Regarding Electric Energy Savings, Power Factors, and Carbon Footprints: A Primer

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Abstract – A short primer is presented which describes the underlying physical theory of certain devices that reduce the current drawn from power distribution lines by improving the power factor of residential electric circuits. A brief discussion is provided of the associated energy savings, change in power factor, and reduction of "carbon footprint."

Introduction

"Power factor correction devices improve power quality but do not generally improve energy efficiency (meaning they won't reduce your energy bill)". This remark is taken from the Energy Star website [1] as part of a discussion on saving the cost of electrical energy for residences. We present a short primer on the physical theory underlying such devices and provide additional information related to the above statement. The devices that we consider introduce electrical capacitance in parallel with activated electrical loads that typically contain both resistance and inductance, e.g., an electric motor or transformer. The question of whether providers and users experience savings for the cost of electric energy or just reduce their carbon footprint is briefly discussed.

The Model

For our model we consider a simplified version of an electric motor as our electrical load. The motor is represented as a resistor R in parallel with an inductor L as shown in Figure 1(a). The wires connecting the two elements are assumed to have no resistance. Energy is dissipated in the resistor, but not in the inductor which provides a magnetic field for operation of the motor.

Further details of the electric circuit are shown in Figure 1(b) with the addition of a sinusoidal alternating voltage source, i.e., the power distribution line with voltage $V_o\sin(\omega t)$ and the watt-hour meter which utilities use for billing purposes. Our model considers watt-hour meters that monitor the product of voltage, current, power factor (PF), and time to determine the cost of the electric energy. V_o is the voltage amplitude, t is the time, and ω is the angular frequency $2\pi f$, where f is the power frequency 60 Hz. For our purposes, the PF can be found by calculating the cosine of the phase difference between the current and voltage waveforms. The "energy savings" device is shown as the capacitor C, but initially is not connected into the circuit. Introduction of the capacitor increases ("corrects") the PF and, as shown below, less current is drawn from the power line. We assume the capacitor is disengaged automatically when the load is turned off**.** Otherwise the capacitor would continue to draw current from the power line and a potential would exist for adverse effects on the distribution system during light load periods. Ground wires are not indicated as we also assume no leakage currents.

Calculations

Mesh analysis for determining currents in circuits is described in many texts on circuit analyses and is not reviewed here [2, 3]. We use the approach to set up simultaneous equations that are solved to determine circulating currents I_1, I_2, and I_3 in the separate loops shown in Figures 1(b) and 1(c). The simultaneous equations that are developed and their solutions are presented below. We choose specific values of the circuit elements (R, L, C) to demonstrate how the current from the power distribution line is affected with and without the inclusion of the capacitance. Since the voltage remains essentially constant, changes in the current mainly influence the amount of electric energy provided by the power line. Because the number of equations will be few, they can be solved by the substitution method or by using matrices and Cramer's Rule.

Mesh analysis for the circuit in Figure 1(b) yields the equations

$$I_2 Z_L - I_3 Z_L = V_o \sin(\omega t) \text{ and}$$
$$-I_2 Z_L + I_3(Z_R + Z_L) = 0,$$

where the impedances $Z_L = \omega L i$ and $Z_R = R$; $i = (-1)^{0.5}$.

Assuming the appliance is a central air conditioner with values for L, R, and V_o of 0.065 henries, 10.4 ohms, and 325 peak amplitude volts (230 volts rms) respectively, the solutions for the currents in amperes (A) are

$$I_1 = I_2 = 31.25 \sin(\omega t) \text{A} - 13.263 \sin(\omega t) i \text{ A} \text{ and}$$
$$I_3 = 31.25 \sin(\omega t) \text{A}.$$

The phase difference between the current and voltage waveforms from the power line is determined by examining the angle associated with the tangent of I_1 as represented in the complex plane, i.e., -23 degrees. Therefore the PF is cos(-23°) or 0.92. The amplitude of the current from the power line is $[(31.25)^2 + (13.26)^2]^{0.5}$ or ~34 A, and the current through the inductor, I_L, is just $I_2 - I_3$ or -13.263sin (ωt)i A.

Mesh analysis for the circuit that includes the capacitor C [Fig. 1(c)] yields the equations

$$I_1 Z_C - I_2 Z_C = V_o \sin(\omega t),$$
$$-I_1 Z_C + I_2(Z_L + Z_C) - I_3 Z_L = 0, \text{ and}$$
$$-I_2 Z_L + I_3(Z_R + Z_L) = 0,$$

where the impedance $Z_C = -i/\omega C$.

We assume a value for capacitance that will demonstrate the maximum effect for reducing the current from the power line. That is we choose a value of C that will satisfy the resonance condition with the inductor, i.e., $C = 1/\omega^2 L = $ ~108.2 μF.

