A6,2X,2A6,18X,2A6,2X,2A6)
B     FMT(1)=740130606060
B     FMT(2)=210673606060
      NFMT(3)=NFRMT(A,N)
      NFMT(4)=NFRMT(B,N)
B     FMT(5)=010067736060
      FMT(6)=FMT(2)
      NFMT(7)=NFMT(3)
      NFMT(8)=NFMT(4)
B     FMT(9)=016734606060
      ND2=N/2
      J=N-ND2
      DO 8 K=1,ND2
      KX=K+J
    8 PRINTNFMT,LABEL(K),A(K),B(K),LABEL(KX),A(KX),B(KX)
      IF(J-ND2)9,10,9
    9 PRINTNFMT,LABEL(J),A(J),B(J)
   10 RETURN
      END

      SUBROUTINE PRL3(LABEL,A,B,C,N,LA,LB,LC)
C      PRINTS 3 COLUMNS HEADED BY LA,LB AND LC CONSISTING OF N VALUES
C          EACH OF A, B AND C WITH THEIR RESPECTIVE LABELS
C     NFMT(1)=(1H (2)=A6,  (6)=10X, (7)=A6,  (11)=1X)
      DIMENSION LABEL(100),A(100),B(100),C(100),NFMT(11),FMT(11)
      EQUIVALENCE (NFMT,FMT)
B     FMT(1)=740130606060
B     FMT(2)=210673606060
      NFMT(3)=NFRMT(A,N)
      NFMT(4)=NFRMT(B,N)
      NFMT(5)=NFRMT(C,N)
B     FMT(6)=010067736060
B     FMT(7)=210673606060
      NFMT(8)=NFMT(3)
      NFMT(9)=NFMT(4)
      NFMT(10)=NFMT(5)
B     FMT(11)=016734606060
      ND2=N/2
      I=N-2*ND2+1
      J=N-ND2
      PRINT 1,LA,LB,LC,LA,LB,LC
    1 FORMAT(1H04X,3(8X,A6),16X,3(8X,A6))
      DO 2 K=1,ND2
```

```
      KX=K+J
    2 PRINT NFMT,LABEL(K),A(K),B(K),C(K),LABEL(KX),A(KX),B(KX),C(KX)
      GO TO (4,3),I
    3 PRINT NFMT,LABEL(J),A(J),B(J),C(J)
    4 RETURN
      END

      SUBROUTINE PRL5(LABEL,A,B,C,D,E,N,LA,LB,LC,LD,LE)
C       PRINTS 5 COLUMNS HEADED BY LA, LB, LC, LD AND LE CONSISTING OF N
C           VALUES EACH OF A, B, C, D AND E WITH THEIR RESPECTIVE LABELS
C     NFMT(1)=(1H  (2)=A6,    (8)=1X)
      DIMENSION LABEL(99),A(99),B(99),C(99),E(99),NFMT(8),FMT(8)
      EQUIVALENCE (NFMT,FMT)
B     FMT(1)=740130606060
B     FMT(2)=210673606060
      NFMT(3)=NFRMT(A,N)
      NFMT(4)=NFRMT(B,N)
      NFMT(5)=NFRMT(C,N)
      NFMT(6)=NFRMT(D,N)
      NFMT(7)=NFRMT(E,N)
B     FMT(8)=016734606060
      PRINT 1,LA,LB,LC,LD,LE
    1 FORMAT(1H04X,5(8X,A6))
      DO 2 I=1,N,5
      J=XMINOF(I+4,N)
      PRINT NFMT,(LABEL(K),A(K),B(K),C(K),D(K),E(K),K=I,J)
    2 PRINT NFMT
      RETURN
      END

      SUBROUTINE PRLI1(L,INT,A)
C       PRINTS 1 LINE OF LABEL, INTEGER AND 1 FLOATING POINT NUMBER
C     NFMT(1)=(1H  (2)=A6,I6,    (4)=1X)
      DIMENSION NFMT(4),FMT(4)
      EQUIVALENCE (NFMT,FMT)
B     FMT(1)=740130606060
B     FMT(2)=210673310673
      NFMT(3)=NFRMT(A,1)
B     FMT(4)=016734606060
      PRINT NFMT,L,INT,A
      RETURN
      END

      SUBROUTINE PRLI2(L,INT,A,B)
C       PRINTS 1 LINE OF LABEL, INTEGER AND 2 FLOATING POINT NUMBERS
      DIMENSION NFMT(5),FMT(5)
      EQUIVALENCE (NFMT,FMT)
C     NFMT(1)=(1H  (2)=A6,I6,  (5)=1X)
B     FMT(1)=740130606060
B     FMT(2)=210673310673
      NFMT(3)=NFRMT(A,1)
      NFMT(4)=NFRMT(B,1)
B     FMT(5)=016734606060
      PRINT NFMT,L,INT,A,B
      RETURN
      END

      SUBROUTINE PRLI3(L,INT,A,B,C)
C       PRINTS 1 LINE OF LABEL, INTEGER AND 3 FLOATING POINT NUMBERS
      DIMENSION NFMT(6),FMT(6)
      EQUIVALENCE (NFMT,FMT)
B     FMT(1)=740130606060
B     FMT(2)=210673310673
B     FMT(6)=016734606060
      NFMT(3)=NFRMT(A,1)
      NFMT(4)=NFRMT(B,1)
      NFMT(5)=NFRMT(C,1)
      PRINT NFMT,L,INT,A,B,C
       RETURN
       END
```

```
      SUBROUTINE PRN1(A,M,LA)
C      PRINTS COLUMN OF DATA A LABELED WITH NUMBERS M - 1,2,ETC IN
C        CONSECUTIVE ORDER AND BY HEADING LA
      DIMENSION A(100),FMT(7),NFMT(7),LA(2),MA(2)
      EQUIVALENCE (NFMT,FMT)
B     FMT(1)=740130606060
B     FMT(2)=310673606060
      NFMT(3)=NFRMT(A,M)
      CALL ORDLAB(LA,MA)
      PRINT 1,MA,MA
    1 FORMAT(1H07X,2A6,18X,2A6)
B     FMT(4)=010067606060
      IF(M-5)2,2,3
    2 NFMT(5)=NFMT(7)
      PRINT NFMT,(I,A(I),I=1,M)
      GO TO 6
    3 NFMT(5)=NFMT(2)
      NFMT(6)=NFMT(3)
B     FMT(7)=016734606060
      MD2=M/2
      J=M-MD2
      DO 4 K=1,MD2
      KX=K+J
    4 PRINT NFMT,K,A(K),KX,A(KX)
      IF(J-MD2)6,6,5
    5 PRINT NFMT,J,A(J)
    6 RETURN
      END

      FUNCTION NFRMT(A,N)
C      CALCULATES A FORMAT TO PRINT A COLUMN OF N NUMBERS WITHIN
C          ANOTHER PRINT SUBROUTINE
      DIMENSION A(500)
      EQUIVALENCE (XNE,NE),(XNF9,NF9),(XND,ND)
B     XNE=250104330673
B     XNF9=260104331173
B     XND=000000000100
      NFRMT=NF9
      A1=.0001
      A2=1.
