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Technical Note 151

MODE CONVERSION IN THE EARTH-IONOSPHERE WAVEGUIDE

JAMES R. WAIT

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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MODE CONVERSION IN THE EARTH-IONOSPHERE WAVEGUIDE

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Abstract

An approximate theory for conversion of modes in an earth-ionosphere waveguide is propounded. The model is two concentric spherical reflecting boundaries which have prescribed surface impedances. The localized irregularity at either the ground or the ionosphere is idealized by a "black screen" which effectively blocks the cross-section of the waveguide over a portion of its area. In this sense, the method is a union of approximate Kirchoff diffraction theory and rigorous mode theory.

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1. Introduction

In the theory of VLF propagation it is usually assumed that the ionosphere is concentric with the surface of a spherical earth. In many actual cases, this is often an excellent approximation. In fact, even if the ionospheric height changes slowly in a horizontal direction the modes do not change their individual character. However, if the changes are abrupt, and occur in a very localized region, conversion of energy from one mode to another may result [Wait, 1962a].

In this paper the general problem is studied from a fresh viewpoint. A localized irregularity is imagined to be equivalent to blocking the aperture which is the cross-section of the waveguide. The method is applicable to irregularities at both the ionospheric reflecting layer as well as at the earth's surface. An example in the last category would be a mountain ridge which is transverse to the propagation path.

The method is based on a union of approximate Kirchoff diffraction theory and rigorous mode theory. To simplify the discussion, the problem is considered to be two-dimensional in nature and, thus, the equivalent source is a line dipole.

2. Formulation

We initiate the analysis with a simple model. The earth's surface and a reflecting layer are represented by two concentric cylindrical surfaces with curvature radii of a and $a + h$, respectively. In terms of cylindrical coordinates (r, θ, z) concentric surfaces are $r = a$ and $a + h$. The situation is illustrated in Fig. 1. The surface impedances at the lower and upper surface are denoted by Z_g and Z_i, respectively. The fields are assumed to vary according to $\exp(i \omega t)$.

The fields in such a region can be expressed as a superposition of transverse electric (TE) and transverse magnetic (TM) modes. We will just consider the TM modes since they are of greatest practical interest at VLF. The analysis for the TE modes is almost identical.

For the TM modes the magnetic field has only an axial component H (i. e., out of the paper in Fig. 1). The electric field has only r and θ components and they can be obtained simply by differentiating H.

We now imagine that the fields at the aperture plane $\theta = \theta_o$ are known. For example $H(\theta_o, r)$ is specified over the interval $a < r < a + h$. Thus, for the region $\theta > \theta_o$ the field will consist of TM modes travelling in the positive θ direction. Furthermore, these modes must satisfy the boundary conditions

$$E_\theta = -Z_g H \quad \text{at} \quad r = a \tag{1}$$

$$E_\theta = Z_i H \quad \text{at} \quad r = a + h . \tag{2}$$

Under the assumptions

$$ka >> 1 ,$$

$$h/a << 1 ,$$

$$\theta << 1 ,$$

it was shown previously that [Wait, 1961] the field could be represented in the form

$$H = H(x, y) = \sum_{n=1,2,3} A_n \Phi(t_n, y)\, e^{-i(x-x_o)t_n} \tag{3}$$

where

$$x = \left(\frac{ka}{2}\right)^{\frac{1}{3}} \theta, \quad x_o = \left(\frac{ka}{2}\right)^{\frac{1}{3}} \theta_o, \quad y = \left(\frac{2}{ka}\right)^{\frac{1}{3}} k(r-a), \quad \text{and} \quad y_o = \left(\frac{2}{ka}\right)^{\frac{1}{3}} kh .$$

