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AVERAGE POWER DISSIPATED IN A DIODE SWEPT ALONG ITS REVERSE CHARACTERISTIC

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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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List of Symbols in Order of Appearance

V_p	- maximum voltage across the device.
I_p	- current through the device at the voltage V_p.
λ	- positive constant less than or equal to unity and serves to characterize a hypothetical device.
λV_p	- voltage across the hypothetical device above which the current is non-zero.
$P_{av}(\lambda)$	- average power dissipated in a hypothetical device, characterized by λ, swept along its reverse characteristic using a full-wave rectified sinusoidally varying voltage. When λ=1 the symbol $P_{av}(1)$ is used.
R_L	- current limiting resistance.
η	- defined at $R_L I_P/V_P$, the ratio of the maximum voltage across the current limiting resistor to that across the device.
ϕ	- defined as $\sin^{-1}(\lambda/1 + \eta)$.
ν	- frequency of the sinusoidally varying source voltage.
E	- energy dissipated in the device during one sweep cycle of duration $1/2\nu$.
P_i	- instantaneous power dissipation in the device.
t_1	- time during the sweep cycle at which the voltage across the hypothetical device is first equal to λV_p
t_o	- time at which $P_i(1) = P_i(\lambda)$ for the first time in the sweep cycle.
A_+, A_-	- areas as indicated in Fig. 2b with dimensions of power.

λ_o, λ'	-	constants such that $\lambda_o \leq \lambda' \leq 1$ and where λ_o refers to the hypothetical characteristic that is required to enclose the real characteristic over the region of significant power dissipation.
$P_{av}[R]$	-	average power dissipated in a real device.
S	-	area as indicated in Fig. 4a with dimension of power and described in Appendix D.
T	-	defined as $\lvert P_{av}(1) - P_{av}(\lambda')\rvert_{maximum}$.
V_M	-	maximum voltage of the voltage supply.
A,B,C,D,F,L	-	areas as indicated in Fig. 4a with dimensions of power.
$P_{avP}(\lambda')$	-	average power dissipated in a hypothetical device with a perturbed characteristic.

AVERAGE POWER DISSIPATED IN A DIODE SWEPT ALONG ITS REVERSE CHARACTERISTIC*

Harry A. Schafft

The commonly used method of sweeping to a fixed power in order to compare the reverse characteristics of a group of similar diodes is found lacking under conditions for which an average temperature is meaningful. It is shown how a determination may be made of the average power dissipation in a diode (or in any device with a similarly shaped characteristic) when it is swept along its reverse characteristic by a full-wave rectified sinusoidally varying voltage in series with a resistive load. The uncertainty in the determination due to the variability of the characteristic is for most cases less than ±5% if the ratio of the maximum voltage drop across the current limiting resistor to the maximum voltage across the device is larger than or equal to two. Errors introduced by small uncertainties in the pertinent parameters are also presented. Finally it is shown how the reverse characteristics of similar diodes can be examined under relatively equivalent heating conditions.

INTRODUCTION

The reverse characteristic of a diode, as well as the open base characteristic of a transistor, is often examined on an oscilloscope screen while the device is swept by a full-wave rectified sinusoidally varying voltage in series with a current limiting resistor. One is usually concerned about the power being dissipated in the device, not only because of the existence of a power rating, but also because these characteristics are temperature-dependent.

It will be shown how a determination may be made of the average power dissipation in any device which has a characteristic that can be enclosed over regions of significant power dissipation by a right triangle in the way drawn in Fig. 1 to enclose the curve R. The approach to the problem involves the calculation of the average power dissipated in a hypothetical device having a characteristic with an adjustable parameter. The calculation is used to develop an upper and lower bound on the value of the average power dissipated in a real device.

When dealing with devices with thermal time constants that are large enough compared with the period of the power supply to make an average temperature meaningful a method is presented which will make it possible for the characteristics of a group of similar devices to be examined under approximately the same heating conditions. This method consists of extending the swept characteristics to a calculable curve in the voltage-current plane which may be drawn on the cathode ray tube.

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It is common practice to use a curve determined by some fixed value for the product of the peak voltage and peak current. It will be shown that the use of such a curve can result in unnecessarily large variations in the heating of the devices being examined. When measurements are made under adverse operating conditions, which are defined in the paper, then the use of any curve can result in very large differences in heating.

DESCRIPTION AND DISCUSSION

Consider a hypothetical device which has a characteristic represented by H in Figure 1. Here no current is drawn until a voltage λV_p is reached. V_p is the voltage to which the device is ultimately swept and λ is a constant where $0 \leq \lambda \leq 1$. As the voltage increases beyond λV_p the current increases linearly with the voltage across the device until it reaches a current, I_p, corresponding to the voltage V_p.

Hypothetical devices with different λ's can, in the extreme cases, take on the characteristic of a resistor for which $\lambda = 0$ or the characteristic of a diode with an "ideal" reverse characteristic for which $\lambda = 1$. The average power, $P_{av}(\lambda)$, dissipated in such a device while it is being swept along its breakdown characteristic using a full-wave rectified sinusoidally varying voltage in series with a current limiting resistor R_L, is shown in Appendix A to be given by:

$$P_{av}(\lambda) = \frac{2V_pI_p\lambda}{\pi(1+\eta-\lambda)} \left\{(1+\eta) \cos \phi - \lambda(\frac{\pi}{2} - \phi)\right\}$$

$$+ \frac{2V_pI_p(1-\lambda)}{\pi(1+\eta-\lambda)^2} \left\{\frac{(1+\eta)^2}{4} (\pi - 2\phi + \sin 2\phi)\right.$$

$$\left. -2\lambda(1+\eta) \cos \phi + \lambda^2(\frac{\pi}{2} - \phi)\right\} , \qquad (1)$$

where $\eta = R_L(I_p/V_p)$, and $\phi = \sin^{-1}(\lambda/1 + \eta)$. More generally, the energy, E, dissipated during one sweep cycle of duration $1/2\nu$, where ν is the frequency of the sine wave, is related to $P_{av}(\lambda)$ by the expression $2\nu E = P_{av}(\lambda)$.

