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NUMERICAL CALCULATIONS FOR REFLECTION OF ELECTROMAGNETIC WAVES FROM A LOSSY MAGNETOPLASMA

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U. S. DEPARTMENT OF COMMERCE
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Extensive numerical results are presented for the reflection coefficient of a horizontally stratified ionized medium. The profiles of electron density and the collision frequencies are both taken to be exponential functions. The d-c magnetic field is taken to be horizontal and transverse to the direction of propagation. The specific results described are applicable to the oblique reflection of VLF radio waves in the D layer of the ionosphere for propagation along the magnetic equator. It is confirmed that the reflection coefficient is non-reciprocal in both amplitude and phase. For a wide range of the parameters, the magnitude of the reflection coefficient is greater for west-to-east propagation than for east-to-west propagation.

The extensive graphical data in the present paper are to be regarded as supplementary to the paper "Reflection of Electromagnetic Waves from a Lossy Magnetoplasma" which contained only a small sample of such calculations.

1. Introduction

The lower ionosphere is primarily responsible for the propagation of VLF radio waves to great distances. In theoretical treatments of this problem, it is often assumed that the lower edge of the ionosphere may be represented by a sharply bounded and homogeneous ionized medium. When the earth's magnetic field is included, the medium is rendered anisotropic. If the vertical inhomogeneity (or horizontal stratification) of the ionosphere is also considered simultaneously, the situation becomes very complicated indeed.

In this technical note a special case of a horizontal stratified and anisotropic ionosphere is considered. Specifically, the earth's magnetic field is assumed to be purely transverse to the direction of propagation. Strictly speaking, this is applicable only to the situation when the path of propagation is along the magnetic equator. However, the characteristics in this special case prevail at other latitudes if the transverse component of the field is appreciable. At least this is borne out by a numerical study of the sharply bounded ionosphere for an arbitrary magnetic dip angle [Johler, 1961]. In any case, the resulting simplicity of the differential equations for the limiting case of a purely transverse magnetic field encourages one to consider this situation in more detail. In particular, it is desirable to investigate the influence of gradient of both the electron density and collision frequency. In much of the previous work on this subject the collision frequency has been assumed constant.

In a study of the recent literature [e.g., Belrose, 1963], it is found that both the electron density $N(z)$ and the collision frequency $\nu(z)$ vary approximately in an exponential manner with height z. For example, in the undisturbed daytime ionosphere we may assume that

$$N(z) = N_o \exp(bz) \quad , \tag{1}$$

and

$$\nu(z) = \nu_o \exp(-az) \ , \tag{2}$$

where a and b are positive constants and z is some specified level in the ionosphere. From a study of the experimental data [Belrose, 1963], it appears that, if the reference level is 70 km above the earth's surface, $N_o \sim 10^2$ electrons/c.c. and $\nu_o \sim 10^7 \text{ sec}^{-1}$. The gradient parameters are then expected to be given approximately by $b \cong 0.15 \text{ km}^{-1}$ (± 0.1) and $a \sim 0.15 \text{ km}^{-1}$ (± 0.02). The quoted values of these constants must be considered tentative and certainly subject to change. Furthermore, it must be understood that significant departures from the exponential shape are to be expected under disturbed conditions.

2. The Model

The situation is shown explicitly in figure 1a. A vertically polarized plane wave is incident at angle θ on to a horizontally stratified ionosphere. The z axis is taken to be positive in the upward direction. At the reference level $z = 0$, the electron density and the collision frequency have values designated by N_o and ν_o, respectively. The "scale height" which is equal to $1/b$ or $1/a$, as indicated in figure 1b, is of the order of 6 km for both of these profiles.

As mentioned above, these are typical of the daytime D layer for both the N and ν profiles. For this model, the reflected wave is also vertically polarized and thus the reflection coefficient is described by a single (complex) quantity R.

The lower ionosphere, which is idealized here as a stratified ionized medium, may be regarded as an electron plasma. The (angular) electron plasma frequency ω_o is thus given by

$$\omega_o^2 = 3.18 \times 10^9 \times N , \tag{3}$$

where N is the electron density in electrons per c.c. and ω_o has dimensions of radians per second.

The continuous profiles of N(z) and $\nu(z)$ are replaced by a very large, but finite, number of steps. In other words, the inhomogeneous medium is replaced by a stack of thin homogeneous layers. For purposes of discussion, there shall be P such layers while a typical layer is the p'th layer. Thus, p ranges from 1 to P through integral values. Somewhere at a sufficiently negative value of z, the medium may be regarded as free space. This level is denoted $z = -z_o$.

The problem may now be solved by an application of non-uniform transmission line theory [Schelkunoff, 1943; Wait, 1962]. Thus, the reflection coefficient for vertically polarized waves, referred to the lower edge of the bottom slab, is given by

$$R_o = \frac{C - \Delta}{C + \Delta} , \tag{4}$$

where $\Delta = Z_1/\eta_o$ and where Z_1 is the input impedance at the bottom of layer number 1. Now, Z_1 may be expressed in terms of Z_2 which, in turn, may be expressed in terms of Z_3. The process is continued until the topmost layer is reached where Z_p is assumed known. The details of this derivation are given elsewhere [Wait, 1962; Wait and Walters, 1964].

The required number of layers is best determined by studying the stability of the solution as the number is increased. Because of the relatively long wavelength involved, and because of the finite losses in the medium, the solution converges nicely as the number of layers is increased.

A typical profile with its layer approximation is shown in figure 1c. The conductivity parameter $\frac{N(z)}{\nu(z)} = \frac{N_o}{\nu_o} \exp[(b+a)z]$ is is plotted against the vertical distance z above or below the reference level at $z = 0$. Between $z = -z_o$ and $z = T$, the upper edge of the top layer, the medium is divided into P homogeneous layers of width h_1, h_2 --- h_p --- h_{P-1}, h_P. The quantity $\frac{N(z)}{\nu(z)}$ is replaced in each layer by a constant value. The values of T and the h_p's must be arbitrarily chosen for the computations, and the selection of these constants for various given parameters λ, the free space wavelength, C, the cosine of the angle of incidence, a, and b is discussed in the appendix.

3. Presentation of Results

The final results of the numerical calculations are presented in such a fashion that the phase of the reflection coefficient R is referred to the level $z = 0$. Thus, by definition

$$R = [R_o \exp(i\,2kC\,z_o)]_{z_o \to \infty} \quad . \tag{5}$$

Physically, this means that the observer at $z = -z_o$ is sufficiently far below the ionosphere that the medium may be regarded as free space. In practice z_o is chosen to be large enough that the phase of R does not vary with further changes in z_o. For the case here, z_o was of the order of 40 km. This particular normalization of the phase has been used on previous occasions [Wait and Walters, 1963].

