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Technical Note 158

EFFICIENT USE OF THE RADIO SPECTRUM

KENNETH A. NORTON

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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ABSTRACT

Methods are given for determining the transmitter power required for satisfactory operation in the presence of noise on any given telecommunications link. It is then suggested that sufficiently high power should be used so that a satisfactory grade of service will be achieved at all times. The above conclusion has reference only to the problem of reception in the absence of interference from other telecommunications systems. It is then shown that optimum use of the spectrum can be achieved only when interference from other signals rather than from noise provides the ineluctable limit to satisfactory reception. The fact that interference, rather than noise, should provide the ineluctable limit to satisfactory reception indicates that greater stress should be placed on the use of various techniques for making systems free of mutual interference rather than designing them simply with the objective of overcoming noise.

Most of the report deals with statistical methods of using the concept of transmission loss on the propagation paths in order to achieve the above described optimum use of the spectrum.

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(continued)

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by

Kenneth A. Norton

1. INTRODUCTION

The radio spectrum is a prime natural resource which is available for use by the citizens of all nations throughout the world and, indeed, by the inhabitants of all planets throughout the universe. The extent to which the spectrum is actually used in various countries is currently limited primarily by the extent of the economic development of their resources. However the use of certain parts of the spectrum is currently limited in many regions of the world by mutual interference between two or more radio transmissions simultaneously occupying the same frequency bands. This simultaneous occupancy of a given band of frequencies will be called frequency sharing.

In the United States the responsibility for the efficient use of the spectrum is shared by the Director of Telecommunications Management, who reports directly to the President, and the independent governmental agency, the Federal Communications Commission. Internationally this responsibility rests with the International Telecommunications Union. Up to the present time the attitude towards efficient use of the spectrum may be described somewhat as follows. First, various uses for the spectrum have been considered socially desirable, and then frequency bands have been allocated to these uses. Currently allocations have been made throughout the range from 10 kc/s to 40 Gc/s, i.e. to all of bands 5 to 10 and including parts of bands 4 and 11*. Internationally, the approved uses for the spectrum are given in the Radio Regulations of the I. T. U. [1959]. These allocations may be different in the different regions of the world. Within the United States frequency allocations conform to those in the Radio Regulations but are further divided between government and non-government users by joint agreement between the Director of Telecommunications Management and the F.C.C. Currently approved allocations are given in the Rules and Regulations of the Federal Communication Commission [Pike and Fischer]. The D. T. M. and F.C.C. also have the responsibility for assigning frequencies, transmitter powers, antenna locations and heights, and modulation characteristics to individual users of the spectrum. Recently the F. C.C. has also requested authority to control television receiver characteristics as well.

Opportunities for making more efficient use of the spectrum arise either in the method of transmission or in the assignment of a specific band of radio frequencies for use by a particular radio transmitter for communication from location A to locations B, C, D, etc. A fixed service is characterized by a single communications link from an antenna at a fixed geographical location A to an antenna at a fixed geographical location B. A broadcasting service is characterized by a single transmitting antenna and multiple receiving locations. A mobile service is characterized by transmitting and, or, receiving locations which may change with time. However, all of the above types of service may be characterized as consisting simply of one or more point-to-point communication links with a transmitter at one terminal and a receiver at the other terminal. The purpose of the communication link is to reproduce at the receiver, with a given

*Band n extends from 0.3×10^n to 3×10^n cycles per second.

degree of fidelity, the information available at the source. The difficulty of achieving this reproduction depends on (a) the nature of the information to be transmitted, (b) the rate at which the information is to be transmitted, and (c) the fidelity of the received signal. Precise definitions of these three aspects of a transmitted signal are available in the literature on information theory. Information theory provides a very fundamental and formally satisfactory basis for describing the efficiency of use of a single communications link. However, this rather sophisticated theory will not be used in the following elementary discussion since this would unnecessarily complicate the exposition of other fundamental technical principles which must be applied in order to achieve efficient use of the spectrum. The application of information theory indicates that presently used methods of transmitting various types of information over a particular link are often quite inefficient. However, it is also generally recognized that a substantial improvement in the efficiency of transmission of a given type of information will usually involve the development of more complex or more powerful equipment. Consequently the extensive use of these more efficient methods of transmission will largely be limited by economic considerations. The technical principles described below are concerned with the efficient, joint, simultaneous use of all of the individual communication links which occupy the same or adjacent frequency bands. Thus the application of these technical principles can improve the efficiency of use of the spectrum to a large extent independently of the efficiency of use of the individual links and may thus be used to advantage without resort to more complex or more powerful equipment. The achievement of this more efficient use depends only upon good spectrum management, i.e. upon the proper assignment of currently available facilities to the individual communication links.

Thus optimum use of the spectrum involves not only the efficient use of the individual communication links but also the efficient joint use of many such links either by time sharing or by frequency sharing of the same frequency bands. This report proposes as figures of merit for a receiving system the protection ratios r_u (between the hourly median wanted and hourly median unwanted signal powers available from a loss-free receiving antenna which is otherwise equivalent to the actual receiving antenna) which are required for the satisfactory reception of the information carried by a specified wanted signal in the presence of specified kinds of unwanted signals. The use of receiving systems having the smallest values of r_u will permit the same portions of the spectrum to be used simultaneously by the maximum number of users and thus the minimization of these protection ratios represent an important means for better spectrum utilization. On the other hand, economic considerations, as contrasted to spectrum conservation considerations, indicate that radio receiving systems should be designed so that the minimum possible transmitter power is required for the satisfactory reception of the wanted signals in the presence of noise. This report proposes as an additional figure of merit for a receiving system the hourly median wanted signal power p_n which is required at the terminals of the equivalent loss-free receiving antenna for the satisfactory reception of the information carried by the wanted signals in the presence of noise but in the absence of any other unwanted signals.

For optimum use of the spectrum by the maximum number of simultaneous users, the transmitting and receiving systems of the individual links should be designed with the primary objective of minimizing the various values of r_u involved and then sufficiently <u>high</u> transmitter powers should be used so that the hourly median wanted signal power p_m exceeds p_n for a sufficiently large percentage of the hours during the intended period of reception at each receiving location. This approach to frequency assignment problems will be unrealistic in a few cases such as the cleared channels required for radio astronomy, but these rare exceptions merely serve to prove the otherwise general rule that optimum use of the radio spectrum can be achieved only when interference from other signals rather than noise provides the ineluctable limit to satisfactory reception.

2. CRITERIA FOR SATISFACTORY SIMULTANEOUS USE OF TWO OR MORE COMMUNICATION LINKS

If two or more communication links are operated anywhere in the spectrum they can, in principle, mutually interfere. Naturally this interference is usually greatest when these links are operated in the same or in nearly the same frequency bands but may be significant even for operations in widely separated frequency bands by virtue, for example, of harmonic radiation. By virtue of this mutual interference the fidelity of the information received on each of the links will be deteriorated not only by noise but also by the cross talk arising from the operation of the other links. For example, consider the joint operation of n links with transmitters having radiated powers P_i, $i = 1$ to n, expressed in decibels above one watt. Let R_{ii} denote the wanted signal-to-operating noise ratio at the input to the receiving system which is required for the satisfactory reception of the wanted signal in the absence of all other kinds of interference. A precise definition of R_{ii} is given in Appendix I where it is designated R_n. Note that R_{ii} will depend not only on the nature of the noise and the nature of the information transmitted over the i^{th} link, together with its rate of transmission, but also depends on the grade of service, i.e. the degree of fidelity of the received signal which is considered to be satisfactory. Let $N_i(t)$ denote the hourly median operating noise power level at the time t referred to the loss-less receiving antenna terminals of the i^{th} link:

$$N_i(t) \equiv F_i(t) + B - 204 \text{ dbw} \cdot \quad (1)$$

In the above $F_i(t)$ denotes the hourly median operating noise factor at the time t of the receiving system expressed in decibels, $B \equiv 10 \log_{10} b$ where b is the effective noise bandwidth of the receiving system expressed in cycles per second and $204 = -10 \log_{10}(kT_o)$ where k is Boltzman's constant and the reference temperature $T_o = 288.39°$ Kelvin. Precise definitions for the operating noise factor and effective noise bandwidth of a receiving system are given in Appendix I. Let $L_{ij}(t)$ denote the hourly median transmission loss [Norton, 1953, 1959 and CCIR 1959] between the receiving antenna of the i^{th} link and the transmitting antenna of the j^{th} link at the time t. Let $P_{ij}(t)$ denote the hourly median signal power in dbw which is available at the loss-less receiving antenna terminals of link i at the time t from the transmitter of link j. A definition of signal power is given by (1) in Appendix I. $P_{ij}(t)$ denotes a wanted signal power when j=i and an unwanted signal power when $j \neq i$. The following equation defines the transmission loss $L_{ij}(t)$:

$$L_{ij}(t) \equiv P_j - P_{ij}(t) \ . \quad (2)$$

Finally let R_{ij} denote the ratio of the i^{th} wanted hourly median signal power to the j^{th} unwanted hourly median signal power, expressed in decibels, which is required for the satisfactory reception of the i^{th} wanted signal in the presence of the j^{th} unwanted signal alone. A precise definition of R_{ij} is given in Appendix I where it is designated R_u. Note that R_{ij} will depend not only upon the nature and rate of transmission of information over the links i and j and the degree of the spectrum overlap between the channels i and j but also upon the nature of the within-the-hour fading of both the wanted and unwanted signals and the degree of fidelity of the i^{th} wanted signal which is considered to be satisfactory. It follows from the above definitions that it is necessary that the following inequalities be satisfied if the i^{th} received signal is to be considered a satisfactory replica of the i^{th} transmitted signal over a period of one hour at the time t:

$$P_{ii}(t) - N_i(t) \equiv P_i - L_{ii}(t) - N_i(t) > R_{ii} \quad (3)$$

$$P_{ii}(t) - P_{ij}(t) \equiv P_i - L_{ii}(t) - P_j + L_{ij}(t) > R_{ij} . \quad (4)$$

The second inequality above must be satisfied for j=1 to n excluding the case j=i which is covered by the first inequality. The above inequalities would also be sufficient conditions for the satisfactory reception of the i^{th} wanted signal at the time t if it were correct to assume that several sources of interference present simultaneously do not deteriorate a wanted signal more than it would be deteriorated by each of the individual sources of interference acting alone. This assumption is clearly not precisely correct, however, and a practical method is given by Norton, Staras and Blum [1952] for making allowance for interference present from several sources simultaneously. For the purpose of the present report however, we will consider the effects of the interference from each source independently and the reader should remember that this assumption will always be at least to some extent unrealistic and in some cases completely inappropriate. The use of this assumption will, however, greatly simplify our subsequent analysis and it will not vitiate the important fundamental principles of frequency assignment which we expect to establish.

We will now show how (3) and (4) may be used to determine the percentages of time p_{ij} ($j=1$ to n) that the i^{th} link will provide a satisfactory signal in the presence of the j^{th} source of interference alone; when $j=i$, p_{ii} is the percentage of time that the i^{th} link will provide a satisfactory signal in the presence of noise. The percentage of time p_{ij} is called the time availability of satisfactory service on the i^{th} link in the presence of the j^{th} source of interference. Consider the cumulative distribution of the random variable $L_{ij}(t)$. Let $L_{ij}(p)$ denote the hourly median transmission loss which is not exceeded during p percent of all hours within a given time period, e.g. all of the hours in the winter or all of the hours in a year. Now we may define $y_{ij}(p)$ as follows:

$$y_{ij}(p) \equiv L_{ij}(50) - L_{ij}(p) \,. \tag{5}$$

Note that $y_{ij}(p)$ is positive for $p < 50\%$ and negative for $p > 50\%$. Similarly for the random variable $N_i(t)$ we may define $z_i(p)$ as follows:

$$z_i(p) \equiv N_i(50) - N_i(p) \,. \tag{6}$$

In the above $N_i(p)$ is the hourly median noise level exceeded for p percent of the hours in the same time period used for defining $y_{ij}(p)$. The random variables $L_{ij}(t)$ and $N_i(t)$ will tend to be normally distributed and we may let ρ_{ij} denote the correlation coefficient between $L_{ii}(t)$ and $L_{ij}(t)$ and ρ_{ii} the correlation coefficient between $L_{ii}(t)$ and $N_i(t)$.

