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Technical Note No. 208

CALCULATED DIFFRACTION EFFECTS AT VLF FROM A LOCALIZED IONOSPHERIC DEPRESSION

JAMES R. WAIT

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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CALCULATED DIFFRACTION EFFECTS AT VLF FROM A LOCALIZED IONOSPHERIC DEPRESSION †

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Propagation of VLF radio waves in the earth-ionosphere waveguide of non-uniform width is considered. The disturbed region is permitted to be of finite extent. It is assumed that the height variations may be locally represented in terms of a propagation function $S(x, y)$ which is a function of both x and y. Using first-order scattering theory, calculations are presented for a disturbed region which is approximately rectangular in the horizontal plane.

1. Introduction

It is now becoming recognized that the propagation of VLF radio waves is influenced by ionospheric disturbances which occur along all or part of the great circle path between transmitter and receiver. However, it is not always appreciated that ionospheric perturbation lying <u>off</u> the great circle path may also have a significant effect. A quantitative estimate of these off-great-circle disturbances is of particular importance in connection with the detection of nuclear detonations. It is known that the x-ray radiation resulting from the explosion will produce additional ionization at D- and E-region heights of the ionosphere. In this note, the results of some calculations are presented which bear on this important problem. The detailed derivation of the required equations were given earlier [Wait, 1963]; however, for purposes of completeness, a brief discussion of the plausibility of the working formulas is given here.

2. The Uniform Guide

In order to achieve tractability, without becoming encumbered in details, some simplifying assumptions are made at this juncture. It is assumed that the space between the earth, of radius a, and the ionosphere is equivalent to a waveguide which supports only several modes of low attenuation. In fact, for distances of the order of 4000 km or greater, it is necessary to consider only one mode for frequencies of the order of 20 kc/s. Thus, at a distance d from the transmitting antenna, the vertical electric field E is given approximately by [Wait, 1962]

$$E \cong \frac{\text{const.}}{[\sin (d/a)]^{\frac{1}{2}}} \exp(-ikdS_1) \quad , \tag{1}$$

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† The research reported here was supported by the Advanced Research Projects Agency under ARPA Order No. 183-62.

if the height h and properties of the ionosphere did not vary. Here, $k = 2\pi/\text{wavelength}$, and thus $k S_1$ is the (complex) propagation constant of the dominant mode. For example, the phase velocity is $(c/\text{Re } S_1)$ where c is the velocity of light and the attenuation rate is $(-k \text{ Im } S_1)$ nepers per unit length. In what follows, the subscript 1 on S is dropped.

3. The Non-Uniform Guide

In a first approach in treating the influence of variable ionospheric heights, one generalizes equation (1) to allow for the dependence of S on the distance x, along the great circle, from the transmitting antenna. Thus, [Wait, 1961]

$$E \cong \frac{\text{const.}}{[\sin(d/a)]^{\frac{1}{2}}} \exp\left(-ik \int_0^d S(x)\, dx\right) . \tag{2}$$

As indicated previously, this is a valid representation for an earth-ionosphere waveguide whose width varies slowly along the great circle path. Obviously, such a simple formula ignores variations in the transverse or y direction. In particular, if the perturbed region lies <u>off</u> the great circle path, the above formula for E would give no information concerning the corresponding modification to the field.

A somewhat heuristic approach to the general problem is based on the concept that the integral in the exponent of equation (2) is the resultant of the individual contributions from the whole x, y plane. S(x, y), which is regarded as a function of both x and y, will contribute to the modification of the field whenever it departs significantly from the undisturbed constant value S^o. This suggests, with a certain amount of hindsight, that the field under disturbed conditions should have the form

$$E = E^o \exp\left(-ik \int_{-x_o}^{x_1} \Omega(x)\, dx\right) , \tag{3}$$

where E^o is the field under undisturbed conditions and $\Omega(x)$ is a function of x, yet to be determined. Here, the coordinates of the transmitter and receiver are $(-x_o, 0)$ and $(x_1, 0)$, respectively. In view of the remarks made above, $\Omega(x)$ must involve an integral over the transverse direction y. Also, the integrand must be weighted by $S(x, y) - S^o$ which is the "local contrast," and furthermore, account must be taken of the different electrical lengths of the paths connecting transmitter and receiver. After some consideration, it is found that [Wait, 1963]

$$\Omega(x) = \left(\frac{i}{\pi}\right)^{\frac{1}{2}} \alpha \int_{y_1(x)}^{y_2(x)} [S(x, y) - S^o] \exp(-i\, \alpha^2 y^2)\, dy , \tag{4}$$

where the y integration extends over the interval y_1 to y_2 which encompasses the ionospheric perturbations. In this equation

$$\alpha^2 = \frac{k S^o}{2} \left[\frac{(x_1 - x_o)}{(x_o - x)(x_1 + x)}\right] . \tag{5}$$

As shown before, [Wait, 1963] in a full analytical treatment of this problem, the factor $-i\,\alpha^2 y^2$ is the correction for the phase which retains only second-order terms in y. Also, it should be mentioned that the form for α, given above, is strictly valid only on a flat earth. However, earth curvature changes this factor by only a small amount, and for present purposes, this correction is ignored.

