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CONFERENCE ON NON-LINEAR PROCESSES IN THE IONOSPHERE DECEMBER 16-17, 1963

EDITORS

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Volume VI

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*Session 4 papers continued from Volume V, Technical Note No. 211.

USE OF RADIO TRANSMITTERS TO DECREASE D-REGION ELECTRON DENSITY*

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A study is made of a method of decreasing the electron density in the D-region. The method involves the heating of electrons by intense electromagnetic radiation. This increases the electron attachment rate and thereby decreases the electron density. The relevant D-layer rate processes are reviewed and incorporated into a detailed analysis of the influence of a perturbing electromagnetic pulse upon the variation in time of the electron density. The theory of conductivity of a weakly ionized gas is reviewed and augmented to include dependency on field strength. Propagation constants are determined and the equations describing the nonlinear radial propagation of a powerful wave in a weakly ionized inhomogeneous medium are derived. Numerical solutions of these equations are obtained which describe the effects due to the vertical propagation of waves for several power levels and frequencies through a heavily perturbed D-region. It is found that the powerful wave modifies the electron density distribution. The combination of high frequency and optimum power level provides a channel of decreased electron density in the D-region through which a wanted wave may propagate with greatly reduced attenuation.

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I. INTRODUCTION

It is well known that high altitude nuclear explosions can cause interference to communication between ground stations operating at hf (3-30Mc/s) [Glasstone (ed.), 1962; DASA 1229, 1961] . Periods of unusual solar activity cause similar difficulties but these disturbances are usually found in the polar regions [Agard Ionospheric Committee, 1961].

The communication interference is due to increased absorption in the D-region (50-80 km altitudes) which is caused by enhanced electron density. The electrons are produced, in the case of the nuclear explosion, immediately and evanescently by soft X-rays from the bomb casing and then subsequently by the more long-lasting sources of β and γ emission from the radioactive debris undergoing slow hydrodynamic dispersal. It is also thought that neutron decay may contribute to the ionization [Crain and Tamarkin, 1961] . The soft X-rays are absorbed in the D-region by the photoelectric effect and thus enhance the ionization. The β's lose their energy in the same region by ionization, and the γ's lose their energy at lower altitudes by Compton recoils. The naturally occurring ionospheric disturbances are thought to be due to the deposition of energy in the lower ionosphere by energetic protons and X-rays from the sun.

The ultimate electron density which can be achieved under the above-mentioned perturbed conditions is controlled by the intensity and duration of the source and the electron removal mechanisms. The main removal mechanism as we shall see in Section II is provided by electron attachment to molecular oxygen. This is quite fast. Thus, the perturbation due to the soft X-rays from the bomb rapidly disappears, to be replaced by the perturbation due to the radiactive decay of the slowly expanding debris.

The communication interference we have spoken about may last for several days and extend over even widely separate areas [Obayashi et al., 1959]. Such interference is troublesome and in emergencies may be fatal. Thus, it has been the subject of some study.

Among the problems which have been studied are the methods by which the enhanced electron density in the D-region may be decreased. It has been suggested that material of high electronegativity be transported from the ground and released at appropriate altitudes to attach and therefore remove electrons [Armour Res. Foundation, No. 61-126, 1961; Munick and Folk, 1960].

The author of this present study suggested in 1958 [Molmud, 1958] that the new data on electron attachment coefficients to O_2 could be utilized to effect a reduction of electron density. The data, as reported by Biondi [1958] showed that the attachment coefficient was monotonic, and increased from at least .04 ev to $\sim$ .1 ev where it maximized. In addition, the maximum in the coefficient was about twice that of its value at .04 ev. It was reasoned, therefore, that if the electrons in the D-layer were maintained at .1 ev average energy then the attachment rate for these electrons would be commensurately higher than for the unperturbed electrons and so the observable steady state free electron density would be lower. The electrons were to be maintained at this abnormally high energy by feeding in energy electromagnetically via powerful radio waves broadcast from the ground. The fact that electrons in the ionosphere can be seriously perturbed in energy by means of powerful ground-based broadcasting stations had already been amply demonstrated through the phenomenon known as the Luxembourg effect or cross-modulation [Mitra, 1952].

The idea of altering the attachment coefficient was presented at a symposium held at Air Force Cambridge Research Center in 1959 [Molmud, 1959a].

The remainder of this paper will be arranged as follows: Section II will discuss the important ionospheric processes occurring in the D-region. In Section III mathematical expressions describing the density of the most important charged components in the D-region will be set up, and analytical solutions will be obtained for these densities following an instantaneous change of electron energy. In Section IV we discuss the theory of conductivity of weakly ionized gases for strong electric fields. We first obtain the coupled drift velocity and average energy of electrons for the case of a Maxwellian gas and elastic collisions only. Then we generalize to the case of air and by an "Ansatz" include inelastic collisions. We obtain time-dependent solutions for velocity and average energy and set up the expressions for the conductivity tensor. Section V is concerned with the problems of analyzing the propagation of electromagnetic waves which are strong enough so that they change the propagation constants of the medium they are traversing. Included are the dependence of electron collision frequency and of electron density on average energy. In Section VI we treat the propagation of powerful waves of several frequencies and power levels through a severely disturbed D-region. We show how these waves are attenuated and how the attenuation depends on the power level and frequency. We also see how the perturbation these waves make in the ionosphere facilitates the passage of a wanted wave.

II. THE IMPORTANT IONOSPHERIC PROCESSES

The processes we shall mention have been discussed already in several review articles [Nicolet and Aikin, 1960; Crain, 1961; Anderson, 1961; Nawrocki and Papa, 1961; DASA 1229, 1961]. The values presented here are taken from these sources.

A. Sources of Ionization

Several different processes contribute to ionization in the normal D-region (50-85 km). Above 70 km ionization of NO by Lyman-α appears to predominate. Below about 70 km cosmic-ray ionization of air begins to become important and below 60 km ionization by keV X-rays ($\lambda < 1$ Å) and cosmic rays predominates. Normally the electron density ranges from $\sim 10^2 cm^{-3}$ at 60 km to $\sim 10^3$ at 75 km. Under disturbed solar conditions [Fig. 4 of Nicolet and Aikin, 1960], the electron density may increase by a factor of 10 at these altitudes. Reid [1961] calculates electron densities produced by solar protons during polar cap absorption events as even greater. Kane [1961], reporting on rocket measurements of D-layer absorption and phase shift during a polar blackout, finds $2 \times 10^3 cm^{-3}$ at ~ 57 km and $6 \times 10^3 cm^{-3}$ at ~ 70 km.

High altitude nuclear explosions contribute to the D-layer ionization [DASA, 1961, Glasstone (ed), 1962] by (a) evanescent soft X-rays and (b) long-lasting β and γ rays from radioactive decay of bomb debris. The γ's deposit their energy at about 30 km altitude and the β's at about 70 km. The maximum electron densities due to the debris may be as large as 10^5-$10^6 cm^{-3}$ [DASA,1961].

B. Charge Removal and Exchange Mechanisms

We include electron attachment as a charge removal mechanism since it effectively removes electrons from interacting with electromagnetic radiation.

1. Electron Attachment

This is described by the process:

$$e + O_2 + O_2 \longrightarrow O_2^- + O_2 \quad ,$$

with a rate given by $K(u)\left[O_2\right]^2 n$, where $K(u)$ is the attachment coefficient, a function of u the average electron energy, $\left[O_2\right]$ is the numerical density of O_2 molecules and n the electron density. The functional dependence of K on u is given in Fig. 1, below, and is obtained from Chanin, Phelps, and Biondi [1959]. Nicolet and Aikin [1960] estimate that K at D-layer temperatures has the value $\sim 1.5 \times 10^{-30} cm^6/sec$.

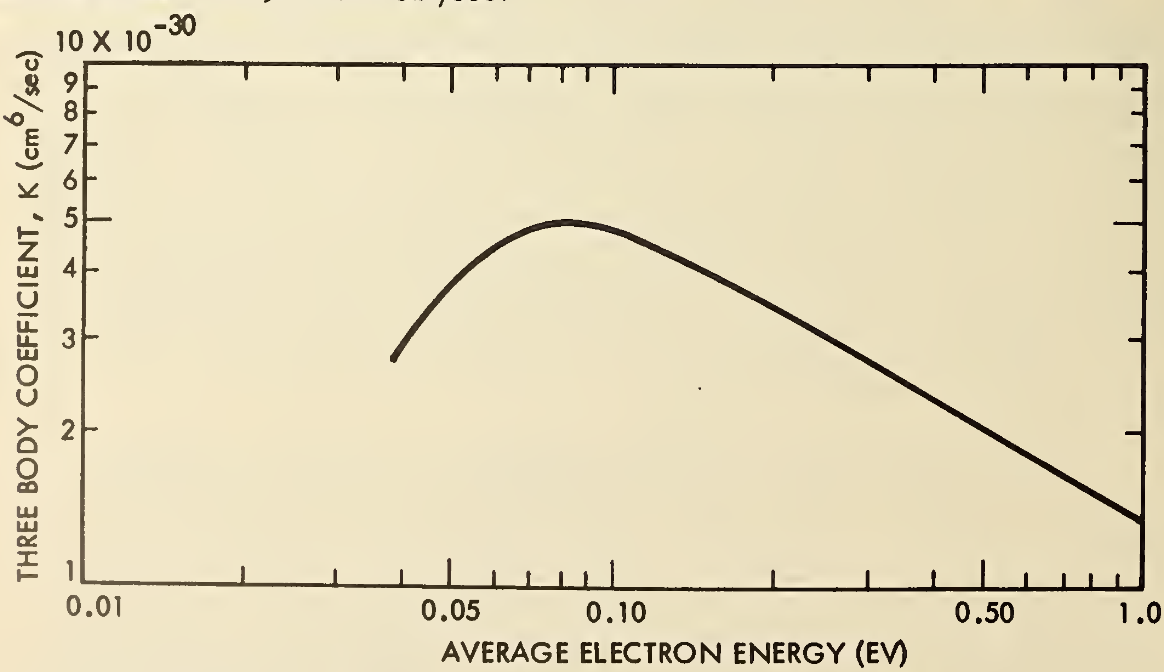

Figure 1. Three-Body Attachment Coefficient vs Average Electron Energy for Oxygen at 300°k [Chanin, Phelps, Biondi, 1959].

2. Photodetachment

This is represented by:

$$O_2^- + h\nu \longrightarrow O_2 + e \quad .$$

Based upon the work of Burch, Smith and Branscomb [1958] the rate coefficient is given [Nicolet and Aikin, 1960] as R = .44 sec^{-1} with a detachment rate of Rn_-, where n_- is the O_2^- density.

3. Collisional Detachment

This is represented by:

$$O_2^- + x \longrightarrow O_2 + x + e \quad .$$

The rate, estimated by Crain [1961] is essentially zero whereas Reid [1961] suggests a detachment coefficient of $\sim 1.5 \times 10^{-17} cm^3/sec$.

4. Electron Recombination

a. $$e + O_2^+ \longrightarrow O + O \quad ,$$

with a recombination coefficient $\alpha = 3.8 \times 10^{-7} cm^3/sec$.

According to Anderson [1961] this coefficient may depend on electron temperature as $T^{-\frac{1}{2}}$

b. $$e + N_2^+ \longrightarrow N + N \quad .$$

$$\alpha = 5.9 \times 10^{-7} cm^3/sec.$$

The magnitude of the coefficients of both (a) and (b) were reported by Kasner, Rogers, and Biondi [1961].

c. $$e + NO^+ \longrightarrow N + O \quad .$$

$$\alpha = 3 \times 10^{-9} cm^3/sec.$$

[cf. Nicolet and Aikin, 1960]

5. Charge Exchange

a. $N_2^+ + O_2 \rightarrow O_2^+ + N_2$.

[Fite et al.,1961] $\alpha = 3 \times 10^{-10} cm^3/sec$.

At 60 km with $[O_2] = 1.5 \times 10^{15} cm^{-3}$ the charge exchange rate is $4.5 \times 10^5 sec^{-1}$. This is so rapid that one doubts whether N_2^+ plays any important role at these altitudes.

b. $O_2^+ + NO \longrightarrow O_2 + NO^+$.

Nawrocki and Papa [1961] give $\alpha = 10^{-10\pm1} cm^3/sec$.

Nicolet and Aiken [1960] give

$[NO] \approx 10^6 cm^{-3}$ at 60 km so $\alpha[NO] = 10^{-4\pm1} sec^{-1}$,

which indicates a very slow charge exchange rate.

6. Ion Recombination

a. $O_2^- + O_2^+ \longrightarrow O_2 + O_2$.

b. $O_2^- + N_2^+ \longrightarrow O_2 + N_2$.

c. $O_2^- + NO^+ \longrightarrow O_2 + NO$.

Crain* has suggested a coefficient of $1 \times 10^{-7} cm^3/sec$ for the above (a,b,and c).

7. Dissociative Attachment

$$e + O_2 \longrightarrow O^- + O$$

See Fig. 2, below [Nawrocki and Papa, 1961 p. 3 - 13 .]

This process becomes important for electrons only above about 1 ev in energy. Thus, some of the electrons produced by ionizing events may be captured by

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* Private communication

this process, but these electrons decay in energy so rapidly that very few can be removed in this way.

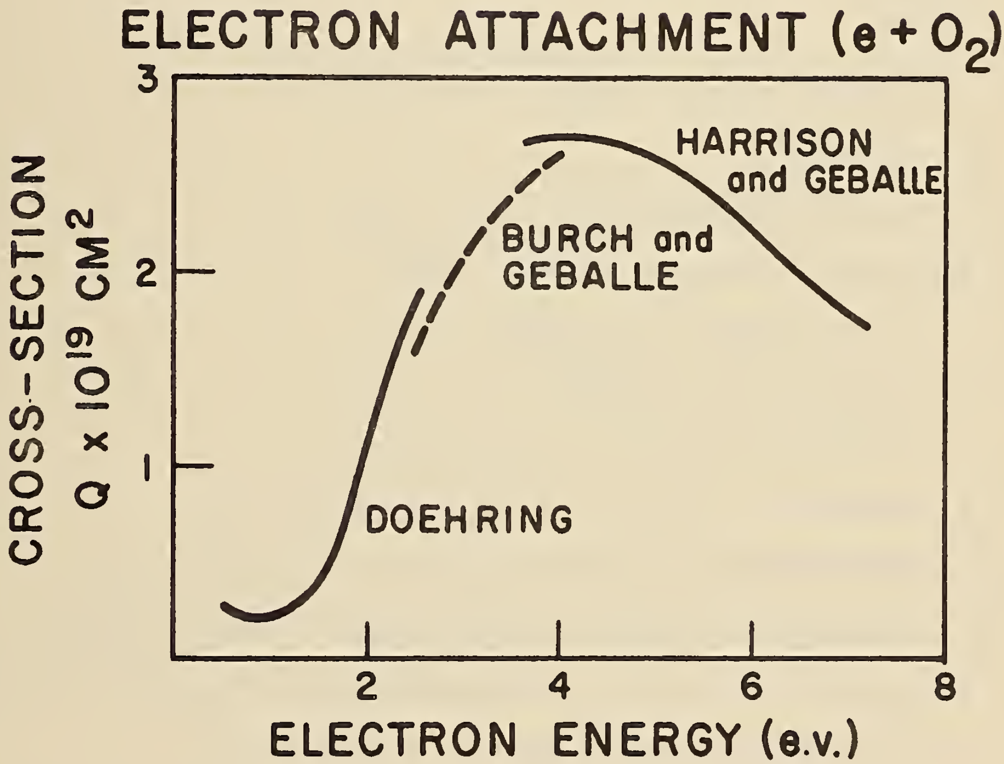

Figure 2. Dissociative Attachment Cross Sections vs Electron Energy

III. AN ANALYTICAL MODEL OF THE D-REGION *

A. The Differential Equations

We now set up the differential equations describing the concentration of the important charged species in the D-region.

Symbols:

n electron density

n_- O_2^- density

n_+ positive ion density

$[O_2]$ particle density of molecular oxygen

q source strength of ion pairs

u average electron energy

o subscript referring to unperturbed state

K attachment coefficient $[e + O_2 + O_2 \longrightarrow O_2^- + O_2]$

R photodetachment coefficient $[O_2^- + h\nu \longrightarrow O_2 + e]$

D collisional detachment coefficient $[O_2^- + X \longrightarrow O_2 + e + X]$

α_e electron recombination coefficient $[e + X_2^+ \longrightarrow X + X]$

α_- O_2^- recombination coefficient $[O_2^- + X_2^+ \longrightarrow O_2 + X_2]$

$$\frac{dn}{dt} = q + (R + D)n_- - K[O_2]^2 n - \alpha_e nn_+ \qquad \text{(III-1)}$$

$$\frac{dn_-}{dt} = K[O_2]^2 n - (R + D)\, n_- - \alpha_- n_- n_+ \qquad \text{(III-2)}$$

$$n_+ = n + n_- \qquad \text{(III-3)}$$

where Eq. (III-3) arises from requirements of local charge neutrality.

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* A slightly less general discussion of the same matters in this Section may be found in Molmud (1963).

Some steady state relationships arising from Eqs. (III-1), (III-2), (III-3) are as follows:

$$\frac{n}{n_+} = \frac{R + D + \alpha_- n_+}{\alpha_- n_+ + R + D + K\left[O_2\right]^2} \tag{III-4}$$

$$\frac{n_-}{n_+} = \frac{K\left[O_2\right]^2}{\alpha_- n_+ + R + D + K\left[O_2\right]^2} \tag{III-5}$$

$$n_+ = \frac{-(\alpha_e - \alpha_-)n + \sqrt{n^2(\alpha_e - \alpha_-)^2 + 4q\alpha_-}}{2\alpha_-} \tag{III-6}$$

$$n = \frac{q + (R + D)\, n_+}{\alpha_e n_+ + R + D + K\left[O_2\right]^2} \tag{III-7}$$

$$q = n_+\left[\alpha_e n + \alpha_- n_-\right], \tag{III-8}$$

The first three equations allow, in steady state, independent solutions for each species n, n_-, and n_+ from cubic equations. For example, the cubic equation for n_+ has the following form:

$$n_+^2\left[\alpha_- + \frac{(\alpha_e - \alpha_-)(R + D)}{\alpha_e n_+ + (R + D) + K\left[O_2\right]^2}\right] - q\frac{\left[\alpha_- n_+ + R + D + K\left[O_2\right]^2\right]}{\alpha_e n_+ + (R + D) + K\left[O_2\right]^2} = 0 \,. \tag{III-9}$$

We shall not concern ourselves with the general solutions of (III-9) but the approximate one where $\alpha_e n_+$ and $\alpha_- n_+$ can be neglected with respect to $R + D + K\left[O_2\right]^2$. Under these circumstances, n_+ has the solution:

$$n_+ = \sqrt{q}\sqrt{\frac{R + D + K[O_2]^2}{\alpha_e(R + D) + K[O_2]^2\alpha_-}} \qquad \text{(III-10)}$$

Equation (III-10) can be combined with (III-4) and (III-5) to yield approximate solutions for n and n_- also.

B. Analytical Solutions of the Differential Equations

We seek solutions of (III-1), (III-2), and (III-3) for the following initial conditions: for $t < 0$, $u = u_o$, and steady state conditions prevail and for $t \geq 0$, $u = u_1$. This is the situation which would obtain if a powerful radio beam were suddenly switched on and propagated through the D-region. The energy of the electrons would rise almost instantaneously (for details see Section IV) and the electron density would change comparatively slowly.

Case 1. $\alpha_e = \alpha_- = \alpha$

We add (III-1) and (III-2) and make use of (III-3), thus obtaining

$$\frac{dn_+}{dt} = q - \alpha n_+^2 \qquad \text{(III-11)}$$

Let us assume, for the moment, that α is independent of u. Then because of the initial conditions (i.e., $dn_+/dt = 0$ at $t = 0$), the only compatible solution of (III-11) is that n_+ is a constant.

Now let α have some dependence on energy (i.e., as $T^{-1/2}$) (see Section II) so that the solution of (III-11) is:

$$\left(\frac{n_{+\infty} - n_+}{n_{+\infty} + n_+}\right)\left(\frac{n_{+\infty} + n_{+o}}{n_{+\infty} - n_{+o}}\right) = e^{-2n_{+o}\sqrt{\alpha_o\alpha_1}\, t} \qquad \text{(III-12)}$$

where $n_{+\infty} = \sqrt{\frac{q}{\alpha_1}}$ and $n_{+o} = \sqrt{\frac{q}{\alpha_o}}$.

We see that the relaxation time, τ, is $1/(2n_{+o}\sqrt{\alpha_o \alpha_1})$. Let us take $n_+ = 10^5 cm^{-3}$ (a severely perturbed condition), $\alpha_o = 5 \times 10^{-7} cm^3/sec$, $\alpha_1 = 5/\sqrt{2} \times 10^{-7} cm^3/sec$ (i.e. $u_1/u_o = 2$ and $\alpha \sim T^{-\frac{1}{2}}$). Then τ = 12 seconds, with corresponding higher times at lower n_+ densities.

Let us provisionally assume that the relaxation time for electrons is much less than that for the positive ions. Then we can solve (III-1), (III-2), and (III-3) under the assumption that n_+ is constant while n is undergoing its substantial rapid changes. The solution is readily obtained:

$$n = n_o e^{-\lambda_1 t} + n_\infty (1 - e^{-\lambda_1 t}) \qquad \text{(III-13)}$$

where $n_o = \dfrac{q + (R + D)n_+}{\lambda_o}$, $n_\infty = \dfrac{q + (R + D)n_+}{\lambda_1}$

and $\lambda_1 = R + D + \alpha_{el} n_+ + K(u_1) [O_2]^2$

$$\lambda_o = R + D + \alpha_{eo} n_+ + K(u_o) [O_2]^2 \ .$$

Here the relaxation time $\tau = 1/\lambda_1$. Its value at 60 km is obtained from $R = .44\ sec^{-1}$, $\alpha_{el} = 5/\sqrt{2} \times 10^{-7}$, $K(u_1) [O_2]^2 = 6.6$ (i.e., $u/u_o = 2$), $D \approx 0$, $n_{+o} = 10^5 cm^{-3}$.

Then $\tau = 1/7$ sec. At 70 km, $K(u_1) [O_2]^2 = 1.26$ and $\tau = .58$ sec; at 80 km, $K(u_1) [O_2]^2 = .06$ and $\tau \sim 2$ sec. Thus our assumption is correct for altitudes of importance and n_+ can be treated as constant while n decays to n_∞. The subsequent time history of n_∞ is controlled by n_+ through (III-12) and (III-4).

Case 2 $\alpha_e \neq \alpha_-$

Here we shall again assume that n_+ is constant while n undergoes its major rapid variations. Then the solution for n is the same as (III-13). The differential equation for n_+ is obtained by adding (III-1) to (III-2) inserting n_∞ for n, and using (III-5) and (III-7)

$$\frac{dn_+}{dt} = q - \frac{q\alpha_{el} n_+}{\lambda_1} - n_+^2 \left[\alpha_{el} \frac{R + D}{\lambda_1} + \alpha_{-1} \frac{K(u_1) \left[O_2 \right]^2}{\lambda_1 + n_+(\alpha_{-1} - \alpha_{el})} \right] . \qquad \text{(III-14)}$$

Ignoring the contributions of $\alpha_{el} n_+$ and $(\alpha_{-1} - \alpha_{el})n_+$ to λ_1, we can readily find the solution to (III-14).

$$\left(\frac{n_+ - n_{+\infty}}{n_+ + b/c + n_{+\infty}} \right) \left(\frac{n_{+o} + b/c + n_{+\infty}}{n_{+o} - n_{+\infty}} \right) = e^{(b + 2cn_{+\infty})t} . \qquad \text{(III-15)}$$

Where $b = - q\alpha_{el}/\lambda_1$, $c = - 1/\lambda_1 \left[\alpha_{el}(R + D) + \alpha_{-1} K(u_1) \left[O_2 \right]^2 \right]$,

and $n_{+\infty}$ is obtained from (III-14) by setting $dn_+/dt = 0$, i.e.,

$$n_{+\infty} = - \frac{b}{2c} - \frac{\sqrt{b^2 - 4qc}}{2c} .$$

Here the relaxation time is given by

$$\tau = \frac{1}{b + 2cn_{+\infty}} .$$

When, for example, at 60 km, $n = 2 \times 10^4 cm^{-3}$, $\alpha_e = 5 \times 10^{-7} cm^3/sec$, $\alpha_- = 10^{-7} cm^3/sec$, $K(u_o) \left[O_2 \right]^2/(R+D) = 7.5$, $K(u_1) \left[O_2 \right]^2/(R+D) = 15$, then by the previous relationships, $q = 3.5 \times 10^3 cm^3/sec$, $n_{+o} = 1.5 \times 10^5 cm^{-3}$, $n_{+\infty} = 2.25 \times 10^5 cm^{-3}$, $\lambda_1 = 7\ sec^{-1}$, $b \approx -3 \times 10^{-4} sec^{-1}$, $2\ cn_{+\infty} = -5 \times 10^{-2} sec^{-1}$. Thus $\tau = 20$ sec. For lower electron densities $n_{+\infty}$ becomes lower and τ larger. For higher altitudes $b \approx -q\alpha_e/(R + D)$ $c \approx \alpha_{e1}/(R + D)$ and $\tau \approx 4$ sec. Thus, the relaxation time for positive ions appears to be sufficiently large that, for all significant altitudes and fairly large perturbations, the assumption that n_+ is constant while n is making its rapid change is warranted. The product $\alpha_e n_+$ is .1 and there is little error in neglecting it in comparison with $R + D + K[O_2]^2 = 7$. At higher altitudes, the last sum may have the value .44 so an error of ~20% may be incurred -- not significant enough to concern ourselves with.

One can readily show that if the energy dependence of the α's is negligible then $n_{+\infty}/n_{+o} > 1$ for $\alpha_e > \alpha_-$ and $n_{+\infty}/n_{+o} < 1$ for $\alpha_e < \alpha_-$. Thus, with reference to (III-13) when $\alpha_e > \alpha_-$, for small values of t, the electron density decreases until $n \sim n_\infty$, then n_+ starts to increase and so does n. Thus, we get a minimum in the electron density as a function of time. This minimum should take several seconds to manifest itself and may be difficult to observe because of ionospheric perturbations.

The equations (III-1), (III-2), and (III-3) have been programmed and solutions for various values of q, α_e, α_-, and $K(u)[O_2]^2/(R + D)$ have been obtained. We now examine one of the cases to see how valid our approximations are. We take $q = .44\ sec^{-1}cm^{-3}$, $\alpha = 3.8 \times 10^{-7} cm^3 sec^{-1}$, $\alpha_- = 3.8 \times 10^{-8} cm^3 sec^{-1}$, with initially $K(u_o)[O_2]^2/(R + D) = 1/2$ and then $K(u_1)[O_2]^2/(R + D) = 1$. The machine (precise) values for n_o and n_{+o} are

$n_o = 8.59 \times 10^2 cm^{-3}$ and $n_{+o} = 1.289 \times 10^3 cm^{-3}$. The approximate values are $n_o = 8.55 \times 10^3 cm^{-3}$, $n_{+o} = 1.28 \times 10^3 cm^{-3}$. The machine value of n at $t = 2.28$ sec is $6.736 \times 10^2 cm^{-3}$. The approximate value is $6.72 \times 10^2 cm^{-3}$, n has a minimum at 9.3 seconds with a value of $6.450 \times 10^2 cm^{-3}$. This minimum is quite broad. For example, at $t = 4.55$ sec, $n = 6.486 \times 10^2 cm^{-3}$ and at $t = 22.5$ sec, $n = 6.455 \times 10^2 cm^{-3}$. The approximate value of n_∞ is $6.45 \times 10^3 cm^{-3}$, close to that which is observed, $\tau_{electrons} = 1.14$ sec, $\tau_{ions} = 1.6 \times 10^3$sec. $n_{+\infty} = 1.454 \times 10^3$ and n_∞ corresponding to $n_{+\infty}$ is $7.2 \times 10^2 cm^{-3}$. At $t = 27$ sec the approximate n_+ is $1.31 \times 10^3 cm^{-3}$ and the machine value is $1.33 \times 10^3 cm^{-3}$.

