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Assessment of Data by a Second-Order Transfer Function

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A newly developed theory for predicting the response of a linear system to an electromagnetic pulse, based only on the measured continuous-wave magnitude, is applied to the problem of possible electromagnetic interferences at a sensitive part of a torpedo. The measured magnitude representing the system's transfer function is deduced first from the measured response at this sensitive point to a known cw source, supplied by the Naval Surface Warfare Center. We derive an analytic expression for the magnitude square of the transfer function to approximate the measured data and obtain a system transfer function in terms of the complex frequency, from which we predict the system's cw phase characteristics and its multiple solutions due to a given impulse source.

Key words: convolution integral; impulse response; linear system; magnitude; phase; transfer function.

1. Introduction

A theoretical method was developed to predict the frequency and time characteristics of an unknown linear system, based only on the measured cw magnitude data, without requiring phase information [1]. The approach is to deduce an approximate analytical expression for the overall system magnitude square from the given measured magnitude data. The essential steps involved in achieving this purpose [1] may be summarized as follows: (i) identify the resonant frequencies and the maximum responses at these frequencies from the data, (ii) approximate each of the resonant regions by a component magnitude square with a simple second-order transfer function, (iii) sum up all of the component magnitude squares to yield an overall magnitude square to represent the unknown system, (iv) derive from it a set of multiple transfer functions in terms of the complex frequency, (v) determine the corresponding phases and impulse functions, and (vi) compute the time responses of the system to a known electromagnetic pulse (emp) source by performing analytically the convolution integral.

Steps (i), (ii), and (iii) are outlined in section 4.1. The approximation for each resonant frequency by a second-order transfer function as required in step (ii) is presented in section 2. A short discussion on frequency transformation for avoiding manipulations with large numbers is given in section 3. The derivation of transfer functions from the approximate magnitude square in step (iv) is detailed in section 4.2. Determination of the system impulse functions and phases required in step (v) are given respectively in sections 4.3 and 4.4. Computation of the emp response of the same system is derived in section 4.5. Related numerical results are presented in figures 4 through 52.

The final frequency and time responses obtained from this method may then be analyzed and compared with the measured data, if available.

2. *Approximation of Resonances by Second-Order Transfer Functions*

The second-order transfer function used to approximate each cw resonance displayed on measured magnitude data takes either of the following two forms:

$$H_a(s) = \frac{A}{s^2 + as + b} , \tag{1a}$$

or

$$H_b(s) = \frac{A(s + c)}{s^2 + as + b} , \tag{1b}$$

where s is the complex frequency, a, b and A are all positive and real numbers with $b > a^2/2$ [1], and c is a real number (which may be positive, 0, or negative).

For the transfer function in eq (1a), the parameters a, b and A are determined by [1]

$$b^2 = \omega_0^4 + \frac{(\omega_2^2 - \omega_1^2)^2}{4} , \tag{2a}$$

$$a^2 = 2(b - \omega_0^2) , \tag{2b}$$

and

$$A = (\omega_2^2 - \omega_1^2)\frac{|H_a(j\omega_0)|}{2} , \tag{2c}$$

where ω_0 is the radian resonant frequency, ω_1 and ω_2 are the half-power frequencies with $\omega_1 < \omega_0 < \omega_2$ and $|H_a(j\omega_1)| = |H_a(j\omega_2)| = |H_a(j\omega_0)| / \sqrt{2}$, and $|H_a(j\omega_0)|$ is the maximum magnitude at the resonant frequency.

All of the four required numbers, ω_0 , ω_1 , ω_2 , and $|H_a(j\omega_0)|$, can be read directly from the measured cw magnitude data. In this case, ω_1 and ω_2 are not independent, but are related by

$$\omega_1^2 + \omega_2^2 = 2\omega_0^2 \tag{3}$$

Thus, it is unimportant if either ω_1 or ω_2 is not available in the measured curve since it can be determined from eq (3). The dimensions of a and b in this case are respectively s^{-1} and s^{-2}. The dimension of A depends on $|H_a(j\omega)|$.

For the transfer function eq (1b), the parameters a, b, c, and A are determined by [1]

$$b^2 = 2\omega_0^4 - \omega_1^2\omega_2^2 , \tag{4a}$$

$$a^2 = \omega_1^2 + \omega_2^2 - 4\omega_0^2 + 2b , \tag{4b}$$

$$c^2 = \frac{(\omega_0^4 - \omega_1^2\omega_2^2)}{(\omega_1^2 + \omega_2^2 - 2\omega_0^2)} , \tag{4c}$$

and

$$A^2 = (2\omega_0^2 - 2b + a^2)\,|H_b(j\omega_0)|^2 . \tag{4d}$$

Again, the required input information, ω_0, ω_1, ω_2, and $|H_b(j\omega_0)|$, can be read directly from the given curve. The dimensions of a, b and c now are respectively s^{-1}, s^{-2} and s^{-1}. The dimension of A depends on $|H_b(j\omega_0)|$.

In eq (4c), we require that [1]

$$\omega_1^2 + \omega_2^2 > 2\omega_0^2 \quad \textit{and} \quad \omega_0^2 \geq \omega_1\omega_2 . \tag{5}$$

If either of the conditions in eq (5) is not satisfied, we use the transfer function in eq (1a).