It is immediately evident, from the form of equation (4), that if $S(x, y)$ is independent of y and if $(-y_1)$ and $(+y_2)$ are sufficiently large,

$$\Omega(x) \cong \left(\frac{i}{\pi}\right)^{\frac{1}{2}} \alpha \int_{-\infty}^{+\infty} [S(x,0) - S^o] \exp(-i\,\alpha^2 y^2)\, dy$$

$$\cong S(x,0) - S^o \; . \qquad (6)$$

Inserting this value into equation (3) gives the required form of the modified field E_∞ when the ionospheric disturbance is <u>effectively</u> infinite in the transverse direction.

In order to present numerical data in a convenient form, it is desirable to normalize the results to the ionospheric depression which has an infinite transverse direction. For example, the E field, under disturbed conditions, may be written

$$\frac{E}{E^o} \cong \exp\left[-ik \int_{-\delta/2}^{\delta/2} \Omega(x)\, dx\right] , \qquad (7)$$

where E^o is the corresponding field for an undisturbed ionosphere and δ is the range of x which encompasses the perturbation. Now, if the function $S(x, y) - S^o$ is independent of y, it follows that

$$\frac{E}{E^o} = \frac{E_\infty}{E^o} = \exp\left[-ik \int_{-\delta/2}^{\delta/2} [S(x) - S^o]\, dx\right] . \qquad (8)$$

The "normalized field anomaly" NFA may then be <u>defined</u> by the ratio

$$\frac{\log(E/E^o)}{\log(E_\infty/E^o)} = \text{NFA} . \qquad (9)$$

It is obvious that NFA approaches one when the perturbation has an infinite transverse dimension. Thus, a quantitative evaluation of NFA gives a good index of the effectiveness of an ionospheric disturbance of finite width.

For purposes of illustration, $S(x, y)$ and S^o may be regarded as real. (In actual fact, they have small imaginary parts.) Furthermore, it is assumed that $S(x, y)$ differs from S^o only over a rectangular region bounded by $y_1 < y < y_2$ and $-\delta/2 < x < \delta/2$ as indicated in figure 1.

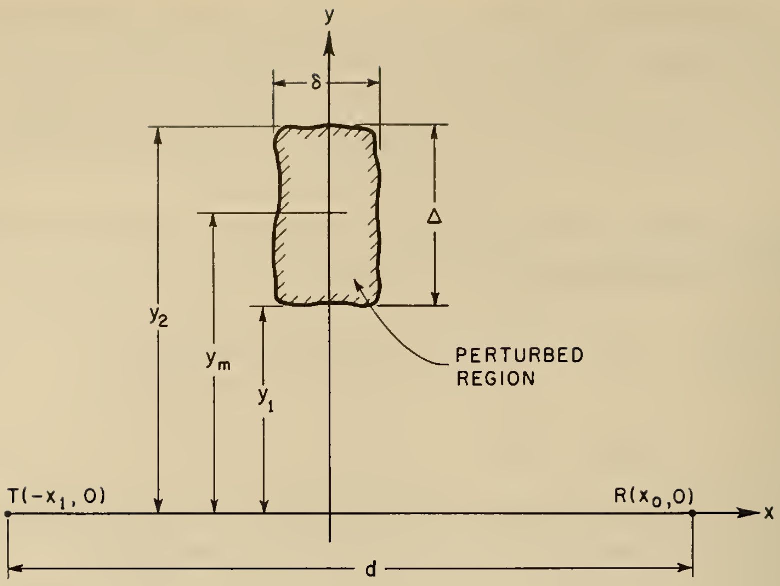

FIGURE 1 - PLAN VIEW OF SIMPLE IONOSPHERIC PERTURBATION SHOWING LOCATIONS OF TRANSMITTING AND RECEIVING ANTENNAS (SCALE EXAGGERATED IN y DIRECTION)

Also, in order to achieve more simplicity, it is assumed that the difference or contrast function $S(x, y) - S^o$ is constant over this rectangular region. It is then found that

$$\frac{\log (E/E^o)}{\log (E_\infty/E^o)} = \mathrm{NFA} \cong \left[\frac{1}{1-i} \int_{Z_1}^{Z_2} e^{-i(\pi/2)\, t^2}\, dt\right] , \tag{10}$$

where $(\pi/2)^{\frac{1}{2}} Z_1 = a\, y_1$,

and $(\pi/2)^{\frac{1}{2}} Z_2 = a\, y_2$,

with $a \cong \left(\frac{k}{2}\right)^{\frac{1}{2}} \left(\frac{x_1 + x_o}{x_o\, x_1}\right)^{\frac{1}{2}}$.

