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Technical Note 187

TRANSMISSION AND REFLECTION OF ELECTRONS BY ALUMINUM FOILS

MARTIN J. BERGER

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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Transmission and Reflection of Electrons by Aluminum Foils

Martin J. Berger

Electron transmission and reflection coefficients for aluminum foils, pertaining both to number and energy, are given for source energies between 0.125 and 2.0 Mev, source obliquities ranging from perpendicular to grazing incidence, and foil thicknesses up to one half of the electron mean range. The results were obtained by a Monte Carlo calculation.

1. Introduction

In the course of an investigation of the Monte Carlo method in application to the transport of fast charged particles, the author has had occasion to compute reflection and transmission coefficients for electrons incident on aluminum foils. These results were obtained in preliminary model studies; they disregard energy-loss straggling, and take into account only the deviation of the electron path from a straight line due to multiple Coulomb scattering.

For foils of small and intermediate thicknesses this approximation is actually quite good. Nevertheless, on account of the preliminary nature of the results, only some of them have been included for publication in [1].[1] It has been pointed out to the author that there is some interest in transmission and reflection data in connection with the shielding of space-vehicles against electrons encountered in the Van Allen belt. There is scarcity of such data, experimental or calculated, and the coverage of the preliminary results is greater than that available from other sources, and includes not only number - but also energy - reflection and transmission coefficients, and their dependence on direction of the incident electron beam as well as on its energy. Accordingly, these results are presented here, with the proviso that they are subject to future revision. Further work of this type,

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[1] Figures in brackets indicate literature references at the end of this paper.

including the effect of energy-loss straggling and the associated bremsstrahlung emitted by the electrons, is currently in progress at the National Bureau of Standards.

2. Calculation

A beam of monoenergetic electrons, with energy E_o, is assumed to be incident in direction θ_o on a plane-parallel foil of aluminum of thickness z. The angle of incidence, θ_o, is the angle between the direction of incidence and the normal to the foil. Thus, the value $\theta_o = 0^o$ corresponds to perpendicular incidence, and the value $\theta_o = 90^o$ to the limiting case of grazing incidence. The azimuth of the direction of incidence does not enter into the calculation.

The following quantities are computed:

1. Number reflection coefficient,

$$R_N = \frac{\text{number of electrons reflected}}{\text{number of electrons incident}}$$

2. Number transmission coefficient,

$$T_N = \frac{\text{number of electrons transmitted}}{\text{number of electrons incident}}$$

3. Energy reflection coefficient,

$$R_E = \frac{\text{energy reflected}}{\text{energy incident}}$$

4. Energy transmission coefficient,

$$T_E = \frac{\text{energy transmitted}}{\text{energy incident}}$$

The calculation of these quantities is achieved by simulating the actual physical multiple scattering process by an (artificially constructed) random walk, each step of which takes into account the combined effect of many successive Coulomb scatterings. The transition probabilities for this random walk are obtained from the Bethe theory of electron stopping power, and the Molière theory of multiple scattering. A detailed description of the procedures used can be found in [1].*

3. Results

It is useful to present reflection and transmission coefficients as functions of the "reduced foil thickness" z/r_0, where r_0 is the mean range of an electron of energy E_0, i.e., the average rectified pathlength it would travel in an unbounded medium from the time it has energy E_0 until it comes to a stop. The reason for the introduction of the reduced thickness is a scaling law, suggested by theoretical considerations and verifiable empirically, which states that in first approximation, particularly for source energies E_0 below 1 Mev, transmission and reflection coefficients are nearly independent of E_0, provided the foil thickness is expressed in units of z/r_0. In other words, the energy dependence of reflection and transmission is largely taken into account by the energy dependence of r_0. This facilitates interpolation of the Monte Carlo results with respect to E_0.

Table 1 contains values of the mean range at the energies treated in this report. Actually, two sets are shown: the first set does not take into account energy loss by bremsstrahlung, and corresponds to the actual input data used in the Monte Carlo calculation; the second set includes the effects of bremsstrahlung, and should preferably be used to convert actual to reduced foil thickness. Actually, there is a significant difference only at the highest source energy listed, 2 Mev.

Tables 2, 3, 4 and 5 contain reflection and transmission coefficients for electron number and energy. These are raw results, as they come off the computer, and could be smoothed and somewhat improved by cross-plotting them against source energy and obliquity, and against foil thickness. Three significant figures are given, but the last is not really significant and has been left in to facilitate cross-plotting and comparisons.