Consider two hypothetical devices, H and H^1, characterized by $\lambda<1$ and $\lambda=1$ respectively. It will be shown that the average power dissipation in a real device can be bounded and so be known to a specifiable degree of accuracy if its characteristic, R, can be bounded by H and H^1 in the way shown in Fig. 1, where the point (I_p, V_p) is common to all three curves. It is assumed that the power dissipation in the real device while its operating point is outside the right triangle defined by the points (I_P,V_P), $(0,V_P)$, $(0,\lambda V_P)$ constitutes a negligible fraction of the average power dissipation.[1] This contribution to the uncertainty in the calculated values is not included in the uncertainties to be quoted later.

A brief examination how of the instantaneous power dissipation, P_i, during one-half of the sweep cycle will aid in obtaining a qualitative understanding of the relative dependence of $P_{av}(\lambda)$ on R_L and V_P. This dependence will be presented graphically later. The examination will also make clearer the way in which the average power dissipation of a real device will be bounded. If a hypothetical device is swept out to a voltage V_P and a current I_P and the instantaneous power is plotted as a function of time, then a curve similar to that shown in Figure 2a will be obtained. The area under this curve is a measure of the energy dissipated during one-half of the sweep cycle or one-quarter of the sine wave cycle. This curve will depend on the λ associated with the hypothetical device and on R_L. For all other variables held constant the dissipation of power in the hypothetical device will begin later in the sweep cycle, i.e. t_1 as shown in Figure 2a will increase, as devices with larger λ's are examined. On the other hand, t_1 will decrease as R_L is made larger for a constant V_P.[2]

A plot of the product of the instantaneous power dissipation and twice the sine wave frequency as a function of time is shown in Figure 2b for the characteristics of two hypothetical devices. The scale of the vertical axis is chosen to make the area under each curve equal to one-half the average power dissipation, i.e. $P_{av}(\lambda)/2$. Curve 1 represents the dissipation in a device with $\lambda = 1$ while curve 2 in one with $\lambda<1$. In general the two curves will intersect not only at a time $t = 1/4\nu$ but also at some time earlier in the cycle. The time at which this other intersection occurs is designated as t_o. The relative magnitude of the areas A_+ and A_-, shown in Figure 2b, determines whether $P_{av}(\lambda)$ will be smaller or larger than $P_{av}(1)$. An outline of the derivation of t_o in Appendix B shows that t_o is a function of λ and η. As η is decreased, t_o increases toward $1/4\nu$ and A_- decreases to zero while A_+ remains finite; thus for small η, $P_{av}(\lambda) > P_{av}(1)$. On the other hand, if η is increased to very large values, t_o will decrease toward zero and A_+ decreases to zero while A_- remains finite; thus for large η, $P_{av}(\lambda) < P_{av}(1)$. At some intermediate value of η, $A_+ = A_-$ and so $P_{av}(\lambda) = P_{av}(1)$. This behavior can be seen in Figure 3 where the normalized average power dissipation, $P_{av}(\lambda)/I_PV_P$, is plotted as a function of η for four different values of λ. An important point to observe in

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1. The product of the voltage and current at the point of enclosure should be much less than V_PI_P.
2. The dependence of t_1 on λ and R_L is given in Appendix A.

the plot is that $P_{av}(\lambda)$ is a monotonic function of λ except in the region $2.0 \lesssim \eta \lesssim 2.6$, where the dependence on λ is small. Thus for any hypothetical device characterized by λ', $P_{av}(\lambda')$ can be bounded in the following way:

$$P_{av}(\lambda_o) \le P_{av}(\lambda') \le P_{av}(1) \text{ for } \eta \gtrsim 2.6$$

$$P_{av}(\lambda_o) \ge P_{av}(\lambda') \ge P_{av}(1) \text{ for } \eta \lesssim 2.0$$

$$\text{and } P_{av}(1) + C \ge P_{av}(\lambda') \ge P_{av}(1) - C \text{ for } 2.0 \lesssim \eta \lesssim 2.6 \text{ ,}$$

where C is some appropriately chosen constant, and $\lambda_o \le \lambda' \le 1$. However, as can be seen in Figure 3, $P_{av}(\lambda)$ decreases as η decreases to zero and the dependence on λ increases sharply. Thus while $P_{av}(\lambda')$ can be indeed be bounded, the uncertainty in $P_{av}(\lambda')$ can soon approach the magnitude of $P_{av}(\lambda')$ as η decreases much below unity.

The reverse characteristics of real devices are, at least, only similar to those of the hypothetical devices at the higher currents. Real devices have characteristics that appear more like that pictured by R in Figure 1. It is shown in Appendix C that the average power dissipated in a real device, $P_{av}[R]$, with an arbitrarily shaped characteristic, can be bounded if the characteristic itself can be bounded over the region of significant power dissipation. Here, as in the case of $P_{av}(\lambda')$, the usefulness of the bound decreases markedly at low values of η. For ease in presentation and ultimate usefulness, Appendix C considers only conditions for which $\eta \ge 1$. It will be seen later in Appendix D that if the uncertainties for the cases examined are to remain tolerably small, then only conditions for which $\eta \ge 2$ should be considered; therefore the data that follows will be presented for such values of η.