In obtaining equation (10), it has also been assumed that x_o and $x_1 >> \delta$, and S^o is replaced by unity.

4. Some Concrete Results

The normalized field anomaly NFA, as defined above, is a complex quantity. Its real part may be defined as the normalized phase anomaly, NPA, while the imaginary part is defined as the normalized amplitude anomaly, NAA. Specifically,

$$NPA = \frac{1}{2}\left[C(Z_2) - C(Z_1) + S(Z_2) - S(Z_1)\right] , \tag{11}$$

and

$$NAA = \frac{1}{2}\left[S(Z_2) - S(Z_1) - C(Z_2) + C(Z_1)\right] , \tag{12}$$

where

$$C(Z) = \int_0^Z \cos\left(\frac{\pi}{2}t^2\right)dt, \ (13) \quad \text{and} \quad S(Z) = \int_0^Z \sin\left(\frac{\pi}{2}t^2\right)dt , \tag{14}$$

are Fresnel integrals [Born and Wolf, 1959].

The quantity NPA is a measure of the modification of the phase of signal ratio E/E^o. One should note that

$$\log E/E^o = \log |E/E^o| + i \arg (E/E^o) , \tag{15}$$

while

$$\log E \ /E^o_\infty = i \arg (E_\infty/E^o) , \tag{16}$$

since $|E \ /E^o|$ is essentially unity when the imaginary part of $S(x)$ is neglected. Thus

$$NPA \cong \frac{\arg (E/E^o)}{\arg (E_\infty/E^o)} , \tag{17}$$

and

$$NAA \cong -\frac{\log |E/E^o|}{\arg (E_\infty/E^o)} \cong \frac{1 - |E/E^o|}{\arg (E_\infty/E^o)} . \tag{18}$$

To show numerical calculations, the following special case is considered; namely, $x_o = x_1 = d/2$, which states that the perturbed region is equidistant from the two ends of the path. Also, the frequency is taken to be 20 kc/s, corresponding to a wavelength, $\lambda = 15$ km.

In figure 2, NPA and NAA are shown plotted as a function of y_1 for the case where $y_2 - y_1 = \Delta \to \infty$ and $d = x_o + x_1 = 4000$ km. It is noted that NPA asymptotically approaches unity for y_1 sufficiently negative. This limit corresponds to an ionospheric perturbation which is effectly infinite in the transverse direction. Then, as y_1 is increased, in the positive direction, NPA oscillates and at $y_1 = 0$ it becomes exactly 1/2. Further increases of y_1 reduce NPA until it asymptotically approaches zero. The latter limit corresponds to removing the ionospheric perturbation an "infinite" distance off the path.

It is interesting to note that the normalized amplitude anomaly, NAA, oscillates about zero with maximum excursions when y_1 is in the vicinity of 0. Within the approximation that $S(x, y)$ is real, conservation of energy demands that the average value of NAA, with respect to y_1, is zero.

The curves of NPA and NAA shown in figure 3 are for the same conditions as those in figure 2 except that $d = 10,000$ km. Qualitatively, the results are very similar. The main difference is that the period of the oscillations and the transition region is stretched out in the transverse direction.

Results of somewhat greater practical interest are shown in figure 4 where NPA is plotted as a function of $y_m = y_1 + (\Delta/2)$ for various values of Δ the width of the perturbed region. (See figure 1 for relevant geometry.) The wavelength is 15 km and the total range d is 4000 km. These curves are <u>even</u> about $y_m = 0$, so only positive values are shown in the abscissa. Also, in order to conserve space, the ordinates are shifted by 0.5 for each curve.

As expected, the curves in figure 4 show that NPA asymptotically approaches zero for values of y_m while it becomes approximately unity when y_m is near zero. Because of the interference from various diffracted waves, the structure of the field variations are rather complex although there is some semblance of a periodicity at larger values of y_m.

Curves in figure 5 are for the same conditions as for figure 4 except that now $d = 10,000$ km. The expected stretching of the abscissa is evident along with other changes in detail.

A set of curves showing NAA for $d = 4000$ km and 10,000 km are given in figures 6 and 7, respectively. These are for the same conditions as the corresponding NPA curves in figures 4 and 5. The NAA curves in figures 6 and 7 are even about $y_m = 0$, and at the same time, the average value of NAA is zero with respect to y_m.