The approximate solutions we have given appear to adequately satisfy the differential equations.

IV. THEORY OF CONDUCTIVITY OF WEAKLY IONIZED GASES

In this Section, we develop the fundamental equations and relationships for conductivity and electron average energy used in the succeeding sections.

A. The Moment Equation

The standard procedure for obtaining expressions for average drift velocity and average energy of electrons in a gas in an electric field is to solve Boltzmann's transport equation [Allis, 1956; Chapman and Cowling, 1939; Margenau, 1946]. The method involves approximations and does not yield transient solutions nor solutions for arbitrarily large fields. Allis [1956] has pointed out that the moment equation, derived from Boltzmann's equations, can be solved exactly for the moments under certain circumstances without knowing the distribution function satisfying the transport equation. Surprisingly enough, this procedure has received very little attention even though the resulting solutions provide very useful information for realistic cases. We shall adopt this procedure for the present and finally, when we can make no more progress, we shall resort to solutions of the transport equation.

We start with the conservation equation on $\bar{X}(v)$ where $X(v)$ is any quantity associated with the electrons in the gaseous medium. The bar means averaging over the distribution function of the electrons. We shall assume spatial homogeneity in what follows:

$$\frac{\partial}{\partial t}(n\bar{X}) - n\overline{\nabla_v \cdot \left[\frac{q}{m}\vec{E} + \omega_b \times \vec{v}\right] X} = \int (X' - X)F(\vec{V})f(\vec{v})\,|\vec{v}-\vec{V}|\sigma(\theta,|\vec{v}-\vec{V}|) \cdot d^2\Omega\, d^3\vec{v}\, d^3\vec{V} \qquad \text{(IV-1)}$$

where n is the density of electrons, $\omega_b = -\frac{qH}{mc}$ $q = -|e|$, X' - X is the change in X due to collision, $F(\vec{V})$ is the distribution function for neutral particles, $f(\vec{v})$ is the distribution function for electrons, $\sigma(\theta,|\vec{V}-\vec{v}|)$ is the differential cross section for scattering an electron of relative velocity $\vec{v}-\vec{V}$ through an angle θ into a solid angle $d^2\Omega$. The integration is over solid angle and velocity space. $\nabla_v \cdot$ means divergence operator in velocity coordinates.

1. <u>Let $X = \vec{v}$</u>

Then, assuming elastic collisions [Allis, 1956, p. 423]

$$\frac{\partial}{\partial t}(n\vec{v}_d) - n\left(\frac{q}{m}\vec{E} + \vec{\omega}_b x\vec{v}_d\right) = \frac{M}{M+m}\int F(\vec{V})f(\vec{v})\ Q(|\vec{v}-\vec{V}|)(\vec{v}-\vec{V})|\vec{v}-\vec{V}|\ d^3\vec{v}d^3\vec{V} \qquad \text{(IV-2)}$$

where $Q(|\vec{v}-\vec{V}|) = \int (1 - \cos\theta)\sigma(\theta,|\vec{v}-\vec{V}|)d^2\Omega$, the cross section for momentum transfer; M is the mass of the molecule, m the mass of the electron and $\vec{v}_d = \int \vec{v}fd^3\vec{v}$, the average drift velocity of the electron.

2. <u>Let $X = \frac{1}{2}mv^2$</u>

Then

$$\overline{\nabla_v \cdot \frac{q}{m}\vec{E}\,\tfrac{1}{2}mv^2} = \int \frac{q}{m}\vec{E}\cdot(\nabla_v \tfrac{1}{2}mv^2)fd^3\vec{v} = q\vec{E}\cdot\int \vec{v}fd^3\vec{v} = q\vec{E}\cdot\vec{v}_d$$

$$\overline{\nabla_v \cdot(\vec{\omega}_b x\vec{v})\,\tfrac{1}{2}mv^2} = \overline{\omega_b x\vec{v}\cdot m\vec{v}} + \overline{\tfrac{1}{2}mv^2\nabla_v\cdot(\vec{\omega}_b x\vec{v})} = \overline{\tfrac{1}{2}mv^2(\nabla_v x\vec{v}\cdot\vec{\omega}_b)} = 0\ .$$

Proceeding as detailed in Allis [1956, p. 424]

$$\frac{\partial}{\partial t}(nu) + \frac{Mm}{(M+m)^2}\int(m\vec{v} + m\vec{V})\cdot(\vec{v}-\vec{V})Q(|\vec{v}-\vec{V}|)|\vec{v}-\vec{V}|f(\vec{v})F(\vec{V})d^3\vec{v}d^3\vec{V} = q\vec{E}\cdot\vec{v}_d n \qquad \text{(IV-3)}$$

where $u = \frac{1}{2}mv^2$.

3. Let $Q(|\vec{v}-\vec{V}|)|\vec{v}-\vec{V}|$ = constant

For the case of $Q(|\vec{v}-\vec{V}|)|\vec{v}-\vec{V}|$ a constant (a Maxwellian gas) Eqs. (IV-2) and (IV-3) may be reduced to the convenient forms,

$$\frac{\partial \vec{v}_d}{\partial t} + \frac{M}{M + m} \nu (\vec{v}_d - \vec{V}_d) = \frac{q}{m} \vec{E} + \vec{\omega}_b \times \vec{v}_d \tag{IV-4}$$

$$\frac{\partial u}{\partial t} + \frac{2Mm}{(M + m)^2} \nu\left[(u - u_o) + (M - m)\, \vec{V}_d \cdot \vec{v}_d\right] = q\vec{E} \cdot \vec{v}_d \quad . \tag{IV-5}$$

Where $\nu = \rho Q(|\vec{v}-\vec{V}|)|\vec{v}-\vec{V}|$

$$\vec{V}_d = \int \vec{V} F(\vec{V}) d^3\vec{V}$$

$$u_o = \int \tfrac{1}{2} M V^2 F(\vec{V}) d^3\vec{V} \quad .$$

In deriving Eq. (IV-5) from (IV-3), it is assumed that $F(\vec{V})$ is Maxwellian centered about a velocity $\vec{V}_d$.

We see that (IV-4) and (IV-5) are two coupled differential equations for $\vec{v}_d$ and u.

The first of these equations has already been obtained by Altshuler [1963] and others [Kihara, 1952]. The second was obtained by Molmud [1956] by a less general procedure. Since they are both linear, exact solutions can be written down immediately for any E(t). We postpone investigating the solutions, however, until later.

4. <u>Let $Q(|\vec{v} - \vec{V}|) = c|\vec{v} - \vec{V}|$</u>, where c is a constant.

It has been observed [Molmud, 1959b; Phelps, 1960] that the momentum transfer cross section for air is most closely represented (for temperatures far from absolute zero) as varying linearly with $|\vec{v}-\vec{V}|$. Under these circumstances, (IV-2) and (IV-3) (neglecting terms of order m/M and V^2/v^2) become:

$$\frac{\partial \vec{v}}{\partial t} + \frac{M}{m + M} \nu \left[\frac{\overline{\vec{v}\frac{1}{2}mv^2}}{u} - \vec{V}_d - 2\tfrac{1}{2}m \frac{\overline{\vec{v}\vec{v}}\cdot\vec{V}_d}{u} \right] = \frac{q}{m}\vec{E} + \vec{\omega}_b \times \vec{v}_d \tag{IV-6}$$

$$\frac{\partial u}{\partial t} + \frac{2Mm}{(M + m)^2} \nu \left\{ \frac{m}{2} \frac{\overline{v^4}}{\overline{v^2}} - \frac{5}{3} u_o + \frac{\frac{(M-m)}{2}\overline{v^2\vec{v}}\cdot\vec{V}_d}{\overline{v^2}} \right\} = q\vec{E} \cdot \vec{v}_d \quad . \tag{IV-7}$$

Again as in the case of the Maxwellian gas, we have two coupled differential equations, but here the coupling is stronger because now ν is a function of the energy.

Equations (IV-6) and (IV-7) are not entirely useful as they stand for they contain averages over the distribution function of the electrons, which is not known. They do, however, contain useful information. For example, in Eq. (IV-6) there is a coupling term between the pressure term of the electrons, $\overline{\vec{v}\vec{v}}$, and the average drift velocity of the neutral molecules. In Eq. (IV-7), if the distribution function is assumed Maxwellian then $\overline{v^4}/\overline{v^2} = (5/3)\overline{v^2}$.
Thus ignoring $\vec{V}_d$ and assuming the isotropic part of the distribution function is Maxwellian, then (IV-6) and (IV-7) become

$$\frac{\partial \vec{v}_d}{\partial t} + \frac{M}{M + m} \, k \nu \vec{v}_d = \frac{q}{m} \vec{E} + \vec{\omega}_b \times \vec{v}_d \tag{IV-8}$$

$$\frac{\partial u}{\partial t} + \frac{2Mm}{(M + m)^2} \nu \frac{5}{3} \left\{ u - u_o \right\} = q\vec{E} \cdot \vec{v}_d \tag{IV-9}$$

where $\vec{v}_d k = \dfrac{\overline{\vec{v} \frac{1}{2} m v^2}}{u}$.

We see that (IV-8) and (IV-9) bear a formal resemblance to (IV-4) and (IV-5), the only differences being in the presence of k and that ν is now a function of energy.

We here concentrate on solutions of (IV-4) and (IV-5) and shall return later to (IV-8) and (IV-9) to see how their solutions may be obtained.

B. The Solution of the Moment Equation for Drift Velocity

1. The Maxwellian Gas

We seek solutions for Eq. (IV-4) under the following conditions: for $t < 0$, $\vec{v} = 0$, $\vec{E} = 0$; for $t \geq 0$, $\vec{E} = \vec{E}_o e^{i\omega t}$.

Let us call $E_\perp$ that component of $\vec{E}$ perpendicular to $\vec{H}$ and E_p the component parallel to $\vec{H}$. Furthermore, the component of the average velocity perpendicular to $\vec{H}$ is broken up into a component $v_\parallel$, parallel to $E_\perp$ and $v_\perp$, perpendicular to $\vec{E}$. Here and in what follows, we omit the subscripts d. The various components may be represented as in Fig. 3.

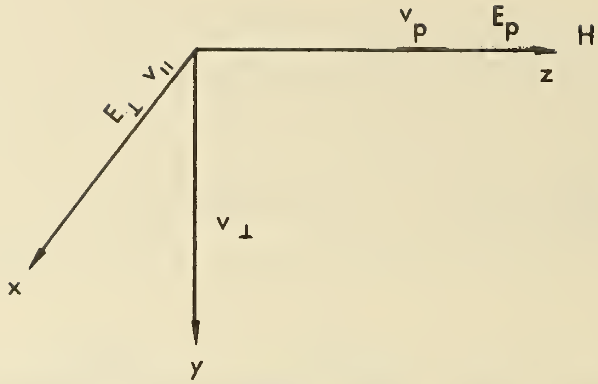


Figure 3. Coordinate System for Eqs. (IV-10,11,12)

The equations to be solved are now:

$$v_p + \nu\, v_p = \frac{q}{m} E_p \tag{IV-10}$$

$$\dot{v}_\parallel + \nu\, v_\parallel = \frac{q}{m} E_\perp + \frac{v_\perp H q}{cm} \tag{IV-11}$$

$$\dot{v}_\perp + \nu\, v_\perp = - v_\parallel \frac{qH}{mc} \quad . \tag{IV-12}$$

The solution of (IV-11) and of (IV-12) will provide the solution of (IV-10) upon setting H = 0. From (IV-11) and (IV-12) we readily obtain the following:

$$\ddot{v}_\perp + 2\nu \dot{v}_\perp + (\nu^2 + \omega_b{}^2)\, v_\perp = \frac{q}{m} E_\perp\, \omega_b \quad , \tag{IV-13}$$

the solution of which is

$$v_\perp = \frac{q}{m}\omega_b e^{(-\nu + i\omega_b)t}\int_0^t e^{-2i\omega_b\tau}\left[\int_0^\tau E_\perp e^{(\nu + i\omega_b)x}dx\right]d\tau \quad . \qquad \text{(IV-14)}$$

Upon substitution of the time dependence of E, decided upon above, we obtain,

$$v_\perp = \frac{q}{m}\frac{\omega_b E_{\perp o}}{\nu + i(\omega + \omega_b)}\left\{\frac{e^{i\omega t}}{\nu + i(\omega - \omega_b)} - \frac{e^{(-\nu + i\omega_b)t}}{\nu + i(\omega - \omega_b)} - \frac{e^{-\nu t}\sin\omega_b t}{\omega_b}\right\}. \qquad \text{(IV-15)}$$

Thus $v_{\parallel}$ becomes:

$$v_{\parallel} = \frac{e}{m}\frac{E_{\perp o}}{[\nu + i(\omega + \omega_b)]}\left\{\frac{1}{\nu + i(\omega - \omega_b)}\left[(\nu + i\omega)\, e^{i\omega t}\right.\right.$$

$$\left.\left. + i\omega_b\, e^{(-\nu + i\omega_b)t}\right] - e^{-\nu t}\cos\omega_b t\right\}, \qquad \text{(IV-16)}$$

and v_p:

$$v_p = \frac{e}{m}\frac{E_{po}\left[e^{i\omega t} - e^{-\nu t}\right]}{\nu + i\omega} \quad . \qquad \text{(IV-17)}$$

The solutions (IV-15) through (IV-17) reach a steady state in a time roughly several times $1/\nu$. The collision frequencies, ν, we shall be dealing with are such that ν is $\sim 10^7 \sec^{-1}$ and thus in 10^{-6} sec steady state conditions for v will be reached.

The conductivity of an ionized medium is defined by setting $j = ne\vec{v} = \sigma\vec{E}$. Since in the present case $\vec{v}$ is not in the same direction as $\vec{E}$, σ is a tensor which we shall denote by $\|\sigma\|$. The conductivity tensor may be written as [Allis, 1956; Molmud, 1962a].

$$\|\sigma\| = \frac{\omega_p^2}{8\pi} \begin{Vmatrix} L + R & i(R - L) & 0 \\ i(L - R) & L + R & 0 \\ 0 & 0 & 2P \end{Vmatrix} , \qquad \text{(IV-18)}$$

where

$$P = \frac{1}{\nu + i\omega}$$

$$L = \frac{1}{\nu + i(\omega + \omega_b)}$$

$$R = \frac{1}{\nu + i(\omega - \omega_b)}$$

$$\omega_p^2 = \frac{4\pi n e^2}{m}$$

and the z axis is chosen to be in the direction of $\vec{H}$.

2. Air

We cannot proceed with the solution of Eq. (IV-8) in the same manner as (IV-4) for two reasons: (a) the value of k depends on the distribution function which has not been determined yet and (b) the value of ν depends on u of Eq. (IV-7). If we restrict ourselves to weak fields so that $u - u_o < \varepsilon$ where $\epsilon \longrightarrow 0$, then (IV-7) is decoupled from (IV-8). Additionally, reasoning by analogy with the results for the Maxwellian gas, we expect that v will achieve steady state in approximately a time of $1/k\nu$. Thus, all that need be considered is the steady state solution of v. For this, we now turn to the transport equation.

The solution of the transport equation for the distribution function of electrons under the influence of an oscillatory electric field has already been determined by Margenau [1946] and Allis [1956]. Margenau has shown that even for $u/u_o = 2$, the isotropic part of the distribution function is Maxwellian. The resulting conductivity tensor has the same form as (IV-18) with somewhat different definitions for P, L and R. These are now (isotropic part of distribution function assumed Maxwellian):

$$P = \frac{8\pi}{3} \beta\left(\frac{\beta}{\pi}\right)^{3/2} \int_o^\infty \frac{v^4 e^{-\beta v^2} dv}{i\omega + \nu(v)}$$

$$L = \frac{8\pi}{3} \beta\left(\frac{\beta}{\pi}\right)^{3/2} \int_o^\infty \frac{v^4 e^{-\beta v^2} dv}{i(\omega + \omega_b) + \nu(v)}$$

$$R = \frac{8\pi}{3} \beta\left(\frac{\beta}{\pi}\right)^{3/2} \int_o^\infty \frac{v^4 e^{-\beta v^2} dv}{i(\omega - \omega_b) + \nu(v)}$$

where $\beta = \frac{m}{2kT}$, $\nu(v) = \rho Q(v) v$. These integrals may be evaluated, and functional forms are displayed in Molmud [1959b] and Phelps [1960] Molmud [1962a, (and for an equivalent treatment, Shkarafsky, 1961)] has shown how to treat these functional forms as though they arose from a Maxwellian gas. The end result of the analysis is that ν in P, L and R of (IV-18) is to be replaced by g, where $g \equiv g_r + ig_i$, and g_r and g_i are functions of ν/Ω where $\nu = \int v \rho Q f d^3 v$ and $\Omega = \omega \pm \omega_b$.

Thus to use (IV-18) for air, we write

$$P = \frac{1}{g_r + i(\omega + g_i)}$$

$$L = \frac{1}{g_r^+ + i(\omega + \omega_b + g_i^+)}$$

$$R = \frac{1}{g_r^- + i(\omega - \omega_b + g_i^-)}$$

where the superscript + or - refers to taking the + or - sign respectively in $\Omega = \omega \pm \omega_b$. The absence of a superscript means that ω_b is to be taken as zero. Figures 4 and 5 below [from Molmud, 1962a] respectively present values of $g_r^{\pm}/\nu$ vs $(\omega \pm \omega_b)/\nu$ and $g_i^{\pm}/(\omega \pm \omega_b)$ vs $(\omega \pm \omega_b)/\nu$.

C. <u>The Solution of the Moment Equation for Average Energy</u>

1. <u>Maxwellian Gas</u>

We now solve Eq. (IV-5) (neglecting V_d) with similar initial conditions as used for v, i.e., $t < 0$, $E = 0$, $u = u_o$; $t \geq 0$, $E = E_o e^{i\omega t}$. However, $\vec{E} \cdot \vec{v}$ in Eq. (IV-5) is to be interpreted as meaning real part of $\vec{E}$ dotted onto real part of $\vec{v}$. Thus

$$\vec{E} \cdot \vec{v} = \frac{e}{m} A_o \cos^2 \omega t + B_o \sin \omega t \cos \omega t + \frac{e}{m} A \cos^2 \omega t + B \sin \omega t \cos \omega t ,$$

where the transient terms in v have been neglected, E_{po} and $E_{\perp o}$ are defined in Section IV-B, and

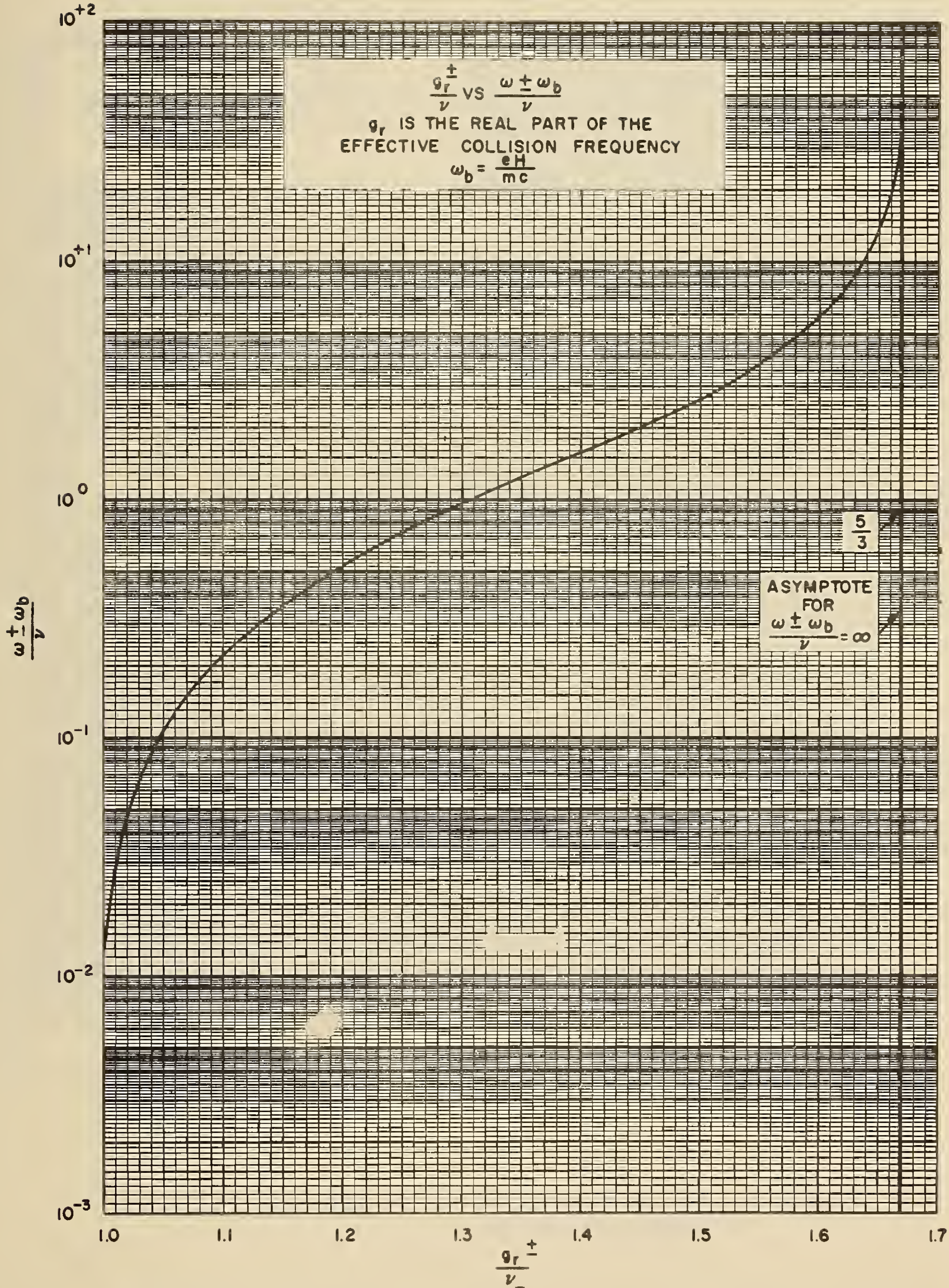


Figure 4. The Real Part of the Effective Collision Frequency vs Ω/ν

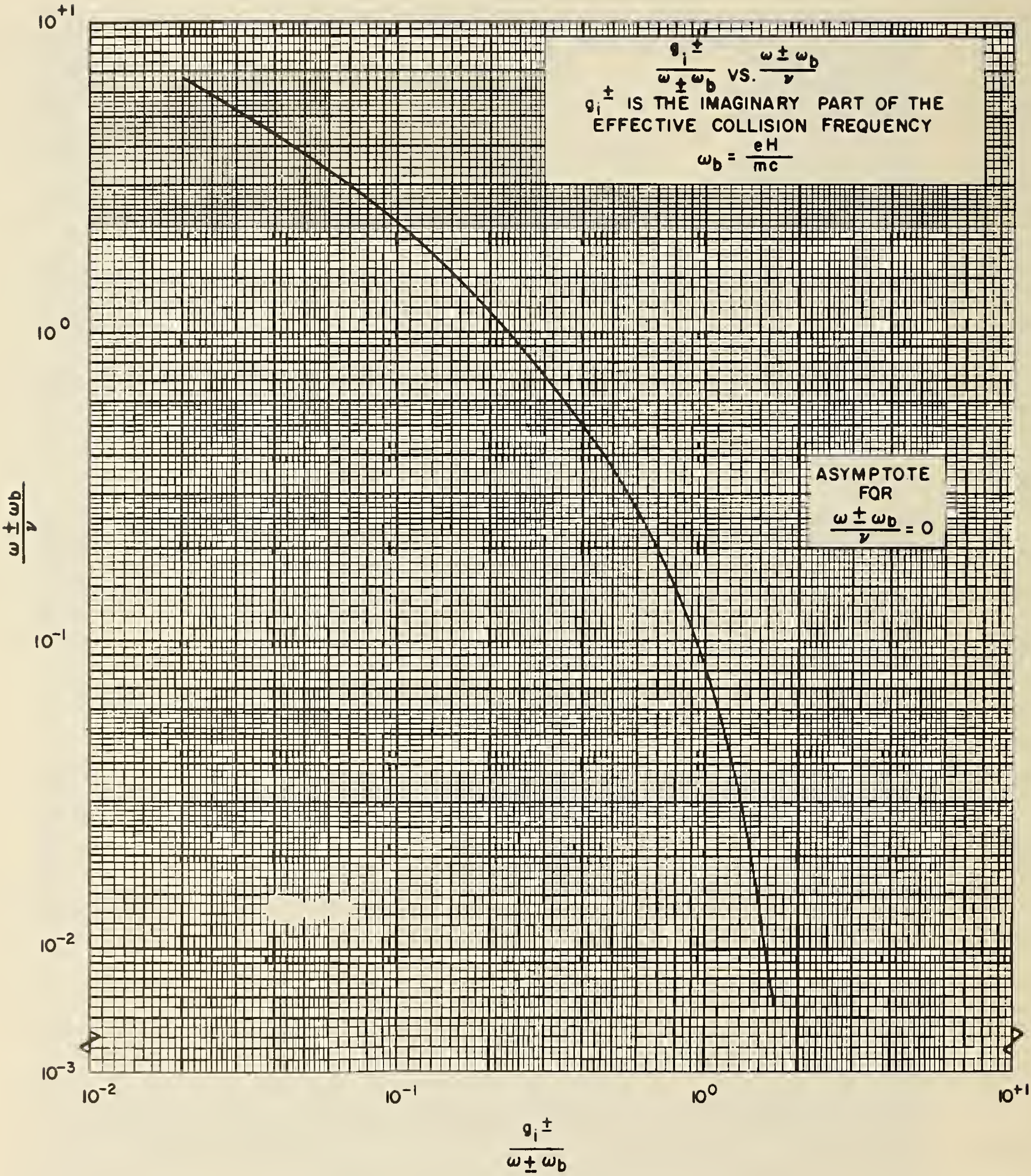


Figure 5. The Imaginary Part of the Effective Collision Frequency vs Ω/ν

$$A = \frac{\nu(\nu^2 + \omega_b^2 + \omega^2)\, E_{\perp o}^2}{(\nu^2 + \omega_b^2 - \omega^2)^2 + 4\omega^2\nu^2} \quad , \quad B = \frac{\omega(\nu^2 + \omega^2 - \omega_b^2)\, E_{\perp o}^2}{(\nu^2 + \omega_b^2 - \omega^2)^2 + 4\omega^2\nu^2}$$

$$A_o = \frac{\nu}{\omega^2 + \nu^2} E_{po}^2 \quad , \quad B_o = \frac{\omega E_{po}^2}{\omega^2 + \nu^2} \quad .$$

The solution of Eq. (IV-5) follows:

$$u = u_o + \frac{e^2}{2m}\frac{1}{G\nu}\left[A_o + A\right]\left\{(1 - e^{-G\nu t}) + G\nu\frac{\left[G\nu\cos 2\omega t + 2\omega\sin 2\omega t - \nu e^{-G\nu t}\right]}{G^2\nu^2 + 4\omega^2}\right\} \tag{IV-19}$$

$$+ \frac{e^2(B_o + B)}{2m(G^2\nu^2 + 4\omega^2)}\left[G\nu\sin 2\omega t - 2\omega\cos 2\omega t + 2\omega e^{-G\nu t}\right] \quad ,$$

and $G = \dfrac{2Mm}{(M + m)^2}$, the fractional energy lost per collision.

