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An Analysis of Efficiency Testing Under the Energy Policy and Conservation Act: A Case Study With Application to Distribution Transformers

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[2] Some elements at Boulder, CO.

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Abstract

The protocols for efficiency testing promulgated by 10 CFR Part 430 and established under the Energy Policy and Conservation Act of 1975 as amended (EPCA) are discussed. The case of distribution transformers, which are covered under the proposed new 10 CFR Part 432, is treated in detail. Model calculations are presented that estimate the probability of demonstrating compliance and the average number of units tested under the 10 CFR Part 430 and Part 432 sampling plans. The results of model calculations for the sampling plan contained in NEMA Standard TP 2 are also presented.

Keywords

distribution transformers; efficiency testing; energy conservation; energy policy; EPCA; operating characteristic; sampling plan

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1 Introduction

This report provides analysis of the protocols for efficiency testing promulgated by 10 CFR Part 430 [1] as established under the Energy Policy and Conservation Act of 1975 (EPCA) as amended [2], and proposed for the new 10 CFR Part 432 [3]. Specifically, the performance of these testing protocols in establishing conformance with a rated efficiency or a rated energy use is addressed. The case of distribution transformers, which are covered under the proposed new 10 CFR Part 432, is discussed in detail. This discussion includes analysis of existing industry standards for transformer efficiencies. These results are presented, in part, to assess the impact of EPCA rule making on the transformer industry.

In the case of distribution transformers, laboratory measurements of efficiency are likely to be used for two purposes: 1) for testing of compliance with a rated efficiency; and 2) for enforcement testing. The objectives of testing for each of these purposes differ in significant ways: Compliance testing is a one-time activity undertaken at the initiation of the program or upon the introduction of a covered product, while enforcement testing would be undertaken when the performance of a specific product or products is contested. Enforcement testing is one of a series of requirements during an enforcement action.

The remainder of this report is organized as follows: The general objectives and constraints for testing under EPCA are discussed in Section 2. Current industry practice regarding efficiency performance and efficiency testing is reviewed in Section 3. The methods and model assumptions used in the evaluation of these testing protocols are presented in Section 4. A discussion of the sampling plans for compliance testing provided in 10 CFR Part 430, the proposed new 10 CFR Part 432, and NEMA Standard TP 2 is presented in Section 5. The Sampling Plan for Enforcement Testing established by 10 CFR Part 430 is presented in Section 6. For the convenience of the reader, each of these sampling plans are provided in appendices to this report: Appendix A contains an example sampling plan from 10 CFR Part 430 and the proposed 10 CFR Part 432 sampling plan for compliance testing. Appendix B contains the sampling plan provided in NEMA Standard TP 2. A Sampling Plan for Enforcement Testing which is adapted form that provided in 10 CFR Part 430 appears in appendix C. Finally, the computational algorithms used to model each of these sampling plans are presented in appendix D.

It should be emphasized that this report is *not* a statement of policy by the U.S. Department of Energy (DOE) and that this report should be regarded only as commentary on these testing protocols. DOE contacts for further information on the rule making for distribution transformers and on EPCA legislation are provided in Section 8.

2 General guidelines

In this section, we summarize briefly the general objectives of testing under EPCA. This material is included to provide ground rules for evaluating these various sampling plans.

A statement of purpose for the EPCA legislation is given in 42 U.S.C. 6312(a):

> It is the purpose of this Part to improve the efficiency of electric motors and pumps and certain other industrial equipment in order to conserve the energy resources of the Nation.

To this end, EPCA establishes energy performance standards that may specify acceptable levels of efficiency or energy use for each covered product. EPCA further requires that any represented energy performance be accurate in 42 U.S.C. 6314(d)(1):

> no manufacturer, distributor retailer or private labeler may make any representation —
>
> (A) in writing (including and representation on a label), or
>
> (B) in any broadcast advertisement,
>
> respecting the energy consumption of such equipment or cost of energy consumed by such equipment, unless such equipment has been tested in accordance with such test procedure and such representation fairly discloses the results of such testing.

EPCA relies on a program of systematic testing to establish that these performance levels are met. The objectives and limitations of testing under EPCA are stated in 42 U.S.C. 6314(a)(2):

> Test procedures prescribed in accordance with this section shall be reasonably designed to produce test results which reflect energy efficiency, energy use, and estimated operating costs of a type of industrial equipment (or class thereof) during a representative average use cycle (as determined by the Secretary), and shall not be unduly burdensome to conduct.

This report assumes that the purpose of EPCA is satisfied if the average efficiency is not less than the efficiency standard established under EPCA and the rated efficiency. In the case of an energy use standard, this report assumes that the purpose of EPCA is satisfied if the average energy use is not greater than the energy use standard established under EPCA and the rated energy use.

To re-cap, for purposes of this analysis, we assume that the performance objectives are met provided that the *average* energy efficiency (energy use) is not less (greater) than the EPCA energy performance standard and the rated value. Compliance with a rated energy performance is established under EPCA by systematic testing; and EPCA stipulates that testing should not be unduly burdensome to conduct. For the purposes of this analysis, two criteria are considered: 1) the assurance provided by a sampling plan that the average performance of each covered product meets or exceeds the rated performance, and 2) the burden placed on industry by testing under that sampling plan.

3 Industry practice

This section includes a brief discussion of current industry standards for distribution transformers. In a rule making, it may be useful to clarify current industry practice and, where current industry practice is consistent with the purpose of EPCA, to harmonize any energy performance standards established under EPCA with these practices.

The energy efficiency and energy consumption of distribution transformers is covered by two standards sanctioned by the National Electrical Manufacturers Association (NEMA): NEMA Standards Publication TP 1-1996, "Guide for Determining Energy Efficiency for Distribution Transformers" [4] and by NEMA Standards Publication TP 2-1998 "Standard Test Method for Measuring the Energy Consumption of Distribution Transformers" [5]. The efficiency requirements for NEMA class-1 designation are established in section 4 of the NEMA TP 1 standard, which applies to both liquid-filled and dry-type distribution transformers. The NEMA TP 1 Standard establishes "minimum efficiencies" for liquid-filled and dry-type single- and three-phase transformers, which are tabulated tabulated in Table 4-1 and Table 4-2, respectively. Dry type transformers are further differentiated by their voltage rating. The NEMA standard TP 1 tables are reproduced in part here in Tables 1 and 2, respectively. NEMA standard TP 1 states that these values are the "minimum efficiencies" for transformers designated as NEMA class 1. For purposes of this discussion, the phrase "minimum efficiencies" will refer to the minimum *average* efficiency of a population. NEMA Standard TP 2 [5] section 7 provides a sampling plan designed to establish compliance with the TP 1 efficiencies. Section 7 of the TP 2 standard is included in appendix B of this report and is discussed below in section 5.2.

Two standards sponsored by the Institute of Electrical and Electronics Engineers (IEEE) are also relevant to this discussion: ANSI Standard 57.12.00-1993 [6], "General Requirements for Liquid-Immersed Distribution, Power and Regulating Transformers" and ANSI Standard 57.12.01, "General Requirements for Dry-Type Distribution and Power Transformers, Including Those with Solid Case and/or Resin-Encapsulated Windings" [7]. These standards cover a broad range of transformer requirements, including recommended tolerances on measured losses: under these standards, the total losses of a single unit shall not exceed 106 percent of the rated value, and the average loss for two or more transformers shall not exceed the rated value. The second requirement, which is on the average loss, is being debated within the sponsoring committee and may be modified or deleted in future versions of these standards. Since the measured losses are directly related to the efficiency, these standards, in effect, establish a tolerance for the measured efficiency and thus provide a level of quality assurance for efficiency.

4 Methods of analysis

Two figures-of-merit provide the basis for the evaluation of these testing protocols: the *operating characteristic* and the *testing burden.* The operating characteristic of a sampling plan is the probability of demonstrating compliance when testing a specific distribution of efficiencies under that sampling plan. This quantity provides an estimate of the probability or risk that an acceptable product could fail by chance or that an un-

Table 1: NEMA Class 1 efficiency levels for liquid-filled distribution transformers. (Adapted from NEMA Standards Publication TP 1, Table 4-1 [4].)

kVA	Single Phase Efficiency	kVA	Three Phase Efficiency
10	98.3	15	98.0
15	98.5	30	98.3
25	98.7	45	98.5
37.5	98.8	75	98.7
50	98.9	112.5	98.8
75	99.0	150	98.9
100	99.0	225	99.0
167	99.1	300	99.0
250	99.2	500	99.1
333	99.2	750	99.2
500	99.3	1000	99.2
667	99.4	1500	99.3
833	99.4	2000	99.4
		2500	99.4

Table 2: NEMA Class 1 efficiency levels for dry-type distribution transformers. (Adapted from NEMA Standards Publication TP 1, Table 4-2 [4].)

	Single Phase Efficiency			Three Phase Efficiency	
kVA	Low Voltage	Medium Voltage	kVA	Low Voltage	Medium Voltage
15	97.7	97.6	15	97.0	96.8
25	98.0	97.9	30	97.3	97.3
37.5	98.2	98.1	45	97.7	97.6
50	98.3	98.2	75	98.0	97.9
75	98.5	98.4	112.5	98.2	98.1
100	98.6	98.5	150	98.3	98.2
167	98.7	98.7	225	98.5	98.4
250	98.8	98.8	300	98.6	98.5
333	98.9	98.9	500	98.7	98.7
500	—	99.0	750	98.8	98.8
667	—	99.0	1000	98.9	98.9
833	—	99.1	1500	—	99.0
			2000	—	99.0
			2500	—	99.1

acceptable product could pass by chance under that sampling plan. The second figure-of-merit, the testing burden, is the average number of units tested under that sampling plan. The testing burden may be considered in estimating the average cost of testing and in recommending minimum or maximum sample sizes.

These testing protocols are examined by means of model calculations. Two approaches are taken in this analysis: Firstly, where analytic expressions or numerical approximations for the desired figures-of-merit are derived, these expressions are evaluated numerically. A discussion of these algorithms is presented in appendix D and in [8, 9, 10]. Secondly, Monte Carlo [11] simulations are used to estimate these figures-of-merit. These approaches differ substantially, and each should be considered an independent estimate of the figure-of-merit. Comparison of the results obtained by numerical computation and Monte Carlo simulation may thus lend support to the calculated value. Monte Carlo methods have an added advantage for this analysis, in that they may be adapted to evaluate sampling plans where the sample size is not fixed, and may thus provide estimates of the operating characteristic and the testing burden for such plans.

The nature of testing under the sampling plans given in 10 CFR Part 430, the proposed new 10 CFR Part 432, and NEMA Standard TP 2 introduces a computational problem, in that the number of units tested may not be fixed from the outset of testing: For most products tested under the 10 CFR Part 430 sampling plan, a manufacturer could test as few as two units, but may test any larger number. One exception to this minimum sample size of two is lamp ballasts for which not fewer than four units must be tested. Under the proposed new 10 CFR Part 432 for distribution transformers, a manufacturer could test as few as five units of each basic model, but may test any larger number of units, including all units. Under the TP 2 sampling plan a manufacturer may test all units manufactured during a period of 180 days, or may conduct monthly tests over a period of 180 days of no fewer than five units. The scenario in which the sample size is not fixed is difficult to characterize by analytical methods, and we have chosen to first treat these sampling plans in the approximation that the sample size *is* fixed from the outset. Results obtained under this approximation provide a lower bound on the probability of being found in compliance. For example, while testing under the 10 CFR Part 430 sampling plan, a fixed sample of five units would include some cases for which a manufacturer was found in compliance after testing two units and could have stopped testing at that point, but fails after testing five units due to the final three test results. The computed probabilities for a fixed sample size may thus under-estimate the probability of being found in compliance.

Detailed information regarding the distribution of efficiencies is required for these calculations; and we assume that the population of efficiency is normally distributed with mean, μ, and standard deviation, σ. We further assume that the units tested are selected at random.

4.1 Loss representation

We have chosen, for this analysis, to represent transformer energy performance in terms of energy use, i.e. losses. This representation is chosen because it provides a direct comparison with industry standards. Further, the loss representation has the advantage of being independent of efficiency: The tolerance on the measured loss specified in the NEMA and IEEE standards is given as a percentage of the rated loss and is independent of efficiency. Thus the operating characteristic and testing burden are independent of efficiency in the loss representation.

In the discussion that follows, the operating characteristic and testing burden of each sampling plan are represented by contour plots in coordinates of the average loss and the standard deviation of loss, where both are given as a percentage of the rated loss. In all cases, the rated loss corresponds to 100 percent on the loss axis.

Expressed as a percentage, transformer efficiency, E, is given by the following equation:

$$E = \frac{P}{P + L} \times 100,$$

where P is the output power and L is the loss power. In practice, the no-load loss is measured at the rated voltage and the load loss at the rated current and a determination of efficiency involves three critical measurements: power, voltage, and current. Of these, the measurement of loss power is the most difficult resulting in measurement uncertainties that are about an order of magnitude greater than those of voltage and current measurement.

In summary, the measurement uncertainty of the efficiency depends primarily on the uncertainty of the loss measurement. This discussion is provided by way of explanation for the specification of a tolerance on the measured loss in the IEEE and NEMA standards. Since the critical measurement contributing to the measurement uncertainty in the efficiency is the uncertainty in the loss measurement, as a practical matter it is most effective to specify a tolerance for the measurement of loss.

5 Compliance testing

Three sampling plans are discussed: the sampling plan put forward by 10 CFR Part 430 [1], the sampling plan proposed for the new 10 CFR Part 432 [3], and the sampling plan established by NEMA Standard TP 2 [5].

5.1 10 CFR Part 430 and the proposed 10 CFR Part 432 (Method I)

In the discussion that follows, we refer to the sampling plans provided in 10 CFR Part 430 and proposed for the new 10 CFR Part 432 as Method I type plans.

5.1.1 10 CFR Part 430

10 CFR Part 430 establishes specific criteria for each covered product when testing for compliance with a rated energy performance. An example of these criteria, which applies to dishwashers, is provided in appendix A of this report. The general structure of this sampling plan is followed for all other products covered under 10 CFR Part 430. To emphasize its salient features, we paraphrase the Part 430 sampling plan as follows:

(i) Compliance with a rated energy consumption is demonstrated provided;

  (A) The average energy consumption of the sample is not greater than the rated energy consumption, and

  (B) The upper $97^1/_2$ percent confidence limit of the average energy consumption of the entire population divided by 1.05 is not greater than the rated energy consumption.

(ii) Compliance with a rated efficiency is demonstrated provided;

  (A) The average efficiency of the sample is not less than the rated efficiency, and

  (B) The lower $97^1/_2$ percent confidence limit of the average efficiency of the entire population divided by 0.95 is not less than the rated efficiency.

For a given population, the operating characteristic and testing burden of these criteria depend on three factors: 1) the sample size, 2) the statistical confidence in (i)(B) and (ii)(B) and 3) the value of the divisor specified in (i)(B) and (ii)(B). The statistical confidence and the divisors are underlined in the above text. Under 10 CFR Part 430, the statistical confidence specified in (i)(B) and (ii)(B) ranges in value between 90 percent and 99 percent. While the divisor ranges in value between 1.01 and 1.10 in (i)(B) and between 0.90 and 0.99 in (ii)(B).

Under this sampling plan a manufacturer is required to test only as needed to demonstrate the specified statistical confidence. The rule language in 10 CFR Part 430 states that the sample should be of "sufficient size" to ensure a statistical confidence that is not less than $97^1/_2$ percent, in the example cited. Assuming that the statistical confidence is established from test data alone, the minimum sample size under this plan is two. The reason for this minimum value is that the sample standard deviation is not defined for a sample of one.

5.1.2 The proposed 10 CFR Part 432

A Method I type plan is also proposed for distribution transformers in 10 CFR Part 432 [3]. The proposed sampling plan is provided in appendix A of this report. We paraphrase this sampling plan as follows:

> A sample of not fewer than five units must be tested. Compliance with a rated energy efficiency is demonstrated provided:
>
> (a) The mean efficiency of the sample is not less than the rated efficiency, and
>
> (b) The lower 95 percent confidence limit of the average efficiency of the entire population divided by $[1 - 0.03(1 - E_s/100)]$, where E_s is the rated efficiency, is not less than the rated efficiency.

Criteria for the demonstration of compliance with a rated loss are not proposed in 10 CFR Part 432. However, the criteria proposed for compliance testing may be stated in terms of loss; and such loss criteria are fully equivalent to the sampling plan for compliance testing proposed 10 CFR Part 432. These criteria are included here for illustrative purposes and for consideration of users who may wish to analyze intermediate test data in terms of loss power. When stated in terms of loss, the proposed sampling plan may be paraphrased as follows:

> A sample of not fewer than five units must be tested. Compliance with a rated loss is demonstrated provided:
>
> (a) The average loss power of the sample is not greater than the rated loss power, and

(b) The upper 95 percent confidence limit of the loss power of the entire population divided by 1.03 is not greater than the rated loss power.

It should be emphasized that the divisor in criterion (b) above establishes a tolerance on the estimate of the mean loss of the population. This tolerance on the estimate of the mean loss should not be confused with the considerably larger loss tolerance established by ANSI Standards 57.12.00-1993 and 57.12.01 and NEMA Standard TP2: The loss tolerance established by these standards applies to the measured loss of a single unit.

It may be noted that the divisors in criterion (b) differ when the criteria for compliance testing are stated in terms of efficiency or loss. This feature differs from the 10CFR Part 430 sampling plans. In the example provided from 10 CFR Part 430, the divisors applied in the cases of efficiency testing and energy consumption testing are 1+0.05 and 1−0.05, respectively. In effect, these divisors establish a tolerance of 5 percent for both the estimate of the mean efficiency and the estimate of the mean energy consumption. The use of an equivalent tolerance on the estimate of the mean efficiency and mean loss may not apply for devices that are highly efficient such as distribution transformers. The relationship between the uncertainty in the measurement of efficiency and of loss is given by $m = -n(1 - E_s/100)$, where m is the measurement uncertainty in the efficiency, n is the measurement uncertainty in the loss, and E_s is the rated value of efficiency for the product. Clearly, in the limit that E_s approaches 100 percent the measurement uncertainty in the efficiency is much less than in the loss.

5.1.3 Results

The results of model calculations for the Method I sampling plan are shown in Figs. 1–9, where data are presented for a statistical confidence of 95 percent and for values of loss tolerance of 103 percent, 106 percent, and 112 percent. Sampling plans using both fixed samples and variable sizes are treated. The results shown in Figs. 2 to 5 are of particular interest to the transformer case.

For samples of fixed size, a rated efficiency is demonstrated provided:

1. the sample average loss is less than 100 percent, and

2. the upper 95 percent confidence limit of the mean loss is not greater than the loss tolerance.

The condition on the loss tolerance LT is equivalent to the use of a divisor in (i)(B). A sampling plan based on a loss tolerance of 103 percent, for example, exhibits the same behavior as the use of a 1.03 divisor.

Samples of fixed size are first examined. The operating characteristic of the Method I sampling plan for a loss tolerance of 103 percent for samples of 2, 5, 10, 20, and 30 units are shown in Figs. 1–5, respectively. The dependence of the operating characteristic on the loss tolerance is shown in Figs. 6–9, which are for the samples of two and five and for loss tolerances of 106 percent and 112 percent. Monte Carlo simulations were also performed for each of these cases and results obtained by these methods are shown in half-tone in each of these figures. Aside from the random fluctuations, which are inherent to Monte Carlo methods, the two methods appear to give equivalent results.

Examination of these data suggests the following interpretation of the Method I sampling plan: For samples of fixed size, the Method I sampling plan provides assurance at the specified statistical confidence that the average loss does not exceed the loss tolerance. In this case, the statistical confidence is 95 percent and thus the risk of exceeding the loss tolerance is not greater than 5 percent. This behavior is indicated by the asymptotic limit of the 0.05 contours, which in each figure approach the loss tolerance at large standard deviation.

Two additional analyses, which may be modeled by Monte Carlo simulations, are suggested by these results: 1) the operating characteristic and 2) the testing burden under this sampling plan when the sample size is not fixed. In principle, the Monte Carlo simulation should provide a more realistic estimate of the Method I operating characteristic and testing burden. The conditions for a determination of compliance for variable sample size are identical to those used for a fixed sample; however, in this case, testing is continued up to a maximum sample of twenty or until the following condition is satisfied:

$$n \geq \left(\frac{tS}{LT - 100} \right)^2 ,$$

where n is the sample size, LT is the loss tolerance, S is the sample standard error, and t is the fifth percentile of the t-distribution for $n - 1$ degrees of freedom.

The results of these simulations are presented in Figs. 10 and 11, which respectively depict the operating characteristic and the testing burden at a loss tolerance of 103 percent. These operating characteristics suggest that the assurance on loss performance may be somewhat less than for fixed samples, as the asymptotic limit of the 0.05 contour appears to be on the order of 106 percent. These data further suggest that the marginal loss performance required to provide a high probability of demonstrating compliance may be appreciable under this sampling plan, where marginal loss performance refers to the difference between the rated loss and the true average loss. It should be emphasized, however, that the operating characteristics shown in the figure are those for minimal testing and that, under the Method I sampling plan, a manufacturer may be able to reduce the marginal loss by increasing the sample size.