Once the parameters a, b, c [under condition eq (5)], and A are determined, the magnitude square of the component transfer function that approximately represents the resonant frequency under consideration becomes

$$|H_a(j\omega)|^2 = \frac{A^2}{\omega^4 - (2b-a^2)\omega^2 + b^2} , \tag{6a}$$

or

$$|H_b(j\omega)|^2 = \frac{A^2(\omega^2 + c^2)}{\omega^4 - (2b - a^2)\omega^2 + b^2} . \tag{6b}$$

The approximation as presented is good only for the important resonant region being studied and is not meant to match the given measured curve everywhere.

If there are N distinct resonant frequencies in the original cw data, the final approximate magnitude square for the overall system is just the sum of all the component magnitude squares, from which the transfer function in complex frequency $H(s)$ is then extracted [2]. It will be shown later that there may be multiple solutions for $H(s)$. The corresponding solutions for impulse transfer functions $h(t)$ can then be determined in terms of damped sine and cosine functions.

3. Frequency Transformation

To avoid manipulations with large numbers such as the frequency in megahertz (MHz), we use the normalized radian frequency by omitting the factor 10^6. We herein designate ω as the normalized radian frequency and ω' as the actual radian frequency such that

$$\omega' = B\omega \qquad with \qquad B = 10^6 \ . \tag{7}$$

Substituting eq (7) in eqs (6a) and (6b), we obtain respectively the component magnitude square in actual radian frequency,

$$\begin{aligned} |G_a(j\omega')|^2 &= \frac{A^2B^4}{\omega'^4 - B^2(2b - a^2)\omega'^2 + b^2B^4} \\ &= \frac{A'^2}{\omega'^4 - (2b' - a'^2)\omega'^2 + b'^2} \ , \end{aligned} \tag{8a}$$

and

$$\begin{aligned} |G_b(j\omega')|^2 &= \frac{A^2B^2(\omega'^2 + c^2B^2)}{\omega'^4 - B^2(2b - a^2)\omega'^2 + b^2B^4} \\ &= \frac{A'^2(\omega'^2 + c'^2)}{\omega'^4 - (2b' - a'^2)\omega'^2 + b'^2} \ . \end{aligned} \tag{8b}$$

From eq (8a), we see that the parameters a', b', and A' will become respectively aB, bB^2, AB^2 when we convert from normalized frequency to actual frequency. For the transfer function in eq (1b), the parameters a', b', c', and A' in eq (8b) will become aB, bB^2, cB, and AB for the same conversion in frequency.

4. Assessment of Torpedo Measurement Data

The data analyzed in this report came from a test made on a special torpedo by Naval Surface Warfare Center (NSWC). The test was instrumented with current probes in two locations, arbitrarily called C26 and C29, where EMI problems are suspected. The external cw source used in the test for C26 is a magnetic field in amperes/meter (input) as shown in figure 1. The measured probe magnitude response in amperes (output) is given in figure 2. Thus, this input-output configuration constitutes "the system" as far as our particular application is concerned. Both figures 1 and 2, were originally expressed in megahertz. For our analysis, we use the normalized frequency by omitting the factor of 10^6, as discussed in section 3. The transfer function magnitude in meters is obtained by taking the ratio of the response data (fig. 2) to the source data (fig. 1), as shown in figure 3 (solid curve), where the normalized frequency is used. Also, we concentrate only on the portion of data up to 100 Hz for the normalized frequency or 100 MHz in real frequency. Similar analysis for C29 is not included in this report.

4.1. Analytical Expression for Magnitude Squares

From the measured data derived in figure 3 we see many resonant frequencies where the measured magnitudes reach their maxima. For demonstration and considering the storage limitation of our software, as a first step, we identify six resonant frequencies properly numbered in the figure 3. They are : $f_{10} = 5.593$, $f_{20} = 19.609$, $f_{30} = 53.262$, $f_{40} = 56.848$, $f_{50} = 59.373$, and $f_{60} = 72.192$ with their respective maximum values of 0.1151 m, 0.0862 m, 0.3773 m, 0.2709 m, 0.4671 m, and 0.1277 m.

For the first resonant frequency at f_{10} ($\omega_{10} = 35.1431$), we observe the half-power frequencies $f_{11} = 5.077$ ($\omega_{11} = 31.9016$) and $f_{12} = 5.935$ ($\omega_{12} = 37.2876$). Since one of the conditions in eq (5) is not satisfied ($\omega_{11}^2 + \omega_{12}^2 < 2\omega_{10}^2$), we use the transfer function in eq (1a) and obtain $a_1 = 5.2870$, $b_1 = 1.2490(10^3)$, and $A_1 = 21.4460$. Thus, the first component magnitude square becomes

$$|H_1(j\omega)|^2 = \frac{A_1^2}{D_1(\omega^2)} , \tag{9a}$$

where

$$D_1(\omega^2) = \omega^4 - 2.4701(10^3)\omega^2 + 1.5600(10^6) . \tag{9b}$$

The magnitude square in eq (9a) is used to approximate the first resonant region in figure 3.