Using the above definitions we may define factors S_{ii} and S_{ij}, expressed in decibels, which provide measures of the extent to which the inequalities (3) and (4) are satisfied for specified time availabilities p_{ii} and p_{ij}, respectively:

$$S_{ii} \equiv P_i - L_{ii}(50) - N_i(50) - R_{ii} - \left\{ y^2_{ii}(p_{ii}) + z_i^2(p_{ii}) + 2\rho_{ii}\, y_{ii}(p_{ii})\, z_i(p_{ii}) \right\}^{1/2} \qquad (7)$$

$$S_{ij} \equiv P_i - L_{ij}(50) - P_j + L_{ij}(50) - R_{ij} - \left\{ y^2_{ii}(p_{ij}) + y^2_{ij}(100 - p_{ij}) - 2\rho_{ij}\, y_{ii}(p_{ij})\, y_{ij}(100 - p_{ij}) \right\}^{1/2}$$

$$(j \neq i) \,. \qquad (8)$$

The above definitions would be exact if the hourly medians $L_{ij}(t)$ and $N_i(t)$ were normally distributed random variables. For the types of distributions actually encountered in practice (7) and (8) provide entirely adequate approximations.

If we set S_{ii} and S_{ij} equal to zero, then (7) and (8) may be solved for the percentage of the hours p_{ii} that the wanted signal on the i^{th} link is "expected" to be received with satisfactory fidelity in the presence of noise and the percentage of the hours p_{ij} that the wanted signal on the i^{th} link is "expected" to be received with satisfactory fidelity in the presence of the j^{th} interfering signal.

The values of S_{ii} and S_{ij} defined by (7) and (8) may be calculated by means of prediction formulas for the hourly median transmission loss, for the hourly median noise level, and for the long term variabilities of these quantities. Section 6 of this report contains a resume of the formulas available in the literature. Rather comprehensive formulas suitable for the prediction at frequencies above 40 Mc/s of the transmission loss and its long term variability for propagation paths involving only tropospheric modes of propagation have recently been published by Rice, Longley and Norton [1962]. It is convenient to assume that the errors of predicting S are normally distributed with a standard deviation σ_S which may be estimated by the methods given in the above reference. Now we may determine the service probability q_{ii} that the wanted signal on the i^{th} link will be received with satisfactory fidelity in the presence of noise for p_{ii} percent of the hours by setting $(S_{ii}/\sigma_{S_{ii}})$ equal to the standard normal deviate $\lambda(q_{ii})$ which is tabulated in Table I. The inverse function $q(\lambda)$ is defined by the following equation:

$$q(\lambda) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\lambda(q)} dx \exp(-x^2/2). \qquad (9)$$

Alternatively the service probability $q(\lambda)$ may be defined in terms of erf (x), the error function of x:

$$q(\lambda) = 0.5 \left\{1 - \text{erf}\left[\lambda(q) / \sqrt{2}\right]\right\} \quad \left[\text{For } q(\lambda) < 0.5,\ \lambda(q) < 0\right] \qquad (10)$$

$$q(\lambda) = 0.5 \left\{1 + \text{erf}\left[\lambda(q) / \sqrt{2}\right]\right\} \quad \left[\text{For } q(\lambda) > 0.5,\ \lambda(q) > 0\right] \qquad (11)$$

Table I

$q(\lambda)$	$\lambda(q) = (S/\sigma_S)$
0.001	-3.090
0.002	-2.878
0.005	-2.576
0.01	-2.326
0.02	-2.054
0.05	-1.645
0.1	-1.282
0.3	-0.524
0.5	0
0.7	+0.524
0.9	+1.282
0.95	+1.645
0.98	+2.054
0.99	+2.326
0.995	+2.576
0.998	+2.878
0.999	+3.090

Similarly we may determine the service probability q_{ij} that the wanted signal on the i^{th} link will be received with satisfactory fidelity in the presence of the j^{th} unwanted signal for p_{ij} percent of the hours by setting $\left(S_{ij}/\sigma_{S_{ij}}\right)$ equal to the standard normal deviate $\lambda(q_{ij})$.

The above analysis provides quantitative criteria for determining the merit of an assignment plan for the simultaneous operation of n links. These criteria may also be used to determine the merit of adding one more link to those already operating. As an example of how these criteria might be used, consider the problem of adding one more link to a line-of-sight microwave relay system. The first step in the procedure would be to tabulate the characteristics of all of the other communication links which might be expected to cause interference to the proposed additional link. Next terrain profiles should be obtained between the receiving terminal of the proposed link ($i = n$) and the terminals ($j = 1$ to n) of all of the potentially interfering links. Using these profiles predictions are next made of the cumulative distributions of the transmission loss and the noise for each of these paths; the noise on a microwave link may be considered constant so that $z_i(p_{ii}) \cong 0$ for all the values of p_{ii}. Finally, setting S_{ii} and $S_{ij} = 0$ in (7) and (8), these equations may be solved for the expected values of the time availabilities p_{nj} ($j = 1$ to n) of the desired signal on the proposed new link in the presence of $(n - 1)$ interfering signals ($j = 1$ to $n - 1$) and noise ($j = n$).

It is convenient to express these n time availabilities, p_{nj}, in terms of n probabilities $p'_{nj} \equiv (100 - p_{nj})/100$ that a satisfactory signal will <u>not</u> be available on the new link in the presence of the n sources of interference considered one at a time. Using these n probabilities we will now derive upper and lower bounds for the probability p'_n that a satisfactory signal will not be available on the n^{th} link; this probability p'_n will be derived from the n values of p'_{nj} and is thus also based on the simplifying assumption made earlier that several sources of interference present at a particular time do not further deteriorate the wanted signal more than it would be deteriorated by the worst of these n sources of interference acting alone. Thus $(1 - p'_n)$ is simply the probability that no one of the n sources of interference, acting independently, will deteriorate the wanted signal on the n^{th} link so that it is no longer satisfactory.

The general theory of probabilities of combined events [Feller, 1957] may be used for estimating the probability p'_n that a satisfactory signal will not be available on the n^{th} link. Let A_j (j = 1 to n) denote the events of not having a satisfactory signal available in the presence of each of the n sources of interference; thus the probability $P(A_j)$ that A_j will occur is equal to p'_{nj}. Now the probability $p'_n \equiv P(A_1 \text{ or } A_2 \text{ or } \cdots A_j \cdots \text{ or } A_n)$ that a satisfactory signal will not be available on the n^{th} link is given by:

$$P(A_1 \text{ or } A_2 \text{ or} \cdots A_j \cdots \text{or } A_n) = S_1 - S_2 + S_3 - S_4 + - \cdots \pm S_n \qquad (12)$$

where $S_1 = \Sigma P(A_1)$, $S_2 = \Sigma P(A_1 \text{ and } A_j)$, $S_3 = \Sigma P(A_i \text{ and } A_j \text{ and } A_k)$, etc.

In the above $i < j < k < \cdots \leq n$ so that in the sums each combination appears once and only once; hence S_r has

$$\{n(n-1)\cdots(n - r + 1)\} / \{1 \cdot 2 \cdots (r - 1) \cdot r\}$$

terms. The last sum S_n is the probability of the simultaneous realization of all n events. Feller [1957] has shown that S_1 is an upper bound to $P(A_1 \text{ or } A_2 \text{ or} \cdots A_j \cdots \text{or } A_n)$ and that this upper bound will be realized only when the n events A_j are mutually exclusive; thus the smaller of S_1 or 1 is the upper bound to p'_n. Let $p'_{n\ell}$ denote the largest of the n probabilities p'_{nj} and it is obvious that this is the minimum value which p'_n can have. Thus we have established the following bounds for p'_n:

$$p'_{n\ell} \leq p'_n \leq \text{The smaller of } \sum_{j=1}^{n} p'_{nj} \text{ or } 1 \qquad (13)$$

For example, if the fading is well correlated for all of the unwanted signals, the wanted signal will tend to be deteriorated only during that small fraction of the time represented by $p'_{n\ell}$ and p'_n will then approach this lower bound; for uncorrelated fading or for negatively correlated fading p'_n will be larger and may approach the upper bound given by (13). It follows from (13), regardless of the nature of the fading, that the time availability $p_n \equiv 100(1-p'_n)$ of satisfactory reception on the n^{th} link in the presence of n sources of interference is bounded by:

$$\text{The larger of zero or } \left\{100 - \sum_{j=1}^{n}(100 - p_{nj})\right\} \leq p_n \leq p_{ns} \qquad (14)$$

In the above p_{ns} denotes the smallest of the n time availabilities p_{nj}. As an example of the use of the above general formula, we will assume that three sources of unwanted signal interference and noise are present at the receiver of a fourth link and that the wanted signal on the fourth link will have separate time availabilities in the presence of these three interfering signals given by $p_{41} = 99\%$, $p_{42} = 98\%$, $p_{43} = 97\%$ and a time availability as regards noise given by $p_{44} = 98\%$. It follows from (14) that the wanted signal on link 4, regardless of the correlation of the fading of the unwanted signals, is expected to be satisfactory for a percentage of time bounded by:

$$92\% \leq p_4 \leq 97\% \tag{15}$$

The above bounds for the time availability in the presence of n sources of interference should be quite useful in those situations, which often occur in practice, for which it is not known whether the fading of the unwanted signals is completely independent or is positively or negatively correlated. If it is reasonable to assume that the long term variations in the unwanted signals occur completely independently, then specific estimates can be obtained for p'_n and p_n as follows. For independent events $P(A_i \text{ and } A_j \text{ and } A_k) = p'_{ni} p'_{nj} p'_{nk}$ so that

$$S_1 = \sum p'_{ni}, \quad S_2 = \sum p'_{ni} p'_{nj}, \quad S_3 = \sum p'_{ni} p'_{nj} p'_{nk}, \quad \text{etc.} \tag{16}$$

When the above values are substituted in (12) we obtain the following expression for p'_n which is applicable when the long term fading may be considered to be independent for the n sources of interference.

$$p'_n = \sum p'_{ni} - \sum p'_{ni} p'_{nj} + \cdots - \cdots \pm p'_{n_1} p'_{n2} \cdots p'_{nn} \qquad (17)$$

As an example the above formula will be applied to the same special case treated above. For this case $p'_{41} = 0.01$, $p'_{42} = 0.02$, $p'_{43} = 0.03$ and $p'_{44} = 0.02$; $S_1 = 0.08$, $S_2 = 0.0023$, $S_3 = 0.000028$ and $S_4 = 0.00000012$, and thus

$$p'_n = 0.08 - 0.0023 + 0.000028 - 0.00000012 = 0.07772788.$$

Note, however, that the formula (17) for p'_n for independent fading can also be written in the simpler form:

$$(1-p'_n) = (1-p'_{n_1})(1-p'_{n2}) \cdot \cdots \cdot (1-p'_{nn}) \qquad (18)$$

The above formula could have been derived more simply from the fact that the probability $(1-p'_n)$ of receiving a satisfactory signal in the presence of n sources of interference is simply equal to the product of the n probabilities of receiving a satisfactory signal in the presence of each interfering source considered separately when these interfering sources have independent probabilities of causing interference.

For the above example we find that the time availability in the case of these four sources of independently fading interference is given by $p_4 = 92.227212\%$. It is interesting to note that this value of p_4 for independent fading lies quite near the lower bound given by (15). In most applications, however, the fading of the n interfering sources will tend to be positively correlated, rather than independent, and under these circumstances p_n will be somewhat larger. Thus, for most practical purposes, we may consider that the value of p_n determined from (18) will be a more realistic lower bound for p_n than the lower bound determined by (14).

In order to decide whether the fourth link should be added at a particular geographical location it is desirable to determine not only the value of p_4 but also the values of the time availabilities p_1, p_2, and p_3 of satisfactory reception on the other three links both with and without the addition of the fourth link. An examination of these seven values of time availability should provide an objective quantitative basis for deciding (a) whether to add the fourth link at the location initially under consideration, (b) whether to choose some alternate location or (c) whether to abandon the fourth link altogether.

The above development of criteria for the satisfactory use of two or more communication links, although applied in the above example to links in a radio relay system, is actually quite general and provides a quantitative and objective basis for deciding on the desirability of proposed assignment procedures for any kinds of service operating anywhere in the radio spectrum. However, because of the somewhat unrealistic assumption made earlier in this section, the analysis will yield somewhat optimistic values for the time availabilities and thus it may sometimes be desirable to use the substantially more complicated but more accurate method of Norton, Staras and Blum [1952].