      DO 10 J=1,10
      DO 1 I=1,N
      IF(ABSF(A(I))-A1)4,2,2
    2 IF(ABSF(A(I))-A2)1,1,3
    4 IF(A(I)) 11,1,11
    1 CONTINUE
      GO TO 12
    3 A1=A1*10.
      A2=A2*10.
   10 NFRMT=NFRMT-ND
   11 NFRMT=NE
   12 RETURN
      END
```

```
      SUBROUTINE ORDHOL( G, C,M)
C     ORDERS HOLLERITH CHARACTERS  G FROM CALL STATEMENT TO  C FOR
C     PRINTING,M IS DIMENSION OF  C. IF DIMENSION OF  G IS LESS THAN M,
C      C IS FILLED WITH BLANKS
      DIMENSION  G(10), C(10)
B     XMASK=777777777777
      K=1
      DO 1 I=1,M
       C(I)= G(K)
      IF(XMASK-G(K))1,2,1
C     INCREMENT K BY -1
    1 K=K+32767
      GO TO 4
    2 DO 3 J=I,M
B   3  C(J)=606060606060
    4 RETURN
      END

      SUBROUTINE  ORDLAB( G, C)
C     CALCULATES A COLUMN HEADING FOR A TABLE FROM HEADING OF 1 TO 12
C     CHARACTERS (1 OR 2 CELLS) GIVEN IN A CALL STATEMENT
      DIMENSION  G(2), C(2)
B     IF(- G(0))  1,2,1
    1  C(1)= G(1)
       C(2)= G(0)
      GO TO 3
B   2  C(1)=606060606060
       C(2)= G(1)
    3 RETURN
      END
```

APPENDIX C

Functions and Subroutines Used By Program

ABSF	Absolute Value
CALWC	Calculates weights by columns for LMWCOL. Uses LMWCL
COMBIN	Combines into one average 2 or more rows or columns
CORECT	Corrects averages in the form AX + B and standard deviations by factor C
ENDJOB	End of computation
EXPF	Exponential
LINM	Calculates A, B and C for both LMODEL and QMODEL
LMODEL	Fits $y_{ij} = A + BC$ to averages
LMWCL	Calculates weighted values for A, B and C and a table of residuals for LMWCOL and CALWC
LMWCOL	Weights the columns then fits $y_{ij} = A + BC$ to averages
LOGF	Logarithm to base e
NFRMT	Calculates part of a format statement. Used with printing subroutines
OMIT	Omits rows and/or columns, as designated, from analysis
ORDHOL	Orders labels for printing. Used with printing subroutines
ORDLAB	Orders column labels for printing. Used with printing subroutines
PLINE1	Prints 1 line 1 to 48 characters (1 to 8 cells) and 1 number
PRL1	Prints 1 column of values with heading and labels
PRL2	Prints 2 columns of values with their respective headings and labels
PRL3	Prints 3 columns of values with their respective headings and labels
PRL5	Prints 5 columns of values with their respective headings and labels
PRLI1	Prints 1 line consisting of label, integer and a floating point value
PRLI2	Prints 1 line consisting of a label, integer and 2 floating point values
PRLI3	Prints 1 line consisting of a label, integer and 3 floating point values
PRMXLT	Prints a table of values with row and column headings and a title and subtitle
PRN1	Prints 1 column of values with heading and labeled by numbers in consecutive order
QMODEL	Fits $y_{ij} = A_i + B_iC_j + D_iE_j = A_i' + B_i'C_j + D_iC_j^2$ to averages
REVERS	Interchanges rows and columns
SQRTF	Square root

SUM1	Sums over rows $\sum_{i=1}^{M} A_i$ or $\sum_{i=1}^{M} A_{ij}$
SUM2	Sums over columns $\sum_{j=1}^{N} A_{ij}$
SUM3	Sum over rows $\sum_{i=1}^{M} A_i B_i$ or $\sum_{i=1}^{M} A_{ij} B_i$ or $\sum_{i=1}^{M} A_{ij} B_{ij}$
SUM4	Sum over columns $\sum_{j=1}^{N} A_{ij} B_j$
SUM5	Sum of squares over columns $\sum_{j=1}^{N} A_{ij}^2$
TRAFOR	See section 2.42
XMINOF	Chooses the smallest value

The following subroutines are called in the program to facilitate its use as a research tool. They are not included in this note for they are not necessary to the program's use as outlined in the preceding paragraphs.

