NBS

Technical Note 225

OPTICAL SCINTILLATION; A SURVEY OF THE LITERATURE

JURGEN R. MEYER-ARENDT AND CONSTANTINOS B. EMMANUEL

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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Table of Contents

Optical Scintillation;
A Survey of the Literature

Jurgen R. Meyer-Arendt and Constantinos B. Emmanuel

In this Technical Note, main emphasis is placed on providing the reader with an exhaustive survey of the literature, covering the field of effects of atmospheric refraction on the propagation of electromagnetic radiation at optical frequencies.

One may distinguish systematic, regular, or normal refraction on the one side from random refraction on the other. The former can be predicted theoretically, using various types of atmospheric models; the latter requires analysis by statistical methods.

Numerous observational, experimental, and theoretical aspects of random refraction, that is, of scintillation in its widest sense, are discussed. The following topics are dealt with: refraction in plane and in spherically stratified media, refractive index variations, radio refraction, scintillation as a function of aperture size, zenith distance, site location, dispersion, and meteorological conditions, the frequency spectrum of scintillation, terrestrial scintillation, image distortion and contrast reduction, refraction and diffraction theories of scintillation, autocorrelation analyses, radio star scintillation, and coherence problems. Questions of atmospheric scattering, absorption, and depolarization are excluded. Finally, a brief review is given concerning newer experimental methods for the observation, recording, and analysis of optical scintillation, including suggestions as to what further theoretical and experimental efforts should be undertaken.

1. Introduction

All refraction in the atmosphere is based on the fact that the atmosphere of the earth has an optical refractive index that is different from that of a vacuum and, furthermore, that the refractive index within the atmosphere also varies with space (and in part with time). This accounts for a variety of refractive phenomena which for a thorough understanding, as we will see later, require refraction as well as diffraction theory.[1]

We divide refractive phenomena occurring in the atmosphere into two large groups, depending on whether the effect is systematic or random (fig. 1). In a systematic effect we assume that the refractive index of the air not only changes as a function of altitude, but that it does so in a theoretically predictable fashion. Light coming from a distant source and reaching the observer inside the atmospheric envelope of the earth will then not propagate

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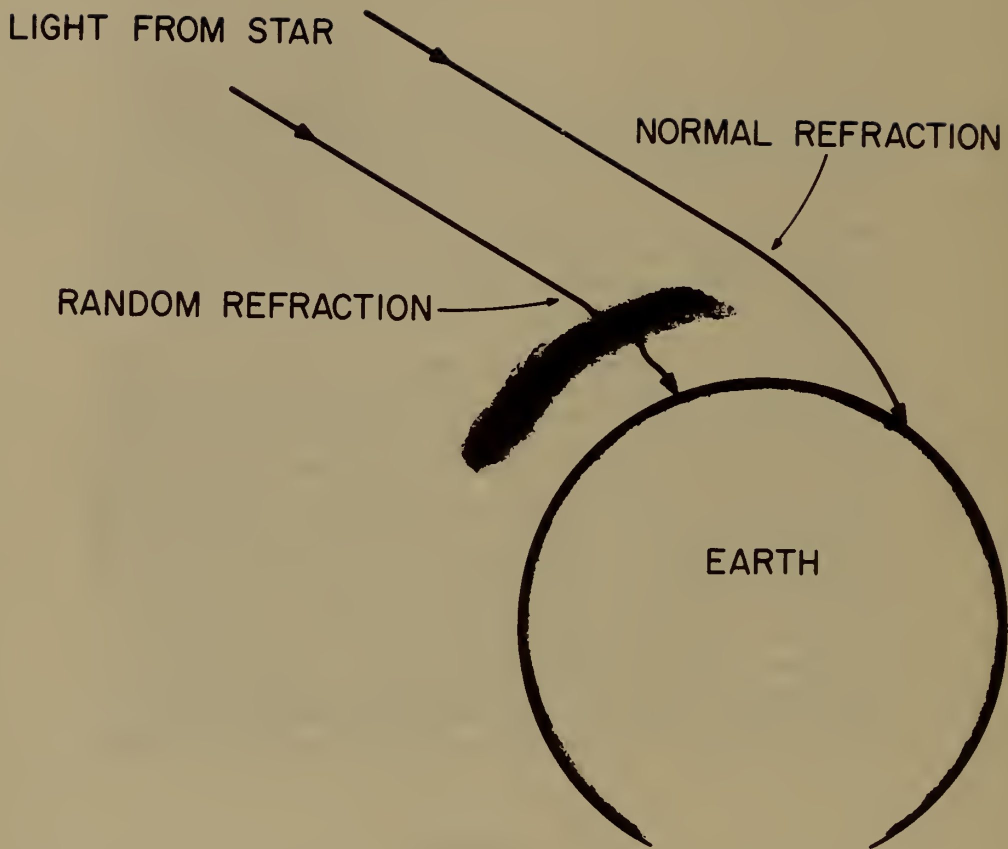

Figure 1

Normal refraction and random refraction of light passing through the atmosphere of the earth.

along a straight line, but in a curved path. This effect, called Regular Atmospheric Refraction, is seen in a number of optical phenomena. A mirage, or fata morgana, and looming are two common examples. Another example is the apparent flattening of the sun at sunrise or sunset, an effect that is observed easily from the ground, but seems to be much enhanced when seen from an orbiting satellite.[2]

In addition to such systematic refractive index variations and to the overall bending of light bundles, there are small scale, random changes present associated with turbulence. These are responsible for the (angular) scintillation of the stars, called twinkling. This is the effect of Random Refraction. Note, however, that the term "random" is being used here somewhat more generally than justified; if these inhomogeneities and the resulting optical effects were completely random, no further conclusions could be drawn, which, as we will see, is not the case.

2. Regular refraction

2.1. Refraction in a plane stratified medium

The refraction of light in a plane stratified medium has been discussed by numerous authors. We mention only Humphreys [1940], Wolter [1956], Stirton [1959], Newcomb [1960], Smart [1960], Wait [1962], and Chen [1964]. We follow, in part, Wolter [1956], and Baker, Meyer-Arendt, and Herrick [1963], and assume that the refractive index n varies in one dimension only, for instance in the vertical direction, so that $n = f(z)$. Consider the medium to be made up of a series of thin horizontal layers, the refractive index in each layer being constant and highest in the layer near the bottom. Let the index in the bottom layer be called n_1, as in figure 2a, n_2 in the next higher layer, n_3 in the third layer, and so on. If we call α_{12} the angle of incidence at the (1, 2) boundary and β_{12} the angle of refraction at the same boundary, we have for the transition from the bottom layer to the second layer

$$n_1 \sin \alpha_{12} = n_2 \sin \beta_{12} = n_2 \sin \alpha_{23} . \tag{1}$$

A similar equation holds for light traveling in the opposite direction. If the light enters the layered medium from an outside medium of index n_o and at an angle of incidence θ, then

$$n_o \sin \theta = n(z) \sin \alpha(z), \tag{2}$$

where z is the height above ground. For a given outside medium and a given initial angle of incidence, then,

$$n(z) \sin \alpha(z) = \text{constant}, \tag{3}$$

2. As reported in 1963 by astronaut L. G. Cooper after his 22-orbit mission.

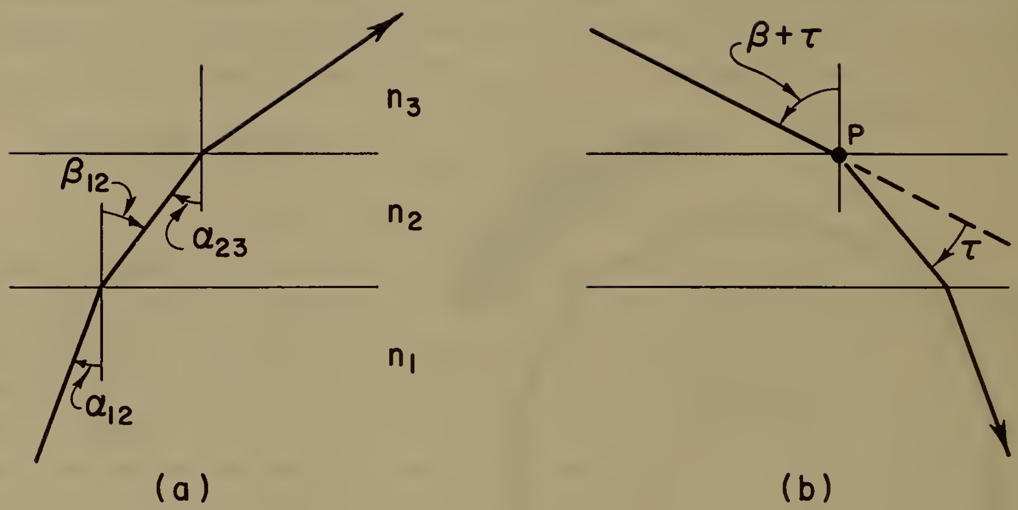

Figure 2

Refraction of light at a boundary.

which means that, if n changes continually with height and monotonically, α will change as well and the light bundle will proceed in a curved path. The shape of this path is a parabola and, at least for linear variations, similar to the path of a projectile in a gravitational field, a similarity which is by no means accidental. In the atmosphere of the earth, the refractive index n decreases with height; therefore, the gradient vector ∇n points downward, toward the center of the earth, and light from a star or other celestial object is bent concavely toward the earth.

Suppose the medium consists of an indefinite number of infinitely thin strata, each of thickness dz. Let P be the point at which a ray intersects the boundary between any two consecutive strata (fig. 2 b). The angle of incidence at P is called $\beta + \tau$, β is the angle of refraction (not shown), and τ is the change of direction which the ray undergoes while passing across the boundary. If we call n and $n + \Delta n$ the indices of refraction of the two boundary layers, we have

$$(n + \Delta n) \sin (\beta + \tau) = n \sin \beta,$$

so that τ, the angle of deflection or schlieren angle, is given as

$$\tau = \frac{\Delta n}{n} \tan \beta . \qquad (4)$$

We now assume that all deviations are small so that the light path approximates part of a circle. The schlieren angle τ and the radius of curvature r of the light beam can then be found from the following considerations: Assume that the light bundle is entering the medium horizontally as shown in figure 3 and that it has a width Δz. The refractive index of the medium decreases in the + z direction, but it does not change in the direction of propagation of the light. Wavefronts in the light bundle are represented by dashed lines. If there are m wavefronts inside the area shown, the arc representing the upper boundary will be

$$\Delta s = m \lambda$$

and the lower boundary will be

$$\Delta s' = m \lambda' .$$

If λ_o is the original wavelength of the light and n and n' are the refractive indices of the medium at two points on the z axis, the lengths of the arcs limiting the field are given by

$$\Delta s = m \lambda = m \frac{\lambda_o}{n} \qquad (5)$$

and

$$\Delta s' = m \lambda' = m \frac{\lambda_o}{n'} = \Delta s \frac{n}{n'} . \qquad (6)$$

The lengths of these arcs depend on the radius of curvature r and on the schlieren angle τ by

$$\Delta s = r \tau \qquad (7)$$

and

$$\Delta s' = (r - \Delta z) \tau . \qquad (8)$$

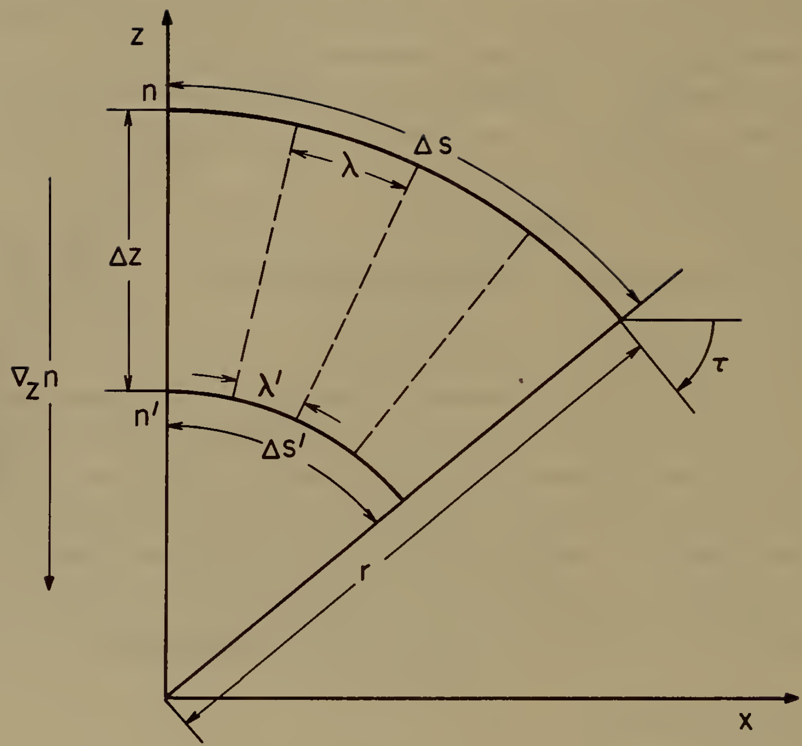

Figure 3

Deflection of light, entering from the left and passing through a stratified medium.

Subtraction gives

$$\Delta s - \Delta s' = \tau \Delta z .$$

Since

$$\Delta s - \Delta s' = m \frac{\lambda_o}{n} - m \frac{\lambda_o}{n'} = m \lambda_o \frac{n' - n}{n n'}$$

and since

$$m \lambda_o = n \Delta s,$$

$$\Delta s - \Delta s' = n \Delta s \left(\frac{n' - n}{n n'} \right) = \left(1 - \frac{n}{n'} \right) \Delta s, \tag{9}$$

so that

$$\tau = \frac{1}{\Delta z} \left(1 - \frac{n}{n'} \right) \Delta s ,$$

$$= \frac{n' - n}{\Delta z\, n'} \Delta s ,$$

$$= \frac{\Delta n}{\Delta z} \frac{1}{n'} \Delta s . \tag{10}$$

For infinitesimally small changes of n, we have

$$\tau = \frac{1}{n} \nabla_z n \, \Delta s . \tag{11}$$

The radius of curvature r of the bent light bundle is obtained by combining (6) and (8) so that

$$\Delta s \frac{n}{n'} = (r - \Delta z) \, \tau .$$

Replacing Δs by (7) gives

$$r \tau \frac{n}{n'} = (r - \Delta z) \, \tau ,$$

and canceling τ and solving for r yields

$$r = \frac{\Delta z}{1 - \frac{n}{n'}} . \tag{12}$$

Again, for infinitesimally small changes of n, we have

$$r = n \frac{1}{\nabla_z n} . \tag{13}$$

If we denote by s the length measured along the (curved) path, then the radius of curvature is given by the differential equation

$$\frac{1}{r} = \frac{\tau}{ds} , \tag{14}$$

where ds is the element of length of the ray. The path of light in a nonhomogeneous medium can also be derived from Fermat's principle. For details see the Appendix of Williams' report no. 3 [1954 a].

We have, so far, followed a two-dimensional convention. Naturally, a light bundle has depth and propagates in three-dimensional space, so that the optical path gradients describing the schliere can point in any arbitraty direction in space. We consider separately the x, y, and z components of any refractive gradient vector ∇n. These are given as

$$\nabla_x n = \hat{x} \frac{\partial n}{\partial x}; \quad \nabla_y n = \hat{y} \frac{\partial n}{\partial y}; \quad \nabla_z n = \hat{z} \frac{\partial n}{\partial z} . \tag{15}$$

If a line is drawn through the gradient field so that along this line the rate of change of the refractive index, dn/dL, is greatest, then the direction of dL must clearly lie in direction of ∇n. The numerical value of dn/dL will then be equal to $| \nabla n |$,

$$\left(\frac{dn}{dL}\right)_{max} = | \nabla n | .$$

If we choose instead to move in a direction perpendicular to ∇n, the cosine of the angle subtended by the direction of motion and the gradient vector will be zero and we find that $dn/dL = 0$ or $n = \text{constant}$.

For any arbitrary orientation in three-dimensional space, the schlieren angle will be, following (11),

$$\begin{aligned} | \tau | &= \frac{1}{n} \left(\hat{x} \frac{\partial n}{\partial x} + \hat{y} \frac{\partial n}{\partial y} + \hat{z} \frac{\partial n}{\partial z} \right) \cdot \Delta s \\ &= \frac{1}{n} \nabla n \cdot \Delta s . \end{aligned} \tag{16}$$

For light propagating along the x axis, it is sufficient to resolve τ into two components, τ_y and τ_z. These are covariant; they can be obtained by rotating the coordinate system about the x axis. Therefore,

$$\tau_y = \frac{1}{n} \nabla_y n \Delta s; \quad \tau_z = \frac{1}{n} \nabla_z n \Delta s . \tag{17}$$

These two component angles are found, following Wolter [1956], to a first approximation by integration over the thickness x_o of the medium:

$$\tau_y = \int_0^{x_o} \frac{1}{n} \frac{\partial n}{\partial y} dx_o$$

and (18)

$$\tau_z = \int_0^{x_o} \frac{1}{n} \frac{\partial n}{\partial z} dx_o .$$

2.2. Refraction in a spherically stratified medium

At larger zenith distances, the atmosphere can no longer be considered stratified in plane layers. Spherically stratified media, in general, may be either of rotational symmetry with respect to one plane, like a cylinder, or they may be rotationally symmetric to any plane, such as a sphere.

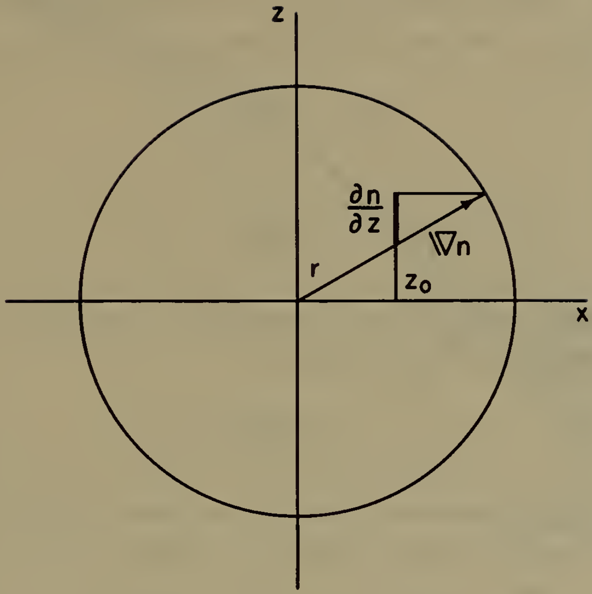

Figure 4

Refractive gradient in a field of rotational symmetry.

We restrict our discussion to media in which the refractive index is highest at the center, gradually and monotonically decreasing toward the periphery and to true refraction phenomena. That is, we exclude scatter, which, among other causes, may also contribute to the bending of a light beam. For further details on the theory of propagation of electromagnetic waves through stratified media of rotational symmetry see Baker [1927], Sweer [1938], Humphreys [1940], Brocks [1949], Liepmann [1952], Hanson [1953, 1958], Sommerfeld [1954], Williams [1954a], Stirton [1959], Newcomb [1960], Smart [1960], Mahan [1962], Wait [1962], and Lawrence, Little, and Chivers [1964].

Let us assume, as stated before, that the refractive index is linearly and inversely proportional to the radius, $n \propto 1/r$, so that the refractive index gradient ∇n will coincide with any one radius. Further assume that the field is rotationally symmetrical with respect to the y axis (fig. 4). Thus, the inhomogeneity has the shape of a cylinder and has refractive indices which are a function of the radius only so that $n = n(r)$. Since the cylinder axis lies in the y-direction,

$$r = \sqrt{x^2 + z^2}$$

and

$$\frac{\partial n}{\partial z} = \nabla_z n = \frac{dn}{dr} \cdot \frac{z}{r} \, . \tag{19}$$

For a light ray passing through the medium at a distance z from the cylinder axis, the schlieren angle, following Wolter [1956], will be

$$\tau_z = z \int \frac{1}{n} \frac{dn}{dr} \frac{1}{r} \, dx \, . \tag{20}$$

In a field of spherical rotational symmetry, all points of equal distance from a point of origin have the same refractive index. Therefore, rotational symmetry not only exists for the y axis, but for the x and z axes as well. Hence, any cross section through the center contains the characteristics of the whole field. The evaluation of just one cross-section, therefore, suffices and is representative of the whole field.

We had mentioned before the equation of a curved ray in a field of non-uniform and monotonically changing refractivity

$$r \, n \, \sin \alpha = \text{constant}, \tag{21}$$

where α is the angle subtended by the refractive gradient vector ∇n and the tangent at a point P on the ray. Depending on the variations of n, the ray can follow a variety of trajectories. Since

$$|\nabla n| \sin \alpha = s,$$

where s is the distance from the center of the sphere to the tangent, (21) can be written as

$$r \, n \, \sin \alpha = n \, s = \text{constant}, \tag{22}$$

which is called Bouguer's formula.

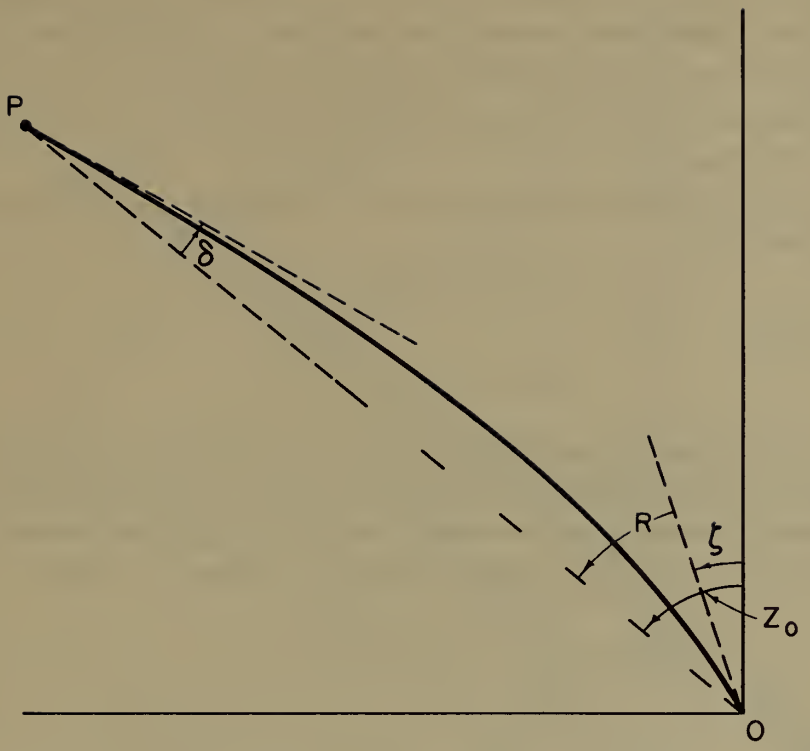

Figure 5

Apparent (ζ) and true zenith distance (Z_o) of an object at finite distance from the earth. Note definition of angle δ.

Let us consider in more detail the refraction of light through the atmosphere, taking into account only large-scale variations.[3] We assume first, that the atmosphere is made up of thin spherical layers which are concentric to the earth's surface. Let Z_o denote the angle of incidence, at the uppermost boundary of the atmosphere (the fictitious boundary beyond which no significant refraction takes place), of a ray from a star that finally reaches an observer at the surface of the earth. Then Z_o is called the true zenith distance of the star, the true zenith distance being defined as the angle which light from the star makes with the vertical of the observer's station before it enters the atmosphere. If there were no atmospheric refraction, the object (here at a finite distance) would be seen by the observer in the direction OP (fig. 5).

If n_o is the refractive index in the uppermost layer and n_G the index near the surface of the earth, then

$$n_o \sin Z_o = n_G \sin \zeta .$$

But $n_o = 1$, so that

$$\sin Z_o = n_G \sin \zeta , \tag{23}$$

where ζ is called the observed zenith distance. The observed or apparent zenith distance of a celestial object is that of a light ray coming from the object when it reaches the observer. Note that the observed zenith distance is always less than the true zenith distance. The angle $Z_o - \zeta$ is called terrestrial or atmospheric refraction,[4] denoted R. Then

$$\sin(\zeta + R) = n_G \sin \zeta$$

or

$$\sin \zeta \cos R + \cos \zeta \sin R = n_G \sin \zeta .$$

Now R is a small angle and we can write $\cos R = 1$ and $\sin R = R$ (R being expressed in radians). Thus

$$\sin \zeta + R \cos \zeta = n_G \sin \zeta$$

or

$$R = (n_G - 1) \tan \zeta . \tag{24}$$

---

3. Small-scale variations, which may also be called micro-structure of the atmosphere, are considered those which are due to random variations of the refractive index. These will be discussed later.
4. Since this effect is by no means restricted to the planet earth, the term "atmospheric" is preferred. Note also that calling R "angle of refraction" is misleading since this term, in geometric optics, is commonly assigned to the angle subtended by a refracted ray and the normal to the boundary at the point of refraction.

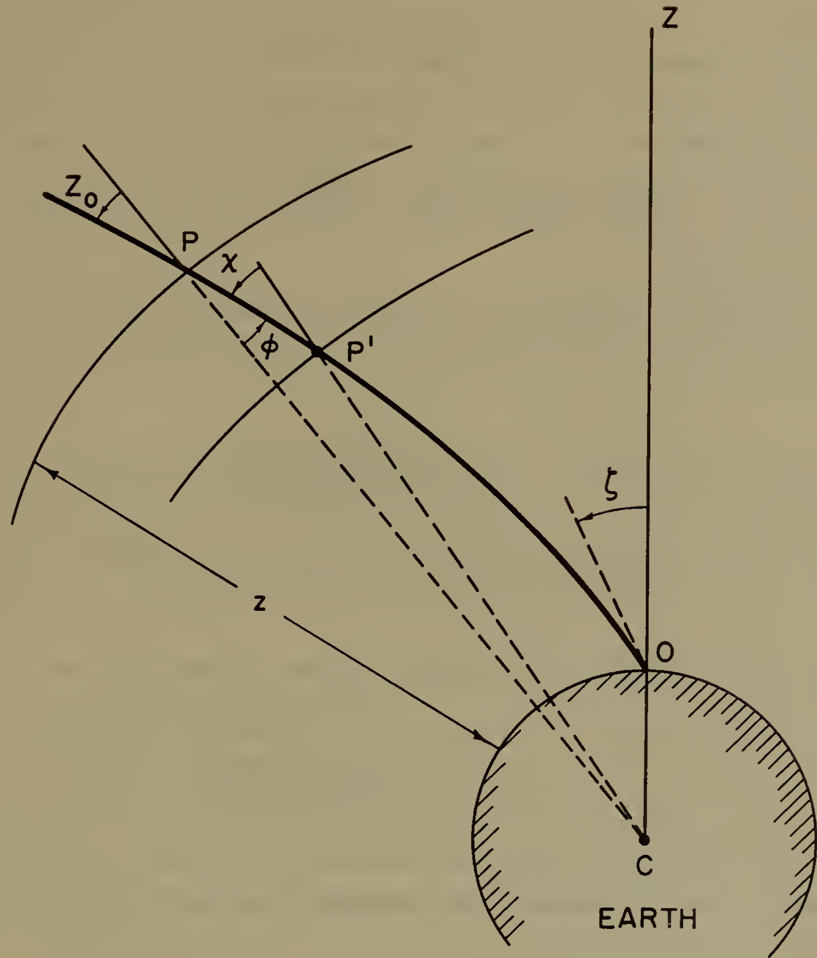

Figure 6

Refraction of light in a spherical model.

We see, thus, that at least near the zenith, the atmospheric refraction is proportional to the tangent of the observed zenith distance. For an apparent zenith distance of $\zeta = 45°$, term R is called the "constant of mean atmospheric refraction", which has an approximate value of

$$R \approx 58''.2 \tan \zeta .$$

Bouguer's formula describes, at least approximately, the paths of light rays and radio waves in the atmosphere. On the horizon the deviation of light due to atmospheric refraction amounts to about half a degree, the angular diameter of the sun. Thus at sunset, the sun is seen still above the horizon when, in the absence of atmospheric refraction it would have just disappeared below it. Another example of a rotationally symmetrical non-uniform medium is the lens in the human eye the refractive index of which is higher at the center than at the periphery.

If (r, θ) are the polar coordinates of a plane curve, then angle Z_o between the radius vector at a point P on the curve and the tangent to the light ray at P (fig. 6) is given by

$$\sin Z_o = \frac{r(\theta)}{\sqrt{r^2(\theta) + \left(\frac{dr}{d\theta}\right)^2}} . \tag{25}$$

It follows that

$$\frac{dr}{d\theta} = \frac{r}{k} \sqrt{n^2 r^2 - k^2} , \tag{26}$$

where k is a constant. The equation of rays in a medium of spherical symmetry may therefore be written in the form

$$\theta = k \int^{r} \frac{dr}{r\sqrt{n^2 r^2 - k^2}} . \tag{27}$$

Smart [1931] has shown that if the refractive index of the atmosphere is a function only of the height z above a spherical earth, then Snell's law can be adapted to a spherically stratified atmosphere and be written as

$$n\, r \sin Z_o = n_G\, r_o \cos \varepsilon . \tag{28}$$

Here n is the refractive index at distance r from the center of the earth, n_G is the refractive index at the ground at a distance r_o from the center of the earth, ε is the elevation angle, and Z_o is the angle subtended by the radius vector and the tangent to the light ray, shown before in figure 6.

Let C in figure 6 be the center of the earth, O the observer, and OZ the direction of his zenith. Light coming from a star will assume a curved path and will lie in a plane determined by the incident ray and the normal at point P to the surface of the earth.

The true zenith distance Z_o of the star at P follows from Snell's law as

$$n_a \sin Z_o = n_b \sin \phi , \tag{29}$$

where n_a and n_b are the refractive indices just outside and inside the spherical shell given by point P. In the triangle CP'P, we see from the law of sines that

$$\frac{\sin \phi}{\widehat{CP'}} = \frac{\sin \chi}{\widehat{CP}} .$$

Combining these last two equations gives

$$\sin \phi = \frac{n_a}{n_b} \sin Z_o = \frac{\sin \chi}{\widehat{CP}} \widehat{CP'} ,$$

so that

$$n_a \widehat{CP} \sin Z_o = n_b \widehat{CP'} \sin \chi$$

$$= n_G r_o \sin \zeta , \tag{30}$$

where r_o is the radius of the earth. This means that, for any rotationally symmetrical system, the product of the refractive index, the radius of curvature of the layer boundary, and the sine of the angle of incidence is a constant for each of the (hypothetical and infinitesimally thin) layers. Differentiation, following Valentiner [1901] and Mahan [1962], gives

$$d Z_o = - \tan Z_o \frac{dn}{n} - \tan Z_o \frac{dr}{r} . \tag{31}$$

The increase in the angle of incidence Z_o in going from one layer boundary to the next higher boundary is then made up of a positive part due to the deviation produced by the atmospheric refraction and a negative part due to the rotation of the radius in going from one surface to a higher surface. If we now impose limits corresponding to the complete ray path from the earth out to the star, we see that the atmospheric refraction R at point P introduced by the atmosphere between the star and the observer is

$$R = Z_o - \zeta$$

$$= - \delta + \int_1^{n_G} \frac{n_G r_o \sin \zeta}{\sqrt{n_P^2 (r_o + z)^2 - n_G^2 r_o^2 \sin^2 \zeta}} \frac{dn}{n} . \tag{32}$$

This equation is the refraction integral in its most general form, in which the object and the observer can be at any arbitrary height z above the surface of the earth. For any arbitrary apparent zenith distance ζ, the atmospheric refraction R depends on the refractive index n_G at the position of the observer, on how the refractive index changes with the height above the earth, on the refractive index n_P at position P, and on angle δ (fig. 5) which represents the angular distance between the apparent and the true direction of point O on the earth as seen from the star.

The problem is much simplified if the celestial object is at a distance much larger than the radius of the earth so that the refractive index in the vicinity of the object is unity and if $\delta << 1°$. Then

$$R = \int_1^{n_G} \frac{n_G\, r_o \sin \zeta}{n_P^2 (r_o + z)^2 - n_G^2\, r_o^2 \sin^2 \zeta} \frac{dn}{n} . \tag{33}$$

This is the general refraction integral derived independently by Newton, Bouguer, and Simpson [Mahan, 1962]. This form of the refraction integral has served as the fundamental equation from which essentially all the theories of astronomical refraction have been derived. In order to determine R for a given apparent zenith distance ζ, one needs to know only the refractive index at the position of the observer and how it changes with height or, more precisely, with pressure, density, temperature, and relative humidity of the air. These changes and related problems will be discussed in the next Chapter.