5.2 NEMA Standard TP 2 (Method II and Method III)

The NEMA TP 2 sampling plan for demonstrating the efficiencies established by NEMA Standard TP 1 is next presented. The NEMA TP 2 sampling plan is given in sections 7.0 and 7.1 of the standard, which can be found in appendix B of this report.

NEMA Standard TP 2 provides that the aggregate measured input power of all transformers tested for a period of 180 days be no greater than that allowed by the rated efficiency. The proposed 10 CFR Part 432 contemplates measurement of the average efficiency of a basic model, where a basic model may be distinguished by the significant characteristics of that product such as the kVA rating, voltage rating, and insulation type, for example. The question of whether it is appropriate to aggregate a broad range of product types for the purposes of determining an average energy performance, as is allowed under NEMA Standard TP 2, is not addressed here. For purposes of this discussion, we assume that the TP 2 sampling plan is applied to measure the average efficiency or the average energy use of a basic model. When applied to a basic model, this condition on the measured input power is equivalent to an average measured efficiency that is not less than the rated efficiency and an average measured loss that is not greater than the rated loss. The TP 2 sampling plan thus establishes a condition on the mean efficiency or mean loss of a sample.

The TP 2 sampling plan provides further that no unit may exceed 108 percent of the rated loss. This tolerance on the measured loss may be interpreted in two ways: 1) The tolerance on the measured loss may establish a quality control limit for the NEMA Class I rating, but this tolerance has no bearing on any demonstration of compliance with a rated efficiency. This interpretation of the TP 2 sampling plan places a condition on the mean of the sample. This scenario will be referred to here as a Method II type sampling plan. 2) Another interpretation of the TP 2 sampling plan places a condition on the mean of the sample and on the sample extremum, where the sample extremum is the maximum measured loss of any unit in the sample. This scenario will be referred to here as a Method III type sampling plan. Method III type criteria are used for quality assurance of various packaged goods [12, 13] and have been proposed for use in testing whether electric motors comply with a rated efficiency [9]. An approximate solution for the probability that such conditions on the sample mean and on the sample extremum are satisfied is discussed by Vangel [8].

Two scenarios for sampling are presented in TP 2: all units manufactured during a 180 day period may be tested, or units may be tested on a monthly basis for a period of 180 days. The standard provides that no fewer than five units may be tested during these monthly tests and that the sample size shall be sufficient to ensure a statistical confidence of not less that 95 percent. The statistical confidence is established by the condition

$$n = (tSK)^2,$$

where n is the sample size, S is the sample standard deviation, the value of t is selected according to the number of units tested as tabulated in TP 2, and the coefficient K is given by the equation

$$K = \frac{108 - 0.08SEL}{SEL(8 - 0.08SEL)},$$

where SEL is the standard efficiency level in percent. This condition on the sample size is equivalent to that used under the Method I sampling plan and is that recommended by ASTM Standard E 122-89 [14].

5.2.1 Method II results

The operating characteristics and testing burden of Method II are presented in Figs. 12–17.

Under Method II, one condition must be satisfied to demonstrate compliance with a rated performance:

1. the mean loss of the sample must be no greater than 100 percent.

The operating characteristics for samples of fixed size are shown in Figs. 12–15, which depict samples of 5, 10, 20, and 30 units. Monte Carlo simulations were again performed for each of these cases and the results of these simulations are shown in each of these figures in half-tone. Numerical computation and Monte Carlo simulation appear to yield consistent values.

The case of variable sample size is examined in Figs. 16 and 17, which depict the operating characteristics and the testing burden of Method II, respectively. These data apply to the case of monthly testing under TP 2 where the the minimum units tested is thirty. The conditions for a demonstration of compliance for a variable sample are identical to those for fixed sample size; however, in this case testing is continued until the condition

$$n \geq \left(\frac{tS}{LT - 100}\right)^2$$

is satisfied.

5.2.2 Method III results

The operating characteristics and testing burden of Method III are presented in Figs. 18–23.

Under Method III, two conditions must be satisfied to demonstrate compliance with a rated performance:

1. the mean loss of the sample must be no greater than 100 percent, and
2. no single unit in the sample may exceed 108 percent of the rated loss.

It should emphasized that under Method III the entire population is not in compliance if any unit in the sample exceeds 108 percent of the rated loss. The operating characteristics for samples of fixed size are shown in Figs. 18–21, which depict samples of 5, 10, 20, and 30 units. Monte Carlo simulations were again performed for each of these cases and the results of these simulations are shown in each of these figures in half-tone. Numerical computation and Monte Carlo simulation appear to yield consistent values.

The case of variable sample size is examined in Figs. 22 and 23, which depict the operating characteristics and the testing burden of Method III, respectively. These data apply to the case of monthly testing under TP 2 where the the minimum units tested is thirty. The conditions for a demonstration of compliance for a variable sample are identical to those for fixed sample size; however, in this case testing is continued until the condition

$$n \geq \left(\frac{tS}{LT - 100}\right)^2$$

is satisfied or the sample exceeds twenty, whichever occurs first. It should be noted that a maximum sample size is not stipulated in the TP 2 sampling plan: the maximum sample size of twenty used here was included to simplify the Monte Carlo simulation.

5.3 Discussion

For a fixed sample size, the operating characteristics of Method I, Method II, and Method III are very similar at small standard deviation. This is due to the condition placed on the sample mean loss under these sampling plans: Each of these sampling plans require that the sample mean loss be no greater than the rated loss; thus at small standard deviation, where the determination of compliance is based primarily on the sample mean loss, each of these sampling plans exhibit similar operating characteristics.

It should be noted that the operating characteristic of these sampling plans depends on the sample size. However, these sampling plans exhibit very different behavior with increased sample size. For example, the marginal loss required under Method I to provide a high probability of demonstrating compliance decreases with sample size. Thus the marginal loss may be reduced to an acceptable level under the Method I sampling plan by additional testing. The behavior of the Method III sampling plan is much different. Under Method III, the risk of failing to demonstrate compliance may actually increase with increased sample size. The reason for this behavior is that as the sample size increases the likelihood of sampling from the wing of the distribution of loss increases.

The design performance of a transformer may depend on both engineering and business factors. However, it appears from this analysis that each of these sampling plans favor loss performance below the rated value. Since, if a transformer were designed to perform, and indeed performed at the rated loss on average, the likelihood of demonstrating compliance with that rated performance is not greater than 50 percent under each of these plans.

The Method I, Method II, and Method III sampling plans all provide assurance that the mean performance of a population meets or exceeds a rated value. The Method I sampling plan appears to be perform well in cases where large quantities are available for testing and the cost of testing is low. However, one unique character of distribution transformers is that the lot size can be small. Indeed some transformers are manufactured in extremely limited numbers. In that the marginal loss can be appreciable for small samples, the Method I sampling plan may, in effect, require that certain transformers have average losses that are appreciably less than their rated loss. Alternately the Method III sampling plan may place a high burden on manufacturers when it is applied to large samples.

6 Enforcement testing

The Sampling Plan for Enforcement Testing provided in 10 CFR Part 430 applies to all products covered under Part 430. A similar plan has been proposed in the new 10 CFR Part 431 for electric motors. The proposed 10 CFR Part 432 [3] does not include a Sampling Plan for Enforcement Testing. Its development will be considered at a later date. While some details may change, the general format of the plan is expected to remain the same if and when it is finalize for transformers.

The Sampling Plan for Enforcement Testing provided in 10 CFR Part 430 establishes clearly delineated procedures to be followed under an enforcement action. This sampling plan is based on a well established statistical method, which is due to C. Stein [15], for obtaining a confidence interval on a mean. A discussion of this procedure can also be found in Bickel and Doksum [16]. The Sampling Plan for Enforcement Testing provided in 10 CFR Part 430 covers both testing of efficiency and for energy consumption and it is very general. The sampling plan presented in appendix C has been adapted from 10 CFR Part 430; however, it has been simplified somewhat to clarify the procedure and it includes only the case for efficiency testing.

The sampling plan is based on a t-test. The t-test is well suited to this application as it is known to be insensitive to departures from the assumption of normality: The t-test is a test on a mean, which is an average of independent values obtained by a random sample. Since, sums of arbitrary, independent random values tend to have a distribution that is *almost* normal, the t-test is not strongly influenced by the exact form of the underlying distribution of efficiencies.

Since the test results obtained during enforcement testing may recommend that certain adverse actions be taken against a manufacturer: relabeling of specific products, the cessation of distribution and sale of certain basic models, and/or the assessment of fines, for example; the risk to a manufacturer of a false determination of noncompliance during an enforcement action is set, by design, to a negligible level. The Sampling Plan for Enforcement Testing provided in 10 CFR Part 430 is based on a 97.5 percent statistical confidence, thus the risk to a manufacturer of a chance false determination of noncompliance is not greater than 2.5 percent.

6.1 10 CFR Part 430

An estimate of the true mean efficiency is first obtained by a random sample,

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \tag{1}$$

here X_i is the measured efficiency of unit i, and n is the number of units tested. The uncertainty in this estimate depends on two factors: 1) the size of the sample, i.e. the number of transformers tested, and 2) the underlying variability in the entire population. The sample standard deviation,

$$S = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}, \tag{2}$$

provides an estimate of the population standard deviation σ; and the standard error in the mean,

$$SE(\bar{X}) = \frac{S}{\sqrt{n}}, \tag{3}$$

provides an estimate of the standard deviation of the mean for samples of n units. For a normal distribution with mean μ; the ratio,

$$t = \frac{\mu - \bar{X}}{SE(\bar{X})}, \tag{4}$$

is distributed according to a probability density function that is know in statistics literature as the t-distribution. Now eq. 4 may be rearranged to provide an expression for the mean of the sample:

$$\bar{X} = \mu - tSE(\bar{X}), \tag{5}$$

where the value of t is associated with a specific sample size and statistical confidence. Values of t are readily available and are included in many references on statistics [17].

In a test of compliance with a rated efficiency, RE, we assume, by hypothesis, that the units to be tested are drawn from a population of transformers for which the mean efficiency is equal to or greater than the rated efficiency. If t is the 97.5 percentile of the t-distribution for $n-1$ degrees of freedom, then the probability of obtaining a mean efficiency,

$$\bar{X} \leq RE + tSE(\bar{X}), \tag{6}$$

is not less than 97.5 percent. This procedure recommends the upper control limit,

$$UCL = RE + tSE(\bar{X}). \tag{7}$$

To apply this method, a random sample is tested and the mean and standard error in the mean are calculated. Based on the size of the sample and the confidence desired, the appropriate t-value is selected and the lower control limit calculated. For example, 97.5 percent confidence and a sample size of five units yields a t-value of 3.18. Provided the mean efficiency obtained from the random sample is not less than the lower control limit, as defined by eq. 7, we may assert with a confidence not less than 97.5 percent that the true mean energy consumption of the entire population is not greater than the rated efficiency.

In any statistical test there is some probability of incorrectly concluding noncompliance. By design, the probability that the mean efficiency for a random sample drawn from this population of transformers would fall below the lower control limit, hence, the risk of incorrectly concluding that the basic model is in noncompliance, is not greater than 2.5 percent.

There is some probability that the sample standard deviation may be large and that the lower control limit may be set, by chance, to an exceptionally low value. This circumstance may be avoided by placing a tolerance on the standard error in the mean, $SE(\bar{X})$. The tolerance for the standard error should be chosen to be appropriate for that product and to be supported across the industry.

Choosing, for example, a loss tolerance of 108 percent,

$$\begin{aligned} \mu_{min} &= \frac{P_{out}}{P_{in} + 0.08(P_{in} - P_{out})} \times 100 \\ &= \frac{\mu}{108 - 0.08\mu} \times 100, \end{aligned} \tag{8}$$

where P_{in} and P_{out} are the input and output power. The lower control limit must then satisfy two conditions:

$$UCL = RE - tSE(\bar{X}) \tag{9}$$

and

$$UCL \geq \frac{RE}{108 - 0.08RE} \times 100, \tag{10}$$

where,the second condition is obtained from eq. (8) by setting the efficiency equal to the RE.

6.2 Results

The operating characteristic and the testing burden of the Sampling Plan for Enforcement Testing are shown in Figs. 24–29. Factors that influence the operating characteristic and testing burden include: 1) the size of the initial sample, 2) the statistical confidence, and 3) the tolerance set on the standard error.

It may be noted that the 97.5 percent contour lies along 100 percent loss in each of these figures and that the risk of a false outcome is therefore independent of variability under this sampling plan.

7 Summary

Compliance testing: The operating characteristic of the Method I, Method II, and Method III sampling plans are equivalent in the limit of small standard deviation. This is due to the condition placed on the mean loss of the sample under each of these plans. Since under each of these plans the mean loss of the sample may not exceed the rated loss, each of these plans provide assurance that the mean efficiency of a population is not less than the rated efficiency. When testing small samples under Method I, the marginal loss performance may be appreciable and may, in effect, require that certain transformers have average losses significantly less than their rated values. However, under Method I, a manufacturer may reduce the marginal loss by testing additional units. Alternately the Method III sampling plan appears to require an appreciable marginal loss performance when it is applied to large samples and may, in effect, require that certain transformers have average losses significantly less than their rated values.

Enforcement testing: The Sampling Plan for Enforcement Testing included in 10 CFR Part 430 is based on statistical methods that are widely used and well documented. The sampling plan is robust, in that it is a test on the mean and that it is not highly dependent on the form of the underlying distribution. The sampling plan is designed to protect the interests of the manufacturer, in that the risk of a false outcome against a manufacturer may be limited to some acceptable level. Additionally, the risk of a false outcome is independent of variance.

The analysis presented here is of value primarily as a qualitative evaluation of the operating characteristic and the testing burden of these testing protocols and secondarily as a quantitative estimate of the statistical confidence associated with testing under these protocols. While the agreement noted between results obtained by analytical methods and Monte Carlo simulations is encouraging, it should be noted, since these results may depend on our model assumptions, that a quantitative estimation of statistical confidence is more tenuous than qualitative evaluation.

8 Further information

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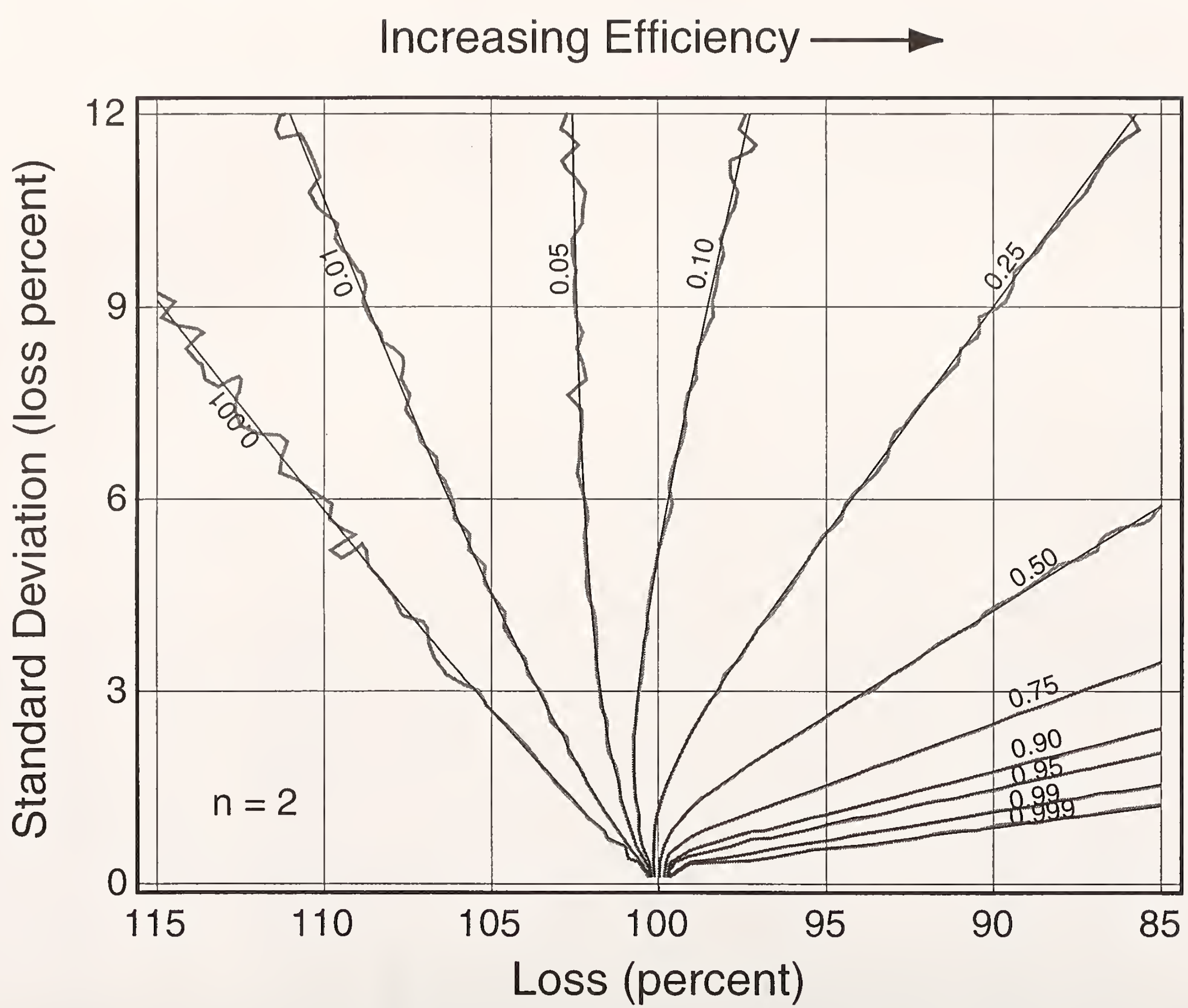

Figure 1: Operating characteristic of the Method I compliance criteria for a fixed sample size of two. The statistical confidence is 95 percent and the loss tolerance is 103 percent. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance. The results of Monte Carlo simulations are shown in half-tone.

Figure 2: Operating characteristic of the Method I compliance criteria for a fixed sample size of five. The statistical confidence is 95 percent and the loss tolerance is 103 percent. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance. The results of Monte Carlo simulations are shown in half-tone.

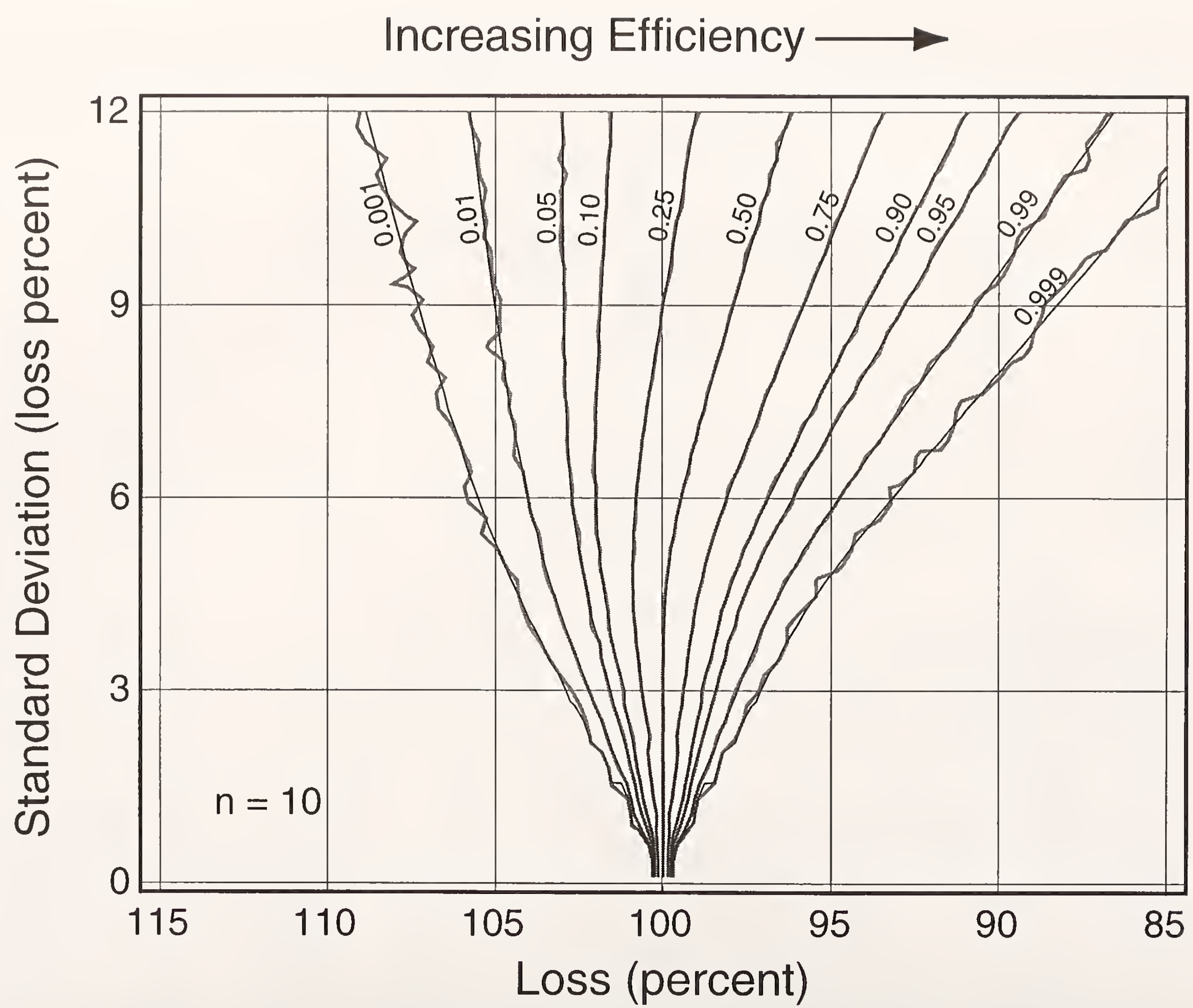

Figure 3: Operating characteristic of the Method I compliance criteria for a fixed sample size of ten. The statistical confidence is 95 percent and the loss tolerance is 103 percent. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance. The results of Monte Carlo simulations are shown in half-tone.