CALWR	Calculates weights by rows
CMODEL	Research model
INCLUD	To include data previously omitted
INTLAB	A complex subroutine for interlaboratory test analysis. A note will be written explaining this process [Reference 2]
LMWROW	Weights the rows then fits y_{ij} = A+BC to averages
MMODEL	Research model
POWER	Research tool
SUB1	Research tool
SUB2	Research tool

APPENDIX D
Variables Used in Main Program

a. Dimensional Variables
(All are included in COMMON)

A(120)	Row averages
AV(120,50)	Averages of original data or original single values per cell
B(120)	Slopes
C(50)	Column main effect
D(120)	Variable in calculations
R(99)	Replicates per cell; with TRAFOR, transformation constants
TITLE(12)	Title to identify run
WFAC(120)	Weighting factor = square of weight
WT(120)	Weight
XC(50)	Column labels

XL(120)	Storage variables
XNS(26)	Instructions in Boolean statements
XR(120)	Row labels
Z(120,50)	Standard deviations for cells with replicate values; residuals from fitting various models

b. Non-dimensional Variables
In COMMON

M	Number of rows
MT	Total number of rows
N	Number of columns
NT	Total number of columns
NR	Original number of replicates per cell
NREP	Number of replicates
NX	Variable integer
XY	Variable label in card columns 7-12

Other

AX	For transforming data
BX	For transforming data
CX	For transforming data
FM	Floating point M
FN	Floating point N
FNRX	Floating point NRX
HOLD	Storage variable for ordering
I	Index for rows
IS	Index for ordering rows
IX	Index for ordering rows
J	Index for columns
JS	Index for ordering columns
JX	Index for column count; for ordering
JXX	Index for column count
K	Index for replicates; for computed go to
MEMORY	Check for missing values
MP1	M + 1
MT1	MT + 1
NP1	N + 1
NRX	Integer in card columns 13-14; number of replicates; number of omissions, etc.