2.3. Optical refractive index and refractive index variations

The refractive index in a vacuum, by convention, is set equal to unity. All material media, then, independent of whether they are solids, liquids, or gases, will have indices of refraction greater than unity.

According to elementary physics, the velocity v at which monochromatic light is propagated in an homogeneous, isotropic, nonconducting medium is

$$v = \frac{c}{n} , \tag{34}$$

where c is the velocity of light in free space and n is the index of refraction of the medium.

For a gas, the refractive index is proportional to the density ρ of the gas. This is expressed by the Gladstone-Dale relation

$$n - 1 = k \rho \tag{35}$$

where k is a constant, depending on the wavelength of the light. Another relationship is given by the Lorentz-Lorenz formula,

$$\frac{n^2 - 1}{(n^2 + 2) \rho} = \text{constant} . \tag{36}$$

The refractive index also depends on the pressure and temperature of the gas:

$$n = 1 + k_2 \frac{p}{T} . \tag{37}$$

Here k_2 is another constant.

For convenience, frequently a scaled-up value called refractivity or refractive modulus, denoted by N, is used:

$$N = (n - 1) \times 10^6 . \tag{38}$$

For a vacuum, $N = 0$. At optical wavelengths,

$$N \approx 79 \frac{p}{T} , \tag{39A}$$

where the pressure p is given in millibar and the temperature T in °K. In units of millimeter[5], H_g, we have

$$N \approx 105 \frac{p}{T} . \tag{39B}$$

Barrell and Sears [1940] have deduced the relation

$$n - 1 = k_3 \frac{p(1 + \beta_T\ p)}{1 + \alpha\ T} , \tag{40}$$

where k_3 is a constant that depends on the wavelength, β_T depends on the compressibility of the air, and α is the theoretical expansion constant for a perfect gas ($\alpha = 0.003661$). The value of β_T was found to depend linearly on T for temperatures from at least 10° to 30° C.

The local refractive index gradient is proportional to a coefficient κ, which in turn, following Brocks [1950], is given by

$$\kappa = 5.03 \frac{p}{T^2} \left(3.42 + \frac{\partial T}{\partial z} \right) \sin \zeta = \kappa' \sin \zeta . \tag{41}$$

The value of 5.03 is a constant which depends on altitude, latitude, and time and 3.42 is the relation between the gravitational constant and the gas constant (and a function of location), p is the air pressure in millibar, T the temperature in °K, $\partial T/\partial z$ is the vertical air temperature gradient measured in °C/100 meter and considered negative when decreasing with increasing height, and ζ is the observed zenith distance.

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5. In this Technical Note, most dimensions will be given in metric units.

1 meter	= 39.3700 inch	= 3.281 ft.
1 kilometer	= 3281 ft.	= 0.62137 mile
1 ft.	= 0.30480 m.	
1 mile	= 1.60935 km.	

For other values, consult a conversion table.

Near the ground, κ depends primarily on $\partial T/\partial z$. It vanishes for a temperature gradient of $-3.42°C/100$ m [Brocks, 1950]. For higher values, κ becomes negative, that is, the light ray will be bent concavely upward, opposite to the curvature of the surface of the earth. For temperature gradients less than $-3.42°C/100$ m and for temperatures which increase with height, the refraction coefficient becomes positive and the light will be bent in the same sense as the earth's surface.

In more general terms, we have, following Brocks [1952],

$$\kappa = \frac{r_o}{n} \frac{\partial n}{\partial z} \sin \zeta , \tag{42}$$

where r_o is the radius of the earth and ζ the zenith distance. For nearly horizontal rays, we have

$$\kappa = c_1 \frac{p}{1000} + c_2 \frac{p}{1000} \frac{\partial T}{\partial z} + c_3 \frac{\partial e}{\partial z} , \tag{43}$$

where e is the humidity, or partial water vapor pressure, of the gas. The constants c_1, c_2, and c_3 depend primarily on the air temperature, and only to a much smaller degree do they depend on the vapor pressure.

The connection between variations of refractive index and temperature is given by a simple formula, following van Isacker [1954], which relates the standard deviations σ_n of the refractive index and of the temperature σ_T, at the 10 km level, by

$$\sigma_n = \frac{6.8 \times 10^{-5} p}{T^2} \sigma_T \approx 4 \times 10^{-7} \sigma_T .$$

<u>Humidity</u>, in the range of optical frequencies, has a rather small influence on the density and the refractive index of air. At radio frequencies, this influence is much greater and, in fact, forms the basis of water vapor determinations by radio refractometry.

In terms of pressure, temperature, and humidity, the density ρ of a gas is given by

$$\rho = \frac{P p}{T Q} \left(1 + 0.377 \frac{e}{p}\right) \tag{44}$$

where Q is the gas constant. The refractive index, following Kazansky [1959], is

$$n = 1 + 1.0485 \times 10^{-6} \frac{p}{T} \left(1 - 0.132 \frac{e}{p}\right), \tag{45A}$$

where p and e are in mm Hg. In units of mb, we have

$$n = 1 + 0.787 \times 10^{-6} \frac{p}{T} \left(1 - 0.132 \frac{e}{p}\right) . \tag{45B}$$

Humidity affects the refractive index in two ways: indirectly, by way of density as just shown, and directly, due to the difference in the light refracting properties of dry air and water vapor.

In general, the partial pressure of water vapor is small. At 20°C, the water vapor pressure is 17.5 mm Hg. With a relative humidity of 50 per cent, the partial vapor pressure is 8.8 mm Hg or about 1 per cent of the total pressure at standard atmospheric pressure, a value which would require a correction of the refractivity by 0.12 per cent. At high moisture contents, the error will be larger and could mean, for geodetic purposes, errors of the order of several seconds of arc. Techniques of humidity determinations have been described by Westwater [1946], Elsner [1955], and Gerrard [1961]. Gutnick [1962] has listed the mean atmospheric moisture profiles up to an altitude of 31 km.

The relative importance of temperature and water vapor pressure for the refractive index of air can be obtained from a ratio of the partial derivatives of the index of refraction with respect to each of these two variables. Calculation shows that a 1°C change in temperature will produce a change in refractive index approximately equal to the change produced by a 28-mb change in vapor pressure.

The composition of the air, likewise, has a very small effect on its refractive properties. Williams [1953], using refractive index values from the Handbook of Chemistry and Physics, obtained a variance of about 2×10^{-16}, so that the standard deviation of the refractive index is about 1.4×10^{-8}. Carbon dioxide is perhaps the only compound that might contribute. The CO_2 content of the atmosphere varies as a function of time and location. The yearly increase in refractive index, due to an increase in CO_2 content, is of the order of 10^{-10}. The CO_2 content over land is higher than over ocean areas, increasing the refractive index by about 6×10^{-8}. Ozone also has some influence. All of these variations, however, are so small that usually they can be neglected. A detailed discussion of the composition of the atmosphere is found in the Pittsburgh University reports [1952-53].

Essen [1953] has determined the refractive indices of water vapor, air, O_2, N_2, H_2, D_2, and He. A complex formula, containing not only the wavelength but also temperature, atmospheric pressure, and concentration of water vapor and of carbon dioxide, has been given by Masui [1957].

The best value used currently for the refractive index of air at $p = 760$ mm, $T = 0°C$, and $\lambda = 5455$ Å (about the center of the visible spectrum) is, according to Stirton [1959],

$$n = 1 + 292.44 \times 10^{-6} . \qquad (46)$$

The preceding discussion has shown that the atmosphere has a variable refractive index which is a function of space (x, y, z) and time (t), so that

$$n = \bar{n} + \Delta n \; (x, y, z, t) . \qquad (47)$$

At optical wavelengths, the fluctuating part $\Delta n \,(x, y, z, t)$ is of the order of 10^{-6} for temperature changes of $\Delta T = 1°C$. The average index $\bar{n}$ also is not constant, but varies slowly depending on the prevalent meteorological conditions. In contrast, Δn varies rapidly.

These small-scale fluctuations are the ones which are responsible for scintillation. They will be discussed in detail later in this Technical Note.

The term dispersion denotes the fact that the magnitude of the refraction of light is a function of wavelength. According to Barrell and Sears [1940],[6] and Williams [1953], the refractive index n of air for monochromatic light of frequency ν is given by

$$\frac{1}{\rho} \frac{n^2 - 1}{n^2 + 2} = \sum_i \frac{k_i}{\nu_i^2 - \nu^2}, \quad (48)$$

where ρ is the density of the air, the ν_i are the resonance frequencies, and the k_i are constants. This equation is similar to the general dispersion equation as it occurs in the Lorentz theory of electrons and is valid for frequencies not too close to the resonance frequencies of the various component gases in air. These frequencies are marked by absorption bands in the air spectrum.

Considerable effort has been made to approximate the factual refractive properties of the atmosphere by theoretical models. In about all cases the model atmosphere is assumed to be spherically stratified. A spherical model derived by Lord Rayleigh as early as 1893 is satisfactory for many purposes but does not account for very small elevation angles (for details see Humphreys [1940]). Harzer [1922-24] has developed a theory of astronomical refraction which for the first time was based on meteorological measurements. The main point in Harzer's theory is that, if one knew the average temperature and the partial pressure of each of the gases including water vapor at all levels in the atmosphere, one can compute the refractive index as a function of height throughout the atmosphere. Harzer did this, through 61 assumed boundary layers, up to a height of 84 km, beyond which the deviations were reduced to less than 1/1000 of a second of arc.

Further refinements were made by Wünschmann [1931], who compared these computational data with results derived from other theories. Wünschmann also took into account large temperature gradients over cities and deviations due to climatic and other meteorological factors.

Certain atmospheric models are based on the concept of the equivalent or effective earth radius which has been found useful in radio propagation. In others, bending of the rays is compensated for by assuming a distorted space where the light travels along straight lines but where the coordinates have been modified.

Extensive work on theoretical models of the atmosphere which cannot be reviewed here in more detail has been carried out by Willis [1941], who has computed astronomical refraction, taking into account pressure, humidity, gravity, height, wavelength, and other

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6. Socher [1951] has recalculated, and given some corrections of, the refraction constants for astronomical use published by Barrell and Sears in 1939.

Table I.

Astronomical Refraction (from Dietze 1957).

Apparent elevation angle	Astronomical refraction	Apparent elevation angle	Astronomical refraction
2°	18.2'	10°	5.3'
2.5°	16.0'	11°	4.8'
3°	14.3'	12°	4.6'
3.5°	12.8'	14°	3.8'
4°	11.6'	16°	3.3'
4.5°	10.7'	18°	2.9'
5°	9.8'	20°	2.6'
5.5°	9.0'	25°	2.0'
6°	8.4'	30°	1.7'
6.5°	7.8'	40°	1.2'
7°	7.3'	50°	0.8'
7.5°	6.9'	60°	0.6'
8°	6.5'	70°	0.4'
8.5°	6.1'	80°	0.2'
9°	5.8'	90°	0.0'
9.5°	5.5'		

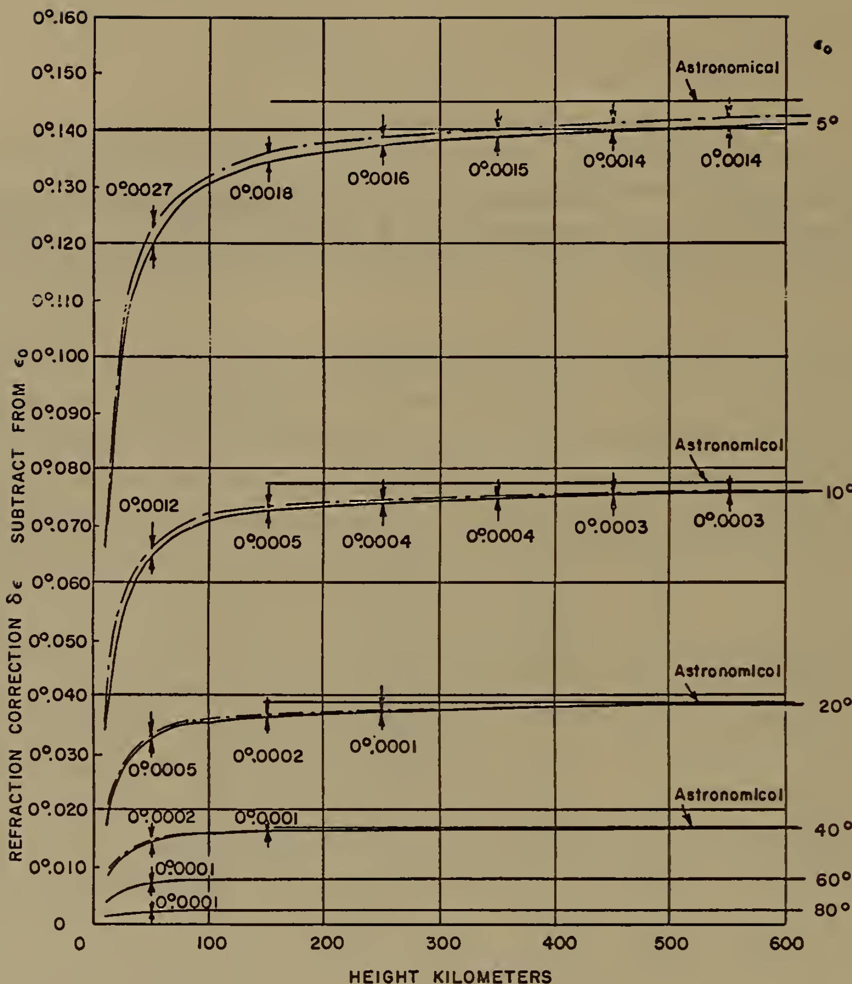

Figure 7

Refraction corrections $\delta\varepsilon$ as functions of height for various apparent elevation angles ε_o [from Stirton, 1959].

parameters. Hagihara [1936] and Sugawa [1955] have computed astronomical refraction from meteorological measurements. Further results published are known as the U. S. Standard Atmosphere [Hess, 1959], the WADC 1952 model atmosphere, the model atmosphere based on the Rocket Panel data, 1952, the ARDC model atmosphere [Minzner and Ripley, 1956], and as the ICAO standard atmosphere. Other reports are those by Hanson [1953, 1958], Dubin [1954], Williams [1954 a, b], Gerrard [1961], Mahan [1962], Sperry Gyroscope Company [1962], Brocks [1963], and Henderson [1963]. Extensive tables are found in Gifford [1922], Haurwitz [1941], Geiger [1957], and Dietze [1957] (see Table I).

Saunders [1963] has computed and tabulated variations with altitude of the density of the atmosphere. Refraction angles are obtained as functions of the altitude and of the apparent zenith distance of a luminous source. At large altitudes and at zenith distances up to 86°, these values agree with Bessel's astronomical values, the maximum discrepancy being 2 out of 726 sec of arc.

Stirton [1959] has published a set of useful formulas which are particularly helpful for use with digital computers. An illustrative plot of the refraction correction $\delta\varepsilon$ as a function of height for various apparent elevation angles ε_o, derived with the help of these formulas, is shown in figure 7. (The refraction correction $\delta\varepsilon$ is the angle between the apparent ray path and the ray path outside of the effective atmosphere, $r_1 \geqq r_o + 100$ km.)

As we see, extensive work has been done in this area. However, any theory is bound to be somewhat deficient until the influence of <u>horizontal</u> refractive gradients is taken into account as well. True, these horizontal gradients are much smaller than vertical gradients, but explicit solutions of the equations of a ray path through a model atmosphere can be obtained only under restrictive assumptions as long as the theoretical predictions are not supplemented by empirical observations. This argument will be discussed further in Chapter 3.18.

2.4. Radio refraction

The refraction of centimeter and radio waves passing through the atmosphere has been the subject of numerous investigations. We discuss this subject here only insofar as it is relevant to the optical problem on hand.

In the temperature range from -50° to +40°C, the refractive index n for radio waves in the centimeter range can be expressed with negligible error as a function of the pressure p in the atmosphere, the absolute temperature T, and the water vapor pressure e as

$$n - 1 = 10^{-6}\left(a\frac{p}{T} + b\frac{e}{T^2}\right). \tag{48A}$$

For the radio refractivity [defined as before as $N \equiv (n-1) \times 10^6$] we have

$$N = (n - 1)\,10^6 = a\,\frac{p}{T} + b\,\frac{e}{T^2}$$

$$= 77.6\,\frac{p}{T} + 3.73 \times 10^5\,\frac{e}{T^2}, \qquad (48B)$$

where a and b are constants which have the numerical values shown, p is the total atmospheric pressure in millibars, T is the absolute temperature in degrees Kelvin, and e the partial water vapor pressure or relative humidity, also expressed in millibars. This equation, in this form, had been originally derived by Schulkin [1952]. Other authors who used this and similar relationships were Essen and Froome [1951], Birnbaum and Chatterjee [1952], Harrison and Williams [1955], J. R. Williams [1955-f], Bean and Cahoon [1957], N. R. Williams [1959], Bean [1960], Bean and Horn [1961], Tatarski [1961], McGavin [1962], Bean and McGavin [1963a], Norton [1963], and others. Gardiner [1964] has summarized the bending of radarbeams in the lower atmosphere, also giving numerous equations, tables, correction curves and nomograms.

One sees that the refractivity is composed of two terms[7], a dry term D which for standard conditions is about $(77.6p)/T \approx 280$ and a wet term W, or $(3.73 \times 10^5\, e)/T^2 \approx 40$. Each term can be considered separately as a function of height. The first term applies to optical as well as to radio frequencies; the second, or explicit water-vapor, term is required only at radio frequencies. The refractivity equation is frequently written in the equivalent form

$$N = (n - 1)\,10^6 = \frac{77.6}{T}\left(p + 4.81 \times 10^3\,\frac{e}{T}\right). \qquad (48C)$$

Another approach is to specify atmospheric refraction in terms of the angle through which a radio ray turns as it passes through the atmosphere. This angle, called bending and designated τ, is the angle subtended by the two tangents to the ray at the two points under consideration and is essentially identical with the schlieren angle. The bending $\tau_{1,2}$ of a radio ray passing between two layers in the atmosphere which have indices of refraction n_1 and n_2 can be derived from Snell's law for a spherically stratified atmosphere,

$$n_2\, r_2 \cos \varepsilon_2 = n_1\, r_1 \cos \varepsilon_1, \qquad (49)$$

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7. The more complete equation, according to Harrison and Williams [1955] and Bean [1962] is

$$N = (n - 1)\,10^6 \doteq 77.6\,\frac{p}{T} + 72\,\frac{e}{T} + 3.75 \times 10^5\,\frac{e}{T^2}.$$

Since the second term on the right-hand side of this equation, ordinarily, is about 1% of the third term, the two-term expression given before is sufficient for most purposes. It gives the values of N to within 0.02% of the three-term equation for the temperature range of -50° to +40°C. The relative contributions of the first and third terms are approximately in the ratio of one to e/60, with e measured in millibars.

where ε_1, ε_2 are the elevation angles at different heights above the ground. Smart [1931] has shown that

$$\frac{dr}{r} \operatorname{ctg} \varepsilon = \frac{r d\phi}{r} = d\phi$$

and

$$d\tau = - \frac{dn}{n} \operatorname{ctg} \varepsilon . \qquad (50)$$

Integration gives

$$\tau_{1,2} \approx \int_{n_1}^{n_2} \frac{dn}{n} \operatorname{ctg} \varepsilon , \qquad (51)$$

where ε is the angle of elevation subtended by the tangent to the radio ray and the tangent to a sphere concentric with the earth's surface. The bending τ, by definition, is called positive when downwards [Bean and Thayer, 1959c; Bean, 1960, 1962].

Model atmospheres for radio refraction. The principal difference between light-optical and radio refraction is the by far larger contribution of water vapor to the refractive index of air at radio frequencies. Otherwise the same reasons, and restrictions, govern the use of atmospheric models for a prediction of radio ray bending.

Still, the assumption is made that the atmosphere is horizontally homogeneous. In fact, Bean and Cahoon [1959] found that azimuthal, or horizontal, bending at heights greater than one kilometer above the ground is almost absent. Even in air masses near weather fronts and land-sea interfaces where horizontal bending would likely be most pronounced, the effects were rather small and, in fact, essentially negligible for all except the lowest elevation angles ($\varepsilon < 2°$).

After Schulkin's [1952] derivation of the fundamental radio refraction equation, it has become almost customary to present the refractivity N in terms of an "effective earth radius" which differs from the true radius by a factor k. For low angles of elevation,

$$k = \frac{1}{1 + \frac{r'}{n} \frac{\partial n}{\partial z}} , \qquad (52)$$

where r' is the local earth radius, often assumed to be 6380 km. Normally, $\partial n / \partial z$ is negative and of the order of $-40 \times 10^{-6} \text{km}^{-1}$. Furthermore, if we assume that this gradient is constant with height, a numerical value of $k = 4/3$ is obtained [for a discussion see Bean and Thayer, 1959c; Bean, 1962].

Further work has shown, however, that the 4/3 earth model has serious shortcomings especially with regard to the assumption that $\partial n / \partial z$ is constant with height, which would mean that n decreases linearly with height. In fact, at low altitudes, up to about 16 km, the 4/3 model gives too little bending, while at high altitudes it gives too much. A better representation is given by an exponential reference atmosphere [Bean and Gallet, 1959; Bean and Thayer, 1959; Bean, 1961; and Thayer, 1961]. Here the radio refractivity N, as a function of height z, can be predicted from an equation

$$N(z) = [n(z) - 1]\, 10^6 = D_o \exp\left\{-\frac{z}{H_d}\right\} + W_o \exp\left\{-\frac{z}{H_w}\right\}. \tag{53}$$

The first term on the right-hand side of this equation, D, is the component of the refractivity due to oxygen, and the second term, W, is the water vapor component. The values of the dry and wet components at the earth's surface are D_o and W_o. The scale heights of D and W, respectively, are H_d and H_w. The term scale height is defined as the height at which the magnitude of a particular atmospheric parameter has decreased to 1/e of its surface value. The scale heights H_d and H_w are sensitive indicators of climatic differences, and maps of each are given for the United States for both summer and winter. Bean [1961] has listed typical values for arctic, temperate, and tropical locations (table 2).

Table 2. Typical average values of the dry and wet components of the radio refractivity N [after Bean, 1961].

Climate	D_o	W_o	N
Arctic	332.0	0.8	332.8
Temperate	266.1	58.5	324.6
Tropic	259.4	111.9	371.3

One sees that the contribution of the wet term W_o to the total value of N is nearly negligible in the arctic, but becomes greater as one passes from temperate to tropical climates. This is easily understood if we consider that the higher temperatures of the temperate and tropical climates, as compared with the arctic, provide a greater water vapor capacity, with the result that W_o will contribute more to the total N.

Further progress has been made by finding that there exists a relationship, to a very good degree of approximation, between the variation of n with height above the surface and the surface value n_G of the radio refractive index [Bean and Thayer, 1959a, b; Bean, 1960; Bean and Thayer, 1963; Iliff and Holt, 1963]. From there, an exponential model of the atmosphere has been developed and adopted for use within the National Bureau of Standards which is known as the Central Radio Propagation Laboratory (CRPL) Exponential Reference Atmosphere. The reference atmosphere is a function of a single, easily measured variable, N_G, the surface value of the radio refractivity, and provides a useful solution to the problem of the prediction of atmospheric radio refraction. The accuracy of this model for predicting average radio refraction effects is illustrated by figure 8, where values of angular ray bending are plotted against height for rays starting at zero elevation angle. Further comparisons with precisely determined absolute radio refractive indices in the lower atmosphere have been made by Bean and Thayer [1963].

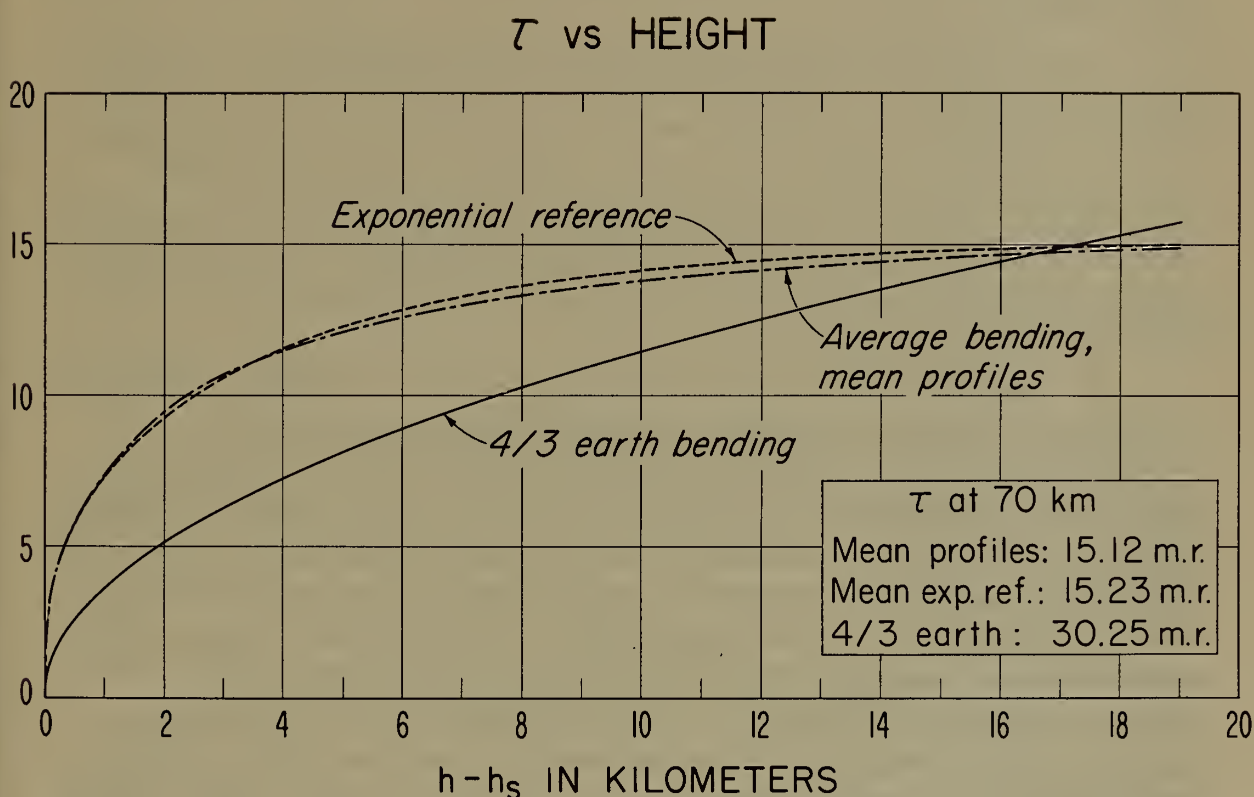

Figure 8

Comparison of observed long-term radio refraction with the CRPL Exponential Reference Atmosphere and with values derived from the 4/3 atmospheric model [from Bean and Thayer, 1959c].

The total bending τ for a radio wave passing through the atmosphere is linearly related to the surface value n_G of the refractive index of the atmosphere and to the initial elevation angle ε_o by

$$\tau = a(\varepsilon_o) + b(\varepsilon_o) N_G . \tag{54}$$

This holds for $\varepsilon_o > 10$ milliradians and average meteorological conditions in a temperate climate. We know that 95 per cent of the total bending occurs in the first 10 km above the earth's surface, that 60 per cent occurs in the first 1 km, and that at least 30 per cent is accomplished in the first 100 meters of the atmosphere for a ray leaving the earth tangentially, that is, where $\varepsilon_o = 0$.

For very small initial elevation angles, say $\varepsilon_o < 10$ milliradians, the initial gradient of the refractive index will have a predictably larger influence on bending. Thus, whenever the magnitude of this initial gradient can be ascertained, it is desirable to make use of this information in addition to the surface refractivity for predicting the bending. Bean and Thayer [1959b, c] have developed a method for using this initial gradient information for prediction and Bean, Thayer, and Cahoon [1960] have shown that the rms error in predicting τ is reduced substantially by the use of the initial gradient in the first 100 meters of the atmosphere [Norton, 1963].

Most of the preceding results are based on theoretical calculations. Fortunately, there exist radio methods and equipment by means of which the radio refraction can easily and directly be determined. Radio refractometers have been developed, for instance, by Birnbaum [1950], Crain [1950, 1955], Crain and Deam [1952], and by Vetter and Thompson [1962]; a radio sextant has been designed by the Collins Radio Company [Anway, 1961, 1963; Bean and Thayer, 1963]. A review article covering techniques for measuring the radio refractive index has been written by McGavin [1962]. An analog computer for conveniently solving the radio refractive index equation has been constructed by Johnson [1953]. The principle of most radio refractometers is based on the fact that the resonant microwave cavity of fixed dimensions is determined by the refractive index of the material contained within the cavity. Consequently, a measurement of the resonant frequency can be considered a measurement of the refractive index of the sample.

3. Random refraction

3.1. Light-optical scintillation, definitions

When a star is observed with the unaided eye or with a telescope of small aperture, the image of the star, instead of being steady, appears to continuously dance about and to fluctuate in intensity. These variations, frequently, are called twinkling or "scintillation". The term scintillation, unfortunately, is used in different ways. Its use should be restricted to describe only a certain class of effects which will be defined shortly. So much, at least, is generally agreed upon: Scintillation and similar terms are reserved for variations on a fairly short term basis.

All of these phenomena are due to irregularities in the structure of the atmosphere, of various dimensions and at various heights above the ground. These irregularities, regardless of how they produce the optical effects to be discussed, hamper accurate observations of, and measurements on, stars and other celestial objects. They may make the image vary in brightness, called twinkling; they may displace the image from its "normal" position, an effect called "quivering"; they may smear out the diffraction image of a star into an irregular "patch" or "tremor disk"; or a point image may "dance about", "wander", "pulsate"; or it may show "focus drift", "image motion", "shimmer", and "atmospheric boil". The color of the image may continuously vary, an effect called "spectral drift" or "color scintillation". Small details of a more extended object may become lost, due to "optical haze", "blur", "boil", or "distortion". All of these contribute to what is called "seeing", a condition which limits many astronomical observations and which by no means is synonymous with scintillation in its more restricted sense.

The term "seeing", likewise, is not well defined. Some authors call seeing the overall effect of dancing, pulsation, and focus drift plus brightness and color scintillation. Others mean by seeing just changes in the position, shape, or size of a star, or of a detail in an extended object, and exclude all brightness variations as such. In some of the European literature, seeing and scintillation are used synonymously. At least some correlation seems to exist between them.

Image motion or dancing is defined as a continuous movement of a star image about a mean point. Image motion is seen best through a telescope of small aperture (less than 10 cm) and high magnification. With increasing aperture, the effect becomes less pronounced. Continuous records of image motion can be obtained by means of an oscillograph camera which allows 35mm film to be moved in the focal plane of a telescope and by interposing a chopper in the light path [Gardiner and others, 1956].

Particularly slow oscillatory motions are called Wandering. Their period is of the order of about one minute of time and the angular excursions about one or a few seconds of arc [Schlesinger, 1927]. Wandering has been recorded even near the zenith and under good seeing conditions. Dancing occurs at frequencies similar to those of scintillation and has an amplitude which ranges from 1, for average seeing conditions, to 10 seconds of arc or more at times of exceptionally poor seeing. It seems likely that all image motion, dancing as well as wandering, in contrast to scintillation, arises from local, near-by turbulences. The different frequencies seem to be a function of the size of the air masses or eddies intercepting the path of light. Wandering may be due to large wedge-shaped air masses of a width of perhaps 100 m. Dancing and pulsation (see later), on the other hand, probably have their origin in small-scale turbulences of about 10 cm size.

Telescopes of large aperture do not show image motion. Instead, they give a time-space integrated result of image motion so that an originally point-like star image is seen as a diffuse, enlarged Tremor Disk. A tremor disk, needless to say, cannot be corrected by focusing; its presence is additional to any aberration which might be inherent in the instrument. The diameter of the tremor disk gives a measure of the amplitude of image motion.

The term pulsation (or breathing) refers to a (fairly rapid) change of size of the star image seen. It is due to a "lens effect" of air masses drifting by close to the telescope objective, continuously de-focusing and re-focusing the image seen in the eyepiece. Pulsation occurs at about the same frequencies as scintillation and dancing. Slow expansion and contraction of a star image at frequencies comparable to those of wandering are called focus drift.

Scintillation in its strictest sense refers only to (rapid) fluctuations in intensity.

