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DESIGN OF SINGLE FREQUENCY FILTERS

BY FORREST F. FULTON, JR.

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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ABSTRACT

Efficient procedures are shown for designing filters formed by a number of identical resonant circuits loosely coupled together, and which are required to accept one narrow band of frequencies and reject another narrow band somewhat removed in frequency, without any special requirements on the shape of the attenuation curve in between these regions. The design is based on using a large number of sections.

INTRODUCTION

A convenient form of filter for some applications is that formed by a number of resonant circuits loosely coupled together, having all circuits tuned to the same frequency. One application is in the suppression of the spurious responses of a superheterodyne receiver having a narrow i-f system, where the necessary filtering of the r-f signal is merely to reject by some minimum attenuation all frequencies separated from the signal by a fixed amount, without any special requirements on the shape of the attenuation curve between the signal frequency and the frequencies to be rejected. In general, any application where the desired signal is much narrower than the separation of the signal and the frequencies to be rejected can conveniently use this type of filter. Some typical response curves obtained for this type of filter with different values of a coupling parameter, n, are shown in Fig. 1.

The advantages of this type filter are that it can be properly adjusted while it is in the equipment simply by maximizing the signal transmission through it, that it is built of identical sections, and that the characteristics are expressed simply in terms of the number of sections, the circuit Q, and a coupling parameter.

It is the purpose of this paper to show two methods of efficiently designing these filters to meet accurately different kinds of specifications; a design for minimum insertion loss, and a design for the minimum number of sections. The designs are based on the assumption that a large enough number of sections will be used that the performance can be calculated from the per-section characteristics of an infinite string of sections.

BASIC RELATIONSHIPS

There are many configurations that can be used for this type of filter with only minor changes in the calculations. Fig. 2 shows the configuration used throughout this discussion; Fig. 3 shows some of the alternative configurations and the changes in parameters to be used with them.

The propagation through one section of a network that consists of an infinite string of identical sections may be described by a complex propagation constant P, which is the principal natural logarithm of the ratio of the input and output currents of the section[1]. The real and imaginary parts of P describe the attenuation and phase characteristics respectively. For sections of the type shown in Fig. 2, we find that

$$\cosh P = \frac{C_m + 2C_1}{2C_1}\left[L\left(\frac{C_1C_m}{C_m + 2C_1}\right)p^2 + R\left(\frac{C_1C_m}{C_m + 2C_1}\right)p + 1\right] \qquad (1)$$

where p is the complex frequency variable. Normalizing to put the signal frequency at one radian per second, and making some definitions to bring in useful parameters, this can be written for real frequencies as:

$$\cosh P = \cosh(\alpha + j\beta) = \frac{Q}{n}\left[1 - \omega^2 + j\frac{\omega}{Q}\right] \qquad (2)$$

where $j\omega = \frac{p}{\omega_o}$, $\omega_o^2 = \frac{1}{LC}$, $C = \frac{C_1C_m}{C_m + 2C_1}$

and defining $n = \frac{2}{\omega_o C_m R} = Q\frac{2C}{C_m}$ for $Q = \frac{\omega_o L}{R}$.

To obtain the insertion loss at the signal frequency this expression is evaluated at $\omega = 1$, giving

$$\cosh P_o = \cosh(\alpha_o + j\beta_o) = j\frac{1}{n} \qquad (3)$$

and

$$\sinh \alpha_o = \frac{1}{n} \text{ for } \alpha_o \text{ nepers} \qquad (4)$$

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[1] Shea, T. E., Transmission Networks and Wave Filters, pp. 76-79, D. Van Nostrand Company, Inc., New York.

The off-band loss can be approximated by

$$\alpha \cong \cosh^{-1} \frac{Q}{n} \left|1 - \omega^2\right| \text{ in nepers} \tag{5}$$

the approximation being satisfactory for off-band losses greater than three times the signal frequency loss.