Following the usage in previous papers [Wait and Walters, 1963], the quantity $\omega_r = \omega_o^2(0)/\nu(0)$ is specified. In particular, $\omega/\omega_r = 1/2$ at 15 kc/s, or $\omega_r = 6\pi \times 10^4 \text{ sec}^{-1}$. The "effective" conductivity σ_e of the medium at this level $z = 0$, is then given by $\sigma_e = \epsilon_o \omega_r \sim 1.7 \times 10^{-6}$ mhos/meter. With exponential-type profiles, the fixing of the parameter of ω_r is not an essential restriction. It is a simple matter to shift the reference level from $z = 0$ to any other value if desired.

The parameters of the problem are thus λ, C, b, a, and $\Omega = \omega_T/\nu_o$ where ω_T is the (angular) gyrofrequency. In order to display the relative influence of these quantities, it is desirable to plot the amplitude and phase of R as a function of ω_T/ν_o from -3 to +3 for a range of values of λ, C, b, and a. It should be noted that λ is in km, C is dimensionless, while b and a have dimensions of km^{-1}. Consequently, the scale length in the present problem is the kilometer. By changing this scale, the results may also have significance at higher frequencies.

In figures 2a and 2b the magnitude of the reflection coefficient $|R|$ and the phase of R are plotted as a function of ω_T/ν_o. Negative values of the abscissa correspond to propagation from west to east along the magnetic equator. The cosine of the angle of incidence is fixed at 0.1. Thus, the angle is highly oblique, being only 5.7° from grazing.

For long-distance propagation of VLF radio waves, such highly oblique conditions prevail.† For the curves in figures 2a and 2b, the collision profile is chosen so that the collision parameter $a = 0.15\ \text{km}^{-1}$ and the wavelength $\lambda = 15$ km correspond to a frequency of 20 kc/s. For these curves, the electron density parameter b takes the values in the range from 0.1 to $0.5\ \text{km}^{-1}$. It is evident that for $\omega_T/\nu_o \cong 0$, the steep gradient of electron density is associated with maximum amplitude of reflection. However, when ω_T/ν_o is finite, this may no longer be the case. In fact, the asymmetry of the curves about $\omega_T/\nu_o = 0$ is a measure of non-reciprocity in the reflection process. As indicated, the reflection coefficient for propagation from west to east is greater than for propagation from east to west. There is also some non-reciprocity in the phase curves, but it is not great. A very similar set of curves is given in figures 3a and 3b where the conditions are the same except that $C = 0.2$.

In figures 4a, 4b, 5a, and 5b, a set of curves shows the influence of varying the collision frequency parameter while keeping the electron density parameter fixed at $b = 0.15\ \text{km}^{-1}$. For these curves, as before, $C = 0.1$ and 0.2, and $\lambda = 15$ km. It is evident that the steeper gradient of the collision frequency corresponds to larger reflection coefficients.

In figures 6a to 8b, $|R|$ and the phase of R are shown as a function of ω_T/ν_o for various frequencies in the range from 6 to 100 kc/s. For the curves in figures 6a and 6b, $C = 0.1$, $b = 0.15$, and $a = 0.15$ while, in figures 7a and 7b, and figures 8a and 8b,

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† In fact, the attenuation of the dominant mode in the earth-ionosphere waveguide at VLF is approximately proportional to $1 - |R|$ for highly oblique incidence [Wait, 1962].

$C = 0.2$ and 0.3, respectively. The tendency is for the reflection coefficient to be diminished at the higher frequencies or shorter wavelengths. In this case, the medium is acting like a good absorber rather than a reflector. It is to be noted that at the steeper angles of incidence (i. e., $C \cong 0.3$), there are some complicated phenomena which are probably related to internal reflections within the medium.

In figures 9a and 9b, $|R|$ and the phase of R are plotted for different values of the angle of incidence. For these curves, $\lambda = 15$ km, $b = 0.15 \text{ km}^{-1}$, and $a = 0.15 \text{ km}^{-1}$. In general, it may be seen that the reflection coefficient is diminished for the steeper angle of incidence. It is rather interesting to note that the asymmetry (or non-reciprocity) in the phase curves is more pronounced at the steeper angles.

Finally, in figures 10a and 10b, $|R|$ and the phase of R are plotted for various values of the collision frequency parameter a for $\lambda = 15$ km and $C = 0.1$. These curves differ from figures 4a and 4b in that here the parameter $\beta = b + a$ is fixed, rather than just a. In other words, the profile of N/ν as a function of z is fixed while the gradient of ν is changed. In the isotropic case, where $\omega_T = 0$, it is interesting to note that R is determined only by the gradient of N/ν. However, for a finite gyrofrequency, the situation is changed significantly. Similar curves are shown in figures 11a and 11b where $C = 0.2$. In general, the non-reciprocity is accentuated when a is diminished. For example, if ν were assumed to be a constant, the dependence of the gyrofrequency is much greater than for a collision frequency which varies with height. In much of the earlier work [e. g, Budden, 1955] on full wave solutions in ionospheric radio waves, it is often assumed that ν can be regarded as a constant. Clearly, such an assumption may lead to very misleading results.

4. Discussion and Concluding Remarks

The numerical results given here should provide some insight into the nature of reflection from an inhomogeneous ionized medium. The nature of the dependence on N, ν, and ω_T is quite complicated. Nevertheless, it appears that the sharper gradients of electron density are usually associated with higher reflection coefficients. The dependence on the collision frequency profile is not so clear-cut.

In nearly every case it may be seen that the presence of the transverse magnetic field causes the reflection coefficient $|R|$ to be non-reciprocal. Furthermore, for a wide range of the parameters, $|R|$ is greater for west-to-east propagation than for east-to-west propagation. This is in accord with experimental data of Round,et al., [1925] who observed that, for propagation over distances of the order of 6000 km, signals from VLF transmitters to the west are received more strongly than from those to the east. This observation has also been confirmed by Crombie [1958] in a series of field strength measurements in New Zealand and by Taylor [1960] who analyzed the wave-forms of atmospherics.