Image distortion is the integrated effect of image motion and pulsation involving many points which form the details of an extended object. It is clear that if such points move independently of, and out of phase with, one another, the details become blurred, lose contrast, and cannot be distinctly recovered from the image.

The term seeing is used to mean the combined effect, and the degree, of the continuous changing of position, shape, and size of an image, generally the image of a celestial point object.

Shimmer is the tremulous appearance and apparent distortion and motion of an object when seen, for instance, through a layer of air immediately above a heated surface.

Boiling refers to the combined effects of distortion, image motion, pulsation, and contrast reduction, more specifically, it means the time-varying nonuniform illumination in a larger spot image.

Actually, all of these effects are not as discrete as the above definitions seem to imply. Instead, they follow each other gradually and frequently overlap. By simply looking at a star, one will often be able to discern only brightness and color changes. Other phenomena usually require photographic or photoelectric recording. We will show later that what appears to be random refraction or scintillation, however, is often not as random as it might seem; in fact, significant information can be retrieved from such fluctuations.

3.2. Stellar shadow bands

Light from a star does not uniformly illuminate the ground. Due to irregularities in the atmosphere, plane wavefronts arriving from a distant star become corrugated and create on the ground a mottled pattern of bright and dark areas, called stellar shadow bands. Such bands are seen easily when a large white sheet is spread out on the ground. Generally, the bands consist of an array of dark shadows, several centimeters wide, which change continuously and move on, thus causing the familiar[8] twinkling of the stars. The velocity of motion is around 1 meter/sec.

8. Seen already by Aristotle [384 - 322 B.C.], Tycho de Brahe, and Isaac Newton who described the "tremulous motion of shadows cast from high towers and . . . the twinkling of fix'd stars". Stellar shadow bands have been observed first by Kepler in 1602 who saw them in the light from Venus falling on a white wall. Since then, these bands are sometimes called Kepler's phenomenon.

An extended source such as the sun does not produce shadow band patterns, just as the sun or the planets generally do not seem to twinkle. This is because light from each point in the extended source will traverse different inhomogeneities in the atmosphere and the overall effect, therefore, will be an integration over many point sources scintillating out of phase with one another. On the other hand, shadow bands can also be seen in the light from the sun, for instance, for a few seconds immediately before and after the total stage of a solar eclipse [Anderson, 1935; Wood, 1956]. They can also be observed during a partial solar eclipse when only a narrow segment of the sun remains unobscured, or when the sun rises or sets behind a distant mountain range [Rozet, 1906]. Shadow patterns can also be seen in the light from the moon when very small and from planets (as Kepler has observed) and sometimes when a flare is dropped from a high-flying aircraft.

A convenient way to see the shadow bands is to point a telescope, best a reflector, at a single star, to remove the eyepiece, and to look directly at the objective mirror. One will then note, if the astronomical seeing is no better than fair, that a complex system of parallel, more or less rectilinear shadows is moving in a direction normal to their lengths. These shadows are the result of the refraction and diffraction of the star light passing through inhomogeneities in the atmosphere.

Such patterns have been observed by numerous authors [Pernter and Exner, 1922;. Gott, 1957] ; excellent photographs of them have been published by Gaviola [1949] , Bowen [1950] , and by Mikesell, Hoag, and Hall [1951] . Gaviola has obtained his photographs using a small camera of 75 cm focal length, placed about 30 cm inside the focal plane of a Cassegrainian reflector of 150 cm focal length and focused on the main mirror. The patterns change very rapidly. Gaviola estimates that any particular fine structure configuration lasts only about 1/1000 to 1/100 of a second. Exposure times of 1/25 or 1/50 sec, as often used, are too long.

The width of the bands is frequently not over 3 or 4 cm. Thus, it may easily happen that one eye is in a dark, while the other is in a bright area at the same time. If we look at a star, following Wood [1956] , with the eyes slightly converging by focusing on some nearby object, a star may appear doubled and the two images will fluctuate incoherently.

More exact analyses require photoelectric means. In most cases, a photo detector, together with a suitable aperture stop (pinhole), is placed in the focal plane of a telescope. In this way, variations in light flux can easily be transformed into varying electric voltages which can be recorded continuously and be subjected to statistical analysis.

3.3. Scintillation and seeing

Stellar scintillation, to a large degree, is independent of what is called Astronomical Seeing. By the same measure, stellar image motion as such, although it enters into, is but only part of seeing. We had defined seeing as the combined effect of continually changing position, shape, and size of a stellar image.

Scintillation alone does not influence the accurate determination of spatial position of a celestial object as much as seeing does. These sidewise oscillations show why a conventional photodetector (in particular, if a narrow aperture stop or pinhole is mounted in front of it) usually is not a very useful means for measuring the degree of seeing. This is the more unfortunate, for seeing often is a much more significant factor than scintillation when it comes to selecting a site for high precision astronomical or tracking purposes.

A perfectly adjusted, high-quality telescope with an aperture of about 3 meter, under perfect seeing conditions, should give star images of a diameter of about 0.04 seconds of arc. In reality, the average star image diameter is about 2.5 seconds of arc [Baum, 1955] . Such image, in addition, will move around, with good seeing conditions, at an amplitude of about 6 sec of arc or approximately 3×10^{-5} radian. Under average seeing conditions, the angular displacement of a star image is 10 sec of arc or approximately 5×10^{-5} radian for stars at the zenith and about 10^{-4} radian for stars at an elevation angle of $\varepsilon = 30°$.

The degree of seeing has since long been estimated by subjective methods. Although, as we have seen, seeing is the integrated result of several parameters, many astronomers use simple, one-parameter scales that run either from 1 to 5 or from 1 to 10, representing by 1 the sharpest image and least image motion[9]. Category 3 in the 1 - 5 scale means a somewhat fuzzy image with at least one diffraction ring still visible and moderate motion of less than 3 seconds of arc. All such scales of seeing [see, for instance, Mikesell, 1955; Baum, 1955; Tombaugh and Smith, 1958] , clearly depend to a great extent on subjective estimates that are affected by the "personal equation" of the observer. There are other subjective methods in use as well. For instance, following Wimbush [1961] , the more rapidly the stars appear to twinkle, or the smaller the zenith distance at which color or brightness scintillation can be seen, the worse the seeing is likely to be.

Objective methods for the evaluation and classification of seeing conditions are, for instance, the Pickering scale [1908] and Hosfeld's [1954] cinematographic Hartmann tests. Another possibility is to determine the diameter of the tremor disk [Anderson, 1942] or to measure the lateral (sidewise) excursions of a star image. Seeing condition monitors providing continuous records have been designed by several authors. The seeing monitor by Bray, Loughhead, and Norton [1959] contains two photocells which receive signals from the east and west limb of the sun. The photocells are adjusted so that with perfect seeing the amount of light falling on each cell is constant. When the seeing is poor, the intensity recorded by each cell fluctuates so that the output contains both a steady and an A. C. component, the latter being a measure of the seeing. One of the instruments described by Giovanelli [1962c] is likewise based on the stability of the image of the solar limb, the other on the sharpness of the granulation of the sun.

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9. Note that Wimbush [1961] uses 0 for very poor and 10 for perfect seeing.

For routine measurements, the light from Polaris is used frequently; a number of semi-automatic devices have been built based on this principle. They were used, for instance, when a proper site was to be selected for the U. S. National Observatory on Kitt Peak [Meinel, 1958]. Photoelectric techniques also make possible to record scintillation in others but the visible range of the spectrum; Paulson, Ellis, and Ginsburg [1962], for instance, found good correlation between scintillation in the visible range and in the infrared.

DeGraffenried's [1963 b] astronomical seeing condition monitor (ASCM) contains a two-dimensional array of optical fibers which is placed in the prime focus of a spherical mirror of 30 cm aperture and 2.5 m focal length. The rear ends of the fibers are connected to a set of 10 photomultiplier tubes. Proper circuits and display components give information about the "center of gravity" of the illuminated area, the standard deviation which is related to the r.m.s. image spread, and the ratio of clarity of the atmosphere vs. image motion and/or distortion which is a measure of the quality of seeing.

It is generally agreed that seeing, in contrast to scintillation, depends primarily on local air disturbances near the telescope [Kassander and co-workers, 1951; Liepmann, 1952; Scott, 1958; Menzel, 1962]. Three distinct regions causing seeing effects have been distinguished. The first is the region close to the telescope, critical in particular for solar observations where the instrument is subject to intense heating. Temperature changes will cause expansion, or contraction, of parts of the telescope, and this may considerably shift the telescope focus. Moreover, if the mirror material does not have a high thermal conductivity, different portions of it will change in temperature at significantly different rates, so that unless the thermal expansion coefficient of the material is very small there will be an appreciable distortion of the mirror surface with a consequent loss of image resolution [Wimbush, 1961]. Sometimes it is advantageous to evacuate part of a system, for instance a spectrograph connected to an astronomical telescope. Much sharper stellar spectra have been obtained in this way [McMath, 1955].

The second critical zone involves temperature convections close to, but outside of, the instrument. The air layer just in front of the mirror of a large reflector is a potent source of trouble [Steavenson, 1955]. White paint, sometimes used for the telescope dome, depending on the pigment contained in it, might be a black body in the infrared, just as snow is, and thus make matters worse. Aluminum is generally best. Another cause can be the greater heat capacity of the building housing the telescope, compared with that of the air. At night the building cools more slowly than the air, giving rise to thermal convections which cause bad seeing [Schmeidler, 1955]. Following DeGraffenried [1963 b], "image motion and distortion are caused by fragment parcels of cold air passing in front of the aperture. These fragments have an apparent random distribution in time, length, and temperature difference. The resultant will therefore execute random motion and random distortion... ." In contrast, O'Connell [1958] has found that the limb of the low sun can be boiling when the distant horizon (about 80 km away) is sharp and steady. This would indicate that there can be image motion even when the lower layers of the atmosphere have no appreciable effect on the seeing [Wimbush, 1961].

The third zone is the higher atmosphere where a variety of meteorological conditions, updrifts and the like, occur. The effect of moist air on seeing is generally overlooked. While humidity has almost no influence on the refractive properties of the air, it gives strong absorption in the infrared, thus producing a "green house" effect which influences the seeing [Meinel, 1960].

Wimbush [1961] relates experiments made by Langley [1903], who tried everything he could think of to keep the air in and around the telescope still and at a uniform temperature, but this had very little effect on the seeing. Then he tried just the opposite and agitated the air in the telescope with an electric fan. The result was a considerable improvement in image resolution. This is in agreement with experimental findings by Ryznar [1963 a], who found that the resolution over a 543 m long path was best under windy and cloudy conditions and worst on clear nights.

Although it is questionable whether there is any correlation between seeing and scintillation, attempts have been made to detect such relationships. Mikesell, Hoag, and Hall [1951] found that when the seeing is good, the high frequency components of scintillation are missing and the low frequency components are much less pronounced than they are under bad seeing conditions. Mikesell [1955] came to the conclusion that the astronomical seeing is roughly correlated with scintillation at frequencies between 10 and 150 Hz. More details are found in publications by Stock and Keller [1960], Hynek, Woo, and White [1962], and in the Proceedings of the Symposium on Solar Seeing held in Rome [1961].

Elaborate equipment has been developed to compensate, at least in part, for the effects of poor seeing. One approach is to use shorter exposure times, either by employing more sensitive photographic material or by means of image intensifiers or television techniques [DeWitt, Hardie, and Seyfert, 1957]. Another way is the use of guidance servo systems, usually photoelectric systems that compensate primarily for dancing. Guidance systems must be capable of tracking rapidly and accurately. Since fast tracking involves large accelerations, it would be impractical to move a large mass such as the entire telescope. Instead, it is much easier to center the star on a beam-splitting pyramid or on the axis of rotation of a knife edge and then to move, by a servo system, only the photographic emulsion or an auxiliary lens mounted in the path.

An eidophor is another kind of a seeing compensator. It is essentially a mirror covered by a thin layer of oil. By conventional cathode ray techniques, electric charges are deposited on the surface of the film which, proportional to the intensity of the incident light and within a few milliseconds, becomes depressed in certain areas, thus compensating for ray deviations produced by the dancing star image [Babcock, 1953]. Babcock [1958] has used a thin, flexible film of low but finite electric conductivity. One side of the film is given a reflecting coating, the other side carries a mosaic of isolated elements that are charged by a cathode ray tube to which the film is the window. The charges then produce electrostatic forces which give rise to localized bending moments in the film and to corresponding deformations, acting like an "optical feedback system".

3.4. Physiological and psychological factors related to scintillation

Some authors have claimed that scintillation may, at least in part, be caused by physiological or psychological factors inherent to the human observer. Hartridge and Weale [1949] and Hartridge [1950], for instance, argue, on the basis of laboratory experiments, that bright stars do not scintillate, while dim ones do. The image of a star will, they say, due to refractive index changes in the atmosphere, continually shift on the eye's retina; hence, if the star is not too bright, different receptor regions in the eye will successively be stimulated. In case of a laboratory source that does not "move", the eye performs minute movements which give rise to the scintillation effect. Hartridge and Weale conclude that while the purely physiological explanation of the scintillation of stars appears to account for all the observed facts, it does not rule out the possibility that true physical variations may also occur; in other words, scintillation is claimed to be primarily a physiological phenomenon and the atmosphere takes a secondary role.

Photographic observations by Gaviola [1949], photoelectric records by a great many authors, and the fact that scintillation does vary significantly with the angle of elevation, however, all indicate that the intensity of the light from a star received by the eye does in fact vary, and hence that physiological factors cannot provide a complete explanation of the phenomenon.

A somewhat different but related problem is the question to which degree a human observer is able to point a telescope accurately at a target. When a telescope is pointed even on a steady terrestrial object, the image formed by the objective and seen through the eyepiece seems to oscillate constantly with respect to the cross hairs mounted in the focal plane of the objective. Bringing the cross hairs into coincidence with the image is often a matter of deciding upon the mean position of the image of the target and setting the cross hairs thereon. The accuracy with which this can be done varies from observer to observer and, in fact, can vary between the left eye and the right of the same observer. According to Washer and Williams [1946], the average value of the probable error of a single setting is about 0.62 sec of arc; it is not a function of the distance of the target. This obviously limits the precision with which a telescope or other instrument can be pointed at a distant object.

3.5. Scintillation as a function of circular aperture size

In this chapter, we shall discuss only <u>circular</u> apertures. Rectangular or slit-like apertures produce other phenomena in addition to those due to the reduction in aperture area. These will be dealt with in Chapter 3.11.

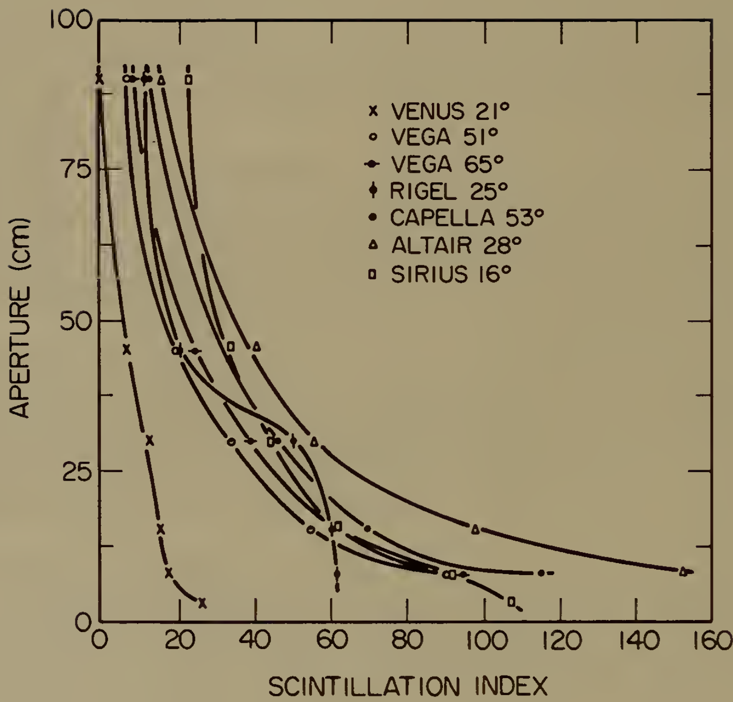

Figure 9

Relation between telescope aperture and amplitude of scintillation for various stars and elevation angles [modified from Ellison and Seddon, 1952].

Assume that a ground-based telescope has a small aperture of, say, 5 to 7 cm. When placed in the region of moving stellar shadow bands, the aperture will receive at any instant of time only narrow, almost parallel bundles of light, the direction of which varies as the shadow band pattern moves past the aperture. Hence a sharp, or nearly sharp, but erratically moving image of the star will be obtained. The peak-to-peak fluctuations of the signal as compared to the mean light level may be of the order of 50 to 150 per cent for stars near the zenith and increase to several hundred per cent for stars near the horizon. In order to retrieve all information on the scintillation spectrum down to the smallest size of the scintillation-causing turbulence elements, apertures as narrow as 3 to 5 cm are required.

On the other hand, with a sufficiently large aperture, the entire range of directions of the light forming the shadow band pattern will be received simultaneously, the average direction being identical with that of the undisturbed bundle above the turbulent zone. Thus, a telescope of large aperture will show less dancing and will give a steady, though blurred and fuzzy image. The dancing of a point source, then, has changed into a diffuse tremor disk the diameter of which can be used as a quantitative measure of the degree of image motion. This shows that scintillation effects can be suppressed by using a larger aperture. The essentially same effect is obtained by retaining a small aperture and averaging over an extended length of time. The probability that the mean resulting from such space- or time-averaging is indeed the true position of the star (excluding the effect of systematic refraction) can be increased at will by making the aperture large enough or the time long enough [in this connection see Hennig and Meyer-Arendt, 1963]. On the other hand, it will be clear that a short exposure, that is, avoiding time averaging, cannot nullify the effect of space integration.

In quantitative terms, it has been found that the amplitude, or width, of the lateral excursions shown by an otherwise distinct star image is inversely proportional to the diameter of the aperture. This, however, seems to be true only within a given range of diameters. Ellison and Seddon [1952], for instance, believe that this relationship does not hold for apertures less than 7.5 cm in diameter. Below this limit, photomultiplier noise becomes prohibitive and, more significant, the scintillation amplitude seems to reach a maximum or saturation level. This suggests that the figure of 7.5 cm for the aperture is not accidentally about the same as the widely accepted magnitude for the width of the shadow bands.

Numerical results from Ellison and Seddon's measurements are reproduced in figure 9. In this figure, the scintillation index is plotted vs. the aperture of the telescope. The plots obtained for various stars and elevation angles show clearly the saturation or "integration" effect discussed before. Even with the smallest aperture used (7.5 cm), the amplitudes never fall below about 30 per cent of the mean level; amplitude variations of 30 per cent or more are often found associated with the most violent movements and boiling of the visual image. Remarkable, and seen only with small apertures, are sudden flashes of increasing light intensity, up to 400 per cent above the mean level (this corresponds to momentary increases in brightness of 1 to 2 magnitudes) and having durations of about 1/100 of a second.

These flashes are most pronounced at an aperture of 7.5 cm and suggest that at this aperture the maximum amplitude of scintillation is reached. This is not definite, however, for at smaller apertures the noise level is high and conclusive experiments have not been made as yet.

We see that larger apertures integrate over the width of at least several shadow bands and smooth out the light variations. With very large apertures, of the order of about 250 cm, scintillation, or better, dancing, according to Ellison and Seddon [1952], should be negligible.

Gardiner and co-workers [1956] are not as optimistic. They found that short term excursions of the star images (dancing) with increasing aperture not only became more diffuse, but sometimes multiple. At 60 cm aperture, as many as three simultaneous distinct images were found often, and at 90 cm, sometimes five simultaneous distinct images could be seen. This suggests that the atmospheric disturbances in question may have dimensions of the order of 15 . . . 20 centimeters.

How circular apertures of different size affect the frequency of scintillation has been shown by Mikesell [1955]. With small apertures, the scintillation amplitude often appears nearly constant at low frequencies. This is the aperture saturation effect described by Ellison and Seddon [1952], and by Megaw [1954], discussed before. With larger apertures, scintillation at high frequencies will increasingly be lost. This again is an aperture smoothing effect. By using a large aperture, therefore, one expects to obtain primarily low frequency data which seem to be more clearly correlated to meteorological conditions. The scintillation amplitude appears to vary inversely with the diameter of the telescope objective only at frequencies around 40 Hz, at least within the range of 10 to 38 cm aperture diameter.

The distribution of the scintillation signal with respect to frequency - that is, its Fourier spectrum - is another interesting parameter [Protheroe, 1961]. In general, for stars near the zenith and for small apertures, the Fourier spectra tend to have constant values at frequencies from zero to around 100 Hz, with decreasing values from there to about 500 to 1000 Hz, where the amplitude becomes zero. On the other hand, when larger apertures are used, the flat part of the spectrum extends to only 10 to 50 Hz, and the zero point is reached at anywhere from 100 to 500 Hz. The decrease in high-frequency components with increase in aperture size is readily explained as an integrating effect across the aperture [Protheroe, 1961]. We may say, in conclusion, that a larger aperture has two effects: it smoothes out the scintillation, and it suppresses the higher frequencies of scintillation: a finite aperture acts like a low-band-pass filter.

These results are in good agreement with theoretical conclusions reached by Tatarski [1961] and Reiger [1962] and with experimental results obtained by Portman, Elder, Ryznar, and Noble [1962].

The random arrival of photons from a star gives rise to fluctuations in the observed light intensity at the focal plane which are called shot noise. Note, however, that the relative importance of scintillation fluctuations and shot noise fluctuations does not depend on the size of the aperture. It depends merely on the brightness of the star and on the optical efficiency

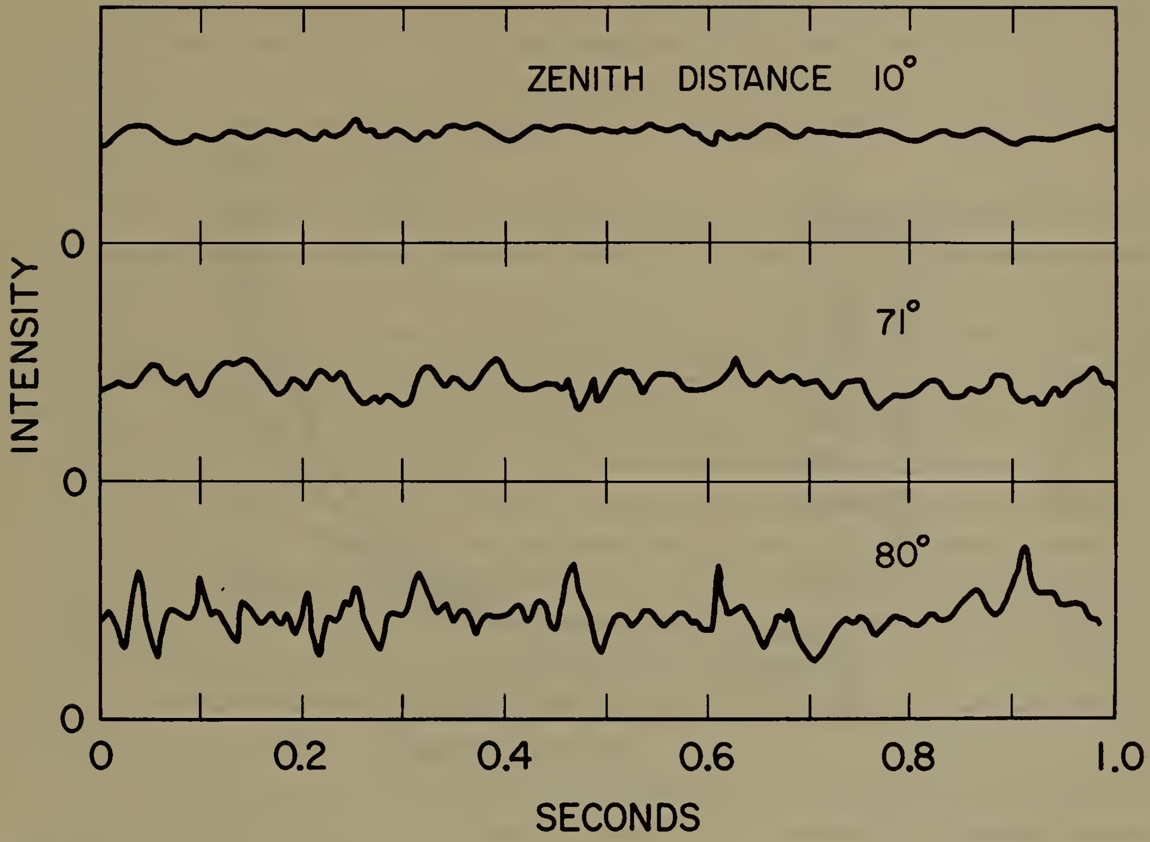

Figure 10

Recordings of scintillation at different zenith distances [after Butler, 1954].

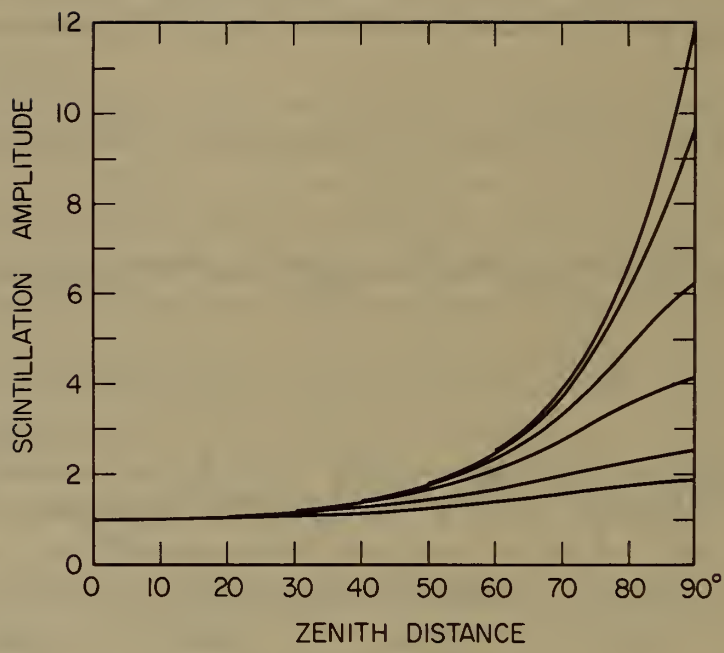

Figure 11

The scintillation amplitude of radio stars (normalized to unity at the zenith) as a function of zenith distance. After Briggs and Parkin [1963].

of the telescope and associated equipment, since increasing either of these increases the number of photons received per second and consequently reduces the ratio of the shot-noise amplitude to the total light received, while at the same time leaving the corresponding ratio for the scintillation unchanged [Stock and Keller, 1960].

3.6. Scintillation as a function of zenith distance

Since Ptolemy noticed that scintillation is more pronounced near the horizon that at the zenith, this relationship is a well established fact. If we look at various stars in the night sky, beginning at the horizon, we note that the degree of scintillation decreases as we approach the zenith[10]. Though often very little scintillation can be seen near the zenith, instruments show that it is still present (fig. 10).

Some residual scintillation is always present, even at the zenith [Schlesinger, 1927; Barocas and Withers, 1948]. Ellison and Seddon [1952] found 5 to 10 percent when recorded with a large (90 cm) aperture telescope and concluded, in agreement with Kolchinski [1952, 1957], that probably there exists a simple proportionality between the amplitude of scintillation and the secant of the zenith distance Z_o. Siedentopf and Elsässer [1954] and Siedentopf [1956], however, concluded from measurements at the Jungfraujoch, Switzerland, at an altitude of 3600 m that the amplitude of dancing in fact seems to increase linearly with sec Z_o, but that the relation between the mean amplitude of brightness scintillation and sec Z_o resembles a saturation curve, the saturation of the high frequencies being more pronounced than that of the low frequencies [Wimbush, 1961]. Mayer [1960] confirms that the relation of dancing with zenith distance is not linear, but is a saturation curve. It is generally agreed that the variation of scintillation with sec Z_o is non-linear when Z_o is large, that is, near the horizon.

Radio stars show an essentially similar behavior (fig. 11), which will be discussed in more detail in Chapter 3.16.

The variation of scintillation with zenith distance of the light source is sensitive to frequency. At about 100 Hz, according to Mikesell [1955], scintillation is constant, but below or above this frequency, it changes greatly with Z_o. For stars at zenith distances from 0° to approximately $Z_o = 45°$, the general form of the distribution of scintillation with frequency stays about the same. This makes possible the use of Polaris as a scintillation source for routine observations. At larger zenith distances, however, more low frequency scintillation (around 100 Hz) is usually present than at higher elevation angles. With increasing elevation angle, the low frequency fluctuations die out while the high frequency component remains, even up to the zenith [Ellison and Seddon, 1952; Butler, 1951a,b; Mikesell, 1951, 1955].

To what degree one is able to predict the dependence of scintillation on elevation angle is still open to question. It has sometimes been suggested that the proper form of this

10. We avoid in this context terms like elevation, altitude, zenith angle, and angle of incidence and use solely the two terms <u>zenith distance</u> and <u>elevation angle</u> which are self-explanatory.

relationship might be used to determine the height of the irregularities causing the scintillation. This in general is not the case. Megaw [1954] found that the r.m.s. brightness fluctuations are a function of zenith distance in the form of $f[(\sec Z_o)^{3/2}]$. This relation, however, seems to hold only for telescope apertures larger than about 15 cm and for elevation angles greater than 30°. For sources nearer to the horizon, the function seems to be more like $(\sec Z_o)^{1/2}$. This might indicate two possibilities: first, that scintillation does not depend on zenith distance for stars near the horizon, and second, that even near the zenith, ray theory might not be sufficient for a proper explanation of the effect. But the contribution from small air packet fluctuations to the observed total brightness fluctuations is made negligible by the aperture integration of all but the smallest astronomical telescopes [Megaw, 1954].

Returning to the somewhat similar situation of radio star scintillation, we note, following Briggs and Parkin [1963], that an increase in zenith distance produces two effects. These are, first, an increase in the magnitude of the phase perturbations and, second, a general change in the geometry of the light paths. The latter means a more marked dependence on diffraction processes, about which more will be said later. It could be that at greater zenith distances anisotropy plays an increasing role, and this might introduce yet another variable. Protheroe [1955], similarly, found that the variations of scintillation modulation with zenith distance and with aperture were interrelated so that the variation with zenith distance depended on the aperture and vice versa.

3.7. Scintillation as a function of site elevation and latitude

Scintillation measurements are carried out frequently for selecting sites for astronomical observatories, tracking stations, and the like. Quite often, such measurements are empirical in nature and consist of recording the brightness fluctuations of a given star such as Polaris, which is chosen frequently because of its relatively fixed position in the sky. More details on site selection are found in Wimbush's report [1961].

If scintillation were due to turbulences in the atmosphere at large, it should tend to become less at high-altitude observation sites. This, to some extent, has been confirmed in observations by Mikesell, Hoag, and Hall [1951] and by Siedentopf and Elsässer [1954] [11]. Mikesell and co-workers, comparing scintillation in Washington, D. C. (86 meters above sea level), and Flagstaff, Arizona (2210 m), found that at the lower elevation the visual image appears fuzzy and pulsating; at the higher elevation the image was relatively sharp but the diffraction pattern is distracting.

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11. In agreement with Sir Isaac Newton, who said that celestial telescopes should be set up on high places, "... for the air through which we look at the stars is in perpetual tremor, as may be seen by the tremulous motion of shadows cast from high towers and by the twinkling of fix'd stars... . The only remedy is a most serene and quiet air such as may perhaps be found on the tops of the highest mountains above the grosser clouds."

In contrast, Anderson [1942] and Irwin [1955] found that the average boiling observed at Barstow, on the floor of the Mojave Valley in California, was not significantly different from that at the Mount Wilson or Palomar Observatories, an observation which, as we will see, seems to contradict a theoretical prediction by Reiger [1962]. Scintillation, in short, tends to decrease with altitude, but the effect of altitude often does not seem to be as great as one might expect, perhaps because mountains can carry up turbulence to great heights.

In particular deep valleys affect the seeing [Williams, 1954d]. This is because cold air, being heavier than warm air, tends to flow down toward the bottom of the valley where it accumulates, undergoing varying undulations and partial mixing, especially during cooling off at night. Above this cold air reservoir, the temperature of the air increases with height, frequently attaining a maximum at a height intermediate between the level of the valley floor and crests of the valley walls. Curiously enough, southern slopes, if at least 50 m above adjoining plains, give good seeing, better than the top of a mountain within a mountainous region or even the top of an isolated mountain [Kiepenheuer, 1962].