DESIGN FOR MINIMUM INSERTION LOSS

In applications where the loss at the signal frequency must be kept as low as possible, the optimum design is that which gives the maximum for the ratio of loss at the frequency to be rejected to loss at the signal frequency. It is shown in Appendix 1 that the condition for this maximum is:

$$\alpha_o \coth \alpha_o = \alpha \tanh \alpha \tag{6}$$

This equation is awkward to use, but the information it provides is given in Fig. 4, which shows the value of α at the rejected frequency that results from the optimum design, plotted against the Q of the coils used and the relative frequency to be rejected. The graph shows that the off-band loss resulting from the optimum design is very nearly constant at 1.20 nepers per section, departing only a few percent from this value toward the limit of the approximation, which occurs only for low Q coils and very narrow filters. It is apparent that no significant error will be incurred by assuming that the most efficient filter is obtained by designing for a loss of 1.20 nepers per section at the frequency to be rejected.

With this parameter determined, the coupling parameter, n, can be determined from equation (5) and the signal frequency loss from (4). The difference between the midband loss, α_o, and the off-band loss, α, gives the rejection obtained per section, and from this and the specified total rejection required at the off-band frequency, the number of sections can be determined. For these calculations, it is convenient to rearrange (5) and put in the value of cosh 1.20, obtaining:

$$n = 0.552\, Q \left|1 - \omega^2\right| \tag{7}$$

Example - A filter for 2.5 Mc, to reject an image at 2.4 Mc by 126 db, built with coils having Q's of 300. For

$$Q = 300 \quad \text{and} \quad \omega = \frac{2.4}{2.5} ,$$

(7) gives an n of 12.99. From (4) the signal frequency loss is 0.077 nepers, giving a rejection of 1.123 nepers or 9.75 decibels per section. For 126 decibels rejection, 13 sections will be required, having a total signal frequency loss of 1.00 nepers or 8.686 decibels.

DESIGN FOR MINIMUM NUMBER OF SECTIONS

If the signal frequency loss that can be tolerated is somewhat greater than the loss given by the minimum loss filter, then the number of sections can be reduced, decreasing the cost and size of the unit. For a filter having the minimum number of sections, the ratio of required loss at the undesired frequency to the allowable loss at the signal frequency is to be precisely equal to the ratio of the per section loss at the undesired frequency to the per section loss at the signal frequency. For this condition

$$\frac{\text{required off-band loss}}{\text{allowable on-band loss}} = r = \frac{\cosh^{-1} \frac{Q}{n} \left| 1 - \omega^2 \right|}{\sinh^{-1} \frac{1}{n}} \tag{8}$$

in which the coupling parameter n required for these sections is the only unknown quantity. With the coupling parameter determined, the number of sections necessary can be determined from the given allowable loss and the loss per section, obtained from (4).

An explicit solution of (8) for the value of the coupling parameter required for these sections is not practical, but fortunately it is also not necessary; in general, the design as outlined above does not come out with an integral number of sections, so the exact solution seldom gives the final design. A more useful procedure is to determine the proper number of sections by using an approximate solution of (8) and round off the number of sections to the next highest integer. The number of sections and the given allowable loss then determine the α_o for the section.

A suitable approximation can be quickly obtained from an iterative process by solving for α_o rather than n, using the relationship (4). Designating the first assumed value of α_o by α_{01}, a correction, $\Delta\alpha_o$, can be obtained from:

$$\Delta\alpha_o = \frac{\alpha_{01}(r_1 - r)}{r_1 - \frac{1}{\alpha_{01}}} \tag{9}$$

where r_1 is the ratio obtained for the assumed value, α_{01}, and may be approximated by:

$$r_1 \cong \frac{1}{\alpha_{01}} \cosh^{-1}\left(\alpha_{01} Q \left[1 - \omega^2\right]\right) \tag{10}$$

Example: A filter for 2.5 Mc, to reject an image at 2.4 Mc by 126 db, having a signal loss of 12 db, using coils having Q's of 300.

The ratio of required loss at the image to allowable signal loss is

$$\frac{126 + 12}{12} = 11.50$$

For an assumed signal loss per section, α_{01}, of 0.2 nepers, the ratio obtained would be approximately 11.15. The first correction, $\Delta\alpha_o$, to the assumed value α_{01}, is -0.011, which is small enough that further iteration would be profitless. For $\alpha_o = 0.189$ nepers or 1.64 db per section, 12 db loss would be obtained with 7.3 sections. Using eight sections with a loss of 1.50 db or 0.173 nepers per section, (4) gives a coupling ratio of 5.76. From (5), the loss at the image frequency is 2.09 nepers, or 18.2 db per section; the total loss at the image is 145 db, giving 133 db rejection. Thus, 8 sections give more rejection than is required, but a smaller number would give too little, and 8 is the minimum that can be used.