Now when $2\omega > G\nu$ only the following terms remain:

$$u = u_o + \frac{e^2}{2mG\nu}\left[A_o + A\right]\left(1 - e^{-G\nu t}\right) \quad . \tag{IV-20}$$

The above inequality usually holds since $G \approx 10^{-5}$ for elastic collisions and $G\nu \sim 10^2$ for regions under consideration. Thus, wave frequencies higher than $10^2 \sec^{-1}$ will satisfy this inequality. Later when we take $G \simeq 10^{-3}$ to account for inelastic collisions, frequencies higher than $10^4 \sec^{-1}$ will be required, but this too will be satisfied for our applications. Thus, we shall deal with the fairly simple form (IV-20) for the average electron energy.

A relaxation time of $1/G\nu \sim 10^{-2}$sec for elastic collisions is obtained when $\nu = 10^7 \text{sec}^{-1}$. When $G = 10^{-3}$ then $1/G\nu \sim 10^{-4}$ sec. The electron density relaxation time is much greater than these relaxation times. Thus, if (IV-20) is representative for electrons in air, then we are justified in assuming in Section III that the electron energy may be changed instantaneously.

Before going on to examine the solution for u for air, let us consider (IV-20) in more detail.

Equation (IV-20) is now written in simpler form:

$$u = u_o + \frac{e^2}{2mG\nu}\left[\frac{\nu}{\omega^2+\nu^2} E_{po}^2 + \tfrac{1}{2}\nu E_{\perp o}^2\left(\frac{1}{(\omega + \omega_b)^2 + \nu^2} + \frac{1}{(\omega - \omega_b)^2 + \nu^2}\right)\right]\left[1 - e^{G\nu t}\right] \quad \text{(IV-21)}$$

We observe that under conditions where $\omega - \omega_b = 0$ and $\nu \ll \omega$, then

$$u = u_o + \frac{e^2}{2mG} \tfrac{1}{2}E_{\perp o}^2 \frac{1}{\nu^2}\left[1 - e^{-G\nu t}\right] \quad , \qquad \text{(IV-22)}$$

allowing enormous value for u, steady state, for very small values of ν. We note for future reference that the magnitude of $e^2/2m$ is 7.9×10^{19} when u is in electron volts and E is in stat volts (esu).

2. <u>$Q = c\,|V - v|$, Air</u>

In the case of Q varying as the velocity, we have seen that in the expression for drift velocity the collision frequency is to be replaced by an effective collision frequency, g, which is complex. In addition, we shall take G, being the fractional energy lost by an electron to include the effects of inelastic collisions (when collisions are elastic, G has the value $2Mm/(M + m)^2$) G is now to be a function of u_o and u and of magnitude determined by experiment. The differential equation for u takes the form suggested by (IV-9).

$$\frac{\partial u}{\partial t} + G(u, u_o)\,\nu(u - u_o) = q\,\vec{E}\cdot\vec{v} \; , \qquad \text{(IV-23)}$$

where $\nu = \nu_o\, u/u_o$.

A good approximate solution to (IV-23) when $\omega \gg G\nu$ and also $\omega > \nu_o$, or $\omega < \nu_o$ and u/u_o not much greater than 2 is:

$$u = u_o + \frac{e^2}{2mG\nu}\left[\frac{g_r}{(\omega + g_i)^2 + g_r^2}\,E_{po}^2 + \tfrac{1}{2}E_{\perp o}^2\left(\frac{g_r^+}{(\omega + \omega_b + g_i^+)^2 + g_r^{+2}} + \frac{g_r^-}{(\omega - \omega_b + g_i^-)^2 + g_r^{-2}}\right)\right]\left[1 - e^{-\int_0^t G\nu d\tau}\right] . \qquad \text{(IV-24)}$$

We shall consider that the u of Eq. (IV-24) can be effected through a ground-based radio transmitter. To determine the power requirements of such a transmitter, we examine the requirements to obtain $u/u_o = 2$ at various levels of the D-region when the frequency of the transmitter is at the gyrofrequency (i.e., $\omega = \omega_b$) and the transmitter is isotropic. We take an experimentally determined value for G, $G = 2 \times 10^{-3}$ [Barrington and Thrane, 1962].* The results are listed in Table IV-I.

* Barrington and Thrane obtain a value for G using an equation similar to (IV-23) in an analysis of cross-modulation experiments.

TABLE IV-I

Ground-Based Power Requirements to Effect $u/u_o = 2$ at Cyclotron Frequencies

Altitude	Power (watts)
60 km	21×10^6
70 km	17×10^6
80 km	4.8×10^5

Thus, fairly large power is required for the low altitudes unless directive antennas are employed. However, it is probably hard to get gains of more than 10 db at cyclotron frequencies.

V. NONLINEAR PROPAGATION IN WEAKLY IONIZED MEDIA

In this Section, we shall concern ourselves with the propagation through an ionized medium of electromagnetic waves so strong that they alter the medium through which they propagate. Thus we shall treat nonlinear propagation problems.

The important alterations of the medium will occur because the electrons become heated, change their collision frequency and also, because of the processes discussed previously, change their density. All these alterations depend on the electric field strength which in turn is based on the manner in which the wave has propagated to the region under consideration. In Part A, below, we set down expressions for the propagation constants which, since they depend on electron density and collision frequency, also depend on the field strength. It will be assumed that steady state conditions prevail so that the propagation constants have no time dependence.

A. Propagation Constants for Weakly Ionized Media

Lorentz's equations for an ionized medium are

$$\nabla \times H = \frac{1}{c}\frac{\partial E}{\partial t} + \frac{4\pi}{c} nev \qquad \text{(V-1)}$$

$$\nabla \times E = -\frac{1}{c}\frac{\partial H}{\partial t} \qquad \text{(V-2)}$$

$$\nabla \cdot E = 4\pi\rho \qquad \text{(V-3)}$$

$$\nabla \cdot H = 0 \qquad \text{(V-4)}$$

where v from Section IV, Eq. (IV-2) is to be employed and n is a function of average energy (Section IV, Eq. (IV-24)) and position.

Now, by the usual manipulation, we form the wave equation and assume harmonic dependence for E. Thus,

$$\nabla^2\vec{E} + \frac{\omega^2}{c^2}\left(1 - \frac{4\pi i\|\sigma\|}{\omega}\right)\vec{E} = \nabla(\nabla\cdot\vec{E}) \qquad \text{(V-5)}$$

and $\|\sigma\|$ is obtained from Eq. (IV-18) of Section IV. $\|\sigma\|$ is also a function of E since σ depends on ν and n which are functions of average energy of the electrons.

Plane wave solutions of (V-5) (neglecting $\nabla(\nabla\cdot E)$) give $E = E_o e^{i\vec{k}\cdot\vec{r}}$, where $\vec{k}$ is the propagation constant. Expressions for $\vec{k}$ for various directions of propagation of plane waves with respect to the static magnetic field have been given by Sen and Wyller [1960], and Molmud [1962a] . Only the case of propagation parallel to the magnetic field is considered here. Then,

$$\begin{Bmatrix} k_o \\ k_x \end{Bmatrix} = \frac{\omega}{c}\left[1 - \frac{\omega_p^2}{\omega}\begin{Bmatrix} L \\ R \end{Bmatrix}\right]^{\frac{1}{2}} \qquad \text{(V-6)}$$

and from Section IV:

$$L = \frac{1}{g_r^+ + i(\omega + \omega_b + g_i^+)}$$

$$R = \frac{1}{g_r^- + i(\omega - \omega_b + g_i^-)} \quad .$$

The wave associated with L (in V-6) is the ordinary wave and is left circularly polarized about the direction of propagation. R is associated with the extraordinary wave which is right circularly polarized.

If $\mathcal{k} = \mu - ik$ where μ is the real part of the refractive index and k is the absorption coefficient, and it is assumed $\omega_p^2/\omega(\omega - \omega_b) << 1$, then

$$\begin{Bmatrix} \mu_o \\ \mu_x \end{Bmatrix} = \frac{\omega}{c} \left\{ 1 - \frac{\omega_p^2}{2\omega} \frac{(\omega \pm \omega_b + g_i^{\pm})}{\left[(\omega \pm \omega_b + g_i^{\pm})^2 + (g_r^{\pm})^2 \right]} \right\} , \tag{V-7}$$

$$\begin{Bmatrix} k_o \\ k_x \end{Bmatrix} = \frac{\omega_p^2}{2c} \frac{g_r^{\pm}}{\left[(\omega \pm \omega_b + g_i^{\pm})^2 + (g_r^{\pm})^2 \right]} . \tag{V-8}$$

Also when $\omega = \omega_b$, then:

$$\begin{Bmatrix} \mu_x \\ k_x \end{Bmatrix} = \frac{\omega_b}{c} \sqrt{ \tfrac{1}{2}\left(1 - \frac{\omega_p^2}{\nu^2}\right) \left\{ \sqrt{1 + \frac{\omega_p^4 \nu^2}{\omega_b^2 (\nu^2 - \omega_p^2)^2}} \pm 1 \right\} } . \tag{V-9}$$

In addition when $\omega = \omega_b$ and $\nu = \omega_p$, then

$$\mu_x = k_x = \frac{1}{c} \sqrt{\tfrac{1}{2} \nu \omega_b} . \tag{V-10}$$

As a numerical example, let us compute the attenuation suffered by a gyrowave. Consider that $\omega = 10^7 \text{sec}^{-1} = \omega_b$ and $n = 5.5 \times 10^3 \text{sec}^{-3}$ so that $\omega_p^2 = \nu^2 = 18 \times 10^{12} \text{sec}^{-2}$ corresponding to 70 km altitude. Then,

$$k_x = 15.5 \frac{\text{nepers}}{\text{km}} = 68 \frac{\text{db}}{\text{km}}$$

This is an enormous attenuation and one which would not allow the extraordinary component of the gyrowave to reach any appreciable height in a disturbed D-region before it is completely absorbed. The attenuation of the ordinary component may be obtained from Eq. (V-8) and is $k_o = .43$ nepers/km = 1.9 db/km, much lower than for the extraordinary wave.

We can now anticipate one of our conclusions, that trying to modify an already disturbed D-region by means of a wave at the gyrofrequency will not succeed, since the extraordinary component will suffer too much attenuation before it can alter any significant depth of the region.

B. The Eikonal Solution for a Radial Strong Wave

The field in a vacuum drops off as 1/r at large distances from a powerful transmitter. The field in a medium whose propagation characteristics depend on the field strength will decay in a more complicated manner, but at least as fast as 1/r. We shall now seek a solution of (V-5) for a radial wave. We expect the resultant solutions to be valid for conditions of little ray bending and over small solid angles with the transmitter at the origin.

If $\nabla(\nabla \cdot E)$ is neglected, (V-5) may be written

$$\frac{\partial^2}{\partial r^2}(rE) + \frac{\omega^2}{c^2}\left(1 - \frac{4\pi i\sigma(|E|)}{\omega}\right) Er = 0 \quad . \tag{V-11}$$

We seek a solution of (V-11) such that $rE = A(r)e^{i\phi(r)}$. Then $A(r)$ and $\phi(r)$ must satisfy the following (from (V-11)):

$$\frac{\partial^2 A}{\partial r^2} - A\left[\frac{\partial\phi}{\partial r}\right]^2 = -\frac{\omega^2}{c^2} A - \frac{4\pi}{c^2}\omega\,\sigma_i A \tag{V-12}$$

$$\frac{\partial^2\phi}{\partial r} + \frac{2}{A}\frac{\partial\phi}{\partial r}\frac{\partial A}{\partial r} = \frac{4\pi\omega}{c^2}\sigma_r \quad , \tag{V-13}$$

where $\sigma = \sigma_r + i\sigma_i$ and σ is a function of the field strength. On the assumption that $\partial^2 A/\partial r^2 << A\omega^2/c^2$, then, if we neglect the effects of magnetic field

$$\phi = \frac{\omega}{c}\int_{r_o}^{r}\left(1 - \frac{\omega_p^{\,2}}{(\omega + g_i)^2 + g_r^{\,2}}\right)^{\frac{1}{2}} dr' \quad . \tag{V-14}$$

Additionally, when $\dfrac{\omega_p^{\,2}}{(\omega + g_i)^2 + g_r^{\,2}} << 1$ and $\partial^2\phi/\partial r^2$ is negligible then

$$\frac{1}{A}\frac{\partial A}{\partial r} = -\frac{\omega_p^{\,2} g_r}{2c\left[(\omega + g_i)^2 + g_r^{\,2}\right]} \quad . \tag{V-15}$$

A similar development and an examination of the sufficiency for the approximation used have been given in an unpublished report [Molmud, 1959c].

The following sequence of transformations puts (V-15) into a more useable form:

$$\frac{1}{A}\frac{\partial A}{\partial r} = \frac{1}{2A^2}\frac{\partial A^2}{\partial r} = \frac{1}{2r^2 EE^*}\frac{\partial}{\partial r}(r^2 EE^*) \quad .$$

Thus, upon writing E^2 for EE^*, we obtain

$$\frac{\partial}{\partial r}\ln(r^2E^2) = -\frac{\omega_p^{\,2} g_r}{c\left[(\omega + g_i)^2 + g_r^{\,2}\right]} \quad . \tag{V-16}$$

It was shown in the previous Section that the average energy of the electrons is a function of E^2 (Section IV) and that the collision frequency, ν, in air varies linearly with average energy. Since g_r and g_i are functions of ν/ω, then they too depend on energy and thus E^2. Furthermore, ω_p^2 is proportional to electron density and it was shown that, that too is a function of average energy (Section III) and thus E^2. Therefore, Eq. (V-16) may be considered as a differential equation for E^2. Similar equations have been solved analytically for homogeneous media [Molmud, 1959c; and Ginsburg and Gurevich, 1960]. No analytic solutions have been obtained for inhomogeneous media.

In order to obtain solutions of (V-16), for an inhomogeneous medium (such as the D-layer) we must resort to programming. The results of such programming will be demonstrated in Section VI where we concern ourselves with the problem of modifying the electron density in the D-region by ground-based powerful radio transmitters.

The propagation problem treated in VI concerns itself with the propagation of a powerful wave vertically through the D-region to an altitude of 100 km. The field strength is to satisfy Eq. (V-16) and $\omega_p^2 = \frac{4\pi e^2}{m} n_\infty$ where

$$\frac{n_\infty(z)}{n(z)} = \frac{R + K(u_o)\left[O_2\right]^2}{R + K(u)\left[O_2\right]^2} , \qquad \text{(V-17)}$$

$n(z)$ is a chosen distribution of electron density and $n_\infty(z)$ is the quasi-stationary electron density due to the alteration of the average electron energy u by the powerful wave (cf. Section III), K(u) is the attachment coefficient (Section II, Fig. 1). In addition, u is given by the following relationship.

$$\frac{u}{u_o} = 1 + \frac{7.9\, E^2 g_r \times 10^{19}}{G u_o(z) \left[(\omega + g_i)^2 + g_r^2 \right]} \tag{V-18}$$

where $u_o(z)$ is the average background energy of the molecules as a function of altitude and is in electron volts, g_r and g_i are functions of $\left|\nu_o(z)/\omega\right| \left|u/u_o\right|$ given by Figs. 4 and 5 of Section IV. Inasmuch as there is no data on the dependence of G on u and u_o in the region of .03 eV < u < .2 eV, G will be assumed constant.

We take advantage of this last simplification by writing (V-16) as

$$\frac{\partial}{\partial r} \ln \left[r^2 \left| \frac{E^2}{G} \right| \right] = - \frac{\omega_p^2 g_r}{c \left[(\omega + g_i)^2 + g_r^2 \right]} \quad , \tag{V-19}$$

which upon combining with (V-17) and (V-18) results in an equation for u/u_o. The wave propagation problem will be uniquely specified by supplying ω and $u(z_o)/u_o(z_o)$. From (V-18) we see that u/u_o is linearly dependent on E^2, which in turn measures the power requirement of the transmitter (assuming no attenuation of the wave up to z_o).

VI. PERTURBATIONS IN THE D-REGION CAUSED BY A POWERFUL WAVE LAUNCHED FROM THE GROUND

A. Details of the Program

In this Section we investigate the modification that can be made of a D-region due to a powerful wave being propagated through it. Our purpose will be to see under what conditions and to what degree this modification will facilitate the propagation of a wanted wave. We also investigate the characteristics of the nonlinear propagation of the powerful wave.

We choose a disturbed D-layer electron density distribution which might be associated with the explosion of a high altitude nuclear device, the electrons being produced by deposition of energy via the β particles from the radioactive debris. Figure 6, below, represents such a distribution and comes from DASA [1961]. Table VI-I gives the necessary atmospheric data. The temperature and O_2 density were obtained from the ARDC model atmosphere [Minzner, Champion, and Pond, 1959], electron density from Fig. 6 (daytime) and collision frequency from Phelps [1960]. *

These data are used to compute the attenuation of a weak 30 Mc/s wave (the wanted wave) propagated vertically from the ground to an altitude of 100 km. We then illuminate this same region by a powerful wave of the same frequency whose power level is determined by specifying u/u_o, the ratio by which electron energy is changed at 45 km by the presence of the wave.** Forty-five km is chosen because the wave will have suffered negligible attenuation

---

* $\nu = 1.04 \times 10^8$ p $\sec^{-1}$ where p is pressure in mm Hg. p is obtained from Minzner, Champion and Pond [1959] using geopotential altitudes.

** We shall assume that the average energy of the unheated electrons, u_o, is the same as the background gases and have the values as given in Table VI-1. For justification of this see Appendix II.

up to that altitude and thus the specification of u/u_o will uniquely characterize the power level of the transmitter.

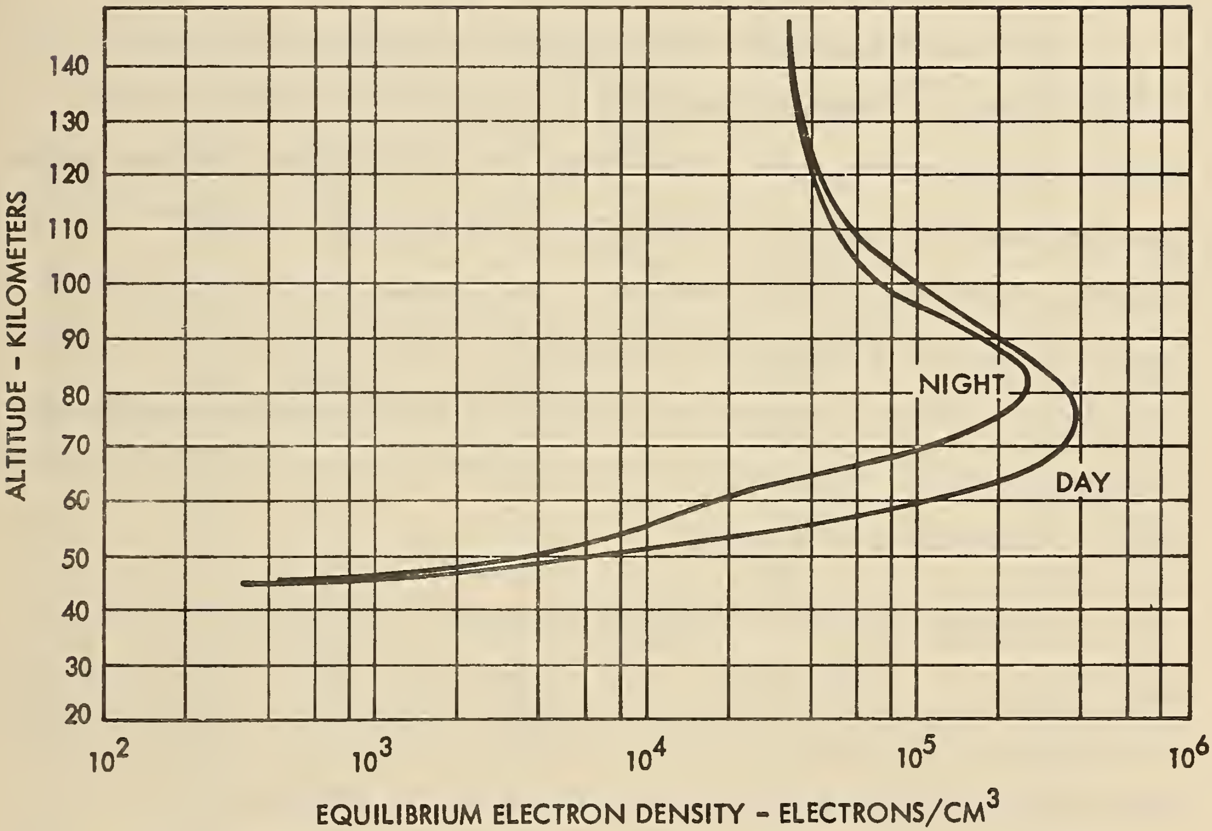


Figure 6. Equilibrium Electron Density due to a High Altitude Nuclear Explosion

The powerful wave will modify the average electron energy and density, n being changed to n_{∞}. It will consequently suffer a different attenuation than the weak wave.

Now when the powerful wave is turned off the electron energy quickly returns to normal so that $u/u_o = 1$ everywhere, but the electron density remains near n_{∞} for a time of the order of a second. We then propagate the weak 30 Mc/s signal through this newly modified ionosphere before the electron density can relax to normalcy. The attenuation of the weak 30 mc wave will differ from that of the strong 30 Mc/s wave because the collision frequency is altered. It will also differ from the attenuation experienced by the weak 30 Mc/s wave sent up before the poweful wave because the electron density distribution is altered.

The above process is repeated for waves of frequencies $40\sqrt{2}$, 80 and 320 Mc/s. For each of these other frequencies we follow the intense wave with a weak 30 Mc/s wave through the perturbed D-region to see how effectively the other frequencies at the various power levels have "swept out" the electrons in the D-region. We do not consider any frequencies lower than these (e.g., gyrofrequency) because of the excessive attenuation they would experience which would render ineffectual their capability of altering ionospheric electron densities.

TABLE VI-I

Parameters of the Upper Atmosphere
Employed in Computational Programs

Alt.km	u_o eV	$\nu_o(sec^{-1})$	$[O_2]$ (cm^{-3})	$\underline{n}$ (cm^{-3})	$n\nu(cm^{-3}sec^{-1})$
40	.0338	2.24×10^8	1.7×10^{16}		
43	.0350	1.52	1.2		
45	.0358	1.22×10^8	8.8×10^{15}	5×10^2	6.1×10^{10}
47	.0366	9.50×10^7	6.8×10^{15}	10^3	9.5×10^{10}
50	.0365	6.55×10^7	4.7	6×10^3	39×10^{10}
52	.0365	5.10	3.7	10^4	51×10^{10}
54	.0360	4.10	3.0	3×10^4	12×10^{11}
56	.0350	3.14	2.4	4×10^4	13×10^{11}
58	.0340	2.42	1.9	8×10^4	19×10^{11}
60	.0330	1.90	1.5	10^5	19×10^{11}
62	.0320	1.43	1.2×10^{15}	1.5×10^5	21×10^{11}
64	.0310	1.03	9.5×10^{14}	2×10^5	21×10^{11}
66	.0300	7.85×10^6	7.4	3×10^5	24×10^{11}
68	.0290	6.00	5.7	3.3×10^5	20
70	.0280	4.30	4.4	3.5×10^5	15
72	.0270	3.24	3.3	3.8×10^5	12
74	.0260	2.36	2.5	3.8×10^5	9.0
76	.0255	1.68×10^6	1.8	3.8×10^5	1.5
78	.0225	1.12	1.3×10^{14}	3.8×10^5	1.0
80	.0214	7.75×10^5	9.2×10^{13}	3.5×10^5	
82	.0214	5.22	6.2	3.4	
84	.0214	3.50	4.1	3.2	
86	.0214	2.34		3.0×10^5	
88	.0214	1.57		2.5×10^5	
90	.0214	1.05		2×10^5	
100	.0257	1.68×10^4		10^5	

The method of computing the propagation of the strong wave was discussed in Section V.

To recapitulate: we are assuming the fractional energy loss on collision, G, is constant; vertical and radial propagation; collision frequency proportional to average energy, and steady state propagation with n_∞ replacing n. The latter assumption requires that the wave be on long enough to achieve n_∞ but not so long that recombination effects play an important role. It was shown in Section III that a wave lasting $\sim \frac{1}{2}$ second will suffice.

K(u) will be obtained by linearly extrapolating the data of Biondi [Chanin, Phelps, Biondi, 1959]. This gives somewhat higher values of K at D-layer temperatures (i.e., $2.8 \times 10^{-30} \mathrm{cm}^6/\mathrm{sec}$) than is estimated by Nicolet and Aiken [1960]. Thus predictions of electron density reduction will be somewhat pessimistic.

We shall specify that u/u_o have the values 1, 2.52, 4.00 and 6.00 at 45 km. The value $u/u_o = 2.52$ was chosen because this maximizes the value of the electron attachment coefficient, K, at 45 km, while $u/u_o = 4.00$ at 45 km maximizes the coefficient at 60 km, the altitude of maximum attenuation per unit length (assuming $1/r^2$ dependence of power and no absorption). The value $u/u_o = 6.00$ allows for some absorption so that the wave may have enough power to reach to higher altitudes and alter the D-region.

Table VI-II present the essential results of the programs.

TABLE VI-II

Attenuation of Waves Sent from Ground through D-Region of Figure 1, Section VI

Frequency Mc/s	Weak Wave Attenuation db	Strong Wave Attenuation db	Weak Wave After Strong Wave. Attenuation db	Weak 30 Mc/s wave After Strong Wave. Attenuation db
$\frac{u}{u_o} = 2.52$				
30	84.77	83.42	78.92	78.92
$40\sqrt{2}$	24.75	26.48	21.46	73.93
80	12.51	14.48	10.19	69.61
320	.79	1.09	.58	62.53
$\frac{u}{u_o} = 4.00$				
30	84.47	85.09	78.35	78.35
$40\sqrt{2}$	24.75	29.49	21.09	72.41
80	12.51	17.36	9.91	67.30
320	.79	1.53	.53	56.78
				52.31 optimum
$\frac{u}{u_o} = 6.00$				
30	84.47	87.28	79.13	79.13
$40\sqrt{2}$	24.75	32.90	21.40	73.13
80	12.51	20.79	10.12	68.23
320	.79	2.58	.60	62.46

The most important result is this: The weak 30 Mc/s (wanted) wave propagating vertically through the D-region of Fig. 6 will suffer an attenuation of 84.47 db. By using the strategy of propagation through this region of a wave of 320 Mc/s so powerful that it causes $u/u_0 = 4.00$ (raises the energy of electrons by a factor of four) at 45 km altitude, we modify the electron density distribution. The 30 Mc/s wave now propagating through this modified region will suffer an attenuation of 56.78 db; a decrease of attenuation of 27.69 db or an increase of power level of 590! This decrease of attenuation is quite close to the optimum which is obtained by keeping u at .09 ev and thus K at maximum throughout the D-region. The optimum attenuation is 52.31 db, down by 32.16 db from the original attenuation. Thus the best we could possibly do by any strategy of heating the electrons by ground-based transmitters is to increase the received power by 1650 (see Part D of this Section, however).

