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A NOTE ON ANTIPODAL FOCUSING

JAMES R. WAIT



U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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A NOTE ON ANTIPODAL FOCUSING

James R. Wait

There has been some interest shown recently in the practical significance of the antipodal focussing in VLF propagation. If the earth and the ionosphere were perfectly concentric spherical surfaces, theory indicates that the amplitude of the field should build up to a maximum at the geographic antipodal point of the transmitter. In the language of optics, this point may be called an axial caustic.

Experimental evidence of some manner of focussing at the antipode was obtained by Round, Tremellen, Eckersley, and Lunnon [1925] in an early series of measurements. Their field strength data were obtained at frequencies in the range from 20 kc/s to 30 kc/s. More recently, Crombie [1958] and Bickel, et al., [1963] have found clear evidence of antipodal focussing. As predicted by theory [e. g., Wait, 1962], the field strength E was found to vary approximately as the inverse square root of the distance from the antipode. There was also some indication that a standing wave pattern existed in the antipodal region. Unfortunately, this pattern did not always exhibit a consistent form and the spacing between the nulls departed considerably from the half-wavelength expected on the basis of simple theory. It has been indicated by Crombie [1963] and by Bickel [1963] that asymmetry in the earth-ionosphere cavity may impair the focussing to some extent. Generally, they used arguments based on geometrical optics.

It is the purpose of the present paper to discuss this matter further. To shed some light on the subject, a very simple perturbation of the ideal model is considered. Essentially, it is assumed that the illumination of the antipodal region is non-uniform.

The region around the geographic antipode is depicted in Fig. 1. For present purposes, the antipodal region may be regarded as a radial transmission line and thus local cylindrical coordinates (ρ, ϕ, z) are convenient. The earth's surface is $z = 0$, while the effective lower boundary of the ionosphere is h . For some concentric region $\rho \leq \rho_0$, it is assumed that the ionosphere height h and its electric properties are constant. Furthermore, the surface impedance of the earth's surface at $z = 0$ is regarded as a constant. This assumption of local uniformity around the antipode greatly simplifies the subsequent discussion.

An appropriate form of solution for the vertical component of the electric field, for a waveguide mode of order n , is given by

$$E^{(n)}(\rho, \phi, z) = \sum_{m=-\infty}^{+\infty} A_m J_m(k\rho S_n) e^{im\phi} f_{m,n}(z), \quad (1)$$

where J_m is the Bessel function of the first kind of order m and argument $k\rho S_n$ in which $k S_n$ is the propagation constant for a waveguide mode of order n . The function $f_{m,n}(z)$ is a height-gain function for mode n and its specific form is of no concern at the moment; however, in general, it depends on both m and n . If the field in the antipodal region were perfectly symmetrical, $\partial/\partial\phi = 0$ and then

$$E^{(n)}(\rho, \phi, z) = A_0 J_0(k\rho S_n) f_{0,n}(z). \quad (2)$$

This particular situation has been discussed extensively in the literature [e. g., Wait, 1958, 1962; Norton, 1959]. In most applications to VLF propagation to great ranges, it is permissible to consider only one mode and thus the affix n may be dropped in what follows.

Equation (2) has a clear physical significance when J_0 is replaced by the first term of its asymptotic expansion. Thus for $|k\rho S| \gg 1$,

$$J_0(k\rho S) \sim \sqrt{\frac{2}{\pi k\rho S}} \frac{e^{ik\rho S} e^{-i\pi/4} + e^{-ik\rho S} e^{i\pi/4}}{2}, \quad (3)$$

which can be regarded as the superposition of two travelling waves.

At the radial distance $\rho = \rho_0$, the field has the form

$$E(\rho_0, \phi, z) = \sum_{m=-\infty}^{+\infty} A_m J_m(k\rho_0 S) e^{im\phi} f_m(z), \quad (4)$$

where the notation has been simplified in accordance with the discussion above. On physical grounds we will now assume something about the variation of $E(\rho_0, \phi, z)$. Then, the resulting form of $E(\rho, \phi, z)$, when $\rho < \rho_0$, will be discussed.

By multiplying both sides of equation (4) by $e^{-im'\phi}$ and integrating with respect to ϕ , over the range 0 to 2π , one is lead readily to the formula

$$A_m = \frac{1}{2\pi J_m(k\rho_0 S)} \int_0^{2\pi} E(\rho_0, \phi, z) e^{-im\phi} d\phi, \quad (5)$$

if one notes that

$$\int_0^{2\pi} e^{i(m-m')\phi} d\phi = \begin{cases} 2\pi & \text{if } m = m' \\ 0 & \text{if } m \neq m' \end{cases} \quad (6)$$

Thus, formally, the coefficient A_m may be found from the specific form assumed for the electric field around the ring $\rho = \rho_0$.