If the characteristic of the real device is bounded by the characteristics of two hypothetical devices, one with $\lambda = 1$ and the other with $\lambda_o < 1$, then from Appendix C:

$$P_{av}(\lambda_o) - 2A_+ < P_{av}[R] < P_{av}(1) + 2A_+ + 2S \text{ for } \eta \gtrsim 2.6$$

and either

$$P_{av}(\lambda_o) - 2A_+ < P_{av}[R] < P_{av}(1) + 2A_+ + 2(S + T)$$

or

$$P_{av}(1) - 2A_+ < P_{av}[R] < P_{av}(\lambda_o) + 2A_+ + 2S \text{ for } 1 \lesssim \eta \lesssim 2.6$$

where $T = \left| P_{av}(1) - P_{av}(\lambda')\right|_{\text{maximum}}$ for $\lambda_o \le \lambda' \le 1$

and S is a complicated function of η and λ described in Appendix D. The constant, λ_o, will refer to that hypothetical characteristic which is required to enclose the real characteristic over the region of significant power dissipation. A discussion of the magnitudes of the terms involved in the bounds for $P_{av}[R]$ as well as how they were determined is presented in Appendix D for $\lambda_o = .6$ and $\lambda_o = .8$. Two values of λ_o were used, with the stated magnitudes, in order to make the calculations adaptable to as wide a variety of devices as seemed feasible. Because both S and T were found to be negligibly small over most of the range of η, the value for $P_{av}[R]$ was chosen to be a value mid-way between the values of $P_{av}(\lambda_o)$ and $P_{av}(1)$ at any specific value of η. This was done so that the uncertainties could be made symmetric about $P_{av}[R]$. The results of Appendix D show that for real devices which have characteristics which can be bounded by a hypothetical characteristic with $\lambda_o = .6$ the uncertainty in $P_{av}[R]$ is within $\pm 5\%$. If a hypothetical characteristic with $\lambda_o = .8$ can be used, the uncertainty in $P_{av}[R]$ is reduced to within $\pm 3\%$. For a given λ_o the dependence on η of the normalized average power dissipation in a real device, $P_{av}[R]/I_P V_P$, can be sufficiently well represented by two simple expressions to make no change in the above specified uncertainties necessary. These expressions are shown below for $\lambda_o = .6$ and $\lambda_o = .8$ and will give the value of $P_{av}[R]$ once the values of I_P, V_P and R_L are specified.

For $\lambda_o = .6$,

$$\frac{P_{av}[R]}{V_P I_P} = .6010 - \frac{.1840}{\eta}\ , \text{ for } 2 \le \eta \le 10 \qquad (2a)$$

and

$$\frac{P_{av}[R]}{V_P I_P} = .6090 - \frac{.2530}{\eta}\ , \text{ for } \eta \ge 10. \qquad (2b)$$

For $\lambda_o = .8$,

$$\frac{P_{av}[R]}{V_P I_P} = .6140 - \frac{.2120}{\eta}\ , \text{ for } 2 \le \eta \le 10 \qquad (3a)$$

and

$$\frac{P_{av}[R]}{V_P I_P} = .6230 - \frac{.2980}{\eta}\ , \text{ for } \eta \ge 10. \qquad (3b)$$

The stated uncertainties do not include, as mentioned earlier, the contribution due to the dissipation while the real device's characteristic is not bounded. It is assumed that λ_o is chosen to make this contribution negligible. Additional sources of error which can become appreciable are the uncertainties in V_P, I_P, and R_L. It is shown in Appendix E that the fractional change in the average power dissipation, $\Delta P_{av}[R]/P_{av}[R]$ introduced by small changes in V_P, I_P, and R_L is given approximately by the relation

$$\frac{\Delta P_{av}[R]}{P_{av}[R]} < \frac{\Delta I_P}{I_P} + \frac{\Delta V_P}{V_P} + \frac{1}{2\eta}\frac{\Delta R_L}{R_L}\ .$$

Thus it is obvious that unless considerable care is used in the measurement procedures the errors introduced in $P_{av}[R]$ can easily be as large as the already specified uncertainties used to take into account the variability of the shape of the characteristic.

Using equations (2) or (3), depending on which λ_o is chosen, a curve in the voltage-current plane can be obtained to which the characteristics of similar devices under test can be swept in order to assure uniform average power dissipation and thus approximately uniform heating conditions. For a specific value of the average power dissipation desired, R_L must be chosen so that η will be larger than or equal to two for all test conditions. The value of η will be a minimum at the maximum peak voltage, V_P, expected for the devices under test. Since η is the ratio of the voltage drop across the current limiting resistor to the voltage drop across the device it is advantageous to adjust conditions so that η is small at the maximum expected peak voltage. Therefore η should be chosen to be equal to two at this voltage and the appropriate expression is then solved for I_P. Knowing I_P, V_P and η the current limiting resistor R_L to be used in the test can be determined. Finally by substituting $I_P R_L/V_P$ for η in the appropriate equations, i.e. (2) or (3), and solving for I_P the other points on the voltage-current plane may be obtained over the desired voltage range and a curve may then be drawn joining the points obtained. The uncertainty in $P_{av}[R]$ when the characteristic of the device is swept to this curve is within either $\pm 3\%$ or $\pm 5\%$, depending on the value of λ_o required. To this uncertainty the errors introduced by the uncertainty R_L and the maladjustment of the peak current should be added. These errors, as shown in Appendix E, contribute a fractional change in the average power dissipation of

$$\frac{\Delta P_{av}[R]}{P_{av}[R]} \lessapprox 1.4 \frac{\Delta I_P}{I_P} + \frac{1}{2\eta} \frac{\Delta R_L}{R_L} .$$

It is often assumed that approximately uniform heating conditions can be obtained by extending the characteristics of the test devices to the curve $I_P V_P$ = constant $\propto P_{av}[R]$. If this assumption is correct then $P_{av}[R]/I_P V_P$ should be a linear function of η. That this is not the case can be seen by the variation of $P_{av}(\lambda)/I_P V_P$ with η shown in Figure 3 and from equation (2) and (3). The departure from linearity is greatest at the lower values of η. The use of such a curve can result in variations in $P_{av}[R]$ of almost 25 percent as similar devices with different characteristics are examined. Of course if measurement conditions are such that $\eta<2$ then the variations of $P_{av}[R]$ can be very much larger.