The function Φ satisfies the differential equation

$$\left(\frac{d^2}{dy^2} - t + y\right) \Phi(t, y) = 0 \tag{4}$$

and t_n are obtained from

$$\left.\frac{d}{dy} \Phi(t_n, y)\right]_{y=0} + q\,\Phi(t_n, 0) = 0 \tag{5}$$

where

$$iq = \left(\frac{ka}{2}\right)^{\frac{1}{3}} \frac{Z_g}{120\,\pi} ,$$

and

$$\left.\frac{d}{dy} \Phi(t_n, y)\right]_{y=y_o} - q_i\, \Phi(t_n, y_o) = 0 \tag{6}$$

where

$$i\,q_i = \left(\frac{ka}{2}\right)^{\frac{1}{3}} \frac{Z_i}{120\,\pi} .$$

The latter two equations are equivalent to the boundary conditions given by (1) and (2).

Solutions of (4) are linear combinations of the Airy integral functions $w_1(t-y)$ and $w_2(t-y)$. In terms of the functions $Ai(\alpha)$ and $Bi(\alpha)$, defined and tabulated by Miller [1946],

$$w_1(\alpha) = \sqrt{\pi}\,[Bi(\alpha) - i\,Ai(\alpha)] \tag{7}$$

and

$$w_2(\alpha) = \sqrt{\pi}\,[Bi(\alpha) + i\,Ai(\alpha)] \,. \tag{8}$$

It is easy to verify that

$$\Phi(t_n, y) = w_1(t_n - y) + A(t_n)\, w_2(t_n - y) \quad , \tag{9}$$

where

$$A(t) = - \left[\frac{w_1'(t - y_o) + q_i\, w_1(t - y_o)}{w_2'(t - y_o) + q_i\, w_2(t - y_o)} \right] \quad , \tag{10}$$

satisfies (6). Furthermore, if

$$A(t_n)\, B(t_n) = 1 = e^{-i 2 \pi n} \tag{11}$$

where

$$B(t) = - \left[\frac{w_2'(t) - q\, w_2(t)}{w_1'(t) - q\, w_1(t)} \right] \tag{12}$$

it is seen that Φ also satisfies (5).

It should be mentioned at this stage that the surface $r = a + h$ can be regarded as a reference surface where the ratio of the tangential fields are specified. In the general case Z_i or q_i may be a function of the eigen-values t_n. However, for VLF it is a good approximation to regard Z_i or q_i as constants. This is equivalent to stating that the surface impedance does not depend on the angle of incidence. In a similar manner Z_g or q can be regarded as constants.

The orthogonality properties of the modes are now studied. We consider two sets of values, t_n and t_m, which satisfy the boundary equations (5) and (6). However, for any value of y these must also satisfy

$$\frac{d^2}{dy^2} \Phi_n - (t_n - y) \Phi_n = 0 \quad , \quad \Phi_n = \Phi(t_n, y) \tag{13}$$

and

$$\frac{d^2}{dy^2} \Phi_m - (t_m - y) \Phi_m = 0 \quad , \quad \Phi_m = \Phi(t_m, y) \; . \tag{14}$$

After multiplying the first of these equations by Φ_m and the second by Φ_n they are subtracted from one another. Both sides of the resulting equations are then integrated with respect to y over the range 0 to y_o. This results in

$$\Phi_n \frac{d}{dy} \Phi_m - \Phi_m \frac{d}{dy} \Phi_n \Big]_o^{y_o} = (t_n - t_m) \int_o^{y_o} \Phi_n \Phi_m \, dy \; . \tag{15}$$

In view of the boundary conditions on Φ_n and Φ_m at $y = 0$ and $y = y_o$, the left-hand side of the preceding equation is zero. Thus, the integral on the right also vanishes if t_n is not equal to t_m. Therefore, we have the important result

$$\int_0^{y_o} \Phi(t_m, y)\,\Phi(t_n, y)\,dy = 0 \quad \text{if} \quad m \neq n . \tag{16}$$

It now follows that, if both sides of (3) are multiplied by $\Phi(t_m, y_o)$ and integrated from 0 to y_o,

$$A_n = \frac{\int_0^{y_o} H(x_o, y)\,\Phi(t_n, y)\,dy}{\int_0^{y_o} [\Phi(t_n, y)]^2\,dy} . \tag{17}$$