For the second resonant frequency at $f_{20} = 19.609$ ($\omega_{20} = 123.2070$), we read $f_{21} = 17.784$ ($\omega_{21} = 111.7402$), and $f_{22} = 20.480$ ($\omega_{22} = 128.6796$). Again, because of $\omega_{21}^2 + \omega_{22}^2 < 2\omega_{20}^2$, we use the transfer function in eq (1a) to obtain $a_2 = 16.4905$, $b_2 = 1.5316(10^4)$, $A_2 = 1.7545(10^2)$, and the second component magnitude square

$$|H_2(j\omega)|^2 = \frac{A_2^2}{D_2(\omega^2)} , \tag{10a}$$

where

$$D_2(\omega^2) = \omega^4 - 3.0360(10^4)\omega^2 + 2.3458(10^8) . \tag{10b}$$

For the third resonant frequency at $f_{30} = 53.262$ ($\omega_{30} = 334.6550$), we have $f_{31} = 52.604$ ($\omega_{31} = 330.5207$), and $f_{32} = 55.124$ ($\omega_{32} = 346.3543$). Since the second condition in eq (5) is not satisfied, that is $\omega_{30}^2 < \omega_{31}\,\omega_{32}$, we also use the transfer function in eq (1a) to obtain $a_3 = 16.0080$, $b_3 = 1.1212(10^5)$, $A_3 = 2.0218(10^3)$, and the third component magnitude square

$$|H_3(j\omega)|^2 = \frac{A_3^2}{D_3(\omega^2)} , \tag{11a}$$

where

$$D_3(\omega^2) = \omega^4 - 2.2399(10^5)\omega^2 + 1.2571(10^{10}) . \tag{11b}$$

For the fourth resonant frequency at $f_{40} = 56.848$ ($\omega_{40} = 357.1865$), where f_{41} does not exist and $f_{42} = 58.324$ ($\omega_{42} = 366.4605$), we also use eq (1a) to obtain $a_4 = 18.7740$, $b_4 = 1.2776(10^5)$, $A_4 = 1.8172(10^3)$, and the fourth component magnitude square

$$|H_4(j\omega)|^2 = \frac{A_4^2}{D_4(\omega^2)}, \tag{12a}$$

where

$$D_4(\omega^2) = \omega^4 - 2.5516(10^5)\,\omega^2 + 1.6322(10^{10}) . \tag{12b}$$

Similarly, for the fifth and sixth resonance at $f_{50} = 59.373$ and $f_{60} = 72.192$, the transfer function in eq (1a) also applies. We obtain $a_5 = 9.6223$, $b_5 = 1.3921(10^5)$, $A_5 = 1.6769(10^3)$; $a_6 = 19.2431$, $b_6 = 2.0593(10^5)$, and $A_6 = 1.1149(10^3)$; which yield respectively

$$|H_5(j\omega)|^2 = \frac{A_5^2}{D_5(\omega^2)}, \tag{13a}$$

with

$$D_5(\omega^2) = \omega^4 - 2.7833(10^5)\omega^2 + 1.9380(10^{10}) , \tag{13b}$$

and

$$|H_6(j\omega)|^2 = \frac{A_6^2}{D_6(\omega^2)}, \tag{14a}$$

with

$$D_6(\omega^2) = \omega^4 - 4.1150(10^5)\omega^2 + 4.2409(10^{10}) . \tag{14b}$$

Adding all the component magnitude squares in eqs (9a) through (14a), we have the overall system magnitude square,

$$|H(j\omega)|^2 = \sum_{i=1}^{6} |H_i(j\omega)|^2 = 1.1473(10^7)\frac{N(\omega^2)}{D(\omega^2)}, \tag{15}$$

where

$$D(\omega^2) = D_1(\omega^2)D_2(\omega^2)D_3(\omega^2)D_4(\omega^2)D_5(\omega^2)D_6(\omega^2) , \tag{16}$$

with $D_i(\omega^2)$, i = 1, 2, ..., 6 given previously in eqs (9b) to (14b), and

$$\begin{aligned} N(\omega^2) = {} & \omega^{20} - 9.3575(10^5)\,\omega^{18} + 3.6725(10^{11})\omega^{16} - 7.8083(10^{16})\omega^{14} \\ & + 9.6766(10^{21})\omega^{12} - 6.9726(10^{26})\omega^{10} + 2.7329(10^{31})\omega^8 - 5.0434(10^{35})\omega^6 \\ & + 4.0175(10^{39})\omega^4 - 8.0753(10^{42})\,\omega^2 + 6.0670(10^{45}) \end{aligned}$$

$$= [\omega^4 - 2.4904(10^3)\omega^2 + 2.1141(10^6)]\ [\omega^4 - 3.0973(10^4)\omega^2 + 2.7749(10^8)]$$
$$* \ [\omega^4 - 2.3947(10^5)\omega^2 + 1.4446(10^{10})]\ [\omega^4 - 2.6894(10^5)\omega^2 + 1.8147(10^{10})] \qquad (17)$$
$$* \ [\omega^4 - 3.9387(10^5)\omega^2 + 3.9452(10^{10})]\ .$$

The final maximum value of $|H(j\omega)|$ in eq (15) at ω_{50} is 0.4921, somewhat larger than the actual maximum value of 0.4671, resulting in an amplification factor of 1.0535. This increase is due to the contributions, although small, from the other component magnitude squares. To enforce $|H(j\omega_{50})| = 0.4671$, we simply divide the final magnitude square in eq (15) by $(1.0535)^2$ to yield a corrected system magnitude square

$$|H_c(j\omega)|^2 = 1.0336(10^7)\frac{N(\omega^2)}{D(\omega^2)}\ , \qquad (18)$$

with $D(\omega^2)$ and $N(\omega^2)$ given in eqs (16) and (17).