Solutions for the currents in Figure 1(c) are

$$I_1 = 31.25\sin(\omega t)\,\text{A} - 5.41\cdot10^{-3}\sin(\omega t)\text{i}\ \text{A} \approx 31.25\sin(\omega t)\,\text{A},$$

$$I_2 = 31.25\sin(\omega t)\,\text{A} - 13.257\sin(\omega t)\text{i}\ \text{A},\ \text{and}$$

$$I_3 = 31.25\sin(\omega t)\,\text{A}.$$

The current provided by the power line, I_1, is reduced, i.e., the amplitude is reduced to ~31.3 A from 34 A. Yet the current I_2 necessary to operate the motor is unchanged. The source of what appears to be "extra" current in I_2, -13.257sin(ωt)i A, can be understood by first examining the other currents in the circuit [Fig. 1(c)]. The current through the inductor, I_L, equals -13.257sin(ωt)i A and is unchanged. However, there is now a current through the capacitor, I_C, equal to I_1 - I_2 or +13.257sin(ωt)i A, which is 180 degrees out of phase with respect to the current in the inductor.

The underlying physics provides an understanding of what is happening with the introduction of the capacitor. When voltage is applied to the circuit in Figure 1(c), the capacitor is instantly charged and this charge oscillates between the capacitor and inductor [4] creating the current through C and L indicated by the mesh analysis. The oscillating charge is the origin of the "extra current" in I_2 when the current from the power line is reduced. Appendix A shows this oscillatory relationship between the charge on the capacitor and current through the inductor as a function of time.

For the ideal (lossless) conditions we have assumed, the power line does not need to provide additional current to charge the capacitor-inductor network. This last point can be verified by performing a mesh analysis of the circuit in Fig. 1(c) with the resistor removed. The currents through C and L will be the same (±13.257sin(ωt)i A), but the current from the power line will be ~zero. The zero current from the power line is also expected since the impedance of the L-C circuit is ~infinite. In a more realistic treatment of the problem there will be some resistive losses in the L-C circuit which will be compensated for by additional charging by the power line (see below).

Also very noteworthy is that under the above conditions the PF has increased to nearly unity from 0.92 because there is essentially phase congruence between the current and voltage waveforms, i.e., the phase of I_1 is the same as $V_0\sin(\omega t)$.

While the above calculations are for very ideal conditions, including the use of a matching capacitor, reductions in power line current will occur even when the capacitor is poorly matched. For example, if the capacitor is ±50 µF different from the matched value (108.2 µF), the amplitude of I_1 will be 31.8 A (versus ~34 A) and the power factor will be 0.98 (versus 0.92). Similarly, introducing small resistances of the connecting wires will not change the results significantly.

Discussion

Residential loads that decrease the power factor include refrigerators, air conditioners, and washing machines. Typically these appliances are operated only intermittently, so that average residential power factors may not be very low. If an energy saving device of the type we have considered is used to increase the power factor, the utility will not have to supply as much current when certain electrical appliances are operated. However, the homeowner's electric bill will not decrease because, as noted above, the cost is determined in part by the product of current, voltage, and power factor. That is, as the current from the power line is reduced by the introduction of capacitance, the power factor is increased and the product, I x V x PF, remains essentially the same. The utility will save a small amount of money (see discussion of possible savings below) because it will not have to supply as much current to the residence, will have smaller resistive losses along the power line, will not emit as much greenhouse gas, etc. Will the homeowners reduce their "carbon footprint", are they only helping the utilities reduce their carbon footprint, or can both take some credit? In addition, should the homeowner consider the CO_2 generated during fabrication of the capacitive device and the associated "payback time"? We leave the answer to these questions up to the reader.

It should be noted that some utilities may charge a penalty for power factors that are considered too low, for example below 0.95 [5]. In such cases there could be some savings by installing devices of the type that has been considered. For example, industrial users of electric energy with large inductive loads might be penalized if steps are not taken to increase their power factor.

Possible utility savings. For our central air conditioner example given above, let us estimate approximately how much energy is saved because of lower resistive losses in the distribution system when the customer installs the capacitor. The utility's total resistance as seen looking back into the distribution system from the watt-hour meter is assumed for our rough calculation to be less than 0.05 ohms. The power consumed in the utility circuits for operating the air conditioner without the capacitor is about 28.8 watts ($I_{rms}^2 R$), where I_{rms} is the root mean square (rms) value of I_1 in Figure 1(b), 24 A. With the capacitor installed, the rms value of I_1 [Fig. 1(c)] becomes 22.1 A and the power consumed by the utility is reduced to about 4.4 watts. Therefore, installation of the capacitor reduces the energy from the utility by 4.4 watts multiplied by the running time of the air conditioner. If the air conditioning unit runs for 12 hours each day, the energy savings will be about 52.8 watt-hours per day. At 20 cents per kilowatt-hour, the money saved by the utility would be approximately 1 cent per day. Since in most parts of the United States air conditioners only operate for less than six months of the year, the utility's annual savings would be about $1.80 for a single residence.