3. THE PRINCIPLE OF A HORIZONTAL INCREASE IN POWER

We will now show that the increase in the powers of n communication links by the same number of decibels will improve the operation of each of these n links by the reduction of the effects of noise but will not, contrary to popular belief, increase their mutual interference at all. This is true for every instant of time t as is evident from (3) and (4) if we interpret the values of $L_{ii}(t)$, $L_{ij}(t)$, $N_i(t)$, R_{ii} and R_{ij} in these equations as being instantaneous values. Thus, if P_i and P_j

are increased by the same number of decibels, the wanted signal-to-noise ratio $P_{ii}(t) - N_i(t)$ will increase on all n links i = 1 to n by this same number of decibels. However none of the wanted-to-unwanted signal ratios $P_{ii}(t) - P_{ij}(t)$ $\left\{ i=1 \text{ to } n \text{ and } j=1 \text{ to } n \right\}$ will change at all since $P_i - P_j$ will not change; this follows since P_i and P_j are both increased by the same amount for all combinations of i and j. If we set S_{ii} and S_{ij} equal to zero in (7) and (8) we will find that a horizontal increase in power will increase the time availability p_{ii} but will not cause any change in p_{ij}. It follows from this and equations (12) or (18) that the time availabilities p_{ia} (i=1 to n) of satisfactory reception on the n links after the horizontal increase in power will necessarily be larger than the time availabilities p_i (i=1 to n) before the horizontal increase in power. In the important special case of independent fading (18) may be expressed:

$$p_i = p_{i1} \cdot p_{i2} \cdot \cdots \cdot p_{in} / (100)^{n-1} . \tag{19}$$

It follows from (19) that:

$$p_{ia} = p_i \left\{ p_{iia} / p_{ii} \right\} . \tag{20}$$

Thus we conclude that the use of higher transmitter power on all communication links will usually lead to a more efficient use of the spectrum for any given assignment of stations. One usually minor technical qualification must be made to this otherwise general rule. The equations (3) and (4) are strictly applicable only to linear systems and non-linearities in the system may, in fact, lead to more harmonic radiation and to this extent, more mutual interference when higher power is used.

A decision relative to the desirability for making a horizontal increase in the powers of several mutually interfering links may be reached simply by deciding whether the larger time availabilities p_{ia} (i=1 to n) represent a sufficient improvement in performance to warrant the additional cost. If the values of p_i are determined largely by interference then p_i will be much less than p_{ii} and there will be little advantage in increasing the power. However, if p_i is determined largely by noise, then p_i will be only slightly less than p_{ii} and the outage time can be reduced by a large factor by increasing the power. This point can be illustrated by an example. Suppose $p_i = 90\%$, $p_{ii} = 99\%$ and $p_{iia} = 99.9\%$; such a change in p_{ii} would require a large increase in power but would only increase p_i from 90% to 90.82%, i.e. a reduction outage time from 10% to 9.18%. On the other hand if $p_i = 98.9\%$, $p_{ii} = 99\%$ and $p_{iia} = 99.9\%$, then the same increase in power would result in an increase of p_i from 98.9% to 99.8% and a reduction in outage time from 1.1% to 0.2%, i.e. by a factor greater than 5.

In most cases it would appear to be better to increase the power by different amounts on each link with the objective of making all values of p_i (i=1 to n) the same and equal to a sufficiently large value so that all links achieve their objectives. In this case all of the probabilities p_{ij} will change and the simple formula (20) is no longer available.

4. THE SERVICE PROBABILITY APPROACH TO FACILITY ASSIGNMENT DECISIONS

Frequently it will be found that a single source of interference will be of dominant importance. In such cases it is convenient to use the concept of service probability for making a facility assignment decision. This concept has already been described briefly in Section 2. To use this concept a particular value is chosen for the desired time availability p_{ij} and S_{ij} is then evaluated by (8). This value of S_{ij} can then be used, together with an estimate $\sigma_{S_{ij}}$ of its standard error, to determine $\lambda(q_{ij}) \equiv S_{ij}/\sigma_{S_{ij}}$. This value may then be used in (9), (10) or (11) to determine the service probability q_{ij} that the signal received on the i^{th} link will be of satisfactory fidelity or better for the desired percentage of the hours p_{ij}. Such probabilities have the advantage of making appropriate quantitative allowance for the effects of any errors in the prediction process and thus form a better basis for making a decision about the desirability of some new assignment.

We may also use the following procedure to obtain an estimate of the service probability q_{ig} of obtaining a satisfactory signal on the i^{th} link for a given percentage of time p_{ig} in the presence of n sources of interference. We first set $S_{ij} = 0$ in (8) and determine the "expected" value of p_i from:

$$p_i = 100(1-p'_i) = 100(1-p'_{i1})(1-p_{i2}) \cdot \cdot \cdot \cdot \cdot (1-p'_{in}). \tag{21}$$

In the above the individual values of $p'_{ij} = (100 - p_{ij})/100$ are obtained from the n solutions of (7) and (8) with j = 1 to n. If the "expected" value of p_i determined by (20) is either larger or smaller than the given value p_{ig}, then it follows that q_{ig} will be either larger or smaller, respectively, than 0.5. If $p_i > p_{ig}$, then (8) should be solved for the n values of p_{ij} obtained by setting all of the S_{ij} equal to some positive value which is to be chosen so that the resulting value of p_i as determined by (20) will be equal to p_{ig}. The proper choice for S_{ij} would have to be made by a process of successive approximation. Using this value of S_{ij}, together with the largest of the σ_{ij} (j = 1 to n), an estimate of q_{ig} can be obtained.

If such estimates of q_{ig} are obtained for the n links (i = 1 to n) and these values are all found to be greater than say 0.95, then the n proposed facility assignments may be considered to be technically feasible.

5. THE EFFICIENT ASSIGNMENT OF FACILITIES TO BROADCASTING SERVICES

It was mentioned in the introduction that all kinds of communications services, including broadcasting, can be considered to consist of a number of point-to-point communication links and thus all of the analysis in the preceding sections is also applicable to broadcasting. However, in the case of broadcasting, particularly in bands 8 and 9, it is more convenient to describe the service in a somewhat different way. Thus, suppose we are at a fixed distance d over land from a 100 Mc broadcasting station. Now consider all of the possible receiving locations at this distance as we change through 360° the azimuth of these locations relative to the broadcast transmitting antenna location. For each azimuth there will be a different terrain profile between the transmitting and receiving antennas and thus, for a given time availability p_{ii}, a different value of S_{iid} may be obtained from (7) for each azimuth. Now S_{iid} for the given distance d may be considered to be a random variable which is approximately normally distributed with a standard deviation σ_b. By setting $(S_{iid}/\sigma_b) = \lambda(q_{iid})$ we may determine the probability q_{iid} that a randomly chosen receiving location at the distance d will be provided with satisfactory service for at least p_{ii} per cent of the time. The magnitude of σ_b will vary with the nature of the profiles between the transmitting and receiving antennas, with the radio frequency, and, to some extent, with the receiving antenna height.

As the distance is increased for a given transmitter power, frequency, and antenna heights, the percentage of receiving locations, 100 q_{iid}, receiving satisfactory service will decrease from a value near 100% at short distances to a value near zero at very large distances.

The prediction of q_{iid} requires a prediction of both σ_b and the expected transmission loss as a function of distance for a given frequency and antenna height. Note that σ_b will be systematically larger than σ_{ii} since σ_b includes an allowance for the variance of the transmission loss at a fixed distance for the varying angular distances and other terrain profile features at the various azimuth angles.

The above analysis applies only to a clear channel broadcasting problem and the problem of mutual interference between two or more stations is substantially more complicated. As an introduction to this problem we will consider a typical pair of co-channel VHF television broadcasting stations. In the example given, the stations have a separation of 220 miles; this separation is typical since about half of the separations between neighboring co-channel stations in the United States are greater than 220 miles. On figure 1 the two irregular dotted curves show the decreasing field intensity of station A when using either 100 KW or 1000 KW of radiated power, and the two irregular solid curves show the decreasing field intensity of station B when using either 100 KW or 1000 KW of radiated power. The fields are shown along a line joining the two stations. Note that, although the fields decrease irregularly with distance because of the influence of terrain irregularities, at every location the fields are exactly 10 db stronger when the radiated power is 1000 KW rather than 100 KW. The fields shown are typical of a particular time of day and, in practice, the fields are known to vary from minute-to-minute and from hour-to-hour somewhat more at the larger than at the smaller distances; however, for any particular time, the 1000-KW fields will always be exactly 10 db stronger than the 100-KW fields.

There are two conditions which must be satisfied by the fields at a particular receiving location in order to provide a satisfactory picture to a television receiving installation: (1) the field of the desired station must be sufficiently strong so that it can overcome the noise level in the receiver, i.e., eliminate the obscuring effects of the video noise which appears on the face of the tube and has the appearance of snow, and (2) the differences in the fields from the desired and any undesired stations must be sufficiently great so that these undesired stations do not produce objectionable bars or other effects on the picture tube which interfere with the proper reception of the desired station. Both of these requirements on the desired and the undesired fields depend upon the excellence of the receiving installation. For example, when an indoor antenna and an inexpensive receiver are used, the field required to override the noise may be about 50 db above one microvolt per meter, and the desired field must

FIELDS EXPECTED ALONG A LINE BETWEEN TWO TYPICAL VHF (100 Mc) TELEVISION BROADCASTING STATIONS

ANTENNA HEIGHTS: 1000 FEET AND 30 FEET

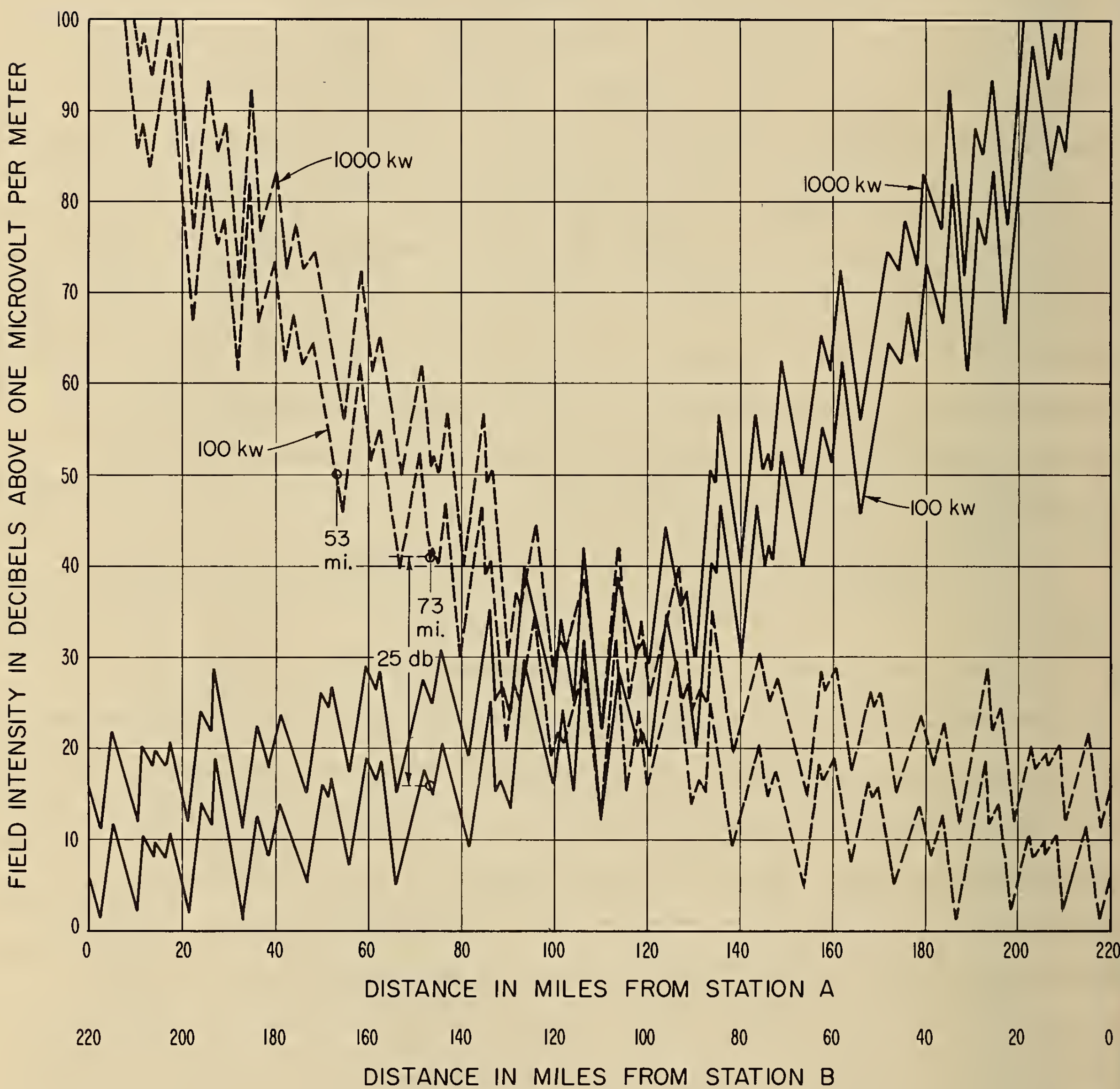

Figure 1

be 25 db* greater than the field from an undesired co-channel offset carrier in order to override the interference. We will call such an installation a grade-B receiving installation. On the other hand, for a grade-A receiving installation where a typical outdoor receiving antenna is used in conjunction with a fairly good (but not necessarily very expensive) receiver, the field required to override the noise will be about 40 db above one microvolt per meter, and the desired field need be only 15 db greater than the field from an undesired co-channel offset carrier in order to override the interference. The typical antenna characteristics used in arriving at the above figures were obtained from a private communication by A. C. Wilson. A 6-db gain and a 10-db front-to-back ratio were assumed; some of the antennas tested indicated more than 12 db gain and more than 20 db front-to-back ratio, so that the values assumed for our grade-A installation are far from being the best available.