The normalizing integral

$$N_n = \int_0^{y} [\Phi(t_n, y)]^2\,dy \tag{18}$$

is now expressed in a more convenient form. This is accomplished by noting that

$$\int [\Phi(t_n, y)]^2\,dy = -(t_n - y)[\Phi(t_n, y)]^2 + [\Phi'(t_n, y)]^2 . \tag{19}$$

This can be proved by differentiating both sides with respect to y and making use of (13). Then using the definition of $\Phi(t_n, y)$ in terms of Airy functions, it follows that

$$\Phi(t_n, 0) = -\frac{2i}{w_2'(t_n) - q\, w_2(t_n)} \tag{20}$$

and

$$\Phi(t_n, y_o) = -\frac{2i}{w_2'(t_n - y_o) + q_i\, w_2(t_n - y_o)} \tag{21}$$

where use is also made of the Wronskian condition

$$w_1(t)\, w_2'(t) - w_1'(t)\, w_2(t) = -2i \tag{22}$$

which is valid for any value of t. Finally, on making use of (19), (20), and (21) along with (5) and (6), it is found that

$$N_n = -\frac{4(t_n - q^2)}{[w_2'(t_n) - q\, w_2(t_n)]^2} + \frac{4(t_n - y_o - q_i^2)}{[w_2'(t_n - y_o) + q_i\, w_2(t_n - y_o)]^2} \,. \tag{23}$$

This can be regarded as a fairly important result.

Equation (17) for the coefficient A_n can be written in the convenient form

$$A_n = \frac{2\Lambda_n}{y_o} \frac{\int_0^{y_o} H(x_o, y)\, \Phi(t_n, y)\, dy}{[\Phi(t_n, 0)]^2} \tag{24}$$

where

$$\Lambda_n = \frac{y_o}{2}\left[(t_n - q^2) - (t_n - y_o - q_i^2)\left(\frac{w_2'(t_n) - q\, w_2(t_n)}{w_2'(t_n - y_o) + q_i\, w_2(t_n - y_o)}\right)^2\right]^{-1} . \quad (25)$$

It is now imagined that the incident field results from an equivalent line magnetic source at $x = 0$ (i.e., $\theta = 0$). Thus,

$$H(x, y) = \sum_m a_m \, \Phi(t_m, y)\, e^{-ix\, t_m} \quad \text{for} \quad x < x_o \quad (26)$$

where a_m is a coefficient which does not depend on x or y. It is assumed that in the aperture plane $x = x_o$ is obstructed in such a manner that the effective aperture is a slit extending from $y = y_1$ to y_2 (i.e., $r - a = z_1$ to z_2). The situation is illustrated in Fig. 2. Thus, within the Kirchoff approximation,

$$\begin{aligned} H(x_o, y) &= \sum a_m \, \Phi(t_m, y)\, e^{-ix_o t_m} \quad \text{for } y_1 > y > y_2 \quad (27) \\ &= 0 \quad \text{for} \quad 0 < y < y_1 \\ &= 0 \quad \text{for} \quad y_2 < y < y_o \, . \end{aligned}$$

In other words, we are assuming that the field within the aperture of the slit has the same value as if the slit were not present. It is known from a study of the rigorous solutions of diffraction by slits that this is an excellent approximation [Born and Wolf, 1959] provided the width of the slit is greater than about a wavelength.

The field in the region $x > x_o$ can now be expressed in the form

$$H(x, y) = \sum_m \sum_n A_n^{(m)} \Phi(t_n, y) e^{-i(x-x_o)t_n} e^{-ix_o t_m} \tag{28}$$

where

$$A_n^{(m)} = \frac{2\Lambda_n}{y_o} \frac{\int_{y_1}^{y_2} \Phi(t_m, y)\Phi(t_n, y)\,dy}{[\Phi(t_n, 0)]^2} a_m . \tag{29}$$

We see clearly that the incident mode of order m excites modes of order n where m and n are positive integers.