Appendix A

The relationship between the charge on the capacitor and the current through the inductor as a function of time can be determined from the results that have been calculated above. We find the charge in coulombs (Q) on the capacitor, Q_C, as a function of time by integrating the current through the capacitor, I_C, i.e.,

$$Q_C = \int I_C dt = 13.257 \int \sin(\omega t) dt = -\frac{13.257}{\omega} \cos(\omega t) Q.$$

The calculated current through the inductor, I_L, is

$$I_L = -13.257 \sin(\omega t) \text{ A.}$$

By plotting both the charge and inductor current as a function of time on the same coordinates (Figure A1), one can see that the charge on the capacitor goes to zero as the current reaches a maximum through the inductor, and reaches a maximum charge as the current through the inductor goes to zero.

Acknowledgement

References

[1]   http://energystar.custhelp.com/cgi-bin/energystar.cfg/php/enduser/std_adp.php?p_faqid=4941&p_created=1204908170

[2]   S.L. Oppenheimer, F.R. Hess Jr., and J.P. Borchers, Direct and Alternating Currents, McGraw Hill Co., New York (1973).

[3]   R.M. Kerchner and G.F. Corcoran, Alternating Current Circuits, John Wiley & Sons, New York (1960).

[4]   D. Halliday, R. Resnick, and J. Walker, Fundamentals of Physics, John Wiley & Sons, Inc, New York (2005).

[5]   DOE Fact Sheet, http://www.eere.energy.gov/; search for "Reducing Power Factor Cost."
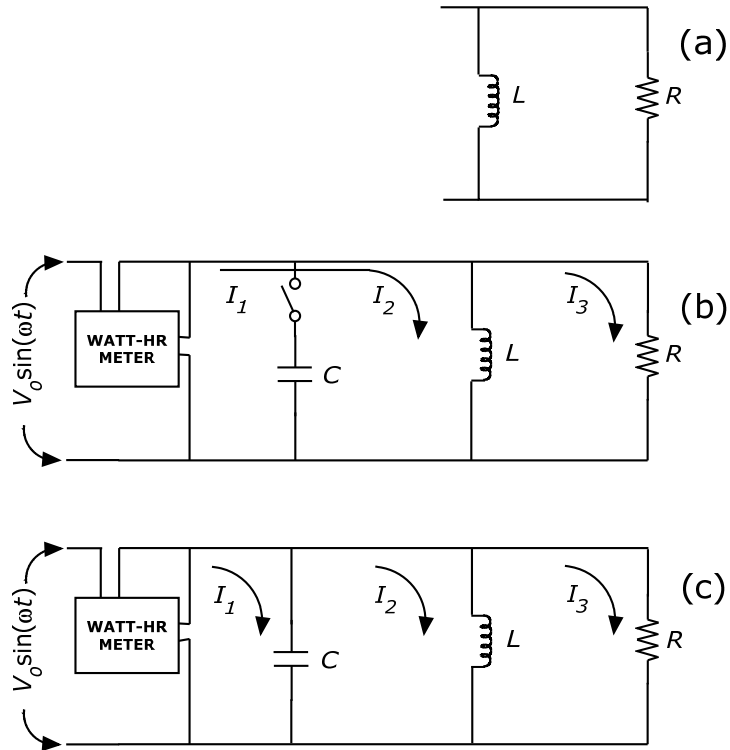
Figure 1. Model circuits for (a) simplified motor, (b) circuit without capacitive device, and (c) circuit with device.
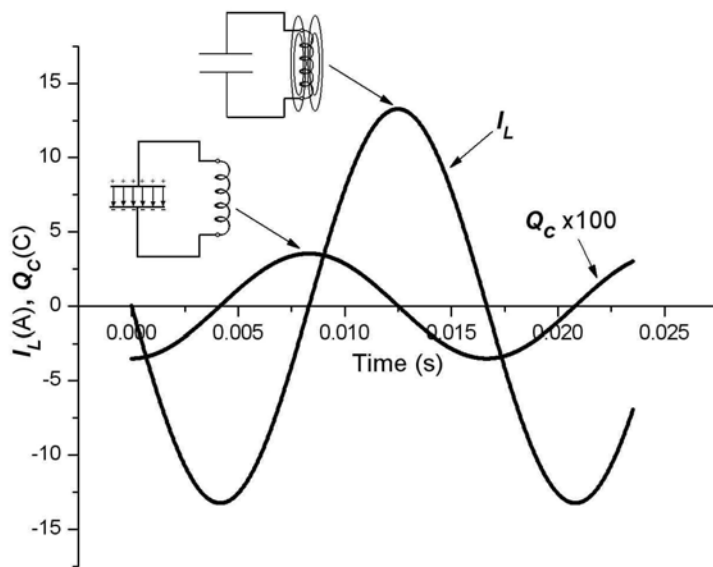

Figure A1. Relation between charge on the capacitor and inductor current. The vertical axis has units of charge and current. Insets are after Ref. [4].