Figure 4: Operating characteristic of the Method I compliance criteria for a fixed sample size of twenty. The statistical confidence is 95 percent and the loss tolerance is 103 percent. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance. The results of Monte Carlo simulations are shown in half-tone.

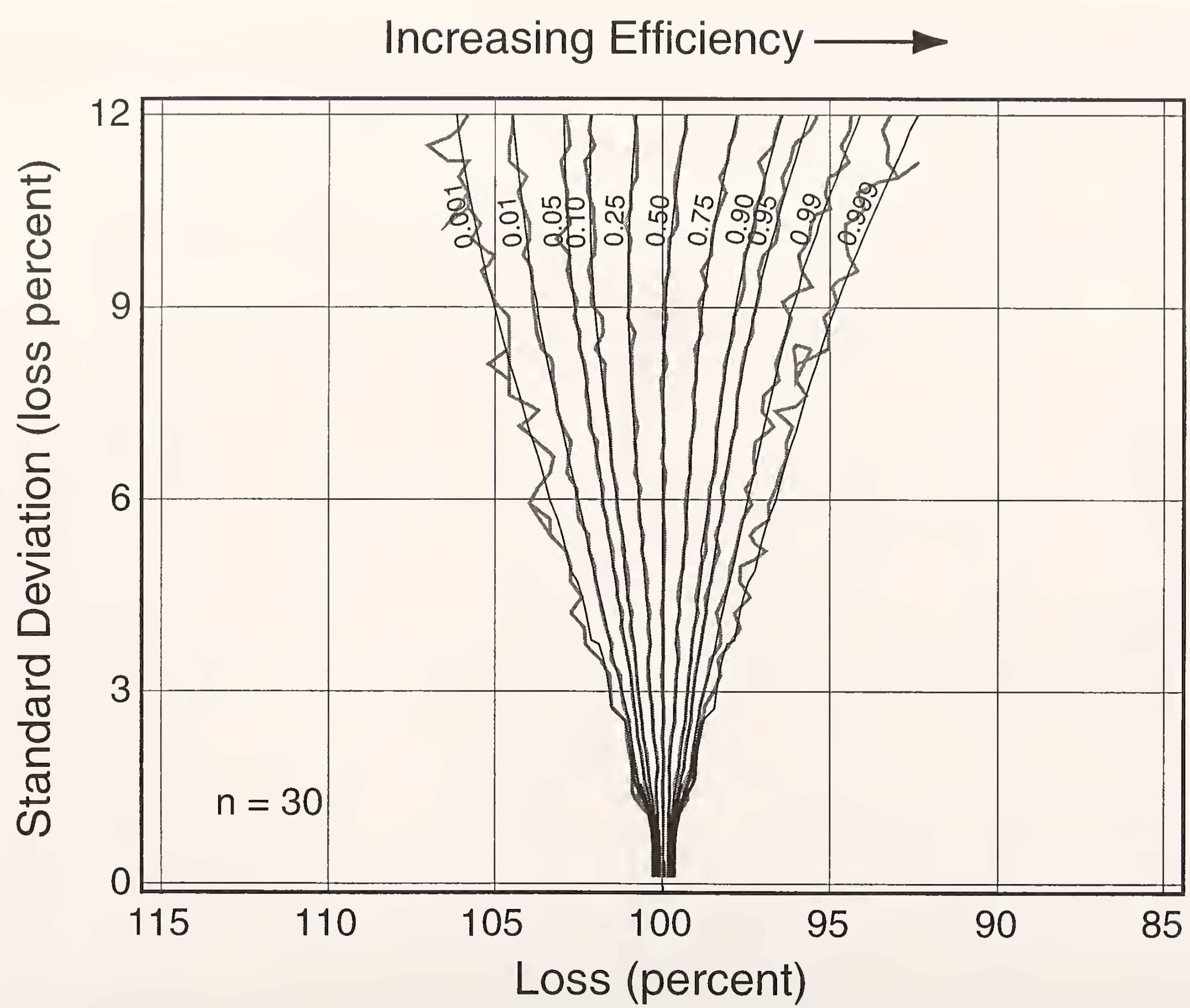

Figure 5: Operating characteristic of the Method I compliance criteria for a fixed sample size of thirty. The statistical confidence is 95 percent and the loss tolerance is 103 percent. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance. The results of Monte Carlo simulations are shown in half-tone.

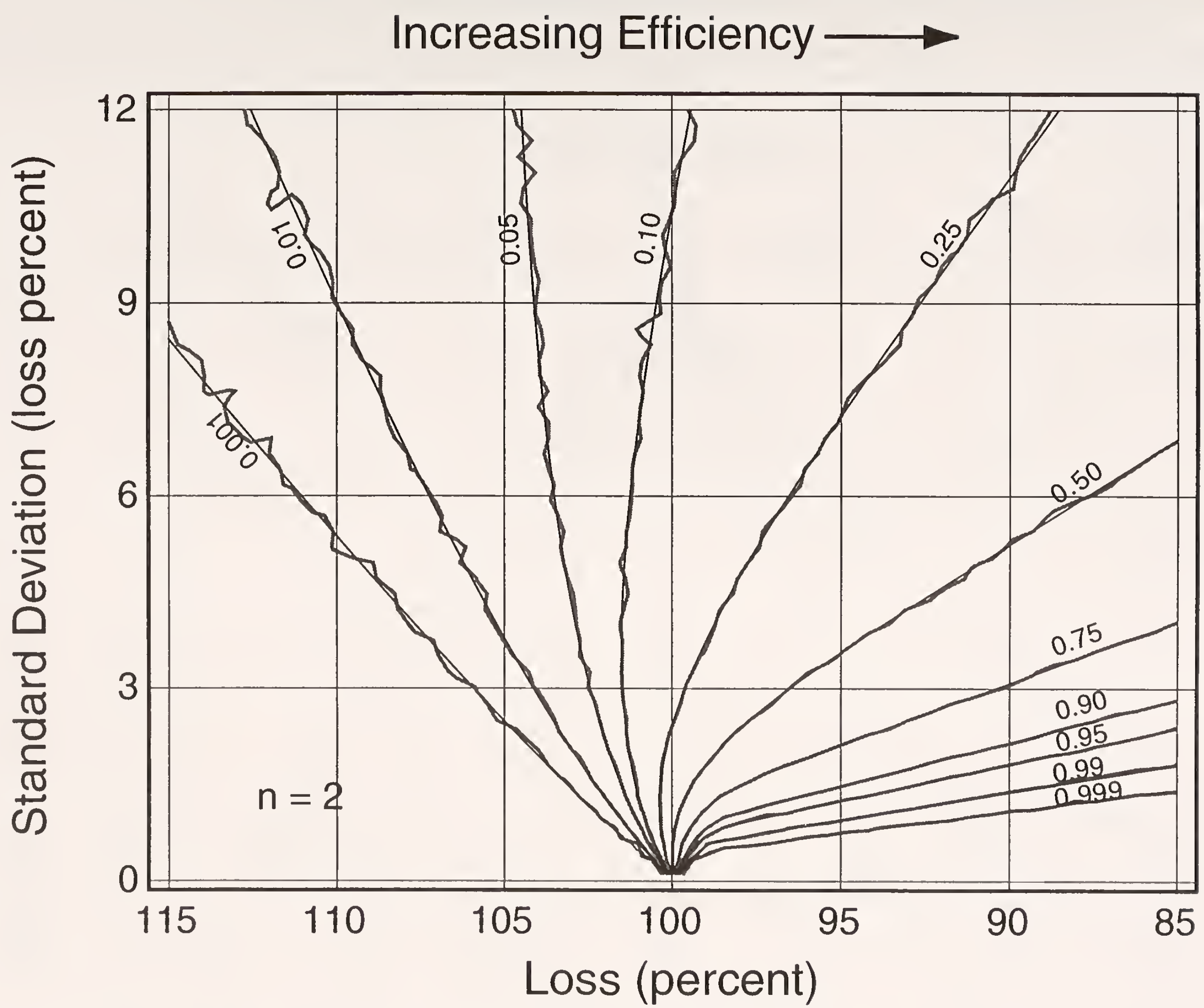

Figure 6: Operating characteristic of the Method I compliance criteria for a fixed sample two. The statistical confidence is 95 percent and the loss tolerance is 106 percent. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance.

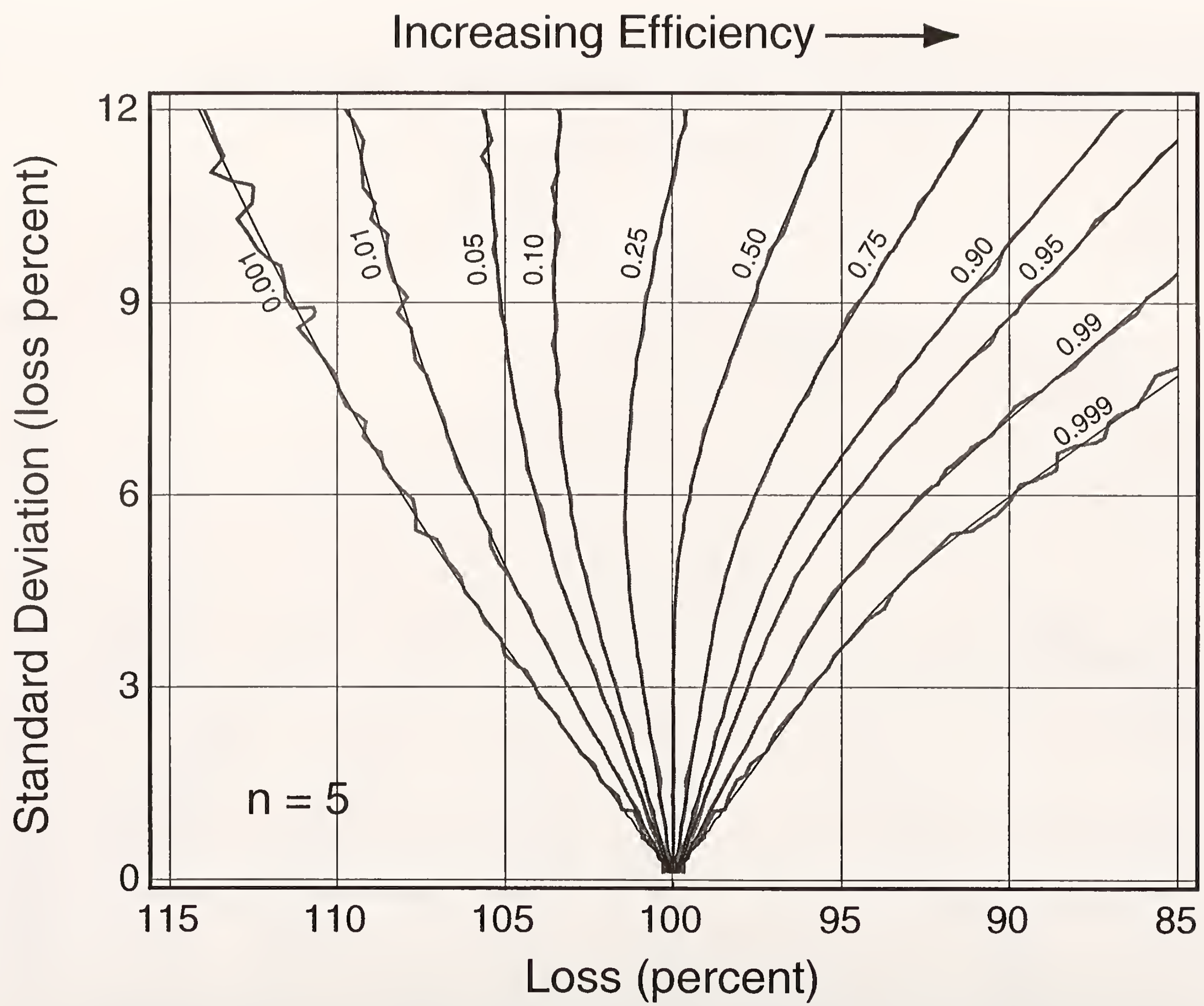

Figure 7: Operating characteristic of the Method I compliance criteria for a fixed sample five. The statistical confidence is 95 percent and the loss tolerance is 106 percent. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance.

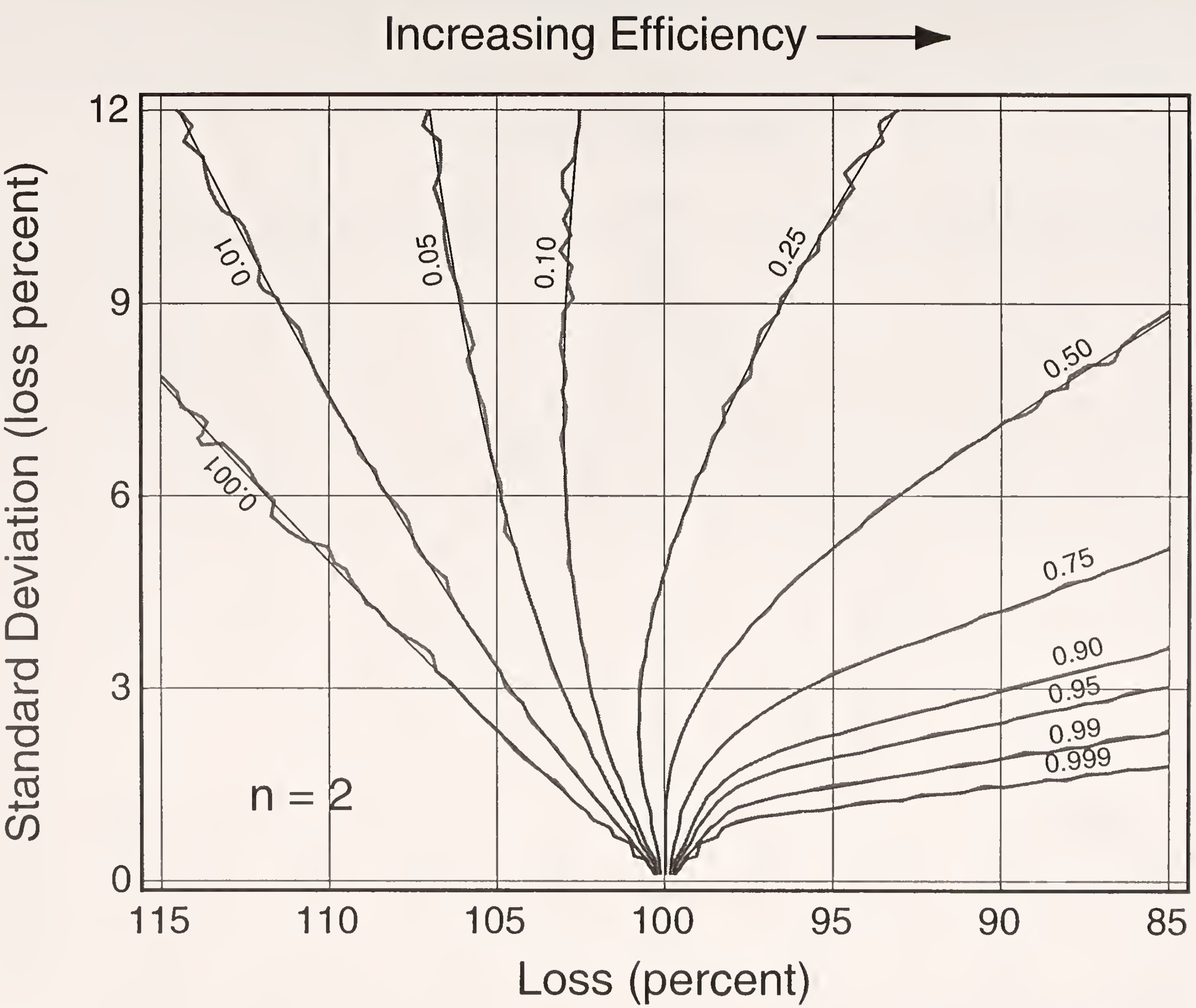

Figure 8: Operating characteristic of the Method I compliance criteria for fixed sample of two. The statistical confidence is 95 percent and the loss tolerance is 112 percent. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance.

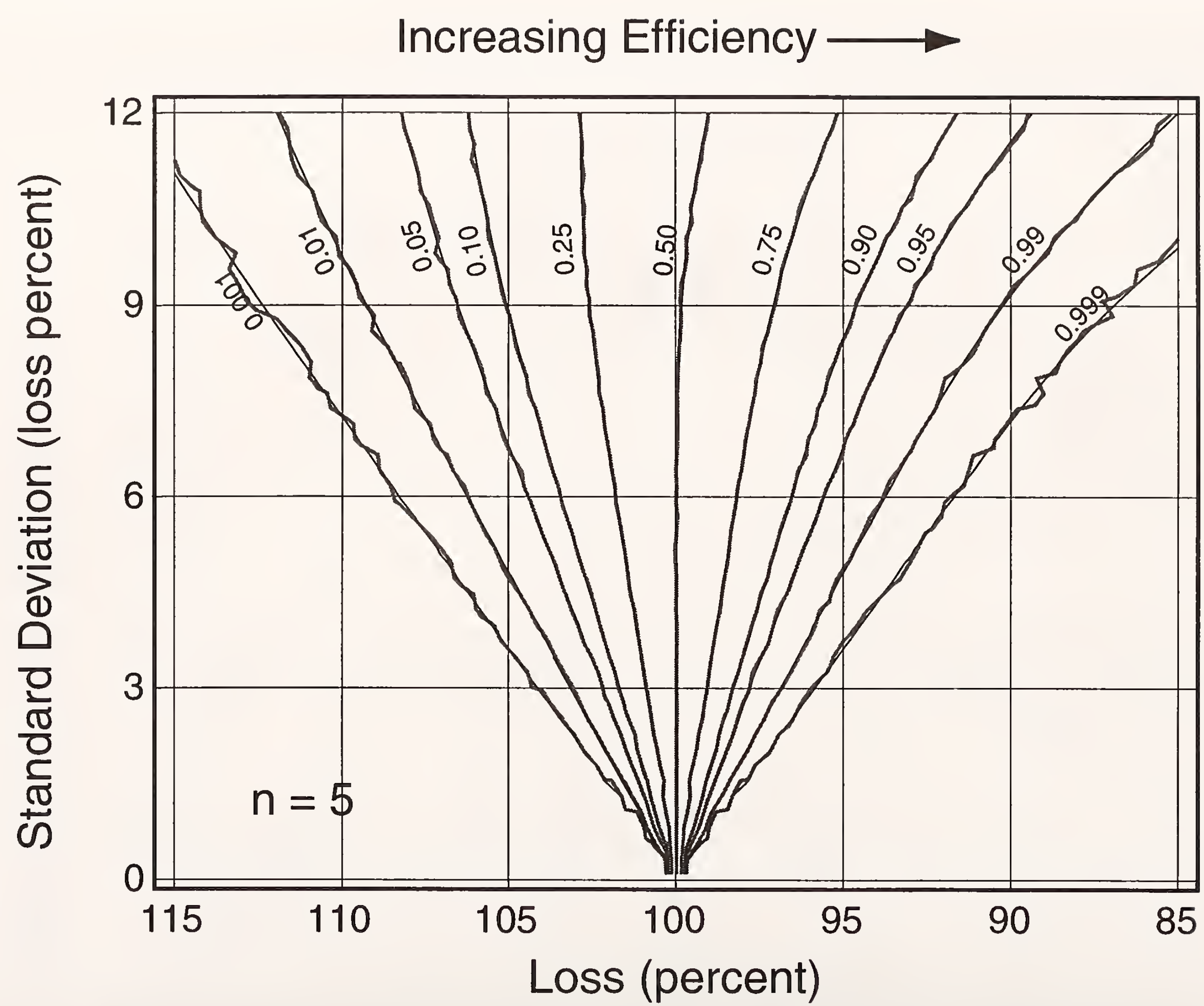

Figure 9: Operating characteristic of the Method I compliance criteria for fixed sample of five. The statistical confidence is 95 percent and the loss tolerance is 112 percent. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance.