It is convenient to write

$$A_n^{(m)} = \left[P_n^{(m)} + Q_n^{(m)}\right] a_m \tag{30}$$

where, for $m \neq n$,

$$P_n^{(m)} = -\frac{2\Lambda_n}{y_o} \frac{\Phi(t_m, 0)}{\Phi(t_n, 0)} \int_o^{y_1} G_m(y) G_n(y)\,dy, \tag{31}$$

and

$$Q_n^{(m)} = -\frac{2\Lambda_n}{y_o} \frac{\Phi(t_m, 0)}{\Phi(t_n, 0)} \int_{y_2}^{y_o} G_m(y) G_n(y)\,dy , \tag{32}$$

where

$$G_n(y) = \frac{\Phi(t_n, y)}{\Phi(t_n, 0)} \tag{33}$$

and

$$G_m(y) = \frac{\Phi(t_m, y)}{\Phi(t_m, 0)} \quad . \tag{34}$$

In obtaining the above forms for $P_n^{(m)}$ and $Q_n^{(m)}$ use has been made of the orthogonality condition given by (16).

When $m = n$, we have

$$A_n^{(n)} = \left[P_n^{(n)} + Q_n^{(n)}\right] a_n \tag{35}$$

where

$$P_n^{(n)} = 1 - \frac{2\Lambda_n}{y_o} \int_o^{y_1} [G_n(y)]^2 \, dy \tag{36}$$

and

$$Q_n^{(n)} = 1 - \frac{2\Lambda_n}{y_o} \int_{y_2}^{y_o} [G_n(y)]^2 \, dy \tag{37}$$

where use has been made of (23).

The integrals over the range 0 to y_1 in the preceding equations can be regarded as the influence of the obstacle on the ground, whereas the integrals over y_2 to y_o are related to the protuberance at the ionosphere. To evaluate these integrals it is desirable to expand the G functions as a power series in y.

Since

$$G_n(0) = 1 \tag{38}$$

$$[d\, G_n(y)/d\, y]_{y=0} = -q \tag{39}$$

and

$$\frac{d^2\, G(y)}{d\, y^2} = (t_n - y)\, G(y) \quad \text{for any} \quad y, \tag{40}$$

it is not difficult to show that

$$G_n(y) = 1 - q\, y + \frac{t_n\, y^2}{2} - \frac{(1 + t_n\, q)}{6}\, y^3 + \ldots \; . \tag{41}$$

Thus,

$$G_n(y)\, G_m(y) = 1 - 2\, q\, y + (t_n + t_m + 2\, q^2)\, \frac{y^2}{2}$$

$$- (1 + 2\, t_n\, q + 2\, t_m\, q)\, \frac{y^3}{3} + \ldots \tag{42}$$

and the expansion for $[G_n(y)]^2$ is obtained by simply replacing t_m by t_n in the preceding result.

The integrations for the P integrals are now readily carried out. They yield

$$P_n^{(m)} = - \frac{2\, \Lambda_n}{y_o}\, g_{m,\,n} \left[y_1 - q\, y_1^2 + (t_n + t_m + 2\, q^2)\, \frac{y_1^3}{6} \right.$$

$$\left. - (1 + 2\, t_n\, q + 2\, t_m\, q)\, \frac{y_1^4}{12} + \ldots \right] \tag{43}$$

where

$$g_{m,\,n} = \frac{\Phi(t_m, 0)}{\Phi(t_n, 0)} = \frac{w_2'(t_n) - q\, w_2(t_n)}{w_2'(t_m) - q\, w_2(t_m)} \tag{44}$$

and

$$P_n^{(n)} = 1 - \frac{2\Lambda_n}{y_o}\Big[y_1 - q\,y_1^2 + (t_n + q^2)\,\frac{y_1^3}{3} - (1 + 4\,t_n\,q)\,\frac{y_1^4}{12} + \ldots\Big]. \qquad (45)$$