APPENDIX: CONDITION FOR MINIMUM INSERTION LOSS

The necessary condition for the minimum insertion loss design is that the ratio of loss at the frequency to be rejected to loss at the signal frequency be a maximum, which means that the derivative of this ratio be zero. The derivative is conveniently taken with respect to the coupling parameter n. This is:

$$\frac{d\left(\frac{\alpha}{\alpha_o}\right)}{dn} = \frac{\alpha_o \frac{d\alpha}{dn} - \alpha \frac{d\alpha_o}{dn}}{\alpha_o^2}$$

and the condition for zero is

$$\frac{\alpha_o}{\frac{d\alpha_o}{dn}} = \frac{\alpha}{\frac{d\alpha}{dn}}$$

since

$$\alpha_o = \sinh^{-1}\frac{1}{n}$$

$$\frac{d\alpha_o}{dn} = -\frac{\alpha}{n}\,\frac{\sinh \alpha_o}{\cosh \alpha_o}$$

$$= \frac{-1}{n \coth \alpha_o}$$

and

$$\alpha = \cosh^{-1}\frac{\Omega}{n}\left|1 - \omega^2\right|$$

$$\frac{d\alpha}{dn} = \frac{-1}{n \tanh \alpha}$$

the condition for minimum insertion loss becomes:

$$\alpha_o \coth \alpha_o = \alpha \tanh \alpha$$

For low values of α_o, the quantity $\alpha_o \coth \alpha_o$ departs slowly from the limiting value of 1.00, reaching 1.05 at $\alpha_o = 0.4$ nepers per section. On the other hand, the quantity $\alpha \tanh \alpha$ has the value 1.00 at $\alpha = 1.20$ nepers, and varies relatively rapidly, reaching 1.05 at $\alpha = 1.24$ nepers.