It is interesting to note that Budden [1955] deduces, from a full wave solution, that the directional dependence is just opposite to this. Although his model is not the same as the one considered here, it is difficult for the authors to accept the validity of his results in this regard. However, it is possible that, because of the complexity of the various phenomena, a reversed trend may emerge for certain special conditions, particularly for the nighttime ionosphere [Rhoads, et al., 1963]. It is also worth mentioning that Budden [1955] has some qualms concerning the accuracy of his numerical data at small values of C.

5. Appendix

The principal object of the calculations is to find Z_1, the normal wave impedance at the lower edge of the bottom layer. The reflection coefficient R_v, for vertical polarization, is then obtained from (4). The numerical method used is briefly outlined here.

The impedance Z_{p-1} is dependent upon Z_p, the impedance of the layer p above it. Therefore, Z_1 is dependent upon all of the Z's of the layers above the first. Because of this property, an iterative process, using a digital computer is appropriate for the computations. In such a process a good starting value of Z is desirable. Then the impedance Z for the layer next to the top one may be expressed in terms of the wave impedances for the upgoing and downgoing waves. These wave impedances can be calculated at once as they are functions of given parameters. Thus, with a starting value of Z available, the impedance for each successive layer down through the first can be found [Wait, 1962].

In these calculations, the quantities T and h_p must be chosen correctly (see figure 1c). The distance T should be chosen such that, if it is increased, the value of the reflection coefficient is unchanged. From economic considerations, however, this distance should be as small as possible consistent with accurate results. Furthermore, the individual layers of thickness h_p must be small enough to well approximate the given conductivity profile. Thus, small intervals are chosen near the top of the medium where the conductivity profile is rapidly varying, while h_p is gradually increased at the bottom of the profile. Again, for the sake of economy, the h_p's should be chosen as large as possible consistent with the required accuracy.

It is also necessary to determine the distance from the reference level to the lower edge of the bottom layer, $z = -z_o$. The lowest layer must be far enough down in the medium to effect convergence. An answer "converges" to the value R at a distance $z = -z_o$ if decreasing z_o leaves R unchanged to the required number of digits. The value of R at this distance is considered the "free space" value of R to the accuracy desired.

When the earth's magnetic field is neglected corresponding to $\Omega = 0$, the problem reduces to the one considered previously by the authors [Wait and Walters, 1963], using the same exponential profile. Because the formulation differed in the two papers, a check was possible when $\Omega = 0$. The results from both types of computation agreed to within five digits wherever they could be compared.

To develop a satisfactory procedure to find R_v using the layer method, the problem under the isotropic assumption (i. e., $\Omega = 0$), was first programmed for horizontal polarization. Because R_h, the reflection coefficient for horizontal polarization, may be expressed in closed form $|R_h| = \exp\left(-\frac{2\pi^2 C}{\lambda \beta}\right)$, it was possible to check the values of R_h obtained by the layer method. Then the values of T and the h_p's were adjusted until the answers calculated by the two methods agreed to five digits.

The program was then written using the step-by-step process for vertical polarization. An important check here utilized the fact that for $C = 1$, the values of R are identical for vertical and horizontal polarization. Extreme values were used for the various given parameters and the paths printed out step by step for vertical and horizontal polarization. By studying these print-outs, it was found that the variation of C, with other given parameters held constant did not greatly alter the choice of the arbitrary constants necessary to effect convergence to the correct value. Thus, the comparison of the case for $C = 1$ with the closed form was used as a criterion to determine T and the h_p's for all the C's. If the value of R_v, using $T = 4$ km, agreed for $C = 1$ to four digits with the closed form answer, this was considered satisfactory. If it did not agree, T was set equal to 10 km and finer intervals of h_p were used. For $T = 10$ km, the values agreed in every instance with the closed form for $C = 1$ to five digits. As a further check, especially for $C \neq 1$, for a few cases, T was set equal to 20 km, and these answers agreed in every instance to five digits with those calculated using $T = 10$ km.

When $\Omega \neq 0$, it was found that the convergence of R_v was at least as rapid as when $\Omega = 0$. Thus, as a starting point, the same choice of T and the h_p's was used for the anisotropic cases. Following is a small table showing the distance z_o required for convergence of R_v to five digits for a typical set of parameters for $\lambda = 15$ km with $\Omega = 0$:

	$C = 0.1$	$C = 0.2$	$C = 1$
β	z_o (km)	z_o (km)	z_o (km)
0.3	50-55	40-45	40-45
0.5	30-35	25-30	20-25
1	10-15	10-15	-
2	5-6	4-5	-

From these results, it was found that if β and λ are constant, the variation of C does not modify the required value of z_o greatly. Because the medium is slowly varying near $-z_o$, large intervals of h_p, requiring few steps, may be used in this region. Thus, if only C of the given parameters varies, it is feasible to use the estimate of the distance z_o suitable for the smallest C for all the C's. Similarly, if only the wavelength is allowed to vary, there is little effect on the required value of z_o. For $\beta = 0.3$ and $C = 0.1$, between $\lambda = 10$ km and $\lambda = 25$ km, the value of z_o varies from about 50 to 55 km, a distance of only five kilometers for the whole range.

As might be expected of the given parameters, the value of β, the gradient of the conductivity change, has the most critical effect on the choice of the arbitrary constants T and the h_p's. For $\beta > 0.3$ and $T = 4$, the initial h_p's were chosen 0.02 km and gradually increased to 0.5 km, corresponding to a total of about 200 layers. The manner of transition of the size of the layers through the medium is not particularly critical. Several different combinations of numbers of various step sizes were tried with the same final results. For $\beta \leqq 0.3$ and $T = 10$ km, the initial h_p's were 0.005 km with steps increasing to 0.5 km, corresponding to about 1300 layers. The choice of the arbitrarily chosen parameters T and the h_p's was probably on the conservative side in some cases, but as the computation was very fast, and accuracy was guaranteed, this seemed a worthwhile procedure.

The general technique described above was also used for perturbed conductivity profiles superimposed on the exponential conductivity profile. There was a provision in these cases for taking smaller intervals of h_p when the effect of the perturbation was encountered and continuing with these until the exponential profile became dominant again [Wait and Walters, 1963]. This technique can be applied to various types of profiles in addition to those referred to with satisfactory results.

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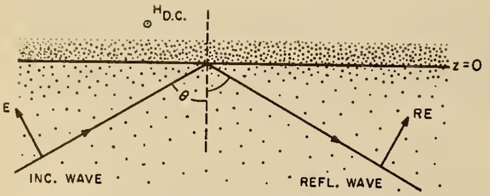

FIGURE 1a - ILLUSTRATING OBLIQUE REFLECTION OF VLF RADIO WAVES FROM AN INHOMOGENEOUS (HORIZONTALLY STRATIFIED) IONOSPHERE WITH A TRANSVERSE D-C MAGNETIC FIELD.