SUMMARY

It has been shown how the average power dissipation in a diode can be calculated when it is swept along its reverse characteristic by a full-wave rectified sinusoidally varying voltage. These calculations will also apply to any similarly shaped characteristic, for example, the open-base characteristic of some transistors. The average power dissipation may be obtained from a simple expression once the maximum voltage across the diode, the corresponding current and the current limiting resistance is specified. The uncertainty arising from the variability of the shape of the characteristic can in most cases be within $\pm 5\%$ if measurement conditions are such that the ratio of the maximum voltage across the current limiting resistor to the maximum voltage across the device is greater than or equal to two. When this ratio is less than two, the average power dissipation in the device becomes very sensitive to the value of this ratio as well as the shape of the characteristic of the device.

Estimates of the fractional change in the average power dissipation introduced by small changes in the pertinent parameters have shown that the uncertainty of the average power dissipation in the device due to poor measurement techniques can easily be larger than the already quoted uncertainty resulting from the variability of the shape of the characteristic.

The ability to determine the average power dissipation in a device permits the construction of a curve to which the characterisitic of each similar diode can be swept and achieve relatively equivalent heating conditions. The use of this curve represents a distinct improvement over the use of a fixed peak power curve.

APPENDIX A

Let the supply voltage be given a $V_M \sin 2\pi\nu t$ for $0 \le t \le 1/4\nu$. Let V represent the voltage across and I represent the current through the device under consideration. From the description of the characteristic of the hypothetical device, it follows that for $0 \le t \le t_1$, $I = 0$ and for $t_1 \le t \le 1/4\nu$,

$$V = \frac{V_P}{I_P}(1-\lambda)\, I + \lambda V_P$$

$$\text{where} \quad t_1 = \frac{1}{2\pi\nu} \sin^{-1}(\lambda/1 + \frac{I_P R_L}{V_P}) \,. \tag{A1}$$

The explicit form of I for $t_1 \le t \le 1/4\nu$ is obtained from the relation

$$V_M \sin 2\pi\nu t = IR_L + \frac{V_P}{I_P}(1-\lambda)\, I + \lambda V_P \,.$$

The solution for I is,

$$-I = \frac{V_M \sin 2\pi\nu t - \lambda V_P}{R_L + V_P(1-\lambda)/\, I_P} \,. \tag{A2}$$

If E is the energy dissipated during one sweep cycle of the full wave rectified sinusoidally varying supply voltage then $P_{av}(\lambda) = 2\nu E$.

But

$$E = 2\int_{t_1}^{1/4\nu} VIdt.$$

Therefore

$$P_{av}(\lambda) = 4\nu \int_{t_1}^{1/4\nu} VIdt. \tag{A3}$$

If (A1) and (A2) are substituted into (A3) and the integration performed then the following result is obtained,

$$P_{av}(\lambda) = \frac{2V_P I_P \lambda}{\pi(1+\eta-\lambda)} \left\{(1+\eta)\cos\phi - \lambda(\frac{\pi}{2} - \phi)\right\}$$

$$+ \frac{2V_P I_P (1-\lambda)}{\pi(1+\eta-\lambda)^2} \left\{\frac{(1+\eta)^2}{4}(\pi - 2\phi + \sin 2\phi)\right.$$

$$\left. -2\lambda(1+\eta)\cos\phi + \lambda^2(\frac{\pi}{2} - \phi)\right\},$$

where $\eta = R_L(I_P/V_P)$, and $\phi = \sin^{-1}(\lambda/1 + \eta) = 2\pi\nu t_1$.

APPENDIX B

The instantaneous power dissipation $P_i(t,\lambda)$ in a hypothetical device is given by $P_i(t,\lambda) = IV$ where V and I are given by equations A1 and A2 in Appendix A. When $P_i(t,\lambda)$ is equated to $P_i(t_1,1)$, a quadratic equation in terms of $\sin 2\pi\nu t$ is obtained. One root of the equation is unity since the two characteristics have a common point at $t = 1/4\nu$. The other root is found to be

$$\sin 2\pi\nu t_o = \frac{(1 - \lambda + \eta)^2}{(1 - \lambda)(1 + \eta)^2} \left[\frac{1}{\eta} - \frac{\lambda^2\eta}{(1 - \lambda + \eta)^2}\right] .$$

APPENDIX C

In Figure 4a the product of twice the sweep frequency and the instantaneous power dissipation for three different hypothetical curves are superimposed and plotted as a function of time. One of these curves is for $\lambda = 1$ while the other two curves are for λ_o and λ' where $\lambda_o < \lambda'$. The size of the areas shown, which have dimensions of power, are exaggerated for ease of identification. These same curves are also plotted in Figure 4b on the V-I plane.

Note that it can be shown that for $\eta > 1$, $\partial t_o/\partial\lambda|_\eta > 0$; so that in Figure 4a the way in which the curves are drawn intersecting the $\lambda = 1$ curve is correct.

From Figure 4a it can be seen that

$$\frac{P_{av}(1)}{2} = A + B + F,$$

$$\frac{P_{av}(\lambda')}{2} = B + C + L + F$$

and

$$\frac{P_{av}(\lambda_o)}{2} = C + D + F \, .$$

Consider now that the λ' curve is perturbed in such a manner that on the V-I plane the slope, dI/dV, decreases in some uniform way as is indicated by the dotted line in Figure 4b. The perturbation is assumed to be such that the $\lambda = \lambda_o$ and $\lambda = 1$ curves on the V-I plane still enclose the perturbed λ' curve.

As the device with the perturbed characteristic is swept in the V-I plane, the instantaneous power dissipation at any time during the sweep cycle is equal to that of some hypothetical device with an unperturbed characteristic with λ such that $\lambda_o \leq \lambda \geq \lambda'$. The average power dissipated, $P_{avP}(\lambda')$, by the device with the perturbed characteristic can be given by the relation,

$$\frac{1}{2} P_{avP}(\lambda') = F + C + a_1 B + a_2 L + a_3 D + S$$

where $0 \leq a_1 \leq 1$, $0 \leq a_2 \leq 1$, $0 \leq a_3 \leq 1$. The nature of S and its magnitude is discussed in Appendix D. Convenient maximum and minimum limits on $P_{avP}(\lambda')$ are

$$\frac{1}{2} P_{avP}(\lambda') \text{ max.} \leq F + C + B + D + S,$$

and

$$\frac{1}{2} P_{avP}(\lambda') \text{ min.} \geq F + C.$$

There are four ways that $P_{avP}(\lambda')$, $P_{av}(\lambda_o)$ and $P_{av}(1)$ may be found to be ordered with respect to their relative magnitudes over the full range of η. This ordering will now be examined to see how bounds on $P_{avP}(\lambda')$ may be constructed for all η.