NT1	NT + 1
PTCONC	Point of concurrence
TRANS	For transformation instruction
XCX	Label in card columns 7-12
XRX	Label in card columns 1-6
XX	Label in card columns 1-6

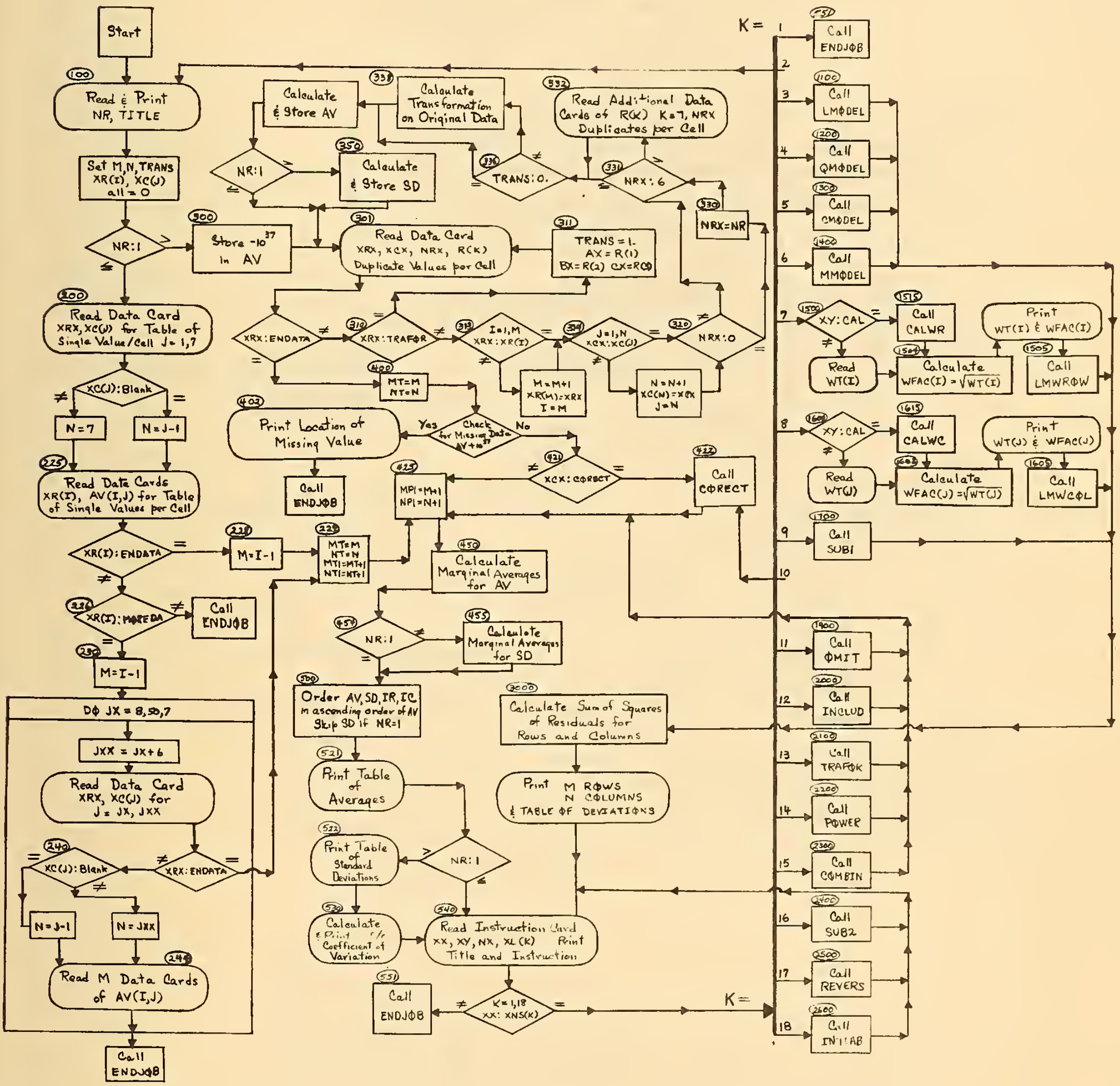
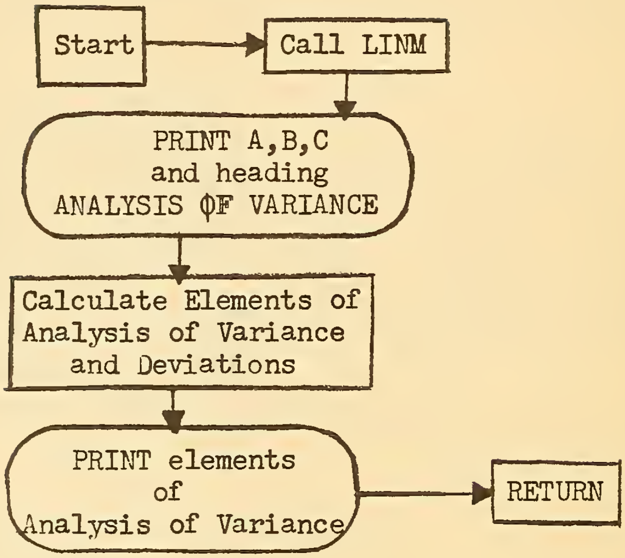


Figure 1. Flow Diagram of Computer Program. Numbers in ovals refer to statement numbers in Fortran program

SUBRØUTINE LMØDEL



SUBRØUTINE LINM

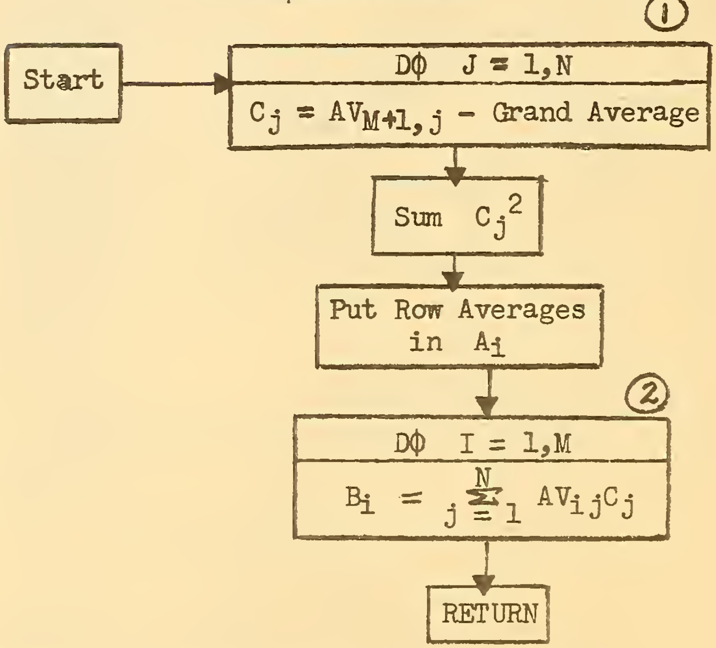


Figure 2. A. Flow Diagram for Subroutine LMODEL
B. Flow Diagram for Subroutine LINM

SUBRØUTINE LMWCL

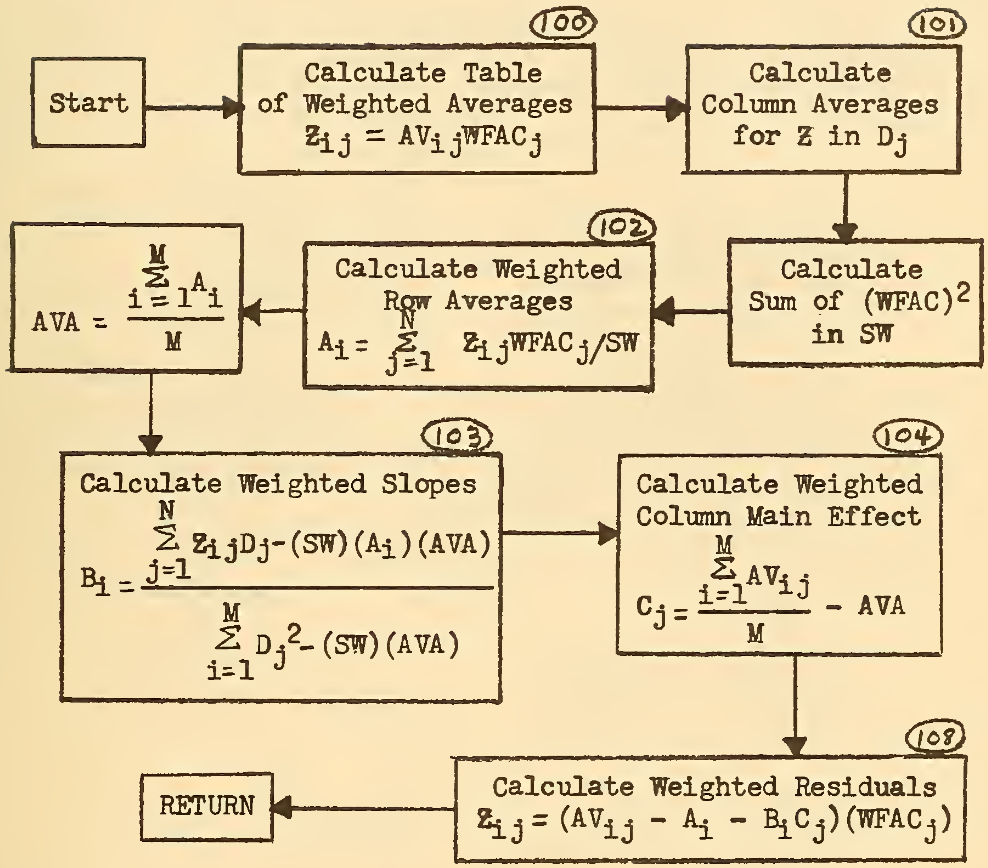


Figure 3. Flow Diagram for Subroutine LMWCL

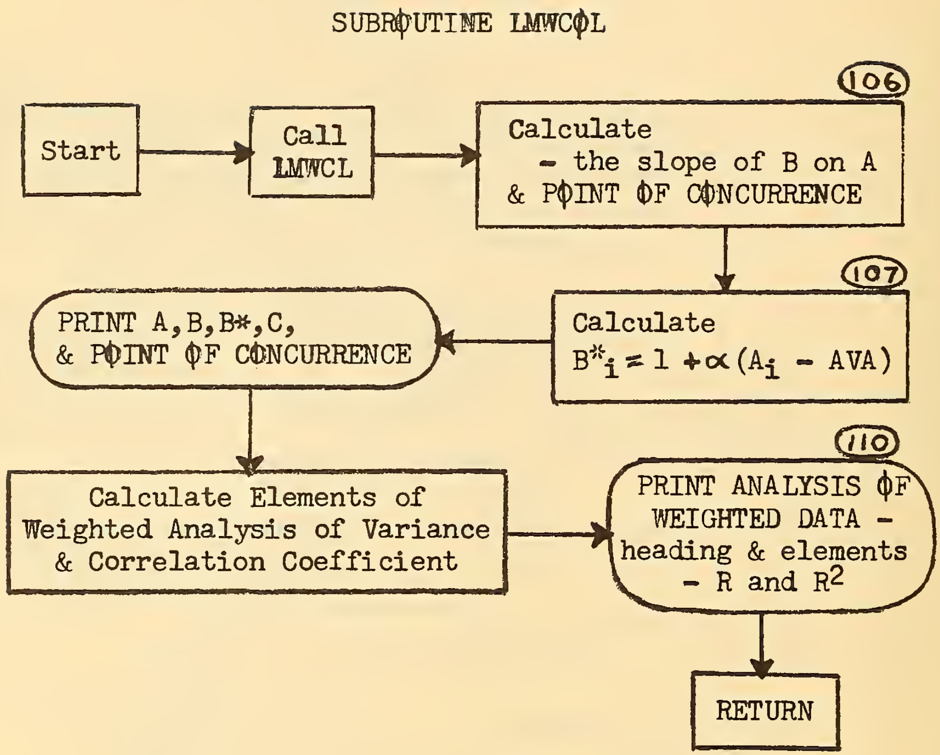
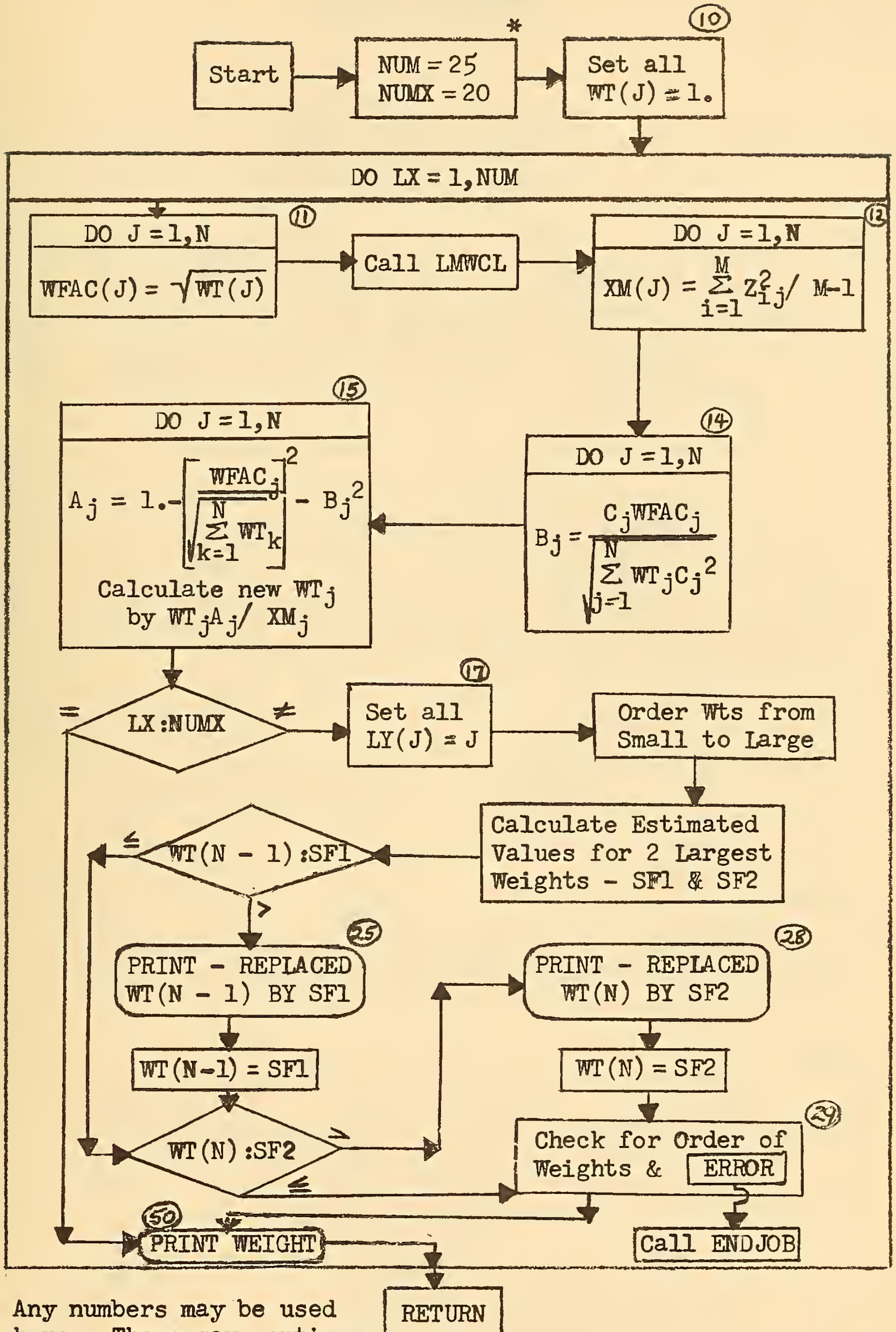


Figure 4. Flow Diagram for Subroutine LMWCOL

SUBRØUTINE CALWC



* Any numbers may be used here. These gave optimum results in repeated experiments.

