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Technical Note No. 221

DEMAGNETIZING FACTORS FOR OBLATE SPHEROIDS USED IN FERRIMAGNETIC RESONANCE MEASUREMENTS

L. B. Schmidt, W. E. Case, and R. D. Harrington

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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Demagnetizing Factors for Oblate Spheroids Used in Ferrimagnetic Resonance Measurements

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Demagnetizing factors for oblate spheroids magnetized along the short axis are given for aspect ratios from 25.0 to 35.0 in increments of 0.1, from 35.0 to 55.0 in increments of 0.2, from 55.0 to 80.0 in increments of 0.5, from 80.0 to 129.0 in increments of 1.0. The values of all demagnetizing factors given in the tables have been rounded off to 6 decimal places and are accurate to ±5 units in the seventh place. The tables are presented in a form convenient for use in ferrimagnetic resonance measurements on disk shaped samples. A brief discussion of the effect of accuracy of demagnetizing factors on measurements of this type is included.

DISCUSSION

It is well known that the applied field H_z required for ferrimagnetic resonance in finite specimens at a frequency, ω, may be obtained from Kittel's [1948] equation,

$$\frac{\omega}{\gamma} = \left([H_z - (N_z - N_x) M_z] [H_z - (N_z - N_y) M_z] \right)^{1/2}, \qquad (1)$$

where γ is the gyromagnetic ratio, M_z is the magnetic moment per unit volume, and N_x, N_y, and N_z are the demagnetizing factors along the corresponding axes. The equation is often used to calculate the gyromagnetic ratio, γ, from resonance measurements made on spherical specimens. For this situation $N_x = N_y = N_z$ and equation (1) reduces to

$$\frac{\omega}{\gamma} = H_z.$$

However, in some situations, it may be more advantageous to use other geometries. For example, low field losses encountered in ferrimagnetic resonance measurements at lower microwave frequencies may be avoided by using thin disk shaped samples with the dc field perpendicular to the face of the disk. The dc and rf field orientations for such measurements are shown in figure 1. For this situation, $N_x = N_y$ and Kittel's equation becomes

$$\frac{\omega}{\gamma} = H_z - (N_z - N_x)\, M_z. \qquad (2)$$

In a typical measurement, H_z is determined experimentally and $(N_z - N_x)$ is calculated from relations developed by Stoner [1945] which express the demagnetizing coefficient as a function of aspect ratio for spheroidal ellipsoids. Stoner expresses the demagnetizing factors as D_x, D_z, where $D_x = N_x/4\pi$ and $D_z = N_z/4\pi$. Equation (2) then becomes

$$\frac{\omega}{\gamma} = H_z - (D_z - D_x)\, 4\pi M_z. \qquad (3)$$

The value of the magnetization, M_z, may be determined independently using a magnetometer.

We have recently carried out a study involving measurements of H_z on several materials at 1100 MHz, using disk shaped samples [Case, Harrington, and Schmidt, 1964]. This study indicated that the measurement of disks of several sizes is often advantageous in obtaining data of