5. Some Final Remarks

While the results in figures 2 to 7 inclusive are for a wavelength of 15 km, the results may be simply scaled to other wavelengths if corresponding changes are made in the other parameters. For example, in a microwave model of this situation, λ might be 3.0 cm leading to a scale factor $p = 15 \text{ km}/3 \text{ cm} = 5 \times 10^5$. All other dimensions must then be divided by p if the diffraction patterns are to be invariant. For example, in figure 5, the total distance $d = 20$ meters and the 4 values of Δ, reading from top to bottom, are 20 cm, 30 cm, 40 cm, and 60 cm.

The correction for earth curvature has been implicitly neglected in all the above discussion. The principal modification is to change the horizontal scale in figures 2 to 7 inclusive. It is not difficult to show that the numerical values of y_1 or y_m should be multiplied by the factor X where

$$X = \frac{\tan(d/2a)}{(d/2a)} , \tag{19}$$

and a is the radius of the earth. For example, this amounts to an increase of about 2 percent and 12 percent, respectively, for $d = 4000$ km and $d = 10,000$ km, respectively. Actually, X is equal to the ratio of the width of a Fresnel zone on a spherical earth to that on a flat earth. It is important to note that in a convenient microwave model such as a parallel plate waveguide, X would be unity and yet it may be used to determine diffraction effects when X is not unity.

The results given in this note, although based on relatively simple theoretical concepts, should give some insight into the behavior of the earth-ionosphere waveguide of non-uniform width. Some of the methods used in this work bear a certain resemblance to an analysis carried out recently by Crombie [1963] who argues from the viewpoint of physical optics. His final results appear to agree with the formulas derived by the author [Wait, 1963] using analytical methods, and his conclusions are compatible with the numerical data given here.

6. References

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7. Acknowledgements

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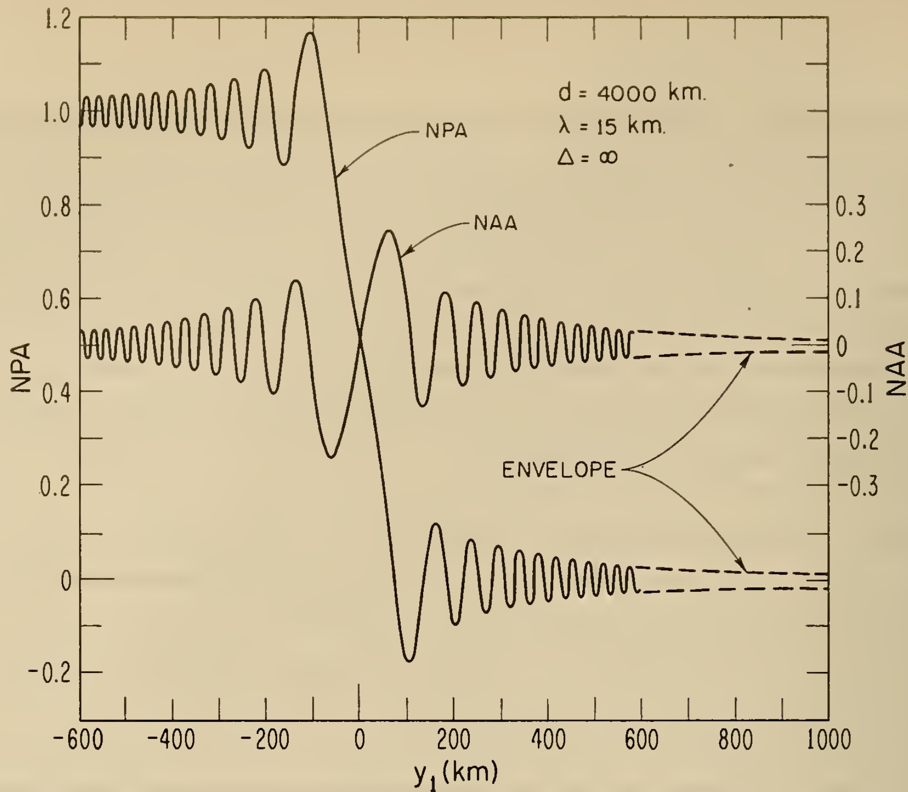

FIGURE 2 - THE NORMALIZED PHASE ANOMALY, NPA, AND THE NORMALIZED AMPLITUDE ANOMALY, NAA, AS A FUNCTION OF y_1 FOR A SEMI-INFINITE TYPE OF PERTURBATION (i.e., $\Delta = \infty$), AND d = 4000 KM