B. Examination of Figures 7 - 17

We now present the curves resulting from the above described series of "Gedanken" experiments. These curves present the dependence of the following on altitude:

Attenuation: defined as

$$10 \log_{10}\left(\frac{E^2}{E_1^2}\ \frac{z_1^2}{z^2}\right)$$

where subscript 1 refers to quantities measured at 45 km and z is altitude.

$\frac{u(z)}{u_o(z_o)}$: where u is the average energy of the electrons and u_o their unperturbed energy

$\frac{n_\infty(z)}{n(z)}$: where n is electron density as obtained from Fig. 1 and Table I and

$$\frac{n_\infty(z)}{n(z)} = \frac{R + K(u_o)\left[O_2\right]^2}{R + K(u)\left[O_2\right]^2} .$$

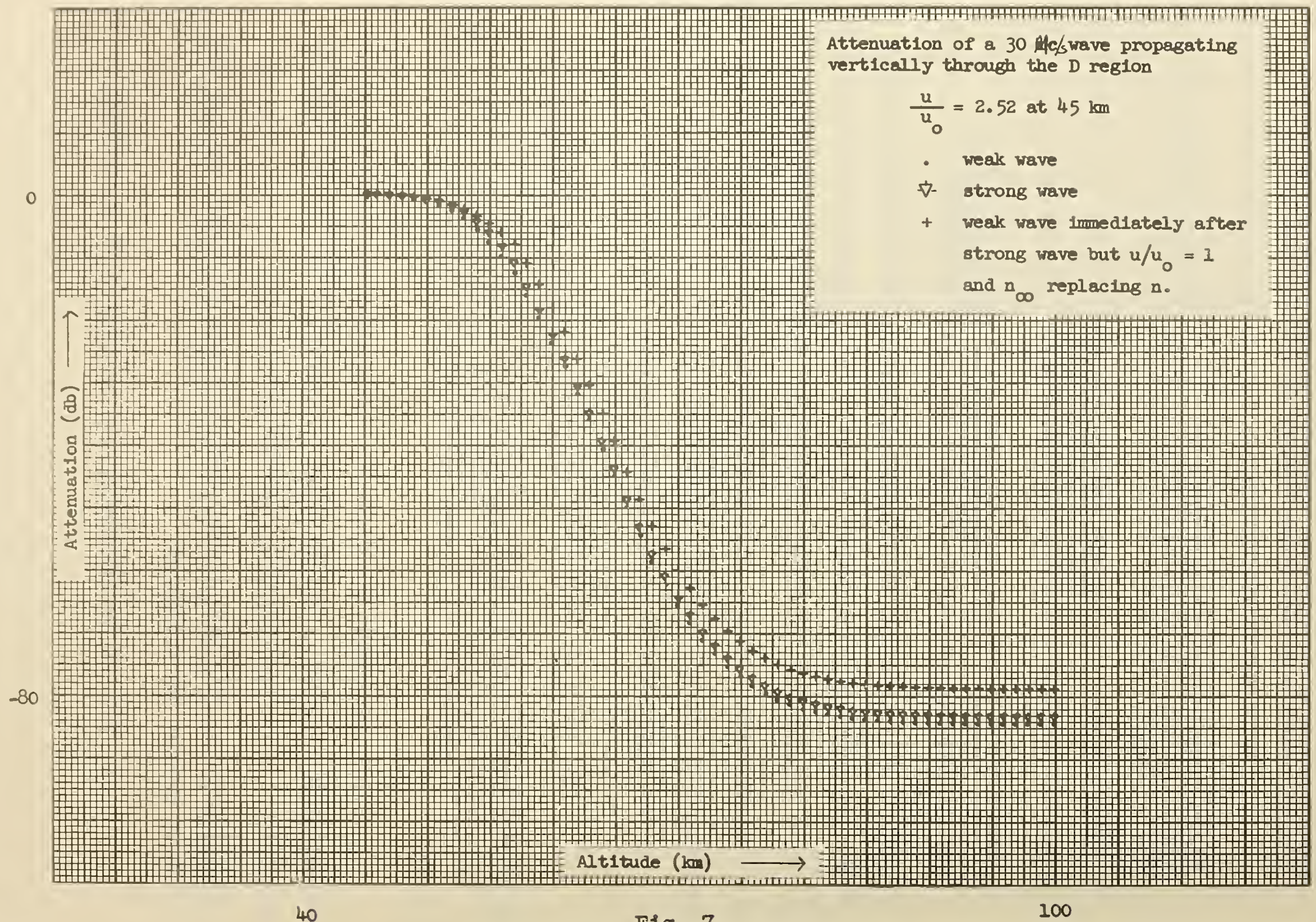
1. Figures 7 - 8. Attenuation of Weak Wave Modified by Previous Passage of Strong Wave of Same Frequency

a. In general the strong wave suffers more attenuation than the weak one. This is made understandable by examining Eq. (V-16) of Section V. There we see that when $\omega > \nu$, the attenuation per unit length varies as the product of $n\nu$. Now at low altitudes $n \propto 1/u$ and $\nu \propto u$. However at higher altitudes n has a weaker dependence on u (Eq. (V-17) of Section V) and thus the attenuation per unit length is larger.

b. There is very little dependence on power of the attenuation of the 30 Mc/s wave. This is made evident by the enormous attenuation experienced by this wave so that very little of the D-region can be perturbed by its initial strength.

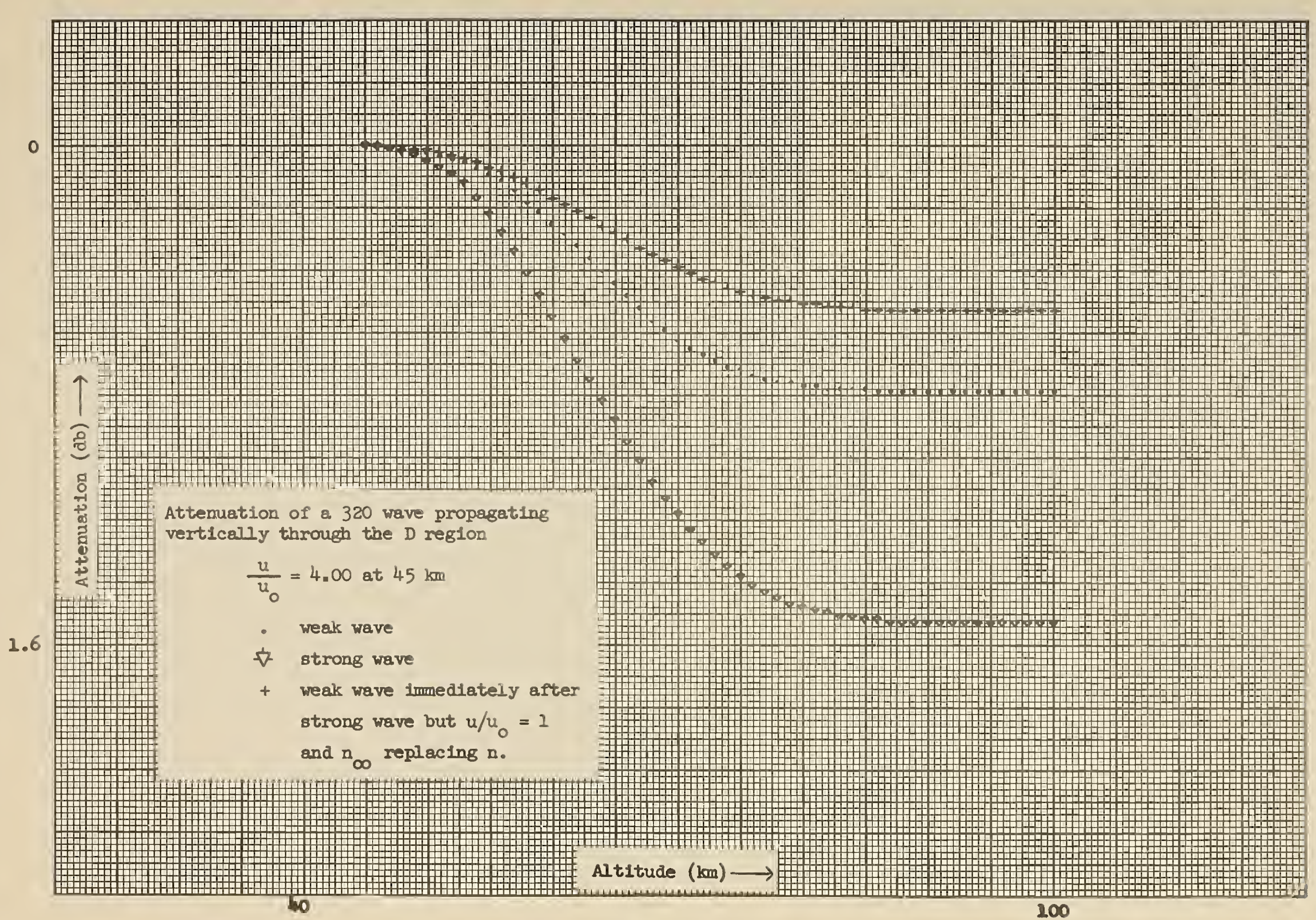
c. The higher the frequency, the greater the dependence of the attenuation on frequency. The important consideration here is the level of attenuation. Since the attenuation varies approximately as $1/\omega^2$, then the higher frequencies will have greater power at higher altitudes and thus will be able to perturb the ionosphere.

d. Most of the attenuation has occurred by 80 km altitude.



Attenuation of a 30 Mc/s wave propagating vertically through the D region
$\frac{u}{u_o}$ = 2.52 at 45 km
. weak wave
∇- strong wave
+ weak wave immediately after strong wave but u/u_o = 1 and n_∞ replacing n.
0
-80
Attenuation (db) ⟶
Altitude (km) ⟶
40
100

Fig. 7



0
1.6
Attenuation (db)
Attenuation of a 320 wave propagating vertically through the D region
$\frac{u}{u_o}$ = 4.00 at 45 km
. weak wave
strong wave
+ weak wave immediately after strong wave but u/u_o = 1 and n_{∞} replacing n.
Altitude (km)
40
100

Fig. 8

2. Figures 9 - 10. **Effects of Power Level on Attenuation of Waves through the D-Region.**

a. The more powerful the wave, the stronger the attenuation. Comment 1(a) above is appropriate here. Additionally, we might expect enhanced electron densities when u becomes greater than .1 eV. This would occur at about $u/u_o > 3$.

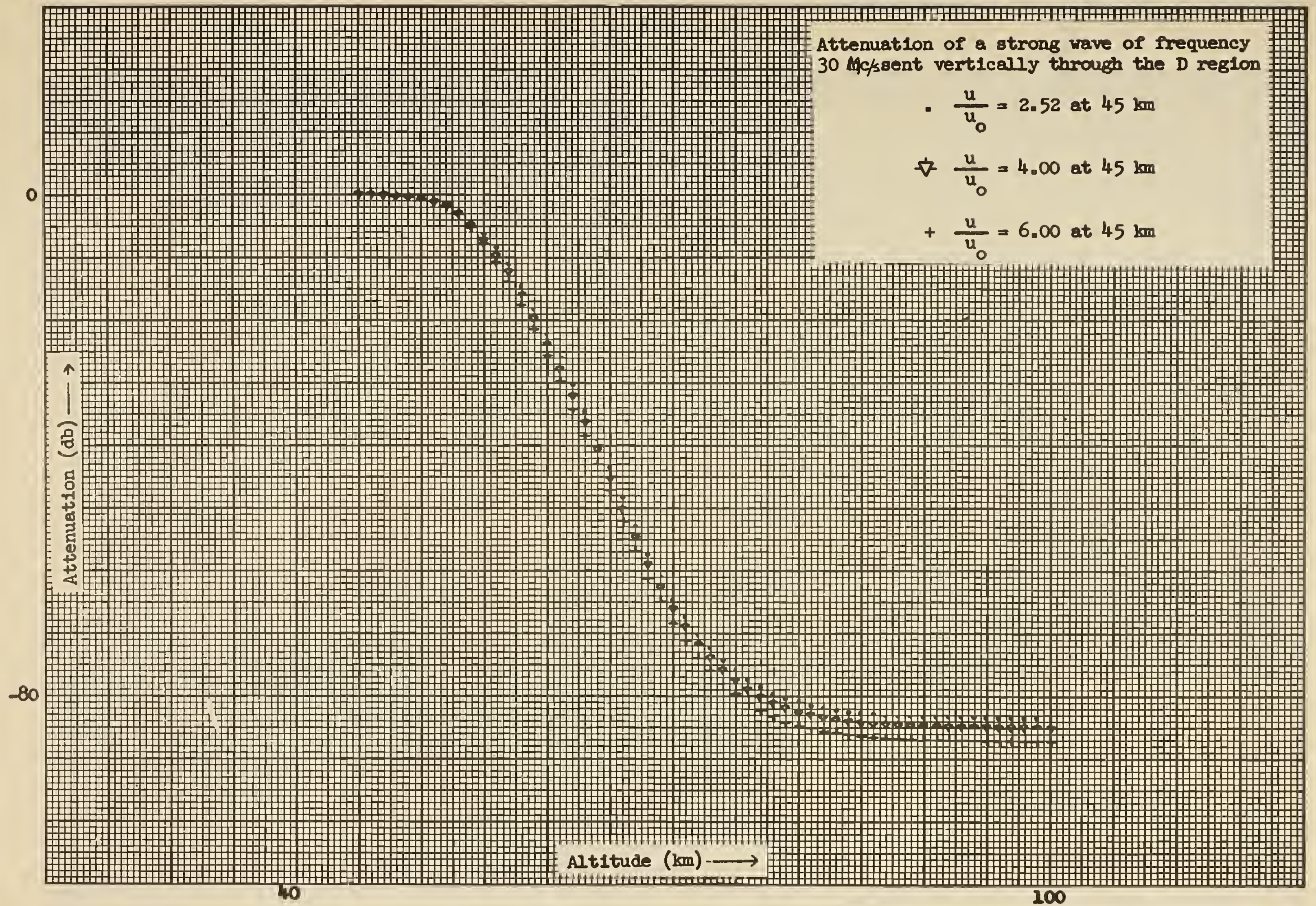

Attenuation of a strong wave of frequency
30 Mc/s sent vertically through the D region
$\frac{u}{u_o}$ = 2.52 at 45 km
$\frac{u}{u_o}$ = 4.00 at 45 km
$\frac{u}{u_o}$ = 6.00 at 45 km
0
-80
Attenuation (db)
Altitude (km)
40
100

Fig. 9

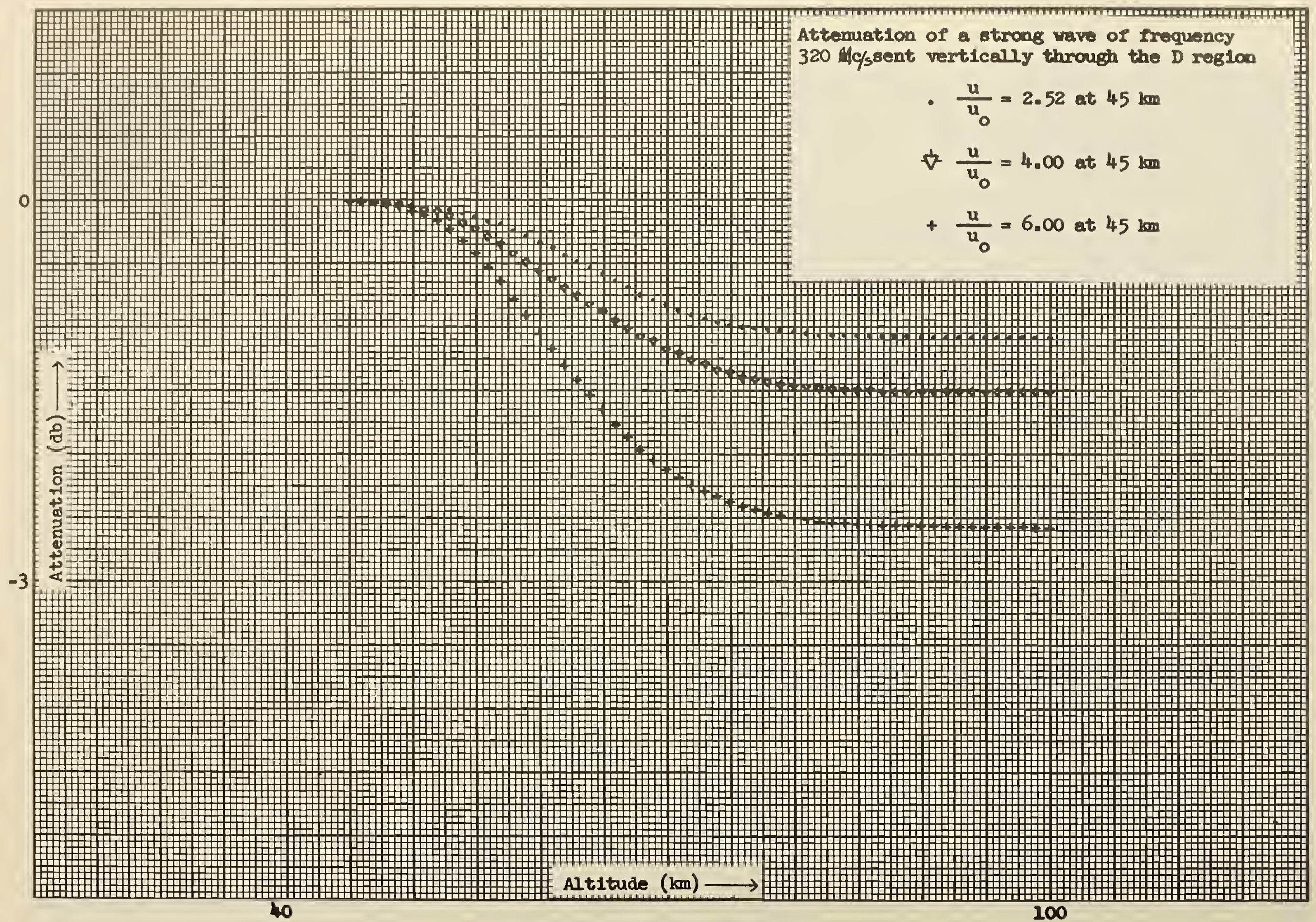

Attenuation of a strong wave of frequency
320 Mc/s sent vertically through the D region
$\cdot$ $\frac{u}{u_o}$ = 2.52 at 45 km
$\frac{u}{u_o}$ = 4.00 at 45 km
+ $\frac{u}{u_o}$ = 6.00 at 45 km
0
-3
Attenuation (db)
Altitude (km)
40
100

Fig. 10

3. **Figures 11 - 12. Attenuation of Weak Wave through D-Region Modified by Previous Passage of Powerful Wave for Several Power Levels**

a. Little dependence on power level is noticed for the powerful wave of 30 Mc/s and $40\sqrt{2}$ Mc/s. This can be attributed to the high level of attenuation suffered by the powerful waves at these frequencies.

b. At 80 and 320 Mc/s $u(z_1)/u_0(z_1) = 4.00$ produces the most reduction of attenuation. $u(z_1)/u_o(z_1) = 6.00$ is not effective as we shall see later because of $n_\infty/n > 1$ at certain altitudes.

The curve labelled "optimum" in Fig. 6, is obtained by maintaining the attachment coefficient, K, at its maximum value at all altitudes.

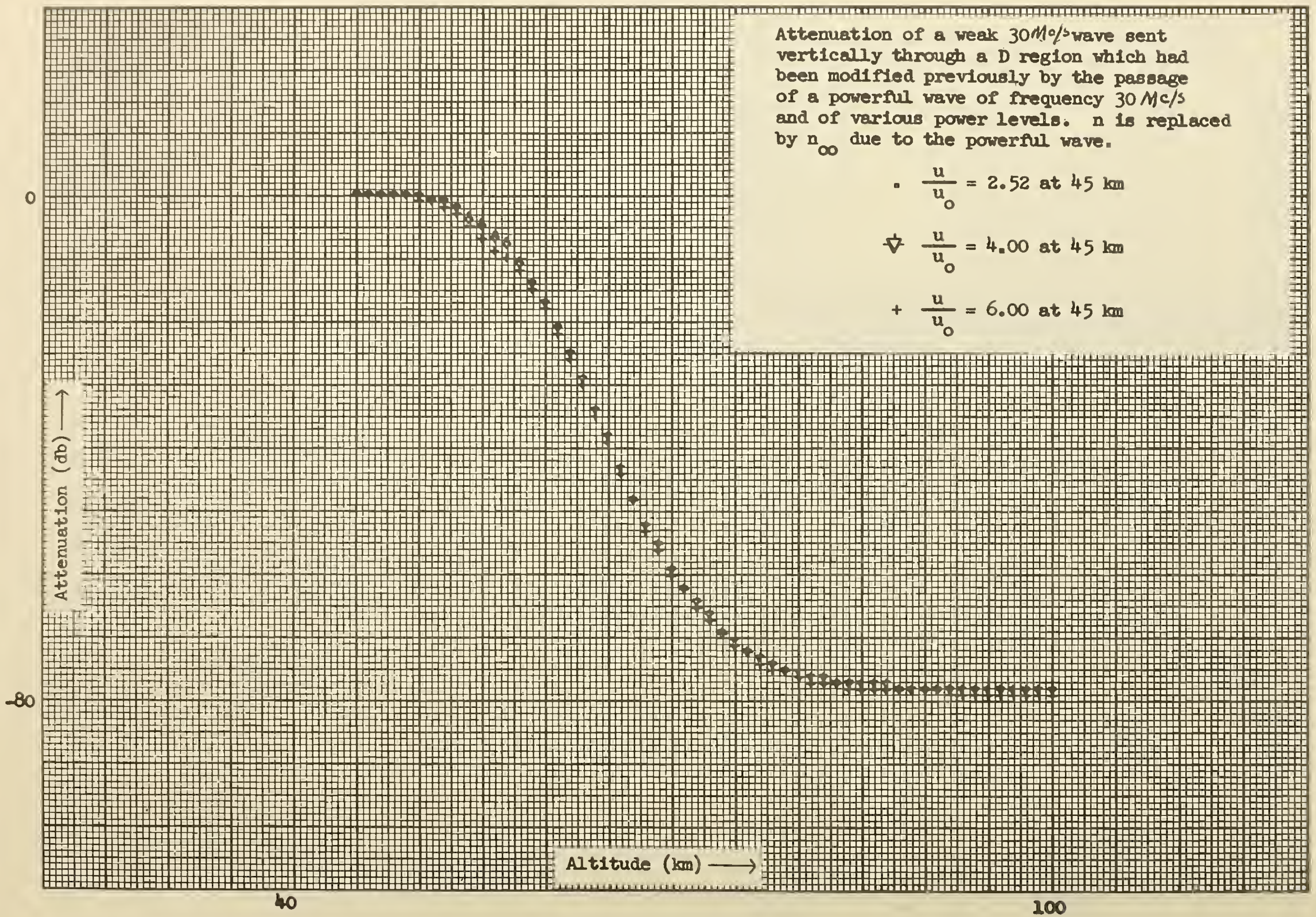

Attenuation of a weak 30 Mc/s wave sent vertically through a D region which had been modified previously by the passage of a powerful wave of frequency 30 Mc/s and of various power levels. n is replaced by n_{oo} due to the powerful wave.
▪ $\frac{u}{u_o}$ = 2.52 at 45 km
▽ $\frac{u}{u_o}$ = 4.00 at 45 km
+ $\frac{u}{u_o}$ = 6.00 at 45 km
0
-80
Attenuation (db) ⟶
40
100
Altitude (km) ⟶

Fig. 11

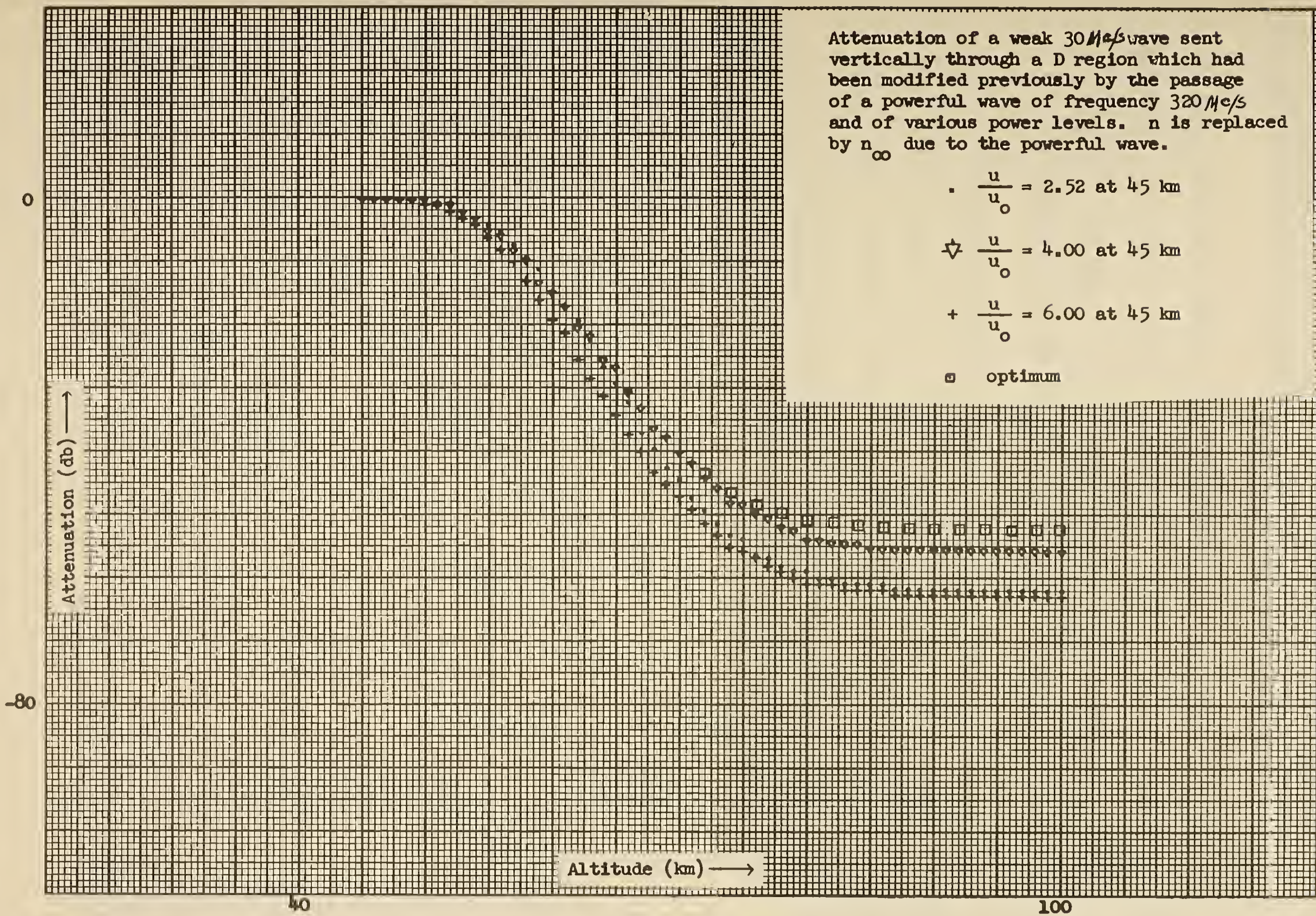

Attenuation of a weak 30 Mc/s wave sent vertically through a D region which had been modified previously by the passage of a powerful wave of frequency 320 Mc/s and of various power levels. n is replaced by n_∞ due to the powerful wave.
· $\frac{u}{u_o}$ = 2.52 at 45 km
$\frac{u}{u_o}$ = 4.00 at 45 km
+ $\frac{u}{u_o}$ = 6.00 at 45 km
optimum
0
-80
Attenuation (db) ⟶
Altitude (km) ⟶
40
100

Fig. 12

4. <u>Figure 13 . Attenuation of Weak Wave through D-Region Modified by Previous Passage of Waves, for Several Frequencies but Same Effective Power Level.</u>

a. The greater the frequency, the greater the reduction of attenuation. This can be attributed solely to the fact that the absorption of the powerful wave decreases as $1/\omega^2$ and thus the higher frequencies can penetrate to higher altitudes with less of their power attenuated.

b. The maximum reduction of attenuation occurs when $u(z_1)/u_o(z_1) = 4.00$. We can see from the slopes of the curves that most of the attenuation occurs between 60 and 70 km. Thus the power level $u(z_1)/u_o(z_1) = 4.00$ which maximized the attachment coefficient at 60 km should cause the greatest reduction of attenuation.

c. The curve labelled "optimum" in Fig. 13 is obtained by keeping K, the attachment coefficient, at its maximum value at all altitudes.