The square root of eq (18) is plotted as the broken curve in figure 3 for comparison. As far as the major regions (resonant frequencies) in figure 3 are concerned, the approximation is very good except that the approximated value at f_{40} is much larger than the actual value.

According to the method known in classical network theory [2], we can extract the transfer function in complex frequency s from the magnitude square in eq (18) by substituting $s = j\omega$. Thus,

$$H(s)H(-s) = |H_c(j\omega)|^2 \qquad (with\ s{=}j\omega)$$

$$= 1.0336(10^7)\frac{N(-s^2)}{D(-s^2)}\ , \qquad (19)$$

where

$$N(-s^2) = N_1(+)N_1(-)N_2(+)N_2(-)N_3(+)N_3(-)N_4(+)N_4(-)N_5(+)N_5(-)\ , \qquad (20)$$

and

$$D(-s^2) = D_1(+)D_1(-)D_2(+)D_2(-)D_3(+)D_3(-)D_4(+)D_4(-)D_5(+)D_5(-)D_6(+)D_6(-)\ , \qquad (21)$$

with

$$\begin{aligned}
N_1(\pm) &= s^2 \pm 20.4343s + 1.4540(10^3)\ ,\\
N_2(\pm) &= s^2 \pm 48.4078s + 1.6658(10^4)\ ,\\
N_3(\pm) &= s^2 \pm 30.1324s + 1.2019(10^5)\ ,\\
N_4(\pm) &= s^2 \pm 21.9290s + 1.3471(10^5)\ ,\\
N_5(\pm) &= s^2 \pm 58.1525s + 1.9862(10^5)\ ,
\end{aligned} \qquad (22)$$

and

$$D_i(\pm) = s^2 \pm a_i s + b_i \qquad i = 1,2,\ldots,6\ . \qquad (23)$$

4.2. Extraction of Transfer Functions

To determine a realizable transfer function $H(s)$ from eq (19), we have to assign all $D_i(+)$ to the denominator of $H(s)$ in order to yield a stable system. Then all $D_i(-)$ automatically belong to the denominator of $H(-s)$. As far as the numerator of $H(s)$ is concerned, we have multiple choices. If all of the $N_i(+)$ are chosen as the numerator of $H(s)$, $N_i(-)$ will be in the numerator of $H(-s)$. In this case, $H(s)$ has no zeros in the right-half s-plane. The system transfer function is said to be at minimum phase. Other choices will produce one or more zeros in the right-half s-plane, resulting in nonminimum-phase transfer functions. There are a total of 31 (from $2^5 - 1$) nonminimum-phase transfer functions for the case under study. The coefficient in eq (19), $1.0334(10^7)$ gives a factor of $3.2146(10^3)$ each to $H(s)$ and $H(-s)$.

For the minimum-phase case, we obtain

$$H_m(s) = 3.2146(10^3)\frac{\prod_{i=1}^{5} N_i(+)}{\prod_{i=1}^{6} D_i(+)} ,$$

$$= \sum_{i=1}^{6} \frac{F_i s + G_i}{D_i(+)} , \tag{24}$$

where

$$\begin{aligned} F_1 &= 0.4305, & G_1 &= 4.5011, \\ F_2 &= 0.8814, & G_2 &= 46.740, \\ F_3 &= 2.5998, & G_3 &= 1196.6, \\ F_4 &= -0.1656, & G_4 &= 950.33, \\ F_5 &= -2.0624, & G_5 &= 926.21, \text{ and} \\ F_6 &= -1.6837, & G_6 &= 93.272. \end{aligned} \tag{25}$$

If we replace one or more $N_i(+)$ in eq (24) by its counterpart $N_i(-)$, the resulting transfer function will be at nonminimum phase, and the corresponding expansion coefficients F_i and G_i will be different from those in eq (25). The values of F_i and G_i for the 31 nonminimum-phase cases can be computed and printed out by the software, when desired.

4.3. Impulse Response

The impulse response for the minimum-phase case in eq (24) is then

$$h_m(t) = \sum_{i=1}^{6}\left[c_i \cos(\beta_i t) + d_i \sin(\beta_i t)\right]e^{-\alpha_i t} , \tag{26}$$

with

$$
\begin{array}{llll}
\alpha_1 = 2.6435, & \beta_1 = 35.243, & c_1 = 0.4305, & d_1 = 0.0954, \\
\alpha_2 = 8.2443, & \beta_2 = 123.48, & c_2 = 0.8814, & d_2 = 0.3197, \\
\alpha_3 = 8.0012, & \beta_3 = 334.75, & c_3 = 2.5998, & d_3 = 3.5126, \\
\alpha_4 = 9.3860, & \beta_4 = 357.31, & c_4 = -0.1656, & d_4 = 2.6640, \\
\alpha_5 = 4.8106, & \beta_5 = 373.08, & c_5 = -2.0624, & d_5 = 2.5092, \text{ and} \\
\alpha_6 = 9.6219, & \beta_6 = 453.70, & c_6 = -1.6837, & d_6 = 0.2129.
\end{array}
\tag{27}
$$