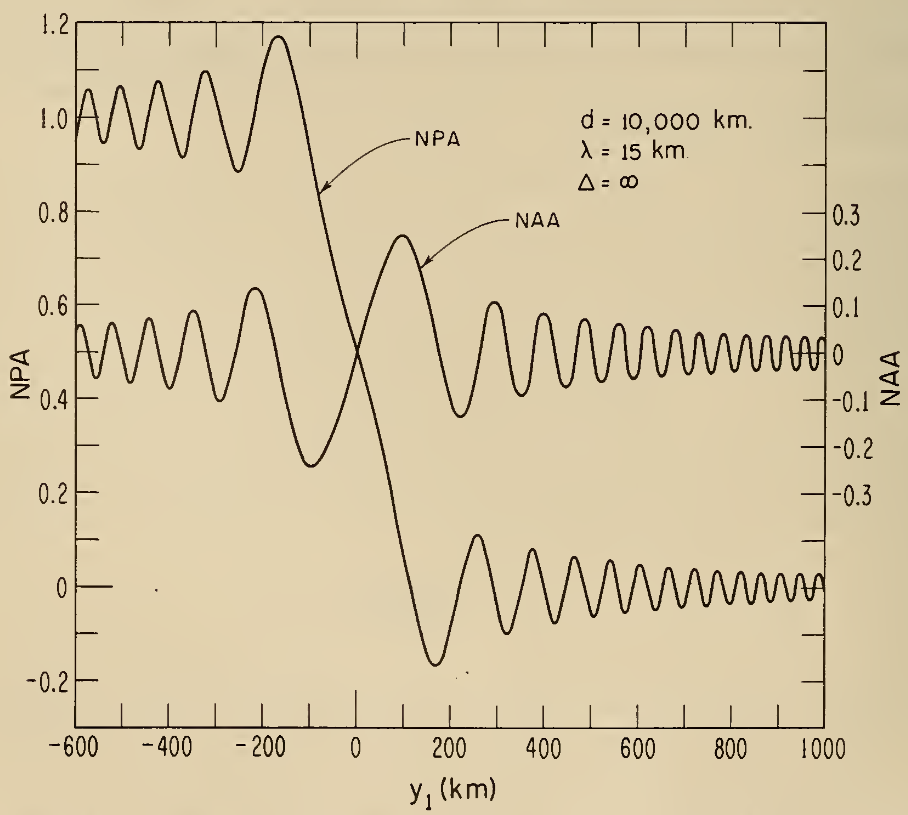

FIGURE 3 - THE NORMALIZED PHASE ANOMALY, NPA, AND THE NORMALIZED AMPLITUDE ANOMALY, NAA, AS A FUNCTION OF y_1 FOR A SEMI-INFINITE TYPE OF PERTURBATION (i.e., $\Delta = \infty$), AND d = 10,000 KM