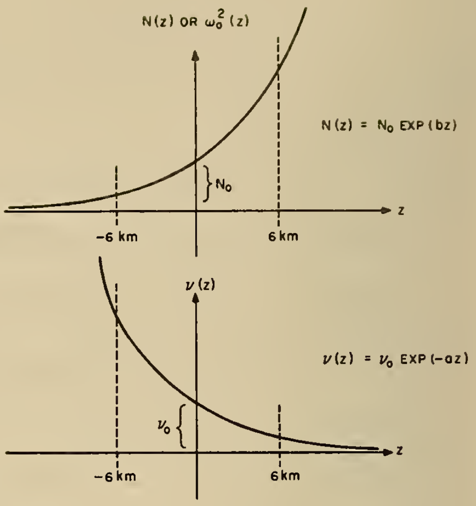

FIGURE 1b - SKETCH OF THE PROFILES OF ELECTRON DENSITY N(z) AND THE COLLISION FREQUENCY ν(z).

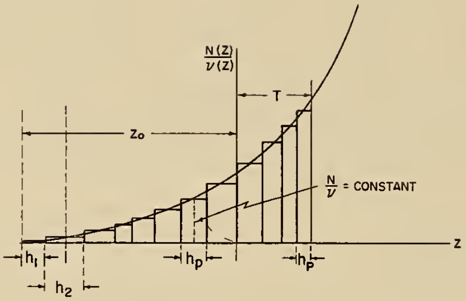

FIGURE 1c - THE STEP APPROXIMATION TO AN EXPONENTIAL PROFILE.

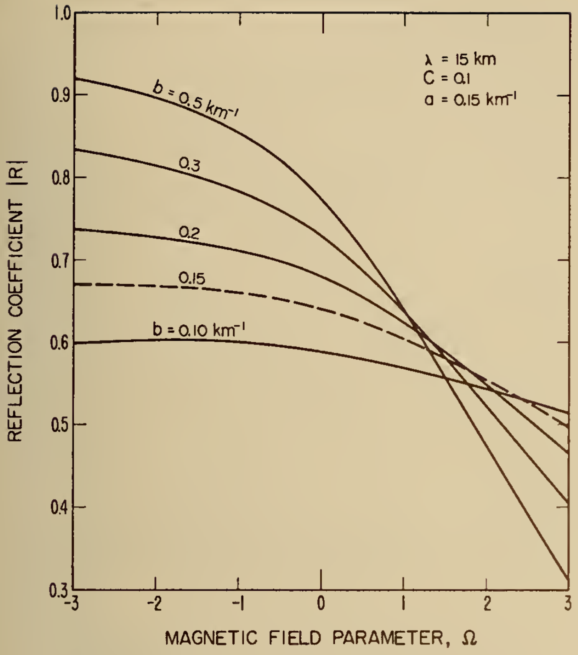

FIGURE 2a - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON ELECTRON DENSITY PROFILE FOR C = 0.1

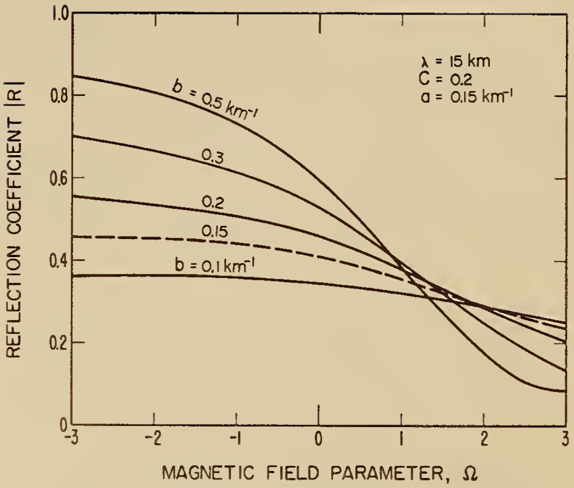

FIGURE 3a - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON ELECTRON DENSITY PROFILE FOR C = 0.2

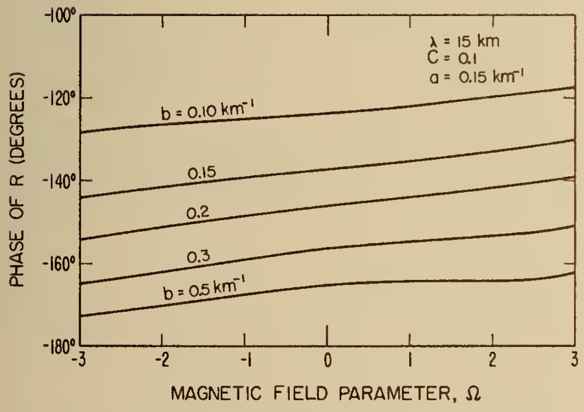

FIGURE 2b - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON ELECTRON DENSITY PROFILE FOR C = 0.1

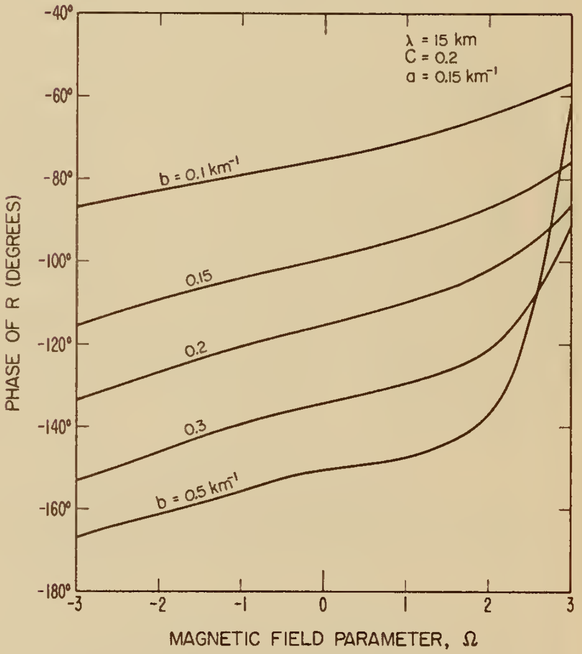

FIGURE 3b - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON ELECTRON DENSITY PROFILE FOR C = 0.2