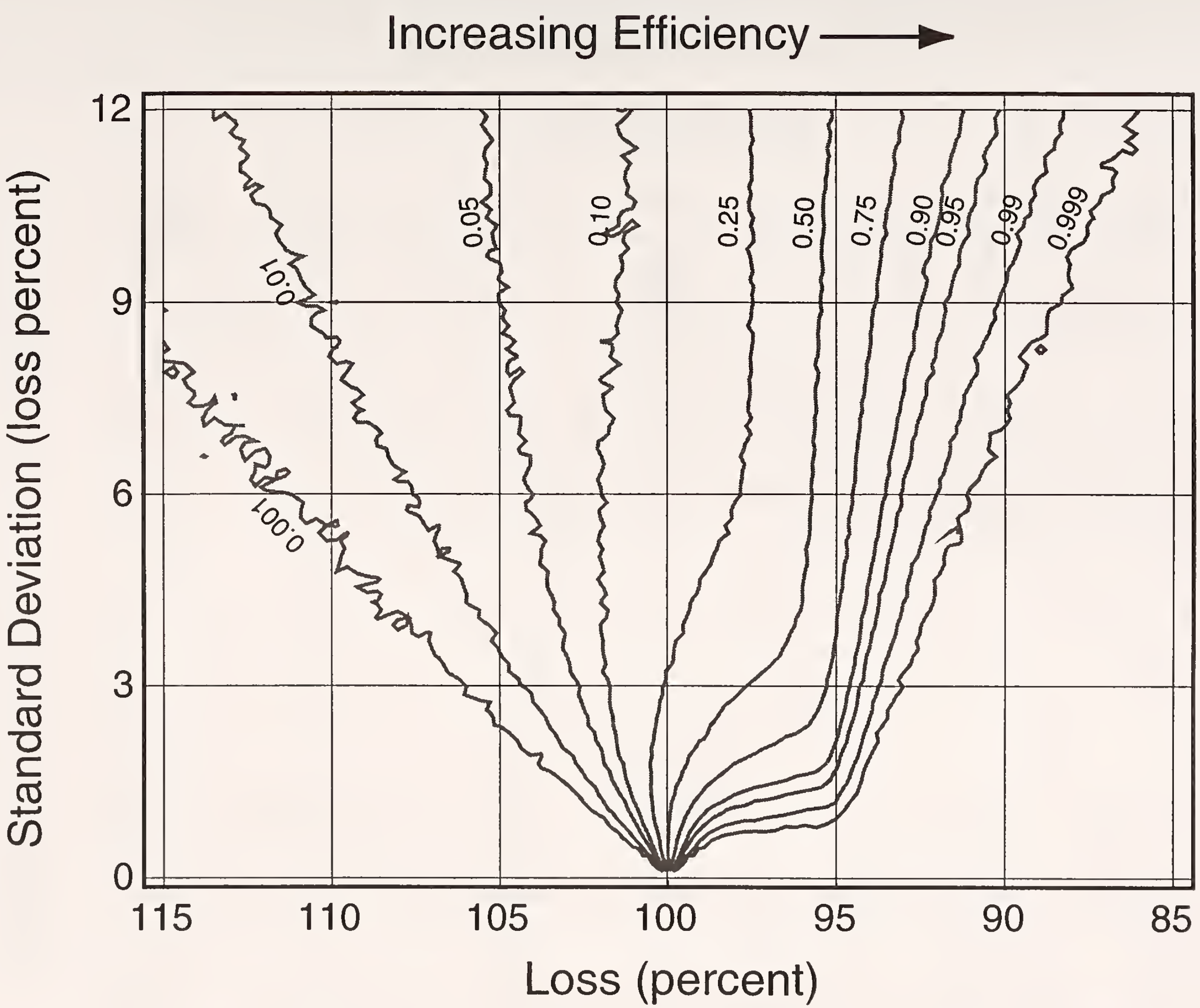

Figure 10: The operating characteristic of Method I. The results of Monte Carlo simulations for an initial sample size of two and a maximum sample of twenty are shown. The statistical confidence is 95 percent and the loss tolerance is 103 percent. The operating characteristic of the Method I compliance criteria. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance.

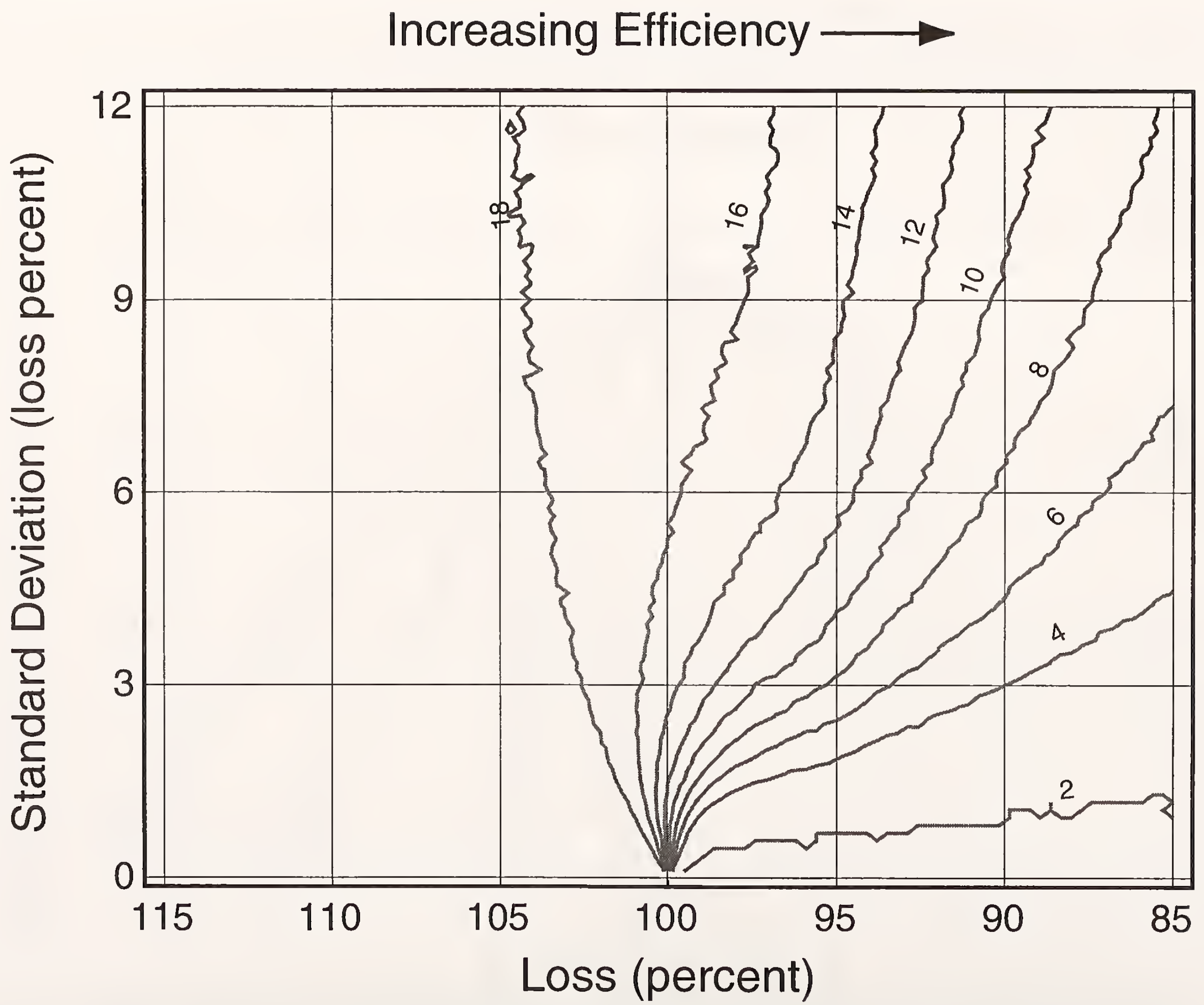

Figure 11: The testing burden of Method I. The results of Monte Carlo simulations for an initial sample size of two and a maximum sample of twenty are shown. The statistical confidence is 95 percent and the loss tolerance is 103 percent. The contours indicate the average sample size at the end of testing.

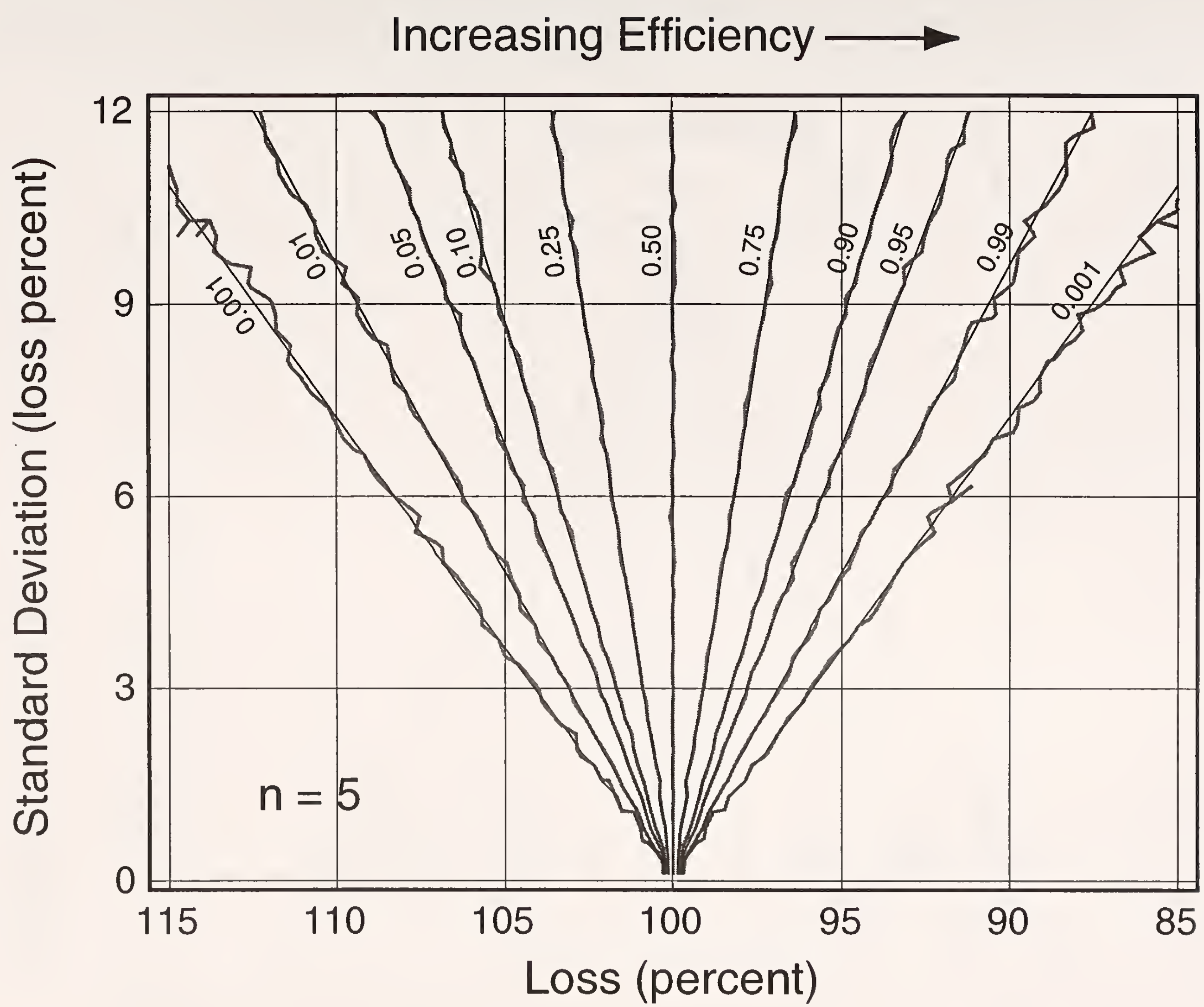

Figure 12: Operating characteristic of the Method II sampling plan for fixed sample of five. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance. The results of Monte Carlo simulations are shown in half-tone.

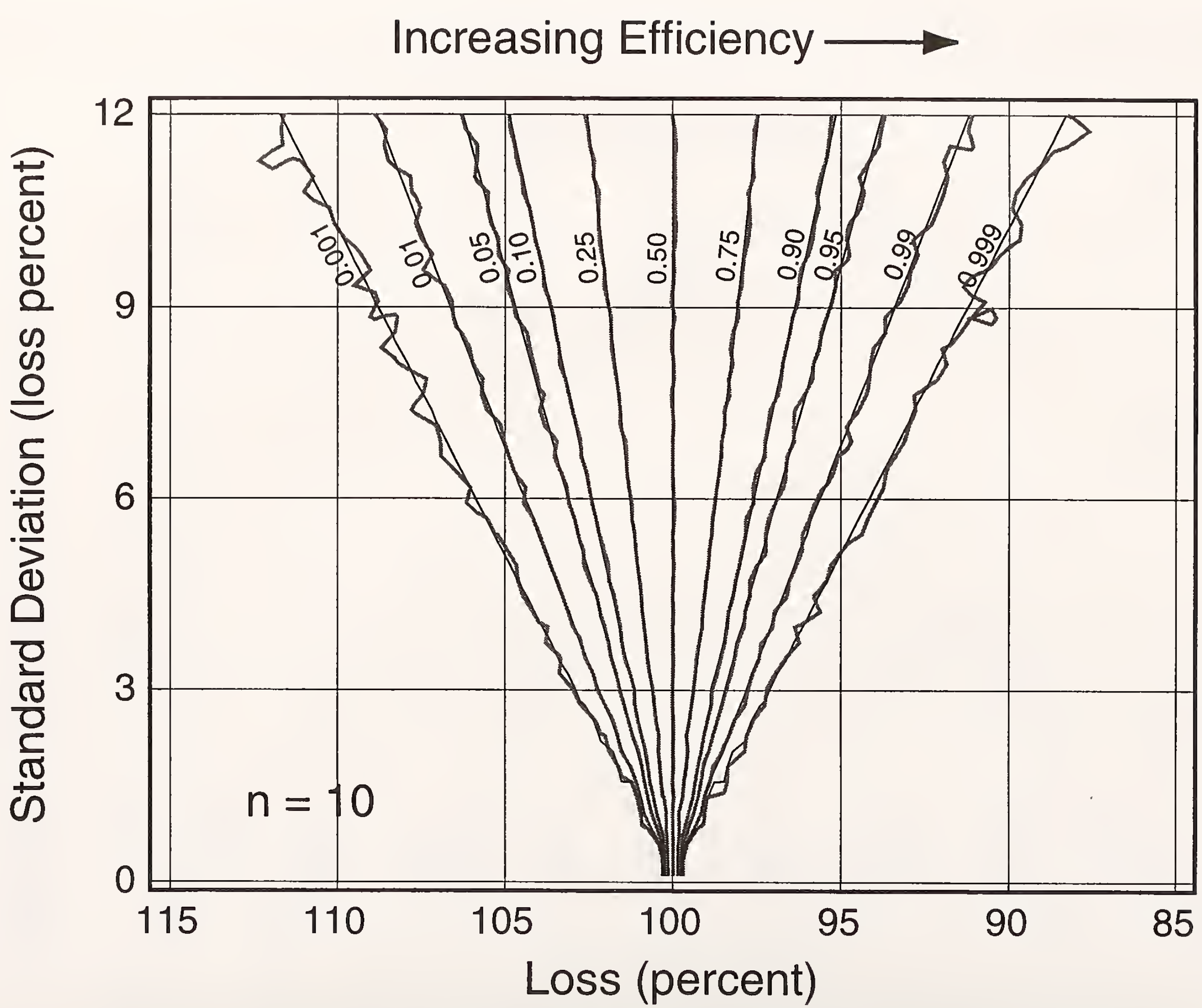

Figure 13: Operating characteristic of the Method II sampling plan for a fixed sample of ten. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance. The results of Monte Carlo simulations are shown in half-tone.

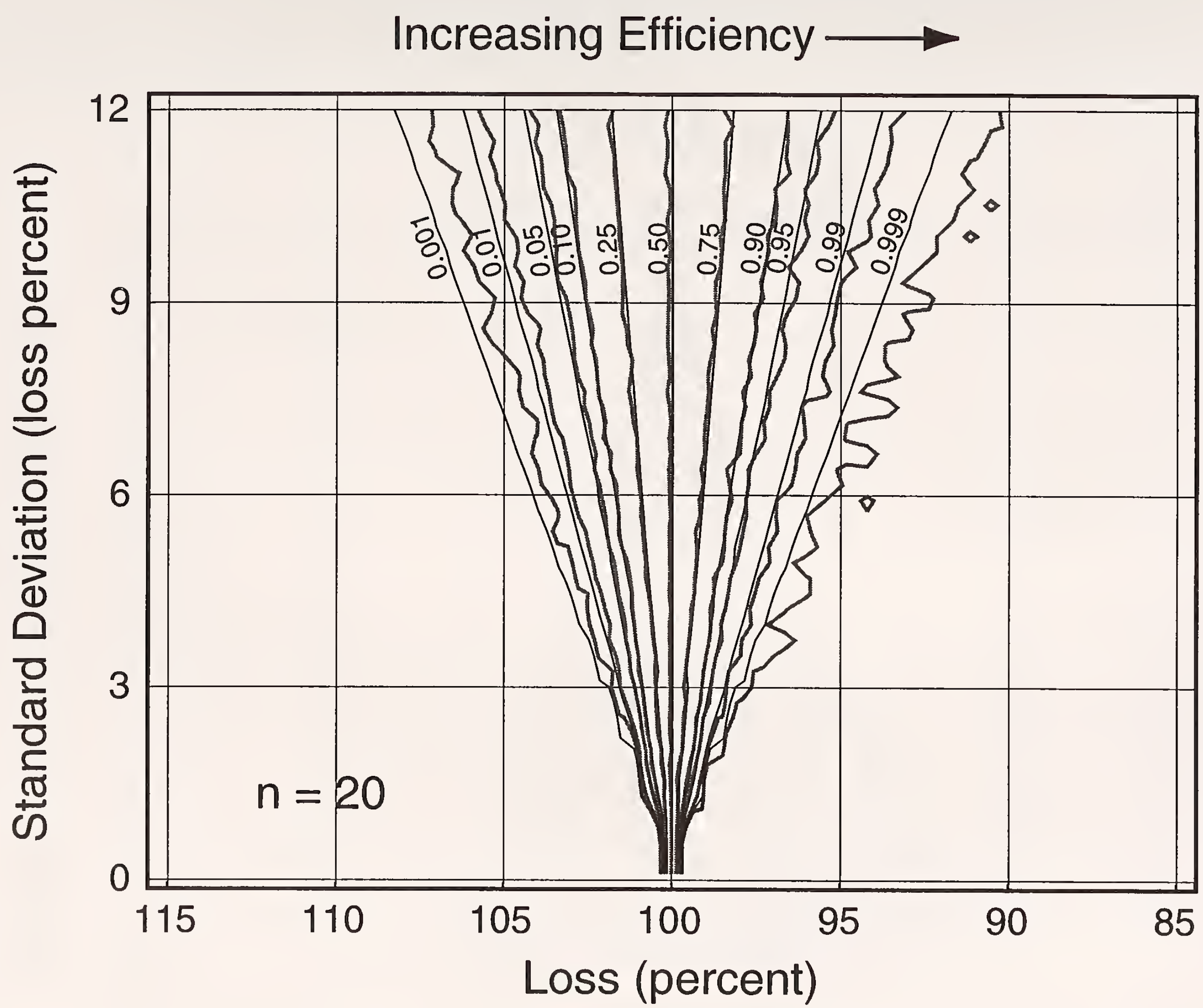

Figure 14: Operating characteristic of the Method II sampling plan for a fixed sample of twenty. The calculations are based on an extremum of 108 percent of the rated loss. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance. The results of Monte Carlo simulations are shown in half-tone.