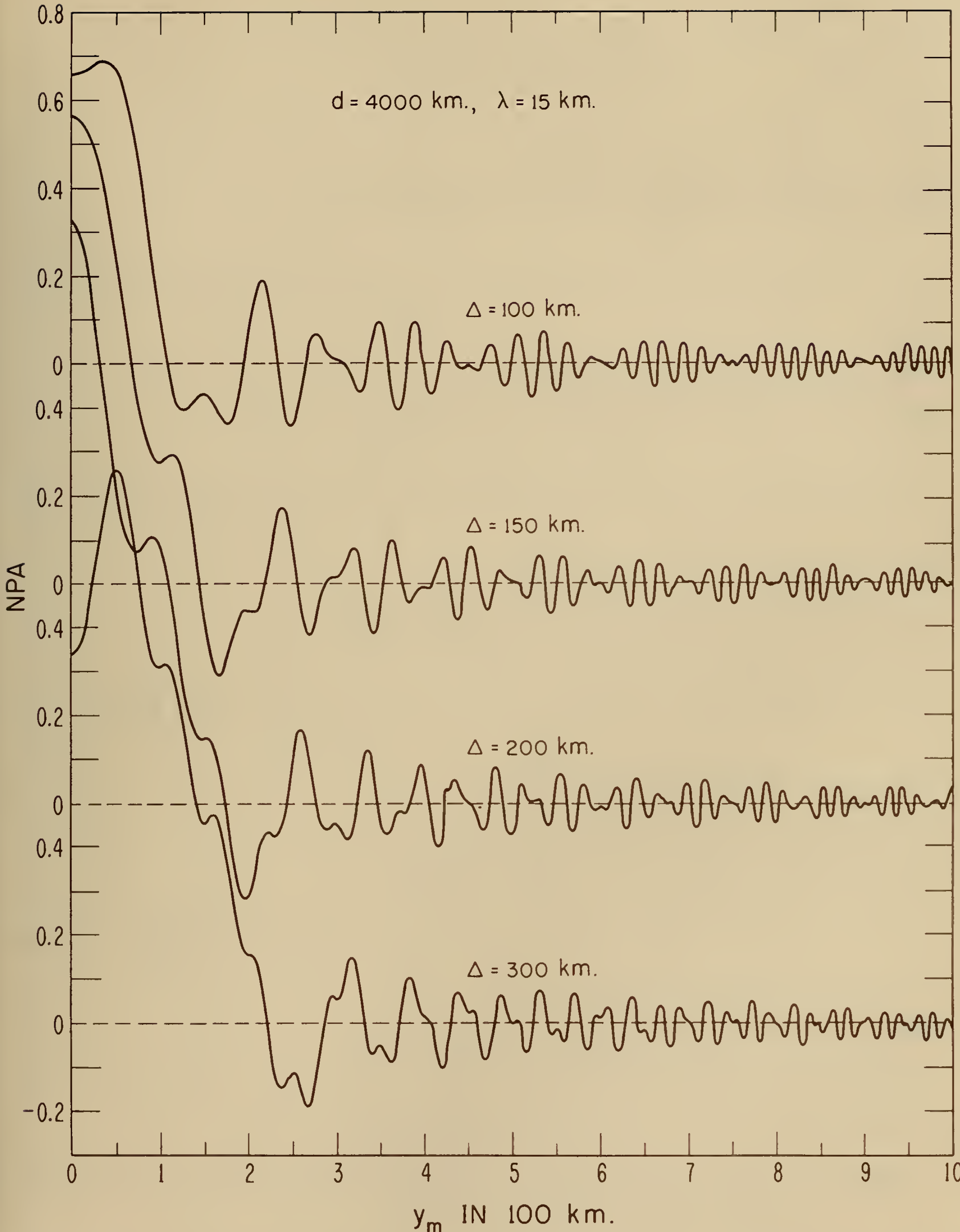

FIGURE 4 - NPA AS A FUNCTION OF y_m MEASURED TO THE CENTER OF A RECTANGULAR SHAPE DEPRESSION FOR d = 4000 KM

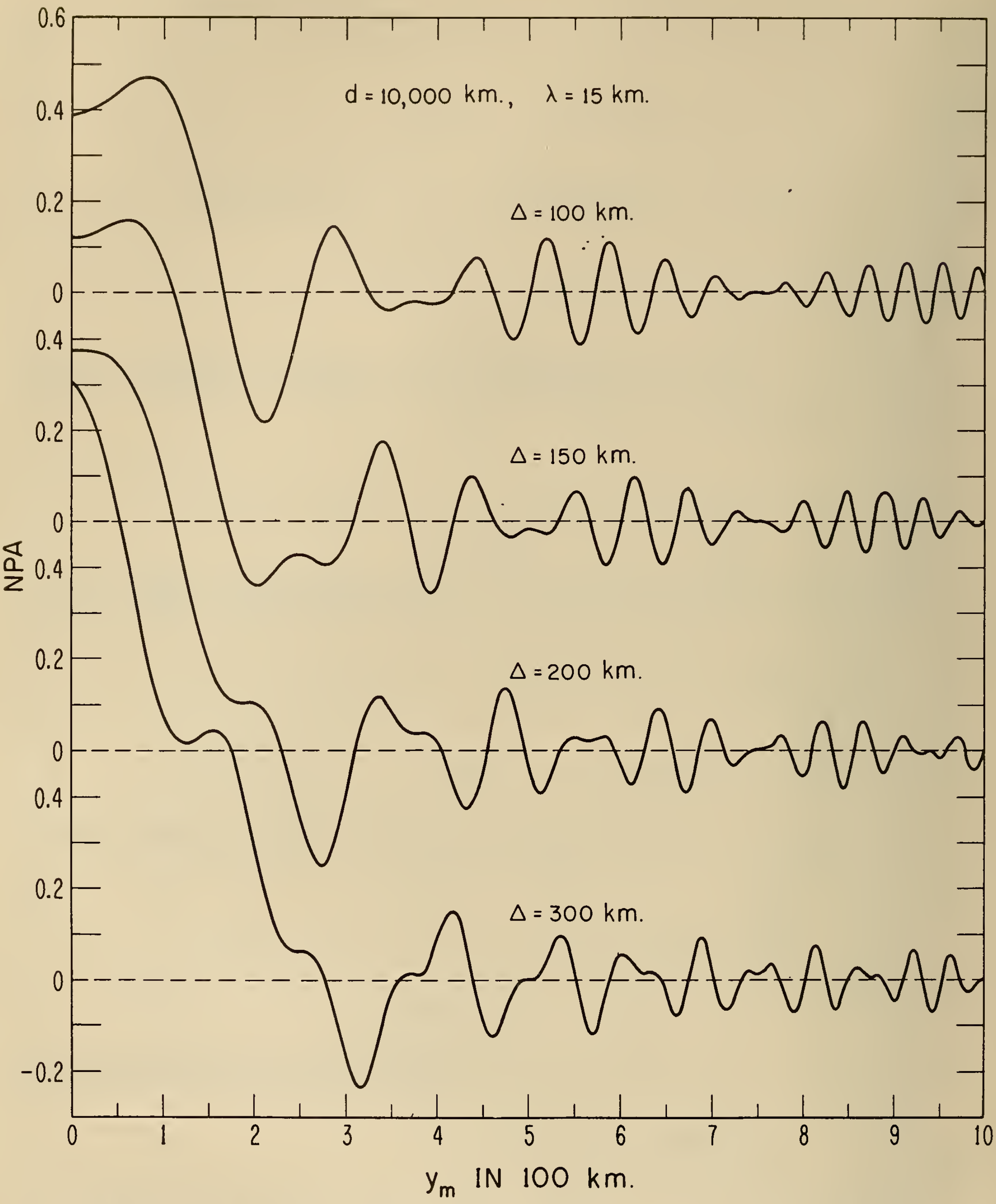