The following table shows the minimum and maximum distances at which the fields satisfy the above two conditions for 100 KW and 1000 KW radiated power, and for grade-A and grade-B receiving installations. The minimum distances represent the nearest locations, as one leaves the transmitter, at which the fields first fail to satisfy one or the other or both of the above conditions, and the maximum distances represent the most remote locations from the transmitter at which the fields still satisfy those conditions. It is clear that the "effective" range of the station lies somewhere between the minimum

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* At this field intensity difference and with a zero front-to-back ratio (typical of an indoor antenna), half of the TASO observers rated the received picture to be somewhere between fine and excellent; see Fig. 41 of the TASO report [1959].

and maximum distances, at which both conditions are satisfied. Appropriate statistical methods have been developed to give a more precise meaning to the term "effective" range, but we need not go into that much detail in order to illustrate the technical principle involved in a horizontal increase in power.

Table I

Service Ranges Along the Line Between the Two Stations of the Accompanying Figure

	100 KW		1000 KW	
	Minimum Distance Miles	Maximum Distance Miles	Minimum Distance Miles	Maximum Distance Miles
Grade B -				
Noise Condition (1)	53	71	78	86
Interference Condition (2)	73	84	73	84
Both Conditions Satisfied	53	71	73	84
Grade A -				
Noise Condition (1)	66	86	88	97
Interference Condition (2)	87	87	87	87
Both Conditions Satisfied	66	86	87	87

It is easy to verify the distances in the above table by reference to figure 1. For example, the minimum distance at which the noise condition is satisfied for a grade-B receiving installation when 100 KW is used is 53 miles; this is the first distance at which the 100-KW field falls below 50 db above one microvolt per meter. As another example, 73 miles is the shortest distance at which the difference between the two 100-KW fields is less than 25 db.

A large number of general conclusions can be reached by an examination of the figures in the above table:

(1) In the first place it is most important to notice that the distance at which the interference condition is satisfied does not depend upon the power level, assuming that the power is changed by the same factor, i.e., by the same number of decibels, for all of the interfering stations.

(2) When higher power is used at all stations, the effective service ranges are increased and the fields at all receiving locations are improved.

(3) It is possible to increase the power to the point where interference, rather than noise, limits the reception. For the example given, the noise condition limits the effective range when 100 KW is used for both grade-A and grade-B receiving installations, while the interference condition limits the effective range when 1000 KW is used for both grade-A and grade-B receiving installations.

(4) The increase in power is more important for the less expensive grade-B receiving installation. For any broadcasting service, it is clearly more economical to use an expensive, high-power transmitting installation than many thousands of expensive receiving installations.

The above conclusions are generally applicable to all broadcasting services. Thus it appears that broadcasting stations should always be encouraged to use the maximum power which is consistent with economic feasibility up to the point at which interference, rather than noise, limits the service. In fact one can only claim to have made really efficient use of the spectrum for a broadcasting service when sufficient power is used for a given geographical configuration of stations so that it is not possible to find a single receiving location at which noise, rather than station interference, limits the reception for any percentage of the time. For typical VHF television broadcasting stations separated by 220 miles, we see by the above example that the technically desirable power is of the order of 1000 KW.

The above example illustrates some aspects of the nature of a broadcasting service. With a fixed separation between stations it appears that efficient use of the spectrum can be obtained only when sufficient power is used so that interference, rather than noise, provides the ineluctable limit to satisfactory reception. On the other hand, if the power is fixed, it has been shown [Norton and Fine 1949, Reference E and Norton 1950, Addendum to Reference E] that there is an optimum separation between the stations for efficient use of the spectrum. Recently Decker [1959 and 1962] has applied these concepts to determine an efficient allocation scheme for an airborne television network in band 9.

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APPENDIX I

ON FIGURES OF MERIT FOR RADIO RECEIVING SYSTEMS IN THE PRESENCE OF NOISE OR UNWANTED SIGNAL INTERFERENCE AND THE CONCEPTS OF OPERATING NOISE FACTOR AND OPERATING NOISE TEMPERATURE

1. Introduction

The optimum use of the radio spectrum requires that radio receiving systems be so designed that the reception of the wanted signals is immune to the greatest degree possible from interference by unwanted radio signals occupying the same or other radio frequency channels. This report proposes as figures of merit for a receiving system the protection ratios r_u (between the hourly median wanted and the hourly median unwanted signal powers available from a loss-free receiving antenna, which is otherwise equivalent to the actual receiving antenna) which are required for the satisfactory reception of the information carried by a specified wanted signal in the presence of specified kinds of unwanted signals. The use of receiving systems having the smallest values of r_u will permit the same portions of the spectrum to be used simultaneously by the maximum number of users.

On the other hand, economic considerations, as contrasted to spectrum conservation considerations, indicate that radio receiving systems should be designed so that the minimum possible transmitter power is required for satisfactory reception of the wanted signals in the presence of noise. This report proposes as an additional figure of merit for a receiving system the hourly median wanted signal power p_n (available at the terminals of the equivalent loss-free receiving antenna) which is required for the satisfactory reception of the wanted signals in the presence of noise, but in the absence of any other unwanted signals.

The figure of merit r_u involves only the ratio between the hourly median values of the wanted and unwanted signal powers, while p_n is a measure of the required magnitude of the hourly median wanted signal power. For optimum use of the spectrum by the maximum number of simultaneous users, the transmitting and receiving systems of the individual links should be designed with the primary objective of minimizing the various values of r_u involved; and then sufficiently high transmitter powers should be used so that the hourly median wanted signal power p_m exceeds p_n for a sufficiently large percentage of the hours during the intended period of reception at each receiving location. This approach to frequency assignment problems will be unrealistic in a few cases, such as the cleared channels required for radio astronomy, but these rare exceptions merely serve to prove the otherwise general rule that optimum use of the spectrum can be achieved only when interference from other signals rather than noise provides the ineluctable limit to satisfactory reception.

2. Definitions of Wanted and Unwanted Available Signal Powers

All signal and noise powers will be expressed in watts. Since it is confusing to use the decibel scale when noise powers are added, the convention will be adopted of using lower case letters to denote power in watts, or power ratios, and capital letters will be used to denote their decibel equivalents. For example, p will denote the wanted signal power available at the terminals of the equivalent loss-free receiving antenna expressed in watts, and $P = 10 \log_{10} p$ dbw.

Let $p_{a\nu} \equiv (dp_a / d\nu)$ denote the wanted signal power density in watts per cycle per second at the radio frequency ν which is available at the terminals of the actual lossy receiving antenna, and p is defined by:

$$p = \int_0^\infty \ell_{rc\nu}\, p_{a\nu}\, d\nu = \ell_{rc}\, p_a \ . \tag{1}$$

where $\ell_{rc\nu}$ is a factor greater than one which provides a measure of the antenna circuit loss at the frequency ν while p_a is the wanted signal power available at the terminals of the actual lossy receiving antenna. Similarly, p_u, $p_{ua\nu}$ and p_{ua} denote corresponding quantities for the unwanted signal. The available power from a generator, in this case the receiving antenna, is the power which would be available at the output of the generator if the output circuit were conjugately matched to the generator impedance. It is evidently desirable to calibrate a signal generator in terms of its open circuit voltage v, and then its available power is given simply by $s_g = v^2/(4\, R_g)$ for a generator with impedance $Z_g = R_g + j\, X_g$. Note that the available power from the generator has the desirable property of being independent of the load impedance, and this desirable property makes this concept especially useful for the definition and measurement of noise factors.

Let the impedance of the load be $Z_\ell = R_\ell + j\, X_\ell$; now it may be shown that the ratio of the available signal power s_g to the power s_ℓ delivered to the load is given by:

$$\frac{s_g}{s_\ell} = \frac{|Z_g + Z_\ell|^2}{4\, R_g\, R_\ell} \equiv \ell_m \geq 1 \ . \tag{2}$$

This mismatch loss factor ℓ_m will be equal to 1, and the delivered power s_ℓ will be equal to the available power s_g when the load and generator impedances are conjugately matched, i.e., when $R_\ell = R_g$ and $X_\ell = -X_g$.

The necessity for referring the wanted signal power to the terminals of an equivalent loss-free receiving antenna rather than to the actual antenna terminals or to the receiver input terminals will be established later but the use of this reference point clearly has the advantage of making the wanted signal powers independent of antenna circuit losses and this provides an added reason for defining the wanted signal powers in this way.

3. The Operating Noise Factor of a Receiving System

An operating noise factor was originally defined in a paper by North [1942], and characterizes the performance of the entire receiving system as contrasted to the receiver noise factor which characterizes only the performance of the receiver itself. Later, Norton [1953 and 1961] gave slightly more general expressions for this factor and designated it as an effective noise figure. This generalized operating noise factor makes appropriate allowance for the external noise picked up by the receiving antenna as well as the noise introduced by the receiver itself, together with the effects of any losses in the antenna circuit and in the transmission line. The purpose of this section is to give this more general formulation for an operating noise factor f of a receiving system, to describe its general properties, and to show how the operating noise temperature T_e of the receiving system may be determined from f. In contrast to the approach to unity of the noise factor f_n of an essentially noise-free network, the operating noise factor f of an essentially noise-free receiving system approaches zero.

Essentially all of the basic concepts used in this discussion originated in the early papers by Burgess [1941], North [1942] and Friis [1944] and in the discussion by North [1945] of the paper by Friis. A precise definition of the noise factor f_n of a four-terminal network was given by Friis [1944], and his definition forms the basis for the following formulation of the operating noise factor of a receiving system. Friis designated his noise factor f_n by the term noise figure, but North's terminology seems preferable since f_n is simply a dimensionless factor.

The available gain, $g_{n\nu}$, of a four-terminal network for a c.w. frequency ν is defined as the ratio of the available signal power, s_o, at the output terminals of the network to the available signal power, s_g, at the output terminals of the signal generator.

$$g_{n\nu} = s_o/s_g \qquad (s_o/s_g \geq 1). \tag{3}$$

Alternatively, the loss factor, $\ell_{n\nu}$, of a four-terminal network is given by:

$$\ell_{n\nu} = s_g/s_o \qquad (s_g/s_o \geq 1). \tag{4}$$

Note that $g_{n\nu}$ and $\ell_{n\nu}$ inherently contain a mismatch factor ℓ_m determined from the signal generator and network input impedances, and thus depend upon the generator impedance as well as upon the characteristics of the network itself. The four-terminal network will have some kind of band-pass characteristic and its effective signal gain or loss will depend upon the spectral distribution of the wanted signal within the pass-band. Two examples will be given to demonstrate the nature of the effective signal gain, g_s.

Consider first a single side band suppressed carrier signal with input available power spectral density $s_{g\nu}$. In this case the effective signal gain for the four terminal network is given by:

$$g_s = \frac{\int_0^\infty s_{g\nu}\, g_{n\nu}\, d\nu}{\int_0^\infty s_{g\nu}\, d\nu} \quad . \tag{5}$$

Note that g_s will be less than or equal to the maximum gain, g_n, within the pass band. In particular, if the receiver pass band is sufficiently detuned relative to the input signal the effective signal gain may even decrease to a value less than unity and its reciprocal would, in such a case, be considered to be the effective signal loss, ℓ_s.

Consider next a carrier, amplitude modulated with a single tone, so that the signal power is all contained in the two side bands with radio frequencies ν_a and ν_b and input r.m.s. voltages v_a and v_b which will be assumed to be in phase. Since the side band voltages will add coherently after detection the following general expression may be obtained for the effective signal power gain in this case:

$$g_s = \frac{\left(v_a \sqrt{g_{n\nu a}} + v_b \sqrt{g_{n\nu b}}\right)^2}{\left(v_a + v_b\right)^2} \quad . \tag{6}$$

For the usual case of equal sideband input voltages $(v_a = v_b)$ the above becomes:

$$g_s = 0.25\, g_{n\nu a} + 0.25\, g_{n\nu b} + 0.5 \sqrt{g_{n\nu a}\, g_{n\nu b}} \quad .$$

If the network is tuned so that $g_{n\nu a} = g_{n\nu b}$, then $g_s = g_{n\nu a}$ and this will usually be somewhat less than the maximum gain, g_n, within the pass band.

The effective signal gain for other types of modulation may be handled in a similar manner and the desirability for having a definition of g_s will be discussed later. However it is more convenient to define the noise factor and the effective noise bandwidth simply in terms of the maximum value, g_n, of $g_{n\nu}$ within the pass band or the minimum value, ℓ_n, of $\ell_{n\nu}$ within the pass band and then to allow separately for the effect on the required signal power, P_n, of the difference between g_s and g_n or the difference between ℓ_s and ℓ_n.