As a simple example, which leads to an embarrassingly simple formula, it will be assumed that the illumination on the ring (at $\rho = \rho_0$) is constant except for a finite sector. Such a situation could be envisaged by imagining that the energy, propagated from the transmitter, passed, in part, over a very lossy region of the earth. Ignoring any diffraction effects, it will then be supposed that the great circle paths in the angular region $\phi_1 < \phi < \phi_2$ are paths of complete absorption. Thus, over this interval on the ring $\rho = \rho_0$,

$$E(\rho_0, \phi, z) \cong \left(\frac{1}{\pi k \rho_0 S} \right)^{\frac{1}{2}} e^{-i(k\rho_0 S_n - \pi/4)} A_0, \quad (7)$$

which is the contribution from the long great circle paths which are propagating from the direction $\pi + \phi_1 > \phi > \pi + \phi_2$ away from the antipode.

Using equation (5), it now follows that

$$\begin{aligned} A_m = & \frac{1}{2\pi J_m(k\rho_0 S)} \left\{ \int_0^{2\pi} J_0(k\rho_0 S) e^{-im\phi} d\phi \right. \\ & + \int_{\phi_1}^{\phi_2} [E^+(\rho_0) - J_0(k\rho_0 S)] e^{-im\phi} d\phi \\ & \left. + \int_{\pi+\phi_1}^{\pi+\phi_2} [E^-(\rho_0) - J_0(k\rho_0 S)] e^{-im\phi} d\phi \right\} \quad (8) \end{aligned}$$

where

$$E^\pm(\rho_0) - J_0(k\rho_0 S) \cong - \left(\frac{1}{2\pi k \rho_0 S} \right)^{\frac{1}{2}} e^{\pm i(k\rho_0 S - \pi/4)} \quad (9)$$

The integrations can be carried out readily to yield, for $m \neq 0$,

$$A_m = -\frac{1}{2\pi J_m(k\rho_0 S)} \frac{1}{(2\pi k\rho_0 S)^{\frac{1}{2}}} \left\{ e^{i(k\rho_0 S - \pi/4)} e^{-im\phi_0} + e^{-i(k\rho_0 S - \pi/4)} e^{im\pi} e^{-im\phi_0} \right\} \Delta\phi \Lambda_m, \quad (10)$$

where

$$\phi_0 = (\phi_1 + \phi_2)/2,$$

$$\Delta\phi = \phi_2 - \phi_1,$$

and

$$\Lambda_m = \frac{\sin(m\Delta\phi/2)}{(m\Delta\phi/2)}.$$

In view of the already imposed condition that $|k\rho_0 S| \gg 1$, the Bessel function J_m in the denominator may be replaced by its asymptotic expansion. Thus, for $m \neq 0$,

$$A_m \cong -\frac{e^{im\pi/2}}{2\pi} e^{-im\phi_0} \Delta\phi \Lambda_m. \quad (11)$$

Within the same approximation, it readily follows that

$$A_0 \cong 1 - \frac{\Delta\phi}{2\pi}. \quad (12)$$

Employing the above explicit formulas for the coefficient A_m , the resulting behavior of the field in the vicinity of the antipode may be deduced from equation (4). To illustrate the nature of the problem, it is assumed that Λ_m can be replaced by unity. This would be well justified if Δ is small compared with unity and if $k\rho S$ is not too large. At the same time, the height-gain function $f_m(z)$ is replaced by unity. With these simplifications,

$$E(\rho, \phi) \cong J_0(k\rho S) - q e^{ik\rho S \cos(\phi - \phi_0)}, \quad (13)$$

where $q = \Delta\phi/2\pi$. It is quite clear that the total field is the superposition of the ideal symmetric antipodal field and a plane wave, of relative amplitude q , incident from the direction $\phi = \phi_0$. The negative sign preceding q is to indicate that the plane wave field is to be subtracted from the ideal or undisturbed antipodal field.