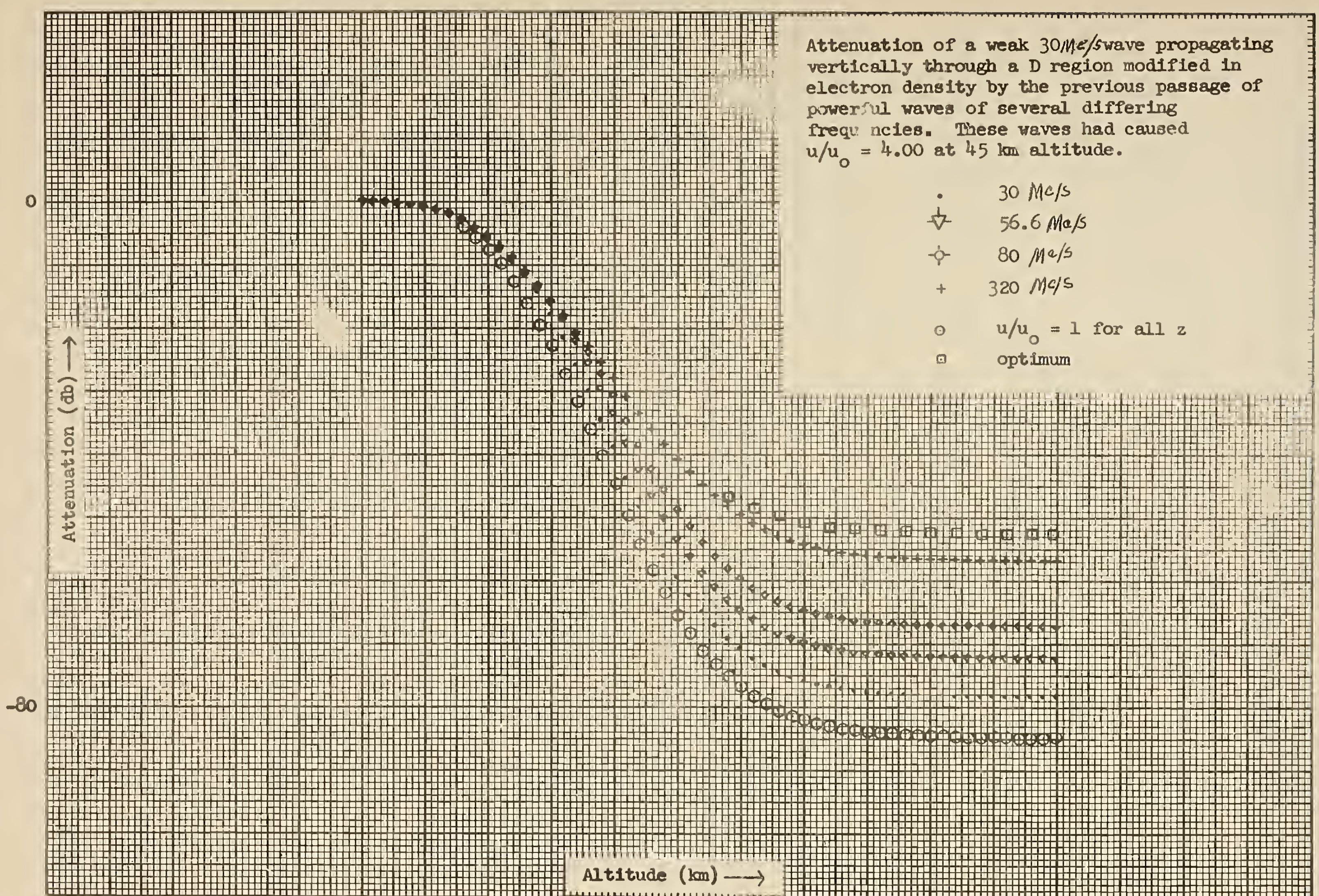

Attenuation of a weak 30 Mc/s wave propagating vertically through a D region modified in electron density by the previous passage of powerful waves of several differing frequencies. These waves had caused $u/u_o = 4.00$ at 45 km altitude.
30 Mc/s
56.6 Mc/s
80 Mc/s
320 Mc/s
$u/u_o = 1$ for all z
optimum
0
-80
Attenuation (db)
Altitude (km)
40
100

Fig. 13

5. Figure 14. **Perturbation of Average Electron Energy in D-Region due to Presence of Waves for Several Frequencies but Same Effective Power Level**

a. We observe a peak in $u(z)/u_o(z)$ at about 51 km altitude for three frequencies. These are explained as follows: $u(z)/u_o(z)$ obeys the equation

$$\frac{u}{u_o} = 1 + \frac{8 \times 10^{19} E^2 \left(\frac{g_r}{\nu}\right)}{u_o G \left[(\omega + g_i)^2 + g_r^2 \right]}$$

At 45 km altitude $\nu_o = 1.22 \times 10^8 \text{sec}^{-1}$ but when $u/u_o = 2.52$ then $\nu = \nu_o \times u/u_o = 300 \times 10^8 \text{sec}^{-1}$.

For $f = 30$ Mc/s, $\omega/\nu = .94$, $g_i/\omega = .25$, $g_r/\nu = 1.3$ and $g_r^2 = 1.2\,(\omega + g_i)^2$. (See Section IV, Figs. 2 and 3.) Now as the altitude increases ν decreases (approximately exponentially), the denominator on the right decreases, g_r/ν approaches 5/3 and g_i approaches zero. Thus u/u_o will increase until the effects of the collision frequency in the denominator is no longer felt. At 51 km the collision frequency is down by about a factor of 2. Beyond 51 km the effects of the inverse square dependence of E^2 on altitude and, more importantly, absorption become apparent (see Figs. 1, 2).

When $f = 80$ Mc/s $\omega/\nu \sim 5/3$, $g_i/\omega = .15$, $g_r/\nu = 1.42$ and $g_r^2 = 1.83\,(g_i + \omega^2)$. Thus u/u_o still depends on ν but not as much as for $f = 30$ Mc/s. This is shown by the lower peak for $f = 80$ Mc/s.

When $f = 320$ Mc/s, $g_r^2 << (\omega + g_i)^2$ and thus there is no dependence on ν and no maximum. However, when $u/u_o = 6.00$, then ν is sufficiently large so as to make itself felt as a small maximum at 48 km. The secondary maximum at 78 km occurs because of local variations of u_o.

b. The greater $u(z_1)/u_o(z_1)$ (i.e., transmitter power level), the higher the altitude at which the maximum occurs. This is evident from ν depending linearly on u/u_o and the previous discussion.

c. The higher the frequency, the lower the altitude at which the maximum occurs. This is a very weak dependence.

d. The dependence of the magnitude of the maximum frequency decreases as $u(z_1)/u_o(z_1)$ at 45 km increases. This is also because of ν varying as u/u_o.

e. $u(z)/u_o(z)$ is greater than one at high altitudes only for the high frequencies. This results from the absorption dependence on $1/\omega^2$.

f. In order to obtain the most efficient reduction of electron density u must be kept at $\sim .09$ eV or $u(z)/u_o(z) \sim 3$ between the altitudes of 60 and 70 km. Of all the examples chosen, f = 320 Mc/s and $u/u_0 = 4.00$ appears to accomplish this best.

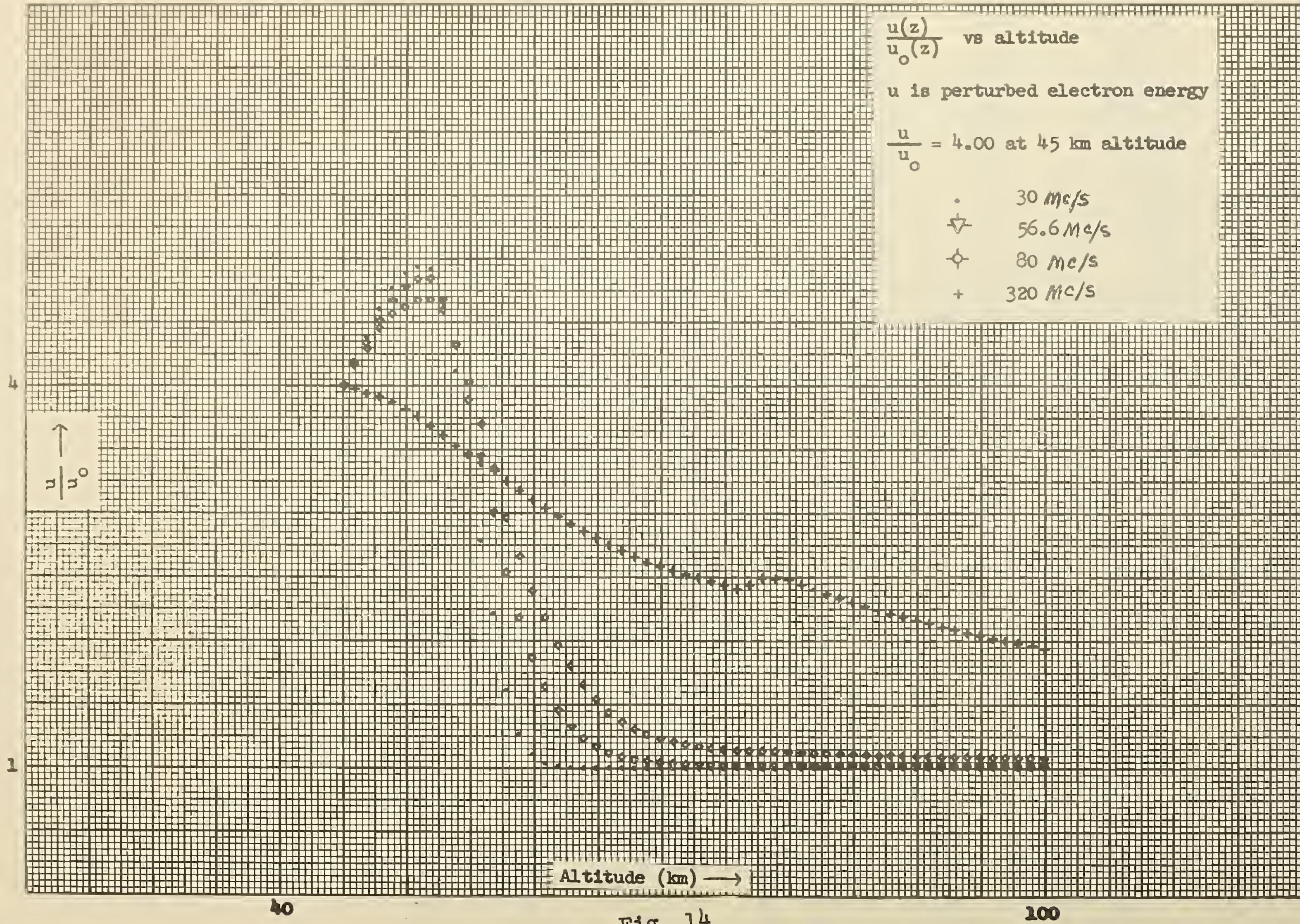
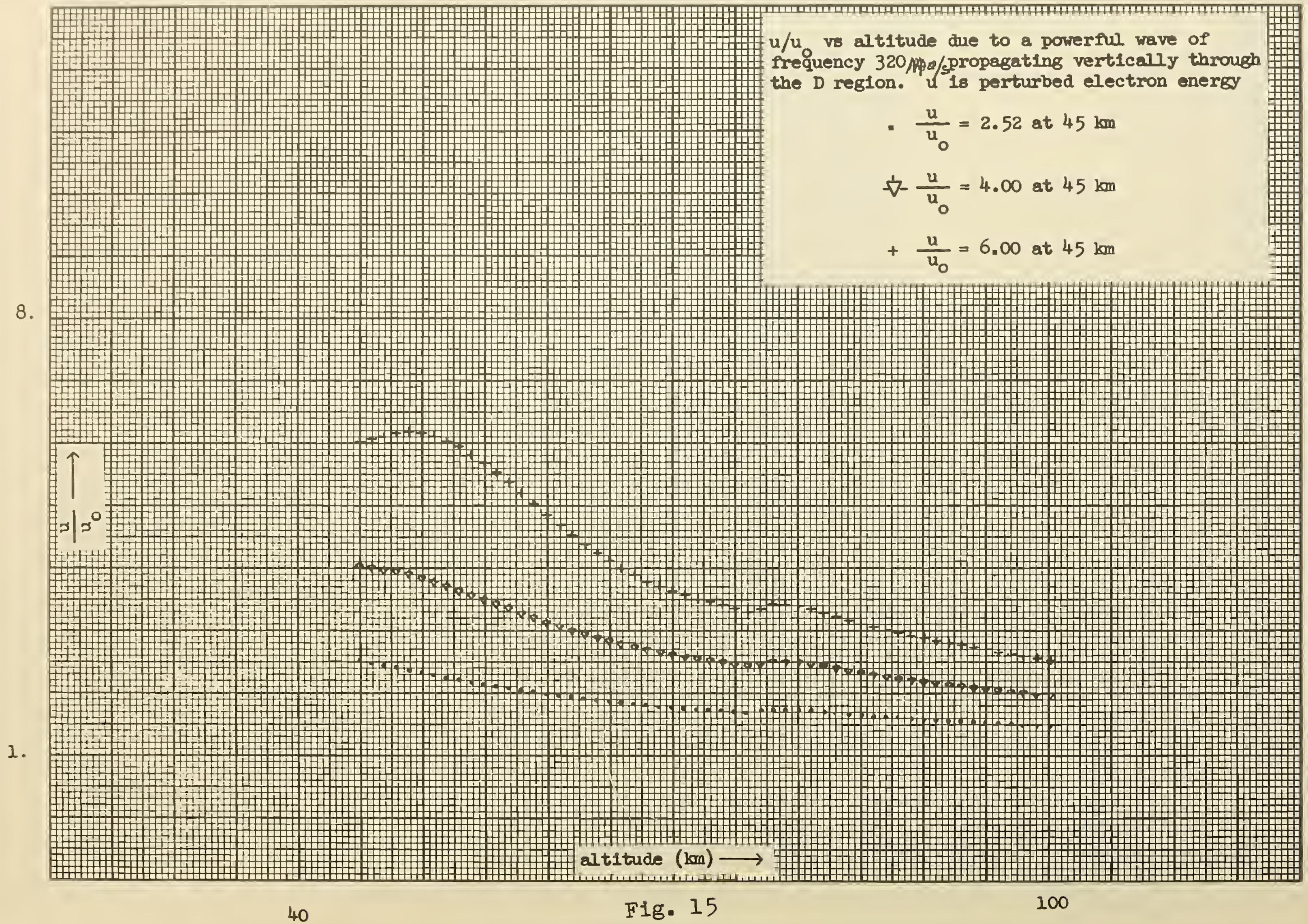

$\frac{u(z)}{u_o(z)}$ vs altitude
u is perturbed electron energy
$\frac{u}{u_o}$ = 4.00 at 45 km altitude
30 Mc/s
56.6 Mc/s
80 Mc/s
320 Mc/s
4
1
$\frac{u}{u_o}$ →
Altitude (km) ⟶
40
100

Fig. 14

6. Figure 15. **Perturbation of Average Electron Energy in D-Region due to Presence of a Wave, for Several Power Levels**

a. At higher altitudes $u(z)/u_o(z)$ is a nonlinear function of $u(z_1)/u_o(z_1)$ at 45 km. $u(z)/u_o(z)$ for f = 320 Mc/s between $u(z_1)/u_0(z_1) = 2.52$ and $u(z_1)/u_o(z_1) = 4.00$ is an exception.



u/u$_o$ vs altitude due to a powerful wave of frequency 320 Mc/s propagating vertically through the D region. u is perturbed electron energy
$\frac{u}{u_o}$ = 2.52 at 45 km
$\frac{u}{u_o}$ = 4.00 at 45 km
+ $\frac{u}{u_o}$ = 6.00 at 45 km
8.
1.
$\frac{u}{u_o}$
altitude (km)
40
100

Fig. 15

7. Figure 16 . Perturbation of Electron Density in D-Region due to Electromagnetic Wave for Several Frequencies but Same Effective Power Level

a. The local maxima in n/n_{∞} at 51 km coincides with and are due to the maxima in $u(z)/u_o(z)$.

b. The magnitude of the minimum value of $n_{\infty}/n \sim .57$ is due to the linear extrapolations of $K(u_o)$ from Biondi's data. Had we used $K(u_o) = 1.5 \times 10^{-30} cm^6$ then n_{∞}/n minimum would have been .33.

c. The D-region is significantly perturbed only by the 320 Mc/s wave. All the others are excessively attenuated.

d. When $u/u_o = 6.00$ at 45 km the local maximum for frequencies less than 320 Mc/s has values greater than 1. This occurs because u/u_o is so high at this place that $K(u) < K(u_o)$.

e. The curve labelled "optimum" is obtained by maintaining the attachment coefficient at maximum value for all z. We see that the 320 Mc/s wave at $u/u_o = 4.00$ approaches this curve remarkable well.

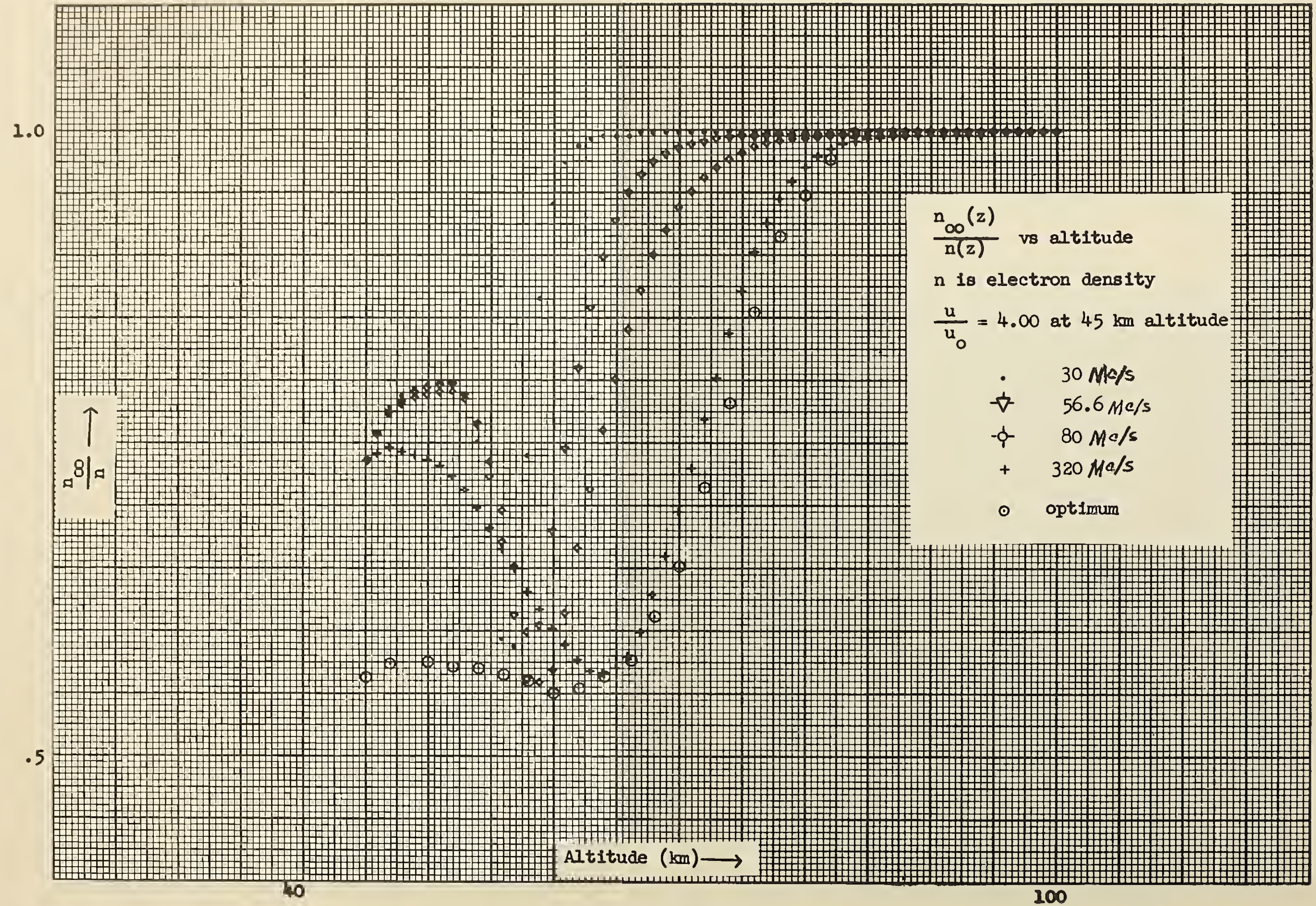

1.0
.5
$\frac{n_\infty}{n}$ →
$\frac{n_\infty(z)}{n(z)}$ vs altitude
n is electron density
$\frac{u}{u_o}$ = 4.00 at 45 km altitude
30 Mc/s
56.6 Mc/s
80 Mc/s
320 Mc/s
optimum
Altitude (km)⟶
40
100

Fig. 16

8. Figure 17. Perturbation of Electron Density in D-Region due to Wave, for Several Power Levels.

a. The greater the power level, the deeper the region which can be affected by the powerful wave.

b. The optimum power level, as is made clear for f = 320 Mc/s, is $u/u_o = 4.00$.

c. See comment in (7-e) above.

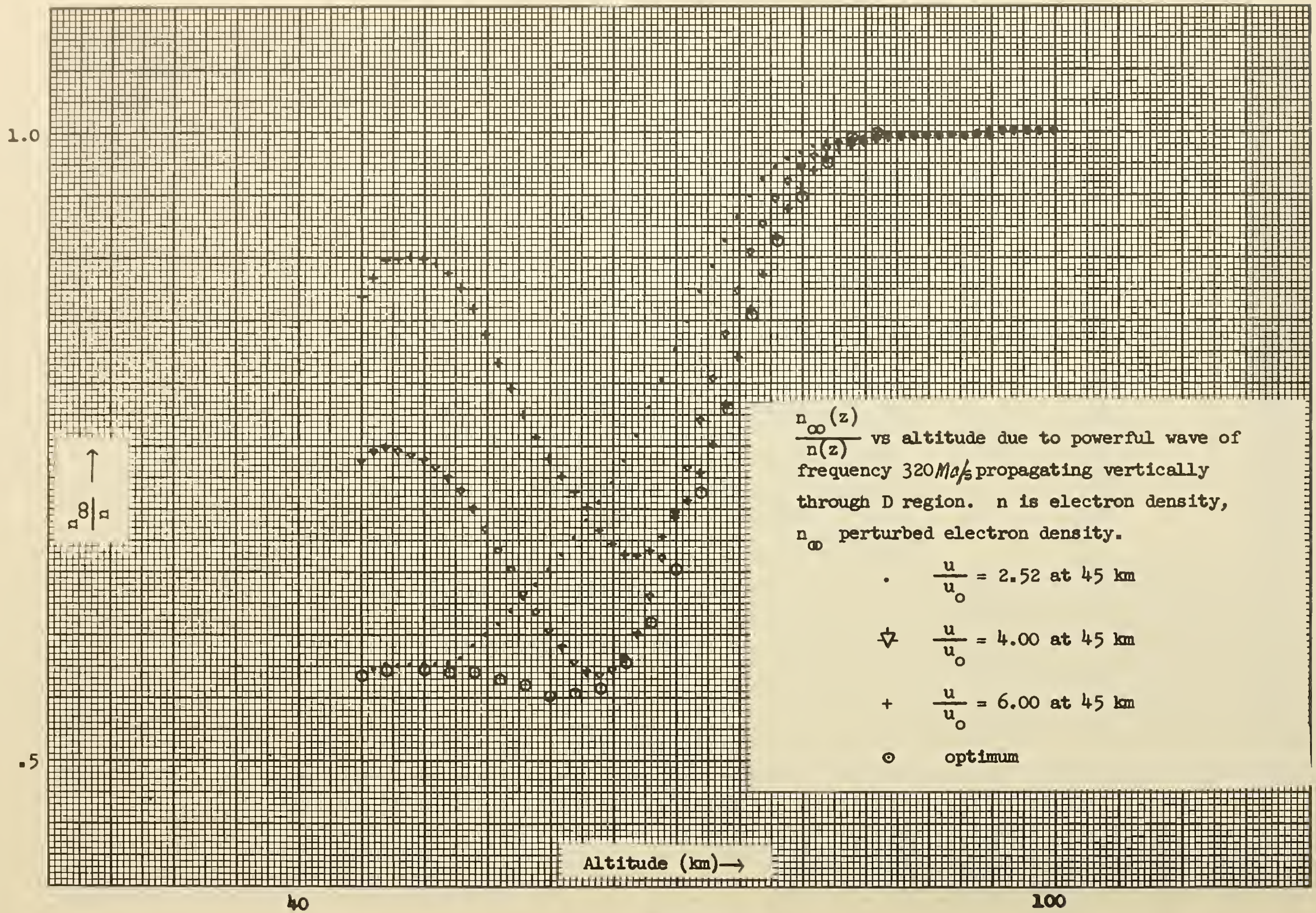

1.0
.5
$\frac{n_\infty}{n}$ →
40
100
Altitude (km)⟶
$\frac{n_\infty(z)}{n(z)}$ vs altitude due to powerful wave of frequency 320 Mc/s propagating vertically through D region. n is electron density, n_∞ perturbed electron density.
· $\frac{u}{u_o}$ = 2.52 at 45 km
▽ $\frac{u}{u_o}$ = 4.00 at 45 km
+ $\frac{u}{u_o}$ = 6.00 at 45 km
⊙ optimum

Fig. 17

C. Qualitative Conclusions Obtained from Examination of Figures 7 - 17.

1. A weak wave propagates in a significantly different fashion than a strong wave.

2. Attenuations vary approximately as $1/\omega^2$. Thus, nonlinear effects just now demonstrated are more spectacular for lower frequencies.