As far as the influence of altitude on a smaller scale is concerned, Biberman [1963] mentions work done by the Naval Ordnance Test Station at China Lake, California. There it was found that missile tracking theodolites in the desert should be 7.5 m high to escape boil. Towers higher than this brought little advantage, and 4.5 m was not high enough. On the other hand, Kiepenheuer [1962] recommends a tower height of more than 20 m above the ground. Becker [1961] showed that a tracking camera 10 m above ground level yields, during periods of atmospheric turbulence, nearly a threefold increase in optical resolution.

Not much is known about the effect of geographical location on scintillation. Anderson [1942] mentions that in southern California most frequently sharp diffraction patterns with two well defined rings were seen; only on a few nights did the observers find the pattern completely blurred out and markedly enlarged. In the East, within 200 miles of Washington, however, clear, sharply defined diffraction patterns could rarely be seen and seldom could one observe more than one or two rings. During the first clear night or two after a storm, the diffraction pattern was always completely blurred out.

In the tropics, Jones [1950] claims most of the stars do not scintillate, at least not when observed with the naked eye.

3.8. Color scintillation

The term color scintillation refers to the apparent, rapid change in color of a star. This phenomenon, sometimes called "spectral drift" of starlight, has been observed already by Kepler, Arago, and Lord Rayleigh. Details are found in publications by Pernter and Exner [1922], Humphreys [1940], Minnaert [1950], Zwicky [1950], and Gardiner and co-workers [1956].

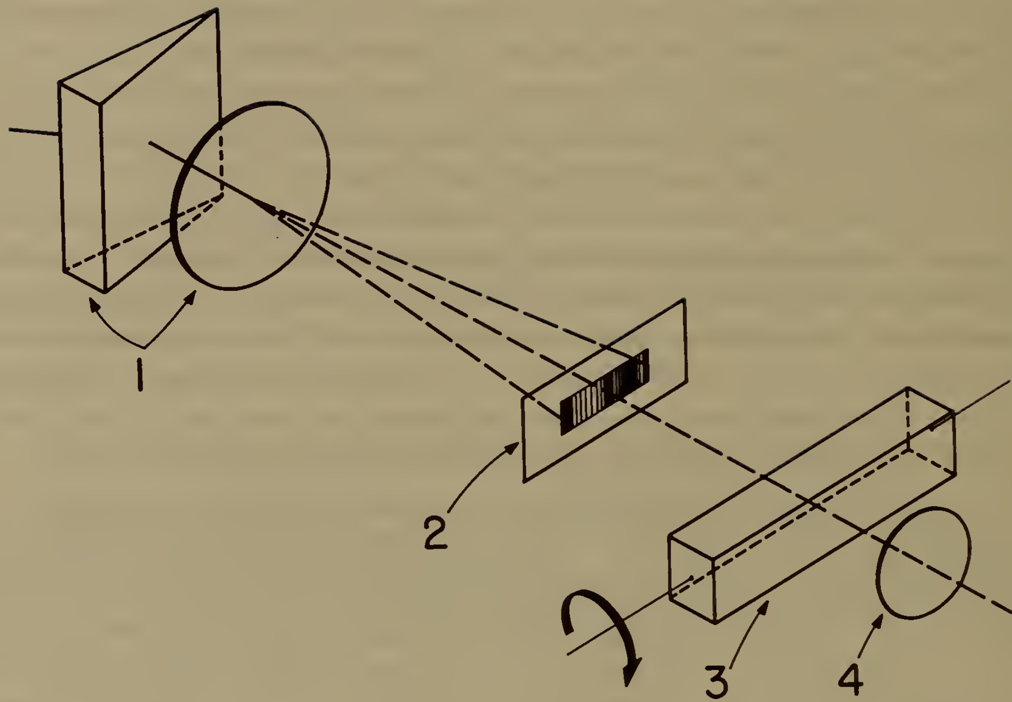

Figure 12

Color scintillometer after Ellison. 1 - prism mounted in front of telescope objective. 2 - star spectrum. 3 - rotating square prism. 4 - eyepiece.

Color scintillation is a common phenomenon. When we look at a scintillating star, either with the unaided eye or through a small telescope, the "white" image which we see is, in fact, composed of a succession of colors changing at such a high frequency that the eye cannot easily distinguish them. However, it is possible to separate these colors by providing some kind of a "time-base". The simplest way, following Varley [1950] and Ellison [1950], is to look at a star through a small, hand-held telescope and to rapidly move the front-end of the instrument, describing a circle. The star image will then likewise be drawn into a circle, and if scintillation is appreciable, the circle will be non-uniform both in brightness and color.

More convenient to use is the Montigny scintillometer, which is simply a thick plane-parallel plate mounted inside the telescope and tilted slightly so that the normal to the plate makes a small angle with the optic axis of the telescope. When the plate is caused to spin about the optic axis at about 5 to 10 rps, the star image is smeared out into a circle and the light is separated into many-colored time elements.

Different is a method originally developed by Respighi [1872], known as the Respighi spectral band phenomenon, and further advanced by Ellison [1950], Zwicky [1950], and Ellison and Seddon [1952]. Ellison and Seddon place a large prism in front of the telescope objective, focus the spectrum of a bright star onto a ground glass screen, and observe the spectrum through a magnifying lens. The time base is provided by vibrating the lens in a direction normal to the optic axis and normal to the direction of dispersion. In this way, the spectrum of the star is broadened, in the direction normal to the dispersion, into an infinite number of time-sequential elementary spectra. Because of color scintillation, however, most of these elementary spectra show some colors missing; some spectra have only the red and violet, others only the yellow and green, and so on. The total spectrum, hence, appears to be crossed by dark bands which, as time goes on, seem to be moving across the spectrum. Respighi, who saw these bands first, already found that sometimes they move from the violet end of the spectrum to the red, and sometimes in the opposite direction, possibly depending on the prevailing wind direction or on other atmospheric conditions.

Ellison [1954], in place of the vibrating magnifying lens, observes the star spectrum through an eyepiece plus a rotating prism, the axis of rotation being parallel to the direction of dispersion as shown by figure 12. The inclination of the dark bands relative to the direction of dispersion gives a measure of the wavelength interval traversed horizontally by a band in a given time interval.

Color scintillation is most pronounced for stars near the horizon and almost absent near the zenith [Respighi, 1872; Montigny, 1874; Pernter and Exner, 1922; Zwicky, 1950; Mikesell, Hoag, and Hall, 1951]. The frequency of the intensity fluctuations at various wavelengths is around 50 Hz, that is, about the same as for ordinary white light scintillation.

Whether or not white light and different colors scintillate in phase is still a matter of controversy. Some authors, for instance Mikesell, Hoag, and Hall [1951] and Ellison and Seddon [1952], feel that there is little difference in, or good coherence between, the intensity fluctuations as recorded at different, nearby wavelengths. On the other hand, the fact that the phenomenon of color scintillation does exist at all shows that little or no phase coherency can exist between scintillations in different wavelengths. As Ellison and Seddon [1952], curiously enough, have pointed out in the same paper just mentioned, the existence of color scintillation implies that various colors must fluctuate in brightness independently of one another in time; for if all wavelengths were to scintillate with equal amplitude and in phase, we should expect to observe changes of apparent brightness in the integrated image but no changes in color.

Sometimes it is assumed that color scintillation occurs because refractive inhomogeneities which cause scintillation would also draw out the images of stars into spectra. This conception, though, is wrong. The displacement of a star, due to random refractive inhomogeneities, is so small that an additional dispersion effect could not be observed at all. Instead, color scintillation is the result of a combined action of random refraction plus regular atmospheric refraction. That means that light of different wavelengths, but coming from the same star and arriving at the same point on the surface of the earth, has followed different paths through the atmosphere. These paths have been influenced independently by local perturbations, and this accounts for the phenomenon of color scintillation. The phenomenon of the Green Flash can be explained in a similar way [Dietze, 1957; Deirmendjian, 1963].

We had seen that the refractive index n, in terms of density ρ, is given by

$$n = 1 + k\,\rho, \qquad (55)$$

where the constant k is a function of wavelength, $k = f(\lambda)$. Values for k for different wavelengths are given in table 3.

TABLE 3

Wavelength dependency of atmospheric refraction [after Brocks, 1948].

Wavelength	k
4000 Å	2.308×10^{-7}
5896 Å	2.262×10^{-7}
7600 Å	2.247×10^{-7}

Various zenith distances, of course, affect the degree of angular separation of light of different wavelengths. For red and blue-green images of a star, we find for a zenith distance of 30° an angular separation of 0''.35, for 60° zenith distance 1''.04, and for 75° zenith distance 2''.24 angular separation. Much more detailed tables, including computed values for the rising sun at very small elevation angles and for wavelengths from the UV to 30μ in the infrared, have been prepared by Treve [1963, 1964].

3.9. Frequency range of scintillation

Scintillating stars, at first sight, seem to twinkle in a completely random manner. We cannot correctly speak of the frequency of such scintillations, but we may estimate that the time interval between moments of nearly maximum brightness is around 1/10 of a second, a value that coincides, not incidentally, with the frequency to which the human eye is most sensitive. If we consider brightness fluctuations averaging about 10 per cent of the mean, then in fact the frequency range will be much wider. It is common knowledge that during nights of good seeing the stars twinkle slowly, up to 20 or 30 times a second: When the seeing is bad, the scintillation frequency centers around 150 Hz. Courvoisier [1950] found periods as long as 60 sec and Ellison [1952] using equipment capable of measuring 10,000 Hz, recorded frequencies mainly around 100 Hz. Most commonly, the fluctuations in intensity vary from 1 to perhaps 500 Hz.

Extensive analyses of the frequency spectrum of scintillation have been made by Mikesell [1951, 1955], Ellison and Seddon [1952], Nettelblad [1953], Butler [1954], and Protheroe [1955]. Mikesell, for instance, could not find a particular frequency which would stand out clearly in the whole spectrum. He states that "even though scintillation energy might have been concentrated for several minutes at one frequency, the fact was not observed because of the length of time spent observing at other settings of the distribution pattern. The procedure furnished no distinction between the short-term dominance of a particular frequency component and a brief enhancement of scintillation at all frequencies." These findings supplement earlier studies by Mikesell, Hoag, and Hall [1951] and by Gifford and Mikesell [1953]. It was found, however, that the contribution to the overall modulation is progressively less for higher frequencies. In general, for stars near the zenith and for small apertures, the Fourier spectra tend to have a constant strength at frequencies from zero to around 75 or 100 Hz, with a decreasing strength from there to about 500 to 1000 Hz where the amplitude becomes zero.

Bellaire and Elder [1960] observed that scintillation during the day is about equal to that during the night, although there is a tendency for lower frequencies to increase during the night and for higher frequencies to increase during the day.

During the winter months, it is found that the amplitudes at all frequencies are larger, but those at the higher frequencies relatively more so, probably because of the higher wind velocities that prevail at that time of year. The scintillation at all frequencies increases with increasing zenith distance, but the relative increase is more marked at lower frequencies [Stock and Keller, 1960; Mikesell, 1960a].

Goldstein [1950], likewise, has made measurements of the frequency spectrum of scintillation in the surface layers of the atmosphere and has related the observed spectrum to gross features of the weather such as average path temperature, relative humidity, and atmospheric transmission properties. He observed that scintillation was greater on clear nights than on hazy nights and less during periods of rainfall.

We had seen before that the aperture size of the telescope has a very definite influence on the frequency spectrum that can be observed. Only if the aperture is smaller than the size of the smallest eddies, about 1 cm, could the highest frequencies be observed which correspond to the region of viscous damping of the turbulent motion of the air. With a telescope of any larger aperture, the effect of small eddies is reduced by the integrating effect of the aperture. According to Megaw [1954], the r.m.s. brightness fluctuation is proportional to $K^{2/3}$ where K is the fluctuation frequency. However, at frequencies large compared with those produced by the translation of eddies of a size equal to the aperture across the line of sight, the effect of this integration is to reduce the index of the basic spectral power law (or r.m.s. fluctuations) by unity, that is, to $K^{-1/3}$. This means that the scintillation spectrum will be proportional to $K^{4/3}$ up to some frequency at which integration over the finite aperture gradually cuts off the higher frequencies.

Siedentopf [1956] and Mayer [1960] found that the amplitude generally increases toward the lower frequencies (0.2 Hz). With increasing zenith distance, the r.m.s. values of the amplitudes of image motion have the character of a saturation curve. For the higher frequencies in the scintillation spectrum, the saturation is more pronounced than for the lower frequencies. Elsässer and coworkers [1959, 1960] have analyzed the r.m.s. values of the angular excursions and the intensity fluctuations in terms of a model atmosphere that consists of a multitude of layers of equal thickness lying on top of each other.

3.10. Terrestrial scintillation

That terrestrial light sources scintillate as well is common knowledge[12]. Experimental studies of terrestrial scintillation are particularly rewarding since the conditions, refractive index variations, wind velocities, and so on, along the path can be monitored rather easily and related to the scintillation measurements as such. Thus, terrestrial experiments can give more complete data than stellar scintillation observations, data which can easily be compared with theories of scintillation.

We omit from the present discussion any large-scale effects of (regular) atmospheric refraction, such as the dip of sea horizon which plays a role in nautical position determinations. The dip of horizon is seen only at low elevation angles of, say, less than 6°; its magnitude is determined by the temperature, pressure, and humidity of the air above the ocean. For a review of this subject and tables of correction factors see Ogura [1926, 1927], and Sone [1926]. For a path 10 km long, vertical refraction errors can amount to about 20 sec of arc. Lateral or azimuthal refraction errors, although by far less serious, exist as well and can affect precise range measurements.

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12. Such scintillation is easily seen when looking down at the lights of a city from a hill. Sometimes, incandescent street lamps are seen clearly to scintillate, but not the mercury arcs.

A rather simple, but instructive example of how shadow patterns can be made visible using a terrestrial light source is given in a publication by Wolfe, Morrison, and Condit [1959]. Their set-up is essentially a shadow schlieren system. Light from a high pressure mercury arc is projected by means of a 180 mm focal length, f: 3.5 telescope lens onto a white projection screen, located across a valley at a distance of 400 meters. The image of the mercury arc at that distance is about 6 m in diameter. The screen thus intercepts only part of the beam.

During a clear, moonless night, a pattern of shadow bands is seen on the screen. The pattern which is in constant turbulent motion obviously is the result of inhomogeneities in the atmosphere which deviate the light striking the screen, resulting either in an enhancement or a reduction of the luminance in a given area of the screen. Photographic records were obtained by placing a photographic film in front of the screen and exposing for 1/25 sec. Similar photographs were obtained by Siedentopf and Wisshak [1948]. Wolfe and co-workers [1959] propose to make this method stereoscopic, using either two light sources and polarizing filters or by moving the light source a given distance so that localized inhomogeneities could be located in space, provided such inhomogeneity would give a clearly identifiable indication on the screen.

A variety of rather similar experiments on terrestrial scintillation has been made by different authors. Keller and Hardie [1954], for instance, have observed a light source through a region of artificially induced turbulence. When unheated air is blown across the optic path, very little change occurs in the image profile. On the other hand, turbulences and related light intensity fluctuations could be produced consistently by dissipating up to 600 watts of electric energy into a slowly moving air stream about 30 cm in diameter, heating it by about 22°C relative to the ambient air. It is interesting to note, although expected, that images seen in red light were often sharper than those seen through the same turbulences but in blue light.

Similar terrestrial observations have been made by Glenn [1962], who used a dark tunnel 30 m in length. Controlled turbulence is induced by a series of 1500 watt electric heaters which produce streams of air, heated to 100°C, about 1.2 m wide and several cm thick. Glenn concludes that the refractive elements constituting the air eddies act as positive spherical and cylindrical lenses with effective focal lengths at least as short as 5 to 6 m and as negative lenses of focal lengths as short as 2 m. The elements themselves have a size of the order of 5 mm.

In a series of experiments, Straub and co-workers [1960, 1962] have investigated the propagation of light in paths 1 to 3 meters above the ground and for path lengths of up to 3.2 km. The light source, a high pressure mercury arc, is placed just in front of the primary focal point of, or slightly farther away than the focal length from, a long focal length objective. The image of the (extended) light source is then adjusted so as to have, at a distance of about 400 m, a diameter equal to that of the aperture of the objective, giving a light

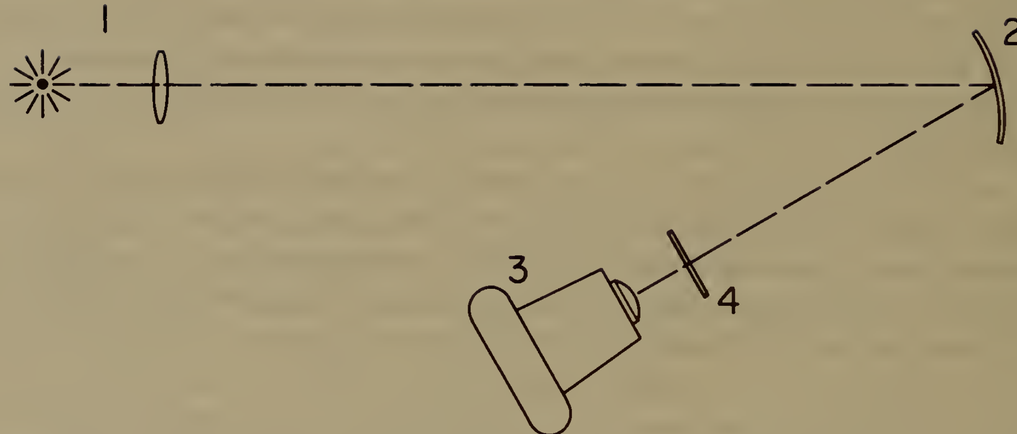

Figure 13

Schematic arrangement for photographing turbulence along a path close to the ground. 1 - light source, 2 - condenser mirror, 3 - movie camera, 4 - neutral density filter. [Modified from Straub, 1962].

beam which, as Straub, Arthaber, and Moore [1960, 1964] call it, is quasi-parallel. The term "parallel", of course, refers only to the outermost rays limiting the bundle; the intensity distribution in any cross-section, though, except close to the lens and to the image plane, is not uniform.

The light bundle is incident on a concave mirror, (2) in figure 13, which images the source into the objective lens of a movie camera (3). The movie camera, in turn, is focused on the mirror. A series of neutral density filters are used to control the exposure. In a similar, but somewhat more sophisticated system, two confocal parabolic mirrors with an aperture stop at the common focal point are used for admitting into the detector end of the system, predominantly, the light coming from the source and rejecting ambient daylight.

When a white cardboard screen is held in the quasi-parallel light beam, shadow patterns will be seen like those described before. The turbulent elements are of varying shape, about 2.5 to 10 cm in diameter, and drift at random through the path. These elements, presumably, indicate eddies of air warmer than the ambient air. Pressure gradients seem unlikely. Motion picture records taken by Straub show that at any one time about 60 per cent of the field is obscured by such shadows. If a slight breeze intersects the optic path, the shadows drift across the field at a velocity proportional to the wind velocity component normal to the light path. Higher wind velocities, however, tend to give a more uniform light distribution, possibly because at higher speeds small scale turbulences in the air are smoothed out.

Besides these small-scale variations, the light beam, as such, was seen to wander about, mostly in the vertical dimension (with an amplitude of about 3 minutes of arc), but some lateral oscillation was also observed, at periods of about 3 to 15 minutes. This phenomenon, likewise, is attributed to larger-scale fluctuations in the density gradient of the near-ground air layers.

Siedentopf and Wisshak [1948] measured, simultaneously, scintillation, wind speed, and solar radiation. The scintillation maxima seemed to coincide with the maximum radiation received from the sun. A secondary maximum occurred during the night with the nocturnal radiation loss. Minima occurred shortly after sunrise and again before sunset. Similar investigations of terrestrial scintillation were carried out by Evans [1955], Croft [1958], Gurvich, Tatarski, and Tsvang [1958], Scott [1958], Tatarski [1958], Tatarski, Gurvich, Kallistratova, Terentera [1958], Mayer [1962], and Walton [1962]. Watson [1957] and Mayer [1962] supplied thermal turbulence artificially by means of a 1.5 m long network of electrically heated wires giving off 3000 watts of heat. The turbulence induced in this way gave an image blur the angular extent of which was estimated to be about 30 microradians. Hinchman and Buck [1964] have observed the fluctuations of a laser beam over a 144 km path.

Bellaire and Elder [1960] have determined which meteorological conditions and other contributing factors produce, and modify, terrestrial scintillation. They used as the light source a 30 cm dc automobile spot lamp mounted on a theodolite, projected the beam over a 600 m path, 1.2 m above the ground into a telescope photometer, and recorded the intensity

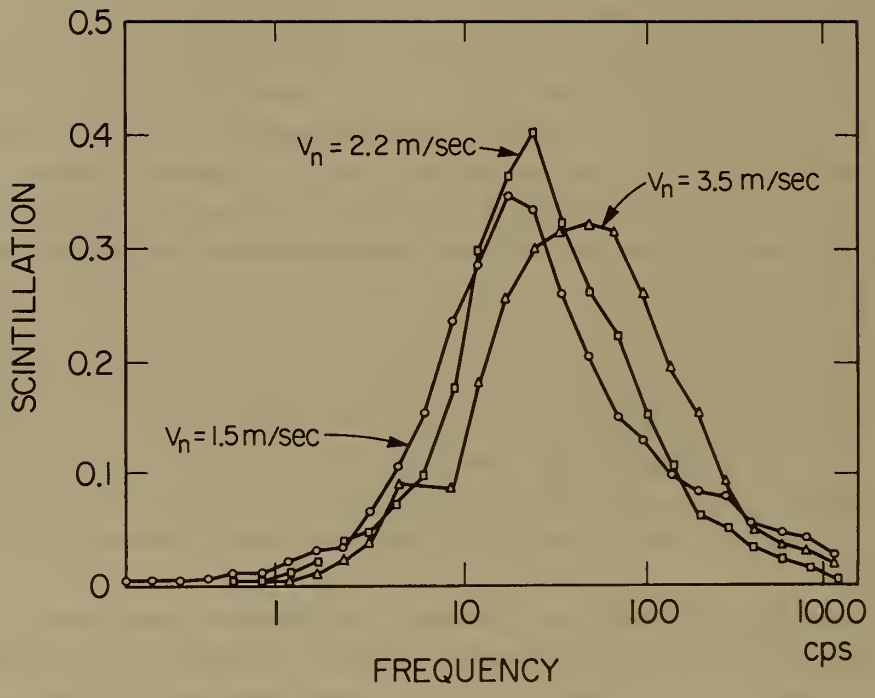

Figure 14

Frequency spectrum of light intensity fluctuations for a 1000 m path 2 m above the ground and for different wind velocities. [Redrawn from Tatarski, 1961].

fluctuations on magnetic tape which was then fed into a frequency analyzer. The main result of the studies is that terrestrial scintillation is primarily a function of vertical temperature gradients. Maxima were observed at midday when they are due to solar heating and at night due to radiation cooling of the ground. Minima occurred at sunrise and sunset. Terrestrial scintillation, although not much different between day and night, tends to decrease during periods of cloudiness when the vertical temperature gradient decreases as well, and in the presence of moderate winds, for the same reason. The best resolution of a terrestrial target, accordingly, was found during cloudy periods and at sunrise and sunset.

Terrestrial scintillation, then, constitutes a major hazard to optical communication, especially when a highly collimated beam and a rather narrow receiver aperture are used [Goodwin, 1963]. On the other hand, optical means can be employed for obtaining meteorological parameters. Johnson and Roberts [1925] have already determined the temperature lapse rate by measuring the apparent vertical displacement of a target viewed through a telescope. For detailed studies of the climate near the ground see Best [1931], Lettau [1939, 1944], Lettau and Davidson [1957], and Geiger [1957].

Tatarski [1961], in supplementing extensive theoretical work which will be discussed later, has observed the scintillation of a terrestrial light source, using path lengths of 250 to 2000 m and average heights of 1.5 to 5 m above the ground. At distances less than 250 m, scintillation was less than the noise inherent in the apparatus, and therefore measurements were not made at such distances. The wind velocity present (better: the component v_n of the mean wind velocity perpendicular to the ray) was calculated from simultaneous meteorological measurements. The frequency spectrum of the fluctuations of light flux was obtained by using a frequency analyzer. Figure 14 shows that when the mean wind velocity increases, the curves shift toward the higher frequencies, which is in good agreement with theoretical considerations.

In a comprehensive study, Portman, Elder, Ryznar, and Noble [1961, 1962], and Ryznar [1963 a] have carried out experiments which again were basically similar to these by Siedentopf and Wisshak [1948] and Tatarski and co-workers [1958, 1961]. Light from 12 volt sealed-beam spot lamps of 12.5 cm aperture was sent over paths 122 to 600 m in length and about 1.5 m above uniform and horizontal grass and snow surfaces. The light beam is received by a telephotometer of 7.5 cm aperture, containing a multiplier phototube. The intensity of fluctuation is measured in terms of the per cent of modulation of the received signal, defined as the ratio of the mean peak-to-peak amplitude to the average, or dc, level. This quantity gives a measure of the intensity of fluctuations independent of changes in brightness due to attenuation by the atmosphere. Continuous records of meteorological variables were made throughout all measurement periods. Wind and temperature profiles, for instance, were measured with sensors at 0.5, 1, 2, and 4 meter above the surface at a single location near the optic path. In agreement with Bellaire and Elder [1960], Portman and co-workers [1962] found that the most striking feature of the scintillation pattern was its correspondence with the absolute value of the vertical temperature gradient.

Continuing studies by the same group involve the use of a CW laser as a light source and a comparison with nearly coaxial measurements carried out with a dc light source [Ryznar, 1963 b].

According to Williams [1954 c], the amplitude of the temperature fluctuations is less than 2.0°C, the r.m.s. value of the fluctuations being approximately 0.7°C. The contribution of ΔT to Δn is approximately

$$-C \frac{p_o \Delta T}{T_o^2} = -(n_o - 1) \frac{\Delta T}{T_o} \approx -10^{-6} \Delta T , \qquad (56)$$

for $n_o - 1 \approx 3 \times 10^{-4}$, $T_o \approx 300°K$. Thus, the value of Δn corresponding to the largest observed fluctuation $\Delta T = 2.0°C$ is about -2×10^{-6}.

The period of the pressure fluctuations is about 5 to 10 minutes. The period of the temperature fluctuations is much smaller. Experimental records show that temperature fluctuations of about 1.0°C may occur during a time interval of 1 second or less.

From the preceding discussion, Williams [1954 c] concludes that the value of Δn, the variation of atmospheric refractive index from the mean value n_o, may frequently be as large as 2×10^{-6}, while the r.m.s. value of Δn is likely to be slightly less than 10^{-6}. Furthermore, fluctuations of about 10^{-6} in the value of the refractive index, having a period of about 1 second, seem likely to be encountered in experiments in the field.

As far as the effective path length in terrestrial scintillation studies is concerned, certain experiments seem to indicate a kind of saturation effect. Generally, it is assumed that the optical effect of turbulence near the ground increases with longer path lengths. Siedentopf and Wisshak [1948], however, found that the relationship between per cent of modulation and length of path had the characteristics of a saturation curve that approached its limiting value at a path length of 1200 meter. Straub [1963] came to a similar conclusion.

The scintillation of balloon borne lights has been studied by Gardiner and co-workers [1956]. These experiments permit a direct evaluation of artificial scintillation as a function of height above the ground and of angle of elevation.

3.11. Scintillation and meteorological conditions

Although scintillation certainly is a nuisance to the astronomer, it can be quite helpful to the meteorologist. Diurnal variations in scintillation, obviously, are best observed over a terrestrial light path. A broad maximum occurs during the noon hours, at the time of maximum solar radiation received. Intermittent secondary maxima are found during the night, a period characterized by frequent changes in temperature difference caused by moderate and variable winds.

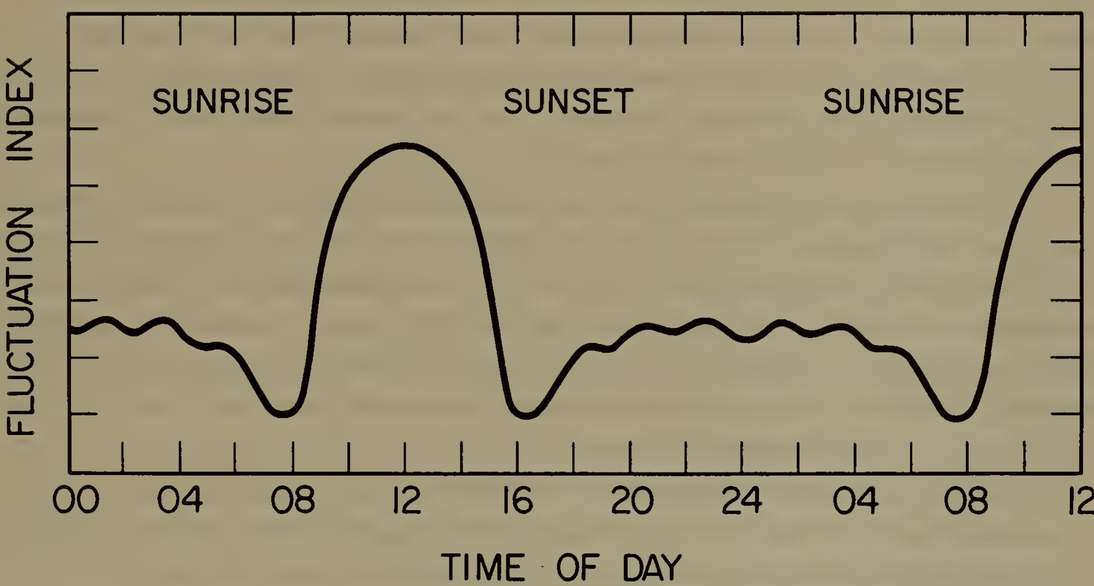

Figure 15

Diurnal cycle of terrestrial scintillation.

As the lapse rate decreases with decreasing solar heating, scintillation drops to a minimum prior to sunset at the time of near-adiabatic conditions. Another minimum is usually observed just after sunrise. These observations, made by Siedentopf and Wisshak [1948], Coleman and Rosenberg [1950], Bellaire and Elder [1960], Portman, Elder, Ryznar, and Noble [1961], Mayer [1962], and others, are summarized in figure 15.

As can be observed easily, the ground temperature and the temperature of the air near the ground ordinarily reach a maximum in the early afternoon, and decrease to a minimum just before sunrise. At night and in clear, calm weather, a temperature inversion as high as 100 meters or more above the ground will occur during all seasons of the year [Williams, 1954 d]. In mountainous regions and in clear weather, a strong inversion is frequently found in the valleys. A warm zone usually exists in an intermediate region on the slopes, with a lapse of temperature with height above the zone. Overcast skies tend to reduce the decrease of temperature during the night, and to counteract the formation of the nocturnal inversion [Geiger, 1957; Williams, 1954 d]. A low-lying layer of thick, heavy clouds may even completely prevent a nocturnal inversion.

Wind, likewise, reduces the amount of nocturnal temperature inversion. The magnitude of the effect depends on the velocity of the wind. As the velocity increases from zero, the motion of air is laminar at first, and the initially stable stratification is relatively undisturbed as if the air layers were caused to slide along the ground. However, as the velocity of the wind continues to increase, the atmospheric motion becomes more and more turbulent in character, that is, eddies are formed, moving in the vertical direction as well as in the horizontal, thus mixing adjacent air layers.

Optical scintillation, connected to such meteorological phenomena, can be observed, and quantitatively evaluated, by various means. It is rather simple to cover the aperture of a large (30 cm) astronomical refractor with a screen with two holes, each about 2.5 cm in diameter, and of variable separation. The stellar shadow band pattern obtained is then compared with the actual wind velocities present, which can be measured by radar tracked balloons. Protheroe [1955] found the best agreement with the wind pattern at the 200 millibar level.

Barnhart, Protheroe, and Galli [1956] have used a tandem of two telescopes pointed at the same star. The light reaching their focal planes is collected by photomultipliers. The horizontal separation of the telescopes and the azimuth of a line joining them can be varied. The two intensity records can thus be compared side by side on a common time basis. Barnhart, Protheroe, and Galli found that at certain azimuth settings no similarity could be detected. When similarity was present, a time shift between intensity maxima existed, indicating that the light and dark areas in the shadow band patterns had crossed first one and then the other of the telescope objectives. When the similarity between the output traces is at a maximum, the line joining the centers of the two objectives seems to be parallel to the direction of motion of the atmospheric inhomogeneities, and the time shift between the traces indicates the sense of motion as well as the speed.