The Q integrals are evaluated in a very similar manner. Thus

$$Q_n^{(m)} = -\frac{2\Lambda_n}{y_o}\,g_{m,n}\,G_m(y_o)\,G_n(y_o)\Big[(y_o - y_2) - q_i(y_o - y_2)^2 + (t_n + t_m + 2\,q_i^2)\,\frac{(y_o - y_2)^3}{6} - (1 + 2\,t_n\,q_i + 2\,t_m\,q_i)\,\frac{(y_o - y_2)^4}{12} + \ldots\Big] \qquad (46)$$

and

$$Q_n^{(n)} = 1 - \frac{2\Lambda_n}{y_o}\,[G_n(y_o)]^2\Big[(y_o - y_2) - q_i(y_o - y_2)^2 + (t_n + q_i^2)\,\frac{(y_o - y_2)^3}{3} - (1 + 4\,t_n\,q_i)\,\frac{(y_o - y_2)^4}{12} + \ldots\Big] \qquad (47)$$

Due to the typically large values of q_i at VLF, the preceding series for the Q functions are probably not useful. It would be better to work directly with equations (32) and (37).

3. Discussion of Formulae

Some of the previous results are now discussed briefly. For purposes of illustration, it is assumed that the ionosphere is a sharply bounded medium whose effective conductivity is $\varepsilon_o\,\omega_r$ where $\varepsilon_o = 8.854 \times 10^{-12}$. Under this condition extensive numerical values of the coefficients t_n satisfying equation (11) are available [Spies and Wait, 1961]. Using these values, the various quantities entering into the formulas for the modal coefficients can be evaluated in a straightforward manner.

It is seen that the modal coefficients, given by equations (43), (45), (46), and (47) all contain the factor Λ_n. This factor is a modal excitation factor and it is a measure of the efficiency of excitation of a given mode from a line or dipole source [Wait, 1961, 1962b]. In the present context it is normalized so that it approaches unity for perfect ground conductivity $(q = 0)$ and a flat earth $(a = \infty)$. In general it is a complex quantity. To illustrate its behavior ω_r is set equal to 2×10^5 and h is taken as 70 km. Furthermore, the ground is assumed to be perfectly conducting. Under these conditions Λ_n for n = 1 has the following complex values for the frequencies indicated:

$$\begin{aligned} \Lambda_1 = \; & 0.95 \,\underline{|3.8^\circ} \text{ (10 kc/s)}, \quad 0.79 \,\underline{|3.0^\circ} \text{ (15 kc/s)}, \\ & 0.59 \,\underline{|4.6^\circ} \text{ (20 kc/s)}, \quad 0.37 \,\underline{|7.4^\circ} \text{ (25 kc/s)}, \\ & 0.19 \,\underline{|10.6^\circ} \text{(30 kc/s)}. \end{aligned} \tag{48}$$

This mode corresponds to the mode of least attenuation. It is characterized by an excitation factor which decreases approximately as the inverse of the frequency. Under the same conditions Λ_n, for n greater than 1, is roughly unity over the same frequency range [Wait, 1962b].

The modal coefficient $P_n^{(n)}$ defined by (45), in the case of $n = 1$, can be written

$$P_1^{(1)} \cong 1 - 2\,\Lambda_1 \frac{h_1}{h} \,. \tag{49}$$

Here, h_1/h is the ratio of the heights of the obstacle on the ground to the height of the ionosphere. This quantity would never be greater than about 0.05 and thus the modification of the first mode by even an extremely high mountain range would be small. This is particularly the case at the upper end of the VLF band where the excitation factor is small.