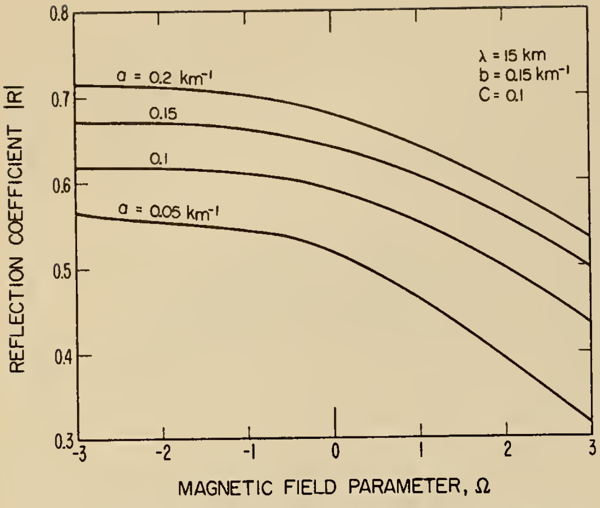

FIGURE 4a - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON COLLISION FREQUENCY PROFILE FOR C = 0.1

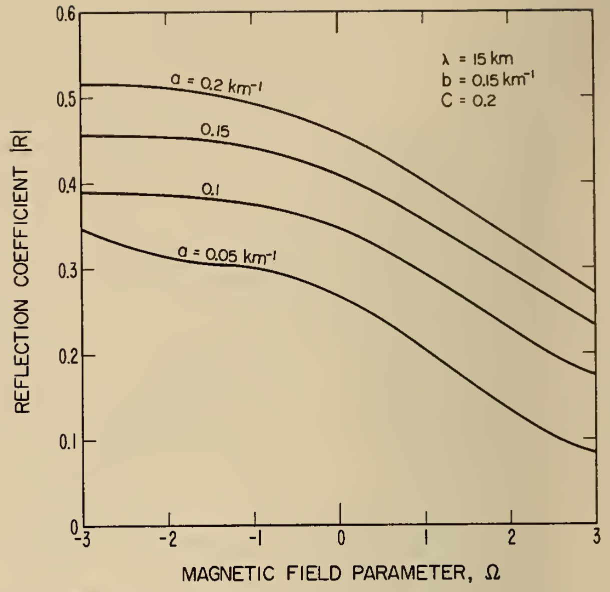

FIGURE 5a - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON COLLISION FREQUENCY PROFILE FOR C = 0.2

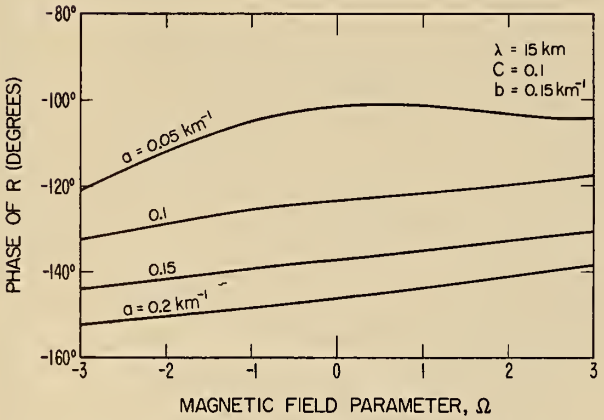

FIGURE 4b - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON COLLISION FREQUENCY PROFILE FOR C = 0.1

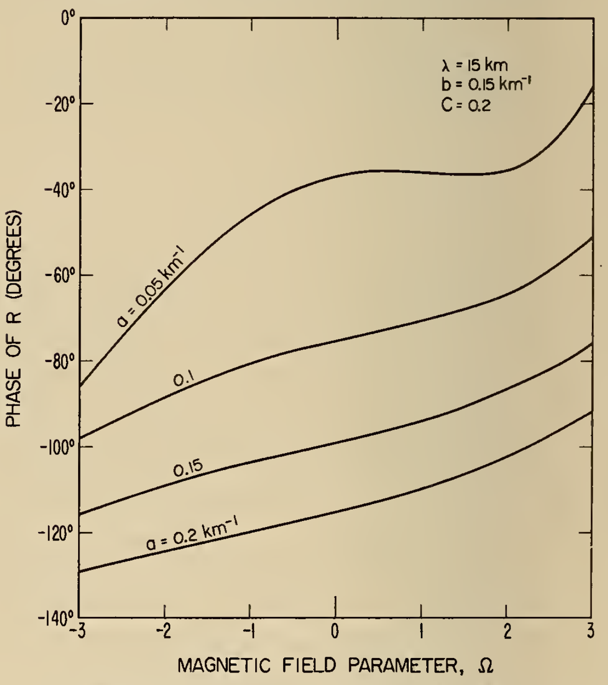

FIGURE 5b - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON COLLISION FREQUENCY PROFILE FOR C = 0.2

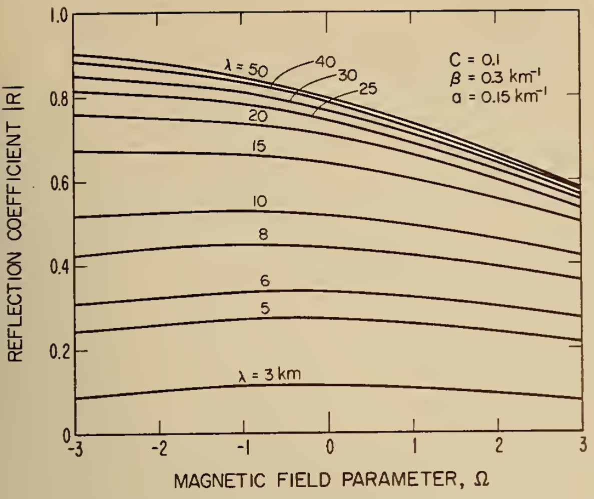

FIGURE 6a - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON WAVELENGTH FOR C = 0.1

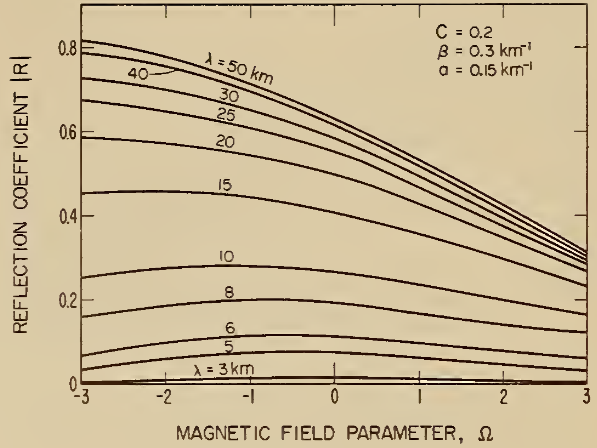

FIGURE 7a - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON WAVELENGTH FOR C = 0.2