Figure 5. Flow Diagram for Subroutine CALWC

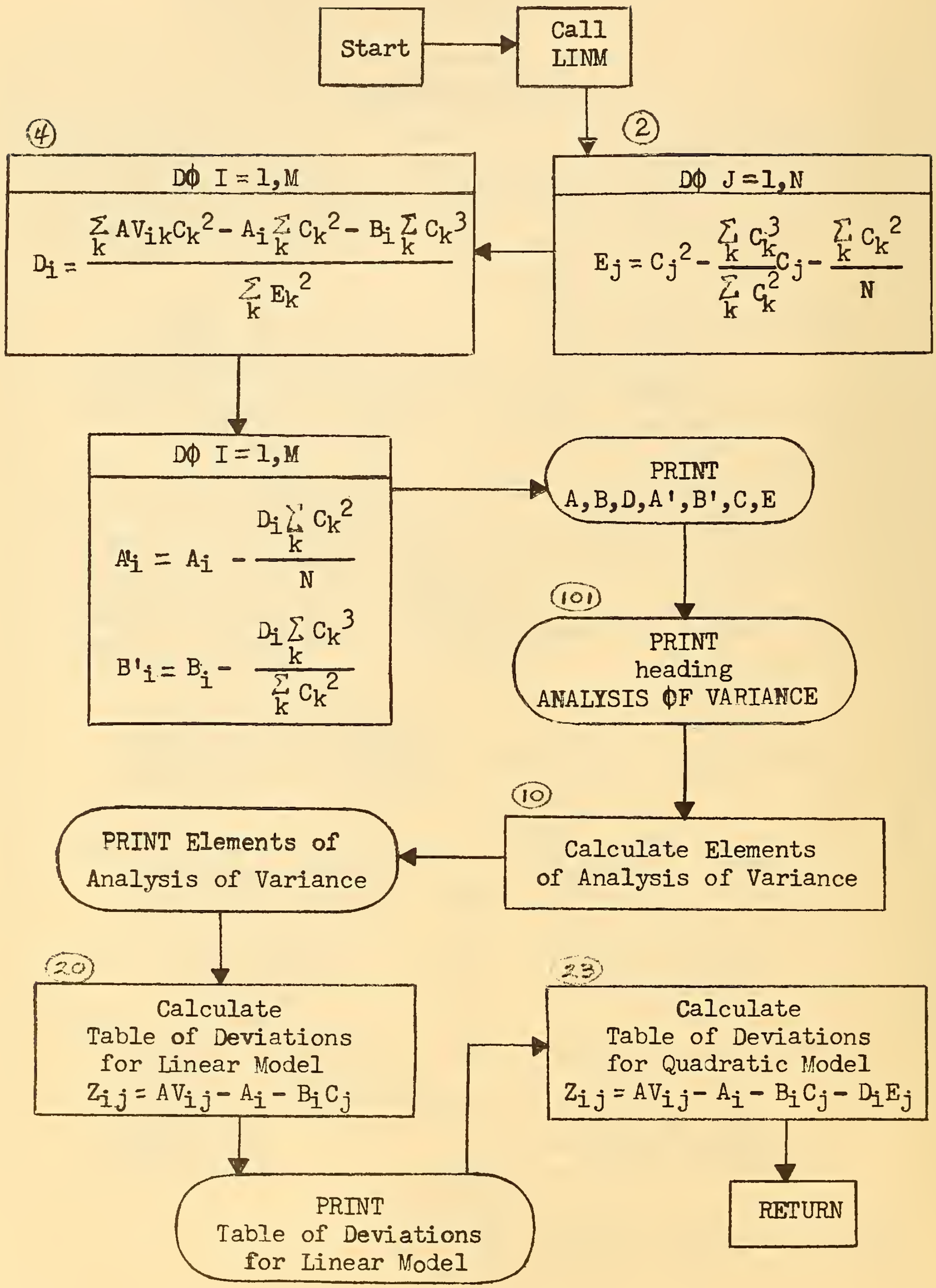


Figure 6. Flow Diagram for Subroutine QMODEL

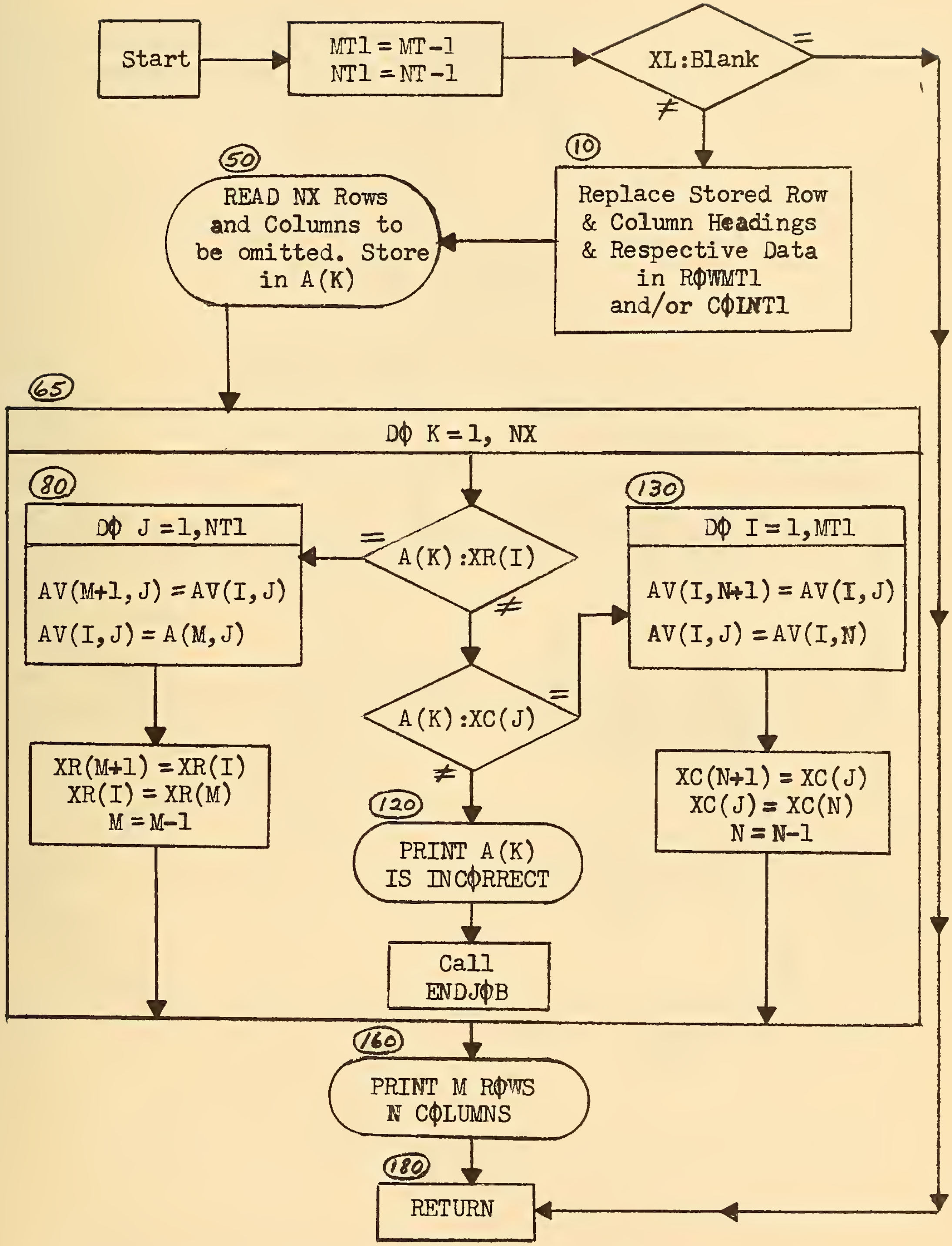


Figure 7. Flow Diagram for Subroutine OMIT