The two-aperture method has also been applied to the observation of double stars. This method of monitoring scintillation, developed by Hosfeld [1954], is used to determine the height of the layers which primarily contribute to scintillation. Hosfeld places two apertures, each 7.5 cm in diameter and separated about 23 cm, over the telescope aperture. If the final image is slightly out of focus, the two apertures form, of a single star, two images which scintillate independently. A double star, in contrast, seen through a single aperture, scintillates coherently. If the two members of a double star are fairly close together so that the light cones coming from them overlap at the height of a scintillation producing layer, then the correlation coefficient will exceed a certain value (0. 707), and, for even closer double stars, approach unity.

When a single slot is placed over the aperture of a telescope objective, one might expect, likewise, that the intensity and character of scintillation will be different for different orientations of the slit. Experiments of this kind have been carried out by Mikesell, Hoag, and Hall [1951], Protheroe [1954, 1955, 1958, 1961, 1964], Keller [1955b], Mikesell [1955], Barnhart and co-workers [1956, 1959], and Protheroe and Chen [1960]. When the slit is parallel to the motion of the shadow band pattern, the scintillation spectrum is comparable to that obtained through a telescope of large aperture. When the slit is perpendicular to this position, the signal is similar to that obtained through a small aperture. In Protheroe's 1961 experiments, a slit 2.5 x 10 cm in size and driven through 190° by a motor was mounted over the objective. The slit is initially oriented in such a way that the zero fiducial of the slit is aligned with the direction of the true north when the telescope is in the meridian. The long axis of the slit is then rotated from the north to the south orientation and back. According to Protheroe, the "ratio of the stellar scintillation signal in a frequency band centered at 300 Hz to the signal in a band centered at 10 Hz generated when a star near the zenith is observed with a 10 cm circular aperture has been shown to be dependent upon the upper-air winds. When the slit is placed over the telescope aperture, the wind direction. . . . may be determined by noting the alignment of the long axis of the slit which gives the minimum 300 Hz signal." Mikesell [1955] found that primarily winds at a height of about 5 to 18 km are responsible for scintillation effects detectable by this method. Protheroe's results were also compared with theoretical results derived by Reiger [1962]. Best agreement was found with curves which correspond to a cut-off eddy size of the order of 10 cm.

Experiments with two slits, carried out by Butler [1954], have cast some doubt on the frequently made assumption that scintillation is the result of the motion of shadow bands perpendicular to their length. The shadow bands for distant street lights often are horizontal. If scintillation records are made with a screen with two vertical slits in front of the objective, some sharp peaks in the trace are doubled, indicating that bright areas have crossed first one slit and then the other. Other sharp peaks were not doubled, which showed that bright areas can appear or disappear in the distance between the slits. If the slits were parallel to the bands, no doubling was seen. This suggests, according to Butler, that the principal motion of the bright and dark areas giving rise to scintillation is along the direction of the bands and not perpendicular to it.

Numerous attempts have been made to link certain parameters of scintillation with certain characteristics of the weather, but never has any correlation been found except with wind. With respect to the frequency spectrum of scintillation, Mikesell [1955] noted that there is no correlation at 10 or 25 Hz with wind speed, but a definite correlation exists at 150 Hz and at even higher frequencies.

As far as the height of the scintillation producing layer is concerned, comparison of scintillation records with wind velocity measurements by a rawinsonde has shown that in nearly all cases the velocity of the pattern agrees with the wind velocity near the tropopause. From this, many authors have concluded that scintillation is caused primarily by a turbulent layer of mixed hot and cold air located at or at least near the tropopause. Since scintillation tends to increase whenever the wind field in the vicinity of the tropopause is strong, scintillation at mid-latitudes in the Northern hemisphere is stronger than summer scintillation [Protheroe, 1961]. Mikesell [1955] noted that the cutoff point of the Fourier spectrum (that is, the point at which the amplitude goes down to zero) occurs at higher frequencies during the winter.

Similarly, Gifford and Mikesell [1953], Gifford [1955], and Mikesell [1955] have observed that at 150 Hz the best correlation exists with winds at heights from 8 to 14 km. This is the region which contains the tropopause. At scintillation frequencies above 550 Hz, however, correlation with winds at other heights and with speeds in excess of 130 km/hr was found. Low frequency (25 Hz) scintillation showed no clear correlation to wind speeds within the first 18 km of the atmosphere.

Somewhat different means of observation were employed by Barocas and Withers [1948], who analyzed the trails of stars near the zenith and the erratics produced in them by random refraction. Ventosa [1890-91] and Burch and Clastre [1948] studied the "boiling", or undulation, of the limb of the sun. These undulations vary with the time of day and seem to be related (a) to the height of the turbulent layers in the atmosphere and (b) to the direction of the wind in these layers.

If we summarize the effects which meteorological conditions have on scintillation, we come to the following conclusions (based on results obtained by Dörr [1915], Bellaire and Elder [1960], Paulson, Ellis, and Ginsburg [1962], Hynek [1963]):

(1) Higher density of the air produces increased scintillation. At lower altitudes, scintillation is more pronounced. At a given location, however, higher barometric pressure may reduce the scintillation.

(2) Temperature alone is not so much of a factor as is the temperature profile, that is, the variation of temperature as a function of height. At high temperature gradients (the typical condition on a sunny day except over a snow surface), scintillation is very marked and increases rapidly with increasing temperature gradient. On the other hand, if the temperature increases with height (representing a stable atmosphere and

the typical conditions on a cloudless night), scintillation increases only slowly with increasing temperature gradient.

(3) Cloudiness during the day as well as at night reduces the magnitude of the vertical temperature gradient, no matter what its direction, and thus reduces the degree of scintillation, sometimes down to zero. Even when the sun only temporarily disappears behind a cloud, scintillation is greatly reduced.

(4) Increasing wind velocity gives more scintillation, but only up to a certain point. Beyond that, scintillation decreases again, even in the hot sun. At night, likewise, moderate wind mixes the lower air layers to a greater extent and height, thus reducing the temperature gradient and diminishing scintillation. Good seeing conditions can prevail even with 50 km/hr winds outside the dome of the telescope.

(5) Fog, rain, and snow-fall greatly reduce (terrestrial) scintillation.

Extensive turbulence zones, according to Stock and Keller [1960], occur, among others, in two typical situations: (a) in a cold front, and (b) in an inversion layer. Inversion layers are particularly typical of dry climates. At night the surface cools rapidly because of radiation, and little of this radiant energy is absorbed by the atmosphere due to lack of water vapor. Thus cold air accumulates close to the ground. The thickness of this cold air layer steadily increases during the night, reaching, in level areas, heights of several hundred meters. In mountainous areas, cool air from the higher slopes drains off into the valleys, rapidly filling these with cold air. Here the inversion may rise during the night to more than 1000 meters above the valley floor. All of these phenomena are almost invariably accompanied by poor seeing conditions.

3.12. Image distortion and contrast reduction

Turbulence in the atmosphere between a point source and its image will cause the image of that point to be degraded in several ways. The image will fluctuate randomly in both intensity (= scintillation) and in position (= dancing), and it will shift in and out of focus (= pulsation). If the object, instead of a point, is an extended object, the image as a whole will change correspondingly and, in addition, it will deteriorate with respect to the distinctness of its details. While it is fairly simple to improve position determinations, either by aperture or by time averaging, it is quite difficult to correct for image distortion induced by atmospheric turbulence.

It will be obvious that if the single points which constitute the image move independently of, and out of phase with, one another, the details in the image become blurred, they lose contrast, and cannot be recovered from the image. Such contrast reduction is caused, mainly, by two processes: (1) by scattering as in haze and fog and (2) by micro turbulences, that is, by small local air masses of different optical density which cross the line of sight. Such turbulences may occur naturally or they may be induced artificially, for instance by an aircraft from which aerial photographs are taken.

Although the proper mechanics and the orders of magnitude are quite different, both phenomena affect the contrast in about the same way. First, light coming from the source is gradually deflected out of its normal path by scattering and attenuated by absorption and thus lost for viewing the source. Secondly, daylight, if present, is scattered into the path and toward the observer. The balance between these two components determines the luminance of, and the contrast reduction in, the image.

Experimental studies concerning these problems have been made by numerous authors. Macdonald [1949], in wind tunnel tests, found the image deterioration about proportional to air velocity. Similarly, the broadening of images from a point source, taken by aerial photography, could be attributed to the degree of turbulence in the intervening air [Macdonald, 1954]. Riggs, Mueller, Graham, and Moté [1947] found that the average deviation, caused by local turbulences, of rays from a point on the target to the camera is in excess of 3 sec of arc over level ground in bright sunshine; the maximum deviation may reach 9 sec of arc.

Bellaire and Elder [1960] have determined, in part using Landolt broken ring charts, the meteorological conditions and other contributing factors which play a role (a) in producing scintillation and (b) thereby reducing visual resolution. Their experiments were carried out over level terrain with a uniform ground cover under relatively uniform weather conditions and at a path length of 600 m. Scintillation appeared to be primarily a function of vertical temperature gradients and of vertical air movements within the ground layers of the atmosphere.

In a series of publications, Duntley and co-workers have derived equations for the visibility and the reduction of contrast due to both primary and secondary scattering, and for such special cases as visibility upwards, downwards, and horizontal visibility [Duntley, 1947, 1948 a]. This has led to an investigation of problems of camouflage under a variety of conditions [Duntley, 1948 b]. In 1957, Duntley and co-workers have given a quantitative treatment of the apparent luminance of distant objects and the reduction of apparent contrast along an inclined path through the atmosphere. The data, taken from an aircraft in flight, showed some correlation between the humidity profile of the atmosphere and its optical transmission properties.

The effect on image transmission of an inhomogeneous medium can be described in terms of the distortion of a point source. Duntley and co-workers [1952, 1957, 1963, 1964] have shown that the probability of receiving light from an object viewed through a turbulent atmosphere follows a normal Gaussian distribution. This follows from a special central-limit theorem due to Liapounoff which shows that the angular probability distribution approaches a normal distribution whose mean is the position on the screen reached by the undeviated ray. If the medium is in what Duntley and his group call an "optical air state" representing an essentially isotropic sum of turbulences, the density of light rays passing through the turbulence and intersecting the image plane follows a probability distribution the spread of which increases as the 3/2 power of the target-image distance. The main result is that the energy flux from any point on the target decreases according to the inverse square law, that is, inversely proportional to the square of the distance. The contrast, on the other hand, and hence the detectability of any target details falls off at a rate inversely proportional to the third power of the distance. In short, fine detail on a receding target is lost at a greater rate than the flux from the entire target. As a consequence, the detail on a target is lost from view sooner than the target itself. The solution for such a reduction of contrast can be given in terms of an infinite series involving incomplete gamma functions. Peterson [1963] has pointed out the similarity of this approach to the problem of finding the probability of hits on a circular target.

As one would expect for turbulent flow conditions, image resolution deteriorates with an increase in the average vertical temperature gradient. Resolution is best in the absence of thermal stratification and worst in very stable thermal stratification with light winds. With a clear sky, resolution deteriorates as wind speed increases up to about 8 km/hr [Ryznar, 1963 a].

The degradation of photographic resolution depends very much on where a circumscript turbulence is located. Experiments relative to this question have been carried out by Smith, Saunders, and Vatsia [1957]. The target, a National Bureau of Standards test chart, was photographed over a 9 m path. Turbulence was generated by a 500 watt electrically heated grid, 5 x 7.5 cm size, with a small fan behind it. It was found that turbulence near the camera is most troublesome and that increasing the range is equivalent to adding turbulence at the far end of the optical path where its influence is relatively small. Keller and Hardie [1954], on the other hand, came to the concludion that the location of the perturbation region has no influence on resolution. At shorter distances the resolving power is affected considerably by the shutter speed while beyond 150 m the exposure time is relatively unimportant. Mikesell [1959], in contrast, could not find any significant brightness scintillation of a terrestrial source seen at a short distance through a wind tunnel at air speeds up to the equivalent of Mach 0.97. The image of the light source became merely blurred and enlarged by the air turbulence.

In the majority of cases, the contrast in an object will be different from the contrast in the related image. Frequently, the image contrast will be lower, although the opposite can occur, for instance when using high-contrast photographic material for reproducing halftone originals. Thus, we conclude that the passage of light through the medium, through the optic system, and through the detector will cause the contrast in the image to be different from that in the object. The degree of difference will depend on the size of the details; more precisely, it will depend on the spatial frequencies of these details. Generally, as the details become smaller, they are reproduced at lower and lower contrast, and at some detail size the system fails to provide sufficient contrast in the image for detection. The description of this ability, or lack thereof, to transfer contrast from the scene to the image is called the Transfer Function.

The transfer function of a complete system is the product of the transfer functions for each component of the system. This means that every link in an image-forming system can be described by its individual, independent transfer function. A multiplication of these functions gives the transform of the image function. For the present study it would be required to determine, if possible, the transfer functions of the atmosphere, the optical system, and the receiver or detector independently of one another.

Such determinations can be made in a variety of ways. Most common is to produce an image of a given test pattern and to measure what spatial frequencies of object details have been, or which have not been, transferred to the image plane. Experimental arrangements for doing so have been described by many authors; the reader is referred only to Perrin [1960], Murata [1960], Meyer-Arendt [1963], and Ingelstam and Hendeberg [1964].

Middleton [1942, 1952] has shown that the silhouettes of various objects, far and near and seen through haze and fog, have the same sharp contours. But the luminance of an object farther away seems to be more similar to the luminance of the background than to an object near the observer; in other words, it has lost contrast. According to Barber [1950], differences in diffusion have no real influence when small angle measurements are made in clear or hazy weather.

The scattering of light, out of the path as well as into the final image, is mainly due to haze and is not a function of different spatial frequencies of the target details, that is, scattering has no influence on the transfer of contrast at different spatial frequencies; only a general reduction of contrast will result [Scott, 1959]. Under ideal atmospheric conditions, the transfer factor is about 0.8 [Eldridge and Johnson, 1958]. Under more typical atmospheric conditions it may be as low as 0.225. This means that scattering plus turbulence can reduce the contrast transfer values by more than 75 %, a fact that makes the interpretation of aerial photographs so hazardous because rarely a simultaneous determination of the transfer function of the atmosphere is made.

Djurle and Bäck [1961] have measured the combined transfer function for the air plus the camera and then subtracted the camera transfer function. Their light source consisted of an array of fluorescent lamps, lined up in a row so as to give a horizontal illuminated area 10 m long and 12 cm wide. A telephotometer is placed about 11 km from the source. The image produced is scanned by a narrow slit, and the output fed into an oscilloscope (some pulses are seen to be more diffuse than others) and further analyzed by Fourier transformation. In a similar way, transfer function measurements have been made by Rosenau [1962, 1963] in order to assess the influence of real atmospheres on aerial photography.

In an experimental study on the transfer characteristics of image motion and air conditions in aerial photography, Hendeberg and Welander [1963] essentially confirm these findings. Air turbulence and haze, then, are the two main factors which influence and impair the quality of aerial photographs. However, only turbulence, not haze, will affect the contrast differently for different spatial frequencies. As might be expected, Hendeberg and Welander

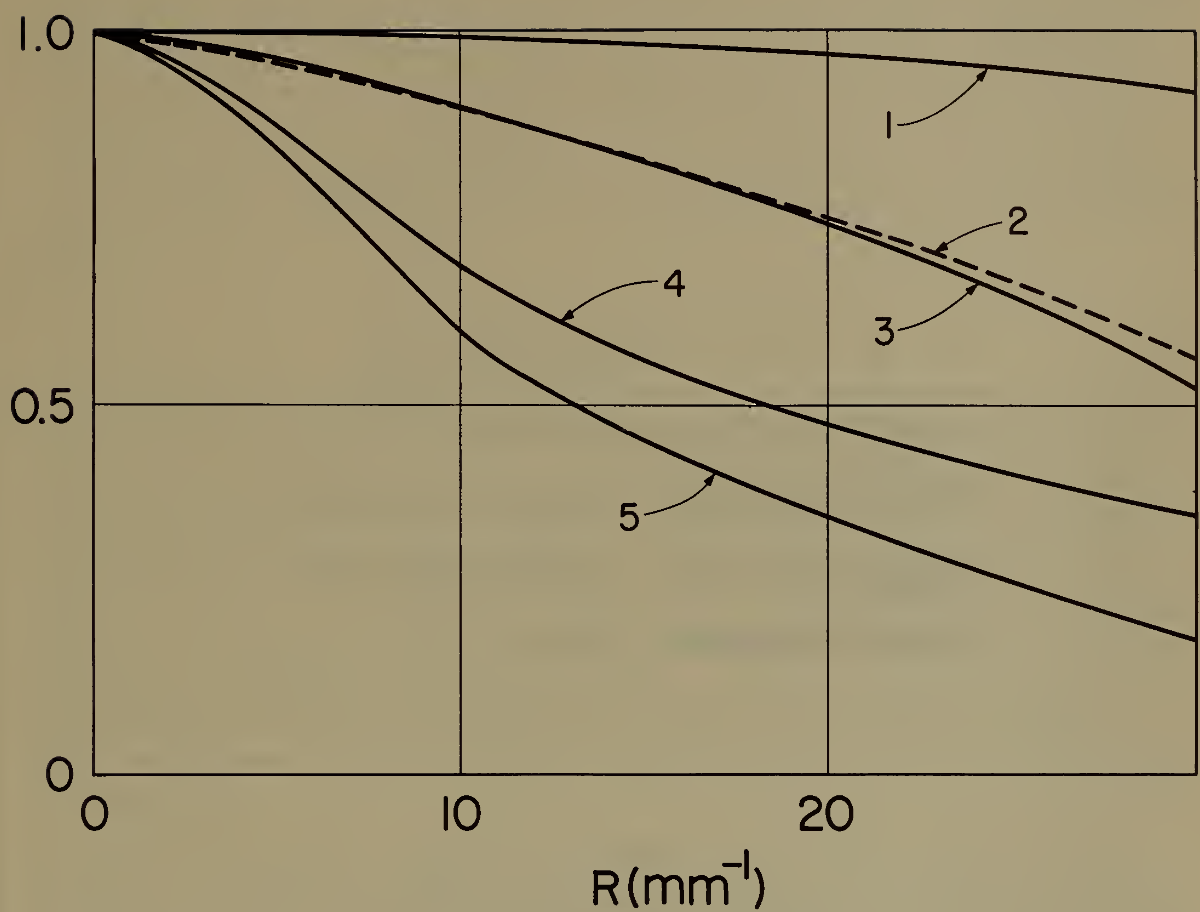

Figure 16

Transfer functions related to image formation through a turbulent atmosphere [after Hendeberg and Welander, 1963]. 1 = theoretical transfer function for an ideal atmosphere and a perfect, aberration-free, diffraction-limited optical system. 2 = transfer function of the optical system alone, determined experimentally. 3 = the same for the photographic emulsion. 4 = transfer function of the optical system plus atmospheric turbulence. 5 = actual transfer function of the complete system.

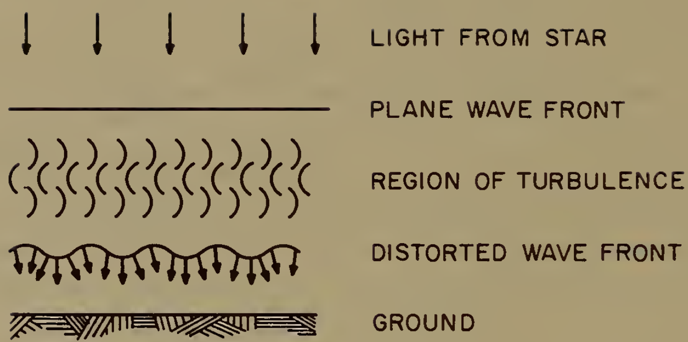

Figure 17
Distortion of a wavefront on passing through a region of turbulence.

found that higher space frequencies are destroyed more than low frequencies, that is, fine details are lost sooner. Some results illustrating this effect are presented in figure 16.

A detailed theoretical study of the image-degrading effects of atmospheric turbulence has been presented by Hufnagel and Stanley [1964]. It is shown that turbulence in the atmosphere between a point object and an optical imaging system causes the image of that point to be degraded in several ways. The authors deduce that the average transfer function is related to the spatial coherence function for the light entering the imaging system. Next an exact closed solution is found for the coherence propagation equation. This yields the desired coherence function in terms of the statistics of the random fluctuations of the atmospheric index of refraction. From here, quantitative predictions of the image degradation in the turbulent atmosphere can be made; these were in good agreement with observed meteorological data.

3.13. The refraction theory of scintillation

The phenomenon of scintillation is most easily explained by refraction in the atmosphere. More specifically, we note that air not only has an index of refraction that is different from that of vacuum, which by convention is set equal to unity, but furthermore that the refractive index of the atmosphere varies as a function of location. Hence, as a plane wavefront of light passes through a region where the refractive index is low, the velocity of the light will be higher and the wavefront will advance farther. When passing through a region of high refractive index, the wavefront is retarded. The result of this is that refractive inhomogeneities in the atmosphere deform plane wavefronts coming from an infinitely distant star into corrugated wavefronts, portions of which are convex in the direction of propagation, while other portions are concave. The concave portions naturally converge, while the convex portions diverge.

Therefore, the light will change in two respects: it will change direction as well as intensity. The intensity on reaching the ground will no longer be uniform (fig. 17). Instead, the light will become concentrated and, if for simplicity we assume cylindrical perturbations, form a pattern of alternately bright and dark bands, the shadow band pattern which we had described before in Chapter 3.2.

These are the essential points of the refraction theory of scintillation which has been proposed first by Robert Hooke[13]. Since then, numerous authors have discussed the action of such "atmospheric lenses", as these refractive inhomogeneities were to be called, among them Respighi [1872], R. Meyer [1955-56, 1956], Queney and Arbey]1955], Wood [1956], Dietze]1957], Becker [1961a], and Glenn [1962].

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13. 1635-1703

At least two distinct regions of refractive anomalies in the atmosphere play a role in scintillation, seeing, and other related phenomena. These are the lower stratosphere and the region near the ground. More specifically, they are the region within 3 km from the level of the tropopause and the first 100 m above the ground. Mirages, for instance, originate close to the ground [Freiesleben, 1951]. Upper air inhomogeneities, in contrast, have little effect on the curvature of the wavefronts as they reach the objective of a telescope and consequently will not cause much of a focus drift or pulsation[14]. But the normals to the essentially plane elemental areas on the wavefronts will oscillate with respect to the optic axis of the telescope [Rösch, 1956; Wimbush, 1961]. This oscillation will cause dancing. With increasing aperture, the transition from image motion to tremor disk will take place at larger apertures than if the turbulent regions were near the telescope [Wimbush, 1961].

Becker [1961b] has described a series of experiments in which these empirical field observations were confirmed experimentally. A vertical column of warm air is introduced in front of the telescope objective lens. The size of a single parcel of turbulence, called a turbulon, was estimated to be less than that of the objective aperture. The effect on telescope resolution was like placing an astigmatic lens in front of the objective. The whole image became displaced as if a negative lens were placed near the objective somewhat off the axis. Since all points in the image were displaced in the same direction, but by varying amounts, the image will then be severely distorted. The magnitude of this sidewise shift varied from 0.25 to 2.5 seconds of arc. No change in displacement was found when the distance of the turbulence from the objective lens was varied from 15 to 180 cm.

The assumption of a divergent-lens effect, obviously, is well justified since the turbulence cells are represented by hot air parcels of lower density and lower index of refraction than the ambient air passing along in front of the telescope objective. Turbulence as it occurs naturally might, however, entail cells of hot and cold air. Such cells would then act as weak positive as well as negative lenses. Experimental work by Glenn [1960] which supports this assumption has been reported before.

It is likely that such turbulence cells, that is, local variations in the index of refraction in the atmosphere, are caused primarily by temperature variations and that variations in water vapor content and of pressure have a negligible effect [Bellaire and Elder, 1960]. For a horizontal path near the ground, scintillation seems even to be entirely dependent upon the vertical temperature gradient along the path.

A good term to describe such irregularities is "blobs", a term coined by Zwicky [1955], who defines aerial blobs as volumes of air of locally different density, temperature, and possibly water content that possess particular optical (refractive) properties. They may be globular, lenticular, or cylindrical in shape and may range in size from millimeters to many

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14. Note, however, that O'Connell [1958] came to the opposite conclusion.

meters. Often hundreds of blobs are fairly regularly spaced and drifting along with the wind at various altitudes up to 50 km or perhaps higher. With decreasing density of the air, it is likely that the elemental size of the blobs becomes larger.

A striking feature of many aerial blobs, according to Zwicky [1955], is their durability and stability. Some of them preserve their shapes for hours, similar to the rather stationary appearance of certain striated cirrus clouds. One may easily see, that if such disturbances settle in the optic path of a telescope, they will defocus and displace the image of stars or change point images which, likewise, may be in or off focus. Depending on the kind and rate of change, the whole spectrum of phenomena such as dancing, wandering, pulsation, focus drift, and scintillation could well originate from refraction in such blobs.

A variety of model experiments have been carried out to show that the refraction theory indeed explains rather well the phenomena of scintillation and seeing. Ellison and Seddon [1952], for instance, have simulated the effect of aerial blobs by surface waves generated in a small water tank by an electrically operated tuning fork (17 Hz) and paddle. A point light source was placed 1. 8 m above the tank, and the light from the artificial star was focused upon the cathode of the photomultiplier through a glass window in the flat bottom of the tank. Waves of small amplitude traveling outwards from the paddle were found to produce brightness scintillation patterns quite similar to the scintillation recorded from a real star at low angles of elevation.

The assumption of refractive inhomogeneities as the cause of scintillation has found thorough theoretical support in Tatarski's [1961] model of a turbulent atmosphere. A further discussion of the theoretical aspects has been given by Reiger [1962]. In fact, it had been discussed as the likely cause of scintillation by as early an observer as Arago.

Experimental methods for determining the radius of curvature of incident wavefronts. We had seen earlier that a large aperture exerts an integrating effect, thus smoothing out image motion and scintillation. When reducing the aperture to about 3 ... 4 cm, however, the twinkling reappears. It is possible, at such small aperture, to determine the radius of the corrugations of the wavefronts received by the telescope. Suppose, following Wood [1956], that at a given instant the wavefront entering the aperture is concave. The light will come to a focus slightly in front of the principal focal plane (the image plane for an infinitely distant object). At another instant, when the aperture is in a dark shadow band where the wavefront is convex, the focal point for this wave will be behind the principal focus. As the dark and light bands sweep across the aperture, the image of the star will alternately appear sharp and blurred. By moving the eyepiece in and out of the tube it is possible to determine the two extreme positions at which sharp point images can still be seen. From there the radii of curvature of the convex and concave portions of the wavefront can be found. Measurements made by Wood [1956] show that the average radius of curvature is about 6 km and that it may range from 1. 8 to 20 km.

If we call a the distance of the focus drift, then the radius of curvature W of the deformed wavefronts, following Pernter and Exner [1922], is given by

$$W = \frac{2 f^2}{a} , \tag{57}$$

where f is the focal length of the objective. Measurements by K. Exner [1882] using a telescope of 4 cm aperture and 170 cm focal length gave values of $0.3 \text{ mm} < a < 3.2 \text{ mm}$. The wavefront deformations then have radii of curvature from 1.8 to 19.3 km with an average of 4.733 km, which is in good agreement with Wood's results.

A similar, but somewhat more sophisticated means of measuring the radius of curvature of the wavefronts is based on the Arago phenomenon. An aperture stop, about 3 to 4 cm in diameter, is placed in front of the telescope objective. If the eyepiece is moved out of focus and farther into the tube, the point image of a star becomes larger, and after a given distance a black distinct spot appears in its center. If the eyepiece is moved still farther into the tube, a fine bright dot appears in the center of the black spot. The eyepiece is left in a position just before the bright spot would appear. If the star does not scintillate, the diffraction figure remains steady as well. In case of scintillation, however, the bright spot in the center will become visible from time to time. Arago counted how often the spot would appear during a given length of time and used this figure as a measure of scintillation.

We may assume that the turbulence elements in the atmosphere have the shape of circular cylinders which differ in temperature by ΔT from the ambient air but have the same pressure p and the same composition. A light ray incident at an angle α would then deviate, following Anderson [1935], by

$$\tau = 0.2 \, (\Delta T) \, (\tan \alpha) \, \frac{p}{p_o} , \tag{58}$$

where p_o is the normal atmospheric pressure. Hence, if $\Delta T = 1°C$, $\alpha = 45°$, and the pressure is about normal, a ray passing through the cylinder would be deviated by 0.2 sec of arc. This is approximately the right order of magnitude, for we have seen that the deviation found under good seeing conditions is of the order of one second, and it is known that temperature discontinuities of more than 1°C are not uncommon.

The changes in refractive index causing stellar scintillation probably arise mainly from convective mixing of air at different temperatures and, to a lesser degree, as far as light waves are concerned, from mixing air parcels of different moisture content. Pressure changes, resulting directly from the turbulent motion, are likewise probably less important except for changes in refractive index on a much larger scale which would be held responsible for such effects as wandering and focus drift. According to Megaw [1954], one might assume, for the lower atmosphere, temperature fluctuations of about 0.1°C, which would give refractive index fluctuations of about 2×10^{-7}. Such interpretation of scintillation in terms of pure refractive index fluctuations has been criticized by Little [1951] as we will see in the following chapter.

3.14. Diffraction theory of scintillation

So far we have assumed that scintillation is due to refractive index gradients present in turbulent air. Little [1951] has shown that this would require rather high density gradients, in excess of 10^{-6}n at distances of a few centimeters, and it also would fail to explain color scintillation. Diffraction theory, in contrast, would require only much lesser gradients.

Little's theory assumes the presence of a more or less distinct turbulence layer which can be compared to an irregular, light diffracting screen which introduces phase variations only. Such a theoretical model has been proposed and analyzed in detail by Booker, Ratcliffe, and Shinn [1950] for the one-dimensional, and by Fejer [1953] for the two-dimensional case.

The principle of the diffraction theory of scintillation is as follows. Wavefronts entering the turbulence zones are essentially plane, but because of the inhomogeneities present, the outgoing wavefronts will be distorted and corrugated, and elemental areas within any one front will be randomly out of phase. The amplitude distribution across a plane immediately below the diffracting screen will be uniform, since the screen was to affect the phase of the wave motion only. As a wavefront propagates some distance farther from the screen, fluctuations of amplitude begin to develop which, of course, are randomly distributed across a plane parallel to the original diffracting layer. The minimum distance at which these amplitude irregularities are fully developed is determined by the size (in all three dimensions of the distortions in the emergent wavefront [Hewish, 1951, 1952; Keller, 1953; Villars and Weisskopf, 1954; Little, Rayton, and Roof, 1956; Wagner, 1962; Briggs and Parkin, 1963; Eckart and Martin, 1963; Lawrence, Little, and Chivers, 1964]).

It can be shown, using phase autocorrelation functions, that the average power spectrum observed behind the diffracting screen is the Fourier transform of the average autocorrelation function of the wave field directly below the diffracting plane. Furthermore, it is found that the width of the autocorrelation function is of the same order as the corrugations in the wavefront, but only if the average distortions in the wavefronts do not exceed about one radian. For an irregular wavefront in which the typical distortion is m radians ($m > 1$), the autocorrelation function is approximately $1/m$ times as wide as the individual irregularities [Little, Rayton, and Roof, 1956] . This means that if the striations seen in a stellar shadow band pattern are 5 cm wide, the irregularities causing them will be of about the same size. Furthermore, if the irregularities in the atmosphere are not isotropic but elongated, probably due to wind, the generalized autocorrelation function of the diffraction pattern at ground level will be a function of the direction across the ground.