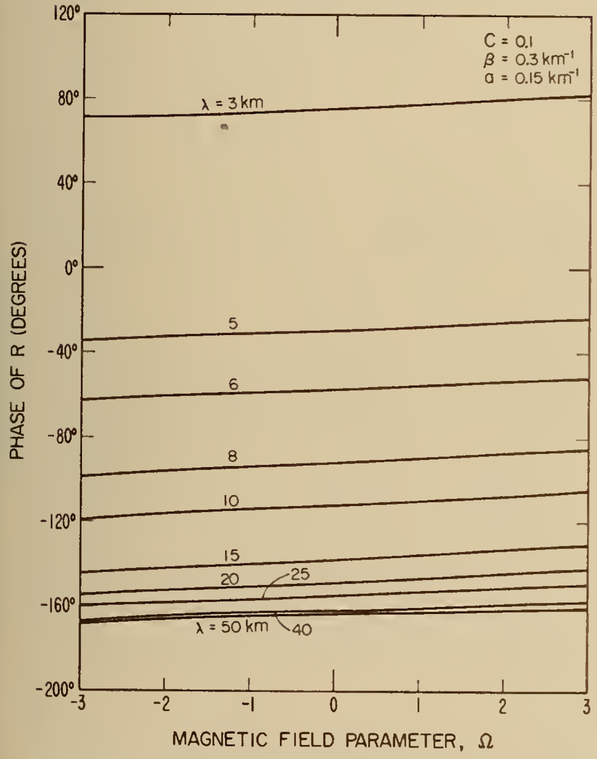

FIGURE 6b - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON WAVELENGTH FOR C = 0.1

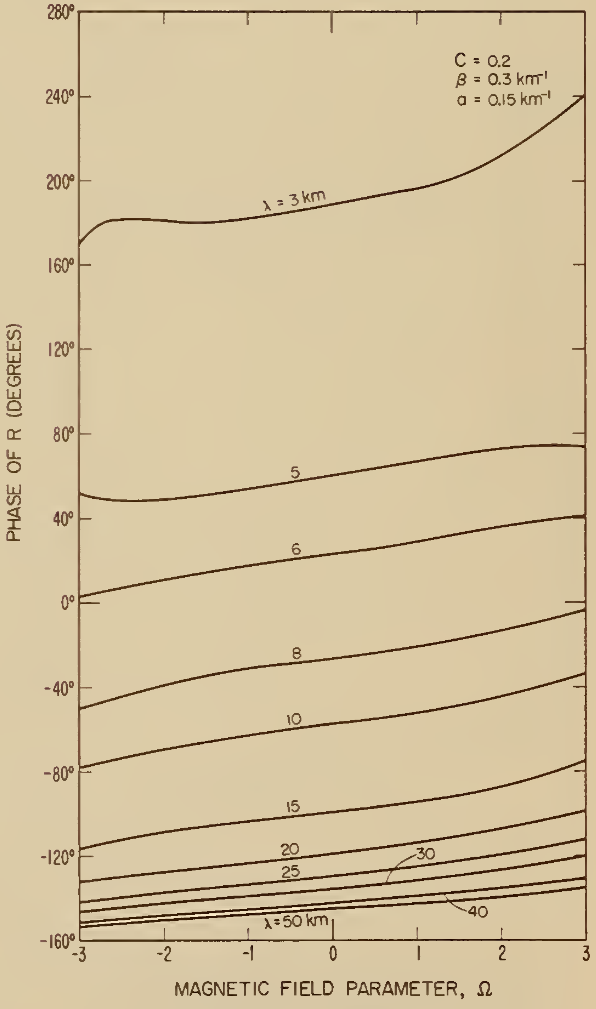

FIGURE 7b - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON WAVELENGTH FOR C = 0.2

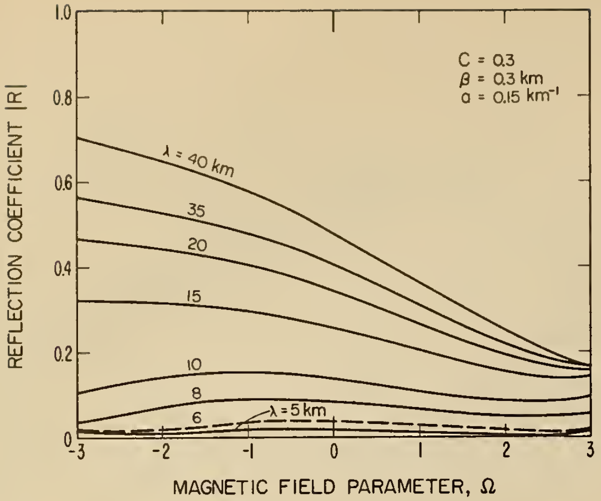

FIGURE 8a - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON WAVELENGTH FOR C = 0.3