The relative conversion of the field from an incident mode of order 1 to a mode of order 2 is obtained from the factor $P_n^{(m)}$ defined by (43) for $m = 1$ and $n = 2$. Approximately, this can be written

$$P_2^{(1)} \cong - 2\,\Lambda_2\, g_{1,2}\, (h_1/h) \,. \tag{50}$$

The complex quantity $g_{1,2}$ is defined by equation (44) for $m = 1$, $n = 2$, and $q = 0$. For the same conditions, its magnitude for the frequencies indicated are given as follows

$$|g_{1,2}| = 1.72 \text{ (10 kc/s)}, \quad 1.47 \text{ (15 kc/s)}, \quad 1.08 \text{ (20 kc/s)}, \quad 0.70 \text{ (25 kc/s)}, \quad \text{and} \quad 0.41 \text{ (30 kc/s)}. \tag{51}$$

Since $|\Lambda_2|$ is of the order of unity, it is thus apparent that the conversion to higher modes may be significant.

The influence of the protuberance on the upper boundary is described by equations (46) and (47). The situation is similar to that of the ground obstacle except that the factors $G_n(y_o)$ appear. Actually, these are the ratio of the field just below the ionosphere reflecting layer to the field at the ground.

Of particular interest is the possibility that, as a consequence of an ionospheric irregularity, a mode of order 1 may be excited by an incident mode of order 2. The magnitude of this first-order mode relative to amplitude of the second-order mode is obtained from equation (46) with $m = 2$ and $n = 2$. Thus, approximately

$$Q_1^{(2)} \cong -2\,\Lambda_1 \, g_{2,1}\, G_2(y_o)\, G_1(y_o) \left(\frac{h-h_2}{h}\right) . \tag{52}$$

The numerical magnitude of the excitation factor Λ_1 has already been discussed. As noted, it may be quite small for frequencies of the order of 25 kc/s. However, in certain cases, the height-gain function $G_1(y)$, is somewhat greater than unity. Thus, the conversion to the lower-order mode may be quite significant. This point can also be demonstrated directly from equation (32) which, in this special case, has the form

$$Q_1^{(2)} = -\frac{2\,\Lambda_1}{y_o}\, g_{2,1} \int_{y_2}^{y_o} G_2(y)\, G_1(y)\, dy . \tag{53}$$

Modes of the "whispering gallery" type [Budden and Martin, 1962], also known as "earth-detached modes" [Wait, 1962b], are associated with a low excitation efficiency (i.e., Λ_1 is small). However, the height-gain function $G_1(y)$ for a "whispering gallery mode" is an increasing function of height. Thus, the product of the integral over y_2 to y_o and the excitation factor Λ_1 may be of appreciable magnitude.

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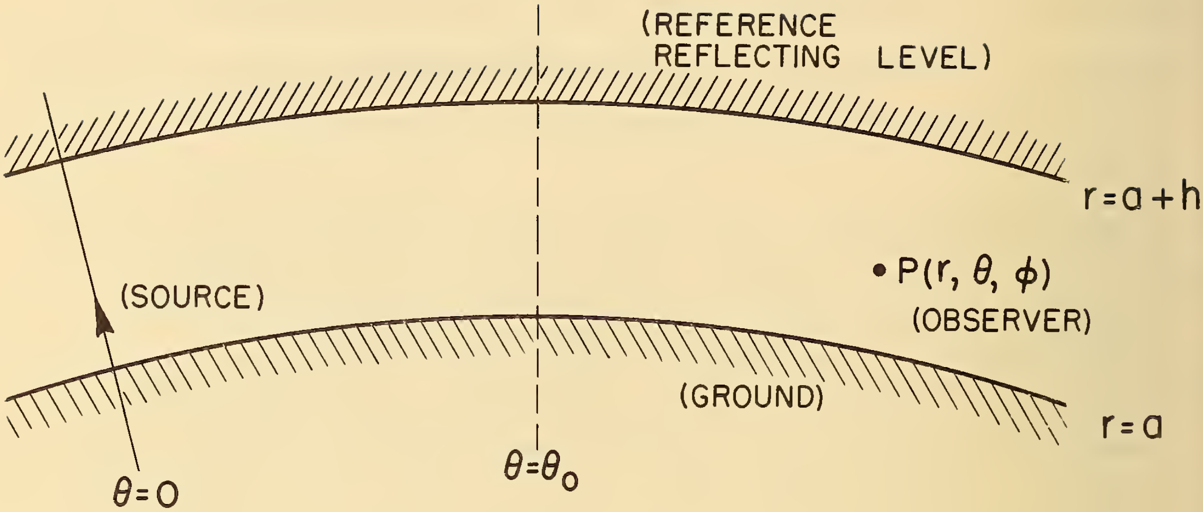