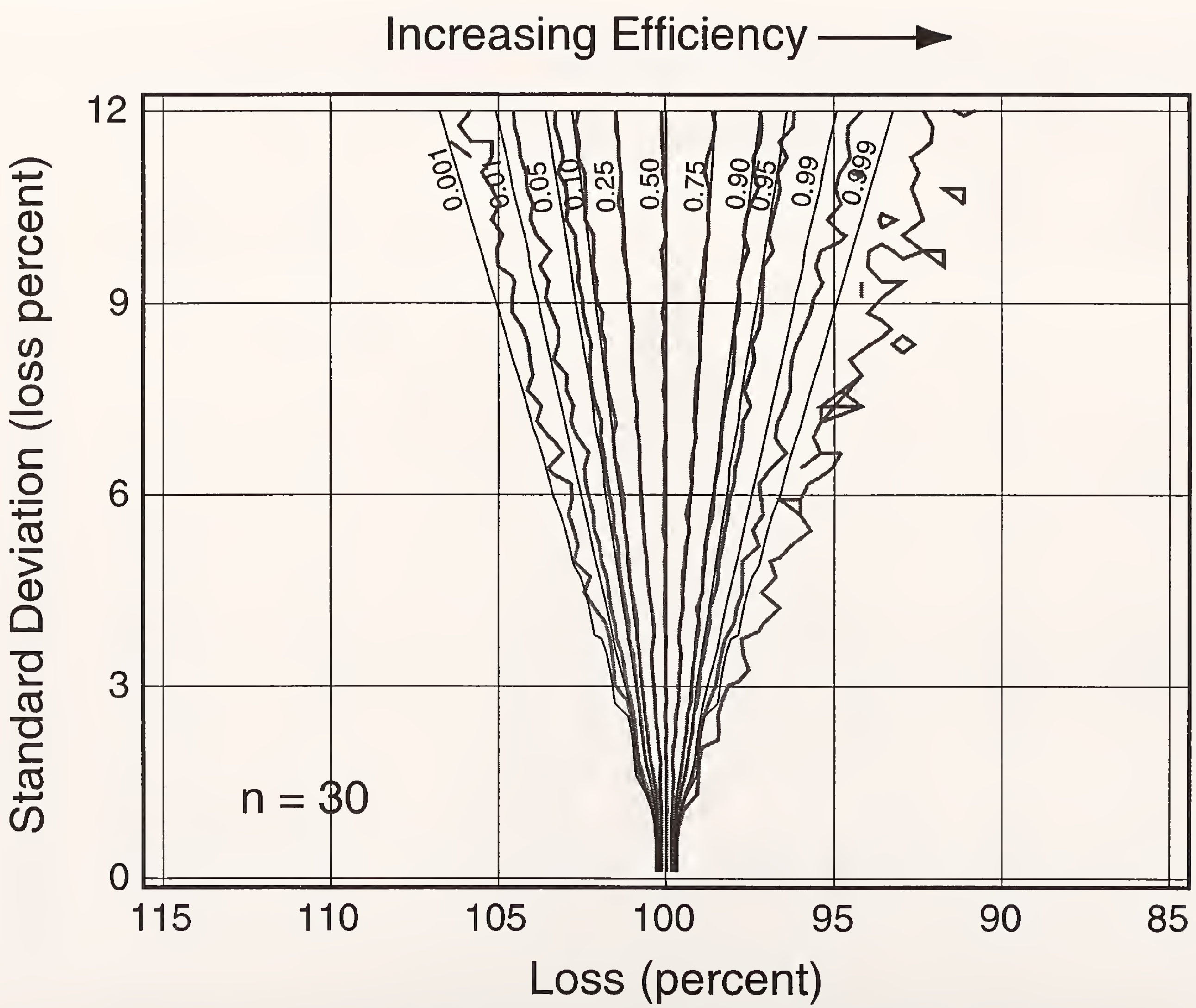

Figure 15: Operating characteristic of the Method II sampling plan for a fixed sample of thirty. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance. The results of Monte Carlo simulations are shown in half-tone.

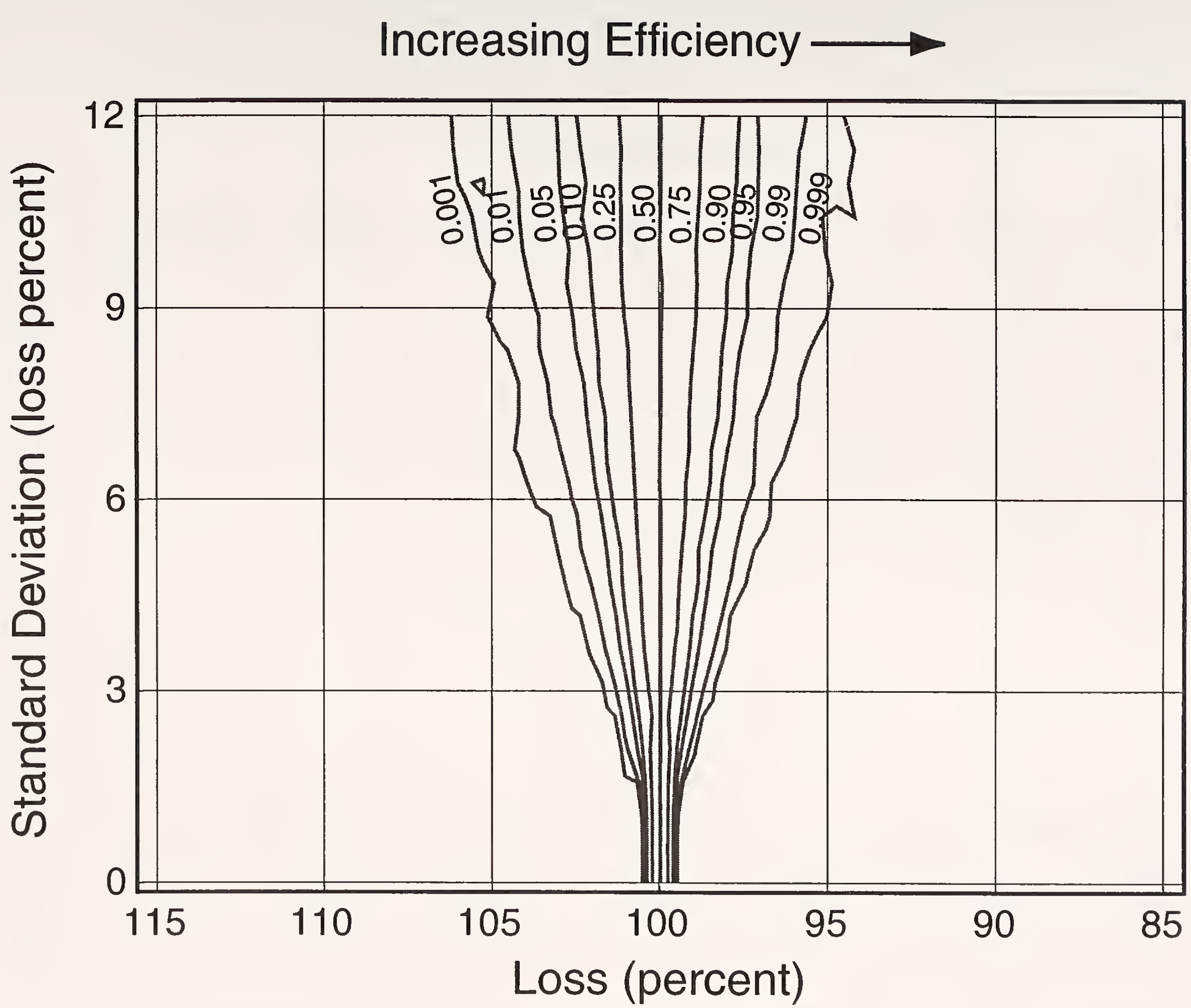

Figure 16: Operating characteristic of Method II sampling plan with variable sample size. Monte Carlo simulation for an initial sample size of five are shown. As in the previous figures, the contours indicate the probability of demonstrating compliance.

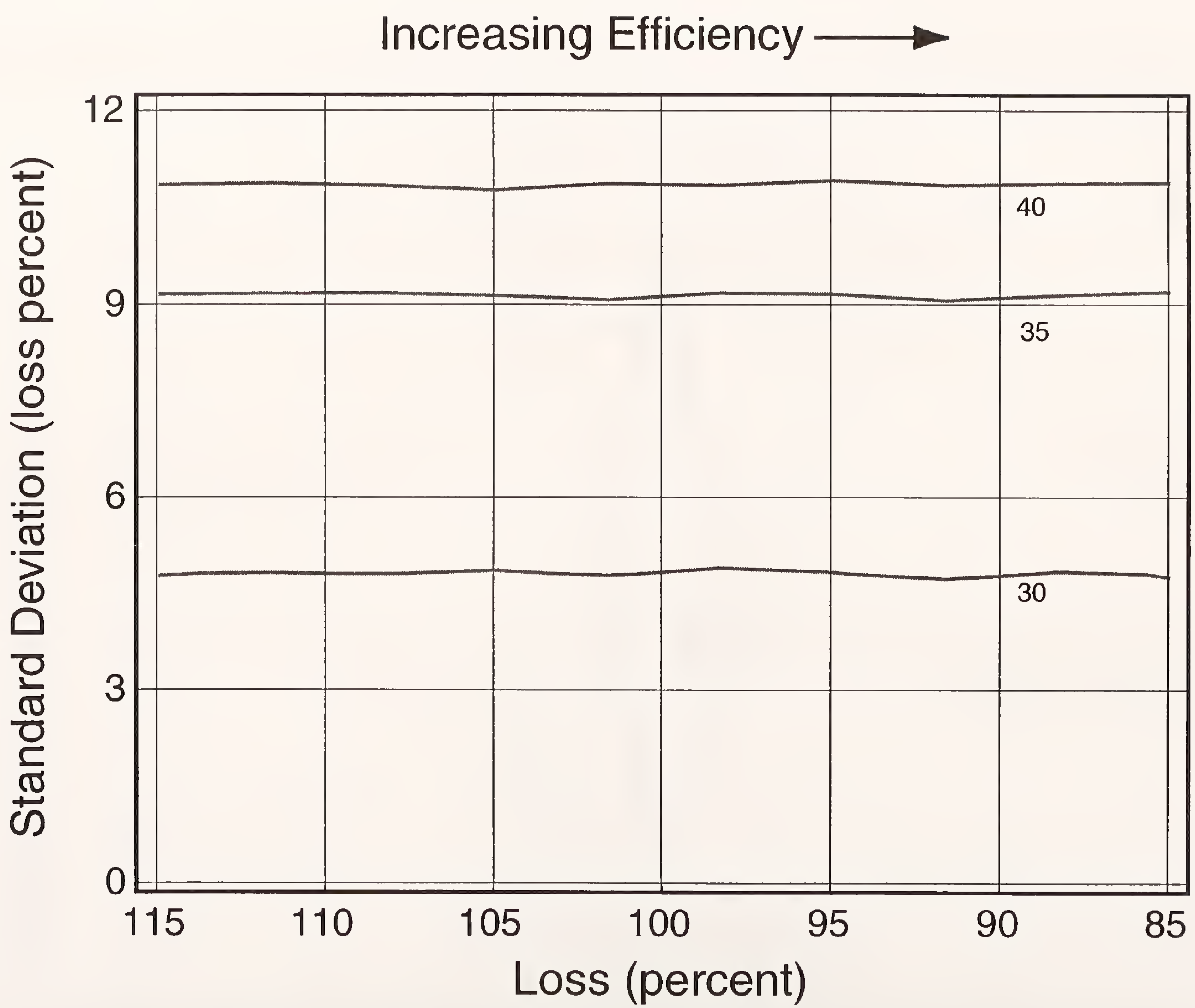

Figure 17: The testing burden of the Method II sampling plan. The results of Monte Carlo simulations are shown. Under the TP 2 sampling plan, compliance with the rated efficiency may be established by monthly tests conducted over a period of 180 days. Each monthly test would consist of no fewer than 5 units. The contours indicate the average sample size at the end of testing, e.g., the 30 contour corresponds to thirty units tested on average.

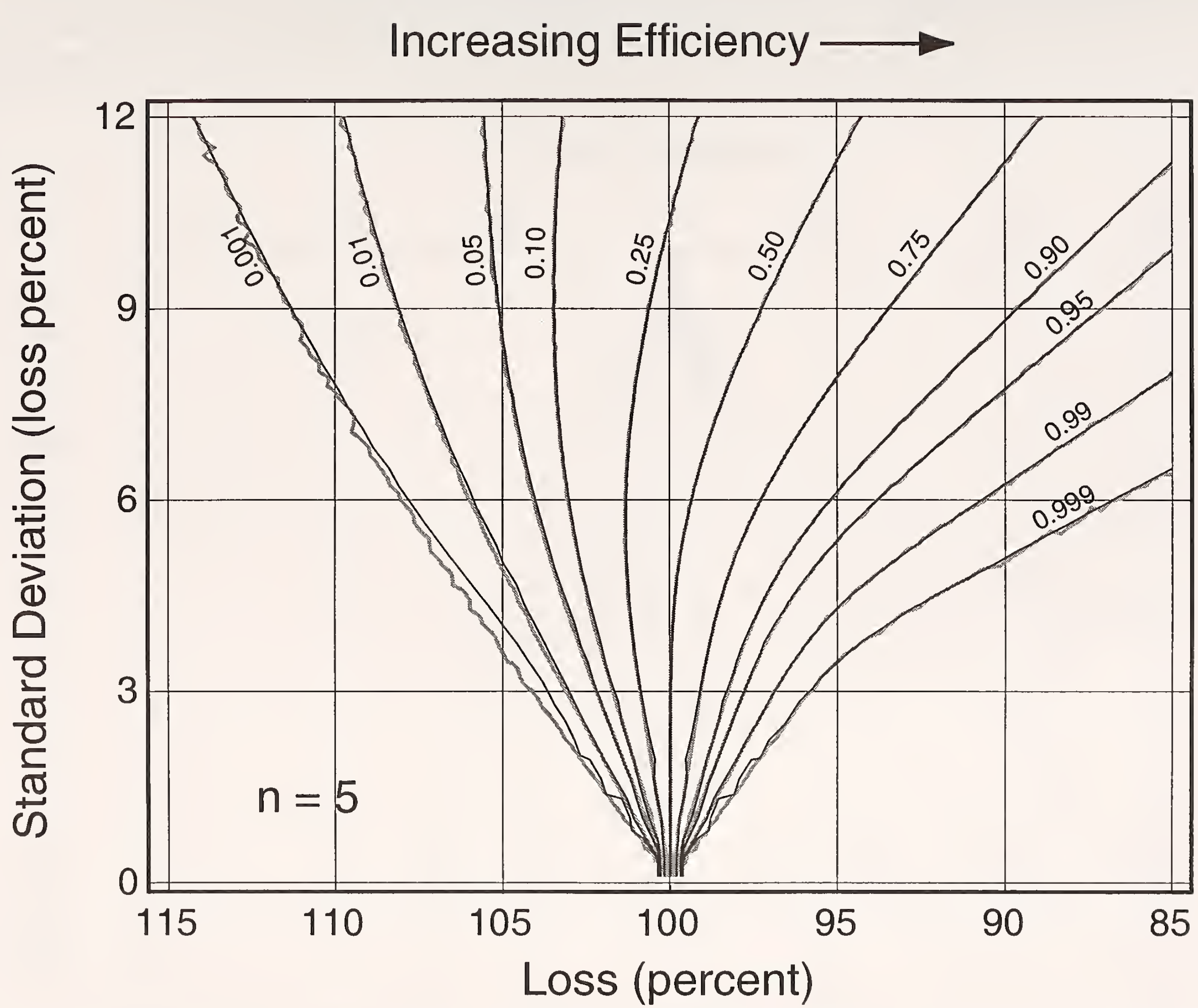

Figure 18: Operating characteristic of the Method III sampling plan for fixed sample of five. The calculations are based on an extremum of 108 percent of the rated loss. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance. The results of Monte Carlo simulations are shown in half-tone.

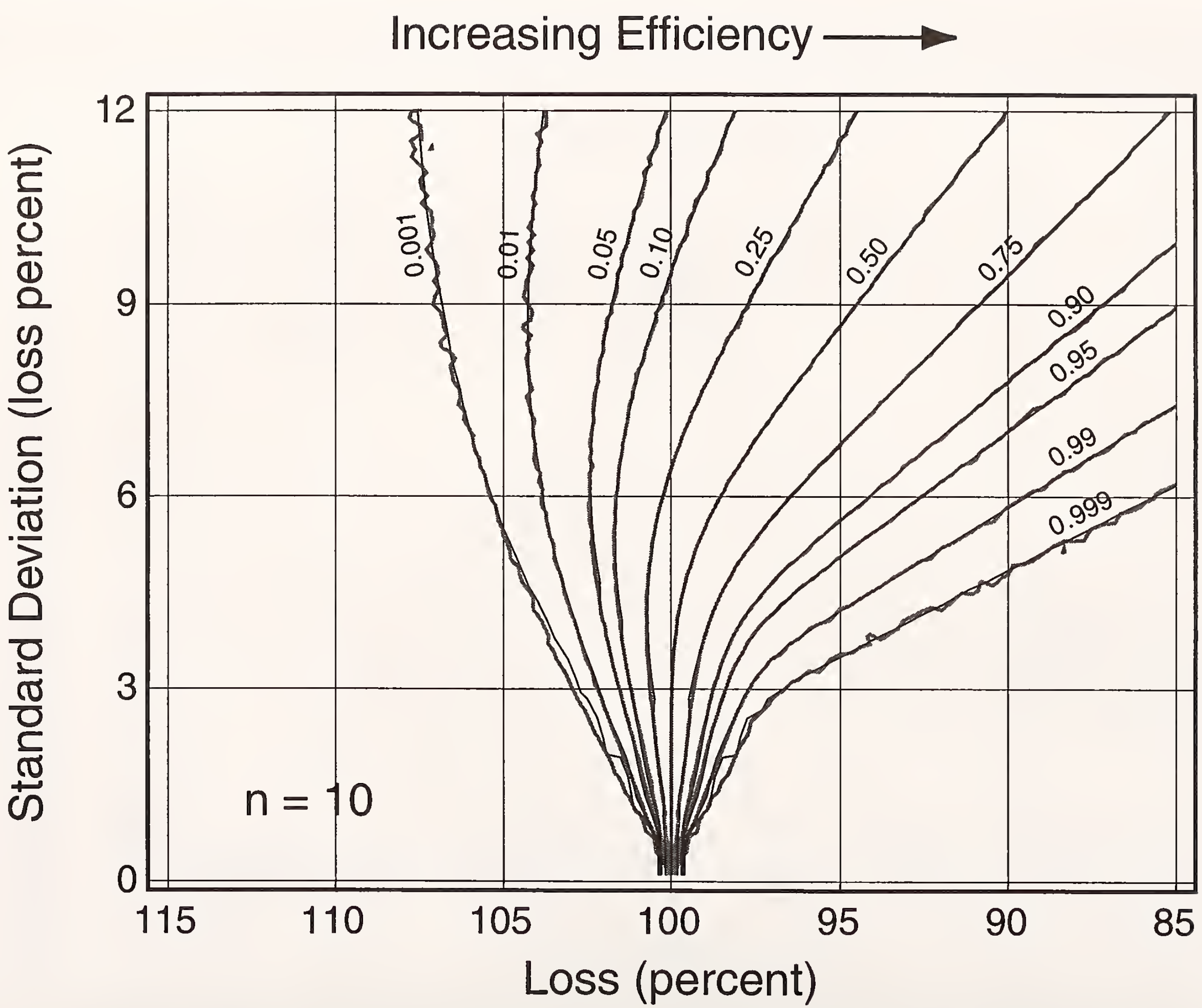

Figure 19: Operating characteristic of the Method III sampling plan for a fixed sample of ten. The calculations are based on an extremum of 108 percent of the rated loss. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance. The results of Monte Carlo simulations are shown in half-tone.

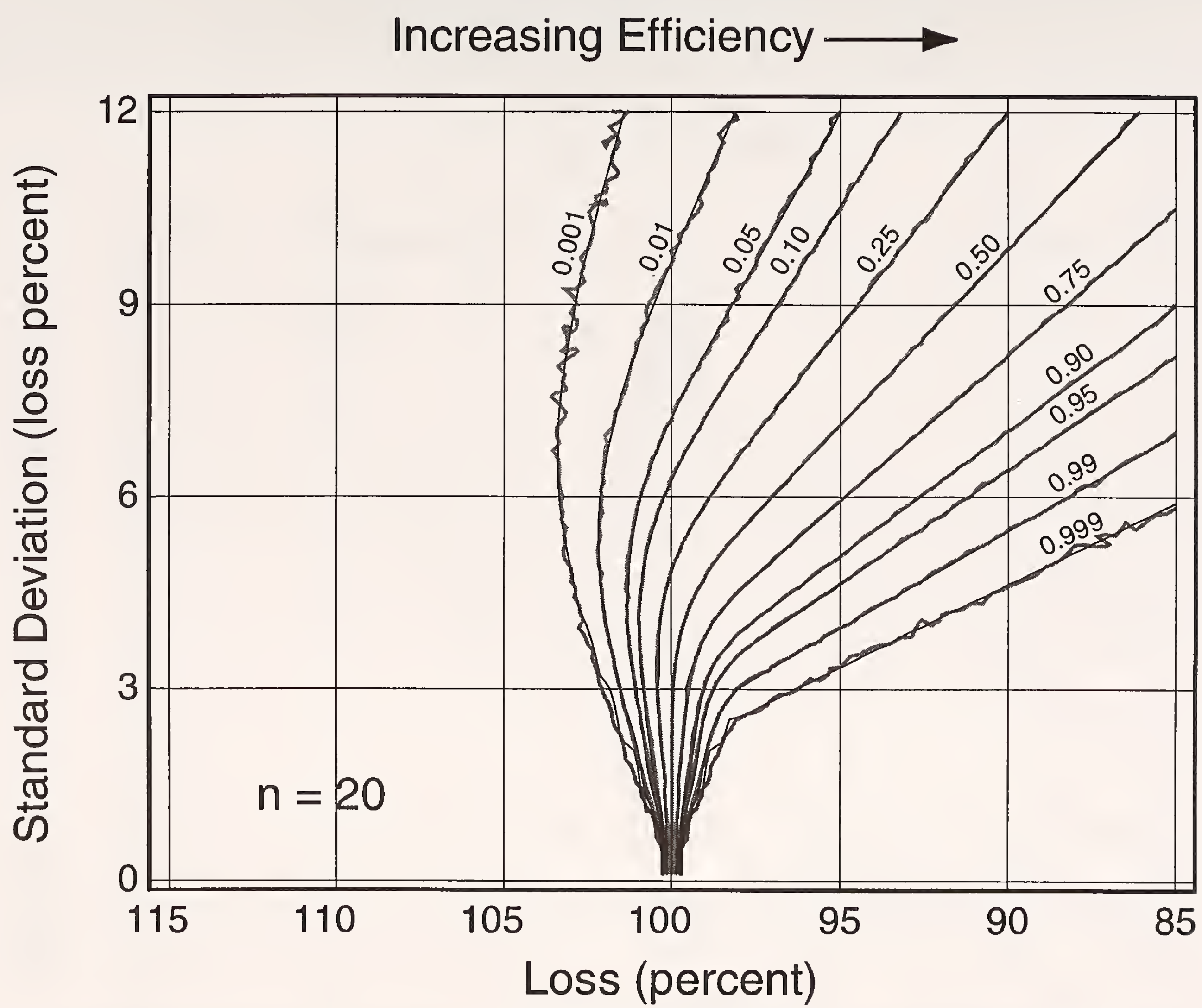

Figure 20: Operating characteristic of the Method III sampling plan for a fixed sample of twenty. The calculations are based on an extremum of 108 percent of the rated loss. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance. The results of Monte Carlo simulations are shown in half-tone.