In a certain sense, the plane wave term can be regarded as the most basic type of perturbation. In fact, a generally perturbed field could be written in the form

$$E(\rho, \phi) \approx J_0(k\rho S) - \sum_s q_s e^{ik\rho S \cos(\phi_0 - \phi_s)}, \quad (14)$$

where the summation over planes incident at various angles $\phi = \phi_s$. Furthermore, the coefficient q_s may be complex to account for relative phase shifts between the perturbing plane waves and the ideal antipodal field.

In order to characterize the amplitude behavior of the resultant field for a single disturbing plane wave, the function

$$E(x) = J_0(x) - q e^{ix \cos \phi}, \quad (15)$$

is considered, where $x = k\rho S = 2\pi\rho/\lambda_e$ in terms of the effective wavelength λ_e . While strictly speaking, S has a small imaginary part [Wait, 1962], it may be regarded as real for the present discussion.

The magnitude of $E(x)$, expressed in db, is plotted in Fig. 2 for $q = 0.1$ for a range of azimuth angles ϕ . Because of certain symmetry properties, the curves are identical at ϕ and $180^\circ - \phi$. Also, although not shown explicitly on the curves, symmetry exists about the direction of $\phi = 0^\circ$. Thus, the results in Fig. 2 are also applicable for negative angles.

The curves, in Fig. 2, for $\phi = 0^\circ$ (and 180°) are almost identical to the ideal antipodal field pattern characterized by the Bessel function $J_0(2\pi\rho/\lambda_e)$ alone. Here, the variation along the radial is in the direction of propagation of the perturbing wave. At other angles, it is apparent that the pattern becomes distorted and the spacing between nulls is modified to some extent.

The effect of varying q , the amplitude of the perturbing wave is shown in Fig. 3. For these curves, ϕ is fixed at 45° . It is evident that for the larger q values, the deep nulls disappear. In this situation the symmetric J_0 antipodal field can be regarded as a perturbation to the plane wave field.

The graphical results shown in Figs. 2 and 3 are intended to indicate merely that the field pattern near the antipodal may become extremely complicated. Many other intriguing patterns can be generated by using other combinations of the parameters. Also, equally interesting, are the resulting phase variations near the antipode which may be calculated from equation (15).

I would like to thank Mrs. C. M. Jackson for carrying out the calculations shown graphically in Figs. 2 and 3.

References

- Bickel, J. E. et al. (1963). Personal communication.
- Crombie, D. D. (1958). Differences between east-west and west-east propagation on VLF signals over long distances, *J. Atmos. and Terrest. Phys.* 12, 110-117.
- Crombie, D. D. (1963). Personal communications.
- Norton, K. A. (June 1959). Transmission loss in radio propagation, Part II, NBS Technical Note No. 12.
- Round, H. J. T., T. L. Eckersley, K. Tremellen, and F. C. Lunnon (1925). Report on measurements made on signal strength at great distances during 1922 and 1923 by an expedition sent to Australia, *J. Instn. Elect. Engrs.* 63, 933-1011.
- Wait, J. R. (1958, 1960). Diffractive corrections to the geometrical optics of low frequency propagation, *Proc. Liege Conference on Wave Propagation (1958)*; also published in *Electromagnetic Wave Propagation*, edited by M. Desirant and J. L. Michiels (Academic Press, 1960).
- Wait, J. R. (1962). *Electromagnetic waves in stratified media*, Chapters VI and VIII (Pergamon Press, London, New York).