Since the gain will vary over the pass band of the network, we will follow North [1945] and define its effective noise bandwidth in cycles per second as:

$$b \equiv \frac{1}{g_n} \int_{\nu_1}^{\nu_2} g_{n\nu} \, d\nu = \frac{1}{hg_n} \int_0^{\infty} g_{n\nu} \, d\nu \; . \tag{7}$$

When we later consider several networks in tandem, b is to be determined using g_n and $g_{n\nu}$ as determined from the values of s_g and s_o available at the input and output, respectively, of this chain of networks:

$$h = \frac{\int_0^{\infty} g_{n\nu} \, d\nu}{\int_{\nu_1}^{\nu_2} g_{n\nu} \, d\nu} \; . \tag{8}$$

In the above, ν_1 and ν_2 are chosen so as to include only the principal response of the network, i.e., a band of frequencies in which the wanted signal power density is a maximum, and which is sufficiently wide so that $g_{n\nu 1}$ and $g_{n\nu 2}$ are negligibly small relative to the maximum value of $g_{n\nu}$ within this band. The factor h in (7) and (8) is a measure of any spurious responses which may be present in the receiver. The following discussion refers particularly to superheterodyne receivers and might have to be modified for other types of receivers. Note that

$h \geq 1$ and may sometimes be greater than 2 for superheterodyne receivers with little or no selectivity at the frequency converter input. In such receivers, spurious responses will be generated by cross modulation of signals or noise on various frequencies appearing on the frequency converter grid of the receiver. The frequency converter mixes the signal $s_{g\nu d}$ on a desired frequency ν_d within the principal response band ν_1 to ν_2 with the oscillator frequency ν_o of the receiver to produce a signal with intermediate frequency $\nu_i = \pm(\nu_d - \nu_o)$. Additional undesired voltages may arise in the intermediate frequency output by virtue of beats between the m^{th} harmonic of the oscillator frequency and the n^{th} harmonic of a spurious signal or noise with radio frequency ν_u not in the band ν_1 to ν_2 to produce the w^{th} subharmonic of the intermediate frequency. The spurious frequencies ν_u which may produce such additional responses in the intermediate frequency output may be determined from the following relation:

$$\pm(n\,\nu_u - m\,\nu_o) = \nu_i / w \tag{9}$$

The above may be solved for ν_u by setting n, m, and w equal to various integers and ν_i equal to some value within the range $|\nu_1 - \nu_o|$ to $|\nu_2 - \nu_o|$; the most important spurious response usually corresponds to $n = m = w = 1$, and the spurious frequencies ν_u for this response differ by $2\,\nu_i$ from the frequencies ν_d within the principal response band. We will assume that the noise voltages in the principal and in the spurious responses are not correlated so that the resulting noise power in the receiver intermediate frequency output is simply the sum of the output noise powers arising from the several responses; this assumption may not be valid in the case of atmospheric or man-made noise, and in such cases the factor h would require a different evaluation.

Note that it would have been possible to define the noise bandwidth by integrating over the entire band, thus obtaining the larger bandwidth hb. However, the intermediate frequency band in which the noise finally appears has the bandwidth b, and this is the motivation for the definition (7). Receivers having spurious responses will have noise factors based on the definition of b given by (7) which are larger by the factor h than the noise factors of corresponding receivers not having such responses. Consequently receiving systems having small values of $H \equiv 10 \log_{10} h$ will be superior to otherwise equivalent systems having larger values of H. H is a numerical measure of the response of a receiving system to spurious signals or noise which may be present outside of its principal response band ν_1 to ν_2, and thus provides a conditional figure of merit for a receiving system.

The noise factor, f_n, of a linear four-terminal network was defined by Friis [1944] as the ratio of the available c.w. signal-to-noise power ratio at the signal generator terminals to the available c.w. signal-to-noise power ratio, $[s_o/n_o]$, at its output terminals, with the available noise at the signal generator terminals set equal to the <u>reference</u> available Johnson noise power, $k\,T_o\,b$, from a resistance at the absolute reference temperature T_o and with the c.w. signal tuned to the maximum response of the network band-pass characteristic:

$$f_n = \frac{[s_g/(k\,T_o\,b)]}{[s_o/n_o]} \qquad \text{(Friis' Original Definition)} \qquad (10)$$

In the above, k is Boltzman's constant. When ℓ_n or g_n is substituted for the ratio of s_g and s_o in (10), the following alternative definitions are obtained for the noise factor of a network in terms of the available output noise power, n_o, and loss factor ℓ_n or gain g_n:

$$f_n = n_o\,\ell_n/(k\,T_o\,b) = n_o/(g_n\,k\,T_o\,b) \qquad \text{(Equivalent Definitions).} \qquad (11)$$

Note that the noise factor f_n depends upon the generator impedance as well as upon the characteristics of the network itself, since $g_{n\nu}$ depends upon the generator impedance. Thus one cannot usefully describe the noise performance of a network in terms of its noise factor alone without also specifying the impedance of the generator used in determining this noise factor. On the other hand the operating noise factor does provide a complete description of the noise performance of a receiving system since the generator impedance is the receiving antenna impedance in this case and this is an integral part of the receiving system.

The above definitions will now be used to derive an expression for the noise factor of the simple network of figure 1 with loss ℓ_n caused by its resistance, R_n, at an absolute temperature T_n. Let the resistance, R_g, of the signal generator be at the reference temperature T_o. The available signal power from the signal generator at the network input terminals is given in terms of its open circuit voltage, v, by:

$$s_g = v^2/(4R_g) \tag{12}$$

and the available signal power at the network output terminals is given by:

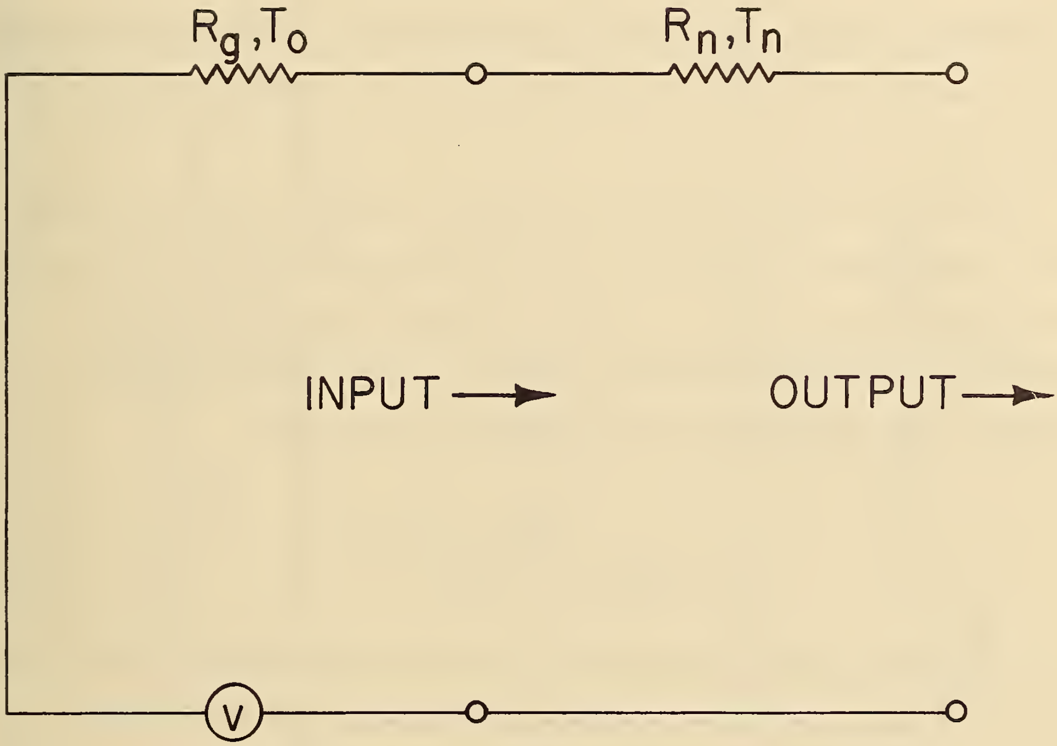
$$s_o = v^2/[4(R_g + R_n)] \cdot \tag{13}$$

Thus, from (6):

$$\ell_n = s_g/s_o = (R_g + R_n)/R_g \tag{14}$$

is obtained.

The available noise power at the output of the network of figure 1 is given by the weighted average of the Johnson noises from the resistances R_g and R_n at temperatures T_o and T_n:



$$f_n = 1 + (\ell_n - 1)(T_n / T_o)$$

$$\ell_n = (R_n + R_g) / R_g$$

Figure 1

$$n_o = kb\left\{\frac{R_g T_o + R_n T_n}{R_g + R_n}\right\} = \frac{kb}{\ell_n}\{T_o + T_n(\ell_n - 1)\}. \qquad (15)$$

When this is substituted in (11), the result is:

$$f_n = 1 + (\ell_n - 1)(T_n/T_o). \qquad (16)$$

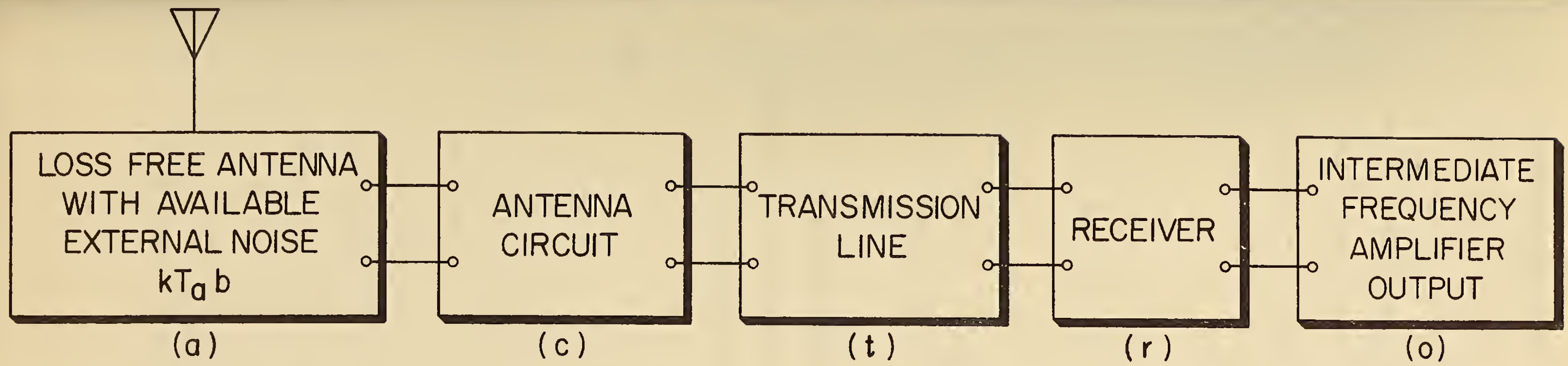
Note that the noise factor of a passive network at the reference temperature T_o is simply equal to its loss factor, i.e., when $T_n = T_o$, it follows from (16) that $f_n = \ell_n$.

Next, (16) will be derived in another way, and it will be shown that it is applicable in general to any lossy passive network with loss ℓ_n and temperature T_n, i.e., having arbitrary input and output impedances. Note that the noise output, n_o, of a linear passive network can be expressed as the sum of the two terms:

$$n_o = kbT_n + \frac{kb(T_o - T_n)}{\ell_n}. \qquad (17)$$

The first term would represent the available Johnson noise power from the network if its source resistance were also at the temperature T_n, while the second term represents a correction arising from the fact that the temperature T_o of the source resistance is different from that of the network, either higher or lower. For example, suppose that $T_o > T_n$; in this case the second term in (17) is the excess noise power $kb(T_o - T_n)$ available at the input reduced by the factor ℓ_n in passing through the network. When n_o, as given by (17), is substituted in the noise factor definition (11), (16) is obtained as before.