Fig. 1 - The waveguide model and the spherical coordinate system (r, θ, ϕ)

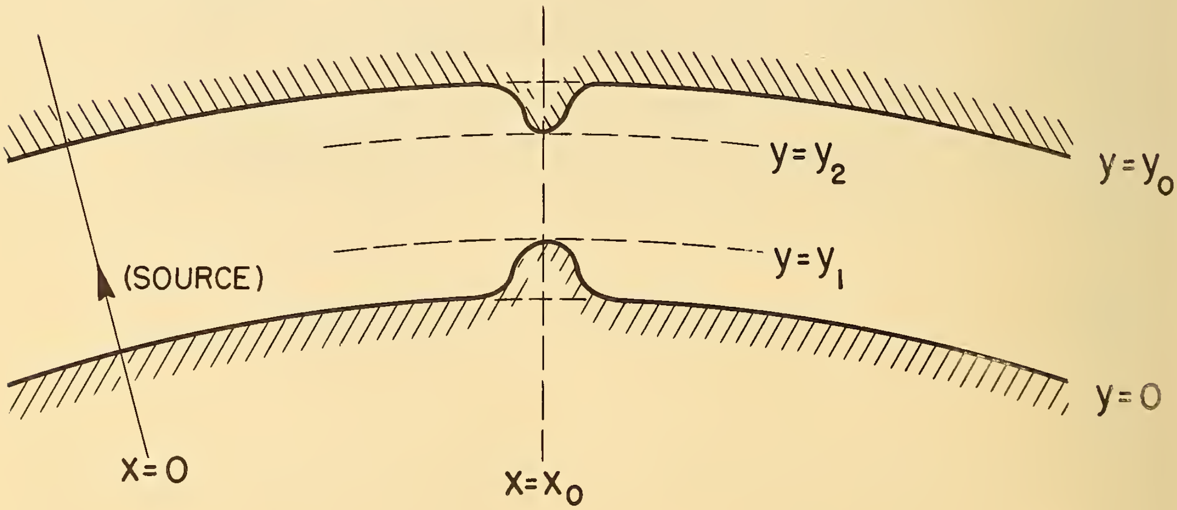

Fig. 2 - The waveguide model showing idealized obstructions in the aperture plane $x = x_o$. Here, the natural coordinate system (x, y) is being used.

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Office of Weights and Measures.

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Cryogenic Engineering Laboratory. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Cryogenic Technical Services.

CENTRAL RADIO PROPAGATION LABORATORY

Ionosphere Research and Propagation. Low Frequency and Very Low Frequency Research. Ionosphere Research. Prediction Services. Sun-Earth Relationships. Field Engineering. Radio Warning Services. Vertical Soundings Research.

Radio Propagation Engineering. Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation-Terrain Effects. Radio-Meteorology. Lower Atmosphere Physics.

Radio Systems. Applied Electromagnetic Theory. High Frequency and Very High Frequency Research. Modulation Research. Antenna Research. Navigation Systems.

Upper Atmosphere and Space Physics. Upper Atmosphere and Plasma Physics. Ionosphere and Exosphere Scatter. Airglow and Aurora. Ionospheric Radio Astronomy.

RADIO STANDARDS LABORATORY

Radio Physics. Radio Broadcast Service. Radio and Microwave Materials. Atomic Frequency and Time-Interval Standards. Millimeter-Wave Research.

Circuit Standards. High Frequency Electrical Standards. Microwave Circuit Standards. Electronic Calibration Center.

NBS