Equation (26) is plotted in figure 4 marked with '1 1 1 1 1', because five N_i (+) are included in the numerator of eq (24) to represent the minimum-phase case. The time scale, in seconds, used here is based on the normalized frequency. The corresponding time scale for the actual frequency will be microseconds. Nonminimum cases are shown in figures 4 to 11 for comparison. The coefficients c_i and d_i for the nonminimum-phase cases will all be different from those in eq (27) although α_i and β_i will remain unchanged. The curve marked with '1 1 −1 −1 1' is interpreted to indicate that $N_1(+)\, N_2(+)\, N_3(-)\, N_4(-)\, N_5(+)$ is chosen for the numerator of the transfer function.

4.4. *Phase Information*

Since we define, in this report, the phase of $H(j\omega)$ as

$$H(j\omega) = |H(j\omega)|\, e^{-j\theta(\omega)} , \tag{28}$$

we have for the minimum-phase case in eq (24)

$$\theta_m(\omega) = \theta_d(\omega) - \theta_n(\omega) , \tag{29}$$

where $\theta_d(\omega)$, due to the denominator factors in $H_m(j\omega)$, is given by

$$\theta_d(\omega) = \sum_{i=1}^{6} \tan^{-1}\left[\frac{a_i\,\omega}{b_i - \omega^2}\right], \tag{30}$$

and $\theta_n(\omega)$, due to the numerator factors in $H_m(j\omega)$, from eq (22), is given by

$$\theta_n(\omega) = \tan^{-1}\left[\frac{20.4343\ \omega}{1.4540\ (10^3) - \omega^2}\right] + \tan^{-1}\left[\frac{48.4078\ \omega}{1.6658\ (10^4) - \omega^2}\right] + \tan^{-1}\left[\frac{30.1324\ \omega}{1.2019\ (10^5) - \omega^2}\right]$$

$$+ \tan^{-1}\left[\frac{21.9290\ \omega}{1.3471\ (10^5) - \omega^2}\right] + \tan^{-1}\left[\frac{58.1525\ \omega}{1.9862\ (10^5) - \omega^2}\right]. \tag{31}$$

For the nonminimum-phase cases, one or all of the terms in eq (31) will change sign to make the total phase larger than that for the minimum-phase case. Phases for the 32 cases (1 minimum phase and 31 nonminimum phases) are presented in figures 12 through 19.

4.5. *EMP Response*

Once the impulse response $h(t)$ is obtained in subsection 4.3, the response $r_{emp}(t)$ at C26 due to an emp source function $emp(t)$ can be derived analytically by performing convolution integrals. That is,

$$r_{emp}(t) = emp(t) * h(t) \ , \tag{32}$$

where $h(t)$ represents either minimum-phase or nonminimum-phase case.

The actual emp source function used in the test is given by

$$emp(t') = K\left[e^{-4(10^6)t'} - e^{475(10^6)t'}\right], \tag{33}$$

where t' is the real time before frequency normalization, and the constant factor K is

$K = \dfrac{52500}{377} = 139.2573.$

The Laplace transform of eq (33) is

$$EMP(s') = K\left[\frac{1}{s' + 4(10^6)} - \frac{1}{s' + 475(10^6)}\right], \tag{34}$$

and its spectrum is given by

$$EMP(j\omega') = K\left[\frac{1}{4(10^6) + j\omega'} - \frac{1}{475(10^6) + j\omega'}\right]. \tag{35}$$

We now apply frequency normalization to convert eq (35) into

$$EMP_N(j\omega) = K(10^{-6})\left[\frac{1}{4 + j\omega} - \frac{1}{475 + j\omega}\right], \tag{36}$$

which yields

$$emp_N(t) = K(10^{-6})\left[e^{-4t} - e^{-475t}\right]. \tag{37}$$

Comparing eqs (33) and (37), we see that the net effects on the time excitation function from the frequency normalization are a dropping of the (10^6) factor from the exponent and a multiplying factor of (10^{-6}) to the coefficient.

Now, the frequency response to the normalized source becomes, from eq (36),

$$|R_N(j\omega)| = |H_c(j\omega)| \frac{471K(10^6)}{\sqrt{(\omega^2 + 4^2)(\omega^2 + 475^2)}} , \tag{38}$$

which is plotted in figure 20. Since there is only one $|H_c(j\omega)|$ for the minimum-phase and non-minimum-phase cases, there is only one frequency response as shown in figure 20.