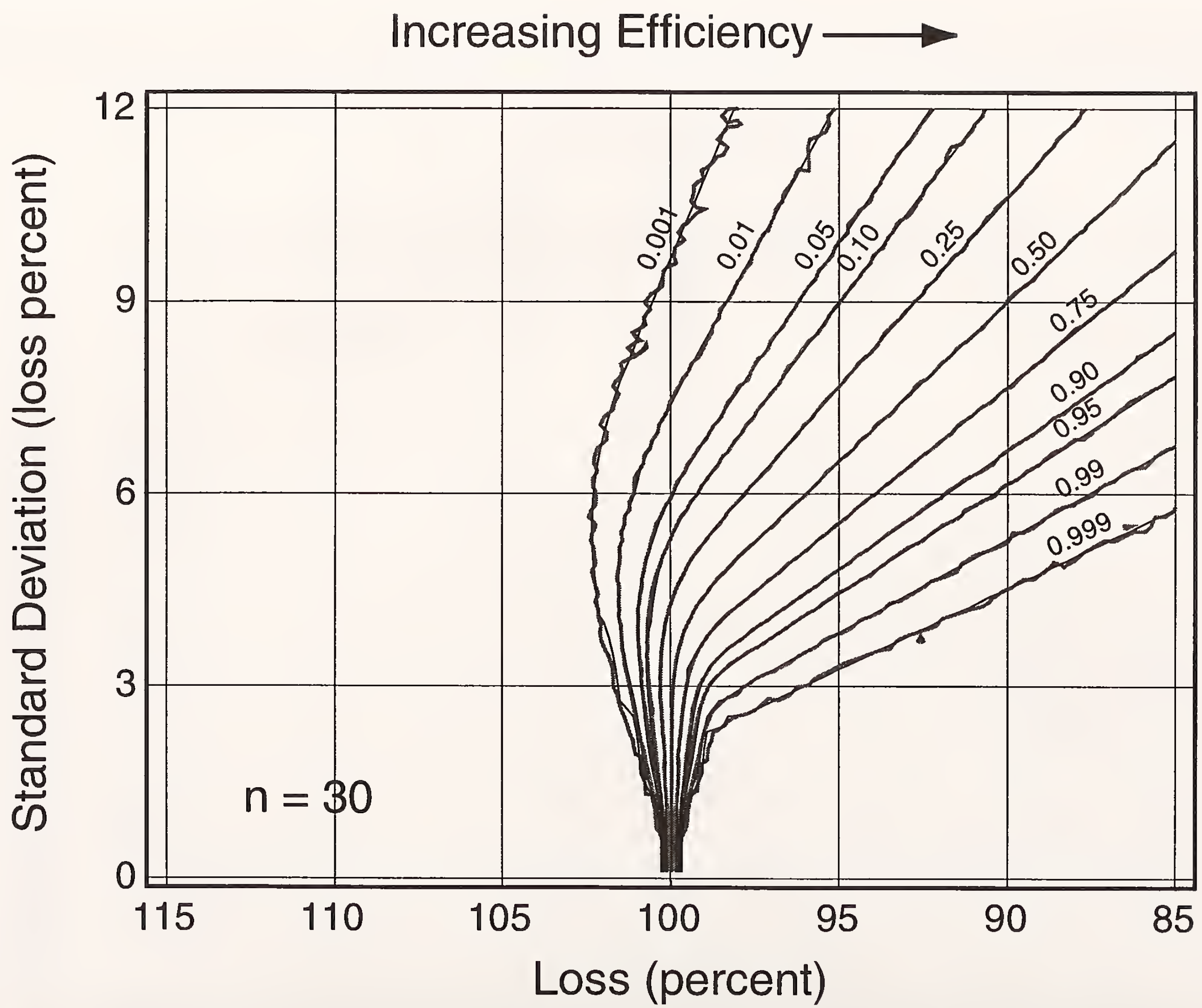

Figure 21: Operating characteristic of the Method III sampling plan for a fixed sample of thirty. The calculations are based on an extremum of 108 percent of the rated loss. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance. The results of Monte Carlo simulations are shown in half-tone.

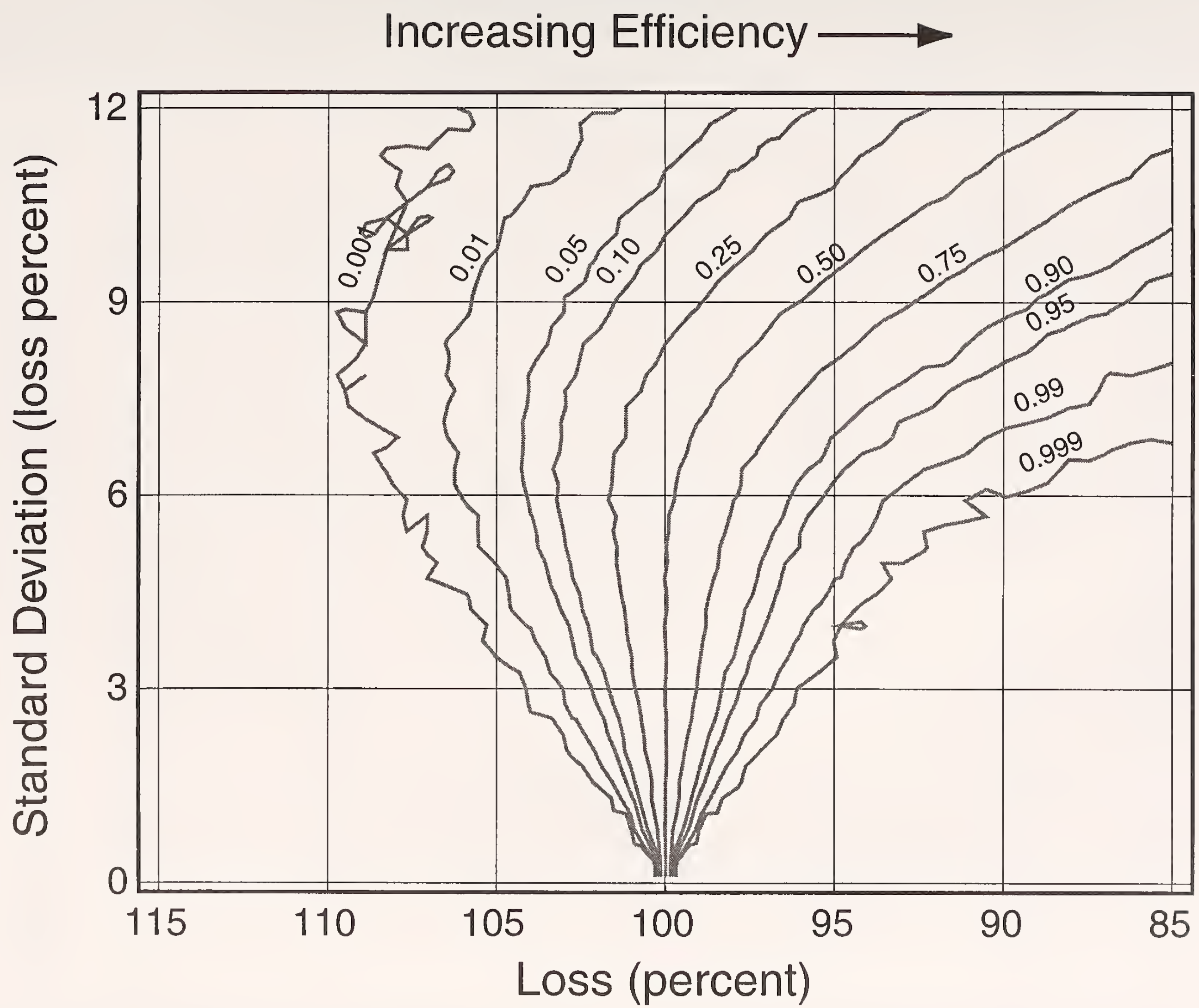

Figure 22: Operating characteristic of Method III sampling plan with variable sample size. Monte Carlo simulation for an initial sample size of five are shown. Based on an extremum of 108 percent of the rated loss. As in the previous figures, the contours indicate the probability of demonstrating compliance.

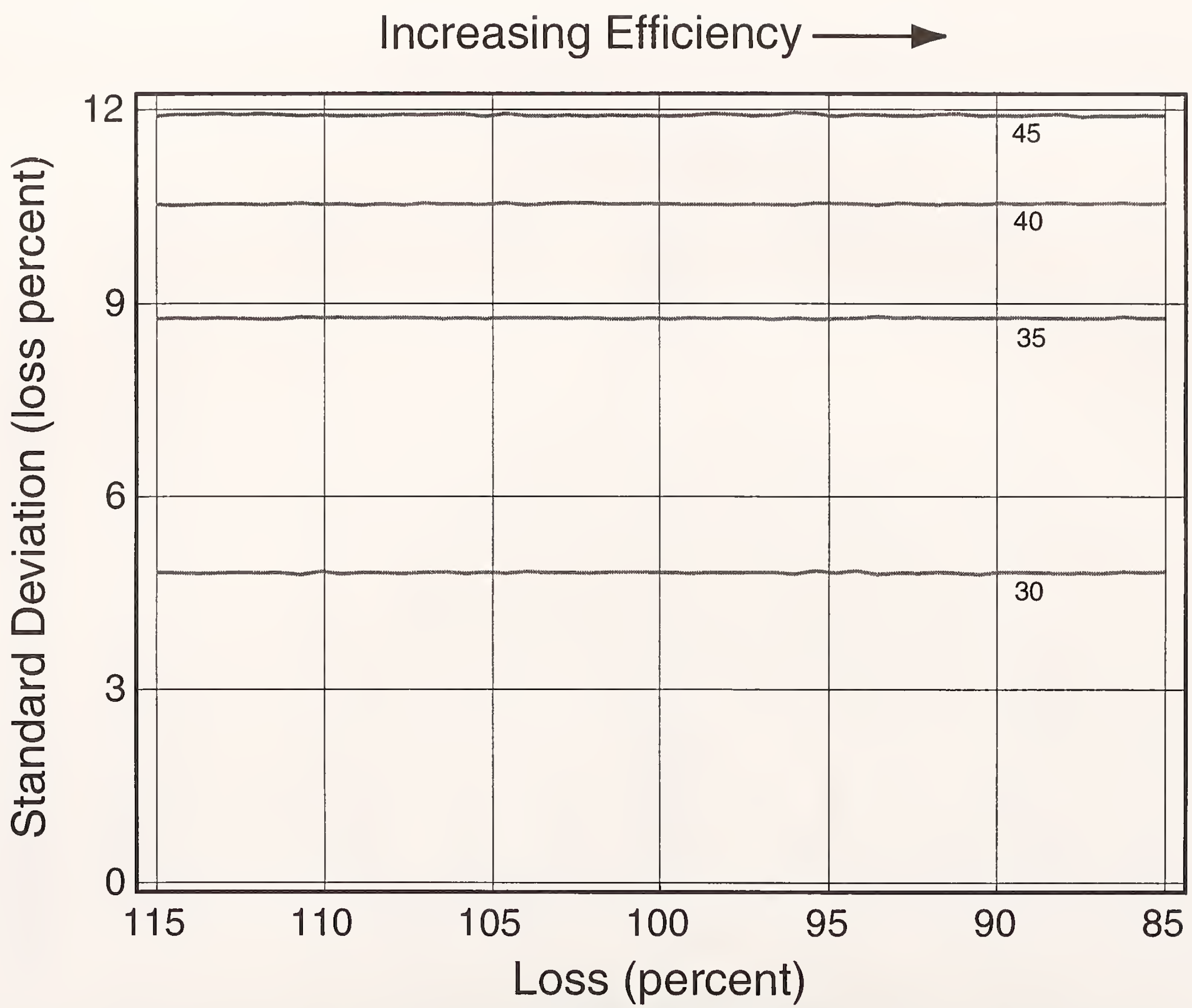

Figure 23: The testing burden of the Method III sampling plan. The results of Monte Carlo simulations are shown. The statistical confidence is 95 percent and the loss tolerance is 108 percent. Under the TP 2 sampling plan, compliance with the rated efficiency may be established by monthly tests conducted over a period of 180 days. Each monthly test would consist of no fewer than 5 units. The contours indicate the average sample size at the end of testing, e.g., the 30 contour corresponds to thirty units tested on average.

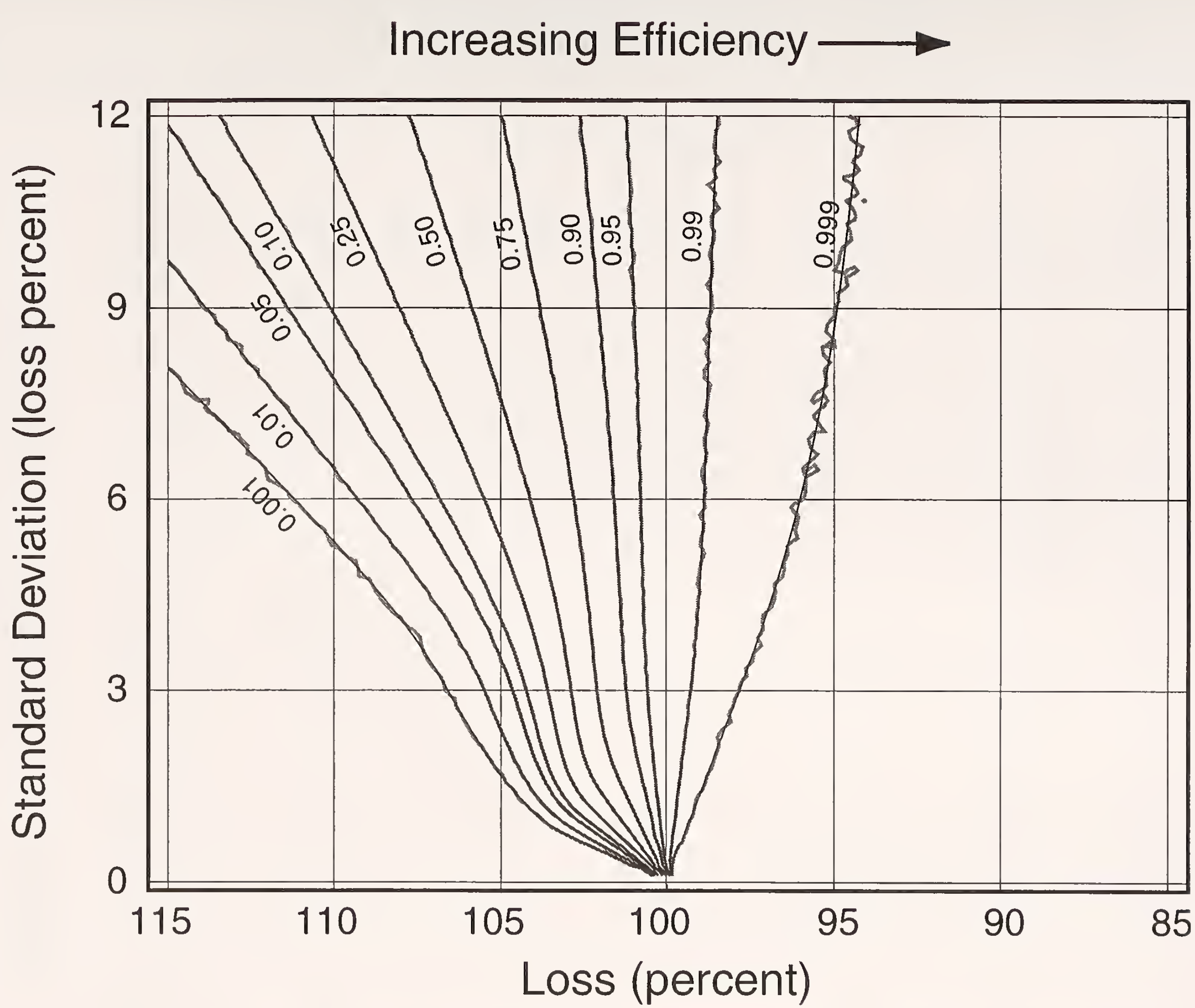

Figure 24: Operating characteristic of the Sampling Plan for Enforcement Testing. Model calculations for an initial sample of four and maximum sample of twenty are shown. The statistical confidence is 97.5 percent and the loss tolerance is 103 percent. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance.

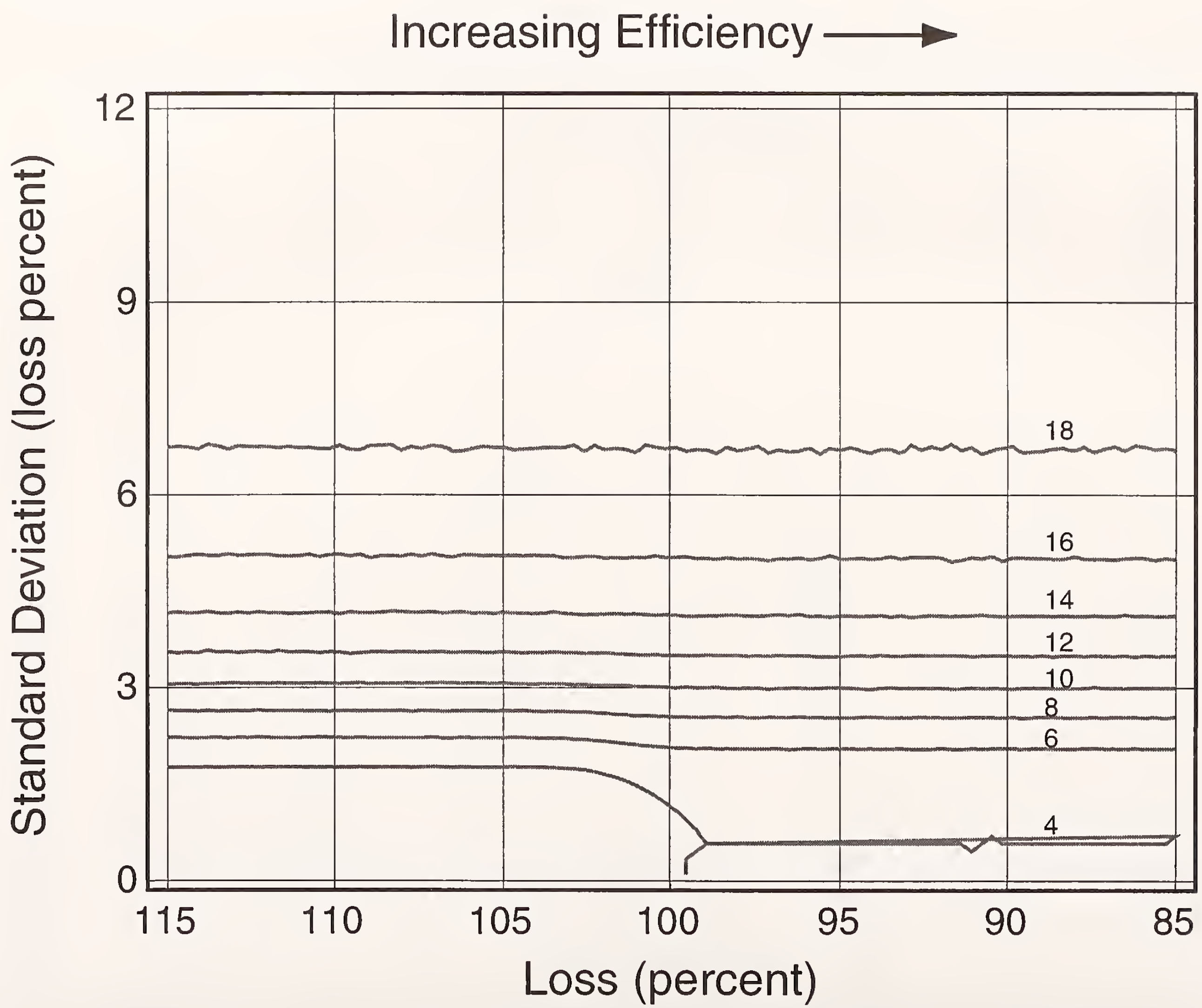

Figure 25: Testing burden of the Sampling Plan for Enforcement Testing. Model calculations for an initial sample of four and maximum sample of twenty are shown. The statistical confidence is 97.5 percent and the loss tolerance is 103 percent. Contours indicate the average sample size at the completion of testing.