1.) $P_{av}(1) > P_{avP}(\lambda') > P_{av}(\lambda_o)$.

This implies that $A > C + L$ or that $A = C + L + \zeta$ where $\zeta > 0$.

Now

$$\frac{1}{2} P_{avP}(\lambda') \leq B + C + D + F + L + S$$

or

$$\frac{1}{2} P_{avP}(\lambda') \leq \frac{1}{2} P_{av}(1) - A + C + D + L + S$$

$$= \frac{1}{2} P_{av}(1) - \zeta + D + S < \frac{1}{2} P_{av}(1) + (C + D) + S.$$

And,

$$\frac{1}{2} P_{avP}(\lambda') \geq C + F = \frac{1}{2} P_{av}(\lambda_o) - D > \frac{1}{2} P_{av}(\lambda_o) - (C + D).$$

Thus

$$\frac{1}{2} P_{av}(\lambda_o) - (C + D) < \frac{1}{2} P_{avP}(\lambda') < \frac{1}{2} P_{av}(1) + (C + D) + S.$$

2.) $P_{av}(\lambda_o) > P_{avP}(\lambda') > P_{av}(1)$.

This implies that $C + D > A + B$ and that $C + D - A - B > C + L - A$ or that $D > B + L$. Now

$$\frac{1}{2} P_{avP}(\lambda') \leq B + C + D + F + L + S = \frac{1}{2} P_{av}(\lambda_o) + B + L + S$$

$$< \frac{1}{2} P_{av}(\lambda_o) + (C + D) + S.$$

And

$$\frac{1}{2} P_{avP}(\lambda') \geq C + F = \frac{1}{2} P_{av}(1) - A - B > \frac{1}{2} P_{av}(1) - (C + D).$$

Thus

$$\frac{1}{2} P_{av}(1) - (C + D) < \frac{1}{2} P_{avP}(\lambda') < \frac{1}{2} P_{av}(\lambda_o) + (C + D) + S.$$

3.) $P_{avP}(\lambda') > P_{av}(1) > P_{av}(\lambda_o)$.

4.) $P_{avP}(\lambda') > P_{av}(\lambda_o) > P_{av}(1)$.

For both cases, $P_{avP}(\lambda') > P_{av}(\lambda_o) - (C + D)$, and

$$\frac{1}{2} P_{avP}(\lambda') \le B + C + D + F + L + S = \frac{1}{2} P_{av}(1) - A + C + D + L + S$$

$$\le \frac{1}{2} P_{av}(1) + T + S + (C + D)$$

where

$$T = \left| P_{av}(\lambda') - P_{av}(1) \right|_{maximum} \ge L - A.$$

Thus

$$\frac{1}{2} P_{av}(\lambda_o) - (C + D) < \frac{1}{2} P_{avP}(\lambda') < \frac{1}{2} P_{av}(1) + T + S + (C + D).$$

For $\eta \gtrsim 2.6$ the first case will hold and depending on λ_o and η, one of the other cases will hold for $1 \le \eta \lesssim 2.6$.

The treatment in this section will also apply to any real device as long as the slope dI/dV on the V-I plane remains constant or decreases in some uniform manner. Therefore, the bounds that apply to $P_{avP}(\lambda')$ will also apply to the $P_{av}[R]$. Accordingly,

$$P_{av}(\lambda_o) - 2A_+ < P_{av}[R] < P_{av}(1) + 2A_+ + 2S \text{ for } \eta \gtrsim 2.6$$

and either

$$P_{av}(1) - 2A_+ < P_{av}[R] < P_{av}(\lambda_o) + 2A_+ + 2S$$

or

$$P_{av}(\lambda_o) - 2A_+ < P_{av}[R] < P_{av}(1) + 2A_+ + 2S + 2T$$

for

$1 \le \eta \lesssim 2.6$ where $A_+ = C + D$.

APPENDIX D

The bounds for $P_{av}[R]$ involve T, S, A_+ and $|P_{av}(1) - P_{av}(\lambda)|$ which depend on λ and η. Estimates and upper bounds for these values were obtained by a number of numerical calculations and graphical constructions.

The value for T was obtained by plotting $P_{av}(\lambda)/I_P V_P$ over a limited range of λ for different values of λ. From the graphic representation it was determined that the contribution of T to the uncertainty of $P_{av}[R]$ was less than 0.3% for $\lambda_o = .8$ and less than 1.6% for $\lambda_o = .6$.

Consider Figure 4a. If curves for all possible λ between 1 and λ_o are also drawn, then the area enclosed by these curves that does not include the area A + B + C + D + F will be equal to or larger than any S that can arise. If $P_i(\lambda)$ is the instantaneous power dissipation in a device with an λ characteristic and if $t_x(\lambda)$ is the time at which $P_i(\lambda+\Delta) = P_i(\lambda)$ where $\lambda + \Delta \leq 1$ and finally if $t_o(\lambda)$ is the time at which $P_i(\lambda) = P_i(1)$ then

$$S < \lim_{\Delta \to o} \sum_{i=1}^{(1-\lambda_o)/\Delta} \int_{t_x(\lambda_o+i\Delta)}^{t_o(\lambda_o+i\Delta)} \{E(\lambda_o+i\Delta) - E(\lambda_o+[i-1]\Delta)\}dt.$$

To estimate the magnitude of S, Δ was set equal to 0.5 and several terms were evaluated. It appears that these terms, over the range of $\eta \geq 2$, are largest for $\eta = 2$ and decrease rapidly as η is increased, and as λ_o increases.