The essential feature of using diffraction theory is that much smaller density gradients are needed to produce much the same effect as in the case of refraction. While for the

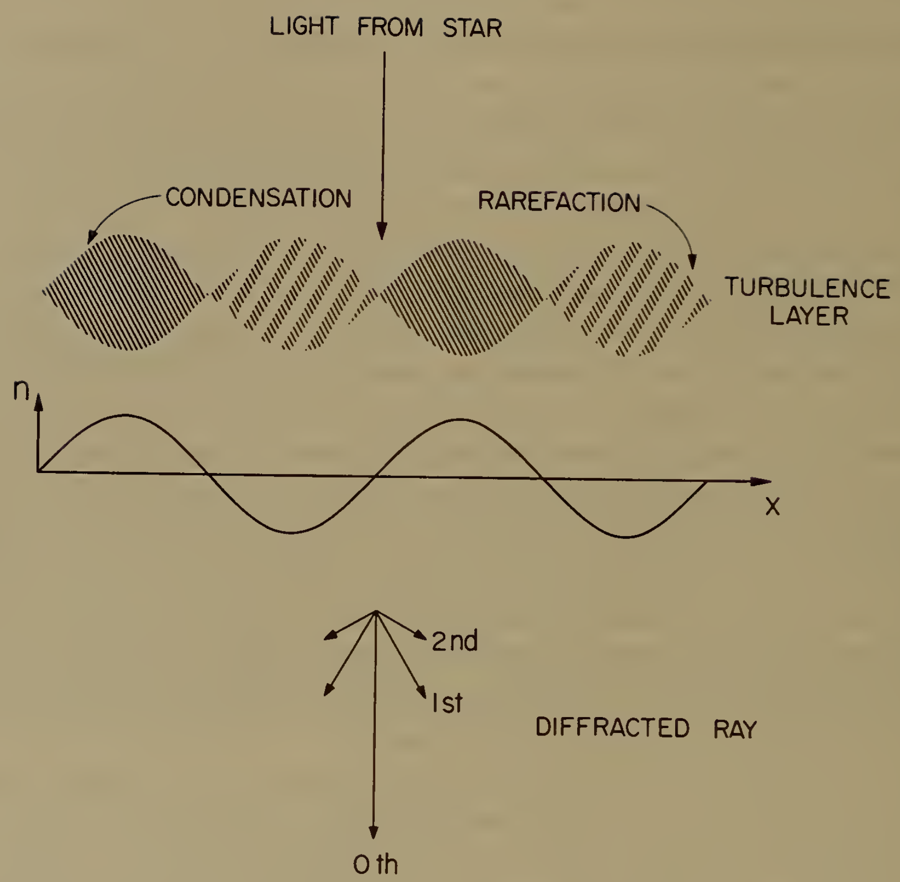

Figure 18

Refractive index variations in the atmosphere acting like a diffraction grating [modified from Keller, 1953].

refraction theory atmospheric density gradients of about 0.5 per cent per cm are required, only about 0.0006 per cent per cm are needed for diffraction. The equivalent atmospheric temperature gradients are 1.5°C cm^{-1} for refraction and 0.002°C cm^{-1} for diffraction [Little, 1951].

For simplicity, we assume first that the turbulent zone in the atmosphere is limited to just one, more or less well defined, layer. We compare such a single layer of random turbulence to a grid consisting of parallel regions of, compared with the average, higher and lower refractive index as shown in figure 18. The sinusoidal plot in the figure indicates these variations in refractive index. The light passing through will be diffracted, producing a series of maxima in much the same way as by a conventional diffraction grating. Only the zeroth, first, and second order maxima are shown.

In a real atmosphere, the diffraction process is not limited to one layer but occurs repeatedly as the light advances. Thus, if there is not just one but many layers through which the ray passes, the intensity of the zeroth order will be much less and more energy will be diffracted out of the original direction. This means that the resulting pattern is broadened and the intensity in the undeviated ray is much reduced.

This total reduction in the intensity of the original "normal" ray can be used as a measure of the sum of the thicknesses of the layers through which the light has passed. When the seeing is good, the diffracting turbulent layer can be compared to a thin, irregular diffracting screen in which the optical thickness of the diffracting elements is so small that most of the incident light passes through the layer in the undiffracted zero-order plane wave. Diffraction in the turbulent layer merely causes faint coronas which surround the star images. Under poor seeing conditions, the images are diffused and broadened. The mean width depends on both the total thickness of the layer and on the mean size of the elements: the smaller the elements, the wider the diffraction pattern. Hence if the pattern width is known and allowance is made for the widening due to the thickness of the layer, the remaining widening is a measure of the turbulent element size [Keller, 1953; Stock and Keller, 1960].

Under average seeing conditions and with a large, 1.5 to 2.5 meter, telescope, the smallest photographic star will be about 2 sec of arc in diameter. This means, following Anderson [1935], that the incident wavefront is so distorted that no appreciable part of it makes an angle greater than about one second of arc with the initial wavefront surface. Theoretically, a star image should be much smaller. If we consider only the diffraction limit of a telescope of aperture D, the angular diameter δ of a stellar image, seen in light of wavelength λ, should be

$$\delta = 5 \times 10^5 \lambda/D . \qquad (59)$$

A good telescope of 1.5 m aperture should, in violet light and under perfect seeing conditions, produce images of stars 0".13 in diameter [Gaviola, 1949]. This almost never can be attained in practice.

The diffraction theory of scintillation and the geometric-optical approach to the same problem are by no means as irreconcilable as they might appear on first sight. There are two principal reasons for this. At first there can be no diffraction without refraction. If the refractive index were the same everywhere, there would be no discontinuities and no diffraction could occur.

Furthermore, there is no refraction in optics, even refraction by a simple lens, which does not, for a rigorous derivation, require diffraction theory. A well known example is the theory of microscopic image formation, a process that only very superficially could be explained by the purely refractive properties of a combination of lenses. Instead, we have to rely on diffraction theory, noting that the structure of the object and the so-called primary interference pattern formed in the back focal plane of the objective lens are "reciprocal" to one another; the one is the Fourier transform of the other. The term reciprocal shows that the diffraction image of the interference pattern, which by itself, likewise, is a diffraction phenomenon, produces an optical image of the (more or less) true shape and structure of the original object. More details on this are found in publications by Abbe [1873], Michel [1950 (pages 203-287)], and Born and Wolf [1959 (pages 417-427)].

In summary, we see, in accordance with Fellgett [1956a, b], that there really is no conflict between the refraction and the diffraction theory of scintillation (and of essentially any other applicable optical phenomenon as well). The validity of the geometric-optical approach or refraction or ray theory lies in its being an approximation to the wave theory. If a correctly derived ray-theoretic result differs sensibly from the corresponding wave result, this would show that the ray approximation is not good enough for the particular problem. For instance, whereas a ray treatment suffices for a discussion of large aberrations, wave optical treatment is needed when the aberrations are small.

Or we may say, following Keller and Hardie [1954], that if the temperature differences among the turbulent elements are large and the elements themselves are large, geometrical refraction dominates and the seeing is poor. If the elements are smaller and of more uniform temperature, the seeing will be influenced by diffraction effects and be better. If there are a great many such elements in the line of sight, however, the seeing may be poor as well.

Booker and Gordon [1950a] have shown that the validity of the geometrical optics approximation can be ascertained by the value of the quantity $s^2/\lambda r$, where s is a representative size of a turbulent element, λ is the wavelength of light, and r is the distance from the turbulence to the observer. If

$$s^2/\lambda r >> 1 , \tag{60}$$

the geometrical optics approximation is sufficient, otherwise wave theory is required.

We follow Tatarski [1961] and assume that an obstacle with geometrical dimensions s be located in the path of a plane wavefront. At a distance L from this obstacle we obtain the image (shadow) with the same dimensions s. At the same time, diffraction of the wave by the obstacle will occur. The angle of divergence of the diffracted (scattered) wave will be of the order $\theta \sim \lambda/s$. At a distance L from the obstacle, the size of the diffracted bundle will be of the order $\theta L \sim \lambda L/s$. Clearly, in order for the geometric shadow of the obstacle not to be appreciably changed, the relation

$$\frac{\lambda L}{s} << s \quad \text{or} \quad \sqrt{\lambda L} << s \tag{61}$$

must hold. When there is a whole set of obstacles of different geometric sizes, it is obviously necessary that this relation be satisfied for the smallest obstacles, that is, the lower limit of the size of the turbulences which size limit we call s_o. Thus, the geometric-optical approach to the problem of scintillation is valid only for limited distances L which satisfy the condition

$$L << \frac{s_o^2}{\lambda} . \tag{62}$$

3.15. Analysis of turbulence element fluctuation

At least certain strata of the atmosphere are in continuous turbulent motion. This motion is caused by, and will produce, at any given instant of time, gradients with respect to pressure, temperature, and possibly other parameters.

We follow Stock and Keller [1960] and express the refractive index difference Δn between a given air parcel and its surroundings as

$$\Delta n = \frac{\Delta \rho}{\rho_o} (n_o - 1) , \tag{63}$$

where ρ_o and n_o refer to the air density and refractive index at normal pressure and temperature. If we assume that all velocities found in natural atmospheric turbulence are far less than the speed of sound, then density differences between adjacent air elements are the result of temperature differences and not pressure differences. Under these conditions, for a perfect gas,

$$\frac{\Delta \rho}{\rho} = \frac{\Delta T}{T} , \tag{64}$$

where T is the temperature. From there it follows that

$$\Delta n = \frac{\Delta \rho}{\rho_o} \left(\frac{\rho}{\rho}\right) (n_o - 1)$$

$$= \left(\frac{\Delta \rho}{\rho}\right) \left(\frac{\rho}{\rho_o}\right) (n_o - 1) , \tag{65}$$

and substitution of (64) into (65) yields

$$\Delta n = \left(\frac{\Delta T}{T}\right)\left(\frac{\rho}{\rho_o}\right)(n_o - 1) \ . \tag{66}$$

If the element is close to the telescope and the latter is at sea level and if $\lambda = 4700$ Å when $(n_o - 1) = 2.9 \times 10^{-4}$, one has

$$\Delta n \approx - 10^{-6} \, \Delta T \ .$$

The turbulence characteristics of a medium can be related to the density gradient by the austausch (meaning: exchange) coefficient K_N which itself is a function of stability, height above ground, and wind speed [Bean and McGavin, [1963b]. Following Stewart [1959] and Tatarski [1961], a fluid is called turbulent if each component of the vorticity is distributed irregularly and aperiodically with time and space, if the flow is characterized by a transfer of energy from larger to smaller scales of motion (eddies), the energy ultimately being absorbed into random molecular motion, and if the mean separation of neighboring fluid particles tends to increase with time. Whether or not a flow is turbulent is not simply a matter of Reynolds number, since the stability of the flow is a criterion of at least equal importance. One can assume that, with the exception of strong inversion layers, the atmosphere is turbulent everywhere, although the intensity of the turbulence varies widely in both space and time. Note that the movement of turbulence does not necessarily mean that the air parcels themselves are moving, at least not at the same velocity.

Kolmogorov's theory, following Hinze [1959], embodies the fundamental concept that the eddy motions are characterized by a wide range of length scales resulting in the following hypotheses: (1) For energy exchange processes at large Reynolds numbers, the small-scale components of the motion depend only on the viscosity (η) and on the mean dissipation of energy per unit mass of fluid (ε). This equilibrium range is called "universal" because the turbulence in this range is independent of external conditions, and any change in the length and time scales can only be a result of the effect of the internal parameters η and ε . (2) When the Reynolds number is infinitely large, the energy spectrum is independent of η and is solely determined by ε . This state of turbulence is called the "inertial subrange".

Dimensional analysis, following Tatarski [1961], shows that the size of the eddies directly influenced by viscosity is of the order of

$$\left(\frac{\eta^3}{\varepsilon}\right)^{1/4}$$

In the lower layers of the atmosphere, ε is about 5 cm^2 sec^{-3} [Sutton, 1955] so that the viscosity-influenced eddies must be very small, of the order of 1 centimeter at the most. The largest atmospheric eddies, on the other hand, are associated with length scales of the order of hundreds of meters or more. Thus the subrange of small eddies in the atmosphere to which Kolmogorov's second hypothesis applies is very large. Tatarski [1961] gives a thorough statistical description of wave propagation in a turbulent medium, employing the Kolmogorov theory of turbulence. The theory has been discussed further in great detail by Villars and Weisskopf [1954].

If we plot the energy contained in a turbulence element as a function of wave number, which is inversely proportional to the wavelength or eddy scale, we obtain a spectrum of turbulence. Under ideal conditions, such a plot will have the shape of a bell-shaped probability distribution curve [Taylor, 1938; Rouse, 1963].

We have discussed before the influence exerted by aerial "blobs" on the propagation of light coming from a celestial source. The characteristics of such blobs may be described most conveniently by the "parcel method", which tacitly assumes that (1) as the parcel moves, no compensatory motions take place in the environment, and (2) the parcel retains its identity, that is, there is no mixing of the parcel with its environment.

As will be obvious, neither of these two assumptions can be rigorously justified. A further assumption, requiring the environment to be in hydrostatic equilibrium is also made. Such an analysis, following Hess [1959], leads to the following expression for the vertical acceleration of the parcel

$$\ddot{z} + \frac{g}{T_o} (\Gamma_d - \gamma) z = 0 \qquad (67)$$

where $\ddot{z}$ is the acceleration of the parcel in the vertical upward direction, g is the acceleration due to gravity, T_o is the absolute temperature of the parcel at $z = 0$, and Γ_d and γ are the environmental and adiabatic lapse rates, respectively.

We now consider the following cases:

i) The coefficient of z is positive and the solution of 67 is a sinusoidal function of time. The parcel will therefore oscillate about its original position with a period given by

$$\tau = 2\pi \left[\sqrt{\frac{g}{T_o} (\Gamma_d - \gamma)} \right]^{-1} \qquad (68)$$

This is the <u>stable case</u>.

ii) The coefficient of z is negative and the solution is given in terms of exponentials of time. In this case the displacement will increase indefinitely. This is the <u>unstable case</u>.

iii) The coefficient of z is equal to zero; the displaced parcel does not accelerate at all. This is the <u>neutral case</u>.

The sign of the coefficient of z in (67) is solely determined by the relative sizes of Γ_d and γ. We then have

$\Gamma_d > \gamma$: Stable

$\Gamma_d = \gamma$: Neutral

$$\Gamma_d < \gamma : \text{ Unstable} .$$

We note here that in case the parcel is saturated, Γ_d is replaced by Γ_s , the moist adiabatic value.

In summary, we have

1) $\Gamma_d > \gamma$: Absolutely stable

2) $\Gamma_s = \gamma$: Saturated neutral

3) $\Gamma_s < \gamma < \Gamma_d$: Conditionally unstable

4) $\Gamma_d = \gamma$: Dry neutral

5) $\gamma > \Gamma_d$: Absolutely unstable .

We shall not, at this place, go further into the details of the aerodynamic and meteorological aspects of atmospheric turbulence. For such details the reader is referred to a comprehensive article by Priestley and Sheppard [1952]. For further details see Kolmogorov [1941, 1958], Obukhov [1941, 1953, 1958a,b, 1962], Sutton [1955], Scheffler [1958, 1959, 1960], Hinze [1959], Tatarski [1961], Pasquill [1962], and Beckmann [1964], and the Air Weather Service bibliography on high-altitude clear-air turbulence in the atmosphere.

The phenomenon of scintillation, as we have seen, may be explained either by geometrical or by wave optics. The former is easier, the latter more rigorous. In any case the treatment must be statistical, because we do not know the actual distribution of densities and refractive indices in the atmosphere nor the actual distribution of the light on the ground. A number of papers have been published which deal with the statistical aspects of this problem. We mention only Liapounoff [1901], Staras [1952], Krasilnikov and Tatarski [1953], van Isacker [1953, 1954], Williams [1954 c], Iudalevich [1956], Kaiser [1956], Tatarski [1956, 1961], Goering [1958], and Beckmann [1964].

Consider the passage of light through a vacuum. The optical path length S will be

$$S = c\,t , \tag{69}$$

where c is the velocity of light and t the time. In an homogeneous medium of refractive index n_o ,

$$S = n_o\, r , \tag{70}$$

where r is the distance from source to receiver. In a turbulent medium, we expect, (70) is no longer valid.

To derive the length of the path in a turbulent medium, we start, following Williams [1954 c], from Huygens' principle. If the surfaces S = constant and $S + dS$ = constant represent adjacent wavefronts, then the separation of the wavefronts is

$$d\sigma = \frac{dS}{n} , \tag{71}$$

where the separation $d\sigma$ is an element of distance measured along the wavefront normals. Equation (71) then means that the normal derivative of the function S is equal to the value of the refractive index. In vector notation, this becomes

$$(\nabla S)^2 = n^2 , \qquad (72)$$

where ∇ denotes the vector gradient operator. This equation is sometimes referred to as the mathematical formulation of Huygens' principle. It also is the equation of the eikonal or wave front and the fundamental equation of geometrical optics [Williams, 1954a].

Equation (72) is a partial differential equation that the variable S, considered as a function of position, must satisfy. We can assume that the refractive index in a given parcel of air in a turbulent atmosphere is

$$n = n_o + \Delta n ,$$

where n_o is constant and Δn represents small variations due to turbulence. We omit intermediary steps (which are found in Williams' report) and state that, with the origin of rectangular coordinates taken at the source, the optical path length from the source to a point P with coordinates (x, y, z) is

$$S = n_o r + \beta \int_0^r n'(\rho \frac{x}{r}, \rho \frac{y}{r}, \rho \frac{z}{r})\, d\rho - \frac{\beta^2}{2n_o} \int_0^r \left[\frac{\bar{r}}{r} \times (\nabla S_1)_\rho \right]^2 d\rho + \ldots \qquad (73)$$

Here,

$$\frac{\bar{r}}{r} \times (\nabla S_1)_\rho = \frac{1}{\rho r} \int_0^\rho \rho_1\, \bar{r} \times (\nabla n')_\rho \, d\rho_1$$

and $(\nabla S_1)_\rho$ and $(\nabla n')_\rho$ denote the values of ∇S_1 and $\nabla n'$, respectively, at a point $(\rho \frac{x}{r}, \rho \frac{y}{r}, \rho \frac{z}{r})$ which lies at distance ρ from the source on the line from the source to point P (x, y, z).

If (73) is written in a simplified, and more general form, one can show, following Bergmann [1946], that the mean optical path length through a randomly inhomogeneous medium is equal to the actual distance traversed by the light:

$$< S'^2 > = 2 * \int_0^r (r - s)\, Q\, (s)\, ds \; . \qquad (74)$$

In this equation, $< S'^2 >$ is the averaged optical path length, the prime denotes a first-order variable, the symbol $* \int_0^r$ refers to integration along a straight line from the source to the point given by the radius vector $\mathbf{r}$, r is the distance between source and receiver, s is the variable of integration, and Q (s) is an autocorrelation function.

The theory of autocorrelation functions and their application to the specific problem on hand have been treated by many authors. We mention only Booker, Ratcliffe, and Shinn [1950], Keller [1952, 1955b], Nettleblad [1953], Wheelon [1959], and Reiger [1962]. We follow Ruina and Angulo [1957, 1963] and consider a point source of radiation in a medium with a random space variation of the index of refraction. We also assume that the spatial autocorrelation function for the refractive index is a function only of the distance between the two points at which the index is measured and not of the location of the points. If the width of the autocorrelation function for the index is r_o (in dimensionless units of wavelengths) which is assumed to be much greater than 1, then the medium can be thought of as consisting of blobs of linear dimensions $\approx r_o$ with statistically independent values for the refractive index. A ray emanating from the point source after traveling through one blob will have a mean square deviation from the mean of its phase front equal to $r_o^2 < \delta^2 >$, where $< \delta^2 >$ is the mean square fluctuation of the refractive index. At a distance R from the source, the ray will have propagated through R/r_o uncorrelated blobs, and the mean square deviation from the mean of the phase front is $Rr_o < \delta^2 >$ [Ruina and Angulo, 1957].

Keller and Hardie [1954] have applied correlation functions to establish the quantitative relationship relating atmospheric turbulence and the distribution of intensity in the seeing image. If we assume that the turbulence is essentially isotropic, then the following two equations were found to hold:

$$\rho(\xi, \eta) = \frac{I}{I_0} \int_{-\infty}^{\infty} d\theta \int_{-\infty}^{\infty} d\phi I(\theta, \phi) \times \exp\left[2\pi i \frac{\xi\theta}{\lambda}\right] \exp\left[2\pi i \frac{\eta\phi}{\lambda}\right] = \exp\{-4\pi^2[S(o,o) - S(\xi, \eta)]\}, \tag{75}$$

and

$$S(\xi, \eta) = \frac{2F}{\lambda^2} \int_0^{\infty} \sigma(\xi, \eta, \zeta)\, d\zeta . \tag{76}$$

Here,

$I(\theta, \phi)$	is the angular intensity of radiation,
λ	is the wave length,
$\rho(\xi, \eta)$	is the autocorrelation function of the electric field strength of the incoming wave front,
$\sigma(\xi, \eta, \zeta)$	is the autocorrelation function for fluctuations in the index of refraction, and
F	is equal to $\sum_\mu (\Delta z)_\mu \delta^2 f_\mu$, where $(\Delta z)_\mu$ is the thickness of the μth turbulent layer and $\delta^2 f_\mu$ is the corresponding mean square fluctuation in the index of refraction.

It was assumed in the experiments that $\rho(\xi, \eta)$ exhibits cylindrical symmetry, so that only one independent variable is needed to define the quantity ρ. We may then write, still following Keller and Hardie [1954],

$$\rho(\xi) \equiv \rho(\xi, o) = \frac{I}{I_0} \int_0^{\infty} d\theta \exp\left\{2\pi i \frac{\xi\theta}{\lambda}\right\} T(\theta) , \qquad (77)$$

where

$$T(\theta) = \int_{-\infty}^{\infty} I(\theta, \phi)\, d\phi .$$

In principle, $T(\theta)$ can be measured in any particular case by measuring the amount of light passing through an infinitely long, infinitely narrow slit placed in the focal plane of a perfect telescope, the length of the slit being parallel to the y direction and the center of the slit being an angular distance θ from the center of the image.

Fejer [1953] has derived a relation between the angular power spectrum of waves emerging from a thin diffracting screen random in two dimensions and the autocorrelation function describing the irregularities of the field as it emerges from the diffracting screen. According to Keller [1955a], there is a close statistical relationship between the pattern of stellar shadow bands seen on the surface of a telescope objective and the scintillation of the total starlight received. The light and dark patches in the shadow pattern are related, at least statistically, with patches of hot and cold air in the upper atmosphere.

It is sometimes instructive, following Stock and Keller [1960], to study the relative amplitudes of fluctuations of various dimensions in the shadow pattern. This information can be expressed statistically as a spectrum function, B, also called power spectrum or power spectral density. This function is essentially the Fourier cosine transform "of the auto-correlation function Q. For the isotropic case,

$$B(w) = 8\pi \langle h^2 \rangle_{Av} (\Delta r)_o^2 \exp\left[-2\pi^2 (\Delta r)_o^2 w^2\right] , \qquad (78)$$

where w is the wave number in cm^{-1} of a Fourier component; $2\pi w B(w)\Delta w$ is the sum of the squares of all the Fourier coefficients of the intensity fluctuation with wave numbers lying in an interval Δw around w; and $\langle h^2 \rangle_{Av}$ is the mean-square amplitude of intensity fluctuation in the pattern. As might be expected, B decreases rapidly for wave numbers larger than $1/(\Delta r)_o$" [Stock and Keller, 1960].

It has been shown by Keller and coworkers [1956] and by Stock and Keller [1960] that in most cases, excluding very severe turbulence, the relation between the spectrum functions for the turbulences $b(w)$ and for the shadow pattern $B(w)$ is

$$\frac{B(w)}{h_o^2} = \frac{16\pi^2 \Delta z\, b(w) \sin^2(\pi z \lambda w^2)}{\lambda^2} , \qquad (79)$$

where λ is the wavelength of the light in centimeters. The formula indicates in a general way the manner in which atmospheric conditions are related to the characteristics of the shadow pattern and hence to the amplitude of scintillation. Large temperature fluctuations cause large differences in the index of refraction, hence large values of b and B. A thick layer produces a larger B than a thin layer. If the star is not at the zenith, then the formula holds if z and Δz are taken to be, respectively, the slant distance along the light-path from the telescope to the layer and through the layer. Thus, at large zenith distances, Δz becomes large, and the $\sin^2$ term increases until it begins to oscillate between 0 and 1. Thus it is not surprising to find, as Mikesell, Hoag, and Hall [1951], Protheroe [1955], and others have observed that scintillation is usually stronger for stars at large zenith distances.

"For stars near the zenith and values of z, λ, and w of practical interest, we usually have $\pi z \lambda w^2 < 1$, and the foregoing formula can be simplified to read

$$\frac{B(w)}{h_o^2} \approx 16\pi^4 z^2 \Delta z\, w^4\, b(w) \quad . \tag{80}$$

In this form two other observational facts are easily understood. First, no dependence on λ appears, and hence the lack of color dependence of the scintillation found by Mikesell et al. and Protheroe follows. Second, the right-hand side of the equation decreases rapidly as w decreases, which accounts for the absence of large element sizes in observed shadow patterns, even though there is reason to believe that large turbulent elements do, in fact, occur in the atmosphere" [Stock and Keller, 1960] .

Barnhart, Keller, and Mitchell [1959] have obtained fairly typical autocorrelation functions for stars near the zenith which can be approximated by the expression

$$Q = \exp - \frac{(\Delta r)^2}{2(\Delta r)_o^2} \quad , \tag{81}$$

where Δr is the separation of the correlated points in centimeters and $(\Delta r)_o$ is of the order of 2 cm. The Q function can be described as shown in (14) in a publication by Keller [1955b].

Liepmann [1952] has calculated the deflection of a light beam passing through a boundary layer of turbulent air. The deflection follows, in general, Snell's law. The curvature of the ray depends on two terms: one due to the gradient of the refractive index fluctuations and one due to the mean gradient interacting with the fluctuations. Root mean square deflection is defined as the diffusion of the ray. This can be expressed in terms of the correlation function of the refractive index gradient. A crude estimate for a typical case is given. For mach 2, Reynolds number 10^7, at sea level, the root mean square deflection is found to be approximately 4×10^{-4} radians.

The signal is fed into a magnetic tape recorder and played back through a frequency analyzer. The output is expressed as the ratio of the equivalent sine wave modulation per unit band width to the mean dc level of the signal. The resultant frequency spectrum can be used to determine the fractional r. m. s. deviation of the scintillation signal over the entire frequency range. By observing the scintillation through a two-hole diaphragm, therefore, one can derive the Fourier spectrum of periodicities in the shadow pattern and the autocorrelation function of the pattern.

In general, the statistical properties of stellar shadow band patterns may be determined photoelectrically by either of two techniques, namely by finding the autocorrelation function of the pattern using a twin telescope of variable separation [Keller, 1955b; Protheroe, 1955; Barnhart, Keller, and Mitchell, 1959], or by finding the spatial power spectrum of the pattern using an optical Fourier analyzer. The latter technique has been described by Protheroe [1961].

An optical Fourier analyzer operates by imaging the shadow band pattern from a given star upon a transmission grating. This grating ideally would have a sinusoidal transmission function in one direction and a constant transmission normal to that direction, although Ronchi grids which have a square wave function can be used. The total light transmitted by the grating will be the product of the grating transmission function and the spatial intensity function of the shadow band pattern. Protheroe's [1961, 1964] Fourier analyzer forms a rather simple attachment for an astronomical telescope. Essentially, it consists of an opaque screen with a small aperture in it, placed in the prime focus of the telescope. The aperture acts as a field stop, admitting light from a single star only. A lens is mounted at some distance behind the pinhole; this lens, actually a set of three lenses of 5, 7.5, and 10 cm focal length which alternately can be brought into the path, produces an image of the telescope objective onto a Ronchi grid used as the analyzer. Two Ronchi grids, with 10 and 20 lines/cm, are mounted on a lever operated slide, such that either one may be placed in the optical path. The slide in turn is mounted in a holder which can be rotated through 360° . A divided scale permits the angular position of the grating to be determined by 5° intervals from -10° to 190°. The light transmitted through the grid is collected by an end-on photomultiplier placed closely behind the grating. The signal from the photocell is amplified, stored in a tape recorder, and then subjected to a frequency analysis in a heterodyne waveform analyzer.

All observations were made with the analyzer grids in two orientations, with the axis of constant transmission parallel to and perpendicular to the shadow pattern motion. The proper orientation was determined by finding the angular position of the grating at which the photocell output has a minimum high frequency content. From a harmonic analysis of the recordings, the peak frequency was found which in turn gives the magnitude of the shadow pattern velocity. The squares of the amplitudes were integrated numerically to find the total power of each signal [Protheroe, 1964].

Correlation functions, in connection with scintillation studies as discussed here, can be determined with respect to space and time. How optical autocorrelation can be measured in two-dimensional random patterns has been described by Kovasznay and Arman [1957]. Keller [1955] has shown that the average root mean square deviation of the combined values of the brightness of a star, observed simultaneously through two telescopes, is directly related to the average spatial autocorrelation function of the shadow band pattern, the distance variable of the autocorrelation function being related to the separation of the telescopes [Protheroe, 1955]. A simpler arrangement is to use a single telescope of large aperture and an objective diaphragm with two holes, about 2.5 cm in diameter[15] and separated by distances variable from 2.5 to 27.5 cm. If the light from the two holes is collected by a single photomultiplier, the output gives the sum of the brightness measure of the star which would be obtained using two independent telescopes.

Tatarski [1961] had assumed that the discrete turbulence discontinuities themselves do not appreciable change while moving through the optic path. This assumption is known as "frozen-in" turbulence. Tatarski's theoretical analysis, furthermore, is based on the assumption that the structure function of the index of refraction is proportional to the 2/3 power of the separation distance and holds for the conditions

$$d_o << (\lambda S)^{1/2} << S_o \tag{82}$$

where d_o is the microscale of turbulence, S_o is the external scale, λ the wavelength of light and S the length of the optical path.

Similarly, Megaw [1954] assumes that the atmosphere behaves as if it were a homogeneously turbulent stream with a uniform mean velocity, large compared with the turbulent velocities, across the line of sight. The relationship between wave number and fluctuation frequency is then

$$k = \frac{2\pi f}{U} , \tag{83}$$

where U is the mean velocity.

It is important to note that even in shear flow, when the turbulence as a whole is markedly inhomogeneous and anisotropic, these considerations will apply to the smaller eddies, provided Reynolds' number of the turbulence is very large, which is likely even locally in a stratified atmosphere. For these fluctuations, the smaller eddies produce the largest effect, which is easily understood in terms of ray optics.

"For a homogeneously turbulent medium the following factors enter as the eddy size under consideration is reduced: (a) more eddies per unit path length ($\propto k$); (b) the transverse gradient of refractive index, for a given 'eddy strength', increases ($\propto k^2$); (c) the 'eddy strength' decreases ($\propto k^{-5/3}$). The powers of k in brackets represent the corresponding dependence, on the wave number, of the spectral density of the mean-square brightness fluctuation, and the net result is $\propto k^{+4/3}$; or, if we prefer to think in terms of root-mean-square (r.m.s.) brightness fluctuation, $\propto k^{+2/3}$. This simple result, subject to the uncertainty of the precise relationship between the refractive-index spectrum and the velocity spectrum, is the basic form of the spectrum of stellar-brightness fluctuation [Megaw, 1954]."

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15. The holes have to be small compared with the size of the turbulence elements in the air.

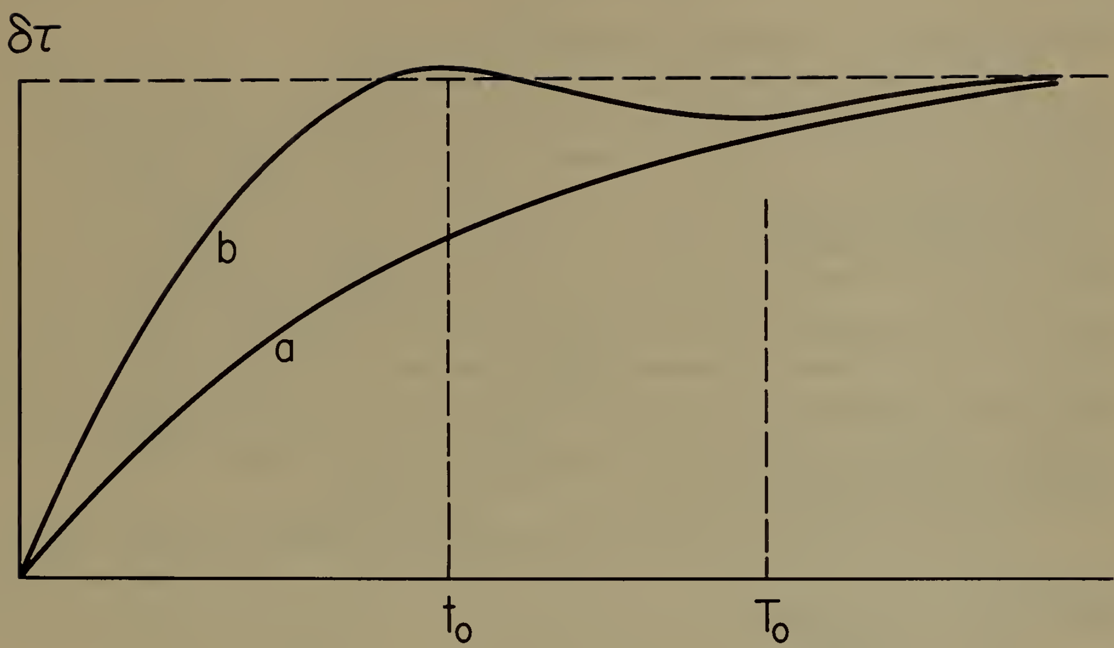

Figure 19

After-effect function (a) for a purely random process, (b) for a process consisting of an almost irregular sequence of pulses of duration t_o and average interval T_o [from Fürth, 1956].