FIGURE 5 - NPA AS A FUNCTION OF y_m MEASURED TO THE CENTER OF A RECTANGULAR SHAPE DEPRESSION FOR d = 10,000 KM

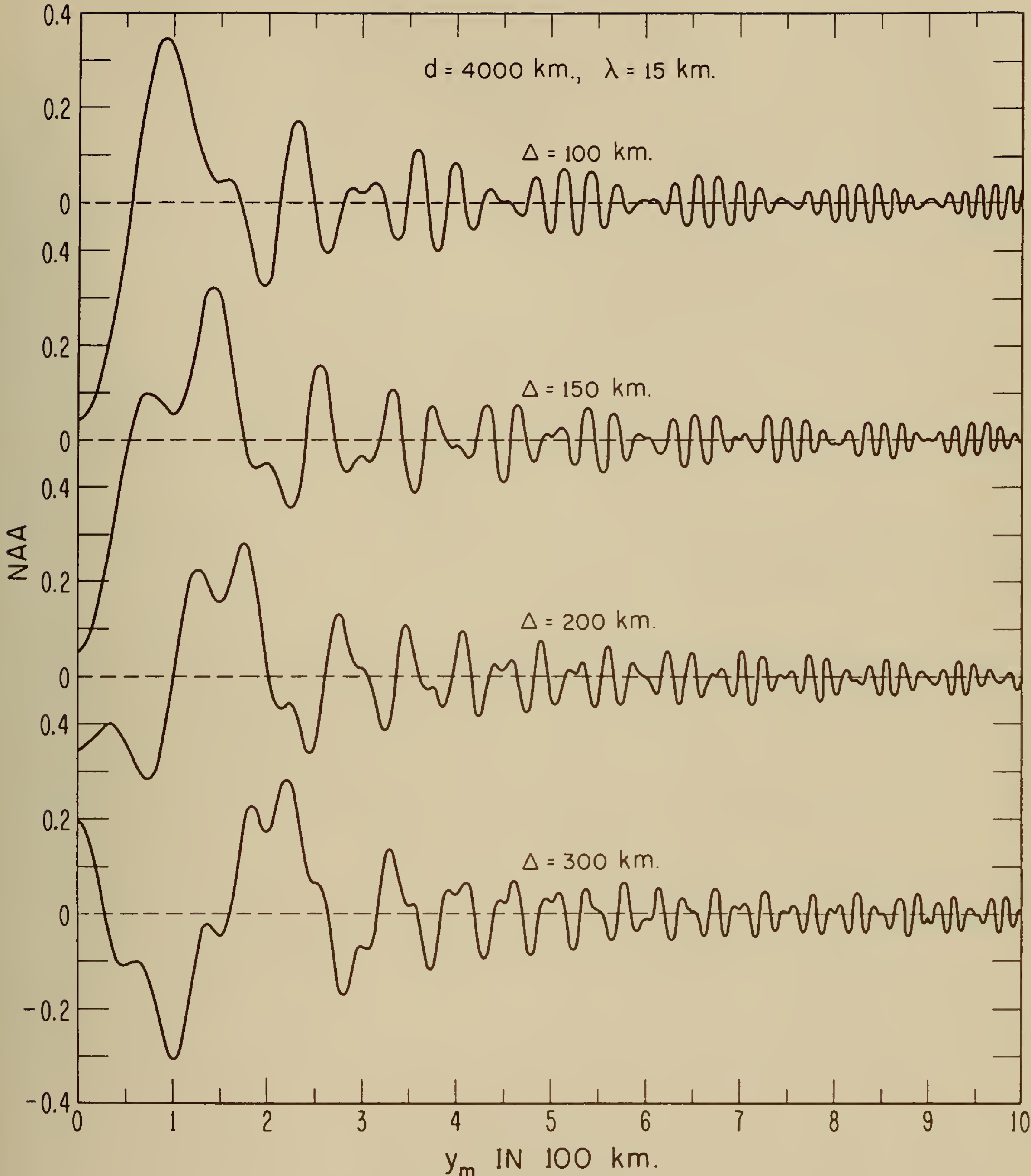

FIGURE 6 - NAA AS A FUNCTION OF y_m MEASURED TO THE CENTER OF A RECTANGULAR SHAPE DEPRESSION FOR d = 4000 KM