At $\eta = 2$ for $\lambda_o = .6$, S is estimated to contribute an uncertainty to $P_{av}[R]$ of less than 0.1% of $P_{av}[R]$.

The values for $A_+/I_P V_P$ and $|P_{av}(1) - P_{av}(\lambda_o)|/V_P I_P$ were calculated for several values of η for $\lambda_o = .6$ and $\lambda_o = .8$. The results are indicated below:

η	$\dfrac{\lvert P_{av}(1) - P_{av}(.6)\rvert + 4A_+}{P_{av}(.6)}$	$\dfrac{\lvert P_{av}(1) - P_{av}(.8)\rvert + 4A_+}{P_{av}(.8)}$
2.0	.065	.041
2.6	.042	.028
4.0	.051	.025
6.0	.060	.030
∞	.095	.045

As can be seen, allowing for T and S, $\pm$ 5% of $P_{av}[R]$ for $\lambda_o = .6$ and $\pm$ 3% of $P_{av}[R]$ for $\lambda_o = .6$ are conservative estimates for the uncertainty of $P_{av}[R]$.

If $P_{av}[R]\eta/I_PV_P$ is plotted against η it is found that to a good approximation the two quantities are linearly related for $\eta \gtrsim 2$. Thus $P_{av}[R]\eta/I_PV_P = P_{av}[R]R_L/V_P^2 \simeq F\eta + G$ where F and G are constants. If both sides of the equation are multiplied by V_P^2/R_L then

$$P_{av}[R] \simeq FI_PV_P + GV_P^2/R_L \ . \qquad \text{(E1)}$$

If small changes in V_P, I_P and R_L are allowed and denoted by ΔV_P, ΔI_P and ΔR_L respectively, then the fractional change in P_{av} is given by

$$\frac{\Delta P_{av}[R]}{P_{av}[R]} = F\frac{V_PI_P}{P_{av}[R]}\frac{\Delta I_P}{I_P} + \left(1 + G\frac{V_PI_P}{P_{av}[R]}\frac{1}{\eta}\right)\frac{\Delta V_P}{V_P} - G\frac{V_PI_P}{P_{av}[R]}\frac{1}{\eta}\frac{\Delta R_L}{R_L} \ .$$

Now $\quad F \simeq .61,\ G \simeq -.25$ and $P_{av}[R]/I_PV_P \gtrsim .51$.

Thus

$$\frac{\Delta P_{av}[R]}{P_{av}[R]} \lesssim \frac{\Delta I_P}{I_P} + \frac{\Delta V_P}{V_P} + \frac{1}{2\eta}\frac{\Delta R_L}{R_L} \ . \qquad \text{(E2)}$$

At currents close to I_P the slope of the real breakdown characteristic should not change greatly, so it can be described by some λ characteristic. This voltage is related to the current by the equation

$$V = \frac{V_P}{I_P}(1-\lambda)I + \lambda V_P \ .$$

For small changes in V_P and I_P,

$$\frac{\Delta V_P}{V_P} = (1-\lambda)\frac{\Delta I_P}{I_P} \ .$$

and equation (E2) for $\lambda \gtrsim .6$ becomes

$$\frac{\Delta P_{av}[R]}{P_{av}[R]} \lesssim 1.4\frac{\Delta I_P}{I_P} + \frac{1}{2\eta}\frac{\Delta R_L}{R_L} \ .$$

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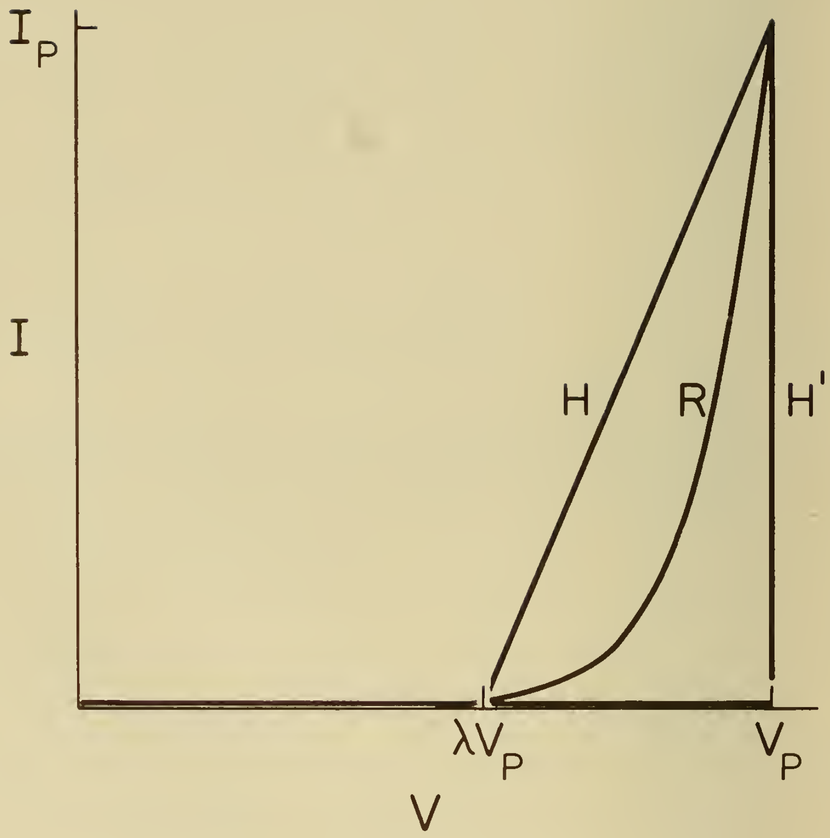

Figure 1 - Voltage-current characteristic of hypothetical devices H'($\lambda = 1$) and H($\lambda < 1$), and real device R.