Reflection coefficients are listed only for $z/r_0 \leq 0.3$ because a further increase of foil thickness does not further increase the amount of reflection. Transmission coefficients are listed only for $z/r_0 \leq 0.5$, because for much greater thicknesses the neglect of energy

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* In the terminology of [1], the calculations presented here are based on Model {I, PL(16, 96), EC, AM, DLT}.

loss straggling would begin to have serious consequences. The transmission coefficients plotted against z/r_0 have an approximately straigh slope between $z/r \doteq 0.3$ and 0.5. A linear extrapolation of this portio of the curve yields a lower limit for transmission, and the intercept, at zero-transmission, is the so-called extrapolated electron range.

For each source energy, the results for all values of θ_0 and z/r_0 were obtained in one run of $n = 1000$ Monte Carlo case histories. The standard deviations of the number reflection and transmission coefficients are

$$\sigma(R_N) = \sqrt{R_N(1 - R_N)/n}$$

$$\sigma(T_N) = \sqrt{T_N(1 - T_N)/n}$$

The standard deviations of the energy transmission and reflection coefficients have not been estimated directly. Experience with gamma-ray Monte Carlo problems indicates that their fractional standard deviations (ratio of standard deviation to the coefficient itself) is approximately 2-3 times greater than those for the number transmission and reflection coefficients.

It can be seen from tables 2-5 that the coefficients, particularly those for transmission, are rather independent of the value of E_0, at least up to 1 Mev, as has been predicted. For many purposes it is reasonable, therefore, to take coefficients averaged over a set of source energies, such as those given in table 6. The ratios T_E/T_N and R_E/R_N, also shown there, represent the ratios of the mean energy of the transmitted (reflected) electrons to the incident energy.

Table 7 compares number transmission coefficients for perpendicular incidence from this paper with results obtained by Perkins [2]. The calculation of Perkins is based on the same general principles as the present work, but differs in procedural details, and includes the effects of energy-loss straggling. The mean ranges assumed by Perkins (1.20 g/cm^2 at 2 Mev, 0.545 g/cm^2 at 1 Mev) differ somewhat from those used by us. In the comparison, his transmission curves, which he plotted against z/r_0, were first converted to an absolute thickness scale and then to a z/r_0 scale using our values for r_0. For $E_0 = 2$ Mev the agreement is quite good, and for $E_0 = 1$ Mev it is fair. The transmission coefficients of Perkins are generally somewhat higher, which at least in part can be ascribed to our disregard of energy-loss straggling. Perkins gives data at 2 Mev which indicate that for $z/r_0 \sim 0.5$ the inclusion of energy-loss straggling may raise the transmission coefficient by about 3-4%. According to a private communication from him, he imposed a certain upper limit on the magnitude of the multiple scattering deflections in his calculation, which, he estimates, may have decreased his reflection coefficients from 10-20%. This of course is accompanied by a corresponding increase of his transmission coefficient and contributes to the discrepancy between his and our results.

In table 8, a comparison is made with experimental transmission data of Agu, Burdett and Matsukawa [3], the agreement being quite close. Further comparisons, as well as indications of the effect of the systematic errors due to the assumed Monte Carlo model of the calculation, will be found in [1]. Finally, it should be pointed out that the error of the energy transmission and reflection coefficients, due to the neglect of energy loss straggling, is likely to be larger than the corresponding error for the number transmission and reflection coefficients.

4. References

[1] M. J. Berger, Monte Carlo Calculations of the Penetration and Diffusion of Fast Charged Particles, to be published in "Methods in Computational Physics", B. Alder, S. Fernbach and M. Rotenberg, Vol. I, Academic Press, New York, N.Y. (1963).

[2] J. F. Perkins, Monte Carlo Calculation of Transport of Fast Electrons, Phys. Rev. 126, 1781 (1962).

[3] B. N. C. Agu, T. Burdett and E. Matsukawa, Transmission of Electrons through Aluminum Foils, Proc. Phys. Soc. London, 71, 201 (1958).