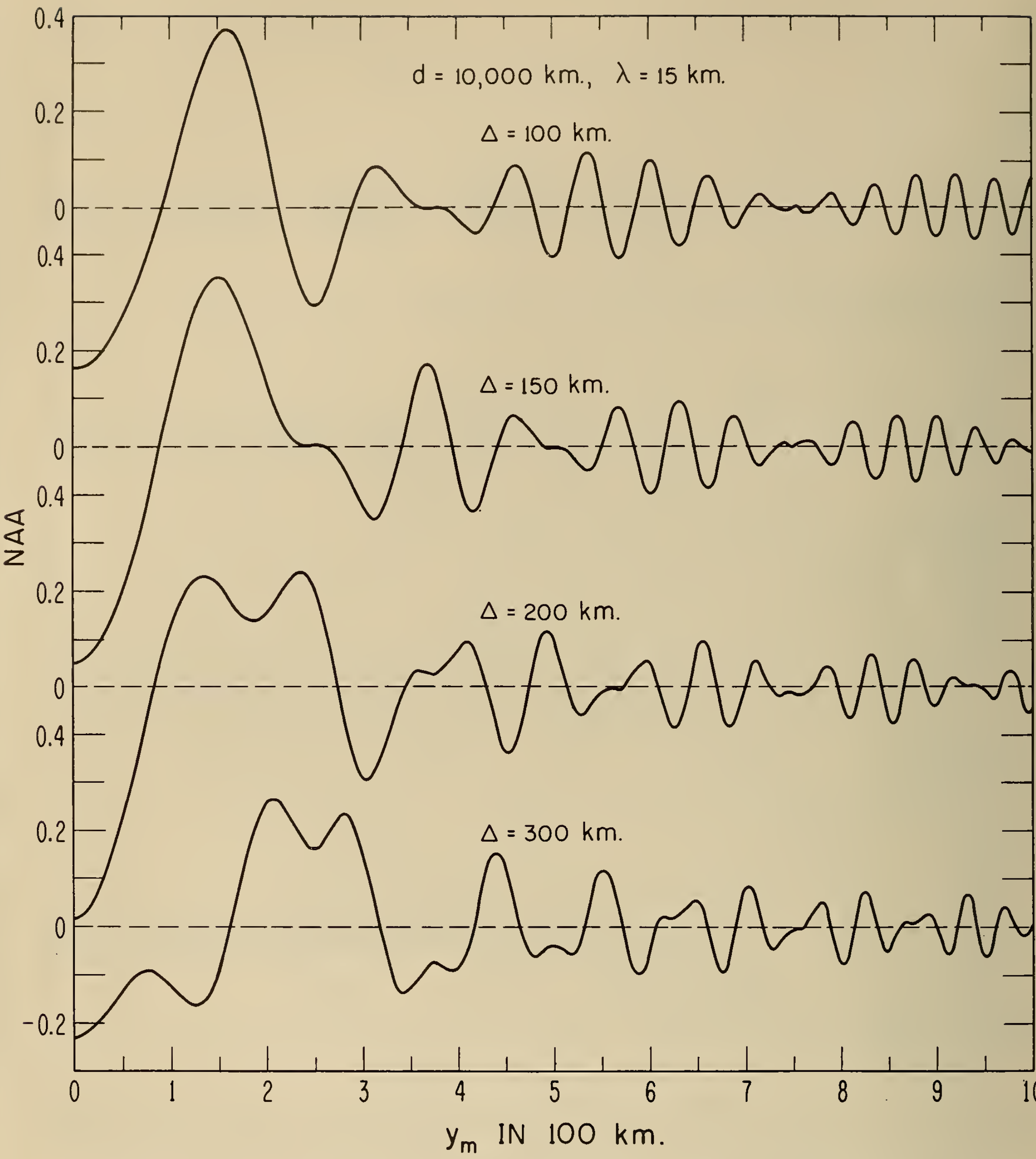

FIGURE 7 - NAA AS A FUNCTION OF y_m MEASURED TO THE CENTER OF A RECTANGULAR SHAPE DEPRESSION FOR d = 10,000 KM

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