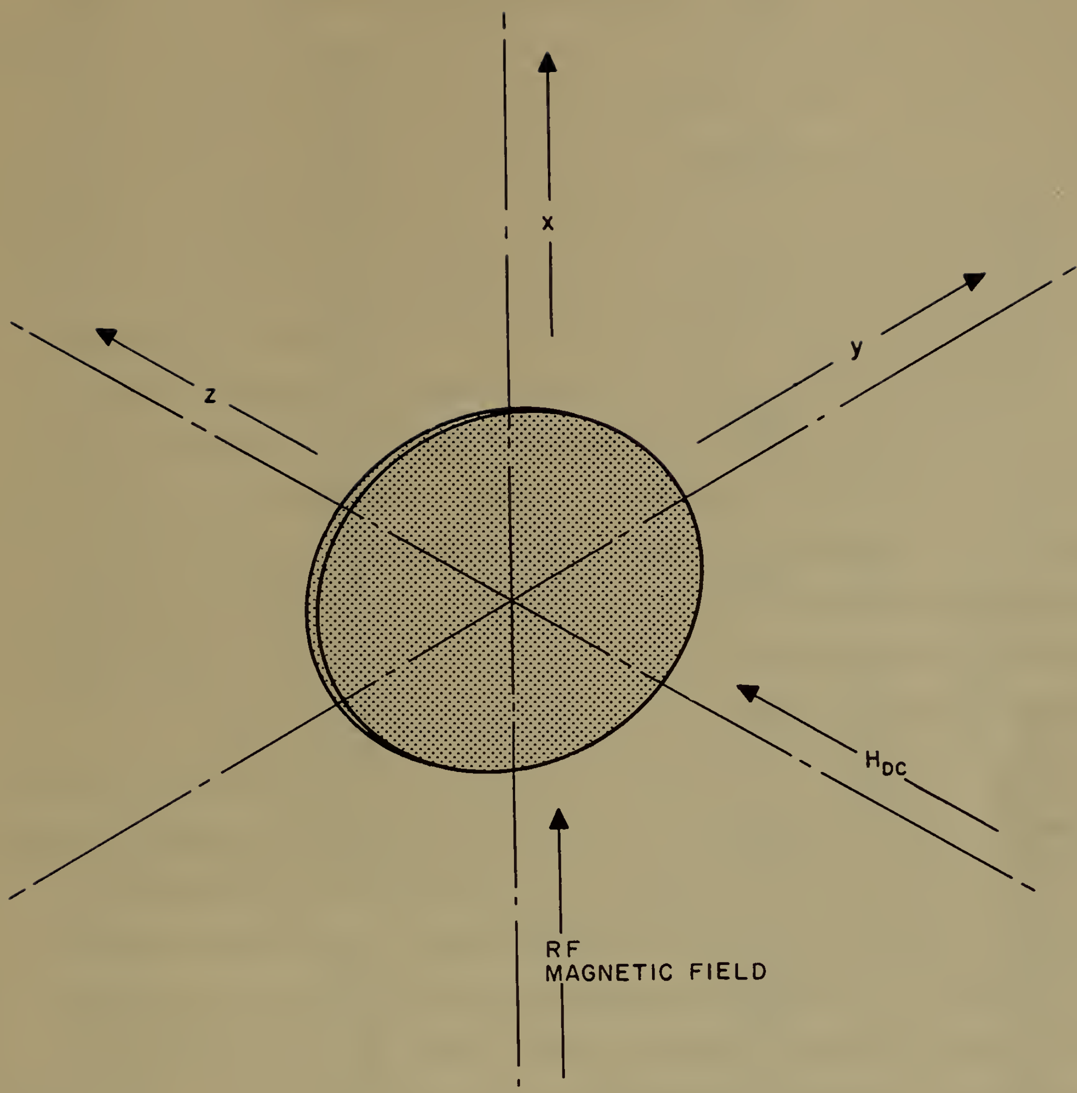

FIGURE 1
Field and Sample Orientations for Ferrimagnetic Resonance Measurements on a Disk.
(Demagnetizing factors in the table are applicable for this geometry.)

this type. Each sample of each material was measured at a number of different aspect ratios (diameter/thickness) by grinding the same diameter sample to successively smaller thickness for each measurement. By using these results a plot of $\frac{\omega}{\gamma}$ as a function of aspect ratio is readily obtained. Figure 2 shows results of this type on one material used in the study. The variation from a smooth curve in this example was less than one percent. It is apparent that $D_z - D_x$ must be known to considerably better than one percent if this quantity is not to contribute to the observed variation in $\frac{\omega}{\gamma}$. In fact, it can be shown that $D_z - D_x$ must be known to a few hundredths of a percent for typical materials if the contribution of any error in $D_z - D_x$ to this one percent variation in $\frac{\omega}{\gamma}$ is to be less than 0.1%. (See Appendix I.) These demagnetizing factors may be calculated to sufficient accuracy using Stoner's equations. When measuring many different samples as mentioned above, it is apparent that tables of demagnetizing factors, given in finer aspect ratio increments than is presently available, would be very convenient.

Stoner has given tables of D_z, but only in increments of 5 and 10 in the region of aspect ratios of 25 to 129. Osborn [1945] prepared graphs of $N/4\pi$ which equals D_z, but again the resolution was not adequate in this region. We have therefore prepared Table I which gives demagnetizing factors that meet the minimum requirements for increments of aspect ratios needed to prepare graphs within the precision represented by figure 2 and the calculations of $\frac{\omega}{\gamma}$ mentioned above. The tables are shown in six decimal places; however, this accuracy is not required for these calculations.

Table I was condensed from tables prepared on a computer, calculated to eight significant figures and rounded off to six significant figures which provided a convenient comparison to Stoner's tables.