Table 1. Mean Range of Electrons in Aluminum

E_o (Mev)	r_o, g/cm^2	
	Bremsstrahlung included	Bremsstrahlung not included
2.0	1.214	1.237
1.0	0.5501	0.556
0.5	0.2246	0.2258
0.25	0.08165	0.08196
0.125	0.02690	0.02706

Table 2. Number Transmission Coefficient, T_N

E_o (Mev)	θ_o (Degrees)	z/r_o				
		0.1	0.2	0.3	0.4	0.5
2.0	0	0.988	0.962	0.847	0.689	0.469
	45	0.922	0.803	0.637	0.459	0.276
	60	0.804	0.648	0.483	0.340	0.161
	75	0.600	0.458	0.328	0.186	0.091
	90	0.237	0.178	0.106	0.064	0.026
1.0	0	0.983	0.925	0.785	0.608	0.373
	45	0.881	0.758	0.582	0.417	0.218
	60	0.766	0.608	0.445	0.283	0.145
	75	0.573	0.429	0.299	0.166	0.065
	90	0.233	0.170	0.114	0.067	0.021
0.5	0	0.972	0.890	0.751	0.559	0.361
	45	0.877	0.747	0.582	0.400	0.202
	60	0.768	0.619	0.451	0.270	0.129
	75	0.564	0.436	0.305	0.165	0.075
	90	0.254	0.197	0.114	0.069	0.023
0.25	0	0.977	0.894	0.738	0.538	0.306
	45	0.863	0.721	0.539	0.349	0.189
	60	0.733	0.575	0.408	0.249	0.122
	75	0.544	0.393	0.268	0.149	0.061
	90	0.237	0.167	0.109	0.064	0.021
0.125	0	0.967	0.880	0.731	0.531	0.321
	45	0.861	0.703	0.543	0.346	0.176
	60	0.744	0.570	0.414	0.242	0.113
	75	0.546	0.404	0.260	0.151	0.063
	90	0.230	0.168	0.116	0.059	0.023

Table 3. Energy Transmission Coefficient, T_E

E_o (Mev)	θ_o (Degrees)	z/r_o				
		0.1	0.2	0.3	0.4	0.5
2.0	0	0.889	0.749	0.546	0.351	0.183
	45	0.780	0.556	0.360	0.205	0.094
	60	0.638	0.412	0.245	0.133	0.051
	75	0.437	0.260	0.148	0.067	0.025
	90	0.165	0.096	0.048	0.022	0.008
1.0	0	0.892	0.727	0.515	0.321	0.154
	45	0.757	0.538	0.343	0.193	0.080
	60	0.622	0.407	0.238	0.121	0.048
	75	0.434	0.259	0.146	0.064	0.019
	90	0.169	0.099	0.053	0.023	0.006
0.5	0	0.890	0.711	0.507	0.313	0.157
	45	0.765	0.546	0.357	0.200	0.081
	60	0.632	0.422	0.257	0.128	0.048
	75	0.442	0.280	0.161	0.070	0.024
	90	0.191	0.119	0.057	0.027	0.007
0.25	0	0.902	0.730	0.511	0.301	0.143
	45	0.757	0.535	0.338	0.181	0.079
	60	0.616	0.402	0.237	0.120	0.047
	75	0.432	0.258	0.144	0.067	0.022
	90	0.183	0.106	0.059	0.027	0.007
0.125	0	0.897	0.728	0.521	0.318	0.158
	45	0.763	0.537	0.354	0.191	0.081
	60	0.630	0.409	0.253	0.127	0.050
	75	0.439	0.274	0.152	0.075	0.025
	90	0.181	0.113	0.064	0.029	0.009

Table 4. Number Reflection Coefficient, R_N

E_o (Mev)	θ_o (Degrees)	z/r_o		
		0.1	0.2	0.3 or larger
2.0	0	0.008	0.015	0.023
	45	0.051	0.078	0.083
	60	0.130	0.159	0.161
	75	0.293	0.313	0.314
	90	0.711	0.719	0.720
1.0	0	0.017	0.061	0.090
	45	0.118	0.198	0.220
	60	0.231	0.318	0.330
	75	0.423	0.489	0.494
	90	0.766	0.792	0.796
0.5	0	0.028	0.087	0.108
	45	0.122	0.209	0.229
	60	0.230	0.304	0.316
	75	0.430	0.485	0.494
	90	0.745	0.774	0.782
0.25	0	0.023	0.069	0.097
	45	0.133	0.216	0.237
	60	0.264	0.349	0.360
	75	0.451	0.523	0.531
	90	0.763	0.796	0.798
0.125	0	0.033	0.093	0.121
	45	0.135	0.228	0.235
	60	0.253	0.339	0.347
	75	0.448	0.514	0.518
	90	0.765	0.785	0.788