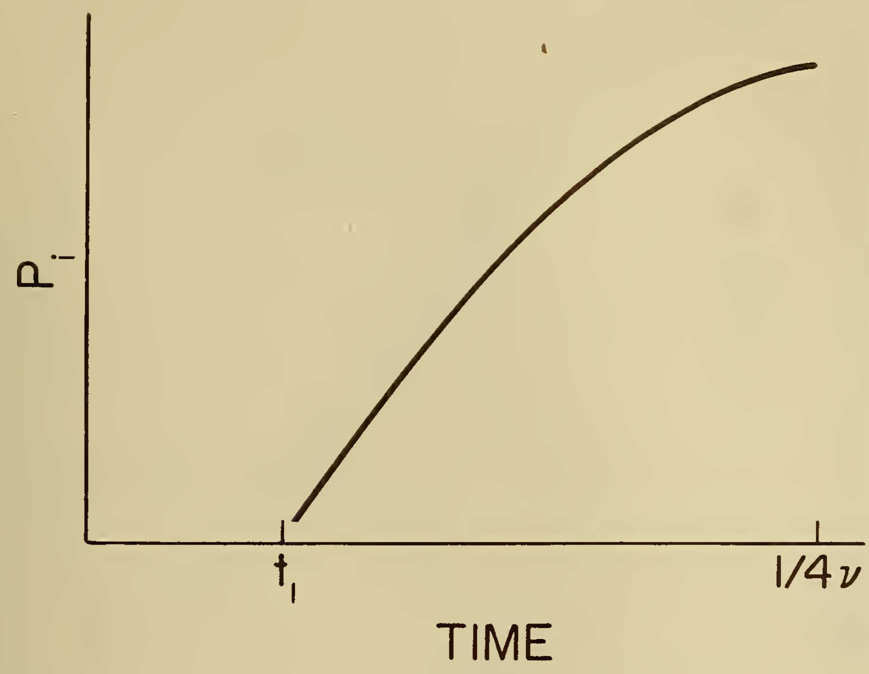

Figure 2a - Qualitative features of the instantaneous power, P_i vs. time characteristic of a hypothetical device.

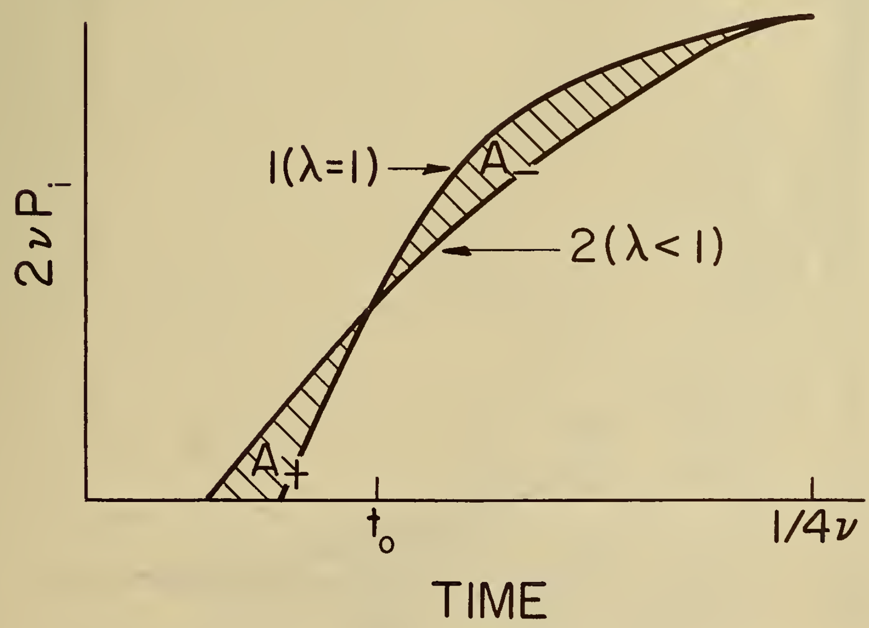

Figure 2b - Product of the instantaneous power and twice the sine wave frequency as a function of time for two hypothetical devices, one with $\lambda = 1$ and the other with $\lambda < 1$.