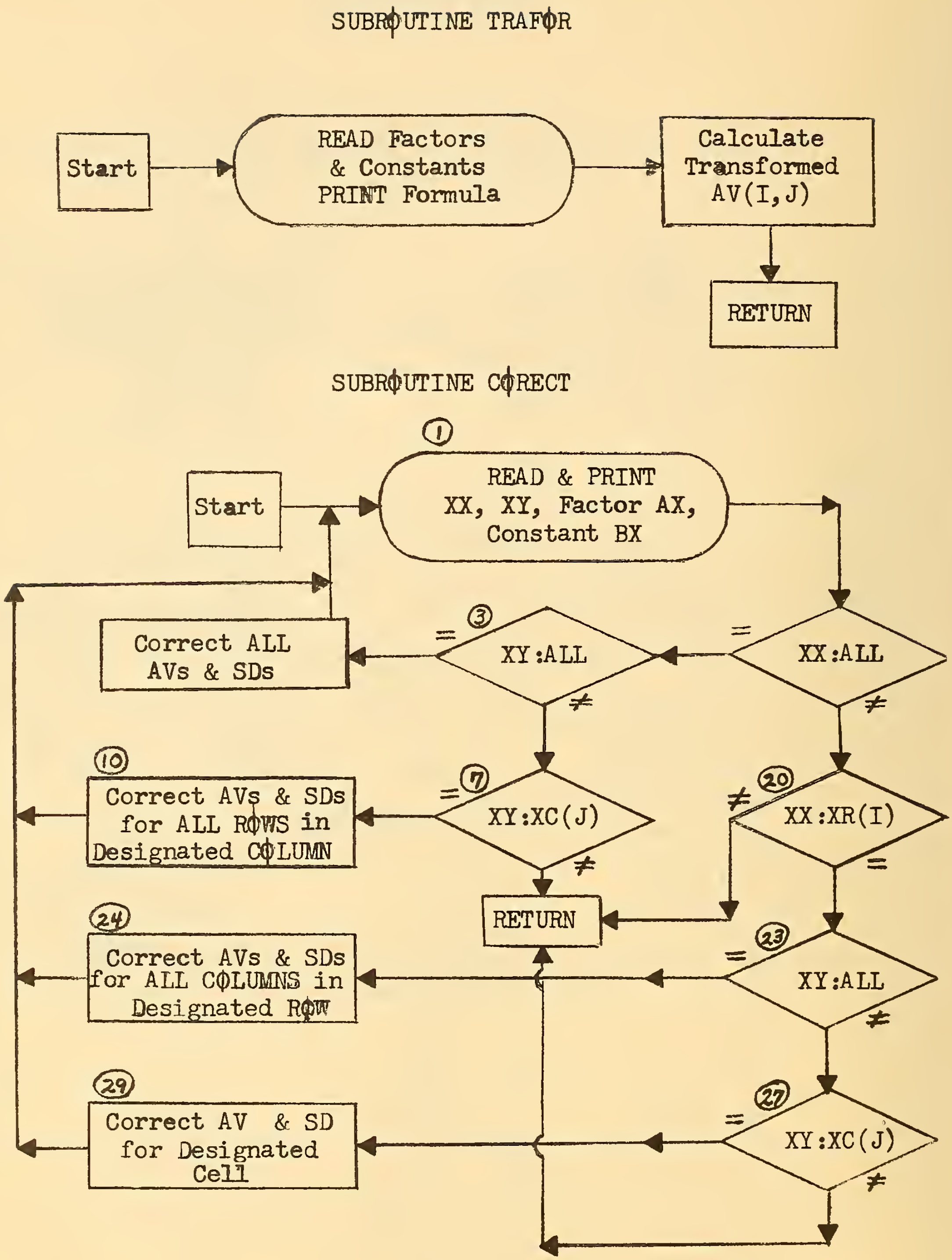
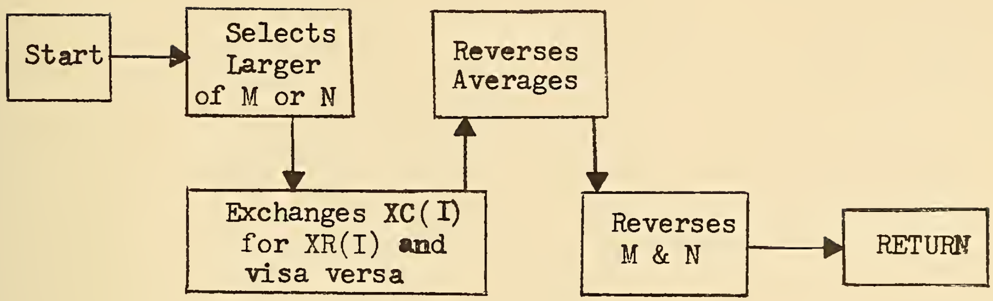


Figure 8. A. Flow Diagram for Subroutine TRAFOR
B. Flow Diagram for Subroutine CORECT

SUBRØUTINE REVERS