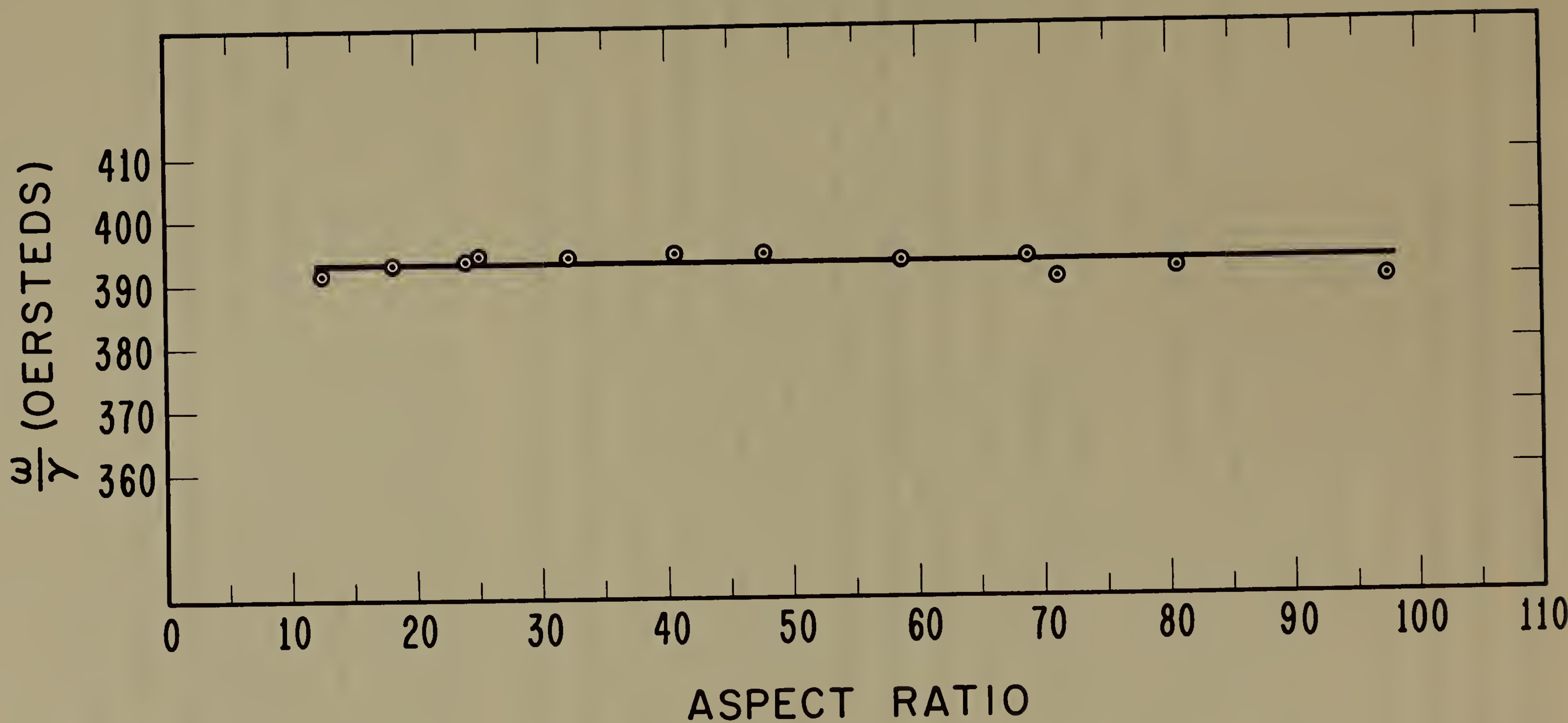

FIGURE 2
Typical Data for ω/γ Versus Aspect Ratio for Disk Samples

Kittel's equation is written in terms of N_x, N_y, and N_z when $N_x + N_y + N_z = 4\pi$. It is more convenient, as noted by Stoner, to express the demagnetizing factors in terms of D_x, D_y, and D_z when

$$D_x = \frac{N_x}{4\pi}, \; D_y = \frac{N_y}{4\pi}, \; D_z = \frac{N_z}{4\pi}.$$

In the tables prepared by the computer, D_z was first calculated using Stoner's equation 4.5 for an oblate spheroid as follows

$$D_z = \frac{1}{1 - m^2}\left[1 - \frac{m}{(1 - m^2)^{\frac{1}{2}}} \cos^{-1} m\right]$$

where $m = 1/\text{aspect ratio}$. Then $D_x = D_y = (1 - D_z)/2$ was calculated to give the values of $D_z - D_x$ which are also given for convenience in using equation (3). The computer program was written to generate its own input in increments of aspect ratio as required. Using this method, the increments could be in integers, in tenths, or in hundredths. In cases where measurements are made on disks with very small changes in aspect ratios, tables in increments of 0.01 are most efficient. The table was readily prepared on the computer in increments of 0.01 from 1.01 to 130.00 to satisfy this requirement and possible future requirements.