3. Attenuations vary as $n(z)$, thus, the nonlinear effects are more spectacular for large $n(z)$.

4. Region of maximum attenuation for a weak wave passing through a perturbed D-layer should be at an altitude where $K_{max}\left[O_2\right]^2/R \gtrsim 1$ in order that we may facilitate propagation through this region.

5. The perturbing wave should be at a high enough frequency so that it itself suffers little attenuation.

6. The perturbing wave should be at high enough power level so that K (attachment coefficient) is maximized at the region of maximum attenuation.

7. The lower the electron density the higher a strong attenuating wave can go and still perturb the electron density. The resulting perturbations depend in minor detail on the magnitude of the electron density, but more on the distribution.

8. There is an optimum power level for the perturbing wave.

9. Inhomogeneities in electron density may be induced, or already existing inhomogeneities may be altered by the presence of a powerful wave.

10. The electron density cannot be reduced by this method above an altitude of $\sim$80 km.

D. Analytical Approximation to the Absorption of the Wanted Wave

We have seen that the best that can possibly be done to facilitate the propagation of a wanted wave is to maintain the attachment coefficient at its maximum value of $5.0 \times 10^{-30} cm^6$. By using this strategy one can reduce the attenuation of a 30 Mc/s wave from 84 db to 52 db. We gave cautioned, however, that such estimates of reduction may be conservative since we have used a rather large value for $K(u_o)$ based on a linear extrapolation of Biondi's data obtained at a higher temperature than prevails in the D-region.

In this Section an analytical approximation to the reduction of attenuation is developed to show how it depends on the assumed value of $K(u_o)$ (or equivalently, $K(u_o)/R$).

The altered attenuation of the wanted wave is given by the following expression:

$$\text{Attenuation} = \frac{1}{c}\int_{o}^{100\text{ km}} \frac{\omega_p^{\,2} g_r}{\left[(\omega + g_i)^2 + g_r^{\,2}\right]} \left[\frac{1 + \dfrac{K(u_o)\left[O_2\right]^2}{R}}{1 + \dfrac{K(u)\left[O_2\right]^2}{R}}\right] dz. \qquad \text{(VI-1)}$$

Now let us assume that u_o is constant over our region of integration and $K(u)$ is at its maximum value. Furthermore, $g_r < \omega$ and $\left[O_2\right] = \left[O_2\right]_1 e^{\frac{-(z - z_1)}{L}}$.

We observe in Table VI-I that $n\nu$ is fairly constant in the region of its maximum values, from 54 km to 74 km. Assuming the constancy of $n\nu$ over this range we can write (VI-1) as

$$\text{Attenuation} = -\frac{\omega_p^{\,2}\nu\frac{5}{3}}{c\omega^2}\int_{z_1}^{z_2}\left[\frac{1 + a\exp\left[-2(z - z_1)/L\right]}{1 + b\exp\left[-2(z - z_1)/L\right]}\right]dz \tag{VI-2}$$

where $a = K(u_o)\left[O_2\right]_1^2 / R$, $b = K(u)\left[O_2\right]_1^2 / R$, $z_1 = 54$ km, $z_2 = 74$ km and L the scale height at 64 km is 7.0 km. Then by simple transformations and integration we arrive at

$$\text{Attenuation} = -\frac{\omega_p^{\,2}\nu}{c\omega^2}\,\frac{5}{3}\,\frac{L}{2}\left\{\frac{(a-b)}{b}\,\ell n\left[\frac{1 + b}{1 + b\exp\left[-2(z_2-z_1)/L\right]}\right] + 2\,\frac{(z_2-z_1)}{L}\right\} \tag{VI-3}$$

The linear extrapolation of Biondi's data yields a value of a = 58.4 and at $K(u)_{max}$, b = 103 at 54 km. By putting in the value of $n\nu$ of $20 \times 10^{11} cm^{-3} sec^{-1}$, one obtains a value of attenuation equal to 51.5 db, remarkably (and probably fortuitously) close to the computed optimum of 52.31 db. Now when we take Nicolet's estimate for $K(u_o)$ of $1.5 \times 10^{-30} cm^6/sec$ then a = 12.3 at 54 km. The attenuation now becomes 30.0 db.

Thus, we see that our predictions of attenuation depend very critically upon the values of the attachment coefficient. Taking the optimistic point of view, we can reduce the attenuation of a 30 Mc/s wave from 84 db to 30 db, a 54 db decrease or an increase of power by 2.5×10^5!

E. Power Requirements

In this Section, we have shown that a powerful wave can alter the electron density in the D-region and modify the propagation of a wanted wave, sometimes decreasing and sometimes increasing its attenuation. The demonstrations were based on a Gedanken experiment where it was assumed that a wave of sufficient power existed already at the altitude of 45 km. We now investigate the power requirements of a ground-based transmitter to produce such effects vertically at 45 km altitude.

We neglect magnetic field effects. Then Eq. (IV-24) of Section IV becomes:

$$\frac{u}{u_o} = 1 + \frac{e^2}{2mG\nu u_o} \frac{g_r E_o^2}{\left[(\omega + g_i)^2 + g_r^2\right]} = 1 + \frac{7.9 \times 10^{19}}{G\nu u_o} \frac{g_r E_o^2}{(\omega + g_i)^2 + g_r^2} \qquad \text{(VI-4)}$$

with u in eV and E_o in esu.

Let us assume $G = 2 \times 10^{-3}$ and $\nu = \nu_o(z)\,\frac{u}{u_o}$; also at 45 km $\nu_o = 1.22 \times 10^8\,\mathrm{sec}^{-1}$; $u_o = .0358$ ev. Now taking f = 320 Mc/s then $\omega/\nu >> 1$ and $g_i << \omega$, $g_r = 5/3$, $\omega^2 = 4.1 \times 10^{18}\,\mathrm{sec}^{-3}$. Then:

$$\frac{u}{u_o} = 1 + 4.5 \times 10^5 E_o^2 \quad . \qquad \text{(VI-5)}$$

When $u/u_o = 4$, then $E_o^2 = 3\ \mathrm{esu}/4.5 \times 10^5 = 6.7 \times 10^{-6}$ esu.

Now $\frac{c}{4\pi} E_o^{\ 2}$ is the power flux at this altitude. The power output of an isotropic radiator to yield such a flux is given by $\frac{c}{4\pi} E_o^{\ 2} \times 4\pi r^2 = cE_o^{\ 2} r^2$. Thus, to obtain $u/u_o = 4$ at 45 km ($r = 4.5 \times 10^6$ cm) and f = 320 Mc/s we need an isotropic radiator of power output $3 \times 10^{10} \times 6.7 \times 10^{-6} \times (4.5 \times 10^6)^2 \times 10^{-7}$ watts $= 4.1 \times 10^5$ megawatts. A 300 ft parabolic antenna would give a gain of 48 db with a beam width of about .75 degrees [SRI, n.d.]. This would cut the power requirement to 6.5 megawatts. The region which would be affected at 45 km would be rather small, however, only about $\frac{1}{2}$ km in diameter. The power requirement for a 100 Mc/s isotropic transmitter would be reduced to about 7×10^4 megawatts. A 400 ft parabolic antenna would provide a gain of 40 db with a beam width of $\sim 1.5^o$. The power requirement would be reduced to 7 megawatts and the region which can be perturbed at 45 km altitude would have the extent of about 1 km.

Thus, in order to perform experiments of the type described in this section, we need high gain CW (pulse length 1/2 second) transmitters of frequency 100 - 300 Mc/s and power output of about 10 megawatts.

Transmitters of nearly such power levels, directivity and frequency are already in existence or being constructed. For example: The ionospheric radar probe being built near Arecibo, Puerto Rico [Gordon and LaLonde, 1961] operates at 430 Mc/s, 2.5 megawatts peak, 150 kW average, 100 kW CW, 60 db gain, maximum pulse length 10^{-2} sec.* The transmitter at Lincoln Laboratories field site at El Campo, Texas, operates at 38 megacycles, 500 kW CW with a $\frac{1}{2}^o$ beam width in one plane and 8^o in perpendicular plane.** The Lima Radar Observatory in Peru is designed to operate at 49.9 Mc/s with 6 megawatts peak, pulse duration of 5×10^{-4} sec, gain of 39.7 db, beam width of 1.07^o by 21.4^o.*** Now the

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* The D region is within the near region of this transmitter so that even when operated CW it can seriously modify electron energies at 60 to 80 km altitude.

** Private communication from Martin Balser.

*** Private communication from Lima, Peru.

power estimates are based upon Eq. (VI-4) where it is assumed that $G\nu \sim u/u_o$. According to the work of Altshuler [1963] it would appear that $G\nu \sim (u/u_o)^{-1/2}$. If this is actually the case, then our power estimates may be overly pessimistic.

F. Communication Facilitation during Blackout Conditions

By a series of *Gedanken* experiments we have shown that the propagation of a wanted wave through a very heavily perturbed ionosphere may be facilitated by modifying the D-region through which the wave passes. The modification is accomplished by illuminating the region with a high frequency powerful wave. In our *Gedanken* experiment the wanted wave was of 30 Mc/s and suffered an attenuation of 85 db when propagated vertically to 100 km altitude. If the D-region is illuminated by an intense 320 Mc/s wave of strength such that at 45 km the energy of the electrons is increased by a factor of 4 then it is modified so that the 30 Mc/s wave will now have an attenuation of 57 db.

The modification is accomplished by heating the electrons, increasing their attachment rate and thus decreasing their density. In our computations we have used very conservative estimates for the attachment coefficient in the D-region. Had we used the currently estimated value then we would have obtained a much lower modified attenuation of the 30 Mc/s wave more like 30 db. Thus the efficacy of the technique described depends critically on exact knowledge of the values of the attachment coefficient.

The computation of transmitter requirements suggests a 6.5 megawatt transmitter with a gain of 48 db. Reducing the frequency would reduce the power requirements by approximately one-half. If the frequency of the powerful wave is reduced too much it may suffer excessive attenuation before it reaches that region of the ionosphere it is to affect. The power requirements are based on a crude theory of inelastic collision processes affecting the average energy of electrons. Based upon recent work by Altshuler [1963] these power requirements may prove too large.

The propagation of the wanted wave is accomplished only in a very short-time interval ($\sim 1/4$ sec) after the powerful transmitter is shut off. Thus a system that would use the above experiment to facilitate communication between two ground points would be able to communicate only within short (1/4 sec) but closely spaced (1/10 sec) intervals. Furthermore if both ground stations were suffering from blackout due to a perturbed ionosphere, then both stations might have to have powerful transmitters. Under these conditions if the two stations wanted to communicate via the E- or F-region, both powerful transmitters would be pointed along the propagation path of the wanted wave and both would have to be turned on and off synchronously. Because of the very high gains of the powerful transmitters, very narrow ducts would be created in the D-region for the propagation of the wanted wave. Thus very accurate positioning and orientation of all transmitters is required.

Our ability to modify the propagation of a wanted wave depends on the distribution in height of the electrons. If, as in our example, the region of maximum attenuation is below 70 km, then we can accomplish a reduction of attenuation (in db) by perhaps a factor of 2 1/2. If the region of maximum attenuation occurs above 70 km, then we can exercise very little control over the propagation.

When $\omega > \nu$ the region of maximum attenuation occurs at the altitude where the product $n\nu$ maximizes. As the frequency is lowered the position of maximum attenuation for the same distribution rises slightly. Thus for lower frequencies the procedure of reducing electron density is at some disadvantage as compared with higher frequencies. However since overall attenuation (in db) varies approximately as $1/\omega^2$ and since the fractional reduction of attenuation is just slightly dependent on ω, then the propagation of low frequency waves is more greatly facilitated than high frequency waves.

Because of the cost of the required powerful transmitters we do not at this moment recommend their employment as a practical means of re-establishing communication during blackout conditions. It has been suggested by Molmud [1963] that they can be employed, however, as diagnostic tools to determine the properties of the lower ionosphere, even during conditions of blackout [Molmud, 1962b].*

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* When this paper was written it was not realized that the D region was within the near zone of the Arecibo transmitter.

VII. SUMMARY

The object of this study is to determine to what extent the propagation characteristics of upper atmospheric plasmas can be ameliorated by powerful ground-based transmitters. The investigation centers on the role of electron energy in determining steady state electron density.

The important processes controlling the electron density in the D-region during daytime are ionization due to U.V., X-rays and protons from the sun, attachment of electrons to O_2 and subsequent photodetachment from the resulting O_2^- and also recombination between electrons and O_2^- and positive ions.

The attachment coefficient for electrons and O_2 is monotonic increasing from .04 ev to $\sim$.1 ev where it peaks. The ratio of the values of the coefficient at these points is $\sim$2. Thus, in an otherwise steady state situation and in a region where attachment rates dominate detachment rates, the increase of the electron energy by a factor of about 2 should decrease the electron density by a similar factor.

In a situation where the electron energy is suddenly changed to a new and constant value, the electron density relaxes to a new value in a characteristic time depending inversely on the sum of the attachment, detachment and recombination rates. The relaxation time depends on the energy of the electrons because the attachment coefficient varies with electron energy. The positive ion density changes also but at a much slower rate.

The electrons in the D-region can be altered in average energy when a high-powered electromagnetic wave is present. This field-induced modification of local conductivity necessitates the solution of the corresponding nonlinear propagation problem.

The conductivity of the medium in which the wave is propagating depends on the electron density and collision frequency and these, in steady state, depend on the field strength. The conductivity is obtained from an expression for the average drift velocity of electrons. The drift velocity and average energy of the electrons obey intimately-coupled nonlinear differential equations. Approximate solutions of these equations show two different relaxation times: about 10^{-7}sec for drift velocity and about 10^{-4}sec for average energy (in the D-region). In contrast the electron density relaxation time is about 10^{-1}sec.

The propagation of an intense electromagnetic wave through the D-region will heat the electrons and therefore alter both the electron density and collision frequency. The optimum reduction in electron density occurs at about 0.1 eV for the electron mean kinetic energy. During blackout conditions a megawatt transmitter with 40 db gain at about 300 Mc/s can decrease the attenuation of a wanted wave by a factor of two to three in db.

APPENDIX I

PERTURBATIONS DUE TO MASS TRANSPORT AND DIFFUSION

We are illuminating a small region of the D-layer by a fairly directive beam of powerful radio waves. The gases in this region may have velocities in the neighborhood of 100 meters/sec [Handbook of Geophys., 1957.] The electrons because of the inhomogeneities in density and energy created by the powerful beam will be subjected to perturbations arising out of these inhomogeneities. In our computations we have not included such effects on the presumption they are negligible especially since the perturbing beam is on only for less than one second. We now compute the effects due to mass transport and diffusion.

Let us rewrite the moment equation including the term allowing for spatial inhomogeneity:

$$\frac{\partial}{\partial t}(n\bar{x}) - \nabla_r \cdot (n\,\overline{vx}) - n\,\nabla_v \cdot \left[\frac{q}{m}\vec{E} + \vec{\omega}_b \times \vec{v}\right] x$$
$$= \int (X' - X)\, F(\vec{V}) f(\vec{v})\, |\vec{v} - \vec{V}|\, \sigma\, (\theta, |\vec{v} - \vec{V}|)\, d^2\Omega\, d^3v\, d^3V \quad . \tag{1}$$

Then the moments: density, velocity and energy become (assuming air):

$$\frac{\partial}{\partial t} n + \nabla \cdot (n\bar{\vec{v}}) = 0 \tag{2}$$

$$\frac{\partial}{\partial t}(n\bar{v}) - \nabla \cdot n\overline{\vec{v}\vec{v}} + \frac{M}{M+m}\,\nu n \left[\frac{\overline{\vec{v}\,\frac{1}{2}mv^2}}{u} - \vec{V}_d - m\,\frac{\overline{\vec{v}\vec{v}} \cdot \vec{V}_d}{u}\right]$$
$$= \frac{qn}{m}\,(E + \vec{\omega}_b \times \vec{v}) \tag{3}$$

$$\frac{\partial}{\partial t}(nu) - \nabla \cdot \overline{nu\vec{v}} + \frac{2Mmn}{(M+m)^2}\nu\left\{\frac{m}{2}\,\frac{\overline{v^4}}{\overline{v^2}} - \frac{5}{3}u_o + \left(\frac{M-m}{2}\right)\frac{\overline{v^2\vec{v}}\cdot\vec{V}_d}{\overline{v^2}}\right\} = q\,\vec{E}\cdot\overline{\vec{v}n} \tag{4}$$

We first observe that $m\overline{\vec{v}\vec{v}} \cdot \vec{V}_d$ for a Maxwellian distribution is just $\frac{2u}{3}\vec{V}_d$. Also, $\nabla \cdot n\overline{\vec{v}\vec{v}} = \frac{3\nabla p}{m}$ where p is the pressure, $1/3\ nm\overline{v^2}$. However, at low altitudes n is perturbed by the powerful wave such that $n \propto 1/\overline{v^2}$ (i.e., $K \propto 1/u$ for low values of u). Thus, the electron pressure is not affected by the powerful wave and the second term of Eq. (3) can be ignored. Now temporarily ignoring the term $\partial n/\partial t$ we can consider (3) an equation in $\vec{v} - \vec{V}_d$ and solve it for this relative velocity. Here $\vec{V}_d$ might be a wind velocity. We will get solutions similar to (IV-15) and (IV-16) of Section IV with the added complication of some additional terms due to the interaction of $\vec{V}_d$ and $\vec{\omega}_b$. These terms are: (to be added to right side of Eq. (IV-16) Section IV)

$$\frac{-V_{dx}\,\omega_b(\nu\sin\omega_b t - \omega_b\cos\omega_b t) - V_{dy}\,\omega_b\,(\nu\cos\omega_b t + \omega_b\sin\omega_b t)}{\omega_b^{\ 2} + \nu^2}\ ;$$

(to be added to right side of Eq. (IV-15) Section IV)

$$\frac{\omega_b\,V_{dx}\,(\nu\cos\omega_b t + \omega_b\sin\omega_b t) - V_{dy}\,(\nu\ \sin\omega_b t - \omega_b\cos\omega_b t)\,\omega_b}{\omega_b^{\ 2} + \nu^2}\ .$$

(Eq. (IV-17) is unchanged.)

We see that $\vec{v} - \vec{V}_d$ might suffer periodic perturbations of magnitude V_d if $\omega_b \approx \nu$. We have not included all the perturbations on the electrons however. The positive massive ions obey essentially the same differential equations as the electrons. However, for them $\omega_b^2 << \nu^2$ in the D-region. Thus their average drift velocity is unimpeded by the magnetic field of the earth. If the electrons underwent any appreciable motion differing from those of the positive ions then charge separation would ensue which would cause large field gradients. These fields thus cause the electrons to move with the same detailed motion as the positive ions. For this same reason the diffusion of electrons out of or into the perturbed region will be seriously modified by the presence of the positive ions.

The diffusion coefficient required to describe the diffusion of a mixture of electrons and positive ions is the ambipolar diffusion coefficient [Allis, 1956] which for our purposes, can be represented as

$$D_a = \frac{m}{M} \frac{\nu_e}{\nu_+} \frac{v}{3\rho Q} .$$

Now since Q is proportional to v for electrons in air, v/Q is insensitive to temperature. However ν_e varies as the temperature of electrons. Thus, in Eq. (2), $n\vec{v} = \nabla D_a n$ but again, since D_a varies as the temperature while n varies inversely as the temperature at low altitudes, then, the diffusion current induced by altering the electron density (due to heating) is negligible.

The term $\overline{nu\vec{v}}$ in Eq. (4) may be treated as $\approx n\bar{u}\vec{v}$ and then $\nabla \cdot nu\vec{v} = u\nabla \cdot (n\vec{v}) + n\vec{v} \cdot \nabla u$. The first term in negligible, from previous considerations. The second term will give time averaged contributions only from the steady drift motion of the electrons (carried by the winds, usually horizontally) and from the component of ∇u in the same direction. Now ∇u in the wind direction will arise mainly from the radiation pattern of the ground-based transmitter and at D-layer altitudes will have magnitudes of the order of electron volts per kilometer. Accordingly the contribution of $\nabla \cdot \overline{nu\vec{v}}$ will be negligible in (4).

The term $\frac{M}{2} \frac{\overline{v^2 \vec{v} \cdot \bar{V}_d}}{\overline{v^2}}$ may be approximated as $\approx \frac{M}{2} \vec{v} \cdot V_d$, which when time-averaged yields $\frac{1}{2} M V_d^{\,2}$, or the average energy associated with the drift of the neutral particles. This energy amounts to .0015 eV for a 100 M/sec wind, a much lower energy than .033 ev the mean energy u_o of the gas particles themselves.

Thus we see that mass transport and diffusion have little weight on our predictions of the effects to be encountered by a momentary (i.e., $\sim$1 second) irradiation of the D-region by a powerful beam of radio waves. The decay of the column of reduced ionization produced by the beam, after the beam is shut off, will be controlled more by photodetachment than by diffusion. This is because of the very small values of D_a and ∇n (i.e., $\partial n/\partial t = D \nabla^2 n$ and at 60 km altitude $D \approx 2 \times 10^4$ cgs, $\nabla^2 n \sim 10^3/10^{10}$ cgs; therefore $\partial n/\partial t \simeq 10^{-3}$ cgs, whereas due to attachment and detachment $\partial n/\partial t \sim 10^3$ cgs).

We are of course assuming that the region illuminated is sufficiently large so that the electrons take a significantly longer time than their density relaxation time to be carried through this region by any wind at the altitude in question.

We have not included the inelastic collision terms in (4). Its dependence on mass transport may presumably be treated by the techniques described by Altshuler [1963].

APPENDIX II

AMBIENT ENERGY OF ELECTRONS IN D REGION

We have assumed in our paper that the ambient electron energy in the D region is the same as the background gas. Sears [1963] and Karzas and Latter [1963] have suggested that electrons when produced are fairly energetic (3 ev from photodetachment, 10 ev from ionization) and therefore that electrons found in the atmosphere may have an average energy significantly different from that of the background gases.

Let us examine the effects these energetic electrons may have on the electron average energy.

We again work with the transport equation and its moments. The transport equation now appears as follows:

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_c + Rn_-\varphi(v) + q\Phi(v) - \nu_3(v)f - \nu_2(v)f \qquad (1)$$

where the nomenclature is the same as in Section IV, $\varphi(v)$ is the distribution function of the just photodetached electrons, q is the ionization strength with $\Phi(v)$ the distribution function of the collisionally (by β particles say) produced electrons, ν_3 is the 3-body attachment rate and ν_2 is the two-body dissociation rate.

We now multiply (1) by u, the kinetic energy of the electron and integrate over all of velocity space. Assuming steady state we obtain (as in IV-23)

$$nG\nu(\bar{u}-u_o) = Rn_-\bar{u}_d + q\bar{u}_i - \overline{u\nu_3}n - \overline{u\nu_2}n \quad , \qquad (2)$$

where $nG\nu(\bar{u}-u_o) = \overline{u\left(\frac{\partial t}{\partial f}\right)_c}$ and $G\nu$ is the energy relaxation rate, empirically determined. $\bar{u}_d$ is the average energy of the nascent photodetached electrons, 3 e v and $\bar{u}_i$ is the average energy of electrons resulting from the ionization process ~ 10 ev.

It is evident, now (from Eq.(2)) that

$$\frac{\bar{u}}{u_o} < 1 + \frac{Rn_-}{n} \frac{\bar{u}_d}{G\nu u_o} + \frac{q}{G\nu n} \frac{\bar{u}_i}{u_o} \tag{3}$$

Also from (III-4) and (III-5), and (III-4) and (III-8)

$$\frac{n_-}{n} = \frac{\nu_3}{R + D + \alpha n_+} \tag{4}$$

$$q = \alpha n^2 \frac{(R + \alpha n_+ + \nu_3)^2}{(R + \alpha n_+)^2} \tag{5}$$

where α refers to the electron-ion and ion-ion recombination coefficient. In the following we ignore $D + \alpha n_+$ in comparison with R, thus strengthening the inequality (3). Thus:

$$\frac{\bar{u}}{u_o} < 1 + \frac{\bar{u}_d}{u_o} \frac{\nu_3}{G\nu} + \frac{\bar{u}_i}{u_o} \frac{\alpha n}{G\nu} \left(\frac{R + \nu_3}{R} \right)^2 \quad . \tag{6}$$

We now make some estimates of $\frac{\bar{u}}{u_o}$ for various levels of the D region perturbed as in Section V, Fig. 6 . We let the 3-body coefficient be $K = 2 \times 10^{-30}$ cm^6/sec, $\alpha = 5 \times 10^{-7}$ cm^3/sec, $G = 2 \times 10^{-3}$ and the values of n, ν, u_o and $[O_2]$ are obtained from Table VI-I. The important parameters are provided in Table I.

TABLE I

Parameters to use in (6)

Altitude (km)	$G\nu$ sec^{-1}	ν_3 sec^{-1}	$\frac{\bar{u}_d}{u_o}$	$\frac{\bar{u}_i}{u_o}$
50	1.32×10^5	44	82.5	275
60	3.8×10^4	4.5	90	300
70	8.7×10^3	.39	108	324

Using these parameters in (6) we can obtain upper bounds on $\frac{\bar{u}}{u_o}$ as in Table II.

TABLE II

Upper bounds on average energy of electrons

Altitude (km)	$\frac{\bar{u}}{u_o}$
50	1.092
60	1.059
70	1.031

The deviations from 1 are negligible and the assumption that the electrons are at the same temperature as the background gases (even for a seriously perturbed D region) are justified.

It is of interest to compare the result (6) with that of Sears [1963]. In his work he did not have to include the ionization process since he dealt with a much less severely disturbed ionosphere than we had. His formulation, equivalent to Eq. (6) but of different form, contains the ratio of the 2-body attachment rate to the energy loss rate. We have the ratio of the 3-body rate to the energy loss rate. In our derivation this ratio comes from the assumption that the main source of n_- is the 3-body process. If the n_- did, in fact, come from the 2-body process then we would have to use that rate in (6). We can resolve this matter by the following: The maximum 2-body rate for electrons at 70 km altitude is $10^4 \sec^{-1}$ at 3 eV and is negligible below 1 eV (see Fig. 2). Now, the energy relaxation rate for these energetic electrons ($>$ 1 eV) is more than 40 times as great [Frost and Phelps, 1962] . Thus the nascent electrons quickly decay in energy below 1 eV with very few of them undergoing any 2-body attachments. Thus the n_- by default comes from the 3-body process. Sears, in his computations, seems to have ignored the electron energy dependence of the energy relaxation rate, and thus obtained excessively high average energies.

Acknowledgement

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Nonlinear Ionospheric Two-Plasma Processes

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The subject of ionized stream interactions and whistler modes has received considerable attention by Gallet and Helliwell, Rydbeck, etc., in which it was postulated that the low-frequency emissions of the exosphere were due to a traveling-wave-type interaction between drifting ionized streams and whistler modes. Several linear theories have been developed to describe the possible interactions in terms of space-charge waves, cyclotron waves and magneto-ionic waves. The solution of the system secular equation yields the wave eigenvalues but no information on the energy exchange process or on the particle distribution functions.