The time response to the normalized source becomes

$$r_{emp}(t) = K(10^6)\left[e^{-4t} - e^{-475t}\right] * h(t) \tag{39}$$

Since the source function is in the form of e^{-pt} and the typical term in $h(t)$ is $[c_i \cos(\beta_i t) + d_i \sin(\beta_i t)]e^{-\alpha_i t}$, we only need to deal with a general formula $e^{-pt} * \left[c_i \cos(\beta_i t) + d_i \sin(\beta_i t)\right]e^{-\alpha_i t}$. One term is

$$e^{-pt} * \cos(\beta_i t)\, e^{-\alpha_i t} = \int_0^t e^{-p(t-\tau)} \cos(\beta_i \tau)\, e^{-\alpha_i \tau}\, d\tau$$

$$= \frac{[(p - \alpha_i)\cos(\beta_i t) + \beta_i \sin(\beta_i t)]e^{-\alpha_i t} - (p - \alpha_i)e^{-pt}}{(p - \alpha_i)^2 + \beta_i^2} , \tag{40a}$$

and the other term is

$$e^{-pt} * \sin(\beta_i t)\, e^{-\alpha_i t} = \int_0^t e^{-p(t-\tau)} \sin(\beta_i \tau)\, e^{-\alpha_i \tau}\, d\tau$$

$$= \frac{[-\beta_i \cos(\beta_i t) + (p - \alpha_i)\sin(\beta_i t)]e^{-\alpha_i t} + \beta_i e^{-pt}}{(p - \alpha_i)^2 + \beta_i^2} . \tag{40b}$$

Substituting eqs (40a) and (40b) into eq (39), we obtain

$$r_{emp}(t) = K(10^6)\sum_{i=1}^{6}\left[m_i' e^{-\alpha_i t} + n_i' e^{-p_1 t} - m_i'' e^{-\alpha_i t} - n_i'' e^{-p_2 t}\right] , \tag{41}$$

where

$$m_i' = \frac{[c_i(p_1 - \alpha_i) - d_i\beta_i]\cos(\beta_i t) + [c_i\beta_i + d_i(p_1 - \alpha_i)]\sin(\beta_i t)}{(p_1 - \alpha_i)^2 + \beta_i^2} , \tag{42a}$$

and

$$n_i' = \frac{-c_i(p_1 - \alpha_i) + d_i\beta_i}{(p_1 - \alpha_i)^2 + \beta_i^2} \qquad (independent\,of\; t)\,, \tag{42b}$$

and m_i'' and n_i'' have the same expressions as m_i' and n_i' but with p_1 replaced by p_2, where $p_1 = 4$ and $p_2 = 475$.

The time responses for all cases are presented in figures 21 to 52, which can be compared with the measured emp response as shown in figure 53. The emp response $r_{emp}(t)$ for the minimum-phase case as shown in figure 21 is almost always nonnegative because the term in eq (42b) is very dominant in this case. Similar solutions for the nonminimum-phase cases can be both positive and negative. Some of them have significant negative parts such as that shown in figure 38. Therefore, it appears that the specific input-output configuration of the torpedo under study constitutes more likely a nonminimum-phase system.

5. Conclusions

We have applied the newly developed theory for predicting the impulse characteristics of an unknown linear system based only on available cw magnitude data to a particular system configuration. By comparing the predicted impulse responses to the measured emp data, we conclude that the system under study is more likely at nonminimum phase. This same procedure can be applied to the analysis of other linear systems and to assess the meaning of the measured data.

We thank the Naval Surface Warfare Center (NSWC) and Naval Sea Systems Command (NAVSEA) for their partial support of this study. We also appreciate the measured data supplied by John Bean and Rich Porter of NSWC.

6. References

[1] Ma, M.T.; Adams, J.W. Characteristics of unknown linear systems deduced from measured cw magnitude. J. Res. Natl. Inst. Stand. Technol., 98:297-319; May-June 1993.

[2] Balabanian, N. Network Synthesis, New Jersey: Prentice-Hall; 1958.