Table 5. Energy Reflection Coefficient, R_E

E_o (Mev)	θ_o (Degrees)	z/r_o		
		0.1	0.2	0.3 or larger
2.0	0	0.008	0.015	0.023
	45	0.051	0.078	0.083
	60	0.130	0.159	0.162
	75	0.293	0.312	0.314
	90	0.711	0.719	0.721
1.0	0	0.013	0.035	0.044
	45	0.083	0.118	0.124
	60	0.168	0.204	0.208
	75	0.337	0.364	0.365
	90	0.714	0.724	0.725
0.5	0	0.021	0.050	0.057
	45	0.091	0.132	0.138
	60	0.174	0.209	0.212
	75	0.354	0.378	0.380
	90	0.702	0.714	0.715
0.25	0	0.018	0.043	0.053
	45	0.099	0.140	0.147
	60	0.204	0.245	0.249
	75	0.381	0.413	0.415
	90	0.720	0.735	0.735
0.125	0	0.026	0.061	0.072
	45	0.105	0.154	0.157
	60	0.201	0.254	0.248
	75	0.380	0.414	0.416
	90	0.725	0.733	0.734

Table 6. Average Transmission and Reflection Coefficients, Obtained by Averaging over the Results for Source Energies E_o = 1.0, 0.5, 0.25 and 0.125 Mev

z/r_o	θ_o (Degrees)	T_N	T_E	T_E/T_N	R_N	R_E	R_E/R_N
0.1	0	0.975	0.895	0.92	0.025	0.018	0.72
	45	0.871	0.760	0.87	0.127	0.095	0.75
	60	0.753	0.625	0.83	0.245	0.187	0.76
	75	0.557	0.436	0.78	0.438	0.363	0.83
	90	0.239	0.181	0.76	0.760	0.715	0.94
0.2	0	0.897	0.724	0.81	0.078	0.047	0.60
	45	0.732	0.539	0.74	0.213	0.136	0.64
	60	0.593	0.410	0.69	0.328	0.226	0.69
	75	0.416	0.268	0.64	0.503	0.392	0.78
	90	0.176	0.111	0.63	0.787	0.727	0.92
0.3	0	0.751	0.513	0.68	0.104	0.057	0.55
	45	0.562	0.348	0.62	0.230	0.142	0.62
	60	0.430	0.246	0.57	0.339	0.229	0.68
	75	0.283	0.151	0.53	0.509	0.394	0.77
	90	0.113	0.058	0.51	0.791	0.727	0.92
0.4	0	0.559	0.313	0.56			
	45	0.378	0.191	0.51			
	60	0.261	0.124	0.47			
	75	0.158	0.069	0.44			
	90	0.065	0.027	0.42			
0.5	0	0.340	0.153	0.45			
	45	0.196	0.080	0.41			
	60	0.127	0.048	0.38			
	75	0.066	0.023	0.35			
	90	0.022	0.007	0.32			

Table 7. Number Transmission Coefficients, T_N, for Perpendicular Incidence. Comparison with the Results of J. F. Perkins, [2]

E_0 (Mev)	z/r_0	This Report	Perkins
2.0	0.2	0.962 ± 0.006	0.96
	0.3	0.847 ± 0.015	0.86
	0.4	0.689 ± 0.015	0.68
	0.5	0.469 ± 0.016	0.48
1.0	0.2	0.925 ± 0.008	0.94
	0.3	0.785 ± 0.013	0.83
	0.4	0.601 ± 0.015	0.64
	0.5	0.373 ± 0.015	0.42

Table 8. Number Transmission Coefficients, T_N, for Perpendicular Incidence. Comparison with the Results of Agu, Burdett and Matsukawa, [3]. The Experimental Results Represent an Average for Various Source Energies between .25 and .75 Mev. The Calculated Results are from Table 6.

z/r_0	Agu *et al* Experiment	Calculation
0.1	0.97	0.975 ± 0.003
0.2	0.88	0.897 ± 0.005
0.3	0.74	0.751 ± 0.007
0.4	0.54	0.559 ± 0.008
0.5	0.34	0.340 ± 0.008

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