In this paper the nonlinear plasma-electromagnetic wave interaction process (multi-dimensional) is examined in a Lagrangian framework for both single-stream and multi-stream configurations. It is shown that nonlinear ionospheric processes may be described by simply generalizing previous microwave electron beam device theories. Arbitrarily oriented static magnetic fields are included and the general theory applies for both $\omega_p/\omega \gtrless 1$ and $\omega_c/\omega \gtrless 1$. Many ionospheric processes are multi-stream in nature and thus a finite-temperature plasma is considered and an arbitrary velocity distribution function may be specified.

In addition to the steady-state solutions given by the above theory, it is possible to obtain the transient solution by solving the nonlinear collisionless Boltzmann-Vlasov equation by an expansion method developed by Dolph and Lomax at the University of Michigan. At present this theory is limited to one-dimensional phenomena but yields an accurate description of hot plasma phenomena and may include the effects of collisions.

INTRODUCTION

In the past few years a great deal of work has been done on both natural and man-made VLF emissions in the ionosphere. Gallet, Helliwell and Rydbeck have made theoretical studies on the basis of the existence of multi-stream electromagnetic-wave interactions like those taking place in klystrons and double-stream microwave amplifiers. Their analyses are linear (small amplitude) and yield only information on the dispersion characteristics of the combined stream-wave system. It is interesting to note that in addition to both forward and backward electromagnetic waves two space-charge waves, two cyclotron waves and two magneto-ionic waves can propagate in the system, yielding an eighth-order system.

Since these authors have neglected second-order and higher terms in their analyses, no information has been obtained on the energy exchange processes. Furthermore, multi-streaming effects and finite-temperature electrons and ions have been neglected in order to simplify the analysis. The complete understanding of Whistler mode generation and propagation in the ionosphere requires a detailed nonlinear theory which accounts for such effects as multi-streaming, large-amplitude oscillations, finite-temperature plasmas, collision effects, and energy exchange between the plasma (or electron stream) and any electromagnetic wave propagating medium. The two nonlinear methods to be discussed briefly in this paper yield such information through a ballistics treatment of the problem and through the solution of the nonlinear Boltzmann transport equation. The former method is developed along similar lines to the nonlinear analysis of the klystron and traveling-wave amplifier made by the author[1] and the latter method is due to Professors C. L. Dolph and R. J. Lomax at the University of Michigan.[2]

LAGRANGIAN BALLISTICS ANALYSIS

The electron stream-plasma interaction is depicted in Fig. 1 wherein the model is composed of two interpenetrating streams surrounded by either an infinite conductivity drift tube or a wave propagating medium. The usual electron stream is virtually a plasma since it is collisionless and has a simple velocity distribution such as a square or Maxwell-Boltzmann distribution and thus may be analyzed on the same basis as a plasma. The drift tube is considered to be cut-off for all frequencies carried by the plasmas so that there is no electromagnetic wave propagation in the region between the streams and the tunnel.

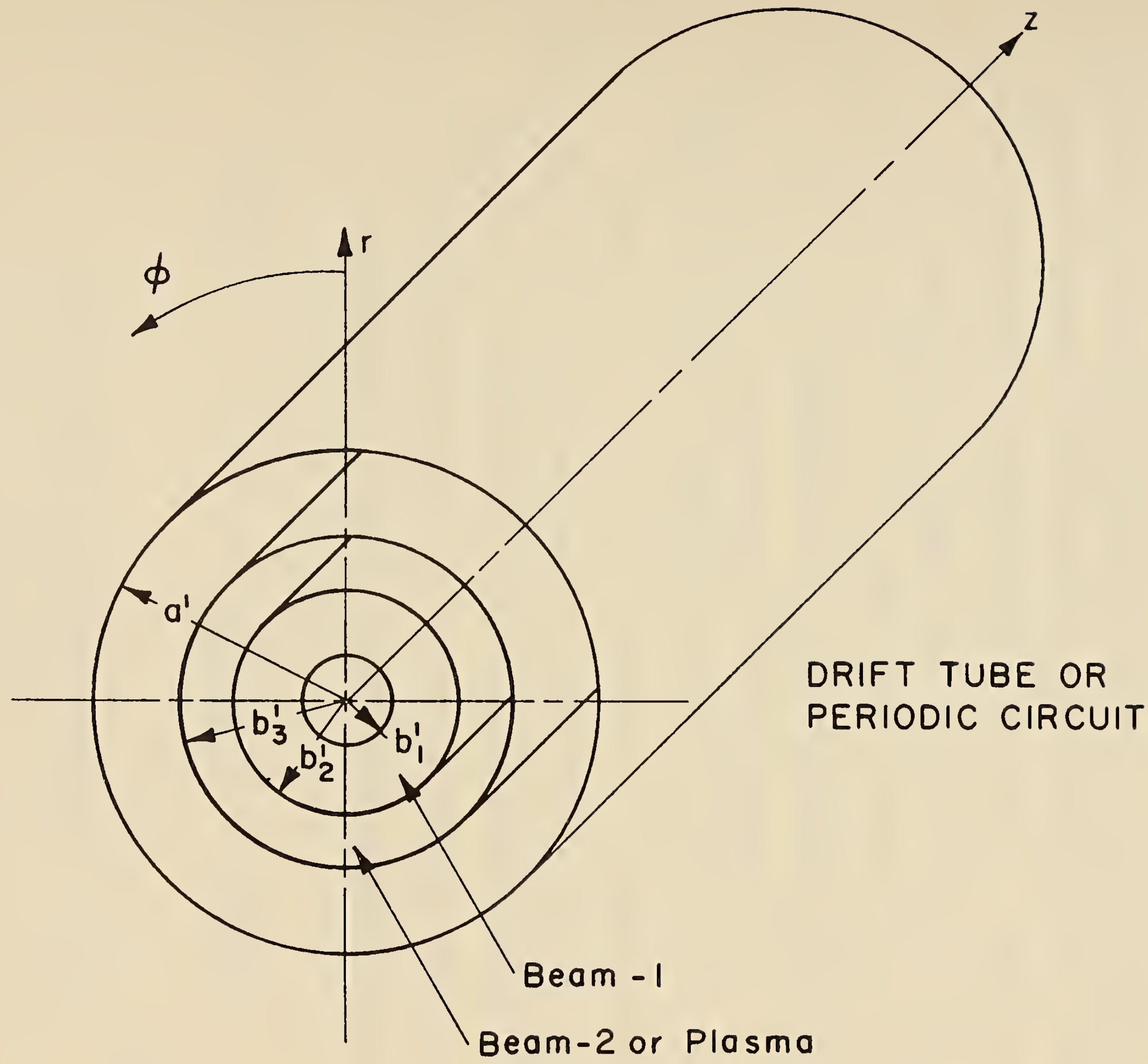

FIG. 1 BEAM-PLASMA-CIRCUIT INTERACTION CONFIGURATION.

The above system is in general a multi-dimensional one and may include axial and radial magnetic fields as well as finite-velocity distributions in the streams. For simplicity only a one-dimensional system will be described here although finite-temperature effects are included. In order to assure one-dimensionality of the system, an infinite axial focusing field (magnetic) is assumed so that $\omega_c/\omega \rightarrow \infty$, and thus both signal frequencies and stream plasma frequencies are well below the cyclotron or gyrofrequency.

In the Lagrangian method we integrate from some initial plane ($z = 0$) along particle trajectories, summing all forces at each displacement plane. In the event that there is a wave propagation medium present it may be treated on an equivalent circuit basis derived from Maxwell's equations. From the $\nabla \times \vec{E}$ equation the following second-order differential equation for the electromagnetic wave equivalent voltage variation with distance may be written

$$\frac{\partial^2 V(z,t)}{\partial t^2} - v_o^2 \frac{\partial^2 V(z,t)}{\partial z^2} = \pm v_o^2 Z_o \sum \frac{\partial^2 \rho(z,t)}{\partial t^2} \tag{1}$$

where

$V(z,t)$ = the wave equivalent voltage amplitude,

$\rho(z,t)$ = the r-f charge density in each stream,

$v_o \triangleq \frac{1}{\sqrt{LC}}$, the equivalent e-m wave phase velocity, and

$Z_o \triangleq \sqrt{L/C}$, the equivalent e-m wave impedance.

The plus sign on the right-hand side is used for forward-wave interaction and the minus sign for backward-wave interaction. In many instances the permeability and permittivity of the medium determine both v_o and Z_o.

The continuity of entering charge

$$\rho_o dz_o = \rho dz \tag{2}$$

and the Lorentz force equation for the one-dimensional system

$$\frac{dv}{dt} = |\eta| \left[E_{c-z} + E_{sc-z} \right] , \tag{3}$$

where

E_{c-z} = the axial component of the e-m wave field, and

E_{sc-z} = the axial component of the Coulomb force field,

complete the system description.

After a considerable amount of mathematical manipulation the following particle force equations evolve:

$$[1 + \alpha\, u_1(y,\Phi_{oj})] \frac{\partial u_1(y,\Phi_{oj})}{\partial y} = \frac{-\epsilon_1 \alpha(1 + \eta')}{2(1 + b)^2}$$

$$\left[\frac{d\, A(y)}{dy} \cos \Phi_1(y,\Phi_{oj}) - A(y)\left(\frac{2}{\alpha} - \frac{d\theta(y)}{dy} \right) \sin \Phi_1 (y,\Phi_{oj}) \right]$$

$$+ \frac{1}{(1 + b)^2} \left(\frac{2\omega_{p1}}{\omega\, \alpha} \right)^2 \int_0^{2\pi} \frac{{}_{11}F\, d\Phi'_{oj}}{[1+ \alpha\, u_1(y,\Phi_{oj})]}$$

$$+ \frac{\epsilon_1 \epsilon_2 (1 + \eta')}{(1 + b)^2 (1 - \eta')} \left(\frac{2\omega_{p2}}{\omega\, \alpha} \right)^2 \int_0^{2\pi} \frac{{}_{21}F\, d\Phi''_{oj}}{[1 + \alpha\, u_2(y,\Phi_{oj})]} \qquad (4)$$

and

$$[1 + \alpha\, u_2(y,\Phi_{oj})] \frac{\partial u_2(y,\Phi_{oj})}{\partial y} = \frac{-\epsilon_2 \alpha(1+ \eta')}{2(1 - b)^2}$$

$$\left[\frac{d\, A(y)}{dy} \cos \Phi_2(y,\Phi_{oj}) - A(y)\left(\frac{2}{\alpha} - \frac{d\theta(y)}{dy} \right) \sin \Phi_2 (y,\Phi_{oj}) \right]$$

$$+ \frac{1}{(1 - b)^2} \left(\frac{2\omega_{p2}}{\omega\, \alpha} \right)^2 \int_0^{2\pi} \frac{{}_{22}F\, d\Phi''_{o1}}{[1 + \alpha\, u_2(y,\Phi_{oj})]}$$

$$+ \frac{\epsilon_1 \epsilon_2 (1 - \eta')}{(1 - b)^2 (1 + \eta')} \left(\frac{2\omega_{p1}}{\omega\, \alpha} \right)^2 \int_0^{2\pi} \frac{{}_{12}F\, d\Phi'_{o1}}{[1 + \alpha\, u_1(y,\Phi_{oj})]} \qquad (5)$$

where

$$b \triangleq \left[1 - \left(u_{o2}/u_{o1}\right) \right] \Big/ \left[1 + \left(u_{o2}/u_{o1}\right) \right],$$ the

stream relative velocity difference,

$$\eta' \triangleq \left[1 - |\eta_2/\eta_1| \right] \Big/ \left[1 + |\eta_2/\eta_1| \right] ,$$

the relative mass ratio,

$$\bar{V}(z,t) = V(y,\Phi) \triangleq \mathrm{Re} \left[\frac{2 Z_o I_o}{\alpha} A(y) e^{-j\Phi} \right] ,$$

the e-m wave voltage,

$y = X \triangleq \alpha \omega z/2 u_o$, normalized distance,

$\Phi(z,t) \triangleq \omega \left(\frac{z}{u_o} - t \right) - \theta(y)$, charge group phase variable,

$$\frac{dz}{dt} \triangleq u_{o1} [1 + \alpha u_1 (y,\Phi_{oj})],$$

$$\frac{dz}{dt} \triangleq u_{o2} [1 + \alpha u_2 (y,\Phi_{oj})],$$

$u_{o1} \triangleq$ average velocity of stream - 1,

$u_{o2} \triangleq$ average velocity of stream - 2,

$\alpha = 2C \triangleq V_g/V_o$, depth of modulation,

$\omega_p \triangleq$ stream radian plasma frequency.

The first terms on the right-hand side of Eqs. (4) and (5) represent the e-m wave field and the integral terms the Coulomb fields. The weighting functions ${}_{11}F$ and ${}_{12}F$ are shown in Figs. 2 and 3 vs. the relative phase positions, $\Phi - \Phi'$. The ${}_{11}F$ weighting function is nearly exponential as expected and does lead to some computational difficulty when $\Phi - \Phi' \to 0$. This may be alleviated to some extent by utilizing a large number of charge groups in both the stream and the plasma.

The above nonlinear equations are solved as an initial value problem on a high-speed digital computer. Some representative solutions for a double-beam system ($\epsilon_1 = \epsilon_2 = -1$) plus an e-m wave are shown in Fig. 4. Here the fundamental r-f current densities are normalized with respect to the d-c current in either stream. The distance variable y may also be expressed in terms of the characteristic Debye length as follows:

$$y = \frac{\pi \alpha}{\sqrt{2}\, \omega_p} \left(\frac{z}{\lambda_D} \right)$$

The efficiency of energy conversion to the e-m wave is shown in Fig. 5 as a function of both the plasma frequency and the mean velocity of the streams relative to the e-m wave.

$$1 + \frac{\alpha p}{2} = \frac{u_o}{v_o}$$

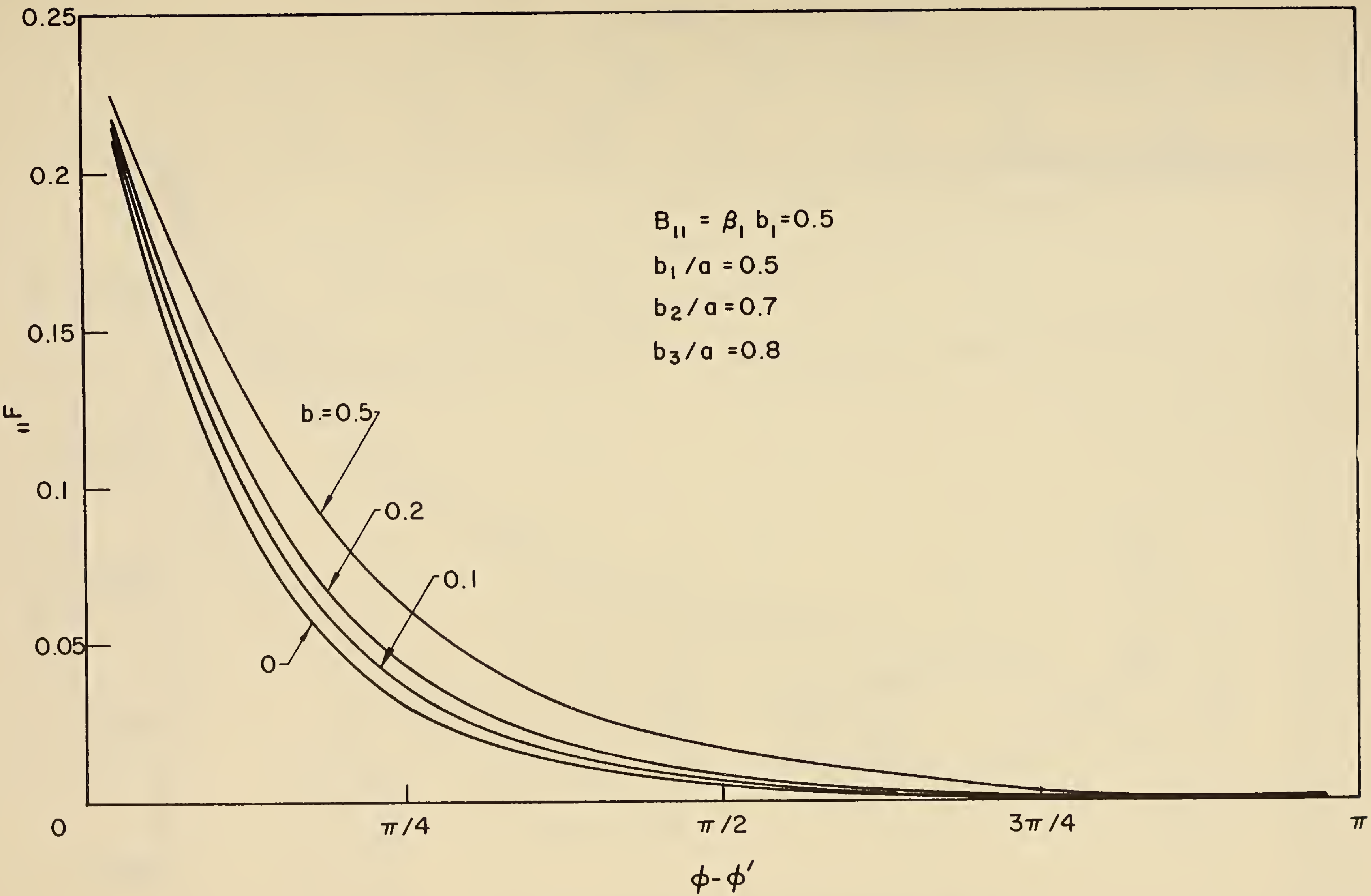

FIG. 2 SPACE-CHARGE-FIELD WEIGHTING FUNCTION $_{11}F$.

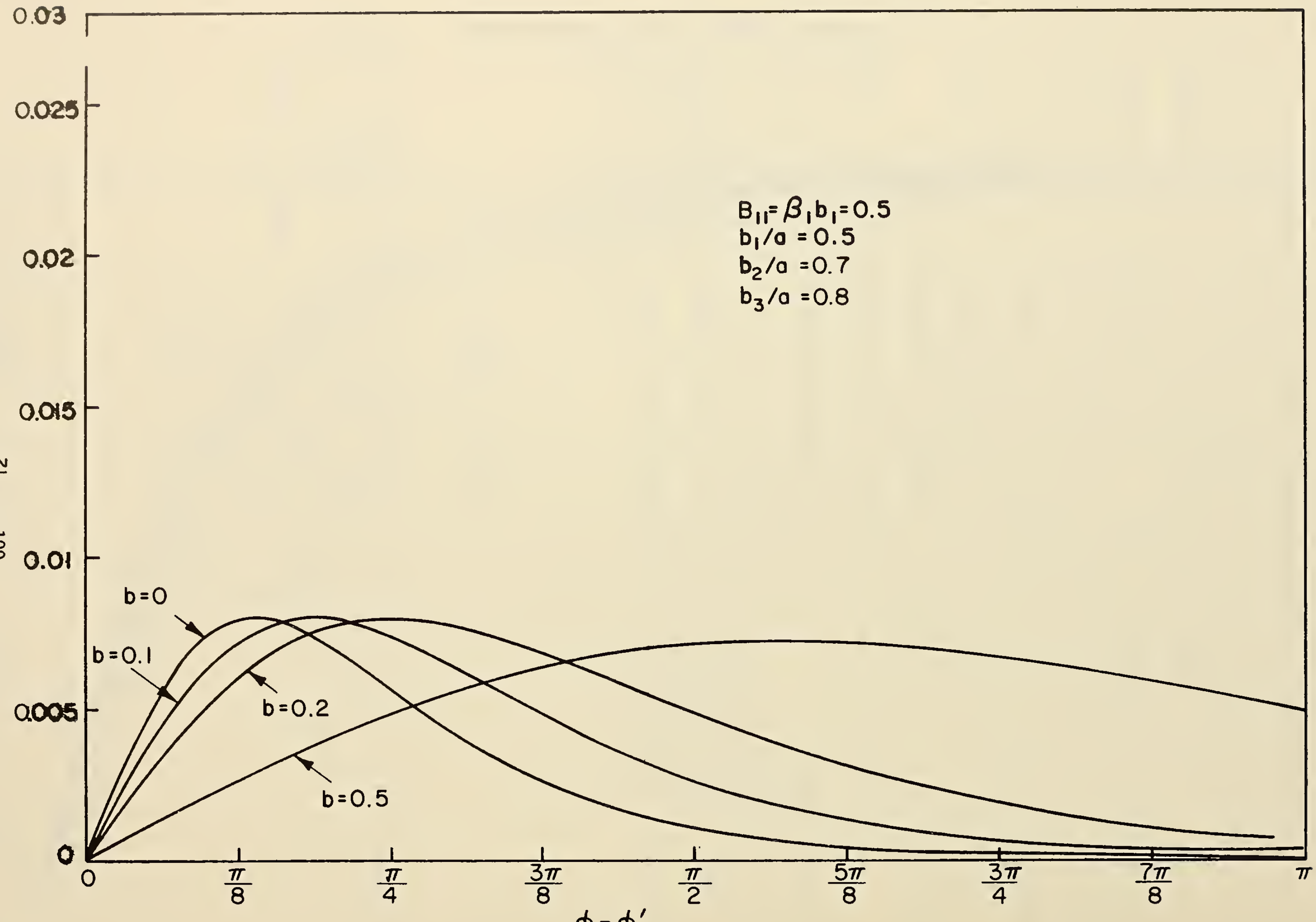

FIG. 3. SPACE-CHARGE-FIELD WEIGHTING FUNCTION ${}_{12}F$.

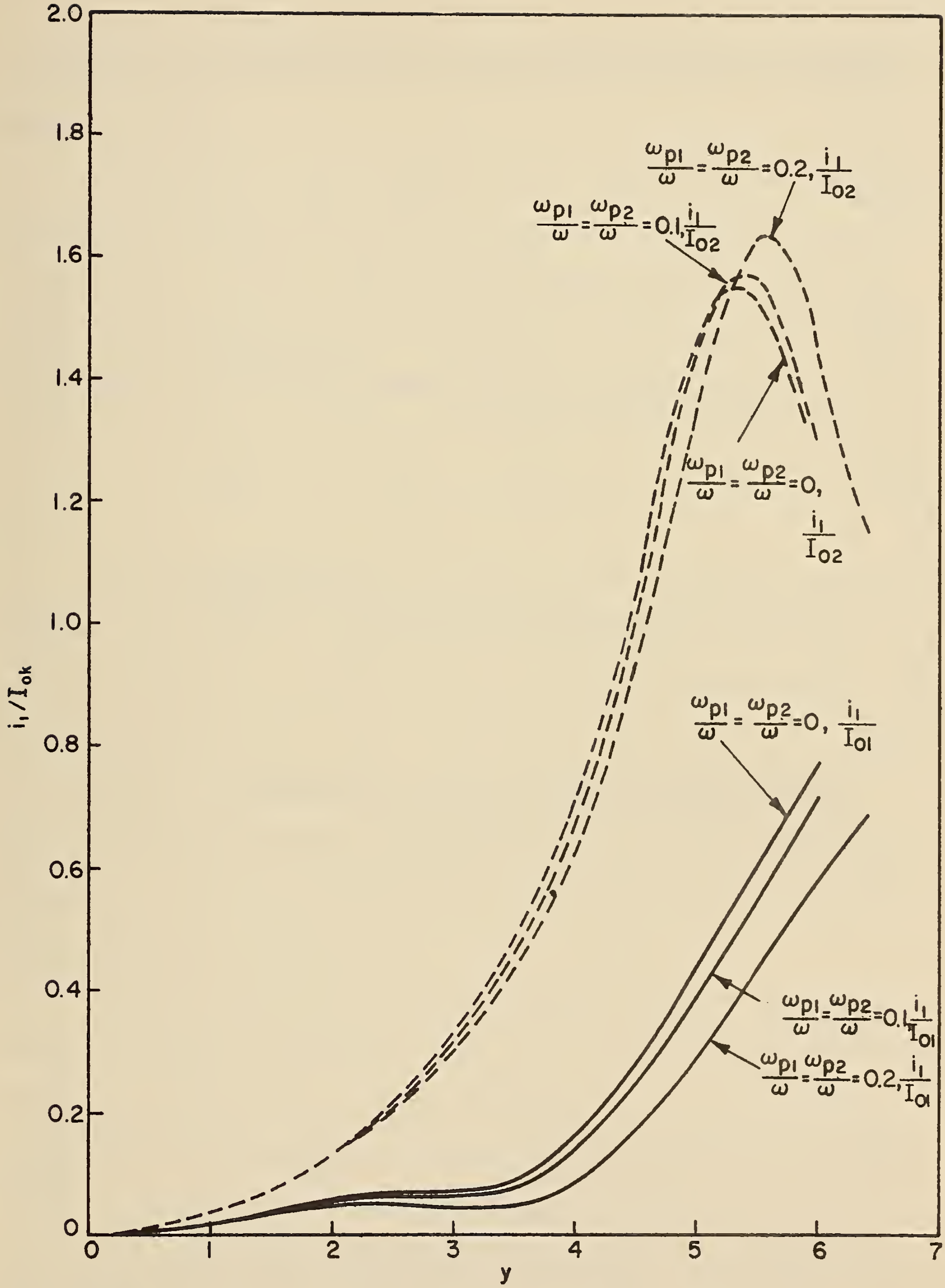

FIG. 4 DOUBLE-BEAM AND CIRCUIT WAVE. FUNDAMENTAL COMPONENT OF BEAM CURRENT VS. DISTANCE AND SPACE CHARGE. ($C = 0.1$, $b = 0.1$, $p = b$, $\epsilon_1 = \epsilon_2 = -1$, $I' = 0$, $\eta' = 0$, $\psi_0 = -30$)

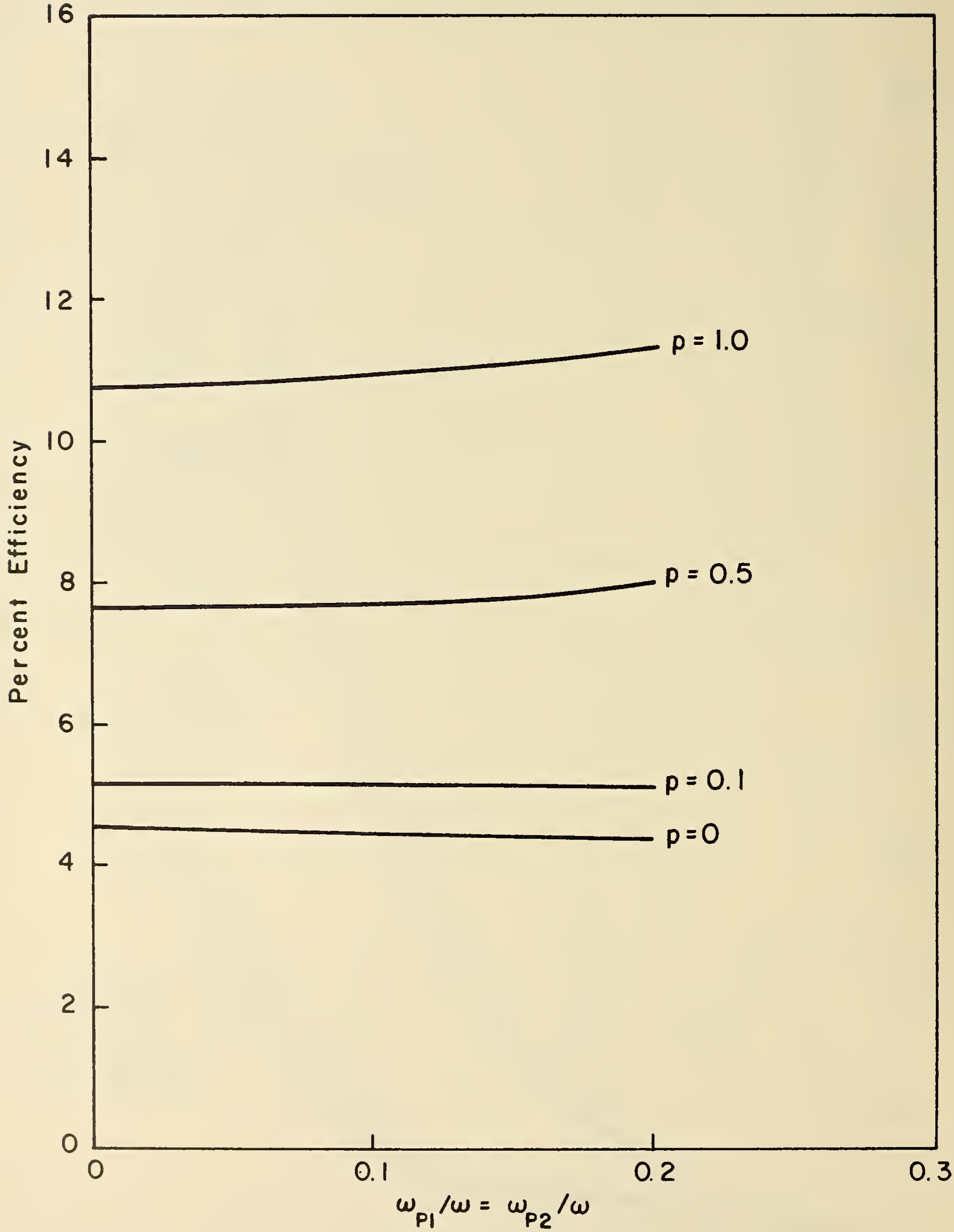

FIG. 5 DOUBLE-BEAM AND CIRCUIT WAVE. EFFICIENCY VS. ω_p/ω AND p. ($C = 0.1$, $b = 0.1$, $\epsilon_1 = \epsilon_2 = -1$, $I' = \eta' = 0$, $\psi_o = -30$)

A value of $p = 1$ corresponds to an excess stream velocity of approximately 10 percent.