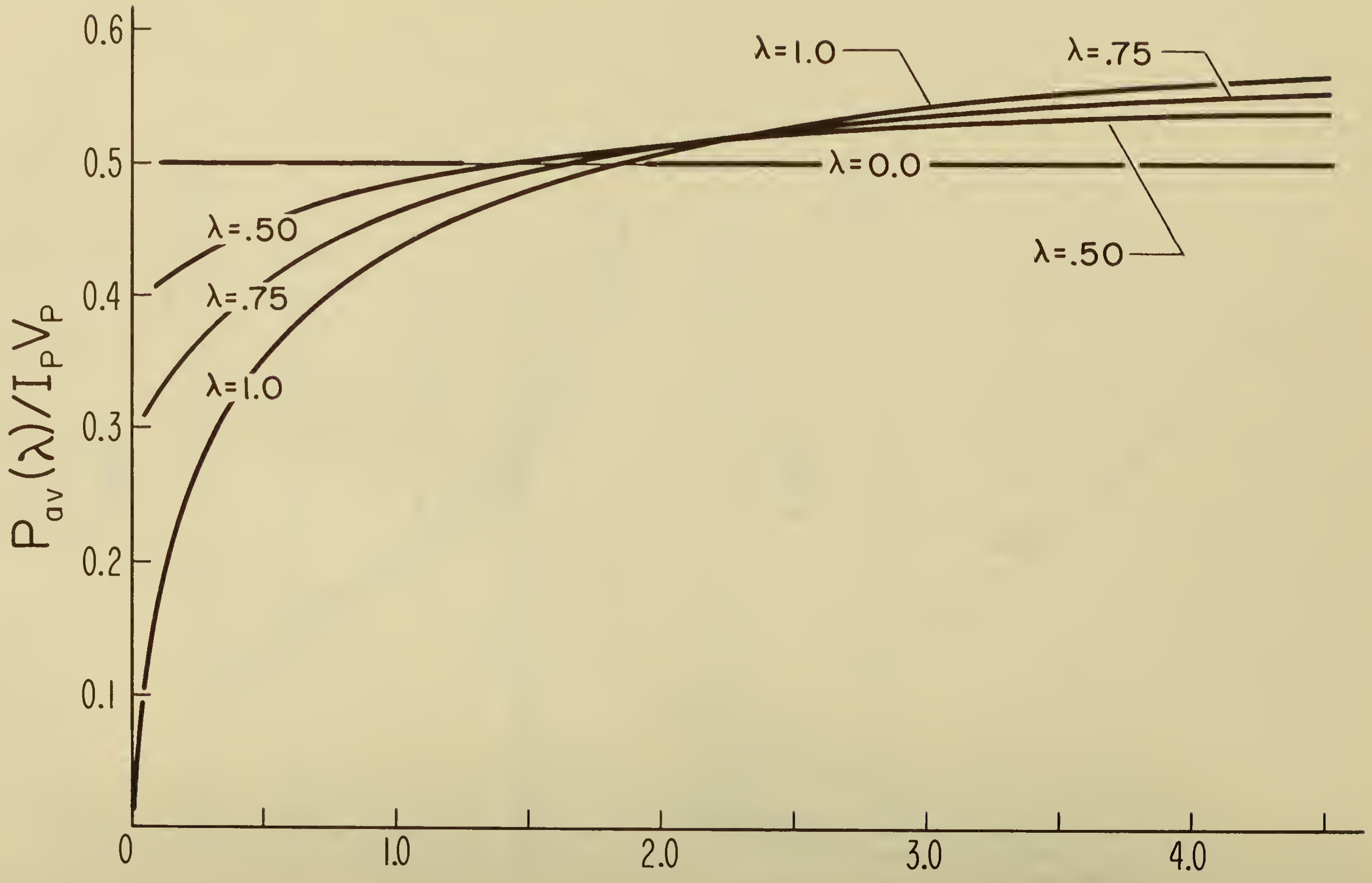

Figure 3 - Dependence of the normalized average power dissipation, $P_{av}(\lambda)/I_pV_p$ on η for four values of λ.

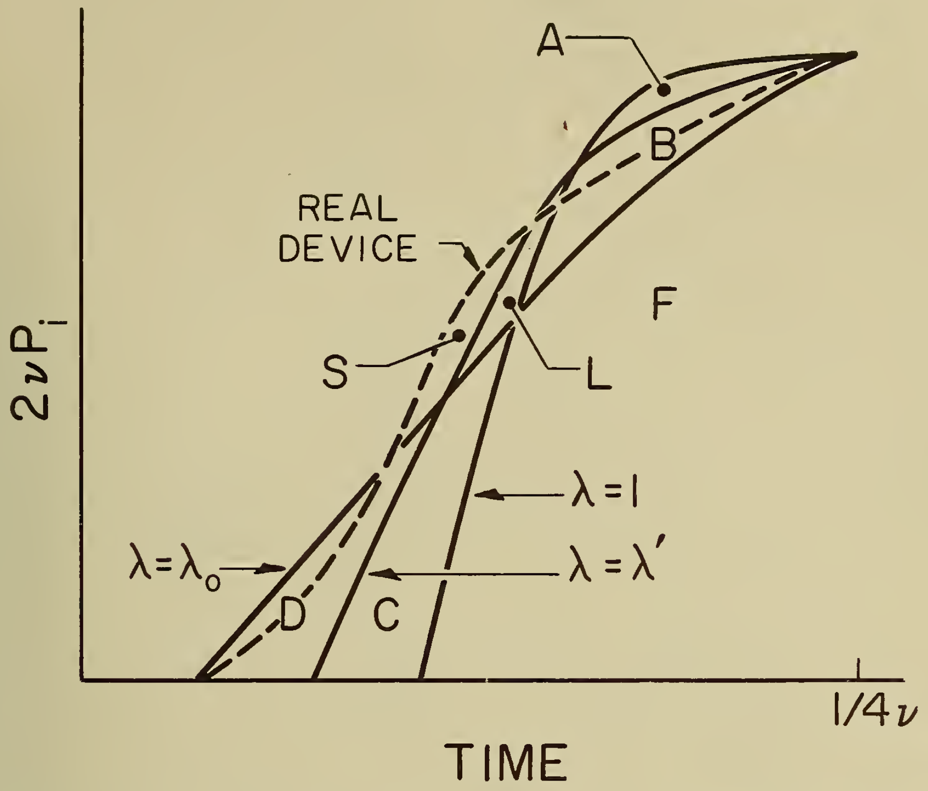

Figure 4a - Qualitative features of the product of the instantaneous power and twice the sine wave frequency as a function of time for three hypothetical devices and one real device.

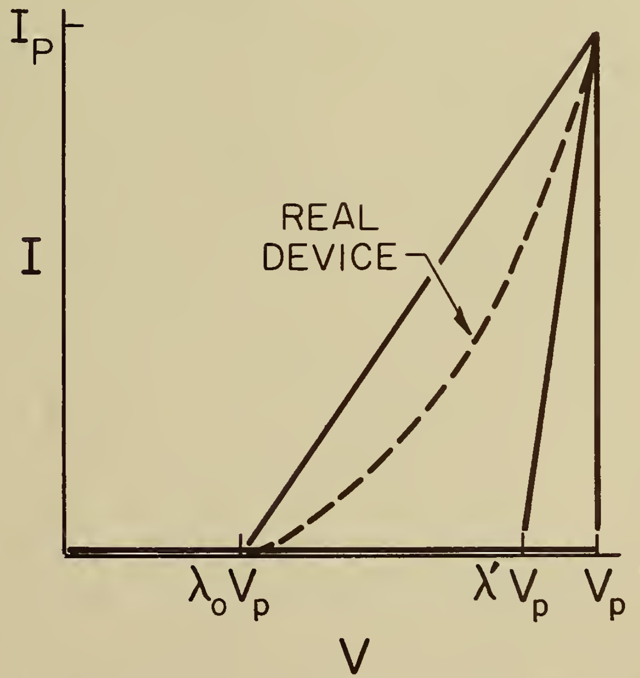

Figure 4b - Voltage-current characteristics of three hypothetical devices and one real device.

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