Where applicable, the values of this table were compared with the corresponding values given in Table II in Stoner's paper. In two instances, the two disagree by one part in the sixth place. Stoner noted that his calculations were carried to the seventh place and rounded to the sixth place. The computer calculated our tables to the eighth significant figure and rounded to the sixth significant figure. At a third place, the aspect ratio of 60, checking by hand calculations confirmed our value, so that Stoner's value appears to be in error.

The accuracy of Table I was spot checked by hand calculations. In addition, the accuracy of the entire table was verified by checking by differences [Miller, 1950]. The values of all demagnetizing factors given in the tables have been rounded off to 6 decimal places and are thus accurate to ±5 units in the seventh place.

The tables were studied for the possibility of using linear interpolation for values between the aspect ratios given. The accuracy of this interpolation was verified by comparing the midpoint between increments in the tables calculated by linear interpolation with the corresponding point in the tables prepared by the computer in increments of 0.01 in aspect ratio. It was found that the values of D_z may be interpolated linearly, accurate to one unit in the sixth place, for all aspect ratios. The values of $D_z - D_x$ may be interpolated linearly, accurate to one unit in the sixth place, except within the range of aspect ratios between 80.0 and 129.0. Within this range, values of $D_z - D_x$ may be interpolated linearly accurate to two units in the sixth place. It was previously noted that this accuracy is more than adequate in the use of equation (3).

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For a fixed value of H_z and M_z, the percent error in $\frac{\omega}{\gamma}$ given by $\frac{\Delta \frac{\omega}{\gamma}}{\frac{\omega}{\gamma}}$ introduced by an error $\Delta(D_z - D_x)$ in $D_z - D_x$ is obtained from equation (2)

$$\frac{\Delta\left(\frac{\omega}{\gamma}\right)}{\frac{\omega}{\gamma}} = - \frac{\Delta(D_z - D_x)}{D_z - D_x} \left[\frac{(D_z - D_x)\, 4\pi M_z}{\frac{\omega}{\gamma}}\right]. \tag{4}$$

The percent error in $\frac{\omega}{\gamma}$ introduced by an error in $D_z - D_x$ is dependent on a factor that is inversely proportional to ω. Any percent error in $(D_z - D_x)$ will thus have a greater influence on the percent error in $\frac{\omega}{\gamma}$ as the frequency decreases. For example, if data were desired at 1100 MHz on a material such as a commercially available substituted garnet we have $4\pi M_z \cong 575$ gauss, $\frac{\omega}{\gamma} \cong 390$ oersteds, and $D_z - D_x = .976735$ at an aspect ratio of 100. If we desire the percentage error in $D_z - D_x$ to cause an error $\frac{\Delta \frac{\omega}{\gamma}}{\frac{\omega}{\gamma}} \leqq 0.1\%$, then substituting in equation (4) we have $\frac{\Delta(D_z - D_x)}{D_z - D_x} \cong 0.07\%$. In many cases, ω would be greater which would increase the allowable percent error of $D_z - D_x$ to hold its contribution of error in $\frac{\omega}{\gamma}$ to 0.1%; however, at the same time, M_z could also be larger to produce the opposite effect. Thus $D_z - D_x$ must be known to a few hundredths of a percent in order to cover all situations.

TABLE 1

Demagnetization Factors of Oblate Spheroids Magnetized Along the Short Axis Versus Aspect Ratio $\left(\frac{\text{Diameter}}{\text{Thickness}}\right)$