A different approach for obtaining the effective mean size of the turbulence elements has been used by Fürth [1955, 1956]. Instead of direct observation of the patterns, Fürth analyzes what is called the "after-effect function", defined by

$$\Delta(\tau) = \overline{f(t+\tau) - f(t)} , \tag{84}$$

where the average is taken over a sufficiently long time. The "reduced after-effect function",

$$\delta(\tau) = \frac{\Delta(\tau)}{\Delta(\infty)} \tag{85}$$

starts from zero at $\tau = 0$ and approaches unity for $\tau = \infty$.

This function is not too difficult to compute from observational data [16] and in many cases gives a rather direct indication as to the type of elementary process from which a stochastic phenomenon is built up. For instance, the after-effect function pertaining to a strictly periodic function f (t) will also be periodic. Thus if the elementary processes consist of a regular succession of "pulses" of duration t_o at equal time intervals T_o, the function $\Delta(\tau)$ is also periodic with period T_o. But any irregularity in the sequence of the pulses will destroy the long range order of the diagram, and it will eventually assume the shape of curve (a) of figure 19, when the frequency has become completely random. An intermediate stage is shown by curve (b) of the same figure with only the first maximum and minimum left. The position of the maximum gives, then, the approximate pulse duration t_o and that of the minimum average time interval T_o between the pulses.

The method of the after-effect function has been successfully used not only on star scintillations, but also for the analysis of records of Brownian movement, density and electric current fluctuations, and others.

Ellison and Seddon [1952] had observed that very small apertures produced very high and steep narrow peaks of durations between 0.005 and 0.01 seconds which they interpret as suddden "flashes". However, the appearance of high and narrow peaks is a common feature of all continuous fluctuation records and can easily be accounted for on the basis of the general theory of fluctuations. Nevertheless, the width of these peaks is a measure for the characteristic relaxation time θ of the fluctuation mechanism.

Conclusions Concerning the Size, Height, and Velocity of Motion of the Turbulence Elements. There can be not much doubt that individual air parcels or turbulence elements cover a very wide range of sizes. In an airplane, one easily feels bumps which are at least many meters in size. Terrestrial scintillation experiments show elements of a size of a few centimeters. R. W. Wood [1956] was able to observe the difference between the scintillation for the left and for the right eye and concluded that most striae are smaller than 7 cm.

16. Fürth has used the data obtained by Ellison and Seddon [1952]

We assume that turbulence elements range in size from molecular dimensions up to kilometers. In any case, they are large compared with the wavelength of light. Large elements, passing across the optical path, will act as prisms, producing the sidewise shifting of a star image known as dancing. Small elements will cause a diffusion and boiling of the image which otherwise, however, will be stationary. The effects of small elements, as we have seen before, will be changed greatly by varying the aperture of the telescope.

Table 4

Estimated sizes of turbulence elements.

Author	Size Estimate	Comments
Wood, 1956	< 7 cm	
Gaviola, 1949	5 cm	
Little, 1951	5 cm	for both refraction and diffraction theory
Chandrasekhar, 1952	10 cm	
Ellison and Seddon, 1952	7...7.5 cm	
Ellison, 1953, 1954	7 cm	
Inoue, 1954	$>$ 1...2 cm	
Megaw, 1954	30 cm	
Keller, 1955b	4...8 cm	$\Delta T \sim 0.05 \ldots 0.2^\circ C$
Keller, Protheroe, Barnhart, and Galli, 1956	20 cm	
Gardiner, et al., 1956	15...20 cm	
Protheroe, 1955, 1961	12.5...25 cm	elongated in direction of motion
Fürth, 1955, 1956	30...40 cm	
Scott, 1959	20 cm	
Meinel, 1960	10 cm	
Reiger, 1962	10 cm	$\Delta T \sim 0.01^\circ C$

In table 4, some numerical results are given for the size of the turbulence elements. If we call s the (horizontal) size of an element, v its velocity, and D the diameter of the aperture of the telescope objective, then the time length t_p of a pulse produced by such an element moving by is

$$t_p = \frac{D}{s\,v} . \qquad (86)$$

This holds for elements that are larger than the aperture.

From these considerations and from Ellison and Seddon's records, Fürth [1956] comes to the conclusion that the linear dimensions of the turbulence elements, under a variety of observational conditions, are very nearly equal; namely, 33...38 cm.

Ellison [1953, 1954] assumes that the irregularities subtend an angle of 3 seconds of arc; from this, one arrives at a mean distance of 5.2 km. This figure, according to Ellison and Seddon [1952], may well be in error by a factor of 2, but it is of interest to note that it is of the same order as the value of 4 km derived by Rayleigh from his discussion of the refraction theory of scintillation which was based upon the early visual observations of Montigny and Respighi. From this average distance of 5.2 km of the turbulent elements which were observed at 30° angle of elevation, the vertical height is found to be 2.6 km.

Gardiner and co-workers [1956] agree that scintillation, or fluctuation in image intensity, is generated predominantly at heights above 6 km. Measurements of the correlation of the scintillation in the components of double stars lead to heights as high as 30 km for low frequency (1-10 Hz) scintillations and from 3 to 12 km for high frequencies (150 Hz). These authors have confirmed these height estimates by observations on balloon borne lights. They found that the lights started to twinkle at a height of 7.5 km. By the time the balloon was 15 km high and about 30° distant from the horizon, well above both the tropopause and the fast winds of the troposphere, its twinkling roughly matched that of adjacent stars. Frequency analyses indicated that the low frequency scintillation (1 - 10 Hz) is generated at heights between 15 and 30 km, while the high frequencies (150 Hz) were produced between 3 and 12 km.

Gardiner and co-workers [1956] also explain why low frequency scintillation has been detected primarily at great heights, whereas high frequencies seem to be associated with levels at or near the tropopause. In the troposphere, in middle latitudes, westerly winds ordinarily prevail at all heights and attain maximum speeds near the tropopause. Above the tropopause, in the stratosphere, a reversal may occur, the winds become easterly and light. "Now since the light path between star and telescope is turning, at about 15° per hour from east to west, density inhomogeneities in an atmosphere at rest (with respect to the earth) will cross the light path from west to east. This effect, negligible at low levels in the atmosphere, becomes appreciable at great" heights. At 30 km, "for example, it is equivalent to the translation that would be produced by a west wind of about 5 mph.

Considering now the strong westerly winds usually found near the tropopause level, and the light easterlies in the stratosphere, it is clear that the effect of translation of lumps of air is to produce relatively high frequency scintillation at the level of the maximum westerlies, and relatively low frequency scintillation in the easterly flow [Gardiner and others, 1956]."

Fürth [1956] estimates the height of the turbulence elements from the following consideration. If diffraction effects were negligible, the schlieren pattern on the ground would simply be a parallel projection of the pattern of irregularities, irrespective of wavelength, and hence complete correlation between the scintillations at different wavelengths would exist. Since this is not the case, diffraction effects must play an essential part in the mechanism of the phenomenon.

It can be shown by diffraction theory that the sizes of the elements, the size g of the shadow striations on the ground, the height z of the disturbing layer, and the wavelength λ are related as

$$\frac{s}{z} \sim \frac{\lambda}{g} \quad . \tag{87}$$

But, following Fürth [1956], s cannot be small compared with g, because this would mean negligible diffraction effects. On the other hand, s also cannot be large compared with g, because then the diffraction pattern on the ground would bear no relation to the pattern of irregularities, and no correlation between the patterns produced with different wavelengths would exist. Hence we must assume that s and g are of the same order of magnitude, that the ray theory equation

$$s = g$$

is approximately valid, and that the height is about

$$z \approx \frac{s^2}{\lambda} \quad .$$

We know from balloon flights (project Stratoscope) that essentially all light-optical scintillation is generated at altitudes less than 24 km. This agrees with an observation by Mikesell [1960b]. A two-man crew in an open gondola balloon was able to see an inversion layer, sharply defined by haze, at an altitude of about 11 km. At nearly 1.6 km above the tropopause, the stars three degrees or more above the horizon appeared steady. As soon as the balloon drifted below the tropopause, they began twinkling.

The velocity at which the elements are moving by, of course, varies within a wide range. As a rule, the translational velocity correlates both in direction and magnitude with atmospheric wind velocities in the neighborhood of the 200 millibar level [Protheroe, 1955]. This pressure corresponds to an average height of 13.0 km. Megaw [1954] and Fürth [1955, 1956] give a drift velocity of about 15 m/sec. The lifetime of the elements, according to Protheroe [1961], may be of the order of milliseconds to tens of milliseconds. The real lifetime, however, may be much longer if the whole atmosphere, not just a single layer, is considered a contributing factor.

That a single layer produces the optical effects under discussion has been suggested as early as 1893 by Lord Rayleigh. Rayleigh, incidentally, wanted to contradict the earlier interference theory of scintillation proposed by Arago (1852). Chandrasekhar [1952] gave the single layer theory further strong support by attributing scintillation to a comparatively thin but highly turbulent layer in the troposphere at an altitude of approximately 4 km. This "seeing layer" was assumed to be about 100 m thick and to have refractive index fluctuations of the order of $\Delta n = 4 \cdot x\ 10^{-8}$ or 0.04 N-units.

A by far thicker turbulent layer had been postulated by Obukhov [1953]. If the wavelength of the light is $\lambda = 5 \times 10^{-5}$ cm and the transverse size or "internal scale" of the atmospheric perturbations is s = 2 cm, then the critical thickness of a turbulent layer is

$$L_{crit} = \frac{(2)^2 \text{ cm}}{\frac{5 \times 10^{-5}}{2}} = \frac{8}{5} \times 10^5 = 1.6 \times 10^5 \text{ cm} = 1.6 \text{ km}. \qquad (88)$$

Therefore, in magnitude, the scale obtained is comparable to the thickness of the tropospheric layer, where intense pulsations of the temperature field and of the index of refraction could be expected.

In contrast to the single layer theory, some arguments can be made which indicate that scintillation and related phenomena may be produced by the whole atmosphere, rather than by a single layer. This has been concluded, for instance, by Butler [1954] and Wimbush [1961]. H. Elsässer and Siedentopf [1959] cite as proof the observed steady increase in scintillation of a balloon-borne light source as the balloon rises up to a maximum height of 12 km. Elsässer and Siedentopf [1959] and Elsässer [1960], from the theoretical point of view, also conclude that the degree of turbulence changes very little as a function of height; some of these conclusions were questioned by Wimbush [1961].

Wimbush [1961] concludes from theory that the layer of the atmosphere which has the greatest influence on brightness scintillation is at a height of about 8 km. Another zone is found near ground level.

Although this ground level layer has no effect on scintillation, it has the greatest influence on image motion, and this is in agreement with Hosfeld's empirical observations.

Likewise, Fürth [1956] finds no evidence for locating these atmospheric perturbations within a narrow layer only. In all likelihood atmospheric disturbances from the top of the troposphere to the D-region of the ionosphere do contribute to the scintillation phenomenon.

Reiger [1962] concludes that the notion of a thin, highly turbulent, seeing layer as the primary cause of starlight scintillation is not compatible with the present experimental evidence. Instead, the entire turbulent atmosphere must be accounted for in a satisfactory theory of starlight scintillation. Thin, highly turbulent layers, when they exist, are responsible only for a fraction of the total scintillation.

3.16. Radio Star Scintillation

The scintillation of radio stars, in some respect, is similar to light optical scintillations; in other aspects it is different. Fluctuations in the intensity of emission from radio stars were first observed by Hey, Parsons, and Phillips [1946]. Since then, a great number of observations have been made, for instance by Price [1948], Hewish [1951, 1952], Little [1951], Villars and Weisskopf [1955], Pawsey [1955], Bracewell and Pawsey [1955], Little, Rayton, and Roof [1956], Lawrence, Jespersen, and Lamb

[1961], Wagner [1962], Chivers [1963], and Lawrence, Little, and Chivers [1964]; a bibliography on radio star scintillation has been compiled by Nupen [1963].

Radio scintillation, in the same way as the scintillation of visible stars, produces "shadow bands" moving across the earth's surface. This can be observed best when comparing the amplitudes of radio waves received from a radio star at two points on the earth about 1 km apart. Results obtained by Hewish [1952] indicate that the variations of phase and amplitude at one point can be ascribed to the steady drift of an irregular wave-pattern over the ground. The irregularities responsible have a lateral extent of 2 to 10 km and are about 400 km above the ground, a figure which has been confirmed by direct measurements from satellites. In addition, much larger irregularities have been observed, extending up to 200 km in width [Lawrence, Jespersen, and Lamb, 1961] and causing what would be called "wandering" in the light optical case.

The irregularities move in a steady, wind-like motion at velocities of about 100 to 300 m/s. Higher velocities are associated with periods of high magnetic activity [Hewish, 1952; Booker, 1958; Harrower, 1963]. Very rapid fluctuations, with a period of 5 to 10 seconds, were also mainly observed during high magnetic activity, slow fluctuations generally under quiet magnetic conditions [Hewish, 1952].

Radio scintillation at frequencies below about 1000 Hz originates in the ionosphere, where differences in ionization density are present which in turn cause differences in the radio refractive index. Radio refraction in the troposphere, where - as we have seen before - most light-optical scintillation originates, is less significant. It accounts mainly for high-frequency radio scintillation.

At meter wavelengths, the fluctuations in the intensity of the radio signals are of the order of 10 percent, and the apparent change in position of a radio star is about 1 to 3 minutes of arc [Ryle and Hewish, 1950; Lawrence, Jespersen, and Lamb, 1961].

Diurnal variations in radio scintillation are not as well defined as in optical scintillation. Maxima seem to be present around midnight and, to a lesser degree, at noon [Little and Maxwell, 1951; Little, Rayton, and Roof, 1956].

The zenith distance has a similar influence as described before in chapter 3.6; that is, intensity fluctuations increase rapidly as the zenith distance of the radio source increases [Little and Maxwell, 1951; Hewish, 1952; Little, 1952; Wagner, 1962]. Because the path length through any layer is proportional to the secant of the zenith distance of the source, one might expect the scintillation index to increase proportionally. Little, Rayton, and Roof [1956], however, found that the scintillation index does not vary appreciable for zenith distances less than 30°. The gradual increase toward the horizon seems to be due to a combination of effects: At lower angles of elevation, not only will the effective thickness of the ionosphere be larger but also the distance of the perturbation from the observer will have increased. In addition, the characteristics of the ionosphere might not be the same at all latitudes.

Hewish [1952], using frequencies from 37 to 214 MHz, showed that the amplitude of scintillation increases as the square of the observing wavelength; the scintillation rate, in contrast, is independent of frequency. This agrees with observations by Chivers [1960], who found that the mean scintillation amplitude varies as the square of the observing wavelength, only, however, up to the point where the lower frequency index saturates at 100 per cent. According to Booker [1958], the scintillation index usually is small at the high-frequency end of the vhf band and increases as the frequency decreases. At the low-frequency end of the vhf band, the scintillation, or fluctuation, index often approaches unity.

As for optical scintillation, several theories have been worked out to explain the mechanism of radio scintillation. The refraction theory is based purely on geometric optics and makes use of differences in radio refractive index in the ionosphere, causing a deviation of elementary radio bundles passing through such areas. It can be shown that for a given electron density, the radio refractive index, with increasing frequency, deviates less and less from unity.

In the diffraction theory, in the same way as for light scintillation, the medium is considered equivalent to a thin diffracting screen. Because the absorption in the ionosphere is negligible for the frequencies normally used for the observation of radio stars, such a screen will produce across the emerging wavefront variations of phase only, with no variations of amplitude [Hewish, 1952; Wagner, 1962; Briggs and Parkin, 1963]. As the wave propagates beyond the screen, fluctuations of amplitude begin to develop which appear as scintillation. The wavefronts at any one point of observation can be considered the sum of the unscattered waves and of waves scattered by the irregularities in the ionosphere. The scattering theory, illustrated in figure 20, has been advanced by Booker and Gordon [1950], Wheelon [1959], Muchmore and Wheelon [1963], and Lawrence and co-workers [1964].

The primary, or undisturbed, plane wavefront induces a given voltage in the receiver, and this sets the phase reference. The sum of all scattered waves then consists of many in- and out-of-phase components, the vector addition of which with the primary wave gives the actual signal received, which varies with time [Muchmore and Wheelon, 1963]. The diffraction as well as the scattering theory seem to be about equivalent; they give identical results as has been shown for a simple case by Booker [1958]. According to Booker, essentially two processes can be distinguished:

(1) Single scattering is due to a thin layer of irregularities in the atmosphere. In this case, the scale of the irregularities at ground level is a projection of the irregularity structure in the atmosphere.

(2) Multiple scattering occurs when the mean-square phase deviation is greater than one radian. In this case the correlation distance on the ground is less than the scale of the atmospheric irregularities.

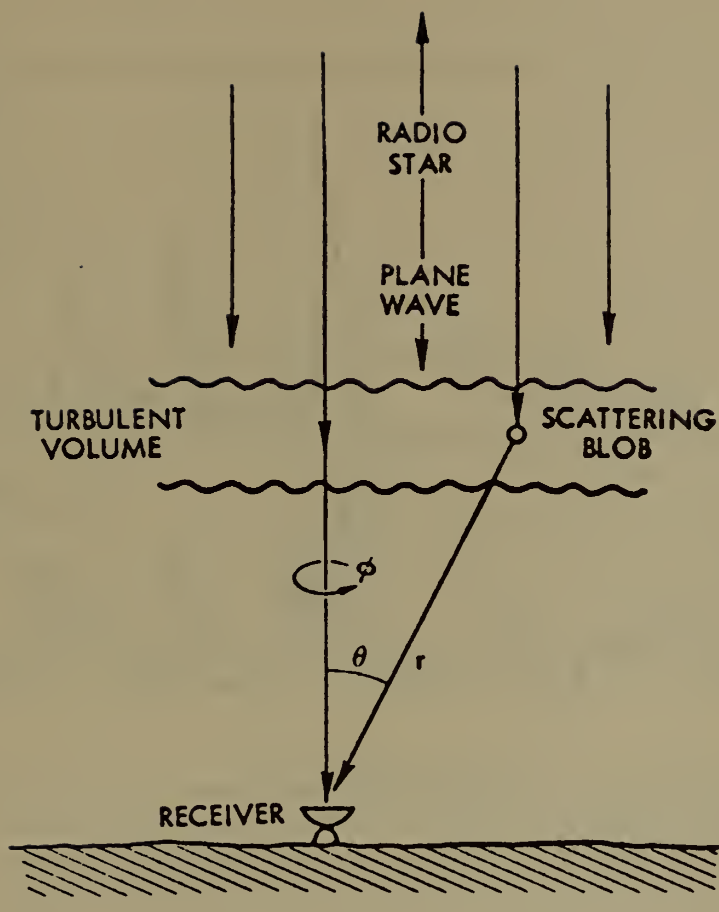

Figure 20

Scattering geometry for radio star scintillation.

[After Muchmore and Wheelon, 1963].

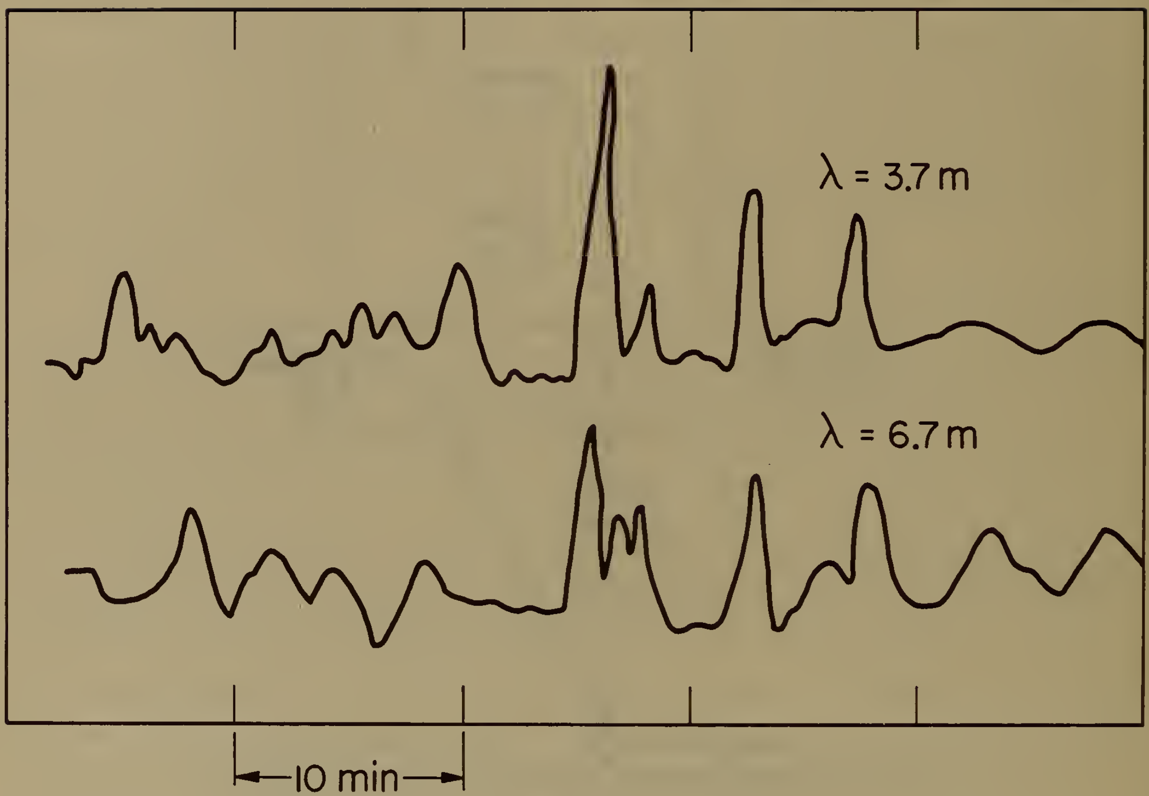

Figure 21

Radio scintillation showing high correlation at wavelengths 3.7 and 6.7 m. [After Smith, 1950].

Both processes result in a redistribution in the amplitude of the radio wave. Besides, they cause irregular fluctuations in the apparent position of a radio source viewed through the ionosphere. As long as the mean phase changes imposed on the incident radio wave are less than one radian, the scale of the pattern on the ground is the same as that in the ionosphere [Booker, 1958; Aarons, 1962; Lawrence, Little, and Chivers, 1964].

A number of studies have been made observing radio stars simultaneously either from different sites or at different wavelengths. Smith [1950] has obtained simultaneous scintillation records from sites 160 km apart, Little and Lovell [1950] at 210 km. It was found that the signals received generally were either steady at both sites or fluctuating at both sites. When fluctuations were present at both sites, there was no correlation between them, which seems to indicate that the origin of these fluctuations must be either in the terrestrial atmosphere or in interstellar space. At a spacing of 100 m, complete correlation was found, and at 3.9 km, the correlation is not complete but high with correlation factors between 0.5 and 0.95. High correlation was also found, looking at the radio star in Cassiopeia, for scintillation at two different wavelengths (fig. 21). On the other hand, at VHF the correlation of the scintillations at two discrete frequencies is generally down to zero if the ratio of the two frequencies is greater than about 3.0.

It is likely that such coherent scintillation extends over a fairly large bandwidth. Similar results have been reported by Hewish [1952], Wild and Roberts [1956], and Chivers [1960]. Aarons [1962], Muchmore and Wheelon [1963], and Warwick [1964], in studies at frequencies ranging from 60 to 3000 MHz, have isolated the tropospheric and ionospheric contributions to short-period radio scintillation. At higher frequencies the shadow pattern on the ground is considered a projection of the irregularity structure of the lower atmosphere. At lower frequencies ionospheric scintillations are dominant, at least at low angles of elevation.

3.17. Coherence.

Radio waves and light-optical waves are generated in different ways. Light waves generally are quantized and discontinuous. Because of this, coherence effects can be investigated more easily and have been studied more extensively in the radio range. A useful method, for instance, for studying the effects of radio refraction is the measurement of the variations of the phase ϕ_r of radio waves received over a radio path relative to their phase ϕ_t at the transmitting antenna [Norton, et al., 1961, 1963]. This phase difference is related to the electrical length of the path, S_e, the radio frequency, ν, and the velocity of light, c, in free space, by the equation

$$\phi_r - \phi_t = \frac{2\pi\nu S_e}{c} \tag{89}$$

Herbstreit and Thompson [1955] developed a method for measuring variations $\Delta\phi$ in the phase difference $\phi_r - \phi_t$ by transmitting precisely controlled cw frequencies ν over line-of-sight paths and in this way obtained accurate measurements of the variations ΔS_e in the

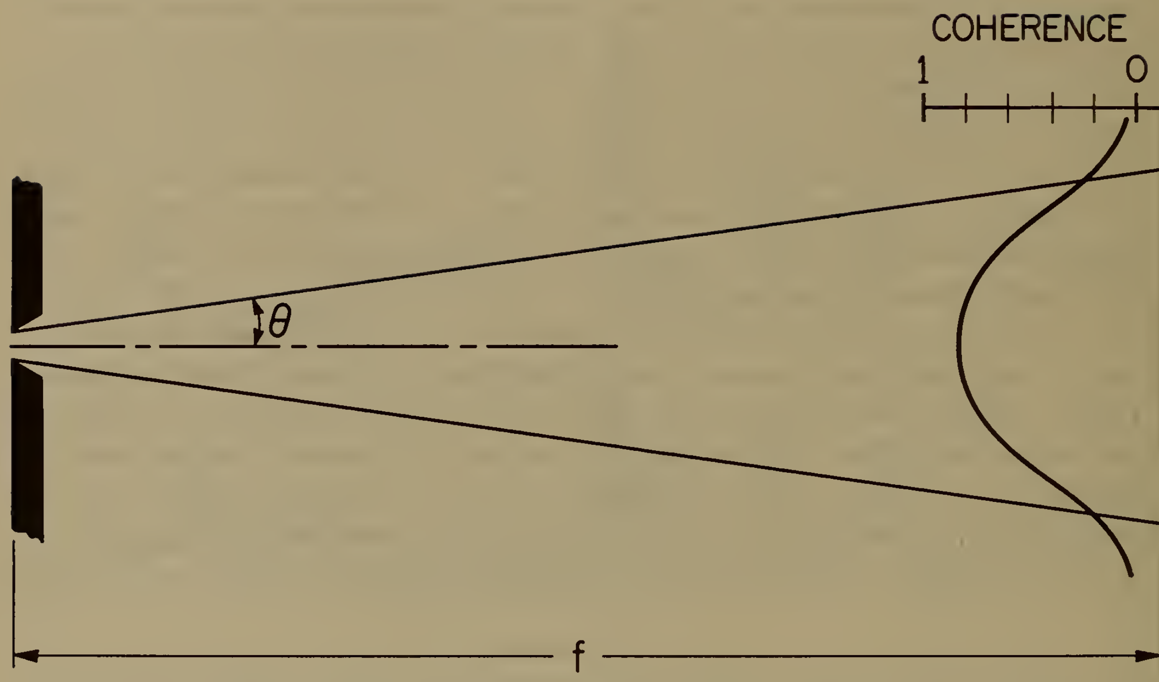

Figure 22

Relationship between bundle limitation and degree of coherence.

electrical length of the propagation path:

$$\Delta S_e = \frac{c}{\nu} \left(\frac{\Delta \phi}{2\pi} \right) \tag{90}$$

In the optical case, Ellison and Seddon [1952], using a 90 cm telescope, found that scintillations coming from points with an angular separation larger than about 3 sec of arc are largely incoherent. For some problems it is adequate to assume that light is either coherent or incoherent. This, of course, cannot be a rigorous solution. However, for demonstrating interference phenomena, we may only want to know within what size of a cone of light sufficiently coherent light will be present. This coherence condition, according to Pohl [1943], is that

$$a \sin \theta << \frac{\lambda}{2} , \tag{91}$$

where a is the diameter of the pinhole, θ is one-half the vertex angle of the light cone emerging from the pinhole, and λ is the wavelength (fig. 22). Only within a cone given by the half angle θ can radiation emitted from an extended source, excluding a laser, be treated like the emission coming from a point source. Since, for small angles,

$$\sin \theta \approx \theta .$$

$$\theta \approx \frac{D}{2f}$$

and

$$\frac{a D}{2f} << \frac{\lambda}{2} ,$$

$$a \frac{D}{f} << \lambda . \tag{92}$$

When a wave train entering an interferometer is divided into two, the wave trains will interfere with one another at the point of observation only if the optical path difference Γ is less than the length Δs of the wave train. Otherwise, one component wave train would have passed the point of observation before the other one arrives. If we call Δt the coherence time, then

$$\Delta t = \frac{\Delta s}{c} , \tag{93}$$

where Δs is the length of the wave train and c the speed of light. The coherence time, being identical with the decay time of excited atomic states, even in a conventional light source, may be as long as 10^{-8} sec. This corresponds to coherence lengths of about 3 meters. In light emitted from a gas laser, the coherence time can be considerably longer, up to about 10^{-4} sec, and Δs may be as long as 30 km. The complex amplitude of light emitted from a conventional source will remain nearly constant only during a time interval δt, which is small compared to the reciprocal of the effective spectral width $\Delta \nu$,

$$\delta t < \frac{1}{\Delta \nu} \quad . \tag{94}$$

The maximal value of δt,

$$\delta t = \frac{1}{\Delta \nu} = \Delta t \, ,$$

is again the coherence time.

In relation to the production of coherent light, it should be remembered that even oscillations of random phase fluctuations contain finite regions, or wave trains, which are rather constant in phase with one another. A definition of coherent light, therefore, should more precisely entail a measure of the length of the regions of coherence in space-time because, as we see, even a natural light field contains small regions of coherence. This length of wave trains or coherence length is given as

$$\Delta s = c \, \Delta t \sim \frac{c}{\Delta \nu} = \frac{\bar{\lambda}_o^2}{\Delta \lambda_o} \, ,$$

where $\bar{\lambda}_o$ is the mean wavelength and $\Delta \lambda_o$ is the effective spectral width in terms of wavelength.

Sufficient coherence will be present, and interference effects will be produced, as long as the path difference Γ between the two arms of an interferometer does not exceed the coherence length Δs, so that

$$\Gamma < \Delta s = c \; \Delta t \sim \frac{c}{\Delta \nu} = \frac{\bar{\lambda}^2}{\Delta \lambda} \quad . \tag{95}$$

It follows that there will be a finite region of coherence around any point P in a wave field. If there are two points P_1 and P_2 in the field, it will be desirable to introduce some measure for the correlation that exists between the vibrations at these two points. This, in fact, is possible by using as a measure C, the degree of coherence. Distinct interference phenomena will result when the correlation is high, that is, when the light at P_1 and P_2 comes from a very small source of a narrow spectral range, and no fringes will be formed if there is no correlation.

A detailed theoretical treatment of problems of coherence, incoherence, and partial coherence has been given by O'Neill [1956], and Beran and Parrent [1964], emphasizing the analogy between optical and electrical filtering. There exist several basic limitations to perfect imagery for a particular system. The first is due to the wave character of radiation itself. If the condition is imposed that the incident and emergent waves in object space must both satisfy the scalar wave equation, there results an inequality in terms of the wavelength of the light and the spacing between details in the object. The second limitation imposed is due to the finite dimensions of the aperture.

3.18. Newer Experimental Methods

In contrast to radio frequencies, the ionosphere is no barrier to optical telecommunication. Some degradation of optical signals, however, is likely to take place in the lower atmosphere. The problem on hand, then, is whether, and if so, how, it may be possible to (a) reduce and/or (b) utilize such degradation which may result from optical, systematic as well as random, refraction.