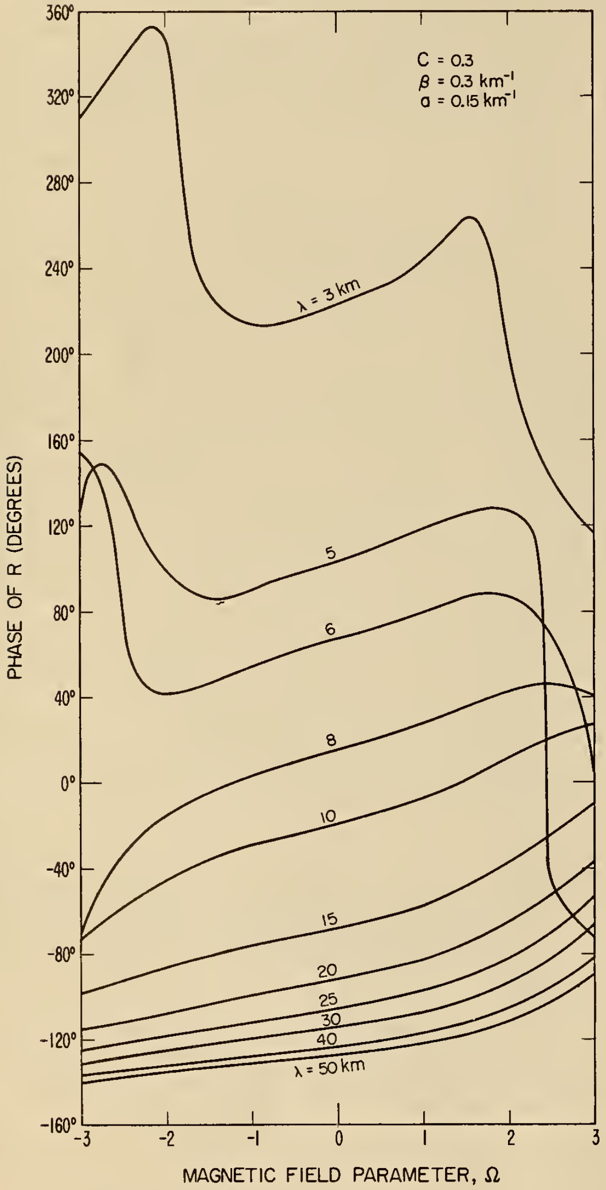

FIGURE 8b - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON WAVELENGTH FOR C = 0.3

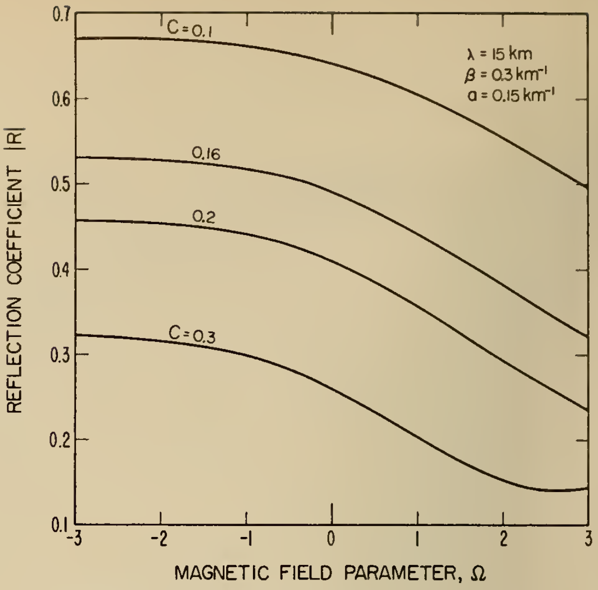

FIGURE 9a - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON ANGLE OF INCIDENCE.

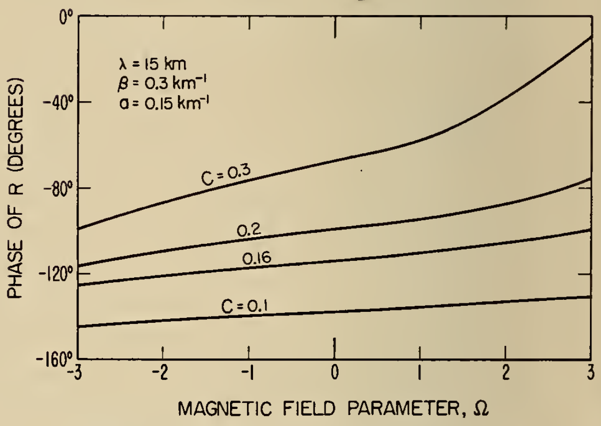

FIGURE 9b - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON ANGLE OF INCIDENCE.

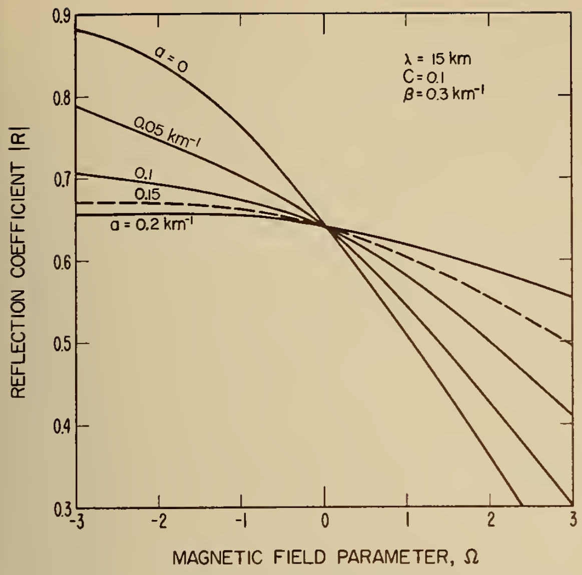

FIGURE 10a - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON COLLISION FREQUENCY PROFILE WHEN THE N/ν PROFILE IS FIXED FOR C = 0.1

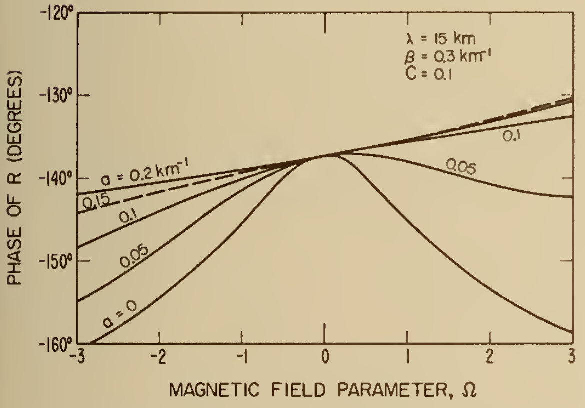

FIGURE 10b - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON COLLISION FREQUENCY PROFILE WHEN THE N/ν PROFILE IS FIXED FOR C = 0.1

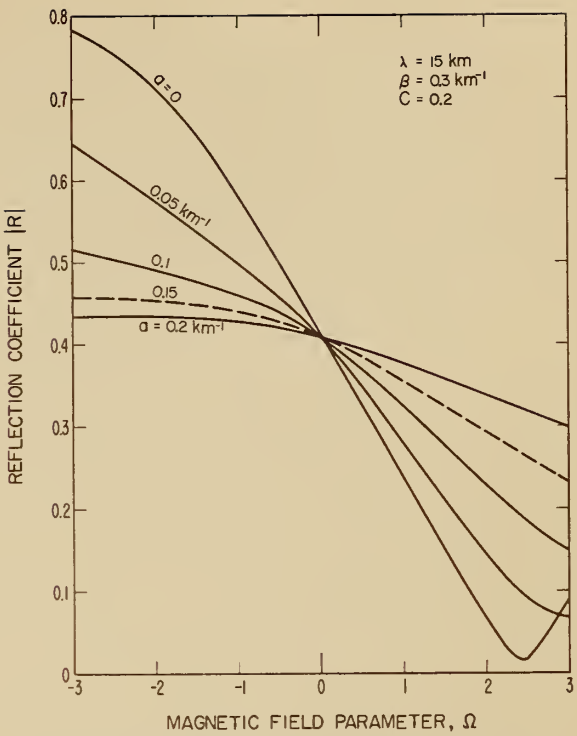

FIGURE 11a - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON COLLISION FREQUENCY PROFILE WHEN THE N/ν PROFILE IS FIXED FOR C = 0.2