SUBRØUTINE CØMBIN

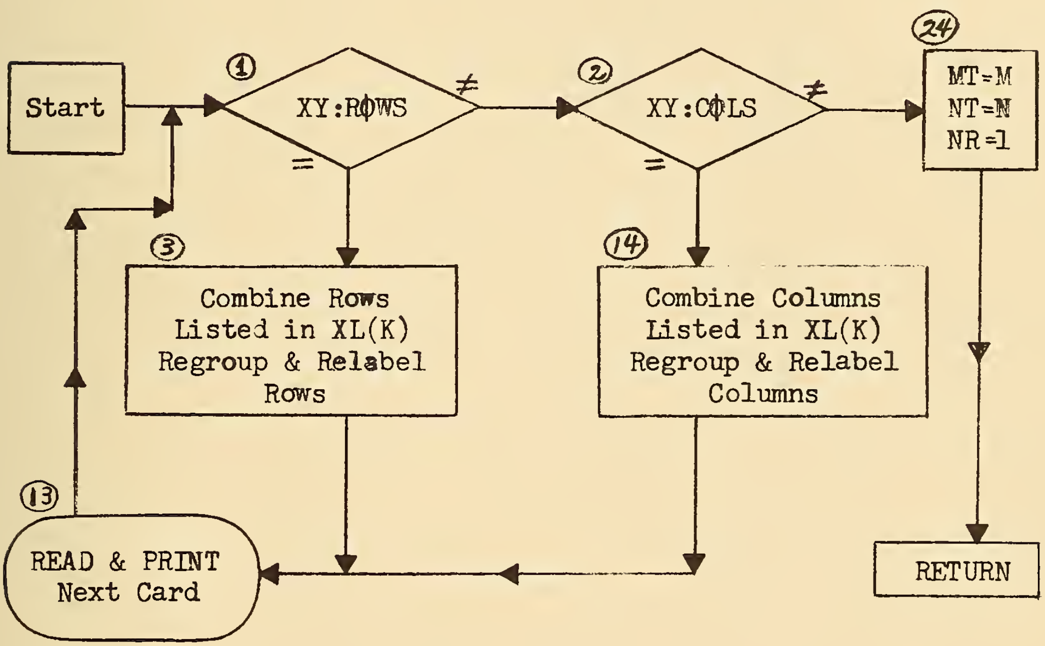


Figure 9. A. Flow Diagram for Subroutine REVERS
B. Flow Diagram for Subroutine COMBIN

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