Using the above definitions and conventions, the operating noise factor f of the linear portion of the receiving system illustrated on Fig. 2 may now be discussed. Let p be the available signal power from an equivalent loss-free receiving antenna, let p_a be the available signal



(a)	(c)	(t)	(r)	(o)
$\ell_a = 1$	ℓ_{rc}	ℓ_{rt}		
T_a	T_c	T_t		
$f_a = T_a/T_o$	$f_c = 1 + (\ell_{rc} - 1)(T_c/T_o)$	$f_t = 1 + (\ell_{rt} - 1)(T_t/T_o)$	f_r	$r = s_o/n_o$

$$f = hf_a + (\ell_{rc} - 1)(T_c/T_o) + \ell_{rc}(\ell_{rt} - 1)(T_t/T_o) + \{\ell_{rc}\ell_{rt}(f_r - 1) - (h - 1)\}$$

$$T_e \equiv f T_o = hT_a + (\ell_{rc} - 1) T_c + \ell_{rc}(\ell_{rt} - 1) T_t + \{\ell_{rc}\ell_{rt}(f - 1) - (h - 1)\} T_o$$

THE LINEAR PORTION OF A RECEIVING SYSTEM

Figure 2

power from the actual receiving antenna, i.e., the available signal power at the antenna circuit output terminals, and let ℓ_{rc} be the loss factor of the receiving antenna circuit; thus $p \equiv \ell_{rc} p_a$. Let T_c denote the effective absolute temperature of the antenna circuit resistance exclusive of its radiation resistance, and (16) may be used to determine the noise factor of this portion of the antenna circuit network:

$$f_c = 1 + (\ell_{rc} - 1)(T_c / T_o). \tag{18}$$

Similarly the noise factor of the transmission line network with absolute temperature T_t and the line loss factor ℓ_{rt} is given by:

$$f_t = 1 + (\ell_{rt} - 1)(T_t / T_o). \tag{19}$$

The noise factor of the receiver itself considered as a four-terminal network will be designated f_r.

Friis [1944] gives an expression for the noise factor of three networks in tandem. This will be used to determine the noise factor of the three-network system (c), (t), and (r) of figure 2:

$$f_{ctr} = f_c + \ell_{rc}(f_t - 1) + \ell_{rc}\ell_{rt}(f_r - 1). \tag{20}$$

It will be convenient to represent the external noise power in the band $d\nu$ which is available at the terminals of the loss-free receiving antenna by $kT_{a\nu}d\nu$ where $T_{a\nu}$ is the receiving antenna radiation resistance noise temperature at the frequency ν. The concept and method of calculation of an effective temperature $T_{a\nu}$ of the receiving antenna have been described by Slater [1942] and by Lawson and Uhlenbeck [1948]. Representative values of $T_{a\nu}$ are given by Crichlow [1955] and by the C.C.I.R. [1956 and 1959] for frequencies $\nu < 10^8$, and are given by Blake [1961] for frequencies within the range $10^8 < \nu < 10^{10}$.

Several useful additional sources of information relative to $T_{a\nu}$ are available in the January 1958 Radio Astronomy Issue of the Proceedings of the I. R. E. and in papers by Hansen [1959] and Hogg and Mumford [1960]. Now the available noise power n_o at the predetection output of the complete receiving system may be represented, with the antenna replacing the signal generator having a reference temperature T_o, as the sum of two terms:

$$n_o = f_{ctr} k b T_o g_{ctr} + k \int_0^\infty (T_{a\nu} - T_o) g_{ctr\nu} d\nu \, . \qquad (21)$$

In practice, $g_{ctr\nu}$ may sometimes best be determined from its components using the relation $g_{ctr\nu} = g_{r\nu} / (\ell_{rc\nu} \ell_{rt\nu})$. The first term in (21) would represent the available noise power at the output of the receiving system if $T_{a\nu} = T_o$ for all values of ν, while the second term represents a correction arising from the difference between $T_{a\nu}$ and T_o. Since the integral in (21) has the limits 0 to ∞, it allows properly for all of the spurious responses as well as for the principal response.

It is useful now to define an effective antenna noise temperature T_a:

$$T_a = \frac{\int_0^\infty T_{a\nu} g_{ctr\nu} d\nu}{\int_0^\infty g_{ctr\nu} d\nu} = \frac{\int_0^\infty T_{a\nu} g_{ctr\nu} d\nu}{h b g_{ctr}} \, . \qquad (22)$$

Using this value of T_a, (21) may be expressed:

$$n_o = f_{ctr} k b T_o g_{ctr} + k h b (T_a - T_o) g_{ctr} \, . \qquad (23)$$

If this value of n_o is substituted in (11), and g_{ctr} for g_n, the following expression is obtained for the operating noise factor of the receiving system:

$$f = h(T_a/T_o) + (f_{ctr} - h) . \qquad (24)$$

Finally, if we follow C. C. I. R. Report No. 65 [1956] and replace (T_a/T_o) by f_a, substitute (20) for f_{ctr}, (19) for f_t and (18) for f_c, the following general expression for the operating noise factor of the receiving system described by the network of figure 2 is obtained:

$$f = hf_a + (\ell_{rc}-1)(T_c/T_o) + \ell_{rc}(\ell_{rt}-1)(T_t/T_o) + \{\ell_{rc}\ell_{rt}(f_r-1)-(h-1)\} . \qquad (25)$$

For the special case of a receiver with no spurious responses so that $h = 1$ and in which the antenna circuits and transmission line are at the reference temperature T_o, it follows that $f_c = \ell_{rc}$, $f_t = \ell_{rt}$, and (25) becomes:

$$f = f_a - 1 + f_c f_t f_r = f_a - 1 + \ell_{rc}\ell_{rt}f_r \qquad (T_c = T_t = T_o \text{ and } h = 1). \qquad (26)$$

The above is the expression for the operating noise factor derived by Norton [1953] and given in C. C. I. R. Report No. 65.

Note that the operating noise factor f for a noise-free receiving system, i. e., with $T_a = f_a = 0$, $f_c = f_t = 1$ and $f_r = h$ is equal to zero. We may establish that $f_r = h$ for a noise-free receiver with spurious responses as follows. The noise n_o available at the output of such a receiver may be expressed:

$$n_o = kT_o \int_0^\infty g_{r\nu} d\nu . \qquad (27)$$

If we substitute this value in (11), we obtain:

$$f_r = \frac{1}{g_r b} \int_0^\infty g_{r\nu} d\nu = h \cdot \tag{28}$$

Some experimenters prefer to specify the performance of low noise receiving systems in terms of an operating noise temperature T_e, and this is related to f and T_o or to h, T_a and f_{ctr} by:

$$T_e \equiv fT_o = hT_a + (\ell_{rc}-1)\, T_c + \ell_{rc}(\ell_{rt}-1)T_t + \{\ell_{rc}\ell_{rt}(f_r-1)-(h-1)\}\, T_o \cdot \tag{29}$$

It appears from the above that the operating noise temperature T_e depends not only upon the effective antenna temperature T_a and the excess noise temperature $(f_r-1)\,T_o$ of the receiver, but also depends upon the losses, mismatch conditions and spurious responses of the receiving system. Thus T_e can be identified with an actual temperature only by virtue of the fact that it has the dimensions of a temperature. This follows from the fact that the operating noise factor f is a dimensionless positive factor which is usually much greater than one, but which may be very much less than one for microwave receiving systems employing low-noise receiving antennas and masers. In fact, practical receiving systems have been developed with operating noise factors f substantially less than unity, so that F is actually negative and $T_e << T_o$; for example, DeGrasse et al [1960] have reported a value of $T_e = 18°K$ which corresponds to F = -12 db.

In summary, just as Friis defined the noise factor of a four-terminal network as the ratio of the input available c.w. signal-to-reference-noise ratio to the predetection output available c.w. signal-to-noise ratio: $f_n = [s_g/(kT_o b)]/[s_o/n_o]$, here the operating noise factor of a receiving system has been defined to be the ratio of the available c.w. signal-to-reference noise ratio from the equivalent loss-less receiving antenna to the predetection receiver output available c.w. signal-to-noise ratio: $f = [p/(kT_o b)]/[s_o/n_o]$. In this expression p denotes the unmodulated c.w. signal power available from the equivalent loss-free receiving antenna and the frequency ν of this unmodulated signal is adjusted so that $g_{ctr\nu}$ is equal to its maximum value g_{ctr}. Note now that $[s_o/n_o] = p/[fk\, T_o b] = p/[k\, T_e b]$ so that the c.w. signal-to-noise ratio $[s_o/n_o]$ at the predetection output is equal to the ratio of the c.w. signal power p to the operating noise power $fkT_o b$ or $kT_e b$ available at the terminals of the equivalent loss-free receiving antenna. As pointed out by Friis [1944], definitions in terms of available signal and noise powers rather than for necessarily matched conditions are advantageous not only because of their greater generality, but also because the use of mismatch conditions in amplifier input circuits often leads to a reduction in the noise factor of networks containing such amplifiers, [Llewellyn, 1931; Haus and Adler, 1958; Haus et al, 1960].

The noise factor concept is useful primarily for receiving systems with adequate predetection gain so that their maximum usable sensitivities are "noise limited" as discussed in C.C.I.R. Recommendation No. 234 [C.C.I.R. 1959]; this report is primarily concerned with such receiving systems but is also applicable to "gain limited" receiving systems as discussed in Section 5.

4. The Operating Signal-To-Noise Ratio

The operating signal gain, g_o, of the linear predetection portion of the receiving system may be defined as the ratio of the wanted signal power available at the predetection output of the receiver to the wanted signal power available at the terminals of the equivalent loss-free receiving antenna:

$$g_o = \frac{\int_0^\infty \ell_{rc\nu}\, p_{a\nu}\, g_{ctr\nu}\, d\nu}{\int_0^\infty \ell_{rc\nu}\, p_{a\nu}\, d\nu} \tag{30}$$

Note that this operating signal gain is exactly the same as the receiving system effective signal gain defined by (5) in the case of a suppressed carrier single side-band system* but will usually be somewhat different for other systems since it includes all of the radio frequency energy and not just that portion carrying the signal information. Now the operating signal-to-noise ratio, r, at the predetection output of the receiving system will be smaller than the tuned c.w. signal-to-noise ratio $[s_o/n_o]$ by the factor $[g_o/g_{ctr}]$ so that $r \equiv [s_o/n_o][g_o/g_{ctr}] = [p/(f\,k\,T_o b)][g_o/g_{ctr}]$. Expressed in decibels, the relation between the wanted signal power, P, available at the terminals of the equivalent loss-free receiving antenna and the operating signal-to-noise ratio R at the predetection output of the receiving system may be written:

$$P = F + B + R + G_{ctr} - G_o - 204 \text{ dbw} . \tag{31}$$

where

$F \equiv 10 \log_{10} f$, $R \equiv 10 \log_{10} r$, $B \equiv 10 \log_{10} b$, and $10 \log_{10} kT_o \equiv -204.00$

when $T_o = 288.39°$ Kelvin and $k = 1.38044 \times 10^{-23}$ [Cohen et al, 1957]. Since T_o must be somewhat arbitrarily chosen in any case, the above

*This follows from (5) when it is noted that the generator input power spectral density available at the terminals of the equivalent loss-free receiving antenna is equal to $s_{g\nu} = \ell_{rc\nu}\, p_{a\nu}$.

value was assigned so that the constant in (31) is equal to 204 for the currently best estimate of k; this choice of T_o is consistent with C.C.I.R. Report No. 65 [1956]. In engineering practice it is more convenient to use this easily remembered even decibel noise level reference -204 dbw or -174 dbm than to adopt the previously proposed values $T_o = 300°$, 290°, or 1° Kelvin as a reference. Note that the use of $T_o = 288.39°$ results in noise factors less than 0.024 db larger than those specified relative to a reference temperature $T_o = 290°$; however, the use of a reference temperature $T_o = 1° K$ would lead to noise factors which would be 24.6 db larger.

The difference $[G_{ctr} - G_o]$ between the tuned c.w. signal gain and the operating signal gain will usually be negligibly small since the receiver pass band will normally be designed to have a width somewhat larger than that ideally required for the reception of the wanted signal. This small difference could be absorbed in the definition of the effective noise bandwidth B but then B would depend upon the wanted signal characteristics as well as the characteristics of the receiving system and this seems undesirable. For this reason this small correction is given explicitly.

A temperature in degrees Kelvin is related to temperatures in degrees Celsius (centigrade) or in degrees Fahrenheit by:

$$T_{Kelvin} = 273.16 + T_{Celsius} = 255.38 + (5/9)T_{Fahrenheit} \quad . \qquad (32)$$

If the signal generator used for receiver noise factor measurements has its impedance at a temperature T_g rather than T_o, then a term $[1 - (T_g/T_o)]h$ should be added to the value so measured to determine f_r. It follows that an error in f_r of less than ± 0.1h will be made if T_g lies within the range 260°K to 317°K, i.e., within the range -13°C to 44°C or within the range 8°F to 111°F. Thus the use of the precise value $T_g = 288.39° K = 15.23°C = 59.42° F$ will be required only in connection with very precise noise factor measurements.

5. The Required Value P_n of Wanted Signal Power

The sensitivity of a receiving system and its over-all merit as regards its ability to overcome noise may conveniently be measured by $p_n = r_n f k T_o b = r_n k T_e b$ watts where r_n denotes the value of the product $r(g_{ctr}/g_s)$ required for the system under consideration to provide the specified grade of service. Note that r_n will depend upon the degree and the nature of the modulation of the wanted signal, as well as the degree to which the receiver pass-band matches and is aligned with the spectral characteristics of the wanted signal. In particular any drift of the pass-band will cause g_s to decrease and then r_n and thus p_n will increase correspondingly. In practice it is not necessary to measure g_s since r_n is most easily determined from directly measured values of p_n and b together with a measured value of either f or T_e by using the relations: $r_n \equiv p_n/(f k T_o b) = p_n/(k T_e b)$. The value of p_n is measured by changing the wanted signal power p, as defined by (1), until the grade of service actually provided is equal to the grade of service specified. For example a curve might be plotted of the error rate for a teletype receiving system versus the wanted signal power p and then p_n will be equal to that value of p corresponding to the value of error rate associated with the specified grade of service.