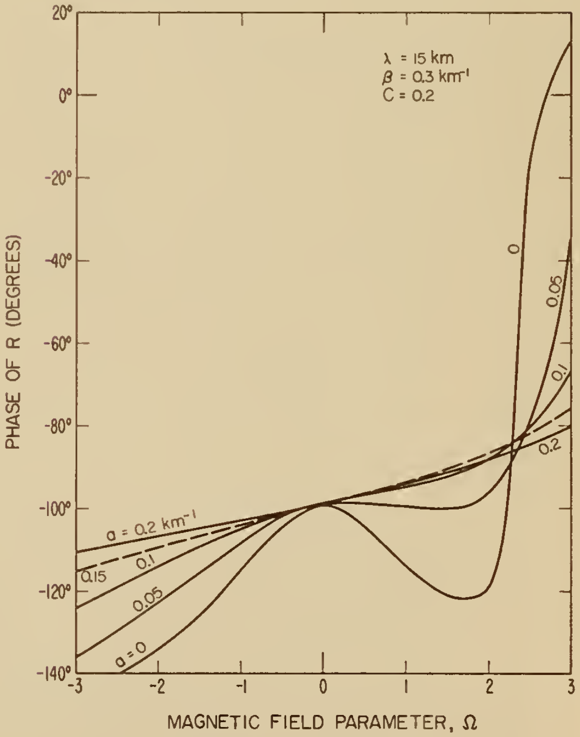

FIGURE 11b - REFLECTION COEFFICIENT CURVES ILLUSTRATING THE DEPENDENCE ON COLLISION FREQUENCY PROFILE WHEN THE N/ν PROFILE IS FIXED FOR C = 0.2

GPO 837-445

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The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

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Electricity. Resistance and Reactance. Electrochemistry. Electrical Instruments. Magnetic Measurements. Dielectrics. High Voltage. Absolute Electrical Measurements.

Metrology. Photometry and Colorimetry. Refractometry. Photographic Research. Length. Engineering Metrology. Mass and Volume.

Heat. Temperature Physics. Heat Measurements. Cryogenic Physics. Equation of State. Statistical Physics.

Radiation Physics. X-ray. Radioactivity. Radiation Theory. High Energy Radiation. Radiological Equipment. Nucleonic Instrumentation. Neutron Physics.

Analytical and Inorganic Chemistry. Pure Substances. Spectrochemistry. Solution Chemistry. Standard Reference Materials. Applied Analytical Research. Crystal Chemistry.

Mechanics. Sound. Pressure and Vacuum. Fluid Mechanics. Engineering Mechanics. Rheology. Combustion Controls.

Polymers. Macromolecules: Synthesis and Structure. Polymer Chemistry. Polymer Physics. Polymer Characterization. Polymer Evaluation and Testing. Applied Polymer Standards and Research. Dental Research.

Metallurgy. Engineering Metallurgy. Metal Reactions. Metal Physics. Electrolysis and Metal Deposition.

Inorganic Solids. Engineering Ceramics. Glass. Solid State Chemistry. Crystal Growth. Physical Properties. Crystallography.

Building Research. Structural Engineering. Fire Research. Mechanical Systems. Organic Building Materials. Codes and Safety Standards. Heat Transfer. Inorganic Building Materials. Metallic Building Materials.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics. Operations Research.

Data Processing Systems. Components and Techniques. Computer Technology. Measurements Automation. Engineering Applications. Systems Analysis.

Atomic Physics. Spectroscopy. Infrared Spectroscopy. Far Ultraviolet Physics. Solid State Physics. Electron Physics. Atomic Physics. Plasma Spectroscopy.

Instrumentation. Engineering Electronics. Electron Devices. Electronic Instrumentation. Mechanical Instruments. Basic Instrumentation.

Physical Chemistry. Thermochemistry. Surface Chemistry. Organic Chemistry. Molecular Spectroscopy. Elementary Processes. Mass Spectrometry. Photochemistry and Radiation Chemistry.

Office of Weights and Measures.

BOULDER, COLO.

CRYOGENIC ENGINEERING LABORATORY

Cryogenic Processes. Cryogenic Properties of Solids. Cryogenic Technical Services. Properties of Cryogenic Fluids.

CENTRAL RADIO PROPAGATION LABORATORY

Ionosphere Research and Propagation. Low Frequency and Very Low Frequency Research. Ionosphere Research. Prediction Services. Sun-Earth Relationships. Field Engineering. Radio Warning Services. Vertical Soundings Research.

Troposphere and Space Telecommunications. Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Spectrum Utilization Research. Radio-Meteorology. Lower Atmosphere Physics.

Radio Systems. Applied Electromagnetic Theory. High Frequency and Very High Frequency Research. Frequency Utilization. Modulation Research. Antenna Research. Radiodetermination.

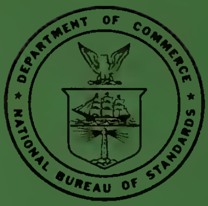
Upper Atmosphere and Space Physics. Upper Atmosphere and Plasma Physics. High Latitude Ionosphere Physics. Ionosphere and Exosphere Scatter. Airglow and Aurora. Ionospheric Radio Astronomy.

RADIO STANDARDS LABORATORY

Radio Standards Physics. Frequency and Time Disseminations. Radio and Microwave Materials. Atomic Frequency and Time-Interval Standards. Radio Plasma. Microwave Physics.

Radio Standards Engineering. High Frequency Electrical Standards. High Frequency Calibration Services. High Frequency Impedance Standards. Microwave Calibration Services. Microwave Circuit Standards. Low Frequency Calibration Services.

Joint Institute for Laboratory Astrophysics - NBS Group (Univ. of Colo.).



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