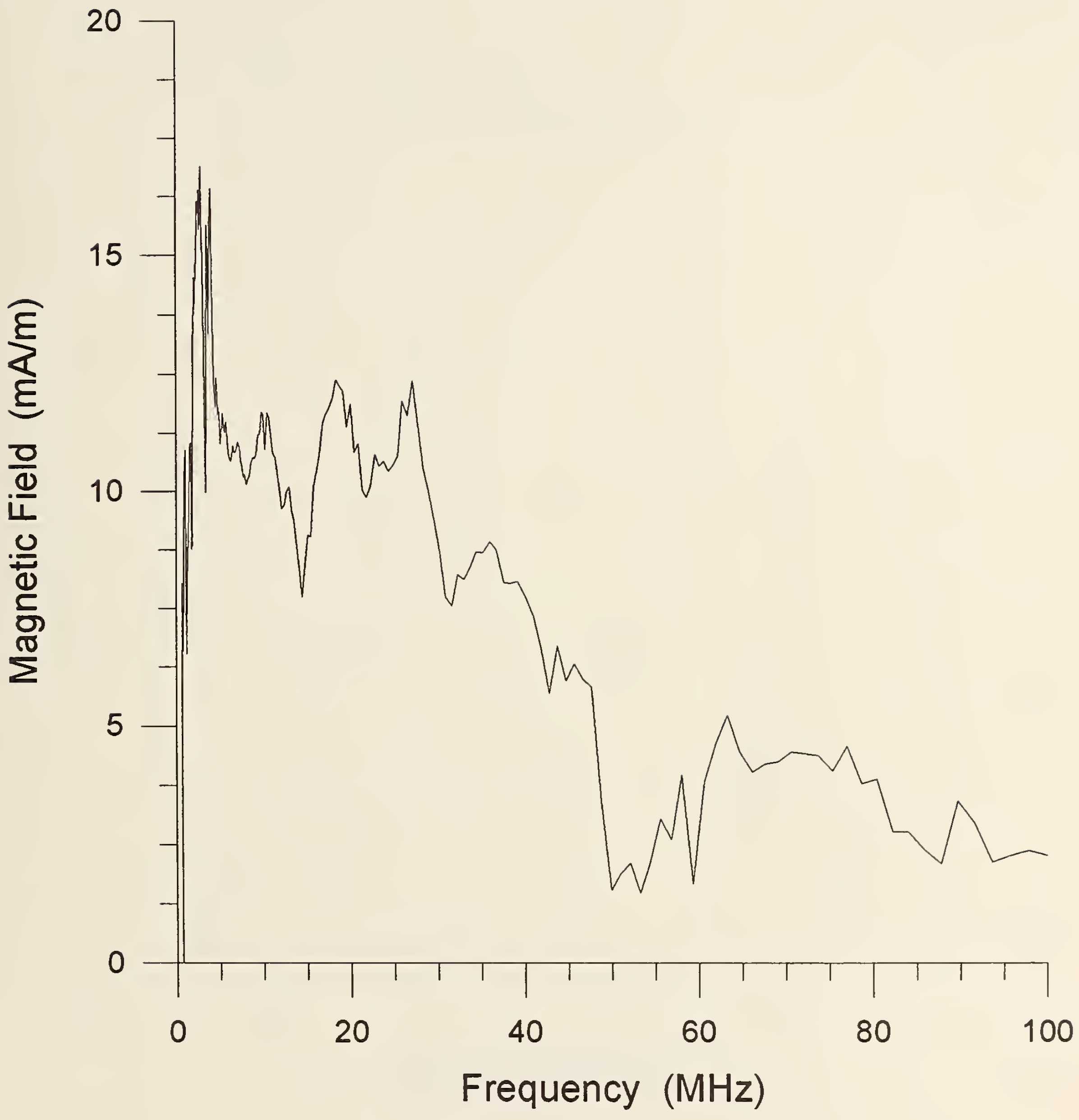

Figure 1. The cw external excitation (magnetic field) used in the experiment.