In many applications, particularly where atmospheric noise is involved, T_a and f_a will be quite variable with time, and in such cases it is useful to consider f and n_o to be random variables and to describe them in terms of appropriate statistical characteristics. In still other applications, such as space-satellite communications, f and n_o will be found to vary with the receiving antenna orientation since T_a, and thus f and T_e will vary as the antenna is pointed in different directions.

Since both the wanted signal and noise power may vary from minute to minute in a random unpredictable fashion, it is convenient to include the effects of these short-term variations of s_o, g_s and n_o in p_n and thus in r_n. Thus p_n should be considered to be the measured median value of p over a short period of time, say one hour, for which the grade of service provided under typical signal and noise fading conditions is just equal to the grade of service specified. To allow for possible effects of receiver drift on p_n, its value measured with the receiver tuned may be increased by the factor (g_{st}/g_{sd}) where g_{st} and g_{sd} are the effective signal gains with the receiver respectively tuned and then detuned by the amount expected to be exceeded under operating conditions for half of the time.

The figure of merit p_n may be expressed in decibels above one watt as:

$$P_n = R_n + F_m + B - 204 \text{ dbw} . \qquad (33)$$

In the above F_m denotes the median value of the operating noise factor F. Suppose now that the value R_r of operating signal-to-noise ratio required to provide the specified grade of service is determined directly at the predetection output of the receiver. In that case it follows from (31) that P_n is given by:

$$P_n = R_r + G_{ctr} - G_o + F_m + B - 204 \text{ dbw} \cdot \qquad (33a)$$

The above expression is applicable only to "noise limited" receiving systems with adequate predetection gain. If the directly determined required value P_n of wanted signal power is greater than the value determined by (33a) then the receiving system is said to be "gain limited". For such systems the concept of operating noise factor is not useful and the value of R_n determined by (33) will be greater than $(R_r + G_{ctr} - G_o)$.

In the special case of an amplitude modulated signal it is usually more convenient to determine the hourly median value of the wanted signal carrier power P_{nc} required to provide the specified grade of service. If R_{rc} denotes the median value of the operating carrier-to-noise ratio required to provide the specified grade of service at the predetection output then:

$$P_{nc} = R_{rc} + F_m + B - 204 \text{ dbw} \cdot \qquad (33b)$$

The above is applicable only for a "noise limited" receiving system tuned to the carrier.

6. Reference Point For the Operating Noise Factor

It has been suggested that the operating noise factor should be referred to the accessible input terminals of the receiver, i.e., at the output of the transmission line, rather than to the input to the equivalent loss-less receiving antenna terminals. The receiver input terminals might appear to have some advantage as a reference point since the signal-to-noise ratio can be directly measured at this point. However, it is easy to show by means of an example that the use of this point of reference will not yield an operating noise factor which provides an appropriate measure of the performance of the entire receiving system and this, after all, was the only purpose for defining this factor. For simplicity in the following example, we may let h = 1 and assume that all of the ambient temperatures are equal to the reference temperature. In this case the operating noise factor f referred to the equivalent loss-free receiving antenna terminals is given by (26). If an operating noise factor f_o were defined to be the ratio of the available signal to reference noise ratio at the output of the transmission line to the predetection receiver output signal to noise ratio, then

$$f_o = f/\ell_{rc}\ell_{rt} = \frac{f_a - 1}{\ell_{rc}\ell_{rt}} + f_r \qquad (T_c = T_t = T_o \text{ and } h = 1). \qquad (34)$$

Now consider two systems with $f_{a1} = 3$, $\ell_{rc1} = 2$, $\ell_{rt1} = 3$, $f_{r1} = 3$, and $f_{a2} = 5$, $\ell_{rc2} = 4$, $\ell_{rt2} = 3$, and $f_{r2} = 3$; for these systems, $f_1 = 20$ and $f_2 = 40$ when referred to the equivalent loss-free antenna terminals, while $f_{01} = f_{02} = 10/3$ when referred to the output of the transmission line. Although the first system is twice as good as the second, since the value of the power p_n required to provide the same signal-to-noise ratio at the output is half as large, one would conclude that the two systems have the same performance if the operating noise factor were referred to the output of the transmission line. It is concluded therefore that the only proper reference point for the operating noise factor and for the corresponding operating noise temperature of a receiving system is at the terminals of the equivalent loss-free receiving antenna.

Using the definitions given in C.C.I.R. Recommendation No. 241 and in C.C.I.R. Report No. 112 [1959] for the transmission loss L, the basic transmission loss L_b and the path antenna directivity gain G_p, the following expressions relate the power P available from the equivalent loss-less receiving antenna to the power P_t' radiated from the transmitting antenna:

$$P = P_t' - L = P_t' + G_p - L_b . \quad (35)$$

The radiated power p_t' is less than the power p_t'' delivered by the transmitter to the transmission line by a loss factor ℓ_{tr} which includes transmission line, mismatch and transmitting antenna circuit losses. Note that a concept of available power from a transmitter is not a useful one and thus $L_{tr} \equiv 10 \log_{10} \ell_{tr}$ is better defined as above and thus simply as the difference in decibels between the power delivered from the transmitter and the power radiated from the transmitting antenna:

$$L_{tr} \equiv P_t'' - P_t' . \quad (36)$$

Now (31), (35), and (36) may be combined to give the following general expression for the transmitter power required to provide a given predetection signal-to-noise ratio R:

$$P''_t = L_{tr} + L_b - G_p + F + R + G_{ctr} - G_o + B - 204 \text{ dbw} . \quad (37)$$

In those cases, particularly at the lower frequencies, where the transmitting and receiving antenna circuit losses are large, it might appear to be advantageous to use the concept of system loss L_s defined in C.C.I.R. Recommendation No. 241 [1959]:

$$L_s \equiv P_t - P_a \tag{38}$$

where P_t is the power input to the transmitting antenna and P_a is the available power from the actual lossy receiving antenna. The power input to the transmitting antenna p_t is less than the transmitter power p_t'' delivered to the transmission line by a factor ℓ_{tt} which may include transmission line and mismatch losses. Thus $L_{tt} = 10 \log_{10} \ell_{tt}$ is simply the difference in decibels between the transmitter power P_t'' delivered to the transmission line and the power P_t delivered to the transmitting antenna:

$$L_{tt} \equiv P_t'' - P_t \; . \tag{39}$$

Now (37) may be expressed:

$$P_t'' = L_{tt} + L_s + F_s + R + G_{ctr} - G_o + B - 204 \text{ dbw} \tag{40}$$

$$F_s \equiv 10 \log_{10} \{(T_{as}/T_o) + (\ell_{rt} - 1)(T_t/T_o) + \ell_{rt} (f_r - 1)\}(h = 1) \; . \tag{41}$$

Here F_s is a noise factor for the receiving system referred now to the terminals of the actual lossy receiving antenna, and $kT_{as}b$ represents the external noise available at these terminals. Note, however, that $(T_{as}/T_o) = \{f_a + (\ell_{rc} - 1)(T_c/T_o)\}/\ell_{rc}$, and thus ℓ_{rc} and T_c must still be known for a proper determination of the performance of the system. For this reason it appears that (40) is no simpler to use than (37) and, of course, these two equations must yield identical results when each of their terms is properly evaluated.

7. Other Considerations

At very high frequencies or at very low temperatures, the available noise power from a source at absolute temperature T will be less than kTb by the factor $(h\nu/kT)/\{\exp(h\nu/kT)-1\}$ as was shown by Nyquist [1928]; here h denotes Planck's constant. Since $(h\nu/kT) = 0.0479932\ \nu(\text{Gc/s})/T$ [Cohen et al, 1957], this correction represents less than 0.1 decibel reduction in the available noise power when $\nu(\text{Gc/s})/T$ is less than 0.955895, i.e., when $\nu < 276$ Gc/s at the reference temperature T_o or when $\nu < 9.5$ Gc/s for a temperature T = 10°K. Balazs [1957] has shown that the Johnson noise power available from a conductor also depends on the shape of the conductor at very high frequencies.

There has been some discussion in the literature [Haus and Adler, 1957 and Siegman 1961] of difficulties with "negative" resistances and their associated "negative" temperatures in some types of amplifiers; such considerations are important, however, only in the design and description of the various internal components of the receiver. In those cases where the receiver output impedance has a negative resistance, r may be re-interpreted to be the signal-to-noise ratio in the load impedance rather than the available signal-to-noise ratio at the intermediate frequency output. With the use of this convention in such cases, the noise factor f_r of any practical stable receiver can still be defined since such a receiver must have positive source and load resistances with positive temperatures at its input and output.

It is sometimes possible to reduce the operating noise factor f of a receiving system and thus to improve its performance by rearranging the order of its component parts. To see how this may be accomplished, we may use the formula of Friis for two networks in tandem. For two networks p and q, with noise factors f_p and f_q, the two networks in tandem with p preceding q will have a noise factor given by:

$$f_{pq} = f_p + (f_q - 1)/g_p \tag{42}$$

Alternatively, if q precedes p, we have:

$$f_{qp} = f_q + (f_p - 1)/g_q \tag{43}$$

From the above we obtain the condition which must be satisfied for $f_{pq} < f_{qp}$:

$$f_p + (f_q - 1)/g_p < f_q + (f_p - 1)/g_q \qquad \text{(For } f_{pq} < f_{qp}\text{)} \tag{44}$$

We will first consider the conditions under which it is advantageous to use a preamplifier p at the antenna terminals preceding the transmission line represented by network q. In this case $g_q = 1/\ell_{rt}$ and $f_q = 1 + (\ell_{rt} - 1)(T_t/T_o)$ so that we may write for the reduction in f with the preamplifier preceding the transmission line:

$$\Delta f \equiv f_{qp} - f_{pq} = (\ell_{rt} - 1)\left\{(f_p - 1) + (T_t/T_o)(1 - \frac{1}{g_p})\right\} \tag{45}$$

Since Δf is inherently positive, it follows that f will always be decreased by having a preamplifier precede the transmission line, and this reduction represents an improvement, expressed in decibels, given by $\Delta F = -10 \log_{10}\{(f_{pq} + \Delta f)/f_{pq}\}$.

Consider next the question of which of two amplifiers in a chain should precede the other. In this case we may subtract 1 from each side of (44) and, provided $g_p > 1$ and $g_q > 1$, we may re-write this inequality in the form:

$$\frac{f_p - 1}{1 - \frac{1}{g_p}} < \frac{f_q - 1}{1 - \frac{1}{g_q}} \qquad \text{(For } f_{pq} < f_{qp} \text{ provided } g_p > 1 \text{ and } g_q > 1\text{)} . \qquad (46)$$

If the above inequality is satisfied, it will be advantageous to have the amplifier p precede the amplifier q.

8. Radiometry

In radiometry, the desired signal is the noise available from the equivalent loss-free receiving antenna, and such a signal will be present in the spurious responses as well. In this case g_{ctr} and b, as defined by (5) and (7) are indeterminate; however, the factor $h\, b\, g_{ctr}$, which is required in (22) for the evaluation of T_a, may be evaluated [See (7)] by:

$$h\, b\, g_{ctr} = \int_0^\infty g_{ctr\nu} d\nu . \qquad (47)$$

The radiometer measures only an <u>average</u> antenna temperature T_a, and it is evident by (22) that the presence of spurious responses over a wide band will not yield a good estimate of $T_{a\nu}$ if this has different values for different receiver responses. Furthermore, the output noise voltages from various parts of the response band must be uncorrelated to the same extent as those from the calibrating white noise source for a valid determination of T_a.