The harmonic current components for a simple two-stream (electron beam-plasma electrons) interaction are shown vs. the distance variable, $X = y = \pi \alpha N_s$, in Fig. 6 where the normalization has been made with respect to the individual stream currents and the mean stream current. The maximum value of i_1/I_o is given by the ballistic value

$$\frac{i_1}{I_o} = 2\,J_1(1.84) = 1.16 \;.$$

The interaction phenomena involved when an electron stream is coupled to the ions of a plasma may be studied by simply setting $\epsilon_1 = -1$ and $\epsilon_2 = +1$. We are primarily interested here in an interaction process in which there is no e-m wave. The results for varying b, η', and ω_{pi}/ω are summarized in Fig. 7 where the stream currents have been considered equal. We note that i_1/I_{ok} can rise considerably above the 1.16 value and furthermore that the results are critically dependent upon b, ω_p/ω, and η'.

Finite-temperature electron and plasma effects are readily treated by giving the charges initial velocity distributions according to Fig. 8. The velocity spread is denoted by $S = (v_2 - v_1)/u_o$ and indicates the deviation from the mean. When S is small there is little effect on the results, as would be expected. Numerous results on the effects of finite S are summarized in Fig. 9 for a two-electron beam interaction.

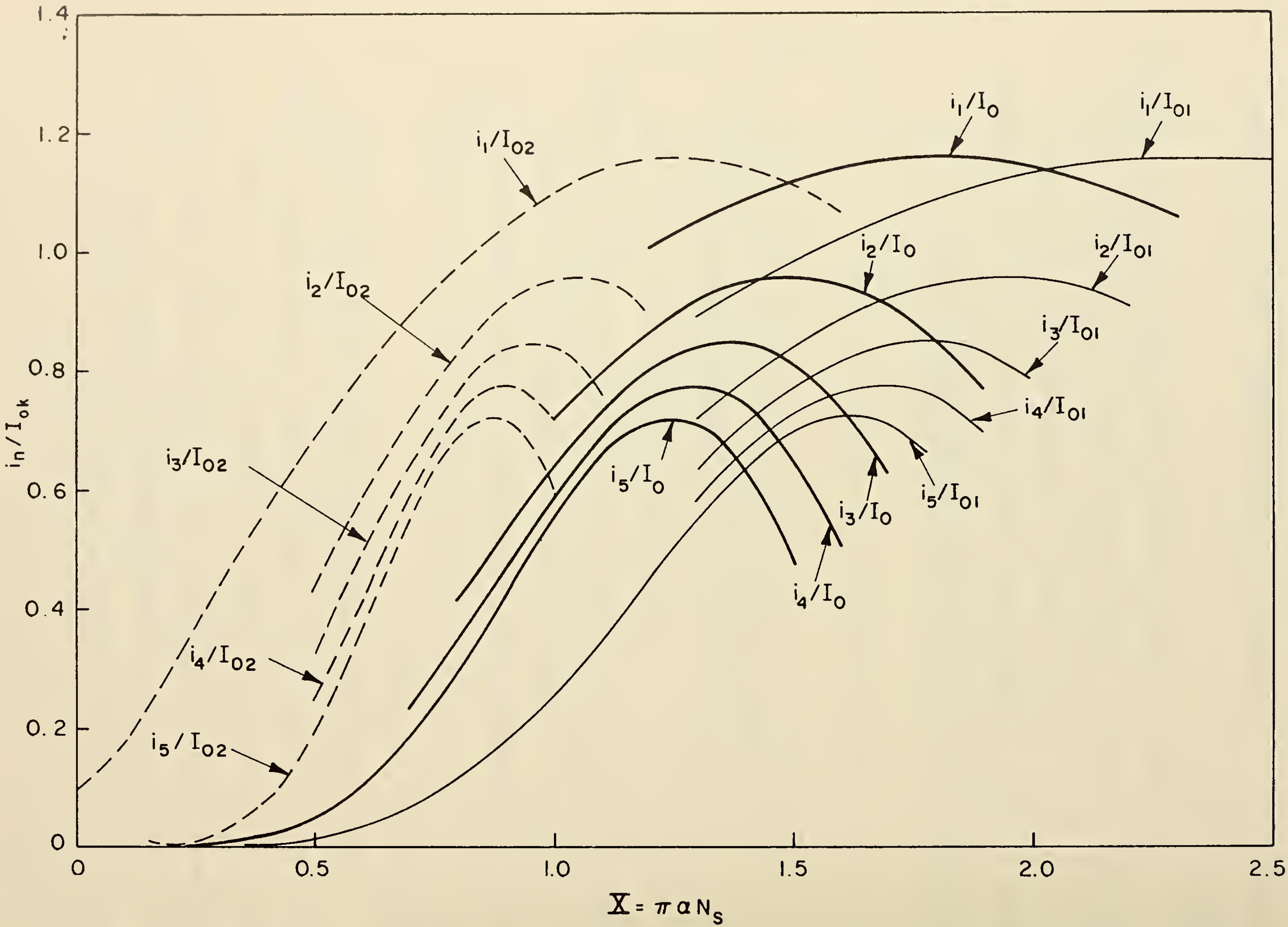

FIG. 6 DOUBLE-BEAM INTERACTION. ($\epsilon_1 = \epsilon_2 = -1$, $\alpha = 0.2$, $I' = \eta' = 0$, $\omega_{p1}/\omega = \omega_{p2}/\omega = 0$)

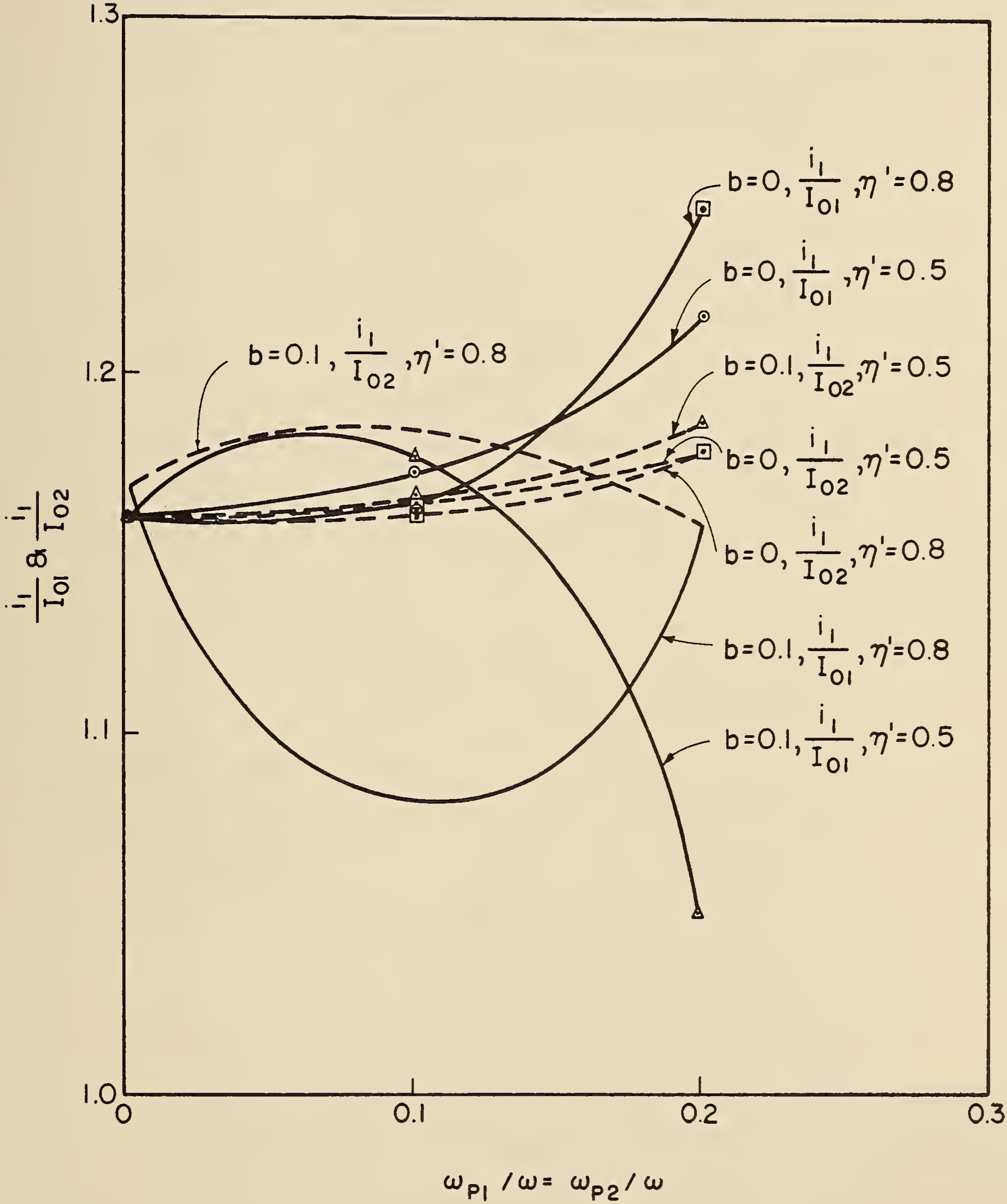

FIG. 7 BEAM-PLASMA INTERACTION. FUNDAMENTAL CURRENT AMPLITUDE VS. ω_p/ω, b AND η'. (ϵ_1 = -1, ϵ_2 = +1, α = 0.2, I' = 0)

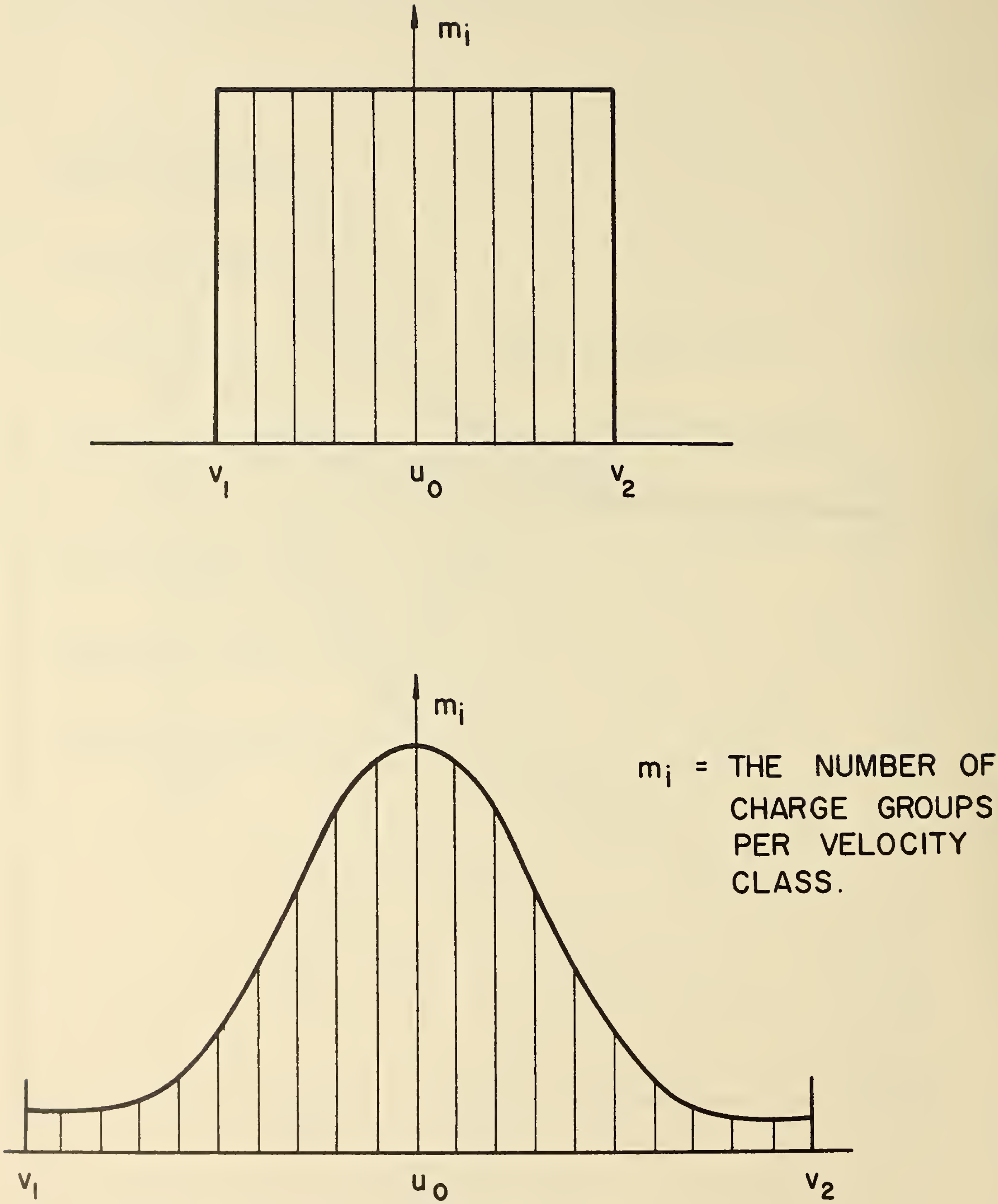

FIG. 8. VELOCITY DISTRIBUTIONS.

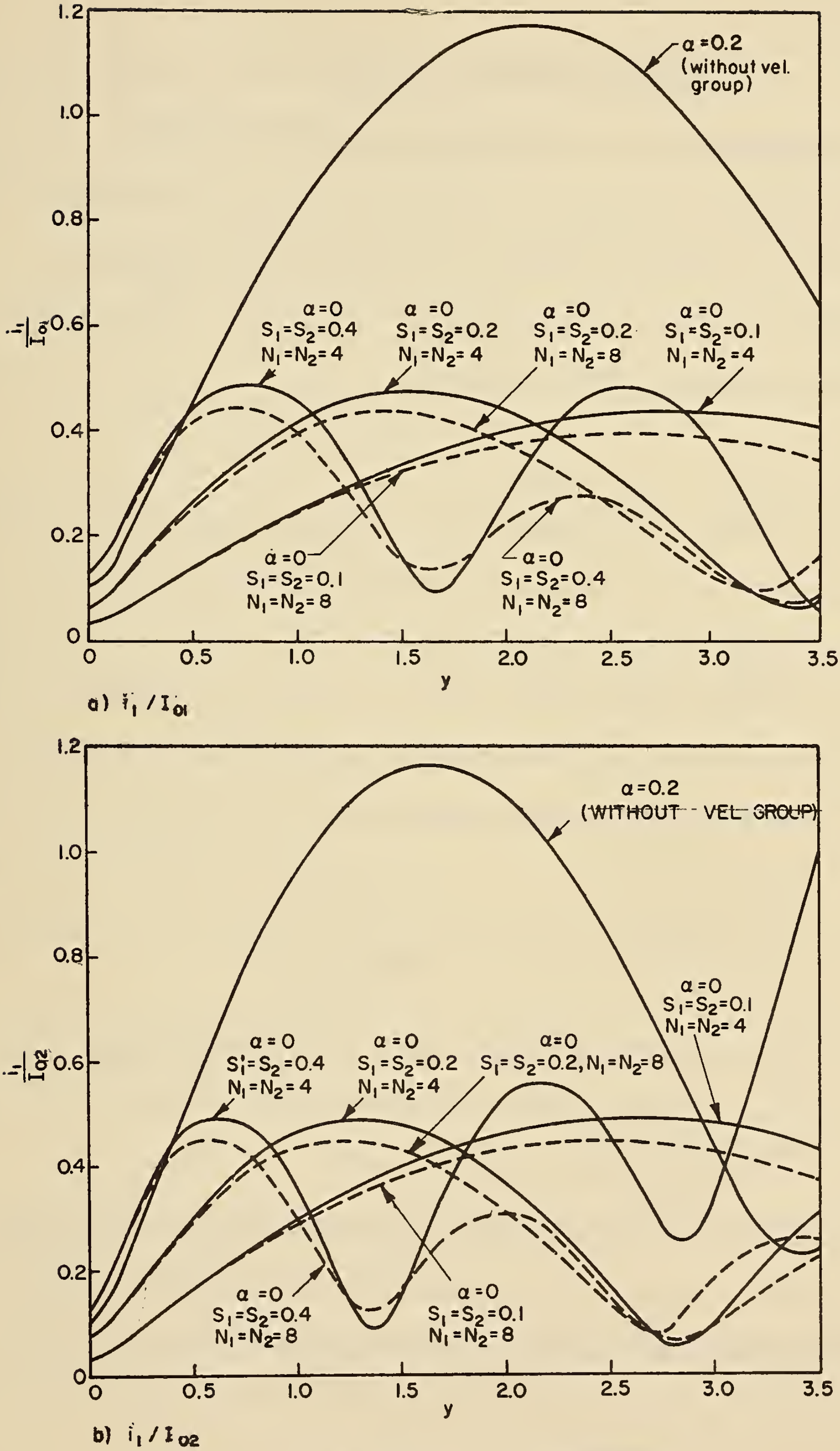

FIG. 9 DOUBLE-BEAM INTERACTION. VELOCITY DISTRIBUTION CHARACTERISTICS. ($b = 0.1$, $\omega_{p1}/\omega = \omega_{p2}/\omega = 0.1$, $\epsilon_1 = \epsilon_2 = -1$, $I' = \eta' = 0$)

An alternate approach to the solution of the nonlinear interaction problem investigated by Professors Dolph and Lomax at the University of Michigan involves the solution of the Boltzmann-Vlasov transport equation by the method of expansions. They solved the following coupled transport equations

$$\frac{\partial f_e}{\partial t} + v \frac{\partial f_e}{\partial x} - \frac{e}{m_e} E \frac{\partial f_e}{\partial v} = 0 \tag{6}$$

and

$$\frac{\partial f_i}{\partial t} + v \frac{\partial f_i}{\partial x} + \frac{e}{m_i} E \frac{\partial f_i}{\partial v} = 0 \tag{7}$$

where

f = the particle distribution function,

$$E = \int_0^x \int_{-\infty}^{\infty} \frac{e}{\epsilon_o} (f_i - f_e)\, dv\, dx' - \int_0^d \int_0^{x'} \int_{-\infty}^{\infty} \frac{e}{\epsilon_o d} (f_i - f_e)\, dv\, dx'\, dx''$$

$- \frac{V}{d} + E_c$, the electric field for a simple diode,

d = diode spacing.

The solution method involves the expansion of the distribution function in terms of orthogonal polynomials in the velocity variable. Half-range expansions are made in terms of Laguerre polynomials $L_k^{\pm}(v)$

which are orthogonal with respect to the weighting functions $\omega^{\pm}(v) = e^{\mp v}$. The actual details of the method involve working with the moments

$$M_k^{+} = \int_0^{\infty} v^k f \, dv \tag{8}$$

$$M_k^{-} = \int_{-\infty}^{0} v^k f \, dv \, . \tag{9}$$

So far this method has been applied to the single and two-stream diode to obtain both the transient and steady-state solutions. The results agree well with other results and indicate promise for the method on more general problems. In the electron-ion problem, the electrons come to equilibrium in a considerably shorter time than do the ions, and therefore the solution time must be scaled to the convenience of the ions.

CONCLUSIONS

It has been established that a nonlinear theory is required to analyze the generation and propagation of VLF ionospheric emissions. Previous theories have been linear and only yield information on the system propagation constants, generally neglecting finite-temperature effects, multi-streaming, and collisions.

One of the nonlinear theories constitutes a Lagrangian ballistics solution of the equations of motion and includes all of the above effects. The effects of collisions are incorporated by including an electric field term which is developed from the appropriate collision model. The other nonlinear theory yields both transient and steady-state solutions and also includes the above effects. Numerous steady-state solutions obtained by the Lagrangian method were presented and the results indicated that the methods are applicable to the analysis of other ionospheric two-plasma phenomena.

List of Figures

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CONCLUDING REMARKS

E. K. Smith, Jr.

National Bureau of Standards

This part of the program has been set aside for final comments. What I will do is to make some general remarks myself and then ask Dr. Donald T. Farley to present a short technical summary of the meeting.

On behalf of Dr. Menzel and myself I would like to thank all of the delegates for their contributions to the success of this meeting. We are particularly grateful for the contributions from our colleagues from overseas Professor V. A. Bailey, a true father of the field, has come all the way from Australia despite poor health. His former student, Dr. R. A. Smith has come a similar distance and has made it possible for an American audience to appreciate the excellent work going on at the University of New England. Professor Cutulo has surmounted the language barrier to present the very interesting work he has carried on for nearly two decades at Naples. We also owe our thanks to Dr. DeBarbieri who has given us a good insight into the fine work of Professor Caldirola and himself at the Universita Di Milano. Finally we need to thank Dr. Belrose for his vigorous participation in the Conference. It would indeed have been awkward to leave the important work which has been going on at Ottawa unrepresented at this Conference.

On your behalf, Dr. Menzel and I would like to convey to Professor Rydbeck our regret that he could not be with us and to wish him a speedy recovery.

I would like to acknowledge the pleasure I have had in working with Dr. Menzel as Co-chairman of this Conference and also the fine efforts of Robert T. Frost and his colleagues on the Local Arrangements Committee.

Finally, Dr. Menzel and I would like to thank the Advisory Committee, made up of Dr. Little, Professor Layzer, Dr. Wait, George Jacobs, and Roger Gallet, for their assistance with the technical program and publication plans, and the Voice of America for their assistance in helping to defray the expenses of this Conference.

CONCLUDING REMARKS

Conference on Non-Linear Processes

D. T. Farley, Jr.
National Bureau of Standards

The effects that have been discussed at this conference can be divided into two broad categories, in one of which collisions are important, while in the other they are negligible. In the first group the non-linearities are created by temperature variations in the medium caused by the electromagnetic waves. This group can be subdivided into weak effects, in which the temperature is perturbed only slightly, and strong effects in which the properties of the medium are changed drastically.

Research has been carried out in the first of these subcategories for many years. Cross-modulation, or the "Luxembourg effect", has been used by several groups to study the D and lower E regions of the ionosphere. The general features of the theory of this effect are quite well understood. It has always been difficult, however, to interpret the measurements unambiguously and obtain from them profiles of electron density and collision frequency as a function of altitude. One of the important results demonstrated at this conference was that it is impossible to uniquely determine these quantities from cross-modulation measurements, even when the data are excellent. An infinite number of solutions, many of which appear quite reasonable, can apparently be made to fit the data. Without an independent measurement of the electron density or collision frequency, it appears that measurements of the Luxembourg effect are of limited use as a diagnostic tool for the lower ionosphere.

When we turn to the subject of strong heating effects, the prospects appear more hopeful. It is now feasible to radiate sufficient power to substantially alter the characteristics of the ionosphere, particularly if the frequency of radiation is near the electron gyro-frequency. It has been appreciated for some time that the ability to raise the temperature of the ionosphere by an order of magnitude or more would be very useful in studying the various chemical and excitation processes in the ionosphere.

From rather simple calculations one can arrive at a qualitative understanding of what will happen. However, as we have heard here, such calculations cannot be expected to give even semiquantitative answers to such questions as how much power is necessary to produce an artificial airglow. The magnitude of most of the excitation effects depends very critically on the shape of the distribution function of the electron energies, as well as on the mean energy. It is the very small minority of electrons with very high energies that we are most interested in. In general the distribution function is far from Maxwellian, and so any calculations which are based on the assumption of such a distribution are likely to be seriously in error. We have heard reports at this conference of some detailed numerical calculations of the distribution function that show great promise of helping us to understand the details of the chemical and photochemical processes in the ionosphere.

Such calculations have been greatly aided by recent laboratory measurements of many of the pertinent cross-sections for the various reactions. These measurements are now being made at energies only slightly higher than those attainable in the ionosphere using very high power transmitters.

To summarize, then, the situation regarding collisional effects in the ionosphere, it appears that the most productive future research may be largely in the realm of very high power experiments. Such experiments, coupled with laboratory measurements and detailed calculations, should give us a much better understanding of the chemistry of the lower ionosphere.

The second class of non-linear phenomena we have considered here do not involve collisions; they can be called coherent non-linearities. These effects occur when charged particles move at approximately the same velocity as the wave. Then the waves and particles can interact strongly, causing either the waves to grow or the particles to accelerate. Since there are very few relativistic particles in the ionosphere, we are concerned mainly with slow waves, such as VLF emissions, magnetohydrodynamic and electrostatic waves, etc. With the possible exception of the resonances observed by the Alouette topside sounder, our understanding of these effects appears to be very limited at present. It is becoming clear, in particular, that the variety of VLF emissions that are observed are doubtless due to a variety of complicated interactions occurring in the exosphere. However, the details of these processes are not at all clear at present. This is another area in which laboratory measurements may eventually be helpful, since many similar effects occur in laboratory plasmas. Hopefully there may be more interaction in the future between these two fields of research than there has been in the past.

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