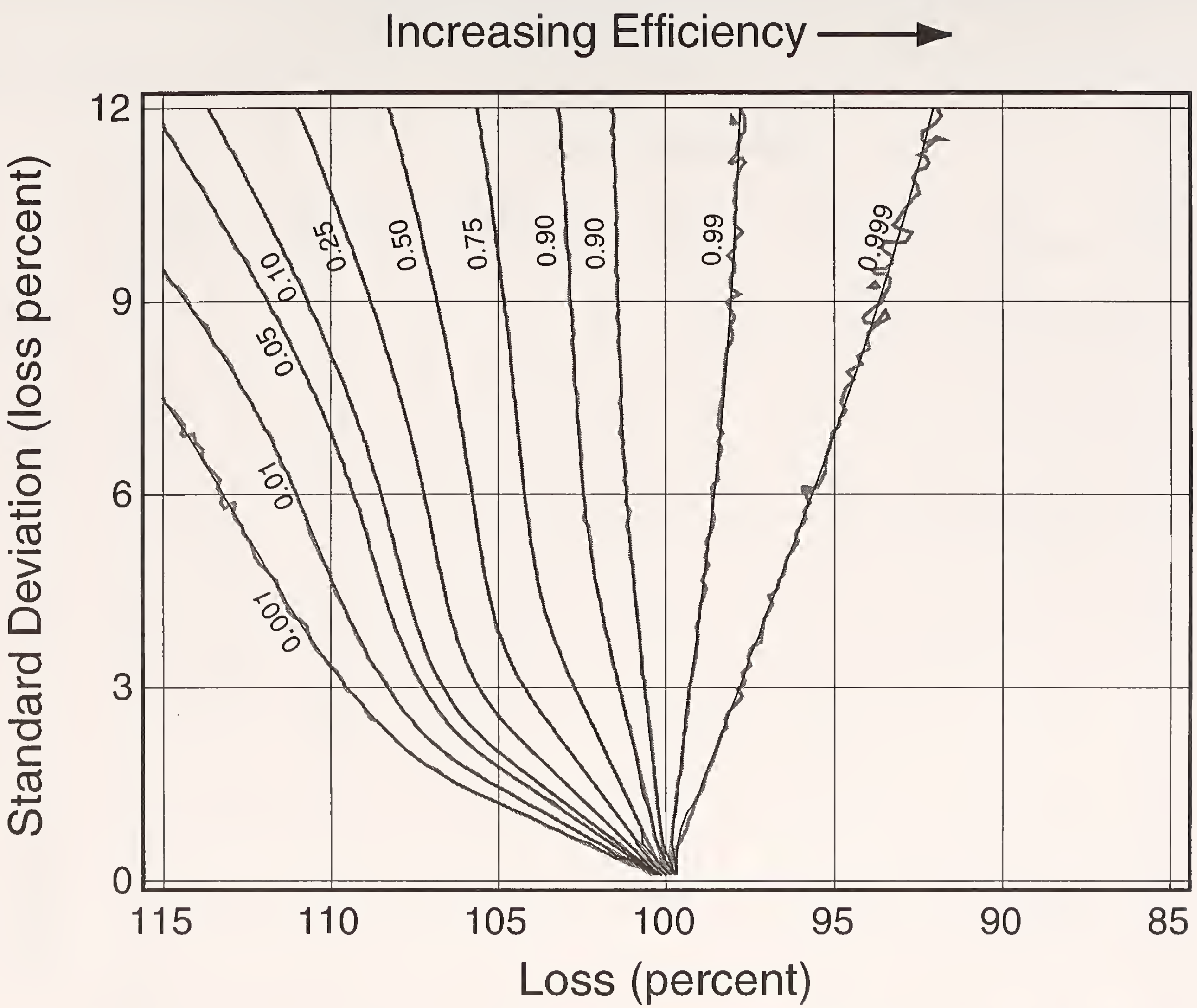

Figure 26: Operating characteristic of the Sampling Plan for Enforcement Testing. Model calculations for an initial sample of four and maximum sample of twenty are shown. The statistical confidence is 97.5 percent and the loss tolerance is 106 percent. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance.

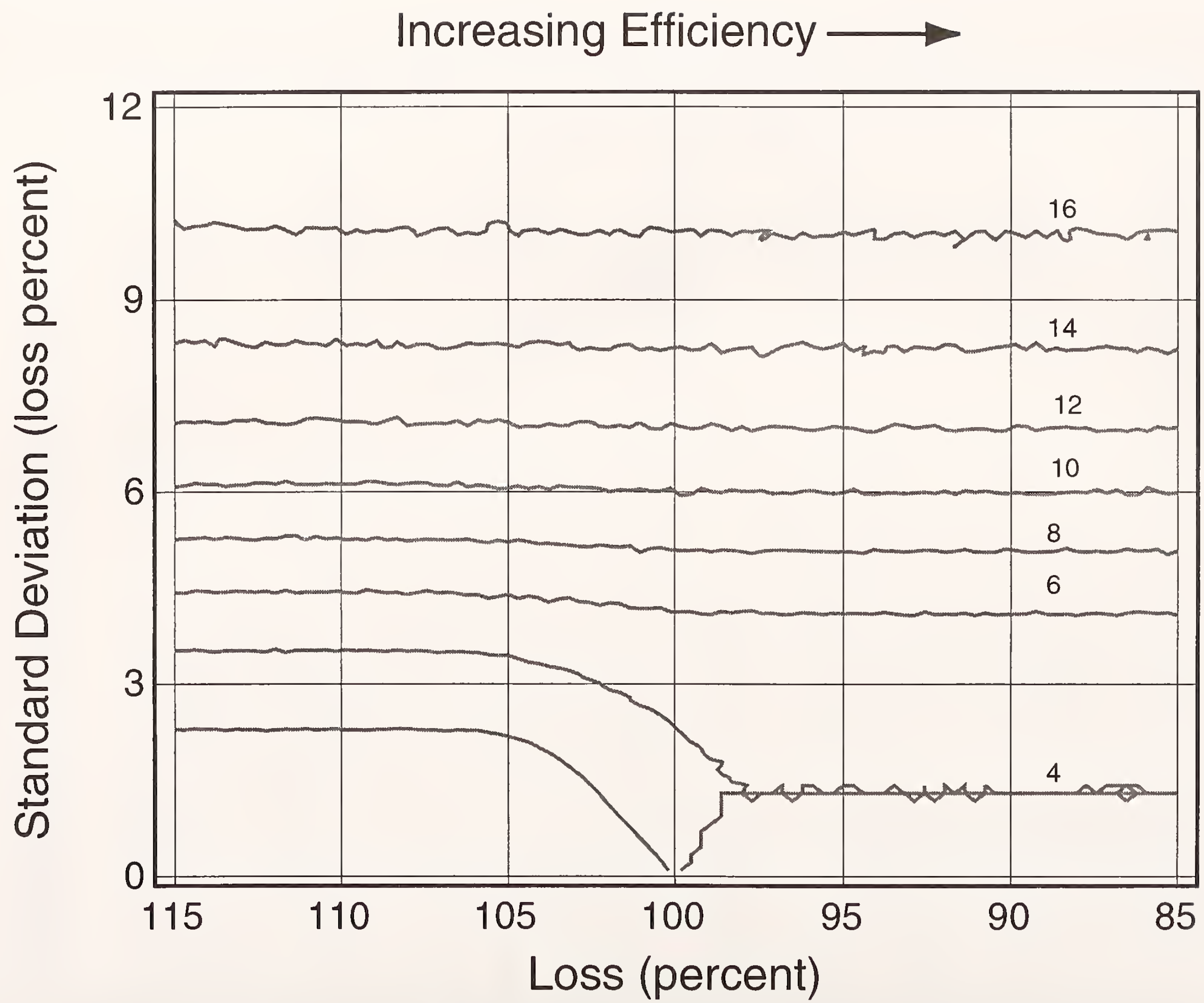

Figure 27: Testing burden of the Sampling Plan for Enforcement Testing. Model calculations for an initial sample of four and maximum sample of twenty are shown. The statistical confidence is 97.5 percent and the loss tolerance is 106 percent. Contours indicate the average sample size at the completion of testing.

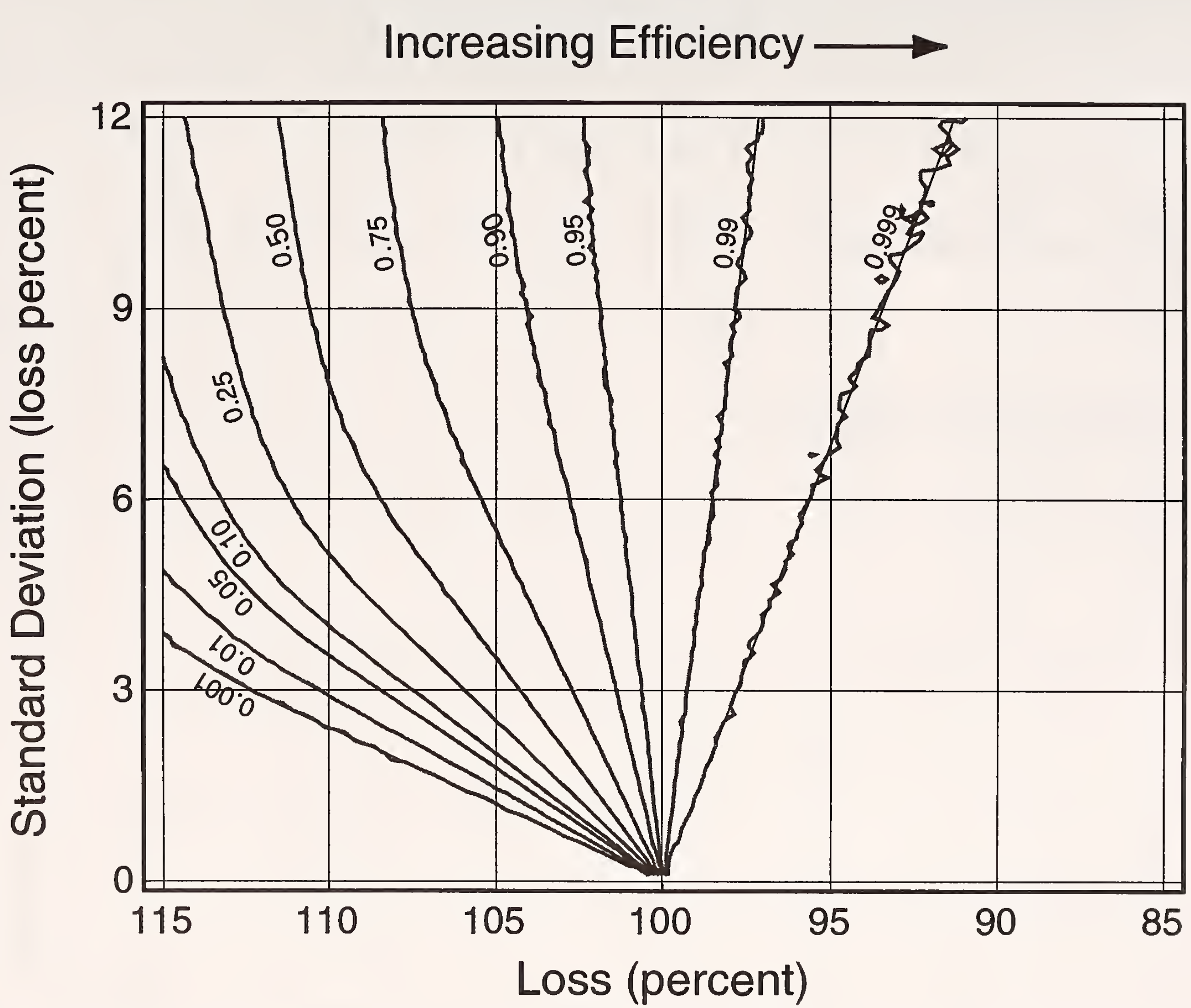

Figure 28: Operating characteristic of the Sampling Plan for Enforcement Testing. Model calculations for an initial sample of four and maximum sample of twenty are shown. The statistical confidence is 97.5 percent and the loss tolerance is 112 percent. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90 percent likelihood of demonstrating compliance.

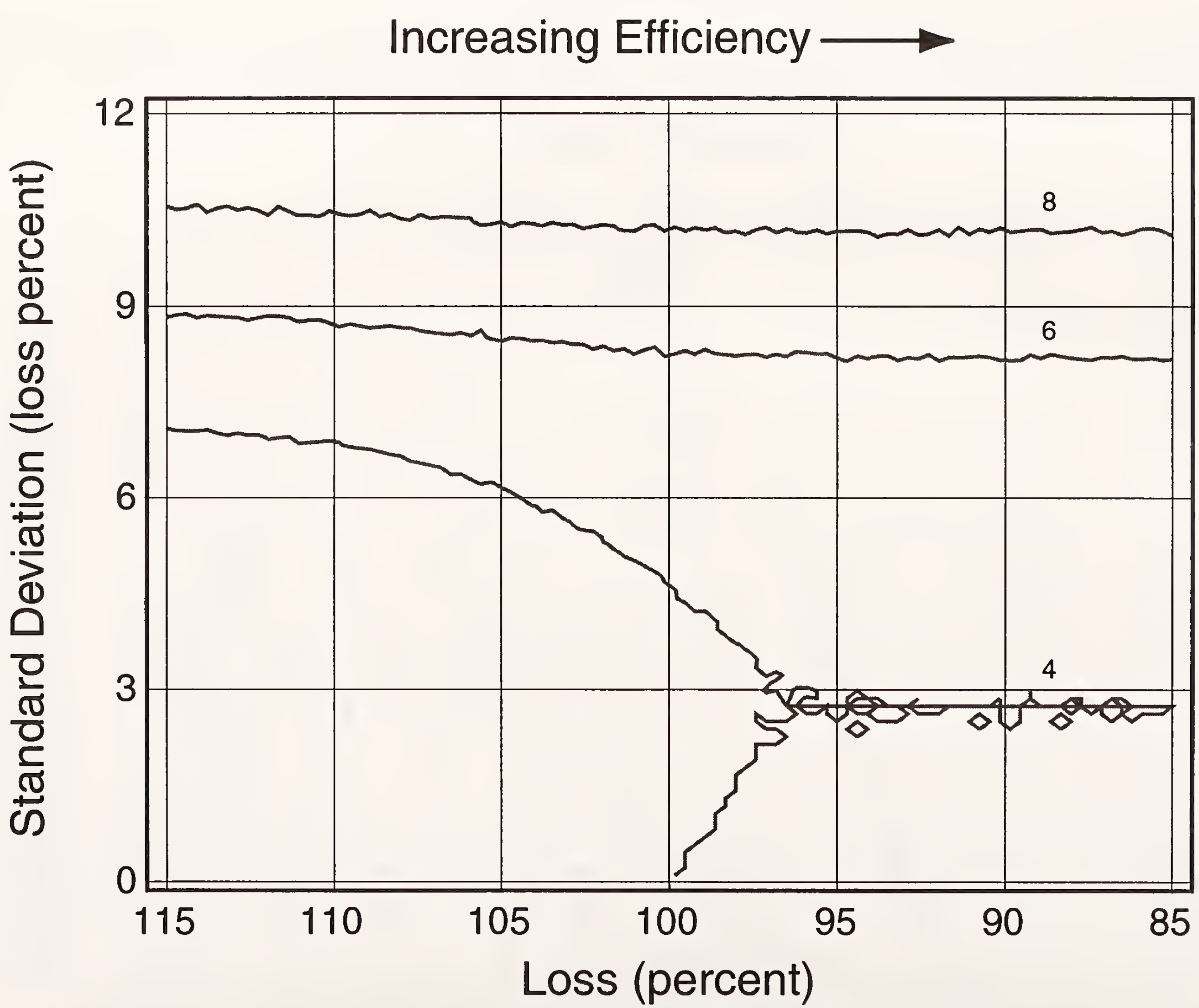

Figure 29: Testing burden of the Sampling Plan for Enforcement Testing. Model calculations for an initial sample of four and maximum sample of twenty are shown. The statistical confidence is 97.5 percent and the loss tolerance is 112 percent. Contours indicate the average sample size at the completion of testing.

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Appendix A

The 10 CFR Part 430 and the proposed 10 CFR Part 432 sampling plans for compliance testing

The following test criteria are presented as an example of compliance testing under 10 CFR Part 430. Similar criteria have been proposed for distribution transformers in the new 10 CFR Part 432 [3] and are also presented.

The 10 CFR Part 430 criteria presented below apply specifically to dishwashers, which is one of the products covered under 10 CFR Part 430. Similar sampling plans are provided in 10 CFR Part 430 §430.24 for all other covered products. The statistical confidence and divisor used in criteria (i)(B) and (III)(B) differ between products and are specific to each product. The values of these coefficients are chosen to be consistent with current industry practice for that product.

§430.24 Units to be tested.

When testing of a covered product is required to comply with section 323(c) of the Act, or to comply with rules prescribed under sections 324 or 325 of the Act, a sample shall be selected and tested composed of units which are production units, or are representative of production units of the basic model being tested, and shall meet the following applicable criteria.

(c)(1) For each basic model of dishwashers, a sample of sufficient size shall be tested to insure that—

(i) Any represented value of estimated annual operating cost, energy consumption or other measure of energy consumption of a basic model for which consumers would favor lower values shall be no less that the higher of (A) the mean of the sample or (B) the upper $97^1/_2$ percent confidence limit of the true mean divided by 1.05, and

(ii) Any represented value of the energy factor of other measure of energy consumption of a basic model for which consumers would favor higher values shall be no greater than the lower of (A) the mean of the sample or (B) the lower $97^1/_2$ percent confidence limit of the true mean divided by 0.95.

The criteria given below are proposed for distribution transformers in the new 10 CFR Part 432. The values of statistical confidence and the formulation of the divisor specified in (b) are chosen to be consistent with current industry practice.

§432.24 Units to be tested.

For each basic model of distribution transformers, a random sample of sufficient size, but no fewer than five productions units, shall be tested to insure that any represented value of efficiency shall be no greater than the lower of the:

(a) Mean of the sample; or

(b) The lower 95% confidence limit of the estimated true mean divided by a number equal to $[1 - 0.03(1 - E_s/100)]$, where E_s is the represented value of efficiency claimed for that particular basic model.

Appendix B

The NEMA Standard TP 2 sampling plan

Sections 7.0 and 7.1 of NEMA Standard TP 2 follow. The reader may note minor differences between this appendix and the NEMA standard TP 1, e.g., the pagination and the reference to the included table in this appendix differ from those in the NEMA standard TP 1.

SECTION 7
Demonstration of Compliance

7.0 General

This section provides a methodology for proving compliance in achieving the specified efficiency levels. It specifies monthly sampling over a 180 day period for the cases where 100% of the units are not tested. This standard requires that no individual transformer shall be considered acceptable if its measured losses exceed the allowance by more than 8%.

For a transformer population of a specific kVA rating, compliance with the energy efficiency standards as defined in Section 4 of NEMA TP 1 shall be demonstrated through measurements of No-Load and Load losses according to the procedures described in this standard. A transformer model is defined as all transformers of the same kVA rating and type as described in the efficiency tables of NEMA TP 1.

According to the IEEE Standards C57.12.00 and C57.12.01 the loss tolerance for an individual unit as related to guarantee is defined as follows:

	Limit Beyond Guarantee
No Load Loss	10% max.
Total Loss at 100% Load	6% max.

At 100% load, the Load Loss is normally four times the No Load loss, suggesting that a Load Loss variability of approximately 5% is allowed. The sum of this allowance and the 10% variability for the No Load loss yields the Total Loss variability of 6%.

Since TP-1 tables reflect loss measurements at 35% or 50% of rated load where No Load Loss is equal to Load Losses, the 6% loss tolerance cannot be used and therefore a new total loss tolerance of 8% shall be applicable at these measurement points. This is consistent with the IEEE Standards C57.12.00 and C57.12.01.

7.1 Number of Units to be Tested

NEMA TP-1 requires that the overall efficiency of the entire population of transformers must meet the specified efficiency standards. This intent is satisfied if the mean efficiency of the entire population satisfies this requirement. The compliance of a group of transformers shall be demonstrated by testing all or randomly drawn samples of these transformers.

7.1.1 Compliance Demonstration Through test on all Transformers:

Manufacturers may choose to test all units of various kVA ratings manufactured during a production period of 180 days to demonstrate compliance with the efficiency standard NEMA TP 1. The intent of this standard is satisfied if the Total Measure kVA Input (TMI) of this batch of transformers is equal to or less than the Total Allowed kVA Input (TAI) calculated based on the measured and specified efficiency levels specified in TP 1. Each individual unit from this production batch must meet or exceed the minimum acceptable efficiency level calculated as follows:

$$\text{Minimum Acceptable Efficiency level} = \frac{SEL}{108 - 0.08SEL} \times 100$$

Where

SEL = the standard percent efficiency level from NEMA Standard TP 1.

Note: The Minimum Acceptable Efficiency level calculation is based on an 8% tolerance on total loss at the load levels considered for the Efficiency levels specified in NEMA TP-1; i.e. no individual unit shall be considered acceptable if its measured losses exceed the allowance by more than 8%.