9. Measurement of the Operating Noise Factor

Normally the operating noise factor of a receiving system would not be measured directly; instead each of its components would be measured, and then (25) used to combine these components. The following systematic procedure may be followed for determining each of these components under the actual conditions of matching or mismatching of the various elements in the receiving system. Let R_a denote the theoretical radiation resistance of the receiving antenna referred to its input terminals, and let R_i be its actual measured input resistance; in this case $\ell_{rc} = R_i/R_a$. A more general discussion of the method of determining ℓ_{rc}, including the effects of insulator losses is given by Crichlow [1955]. At the higher frequencies, ℓ_{rc} will usually be negligibly different from unity; however, in the case of reception with a unidirectional rhombic terminated in its characteristic impedance, ℓ_{rc} may be greater than 2 [Harper, 1941], since nearly half of the received power is dissipated in the terminating impedance and some is dissipated in the ground. Next the transmission line loss factor ℓ_{rt} is measured as the ratio s_g/s_o where s_g denotes the available power from a signal generator at the input to the line with its impedance adjusted so as to be equal to the measured impedance of the receiving antenna, and s_o denotes the resulting available power at the output of the line, i.e., the power which would be delivered to an output impedance conjugately matched to the output impedance of the line with the antenna connected to the input to the line. Finally, the noise factor f_r of the receiver is to be determined by using a c.w. signal generator at its input having the same impedance as the above-defined output impedance of the transmission line. Let the generator impedance have a temperature T_g and let s_d denote the available signal power at the frequency ν which is required to double the receiver output power (signal plus noise) relative to the output noise power with the signal generator turned off; then $f_r = [s_d g_{r\nu}/(g_r k T_o b)] + [1 - (T_g/T_o)] h$. The generator need not be tuned to the receiver in making this noise factor measurement; it is only necessary to use the appropriate values of $g_{r\nu}$ and of the product $g_r b$. The above procedures will yield the appropriate values of ℓ_{rc}, ℓ_{rt} and f_r to be used in (25) for the actual conditions of matching of the transmission line either to the receiving antenna or to the receiver. It will usually be desirable to use non-reflective matching of the line to the receiving antenna, since this will eliminate echoes which could be quite objectionable on long lines; to the extent that the characteristic impedance of the line is real, non-reflective matching will also result in the smallest value for ℓ_{rt}. However, it will sometimes be advantageous to mismatch the transmission line to the receiver in such a way as to minimize f_r; this latter point is discussed in the paper by Llewellyn [1931] and in the I.R.E. Standards [Haus et al, 1960]. Next $g_{ctr\nu}$ is determined over a frequency range which includes all of the receiver responses, g_{ctr} is the maximum value of $g_{ctr\nu}$ and b, h and T_a are then determined by means of their defining equations (7), (8), and (22).

The use of a dispersed signal source [noise generator] has been recommended [Haus et al, 1960] as a simpler method of measurement of the receiver noise factor since the bandwidth need not be separately measured in determining f_r. However, if this method is used in the case of a multiple response receiver, it is essential that the noise density from the generator be constant over a band which includes all of the receiver responses, and the apparent noise factor obtained in this way must be increased by the factor h in order to determine f_r; thus it would still be necessary in this case to measure both b and h in order to determine the noise factor of a multiple response receiver. Furthermore, the ultimate use of f_r is for the determination of P for a given value of R, and it is evident by (31) that its determination requires an equally accurate determination of both B and F. Consequently, the specification of the noise performance of a receiving system (or of a receiver) requires the determination of b and h as well as f (or f_r).

In measuring an available loss or gain of a four-terminal network, it may be inconvenient in some cases to make the signal generator output impedance the same as the output impedance of the preceding networks and the load impedance equal to the network output impedance with its input connected to the preceding networks. There are two cases in which this can be avoided. In case the load impedance is matched to the network output impedance, we may determine ℓ_n or g_n by replacing s_g in (3) or (4) by $s_g \ell_{mnp}/\ell_{mng}$ where ℓ_{mnp} denotes the mismatch factor (2) between the network input impedance and the output impedance of the preceding networks while ℓ_{mng} denotes the mismatch factor between the network input impedance and the signal generator impedance; when the signal generator impedance is made equal to the impedance of the preceding networks, $\ell_{mnp} = \ell_{mng}$ and no correction is involved. In case the signal generator impedance is made equal to the output impedance of the preceding networks, we may determine ℓ_n or g_n by replacing s_o in (3) or (4) by $s_\ell \ell_{mo\ell}$ where s_ℓ denotes the power delivered to the load and $\ell_{mo\ell}$ denotes the mismatch factor between the network output impedance and the load impedance.

Many radio relay systems contain a duplexer between the transmission line and the receiver so as to permit simultaneous transmission and reception from a single antenna. In case this duplexer has the same absolute temperature T_t as that of the transmission line, we may modify (25) as follows to determine the operating noise factor of a receiving system containing such a duplexer: ℓ_{rt} is now measured as the ratio s_g/s_o where, as before, s_g denotes the available power from a signal generator at the input to the line with its impedance equal to that of the receiving antenna, but s_o is now the resulting power available at the output of the duplexer. In addition, the following term is added to (25):

$\frac{\ell_{rc}\ell_{rt}}{g_r k T_o b}\int_0^\infty p_{d\nu} g_{r\nu} d\nu$, where $p_{d\nu}$ is the undesired local transmitter power density available at the output of the duplexer; it is assumed that the receiver output voltages arising from this source from the various responses are uncorrelated.

10. The Measurement of R_u

The value r_u of the wanted-to-unwanted signal power ratio required for a given quality or grade of service will depend upon the nature of the wanted and unwanted signals. Since both the wanted signal power p and the unwanted signal power p_u available at the terminals of the equivalent loss-free receiving antenna will usually vary from minute to minute in a random unpredictable fashion, it is convenient to include the effects of these short-term variations of p and p_u in determining r_u, and thus to consider $R_u \equiv 10 \log_{10} r_u$ to be the value of the ratio of the median wanted signal power p_m to the median unwanted signal power p_{um} required to provide a specified grade of service over a specified period. For example, R_u for Rayleigh fading wanted and unwanted teletype signals could be specified as the value of $10 \log_{10}(p_m/p_{um})$ which will provide a teletype character error rate E equal to 0.001, i.e., 0.1%, over a period of one hour. In case the rate of fading is sufficiently small so that fading durations at signal levels, say 20 db, below the median signal level are large on the average with respect to the duration of a teletype character, then it is possible to determine R_u for Rayleigh fading wanted and unwanted teletype signals and a character error rate E as follows. When p is Rayleigh distributed about a median value p_m while p_u is independently distributed in a Rayleigh distribution about a median value p_{um}, the probability Q that the instantaneous ratio (p_u/p) will be less than x is given by the simple expression:

$$Q[(p_u/p) < x] = \frac{x(p_m/p_{um})}{1 + x(p_m/p_{um})} . \qquad (48)$$

Now let $E_s(x)$ denote the empirically determined probability of error distribution for steady non-fading unwanted and wanted signals as a function of their ratio x. By combining the $E_s(x)$ distribution with the Q distribution of (p_u/p), it is possible to determine the error rate probability distribution E as a function of (p_m/p_{um}).

$$E(p_m/p_{um}) = \int_0^\infty E_s(x) \frac{\partial Q}{\partial x} dx . \qquad (49)$$

The function (49) may be used to determine $R_u = 10 \log_{10}(p_m/p_{um})_E$ corresponding to a teletype error rate E. In determining R_u, the power P_m must be much larger than P_n so that noise does not contribute to the error rate.

Telecommunication systems should be designed to have the smallest values of R_u which can be obtained so as to permit the same portions of the spectrum to be used simultaneously by the maximum number of simultaneous users. An example of large reductions in the required values of R_u obtainable by careful engineering is the use of precise offset carrier frequencies in television. It has been shown [Behrend, 1956 and Television Allocation Study Organization, 1959] that the interference between geographically adjacent television broadcasting stations occupying the same channel can be substantially reduced by the use of precisely offset carrier frequencies. Fine [1960] has made a detailed study of the Television Allocation Study Organization results and found that the ratio of the wanted-to-unwanted carrier levels required to provide a "passable" picture may be decreased from 23.3 db to 18.5 db by using carriers precisely offset by 10,010 ± 5 c/s as compared to the use of a nominal offset of 9985 ± 1000 c/s. Similarly, he found that this required ratio is reduced from 29.3 db to 20.2 db by using a precise offset of 20,020 ± 5 c/s as compared to a nominal offset of 19,995 ± 1000 c/s.

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List of Symbols

$f = (T_e/T_o)$	operating noise factor for the receiving system
f_m	hourly median value of the operating noise factor f
$f_a \equiv (T_a/T_o)$	effective receiving antenna noise factor
f_r	noise factor of the receiver
f_c	noise factor of the receiving antenna circuit
f_t	noise factor of the transmission line to the receiver
f_n	noise factor of a four terminal network
T_o	reference temperature equal to 288.39° Kelvin
$T_{a\nu}$	equivalent loss-less receiving antenna noise temperature in degrees Kelvin at the radio frequency ν
T_a	effective antenna noise temperature in degrees Kelvin for the particular receiving system with effective noise bandwidth b and spurious response factor h
$T_e \equiv f T_o$	operating noise temperature in degrees Kelvin for the particular receiving system
T_c	ambient temperature in degrees Kelvin of the receiving antenna circuit
T_t	ambient temperature in degrees Kelvin of the transmission line to the receiver
p or (p_u)	wanted or (unwanted) radio frequency signal power available at the terminals of the equivalent loss-free receiving antenna
p_a or (p_{ua})	wanted or (unwanted) radio frequency signal power available at the terminals of the actual receiving antenna
$p_{a\nu}$ or $(p_{ua\nu})$	wanted or (unwanted) radio frequency signal power density (watts per cycle per second) available at the terminals of the actual receiving antenna

p_n	hourly median value of the wanted signal power p which is required to provide the specified grade of service in the presence of noise alone
p_{nc}	hourly median value of wanted signal carrier power available at the terminals of the equivalent loss-free receiving antenna which is required to provide the specified grade of service in the presence of noise alone
p_t	power delivered to the transmitting antenna
p_t'	total power radiated from the transmitting antenna
p_t''	power delivered from the transmitter to the transmission line
$g_{n\nu}$	gain factor for the radio frequency ν; ratio of the power available at the output of a four terminal network to the power available at its input
g_n	maximum value of $g_{n\nu}$ within the pass band of the network
$g_{ctr\nu}$	receiving system gain factor for the radio frequency ν; ratio of the power available at the predetection output of the receiver to the power available at the terminals of the equivalent loss-free receiving antenna
g_{ctr}	maximum value of $g_{ctr\nu}$ within the pass band of the receiving system
g_s	effective signal gain factor for a four terminal network; this is the gain factor for the signal bearing components of the wanted radio frequency signal power
g_o	operating signal gain factor for the receiving system; ratio of the total wanted radio frequency signal power available at the predetection receiver output to that available at the terminals of the equivalent loss-free receiving antenna
$\ell_{n\nu}$	loss factor for the radio frequency ν; ratio of the powers available at the input and output, respectively, of a four terminal network

ℓ_{rc} — receiving antenna circuit loss factor

ℓ_{rt} — loss factor for the transmission line to the receiver

$\ell_{tr} \equiv (p_t''/p_t')$ — ratio of the transmitter power delivered to the transmission line to the total power radiated from the transmitting antenna

$\ell_{tt} \equiv (p_t''/p_t)$ — ratio of the transmitter power delivered to the transmission line to the power delivered to the transmitting antenna

b — effective noise bandwidth of a four terminal network; also used to denote the effective noise bandwidth of the linear portion of the receiving system with input terminals at the equivalent loss-free receiving antenna and output terminals at the predetection receiver output.

h — spurious response factor of the receiving system; also used to denote Planck's constant.

k — Boltzman's constant

$fkT_o b = kT_e b$ — operating noise power available at the terminals of the equivalent loss-free receiving antenna; a fictitious power.

r — operating signal-to-noise ratio; ratio of the wanted signal power to the noise power available at the predetection output terminals of the receiver

r_r — median value of r required to provide the specified grade of service

$r_n \equiv p_n/(f_m k T_o b)$ — ratio of the hourly median wanted signal power to hourly median operating noise power required at the terminals of the equivalent loss-free receiving antenna to provide the specified grade of service in the absence of all other sources of interference

r_u — ratio of the hourly median wanted signal power p_m to hourly median unwanted signal power p_{um} required at the terminals of the equivalent loss-free receiving antenna to provide the specified grade of service in the absence of all other sources of interference

$204.00 \equiv -10 \log_{10}(kT_o)$ the signal power at a radio frequency ν in decibels below one watt which is required at the terminals of the equivalent loss-free receiving antenna to produce an operating signal-to-noise ratio $r = 1$ at the predetection output of a receiving system with an effective noise bandwidth $b = 1$ cycle per second and an operating noise factor $f = 1$ or operating noise temperature $T_e = T_o$

$L \equiv 10 \log_{10}(p_t'/p) = L_b - G_p$ the transmission loss over the propagation path between loss-free antennas which are otherwise equivalent to the actual transmitting and receiving antennas

L_b the basic transmission loss; the transmission loss expected over the propagation path between loss-free isotropic transmitting and receiving antennas

$G_p \equiv L_b - L$ path antenna gain between loss-free transmitting and receiving antennas which are otherwise equivalent to the antennas used on the propagation path

$L_s \equiv 10 \log_{10}(p_t/p_a)$ system loss between the actual transmitting and receiving antennas used on the propagation path

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U. S. DEPARTMENT OF COMMERCE
Luther H. Hodges, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

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