The first alternative would mean that ways be sought to enhance the accuracy at which positions can be determined as a function of space and time. This does apply to such tasks as the study of perturbations of the orbits of planets which might indicate an interference by other celestial bodies. Similar requirements for accuracy are given if the position of a space vehicle or other object is to be determined to the highest degree obtainable or if high-precision geodetic measurements are to be carried out. The second alternative, the better utilization of atmospheric phenomena, would relate to weather-forecasting, to the prediction of light and radio propagation through the atmosphere, and to similar tasks.

Several means to achieve these goals follow directly from the preceding discussions. A telescope of large aperture will smooth out the dancing and pulsation of the image of a celestial object, thus giving a space-average of the position of that object. Time-averaging which gives about the same effect is achieved by long exposure times. Observing the object at longer wavelengths such as near the red end of the spectrum (provided the system has no serious aberrations for just such wavelengths) will, in the presence of atmospheric turbulences, enhance the distinctness of a stellar image.

Extensive theoretical work has very much improved our capabilities of accurately predicting, and subsequently correcting for, atmospheric refraction effects which otherwise induce serious errors in the localization of celestial and terrestrial objects. We only refer to the development of models of the atmosphere by Gerrard [1961], to work by Strand [1953] and Hamilton [1956], and to the discussion of atmospheric models in Chapter 2.3 in this Technical Note.

Unfortunately, this approach relates only to systematic changes, and hence to systematic or regular refraction in the atmosphere. Random perturbations, by their very nature, cannot be assessed in this way. But these are just the errors which remain after the usefulness of regular refraction models has been exhausted and which at this state require the major effort of research and development.

Likewise, our knowledge of the actual variations of the physical state of the atmosphere is rather limited, and we are forced to make generalizations by approximation. Whenever actual conditions, prevailing at a given point in space and time, are required to be known, such generalizations, of course, will not suffice for the purpose on hand.

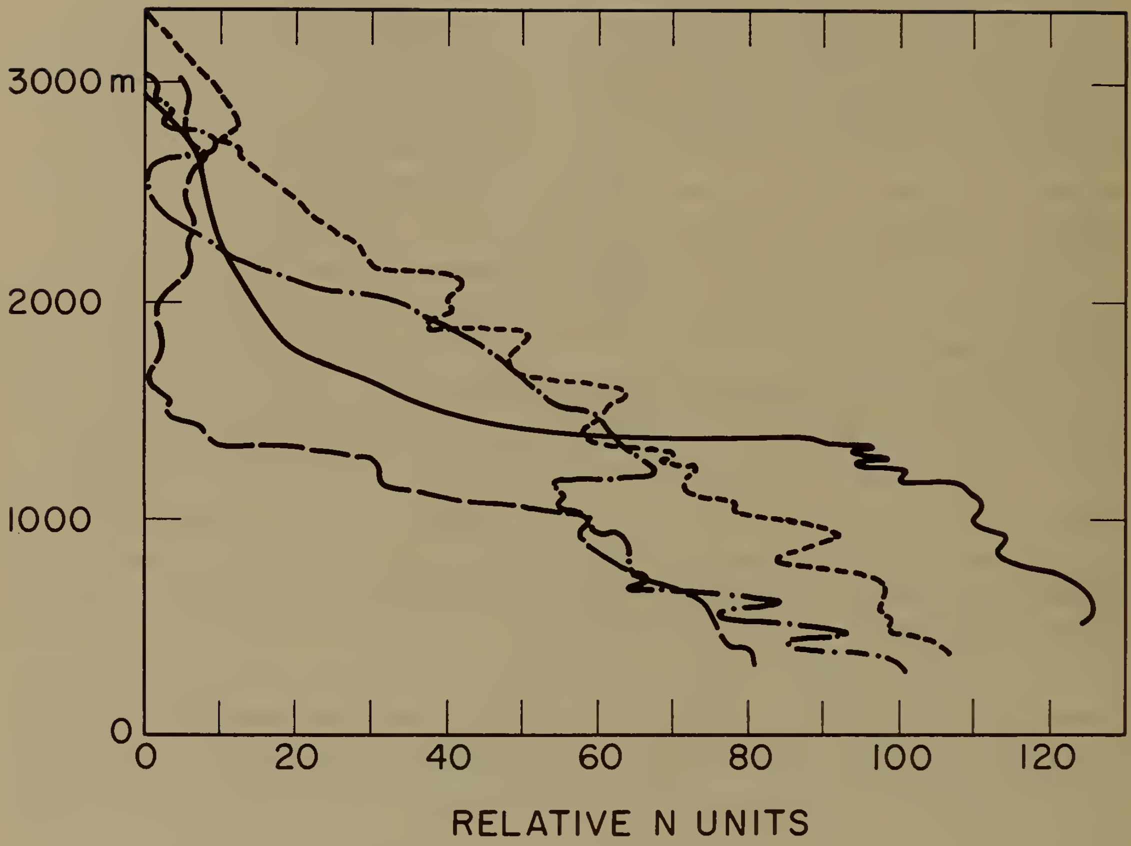

Figure 23

Index of refraction profiles obtained on different days, using a microwave refractometer carried aloft in an airplane. [After Crain, Deam, and Gerhardt, 1953.]

The most direct way of solving this problem is to carry out actual measurements, for instance, of the refractive index along the path of a missile or along the line of sight of an astronomical or geodetic determination. This is not easy, although not impossible, and often rather expensive. Direct measurements from airplanes [Overcash, 1951], from sounding rockets, or from satellites [as proposed by Jones, Fischbach, and Peterson, 1962] are some of the possibilities. Anderson, Beyers, and Rainey [1960] confirm that errors in ranging and tracking through the atmosphere are not primarily due to the instrumentation, but due to meteorological conditions which, at the present time, cannot be monitored and corrected for properly. Figure 23 gives an example of the random nature of the refractive index variations at four different times in cross sections through the lower atmosphere.

Only if we are able to determine actual pressures, humidities, temperatures, and perhaps other parameters in a given cross section can we hope to correct for random perturbations. It is felt, thus, that it is not only better theoretical models that are needed, but also better means for specifically determining the unpredictable fluctuations and other complex parameters which are encountered in the field. To this end, a series of specific research areas are proposed and outlined below:

(1) Analytical Studies of Atmospheric Models. Atmospheric models have been refined to a high degree of usefulness. However, there seems to be a need for models taking into account the wavelength dependency of refraction, that is, dispersion. This is true especially at low elevation angles and in the ultraviolet and infrared. Furthermore, models are needed which not only are based on horizontal stratification, but which as well assume inhomogeneities in the horizontal dimensions. Work of this kind does involve a statistical evaluation of such phenomena.

(2) The Continuous Determination of Water Vapor Pressure in the Atmosphere. Such determinations can be made using a combined radio and light-optical refractometer. The radio component of the system is used to record and to continuously monitor the absolute refractive index and refractive index variations which may change as a function of space and/or time. The light-optical system component would be used to determine air density variations. Subtracting these from the radio data will yield the relative water vapor pressure, without being dependent on any (infrared) absorption phenomena. Studies of this kind should preferably be done in areas of temporarily high humidity, because the microwave data will be affected much by humidity, light-optical data will not.

(3) Scanning Systems for Monitoring the Lateral Excursions of a Light Beam Passing Through a Turbulent Medium. Assume that a highly collimated beam of light is pointed at a given target. The problem is to record continuously in space and time the lateral deviations of this beam which are produced by refractive inhomogeneities.

While photoelectric detectors are most convenient to use, it is not practical to employ an array of photodetectors in the recording plane, for such an array would not allow a space-continuous monitoring of all deflections. Instead, a continuous scanning system has obvious advantages. Such a system has been built by the authors of this Note. It consists of an astronomical telescope with a disk with a radial rectangular slot or a slot with spiral-shaped boundaries, the disk being driven at 6,000 rpm and located in the plane of the star image. The disk has no central axis; it is driven from the periphery similar to a device disclosed in, but not pertinent to, U. S. patent 2,513,367 granted to L. B. Scott [1950].

The incident beam is intersected periodically by the slots moving by. Depending on the distance by which the incident beam is displaced from its normal direction and, hence, from the center of rotation of the disk, a shorter or longer light pulse will be passing through any one slot. These light pulses are converted by a photo detector into electric pulses of varying duration. These then are fed into a pulse width analyzer where times are changed into voltages, the voltages being recorded in a conventional manner. In addition, the rotating disk has a time marking slot which, together with an additional light source and photo detector, serves as a reference in order to detect phase variations with respect to time of the electric pulses. These phase variations give the angular positions θ of the oscillating light beam received.

Such a system provides a simple means of recording the spatial oscillations of a point image in terms of two coordinates, namely, r, the radial distance of the incident beam from the axis of rotation of the reticle, and θ, the phase shift with respect to the pulse produced by the reference mark. From these two coordinates r and θ, the position of the light beam in two-dimensional space can be uniquely determined.

The system, however, has limited capabilities insofar as its rotational velocity and hence scanning rate must exceed the highest scintillation frequency that could occur naturally. A significant advance, then, has been made by R. B. Herrick [1964] who has developed a novel type of a scanning system which incorporates cylinder lenses and grids of a particular design and which has the advantage that it contains no moving parts. The system, currently under construction, will allow the recording of very high-frequency intensity fluctuations as well as of high-frequency spatial displacements of a point image in two-dimensional space.

(4) Use of Schlieren Systems for Optically Probing the Atmosphere. Schlieren systems are well suited for the optical determination of refractive index variations $\partial n/\partial x$ in representative cross sections through the atmosphere. Schlieren systems as such are by no means new. L. Foucault, in 1859, was the first to use a schlieren technique for testing large mirrors (Foucault's knife edge test). A. Töpler [1864] realized their significance. His first schlieren detectors contained a triangle to block out the light. But Töpler soon recognized that only one edge is needed. Since then, a half plane or

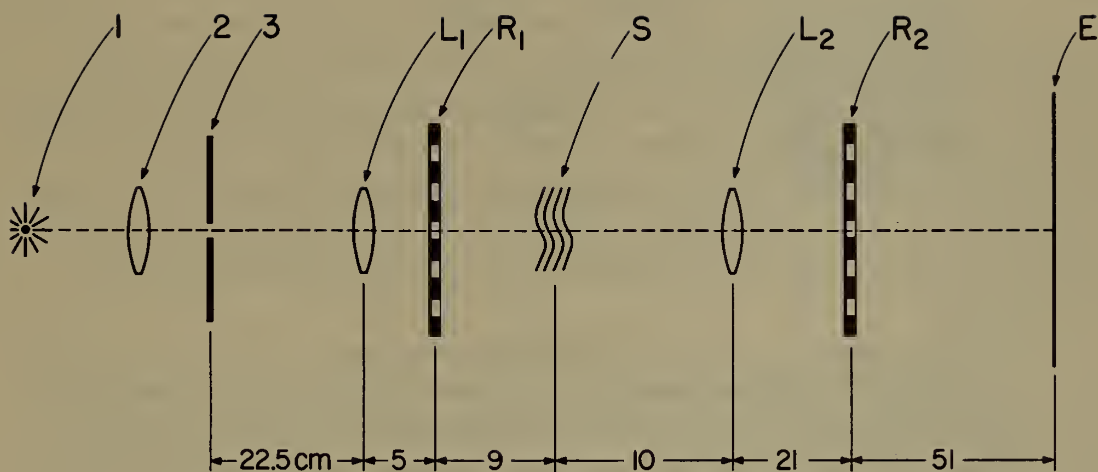

Figure 24
Two-grid, non-complementary, image forming schlieren system.

knife edge is used most commonly as the light stop, but a variety of other elements such as slits, wires, grids, and colored sector plates can be used for the same purpose.

Further advances have been made by many authors, most notably by Schardin [1942], Barnes and Bellinger [1945], Burton [1949], Prescott [1951], and Wolter [1956]. The theory has been treated exhaustively by Gayhart and Prescott [1949], Sommerfeld [1954], Toraldo di Francia [1954], Walter [1956], Rossi [1957], Temple [1957], Smart [1960], Reisman and Sutton [1961], and Sherman [1963].

Quite clearly, it is desirable to obtain the highest contrast from a minimal deflection of light by the schliere. This means that all the light deflected must be completely blocked out by the knife-edge (in a Töpler system) or it must pass by completely free. Thus, in order to obtain maximum sensitivity, the focal length of the schlieren head (the lens system close to the sampling field) must be as long as possible and, furthermore, the diameter of the light source must be as small as possible. From this it follows that a laser makes an ideal light source in a schlieren system. According to Regniere and Giroux [1962], the sensitivity of a Töpler type schlieren system can, by using a laser, be increased by a factor of 50.

We shall not, at this place, review the great variety of schlieren detecting systems which have been designed for different purposes. May it suffice to say that schlieren systems can be built which not only give a faithful, two-dimensional image of refractive or reflective perturbations which constitute "schlieren" and which cause deviations of light, but which at the same time yield directly quantitative schlieren indications that make unnecessary any secondary densitometric evaluation of the schlieren image. One type of such a novel system is what has been called a two-grid non-complementary schlieren system (fig. 24).

Two-grid systems of this kind have definite advantages. They contain, in place of the slit and the knife edge, two Ronchi grids, but these grids need not be complementary to one another; they only have to be parallel [Baker, Meyer-Arendt, and Herrick, 1963; Meyer-Arendt, U. S. Pat. 2, 977, 847]; one-grid systems have been described by Schardin [1942], Bruch and Clastre [1948], Prescott [1951], and Wolter [1956]. This makes the instrument rather easy to aline. Even more important, it allows a by far higher light throughput, making the system easy to operate even with weak light sources.

The effective light source of the schlieren system shown in figure 24 is a narrow slit (3) which is illustrated by a source (1) and a condenser (2). Light from slit (3) is incident through another condenser (L_1) onto a first Ronchi grid R_1. This grid is imaged by means of lens L_2 onto a second Ronchi grid R_2, the rulings of which are parallel or about parallel to that of R_1.

Refractive inhomogeneities present in the schlieren field (S) are projected by lens L_2 onto screen E. From the superposition of the two grids, there results a moiré pattern which is seen, superimposed on a real image of the inhomogeneities, on the screen.

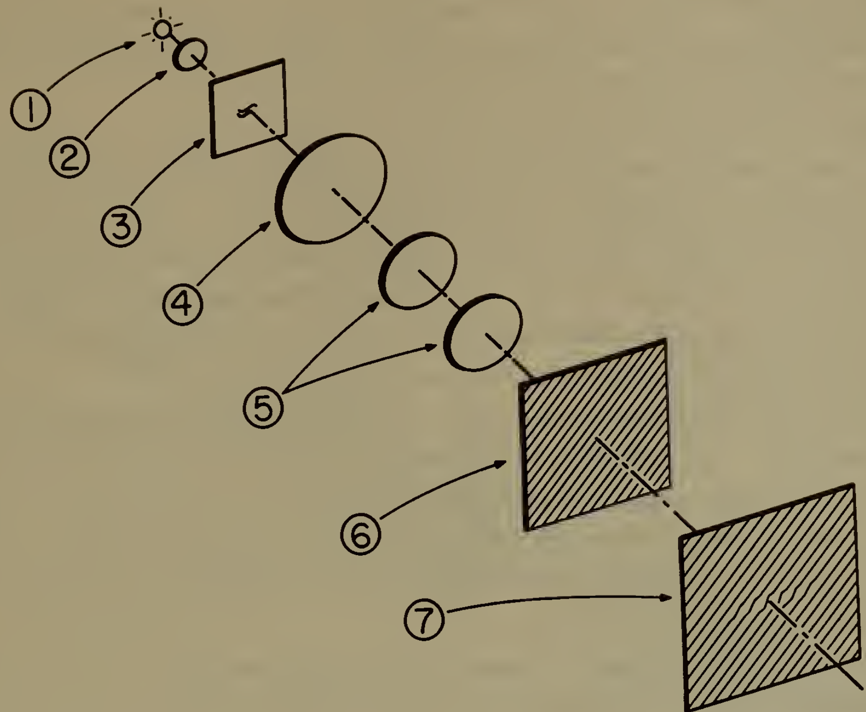

Figure 25

Schematic arrangement of the optical components in a telescopic schlieren detecting system. (1 + 2) = light source + collimator (an artificial star is not needed when observations are made using the sun, moon, or a real star as the light source); (3) = field containing refractive inhomogeneities; (4) = objective lens of telescope; (5) = eyepiece; (6) = Ronchi grid with 53 lines/cm, placed about halfway[17] between eyepiece and photographic film; (7) = image plane showing distortions of the shadow line pattern.

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[17] This position is by no means the only one possible. In addition, two grids can be inserted into the path. At some positions, they produce moiré fringes which seem to enhance the sensitivity of the system.

If no inhomogeneities were present, the moiré pattern would consist of linear parallel lines. But, with refractive index gradients present in (S), the moiré lines become distorted in a characteristic way. A good example for this is the convections in water produced by a temporarily electrically heated wire immersed in the water. It has been found that the pattern distortions are most distinct when both grids subtend an angle of 45° with the advancing front of the hot water, that is, with the refractive gradient vector.

If schlieren techniques are to be applied to an "optical sounding" or probing of the atmosphere, clearly another feature would be highly desirable. In a conventional schlieren system, the field under investigation is located inside the optical system. This requirement makes it difficult to examine refractive index gradients that are located at a great distance from the observer. A new type of a schlieren system has been developed by Meyer-Arendt, Herrick, and Emmanuel [1963], and Meyer-Arendt, Shettle, and Emmanuel [1964] in which the sampling field is outside of the optical system proper.

This new type of a system, which might be called a "forward-looking schlieren detecting instrument" or, for short, a "schlieren telescope", is basically an astronomical telescope with a Ronchi grid placed between the eyepiece and the image plane. Optical inhomogeneities in the sampling field produce distortions of the grid shadow pattern from which the density gradients can be determined. The schematic arrangement of the optical components is shown in figure 25.

A different type of a "schlieren telescope" has been developed by Saunders [1956], Saunders and Smith [1956], A.G. Smith and co-workers [1957, 1958], and Vatsia [1958]. This instrument is basically a conventional Töpler system in which the second knife edge is replaced by a glass plate into which a small slit has been etched. The instrument then acts as a phase schlieren system, in about the same way as a phase contrast microscope. Smith and co-workers have applied this principle to a study of the internal structure of flames and jet streams. They also have made terrestrial scintillation studies, observing convective plumes over house roofs and the like, and investigating the degradation of photographic resolution due to such plumes. The phase system as such, however, still requires intermediary microdensitometry for evaluating the schlieren photographs.

The range of the telescopic schlieren system illustrated by figure 25 depends only on the focal length of the telescope which is part of the system. The sensitivity for measuring refractive gradients is better than 10 N-units/millimeter (the length dimension as measured in the plane of the image) and seems sufficient for detecting refractive gradients in the troposphere. The response time depends on the characteristics of the electronic components necessary and hence is considerably shorter than that of any more mechanical probe which has to be placed in, or moved to, proper positions.

How well the height of the perturbations can be determined using a single instrument can only be inferred from the focal discrimination capability of the telescope, which in turn is a function of the size of the aperture, and from experiments made by Kantrowitz and

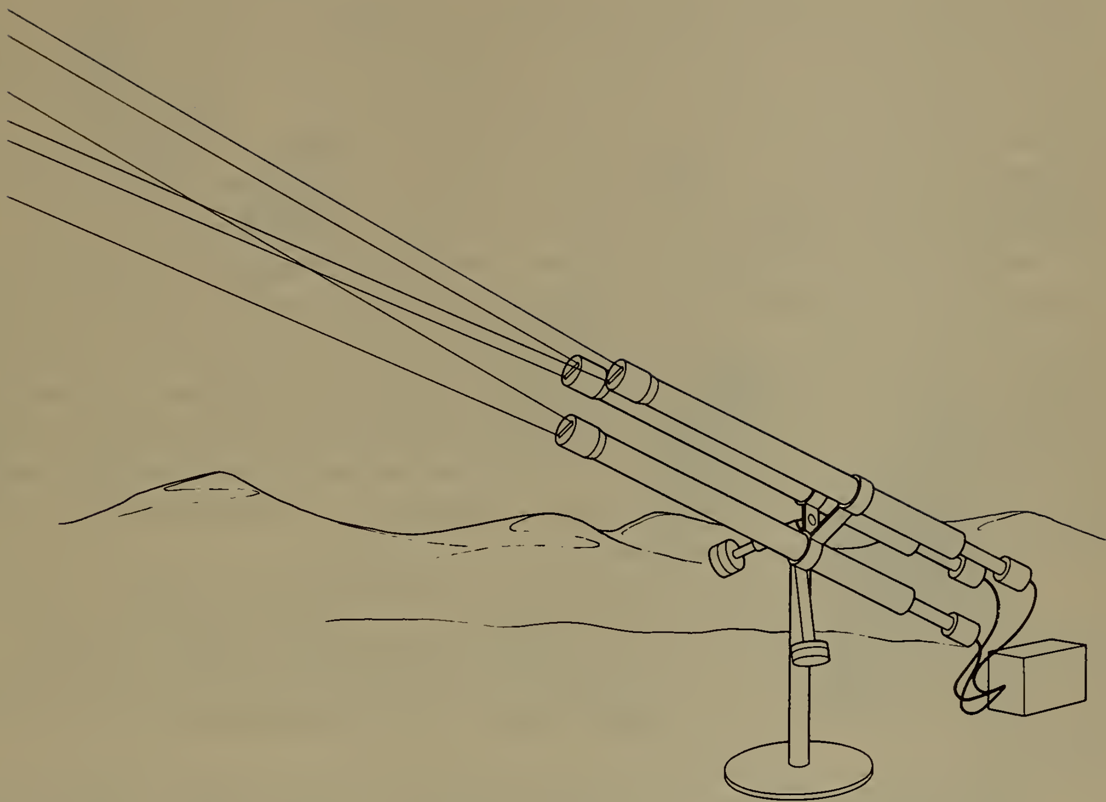

Figure 26

Three-barreled telescope as proposed for height and wind direction discrimination and for cross-correlation analyses of effects of atmospheric turbulences.

Trimpi [1950]. That the sensitivity will be sufficient can be seen from studies by Bruch and Clastre [1948], where the undulation, or "boiling", of the solar limb could be related to the movements of air parcels near the tropopause. Optical soundings of the atmosphere were also described by Bellemin [1923] and Boutet [1950].

(5) Development of a Multiple-Axis Telescope. In chapter 3.11. of this Technical Note, it has been shown how the character of atmospheric turbulences can be assessed with high accuracy by using a dual telescope. Two types of such instruments have been used: (1) large-aperture telescopes screened off except for two small holes or a rectangular slot in front of the objective lens, or (2) a tandem of two relatively small-aperture telescopes. There can be no question that the tandem system is more economical and, if supplemented by a third telescope, completely sufficient to detect all (horizontal) movements of turbulences in space. A height discrimination of the turbulent layers, furthermore, is possible (a) by the focal discrimination provided by the telescope and (b) by using rather narrow acceptance cones for light entering each component telescope. Knowing the finite distance between the three component axes, cross correlation functions between any two telescopes can be obtained, which in turn give the probability that certain inhomogeneities will be at a given height. Figure 26 illustrates an artist's conception of such a system of three parallel telescopes, mounted in an equatorial mount as it is customary for astronomical instruments. Each telescope is provided with a rotatable slot (or Ronchi grid) in front of the objective lens and with a likewise rotatable grid between the eyepiece and the image plane of each telescope. All these rotatable members are driven in phase by synchronous motors.

As long as only time-dependent distortions of the grid patterns are needed, these can be analyzed by cross-correlation functions. It may seem desirable, however, to obtain information of where, in any one of the telescopes, a refractive index gradient is located at any one time. To know this, three image-orthicons or similar detectors can be used to scan the images.

The acceptance cones for each of the telescopes will be varied in size, inserting suitable field stops. This can be done more easily than to vary the mechanical spacing between the telescopes. If then the angular size of the volume seen by any one of the photo detectors is related to the magnitude of the cross-correlation factor, it will be possible to compute the height, direction, speed of motion, and size of the atmospheric turbulences causing scintillation. The latter complex computations will preferably be carried out by means of a high-speed computer. Ultimately, it seems feasible to plot in the form of a map, preferably in three dimensions, contour lines of equal refractive index. One may then assume certain lines of sight, go through a ray tracing program, find the variations of the paths through these inhomogeneities, and thus be able to correct for these variations.

(6) Precision Calibration of Schlieren Systems. It will be obvious that a quantitative interpretation of schlieren patterns requires a reliable calibration method. Schardin [1942] had introduced for this purpose the "normal schliere", that is a lens of known diameter and

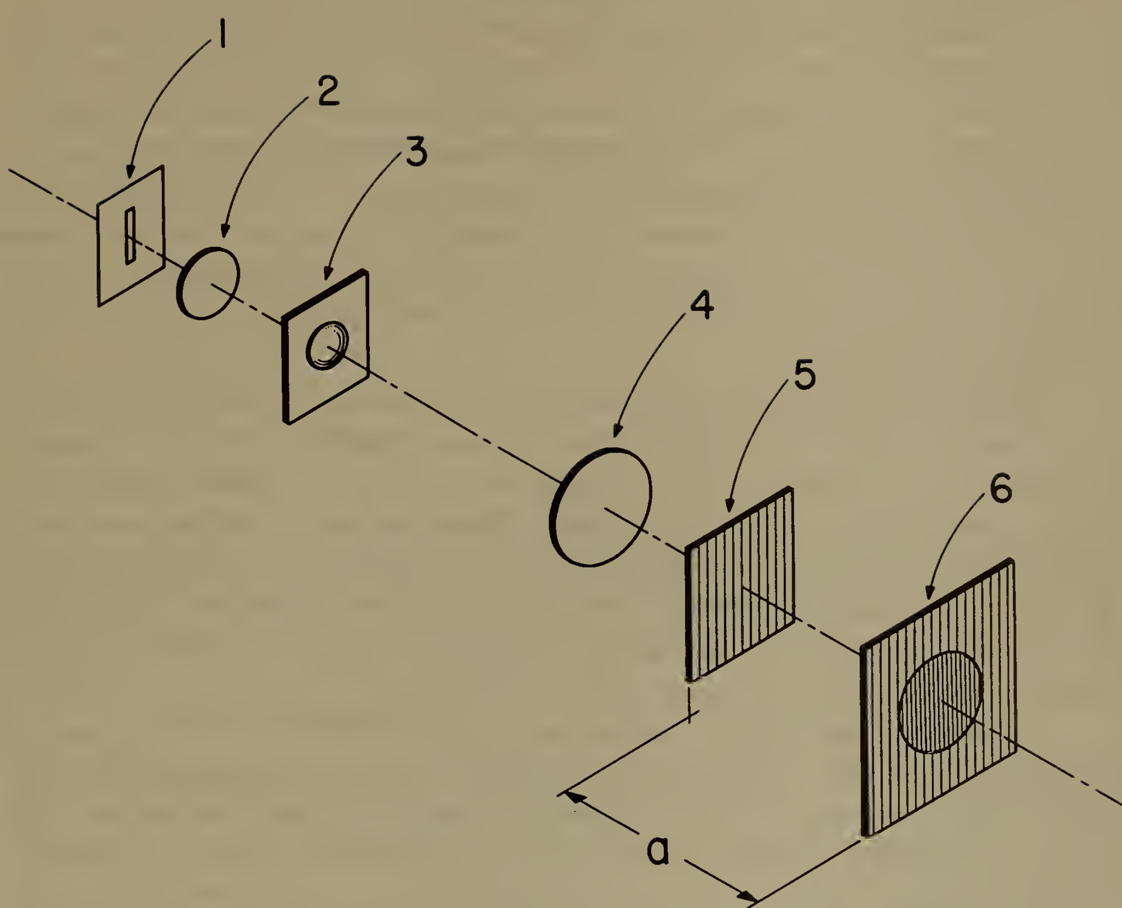

Figure 27

Calibration of a schlieren system by measuring line ratios. 1 = slit; 2 = collimating lens; 3 = standard schliere; 4 = objective lens; 5 = Ronchi grid; 6 = image plane where image of standard schliere is formed. The line ratio is defined as the number of lines inside the circular lens image in (6) divided by the number of lines within the same distance outside the image.

focal length which is placed in the schlieren field. Such standards, and likewise wedges used as standard schlieren, are customarily evaluated by densitometry of the photographic records made or by direct photoelectric photometry.

A different approach, also based in part on Schardin's normal schliere, is to use lenses of different focal lengths as the standards and to evaluate the resulting variation in a Ronchi grid pattern produced by the schlieren system under test [Meyer-Arendt, Shettle, and Emmanuel 1964]. The arrangement is shown in figure 27. The set-up shown is essentially the forward-looking, Ronchi grid schlieren system described before, although any other comparable system can be evaluated in the same manner as well. In the image plane, two fields containing shadow lines of different spatial density can be seen, the "line ratio" between them being a direct measure of the refractive index and refractive index variations in the test field. An additional gain in accuracy can be realized by utilizing Wolter's [1951] principle of "minimum ray definition".

A limitation of this type of calibration procedure is that the test field must be physically accessible. If the normal schliere cannot be placed inside the test field, for instance for evaluating a high-altitude schlieren telescope, another type of a "standard" will be required. It seems feasible to use for this purpose balloon-, rocket- or satellite-borne light sources and to introduce known deviations, for instance shock waves from a controlled high-explosive blast or a lightning discharge. For the latter purpose, an electric spark discharge, powered by a Marx generator, or an exploding wire (see the design by Jones and Earnshaw [1962]) in a cylindrical cavity reflector could be useful.

Since the first observation, in ancient time, of the scintillation of stars, we have certainly gone a long way. Many sophisticated means have been developed to analyze scintillation, to study its impact on telecommunication, to utilize scintillation for weather forecasting, and to even try to eliminate scintillation for certain purposes. As it is customary at the end of a technical survey, we close by saying that much more research in this field is required in the future to even better understand one of the more beautiful optical phenomena in nature.

Twinkle, twinkle, little star...

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4. Symbols

B	=	Spectrum function of shadow patterns
b	=	Spectrum function of turbulences
C	=	Degree of coherence
c	=	Velocity of light in free space
D	=	Aperture; dry component of refractivity
d	=	Distance
e	=	Partial water vapor pressure
f	=	Focal length
g	=	Size of shadow striations
h	=	Amplitude fluctuation
Hz	=	Hertz (c/s)
K	=	Fluctuation frequency
K_N	=	Austausch coefficient
k	=	Constant; circular wave number $(2\pi/\lambda)$
m	=	Number, order number, number of wavefronts, etc.
N	=	Refractivity, refractive modulus; electron density
n	=	Refractive index
n_G	=	Refractive index at ground level
P	=	Point above earth surface
p	=	Pressure
Q	=	Autocorrelation function; gas constant
R	=	Atmospheric refraction
r	=	Radius
r_o	=	Radius of earth
S	=	Optical path length
s	=	Distance; size of element
T	=	Temperature
T_o	=	Absolute temperature at ground level
t	=	Time
v	=	Velocity
v_n	=	Component of wind velocity normal to path of observation
W	=	Radius of curvature of deformed wavefront; wet component of refractivity
w	=	Wavenumber of Fourier element
$\hat{x}, \hat{y}, \hat{z}$	=	Unit vectors
Z_o	=	True zenith distance
z	=	Height above ground

α	=	Angle of incidence; specific volume; gas expansion constant
β	=	Angle of refraction
Γ	=	Optical path difference, retardation
Γd	=	Dry adiabatic lapse rate
Γs	=	Saturated adiabatic lapse rate
γ	=	Lapse rate of temperature; ratio of specific heat
Δ	=	Small quantity
Δs	=	Coherence length
Δt	=	Coherence time
$\Delta \nu$	=	Effective spectral width
δ	=	Angular diameter of object or image
$\delta \epsilon$	=	Refraction correction for elevation angle
∇	=	Operator del
∇n	=	Refractive index gradient vector
$\nabla_z n$	=	Vertical air density gradient
$\partial n/\partial z$	=	Vertical air density gradient
ϵ	=	Elevation angle; mean dissipation of energy per unit mass of fluid
ζ	=	Apparent zenith distance
η	=	Viscosity; ordinate in the image plane
θ	=	Relaxation time
κ	=	Local refraction coefficient
λ	=	Wavelength
ν	=	Frequency
ν_i	=	Resonance frequency
ξ	=	Abscissa in the image plane
ρ	=	Density
σ	=	Standard deviation
τ	=	Schlieren angle, change of direction, bending of radio ray
ϕ	=	Phase difference

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6. Index

Index

(continued)

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