TYPICAL PER-SECTION FREQUENCY RESPONSE CURVES FOR SECTIONS OF THE TYPE SHOWN IN FIGURE 2, USING A Q OF 300

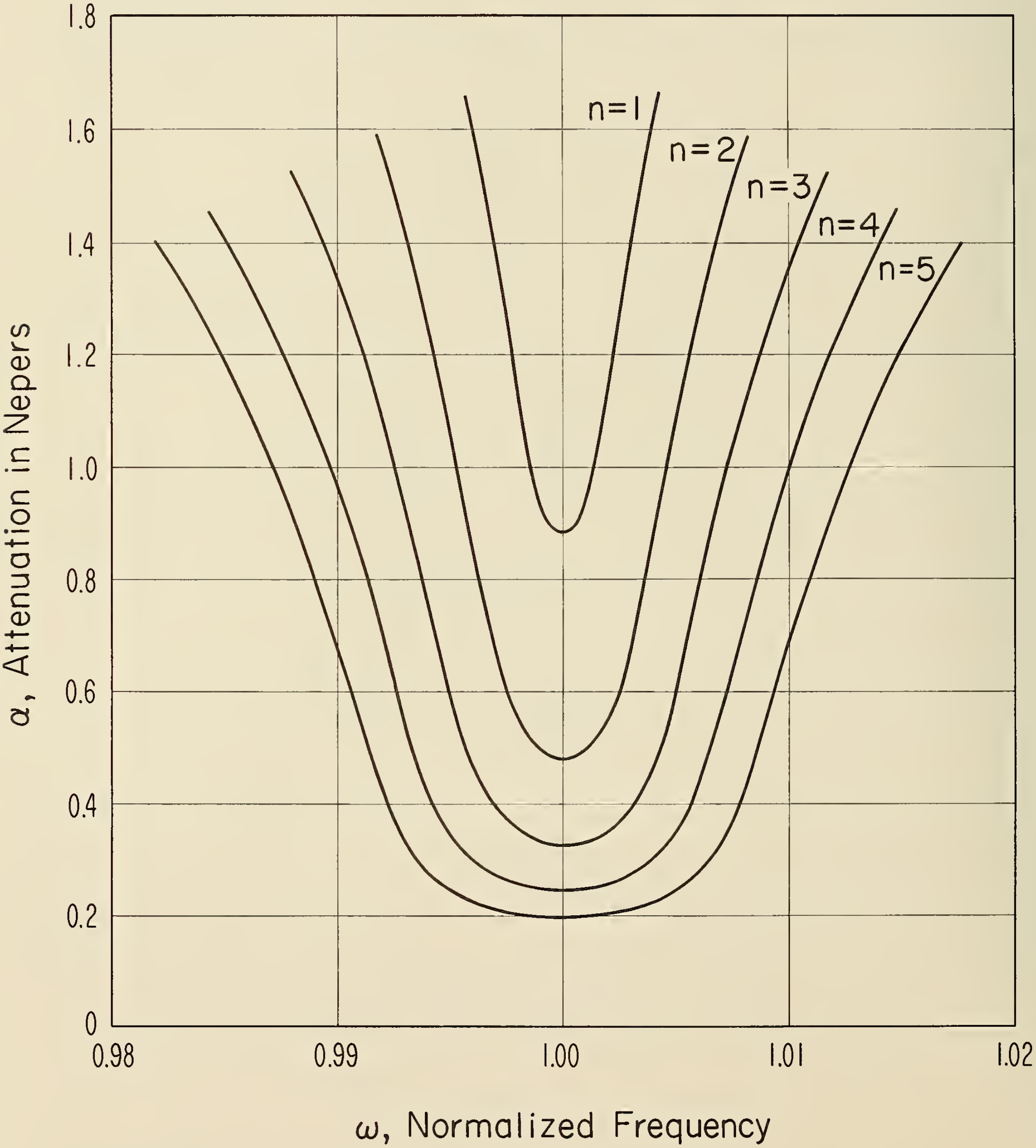


Figure 1

FILTER SECTIONS USED FOR CALCULATIONS

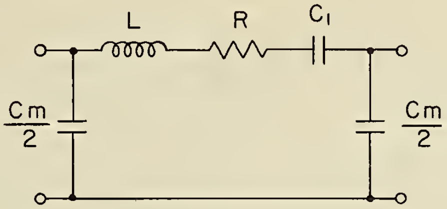
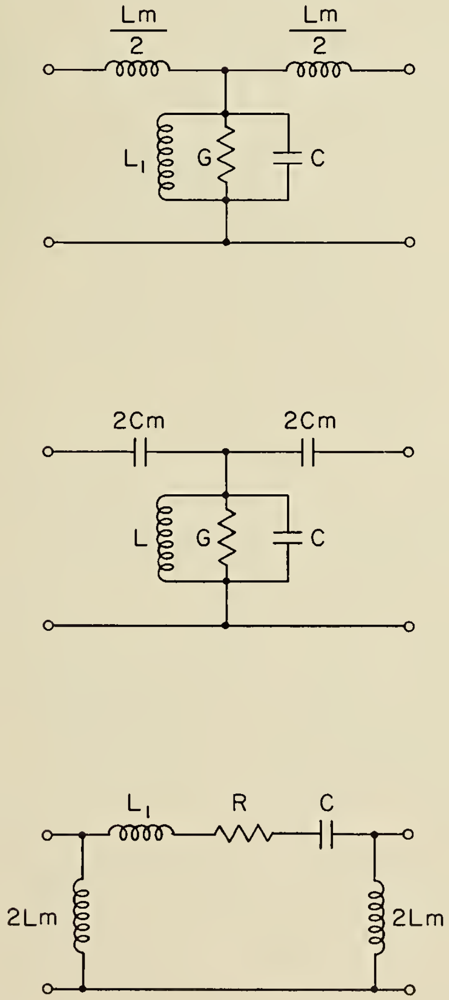