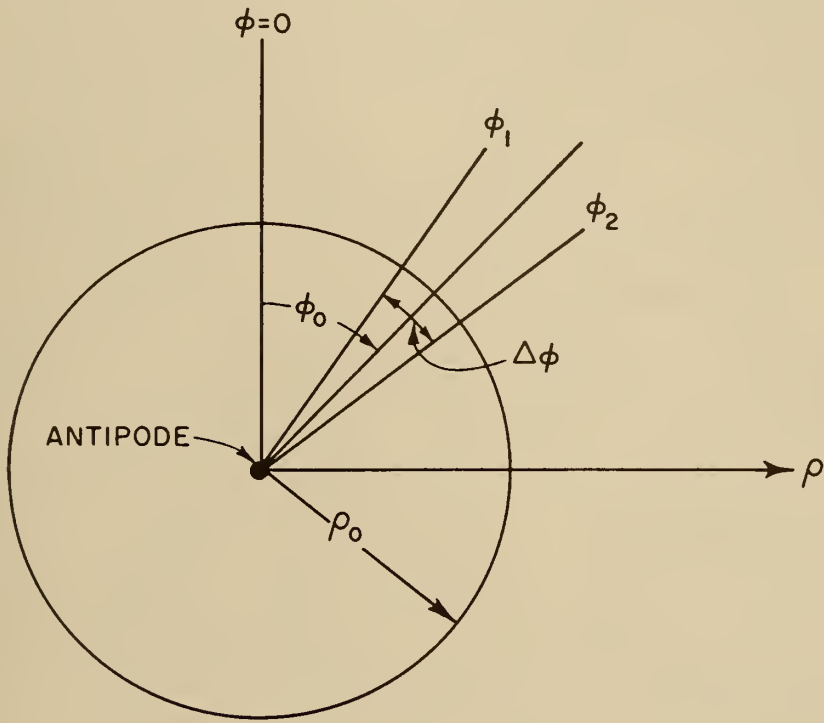


Fig. 1 Coordinate system for antipodal region.