Tables are given for D_z and $D_z - D_x$ where z is along the short axis and

$$D_x = D_y$$

and

$$D_x + D_y + D_z = 1.$$

Aspect Ratio	D_z	$D_z - D_x$	Aspect Ratio	D_z	$D_z - D_x$	Aspect Ratio	D_z	$D_z - D_x$
25.0	.940224	.910336	30.0	.949778	.924667	35.0	.956700	.935049
25.1	.940450	.910676	30.1	.949938	.924907	35.2	.956937	.935405
25.2	.940675	.911013	30.2	.950097	.925146	35.4	.957172	.935758
25.3	.940898	.911348	30.3	.950255	.925383	35.6	.957404	.936106
25.4	.941120	.911680	30.4	.950412	.925618	35.8	.957634	.936451
25.5	.941340	.912010	30.5	.950568	.925853	36.0	.957861	.936792
25.6	.941558	.912337	30.6	.950723	.926085	36.2	.958086	.937129
25.7	.941775	.912662	30.7	.950877	.926316	36.4	.958308	.937463
25.8	.941990	.912984	30.8	.951031	.926546	36.6	.958528	.937793
25.9	.942203	.913305	30.9	.951183	.926774	36.8	.958746	.938119
26.0	.942415	.913622	31.0	.951334	.927001	37.0	.958962	.938443
26.1	.942625	.913938	31.1	.951484	.927226	37.2	.959175	.938763
26.2	.942834	.914251	31.2	.951634	.927450	37.4	.959386	.939079
26.3	.943041	.914562	31.3	.951782	.927673	37.6	.959595	.939393
26.4	.943247	.914870	31.4	.951930	.927894	37.8	.959802	.939703
26.5	.943451	.915177	31.5	.952076	.928114	38.0	.960007	.940010
26.6	.943654	.915481	31.6	.952222	.928333	38.2	.960209	.940314
26.7	.943855	.915783	31.7	.952367	.928550	38.4	.960410	.940615
26.8	.944055	.916083	31.8	.952511	.928766	38.6	.960608	.940912
26.9	.944254	.916381	31.9	.952654	.928981	38.8	.960805	.941207
27.0	.944451	.916676	32.0	.952796	.929194	39.0	.960999	.941499
27.1	.944647	.916970	32.1	.952938	.929407	39.2	.961192	.941788
27.2	.944841	.917262	32.2	.953078	.929617	39.4	.961383	.942074
27.3	.945034	.917551	32.3	.953218	.929827	39.6	.961572	.942358
27.4	.945226	.917838	32.4	.953357	.930035	39.8	.961759	.942639
27.5	.945416	.918124	32.5	.953495	.930242	40.0	.961944	.942916
27.6	.945605	.918407	32.6	.953632	.930448	40.2	.962128	.943192
27.7	.945793	.918689	32.7	.953769	.930653	40.4	.962310	.943464
27.8	.945979	.918969	32.8	.953904	.930856	40.6	.962490	.943734
27.9	.946164	.919246	32.9	.954039	.931059	40.8	.962668	.944002
28.0	.946348	.919522	33.0	.954173	.931260	41.0	.962844	.944267
28.1	.946531	.919796	33.1	.954307	.931460	41.2	.963019	.944529
28.2	.946712	.920068	33.2	.954439	.931659	41.4	.963193	.944789
28.3	.946892	.920338	33.3	.954571	.931856	41.6	.963364	.945046
28.4	.947071	.920607	33.4	.954702	.932053	41.8	.963534	.945302
28.5	.947249	.920873	33.5	.954832	.932248	42.0	.963703	.945554
28.6	.947425	.921138	33.6	.954962	.932442	42.2	.963870	.945805
28.7	.947601	.921401	33.7	.955090	.932635	42.4	.964035	.946053
28.8	.947775	.921662	33.8	.955218	.932827	42.6	.964199	.946299
28.9	.947948	.921922	33.9	.955346	.933018	42.8	.964362	.946543
29.0	.948120	.922180	34.0	.955472	.933208	43.0	.964523	.946784
29.1	.948291	.922436	34.1	.955598	.933397	43.2	.964682	.947023
29.2	.948460	.922690	34.2	.955723	.933585	43.4	.964840	.947260
29.3	.948629	.922943	34.3	.955848	.933771	43.6	.964997	.947495
29.4	.948796	.923194	34.4	.955971	.933957	43.8	.965152	.947728
29.5	.948962	.923444	34.5	.956094	.934142	44.0	.965306	.947959
29.6	.949128	.923692	34.6	.956217	.934325	44.2	.965459	.948188
29.7	.949292	.923938	34.7	.956338	.934508	44.4	.965610	.948415
29.8	.949455	.924183	34.8	.956459	.934689	44.6	.965760	.948640
29.9	.949617	.924426	34.9	.956580	.934870	44.8	.965908	.948863