Figure 2

ALTERNATIVE CONFIGURATIONS FOR FILTER SECTIONS



$$L = \frac{L_1 Lm}{2L_1 + Lm}$$

$$Q = \frac{\omega_o C}{G}$$

$$n = Q\frac{Lm}{2L}$$

$$C = C_1 + 2Cm$$

$$Q = \frac{\omega_o C}{G}$$

$$n = Q\frac{2Cm}{C}$$

Note: frequency scale inverted; $\frac{\omega_o}{p} = j\omega$

$$L = L_1 + 2Lm$$

$$Q = \frac{\omega_o L}{R}$$

$$n = Q\frac{2Lm}{L}$$

Note: frequency scale inverted; $\frac{\omega_o}{p} = j\omega$

Figure 3

OPTIMUM OFF-BAND LOSS FOR MINIMUM INSERTION LOSS, AS A
FUNCTION OF Q AND THE NORMALIZED FREQUENCY ω TO BE REJECTED

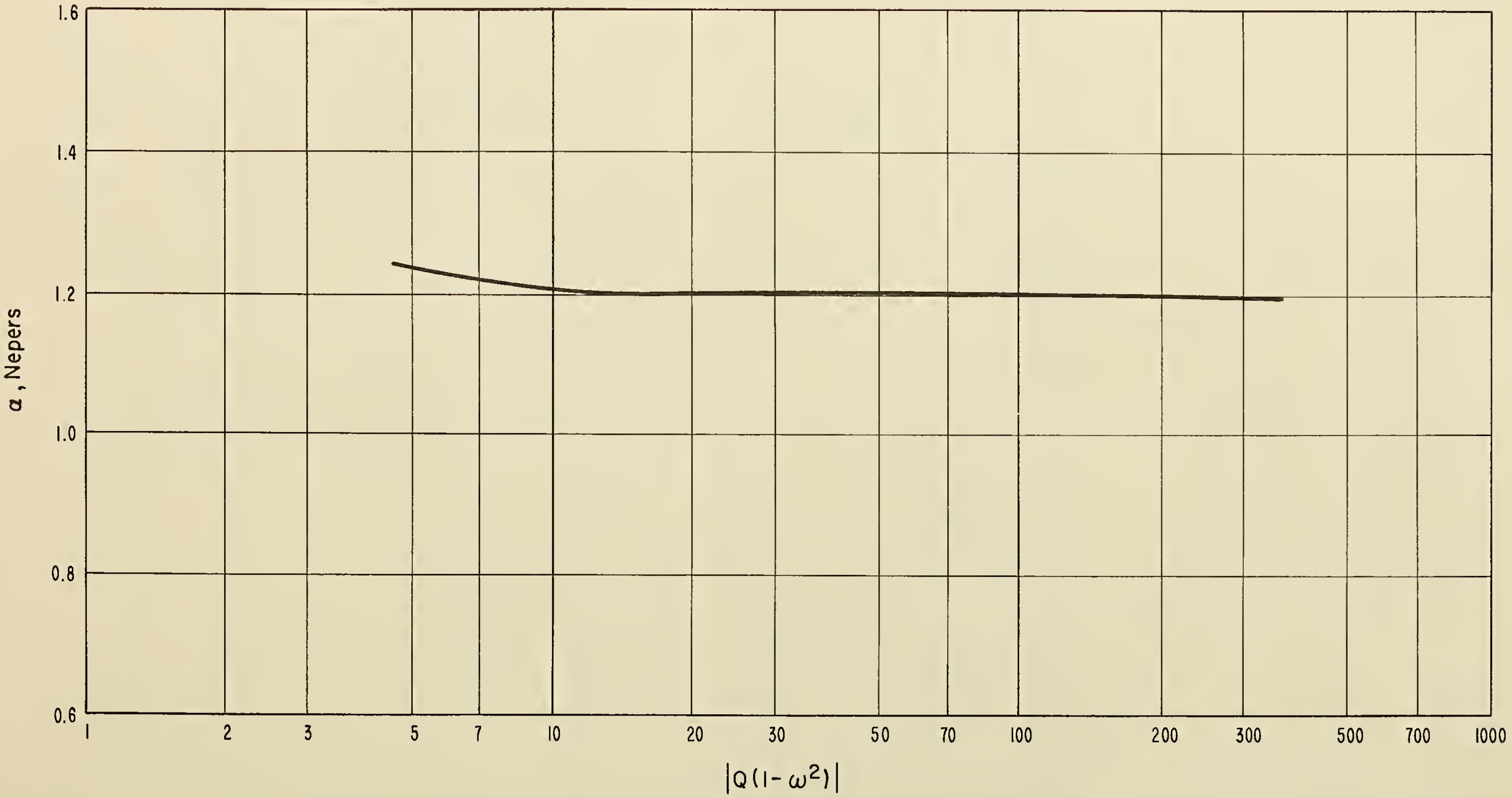


Figure 4

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