RESULTANT ANTIPODAL FIELD FOR $q=0.1$ AND $b=0$

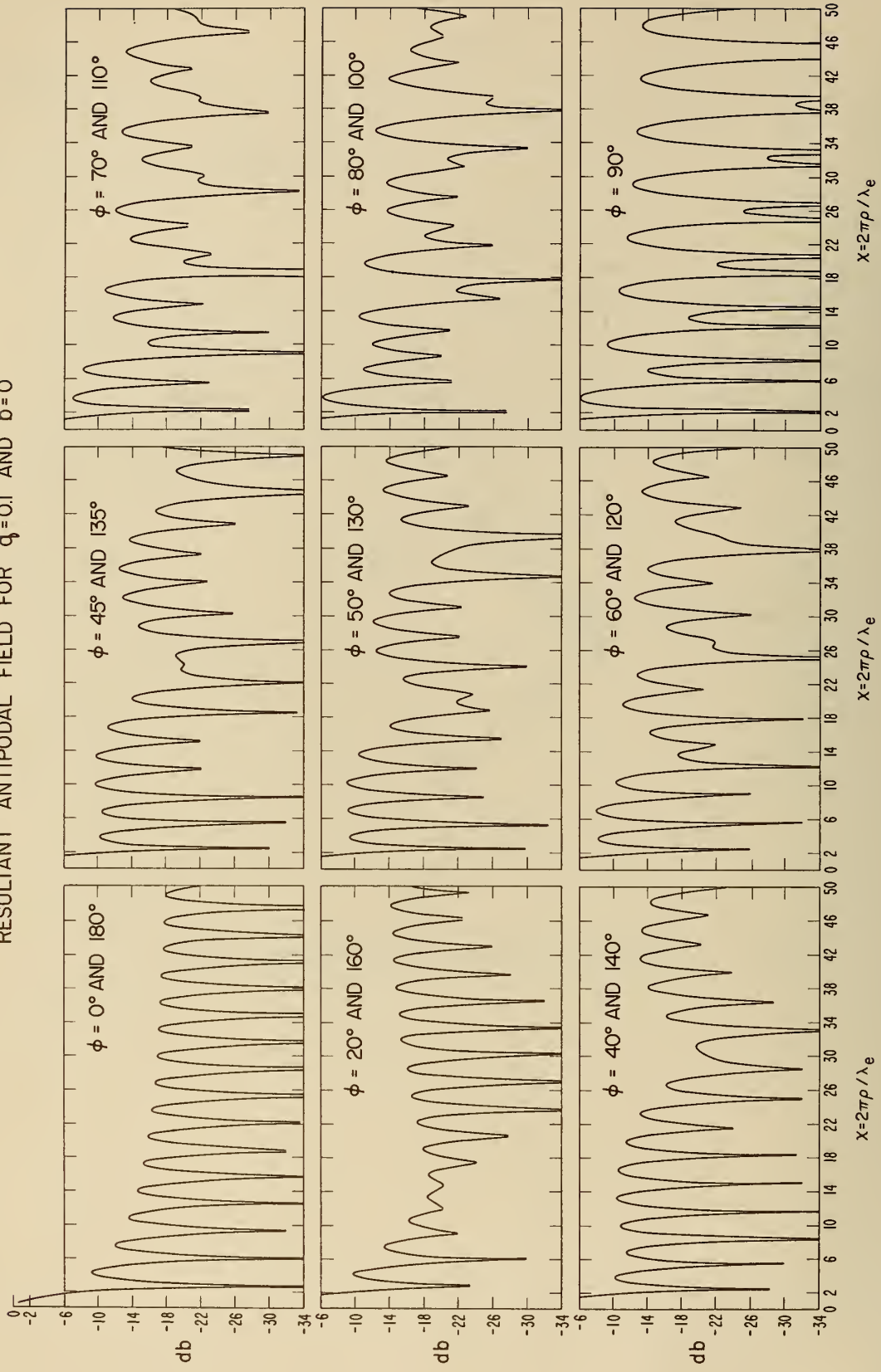


Fig. 2

RESULTANT ANTIPODAL FIELD FOR $\phi = 45^\circ$

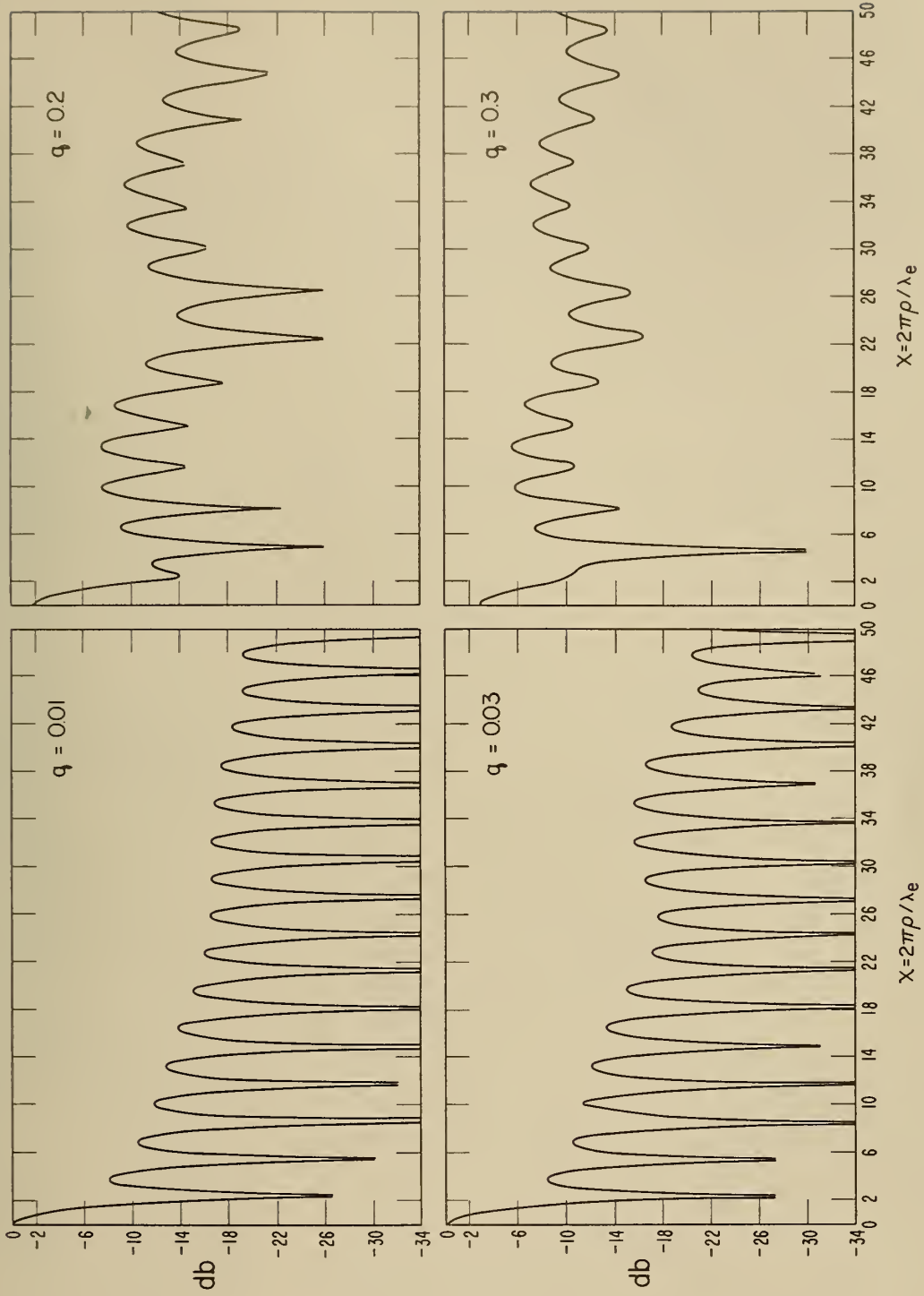


Fig. 3

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THE NATIONAL BUREAU OF STANDARDS

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