Aspect Ratio	D_z	D_z-D_x	Aspect Ratio	D_z	D_z-D_x	Aspect Ratio	D_z	D_z-D_x
45.0	.966056	.949084	55.0	.972087	.958131	80.0	.980673	.971009
45.2	.966202	.949303	55.5	.972333	.958500	81.0	.980908	.971362
45.4	.966347	.949520	56.0	.972575	.958862	82.0	.981137	.971706
45.6	.966490	.949735	56.5	.972812	.959218	83.0	.981361	.972041
45.8	.966633	.949949	57.0	.973045	.959568	84.0	.981580	.972369
46.0	.966774	.950161	57.5	.973275	.959912	85.0	.981793	.972690
46.2	.966914	.950371	58.0	.973500	.960250	86.0	.982002	.973003
46.4	.967053	.950579	58.5	.973722	.960582	87.0	.982206	.973308
46.6	.967190	.950785	59.0	.973940	.960909	88.0	.982405	.973607
46.8	.967327	.950990	59.5	.974154	.961231	89.0	.982600	.973900
47.0	.967462	.951193	60.0	.974365	.961547	90.0	.982790	.974186
47.2	.967596	.951394	60.5	.974572	.961859	91.0	.982977	.974465
47.4	.967729	.951594	61.0	.974777	.962165	92.0	.983159	.974739
47.6	.967861	.951792	61.5	.974977	.962466	93.0	.983338	.975007
47.8	.967992	.951989	62.0	.975175	.962763	94.0	.983513	.975269
48.0	.968122	.952183	62.5	.975370	.963055	95.0	.983684	.975526
48.2	.968251	.952377	63.0	.975561	.963342	96.0	.983852	.975778
48.4	.968379	.952568	63.5	.975750	.963625	97.0	.984016	.976024
48.6	.968506	.952759	64.0	.975936	.963904	98.0	.984177	.976266
48.8	.968632	.952947	64.5	.976119	.964178	99.0	.984335	.976503
49.0	.968756	.953135	65.0	.976299	.964448	100.0	.984490	.976735
49.2	.968880	.953320	65.5	.976476	.964714	101.0	.984641	.976962
49.4	.969003	.953504	66.0	.976651	.964977	102.0	.984790	.977185
49.6	.969125	.953687	66.5	.976823	.965235	103.0	.984936	.977404
49.8	.969246	.953869	67.0	.976993	.965490	104.0	.985079	.977619
50.0	.969366	.954048	67.5	.977160	.965741	105.0	.985219	.977829
50.2	.969485	.954227	68.0	.977325	.965988	106.0	.985357	.978036
50.4	.969603	.954404	68.5	.977488	.966232	107.0	.985492	.978239
50.6	.969720	.954580	69.0	.977648	.966472	108.0	.985625	.978438
50.8	.969836	.954754	69.5	.977806	.966709	109.0	.985756	.978633
51.0	.969952	.954927	70.0	.977961	.966942	110.0	.985884	.978825
51.2	.970066	.955099	70.5	.978115	.967172	111.0	.986009	.979014
51.4	.970180	.955270	71.0	.978266	.967400	112.0	.986133	.979199
51.6	.970293	.955439	71.5	.978416	.967624	113.0	.986254	.979381
51.8	.970405	.955607	72.0	.978563	.967844	114.0	.986373	.979560
52.0	.970516	.955773	72.5	.978708	.968062	115.0	.986491	.979736
52.2	.970626	.955939	73.0	.978852	.968277	116.0	.986606	.979909
52.4	.970735	.956103	73.5	.978993	.968489	117.0	.986719	.980079
52.6	.970844	.956266	74.0	.979133	.968699	118.0	.986830	.980246
52.8	.970952	.956428	74.5	.979270	.968905	119.0	.986940	.980410
53.0	.971059	.956588	75.0	.979406	.969109	120.0	.987048	.980571
53.2	.971165	.956748	75.5	.979540	.969310	121.0	.987153	.980730
53.4	.971271	.956906	76.0	.979673	.969509	122.0	.987258	.980887
53.6	.971375	.957063	76.5	.979803	.969705	123.0	.987360	.981040
53.8	.971479	.957219	77.0	.979932	.969898	124.0	.987461	.981192
54.0	.971582	.957374	77.5	.980060	.970089	125.0	.987560	.981341
54.2	.971685	.957527	78.0	.980185	.970278	126.0	.987658	.981487
54.4	.971787	.957680	78.5	.980310	.970464	127.0	.987754	.981632
54.6	.971888	.957831	79.0	.980432	.970648	128.0	.987849	.981774
54.8	.971988	.957982	79.5	.980553	.970830	129.0	.987942	.981914

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