Table B1:

Sample size n	t at 95% confidence level	Sample size n	t at 95% confidence level
2	6.314	11	1.812
3	2.920	12	1.796
4	2.353	13	1.782
5	2.132	14	1.771
6	2.015	15	1.761
7	1.943	16	1.753
8	1.895	17	1.746
9	1.860	18	1.740
10	1.833	19	1.734
		20	1.729

To demonstrate compliance with the Efficiency standard, proceed as follows:

Step 1. Calculate the Total Allowed KVA input:

$$TAI = \sum \frac{L_i \, kVA_i}{\eta_i}$$

Where
i = 1,2,3,4, ...
kVA_i = kVA ratings of various transformers included in a production batch manufactured in 180 days.
η_i = Specified Efficiency Level in TP 1 for transformer rating kVA_i
L_i = Per unit load at which the efficiency is specified per TP 1

Step 2. Calculate the Total Measured kVA input:

$$TMI = \sum \frac{L_i \, kVA_i}{\eta_{mi}}.$$

Where
i = 1,2,3,4, ...
η_{mi} = Measured Efficiency Level for the transformer.
L_i = Per unit load at which the efficiency is specified per TP 1

Step 3. If TMI is equal to or less than TAI; the compliance of the production batch has been demonstrated.

7.1.2 Compliance Demonstration Through Tests on a Statistically Valid Sample:

The manufacturer may choose to demonstrate the compliance of a plurality of units by a random sampling of the units of each kVA rating produced in a period of 180 days. Statistically valid numbers of units but not less than 5 shall be drawn on a monthly basis from the units of each kVA rating produced during this period for testing. This will assure the randomness of the samples (30 units minimum).

All the units drawn in 180 day period shall be tested for computing the mean efficiency of each kVA rating. None of individual units in a sample shall be considered acceptable if its measured losses exceed the allowance by more than 8%.

It is the responsibility of the manufacturer to assure through adequate quality control procedures and/or random testing that the conformance of various kVA rating transformers is maintained.

For a random sample to be statistically valid, a minimum number of units n in the sample must be tested to assure that the standard deviation of the test results is no more than the standard deviation S of the population with 95% confidence. The minimum sample size shall be determined as follows:

$$n = (tSK)^2$$

where

$$K = \frac{108 - 0.08SEL}{SEL(8 - 0.08SEL)}$$

and t Statistic is determined from Table B1 corresponding to sample size of n at 95% confidence level.

To demonstrate compliance with this standard, proceed as follows:

Step 1. Choose a sample size of n_1 units (5 min.)

Step 2. Compute the mean $\bar{X}_1$ and standard deviation S_1 as follows:

$$\bar{X}_1 = \frac{1}{n_1}\sum X_i,$$

$$S_1 = \sqrt{\frac{\sum(X_i - \bar{X}_1)^2}{n_1 - 1}}.$$

Where

$\bar{X}_1$	=	The average efficiency of the first sample.
X_i	=	The efficiency of the unit i
n_1	=	The number of units in the first set of samples (the subscript refers to the sample number)
S_1	=	The computed sample standard deviation of the first sample

Step 3. Calculate the minimum sample size n as follows:

$$n = (t_1 S_1 K)^2$$

Where t_1 Statistic is chosen from Table B1 corresponding to the sample size n_1.

Step 4. If $n \leq n_1$, the sample size is adequate to yield the acceptable standard deviation. Proceed with Step 6. Otherwise, test additional units and increase the sample size to n_2 such that:

$n_2 \geq n$, where n_2 is the total number of units tested.

Step 5. Repeat Steps 2 and 3 until each sample size for each kVA rating produced is larger than the minimum sample size n.

Step 6. Calculate the average efficiency η_{mi} of all the samples tested in 180 days for each rating kVA_i manufactured within this period.

Step 7. Calculate the Total Allowed kVA input:

$$TAI = \sum \frac{N_i\, L_i\, kVA_i}{\eta_i}$$

Where

i	=	1,2,3,4, ...
kVA_i	=	kVA ratings of various transformers included in a production batch manufactured in 180 days
η_i	=	Specified Efficiency Level in TP 1 for transformer rating kVA_i
L_i	=	Per unit load at which the efficiency is specified per TP 1
N_i	=	Total number of units produced with rating kVA_i

Step 8. Calculate the Total Measured kVA input:

$$TMI = \sum \frac{N_i\, L_i\, kVA_i}{\eta_{mi}}$$

Where

i	=	1,2,3,4, ...
kVA_i	=	kVA ratings of various transformers included in a production batch manufactured in 180 days
η_{mi}	=	Measured efficiency Level for transformers rated kVA_i
L_i	=	Per unit load at which the efficiency is specified per TP 1
N_i	=	Total number of units produced with rating kVA_i

Step 9. If TMI is equal to or less than TAI; the compliance of the production batch has been demonstrated.

Appendix C

A sampling plan for enforcement testing

The following sampling plan is adapted from the Sampling Plan for Enforcement Testing contained 10 CFR Part 430. The plan is similar to that proposed for electric motors [18].

Step 1. The first sample size (n_1) must be four or more units.

Step 2. Compute the mean ($\bar{X}_1$) of the measured energy performance of the n_1 units in the first sample as follows:

$$\bar{X}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} X_i, \tag{C1}$$

where X_i is the measured efficiency of unit i.

Step 3. Compute the sample standard deviation (S_1) of the measured efficiency of the n_1 units in the first sample as follows:

$$S_1 = \sqrt{\frac{\sum_{i=1}^{n_1} (X_i - \bar{X}_1)^2}{n_1 - 1}}. \tag{C2}$$

Step 4. Compute the standard error ($SE(\bar{X}_1)$) of the mean efficiency of the first sample as follows:

$$SE(\bar{X}_1) = \frac{S_1}{\sqrt{n_1}}. \tag{C3}$$

Step 5. Compute the lower control limit (UCL_1) for the mean of the first sample using the applicable rated efficiency (RE) as the desired mean as follows:

$$UCL_1 = RE - tSE(\bar{X}_1). \tag{C4}$$

Here t is the 2.5 percentile of a t-distribution for a sample size of n_1 and yields a 97.5 percent confidence level for a one-tailed t-test.

Step 6. Compare the mean of the first sample ($\bar{X}_1$) with the lower control limit (UCL_1) to determine one of the following:

(i) If the mean of the first sample is below the lower control limit, then the basic model is in noncompliance and testing is at an end.

(ii) If the mean is equal to or greater than the lower control limit, no final determination of compliance or noncompliance can be made; proceed to Step 7.

Step 7. Determine the recommended sample size (n) as follows:

$$n = \left[\frac{tS_1(108 - 0.08RE)}{RE(8 - 0.08RE)}\right]^2 \tag{C5}$$

where S_1 and t have the values used in Steps 4 and 5, respectively. The factor

$$\frac{108 - 0.08RE}{RE(8 - 0.08RE)}$$

is based on a 8 percent tolerance in the total power loss at and fixed output power.

Given the value of n, determine one of the following:

(i) If the value of n is less than or equal to n_1 and if the mean energy efficiency of the first sample ($\bar{X}_1$) is equal to or greater than the lower control limit (UCL_1), the basic model is in compliance and testing is at an end.

(ii) If the value of n is greater than n_1, the basic model is in noncompliance. The size of a second sample n_2 is determined to be the smallest integer equal to or greater than the difference $n - n_1$. If the value of n_2 so calculated is greater than $20 - n_1$, set n_2 equal to $20 - n_1$.

Step 8. Compute the combined mean ($\bar{X}_2$) of the measured energy performance of the n_1 and n_2 units of the combined first and second samples as follows:

$$\bar{X}_2 = \frac{1}{n_1 + n_2} \sum_{i=1}^{n_1+n_2} X_i. \tag{C6}$$

Step 9. Compute the standard error ($SE(\bar{X}_2)$) of the mean efficiency of the n_1 and n_2 units in the combined first and second samples as follows:

$$SE(\bar{X}_2) = \frac{S_1}{\sqrt{n_1 + n_2}}. \tag{C7}$$

(Note that S_1 is the value obtained above in Step 3.)

Step 10. Set the lower control limit (UCL_2) to,

$$UCL_2 = RE - tSE(\bar{X}_2), \tag{C8}$$

where t has the value obtained in Step 5, and compare the combined sample mean ($\bar{X}_2$) to the lower control limit (UCL_2) to find one of the following:

(i) If the mean of the combined sample ($\bar{X}_2$) is less than the lower control limit (UCL_2), the basic model is in noncompliance and testing is at an end.

(ii) If the mean of the combined sample ($\bar{X}_2$) is equal to or greater than the lower control limit (UCL_2), the basic model is in compliance and testing is at an end.

MANUFACTURER-OPTION TESTING

If a determination of non-compliance is made in Steps 6, 7 or 11, above, the manufacturer may request that additional testing be conducted, in accordance with the following procedures.

Step A. The manufacturer requests that an additional number, n_3, of units be tested, with n_3 chosen such that $n_1 + n_2 + n_3$ does not exceed 20.

Step B. Compute the mean efficiency, standard error, and lower control limit of the new combined sample in accordance with the procedures prescribed in Steps 8, 9, and 10, above.

Step C. Compare the mean performance of the new combined sample to the lower control limit (UCL_2) to determine one of the following:

(a) If the new combined sample mean is equal to or greater than the lower control limit, the basic model is in compliance and testing is at an end.

(b) If the new combined sample mean is less than the lower control limit and the value of $n_1+n_2+n_3$ is less than 20, the manufacturer may request that additional units be tested. The total of all units tested may not exceed 20. Steps A, B, and C are then repeated.

(c) Otherwise, the basic model is determined to be in noncompliance.

Appendix D

Computational algorithms

Compliance testing under 10 CFR Part 430 (Method I)

Let $x_1, x_2, \ldots, x_n$ be measured losses, with sample mean $\bar{x}$ and sample standard deviation s. In a test of compliance, we require that

$$\bar{x} \leq k \tag{D1}$$

and

$$\bar{x} + ts/\sqrt{n} \leq (1+\theta)k, \tag{D2}$$

where t denotes the appropriate percentile of a Student-t distribution with $\nu = n-1$ degrees of freedom, and θ is a small positive number (typically, $\theta = .03$). Assume that the data $\{x_i\}_{i=1}^{n}$ are an independent, identically-distributed sample from a normal (Gaussian) distribution with population mean μ and population standard deviation σ.

The probability of compliance is given by the expression

$$p(a,b) = \sqrt{\frac{n}{2\pi}} \int_{-\infty}^{a} e^{-ny^2/2} \chi_\nu^2 \left[\frac{n\nu}{t^2}(a+b-y)^2\right] dy, \tag{D3}$$

where

$$a = \tfrac{k-\mu}{\sigma} \quad \text{and} \quad b = \tfrac{\theta k}{\sigma}. \tag{D4}$$

In this expression, $\chi_\nu^2(u)$ is the probability that a χ^2 random variable V_ν having $\nu = n-1$ degrees of freedom is less than u; that is

$$\Pr(V_\nu \leq u) = \chi_\nu^2(u) = \frac{1}{\Gamma(\nu)2^\nu} \int_0^u z^{\nu/2-1} e^{-z/2} dz, \tag{D5}$$

which is a function easily expressed and calculated in terms of an incomplete Γ-function. Function (D5) can be evaluated numerically using a wide range of public domain software. Gauss-Legendre quadrature is adequate for the calculation of (D3), and it is sufficient to use $\mu - 5\sigma/\sqrt{n}$ for the lower bound of the numerical integral.

The limits of $p(a,b)$ as σ goes to zero or infinity are of particular interest; we consider these in turn

Limit of small σ: As $\sigma \to 0$, $b \to \infty$. If a is allowed to go to infinity as well, then $p(a,b) \to 1$, which isn't useful. So we require that $b \to \infty$ and that a remains finite; that is

$$-\infty < \lim_{\sigma \to 0} \frac{k-\mu}{\sigma} = q < \infty. \tag{D6}$$

If this condition holds, then

$$\begin{aligned} \lim_{\sigma \to 0} p(a,b) &= \sqrt{\frac{n}{2\pi}} \int_{-\infty}^{q} e^{-ny^2/2} dy \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\sqrt{n}q} e^{-z^2/2} = \Phi(\sqrt{n}q) = \alpha, \end{aligned}$$

where $\Phi(z)$ denotes the probability that a standard normal random variable is less than z, and $0 < \alpha < 1$ is a compliance probability which identifies a compliance probability contour, in the (μ, σ) plane, whose asymptotic behavior we are investigating. Hence, if z_α denotes the 100α percentile of a standard normal random variable (that is, $\Phi(z_\alpha) = \alpha$), then

$$q = z_\alpha/\sqrt{n}. \tag{D7}$$

Asymptotically, as $\sigma \to 0$, the α contour of compliance probability is linear; to be specific,

$$\mu \sim k - z_\alpha \sigma/\sqrt{n}. \tag{D8}$$

We could have guessed this result by reasoning that as $\sigma \to 0$, $s \to 0$, and condition (D2) must always hold. So we need only be concerned with condition (D1), and we require that

$$\begin{aligned}
&\Pr(\bar{x} \le k) = \alpha \Rightarrow \\
&\Pr\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \le \frac{k-\mu}{\sigma/\sqrt{n}}\right) = \alpha \Rightarrow \\
&\Phi\left(\frac{k-\mu}{\sigma/\sqrt{n}}\right) = \alpha \Rightarrow \\
&k = \mu + z_\alpha \sigma/\sqrt{n}.
\end{aligned}$$

Limit of large σ: As $\sigma \to \infty$, $b \to 0$. If $a \to 0$, then $p(a,b) \to 0$ if $k > \nu$, $p(a,b) \to 1$ if $k < \mu$, and $p(a,b) \to 1/2$ if $k = \mu$. This is not particularly interesting or realistic, so we require that

$$\lim_{\sigma\to\infty} \frac{k-\mu}{\sigma} = v, \tag{D9}$$

for finite v. It follows that

$$\lim_{\sigma\to\infty} p(a,b) = \sqrt{\frac{n}{2\pi}} \int_{-\infty}^{v} e^{-ny^2/2} \chi^2_\nu \left[\frac{n\nu}{t^2}(v-y)^2\right] dy. \tag{D10}$$

Let $h = v - y$, and note that

$$\begin{aligned}
\lim_{\sigma\to\infty} p(a,b) &= \sqrt{\frac{n}{2\pi}} \int_0^\infty e^{-\frac{n(h-v)^2}{2}} \chi^2_\nu \left(\frac{n\nu}{t^2}h^2\right) dh \\
&= \frac{t}{\sqrt{2\pi\nu}} \int_0^\infty e^{-\left[\frac{tx}{\sqrt{n\nu}}-v\right]^2} \chi^2_\nu \left(x^2\right) dx = \alpha.
\end{aligned}$$

For the limiting behavior of the 100α percent contour in the (μ, σ) plane, we must solve the transcendental equation

$$\frac{t}{\sqrt{2\pi\nu}} \int_0^\infty e^{-\left[\frac{tx}{\sqrt{n\nu}}-v\right]^2} \chi^2_\nu \left(x^2\right) dx = \alpha \tag{D11}$$

for v. The contour will be asymptotically linear, i.e., as $\sigma \to \infty$

$$\mu \sim k - v\sigma. \tag{D12}$$

This asymptotic result could also be derived by reasoning that, in the limit of large σ, s will almost certainty be large and $\bar{x} \le k$ will hold automatically, provided that condition (D2) is satisfied. So the compliance probability is determined in this limit in terms of the second condition alone, which leads to the result developed in this subsection.

Derivation of $\mathbf{p(a,b)}$: Under a normal distribution model, the sample mean and standard deviation, $\bar{x}$ and s, are statistically independent. The sample mean has a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$. The sample standard deviation is proportional to the square root of a χ^2_ν random variable, with proportionality constant $\sigma/\sqrt{\nu}$.

It's easy to see that the probability of compliance can be expressed as

$$\Pr(\bar{x} \le k \text{ and } \bar{x} + ts/\sqrt{n} \le (1+\theta)k) = \Pr\left(\bar{x} \le k \text{ and } s \le \frac{(1+\theta)k - \bar{x}}{t/\sqrt{n}}\right). \tag{D13}$$

Because of independence (which holds *only* under a normal model), s has the same probability distribution whatever the value of $\bar{x}$. So we condition on $\bar{x}$ taking on all possible values $-\infty < x < \infty$, and integrate:

$$\Pr\left(\bar{x} \leq k \text{ and } s \leq \frac{(1+\theta)k - \bar{x}}{t/\sqrt{n}}\right) =$$

$$\int_{-\infty}^{\infty} \Pr\left(k > x \text{ and } s \leq \frac{(1+\theta)k - x}{t/\sqrt{n}}\right) \Pr(\bar{x} = x)dx =$$

$$\int_{-\infty}^{\infty} \Pr(k > x) \Pr\left(s \leq \frac{(1+\theta)k - x}{t/\sqrt{n}}\right) \Pr(\bar{x} = x)dx.$$

Note that we can write the joint probability distribution as a product because of independence. Substituting expressions for the normal and $\sqrt{\chi_\nu^2}$ distributions into the above expressions and simplifying leads to $p(a, b)$ as given in (D3).

Testing under TP 2

Assume that the random variables Y_i are normal with mean μ and standard deviation σ. Since we will assume that μ and σ can be regarded as approximately known from previous data, we can, without loss of generality, employ the standardized sample $X_i \equiv (Y_i - \mu)/\sigma$, having order statistics $X_{(i)} \equiv (Y_{(i)} - \mu)/\sigma$.

Then the probability of being found in compliance under the TP 2 sampling plan can be shown to be

$$p = 1 - \left[F_{X_{(1)}}(t) + F_{\bar{X}}(t) - F_{X_{(1)},\bar{X}}(t_1, t_2)\right]. \tag{D14}$$

The function $F_{X_{(1)}}(t)$ denotes the CDF of $X_{(1)}$, and $F_{\bar{X}}(t)$ denotes the CDF of $\bar{X}$; that is

$$F_{X_{(1)}}(t) = \Pr(X_{(1)} \leq t) = 1 - (1 - \Phi(t))^n,$$

and

$$F_{\bar{X}}(t) = \Pr(\bar{X} \leq t) = \Phi\left(\sqrt{n}t\right),$$

where $\Phi(\cdot)$ is the standard normal CDF, which is defined to be the probability that a normally distributed random variable is less than u, that is

$$\Phi(u) = \int_{-\infty}^{u} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx.$$

This integral may be calculated efficiently and accurately using public-domain routines.

A saddle-point approximation to the bivariate CDF

$$F_{X_{(1)},\bar{X}}(t_1, t_2) = \Pr(X_{(1)} \leq t_1 \text{ and } \bar{X} \leq t_2)$$

is derived by Vangel [8]. Let $\phi(t)$ denote the normal density, and let

$$h(t) = \frac{\phi(t)}{1 - \Phi(t)}$$

be the normal hazard function. The saddle-point approximation $\tilde{F}_{X_{(1)},\bar{X}}(t_1, t_2)$, for $t_1 \leq t_2$, is

$$\tilde{F}_{X_{(1)},\bar{X}}(t_1, t_2) = \frac{\int_{-\infty}^{t_*} \Phi(\sqrt{n}t_2)A(t)dt + \int_{t_*}^{\infty} \Phi\left\{\sqrt{n}\left[t_1 + \frac{n-1}{n}(h(t) - t)\right]\right\} A(t)dt}{\int_{-\infty}^{\infty} A(t)dt}, \tag{D15}$$

$$A(t) = h^{-(n-1)}(t) \exp\left\{\frac{(n-1)^2}{2n}\left[h(t) - t\right]^2 + (n-1)t\left[h(t) - t\right]\right\} \sqrt{1 - h^2(t) + th(t)},$$

where t_* is the (unique) solution to the equation

$$\frac{n-1}{n}\left[h(t_*) - t_*\right] = t_2 - t_1. \tag{D16}$$

Enforcement testing

The probability of demonstrating compliance by testing under the Sampling Plan for Enforcement Testing can be calculated using straightforward numerical integration.

In the following expressions, $n_1 \geq 4$ denotes the minimum sample size specified in the sampling plan, and $n_2 = 20$ is the maximum sample. In order to simplify the equations, $n_1 - 1$ is represented by ν, and t is the 97.5th percentile of the Student-t distribution for the sample size n_1.

The probability of compliance is

$$p = \frac{\left(\frac{\nu}{\sigma^2}\right)^{\frac{\nu}{2}}}{2^{\frac{\nu}{2}-1}\Gamma\left(\frac{\nu}{2}\right)} \sum_{i=n_1}^{n_2} \int_{\kappa_i}^{\kappa_{i+1}} \Phi\left[\frac{(100-\mu)\sqrt{i} - tx}{\sigma}\right] x^{\nu-1} e^{-\frac{\nu x^2}{2\sigma^2}} dx,$$

where the limits of integration are $\kappa_{n_1} = 0$,

$$\kappa_i = \frac{LT - 100}{t}\sqrt{i-1} \text{ for } i = n_1 + 1, \ldots, n_2,$$

and $\kappa_{n_2+1} = \infty$, and the loss tolerance, LT, is a percentage.

NIST *Technical Publications*

Periodical

Journal of Research of the National Institute of Standards and Technology—Reports NIST research and development in those disciplines of the physical and engineering sciences in which the Institute is active. These include physics, chemistry, engineering, mathematics, and computer sciences. Papers cover a broad range of subjects, with major emphasis on measurement methodology and the basic technology underlying standardization. Also included from time to time are survey articles on topics closely related to the Institute's technical and scientific programs. Issued six times a year.

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