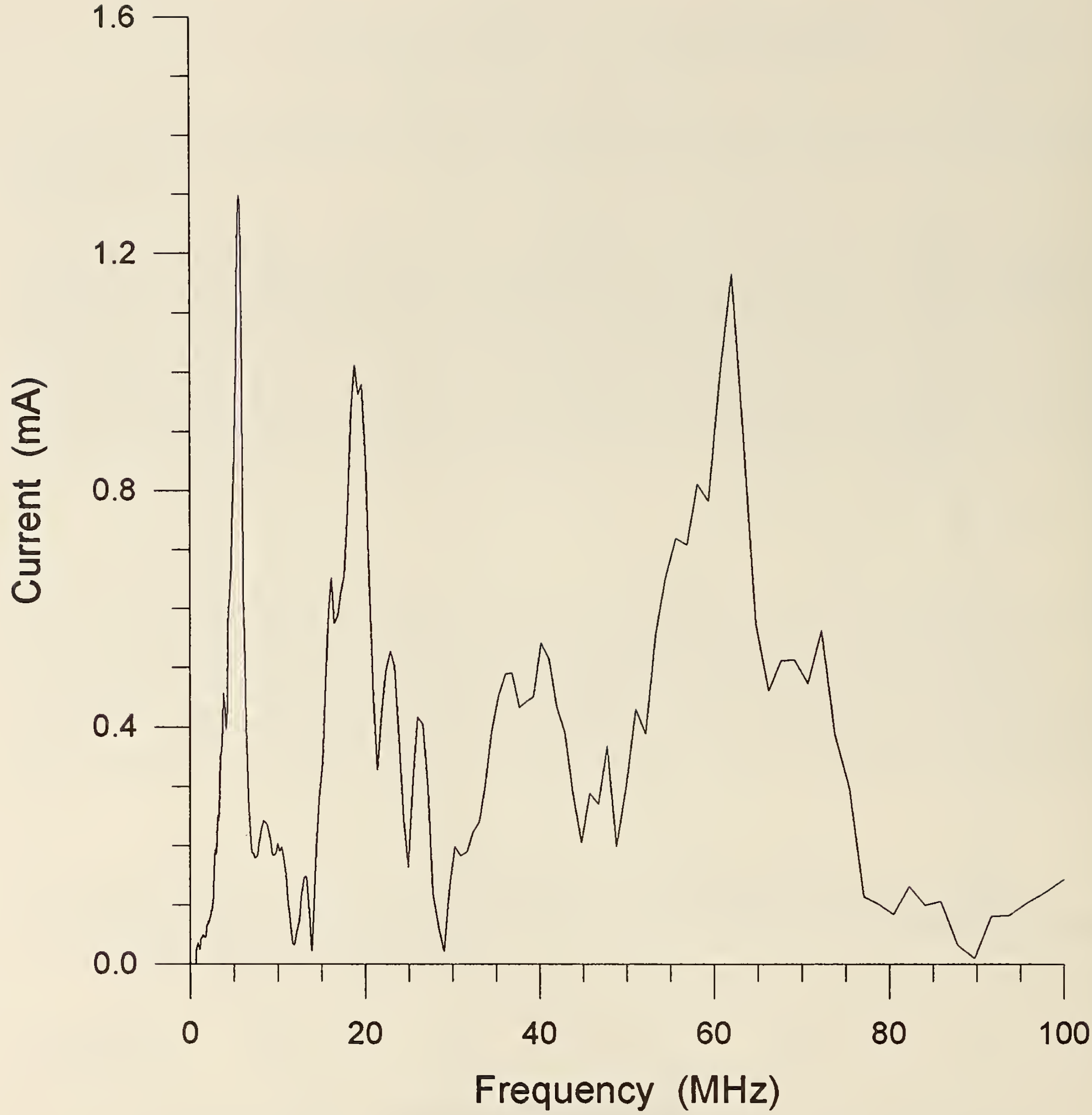

Figure 2. Measured C26 magnitude response to the cw excitation shown in Figure 1.

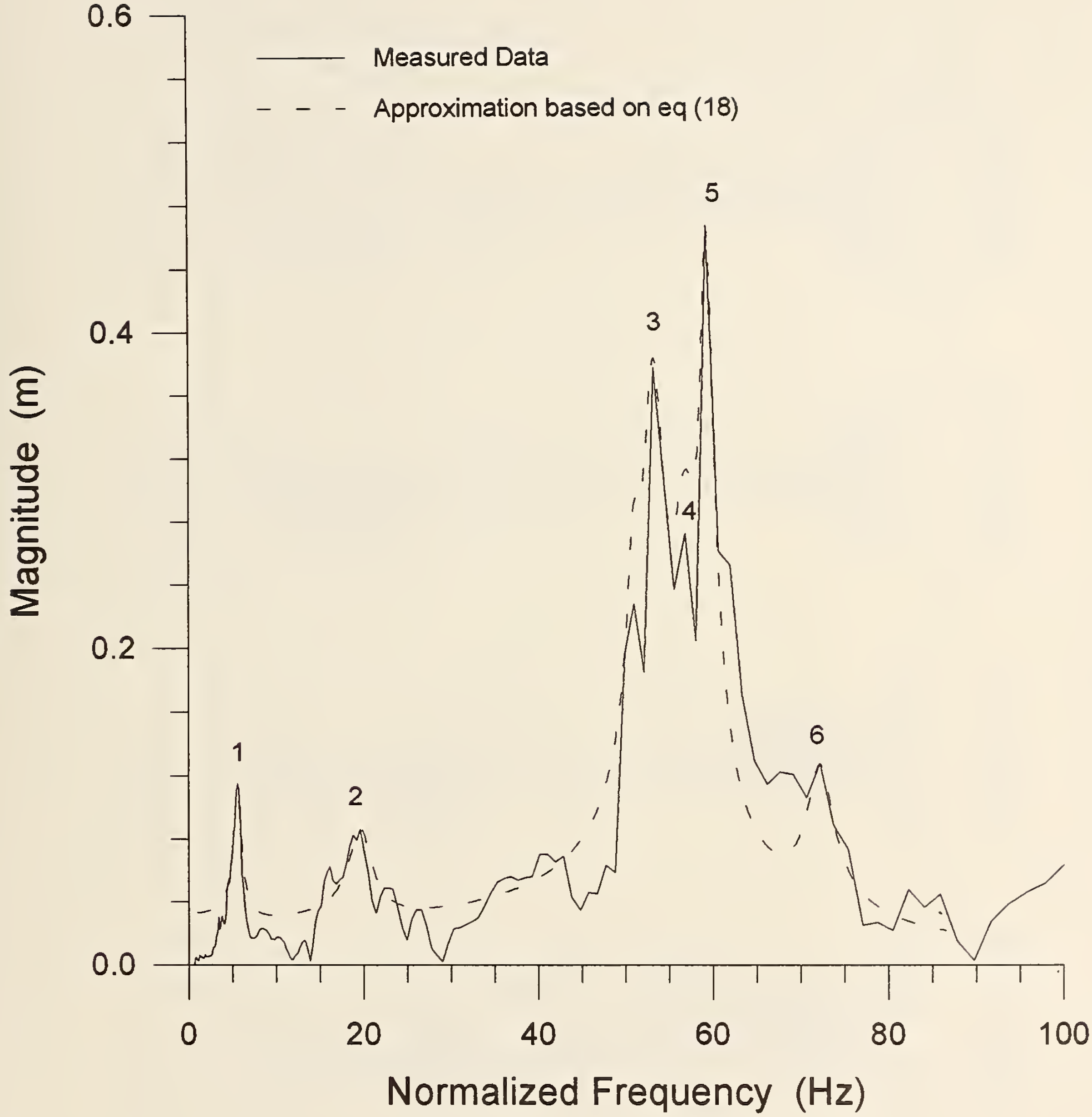

Figure 3. Magnitude of the system transfer function, both measured and approximated based on eq (18).

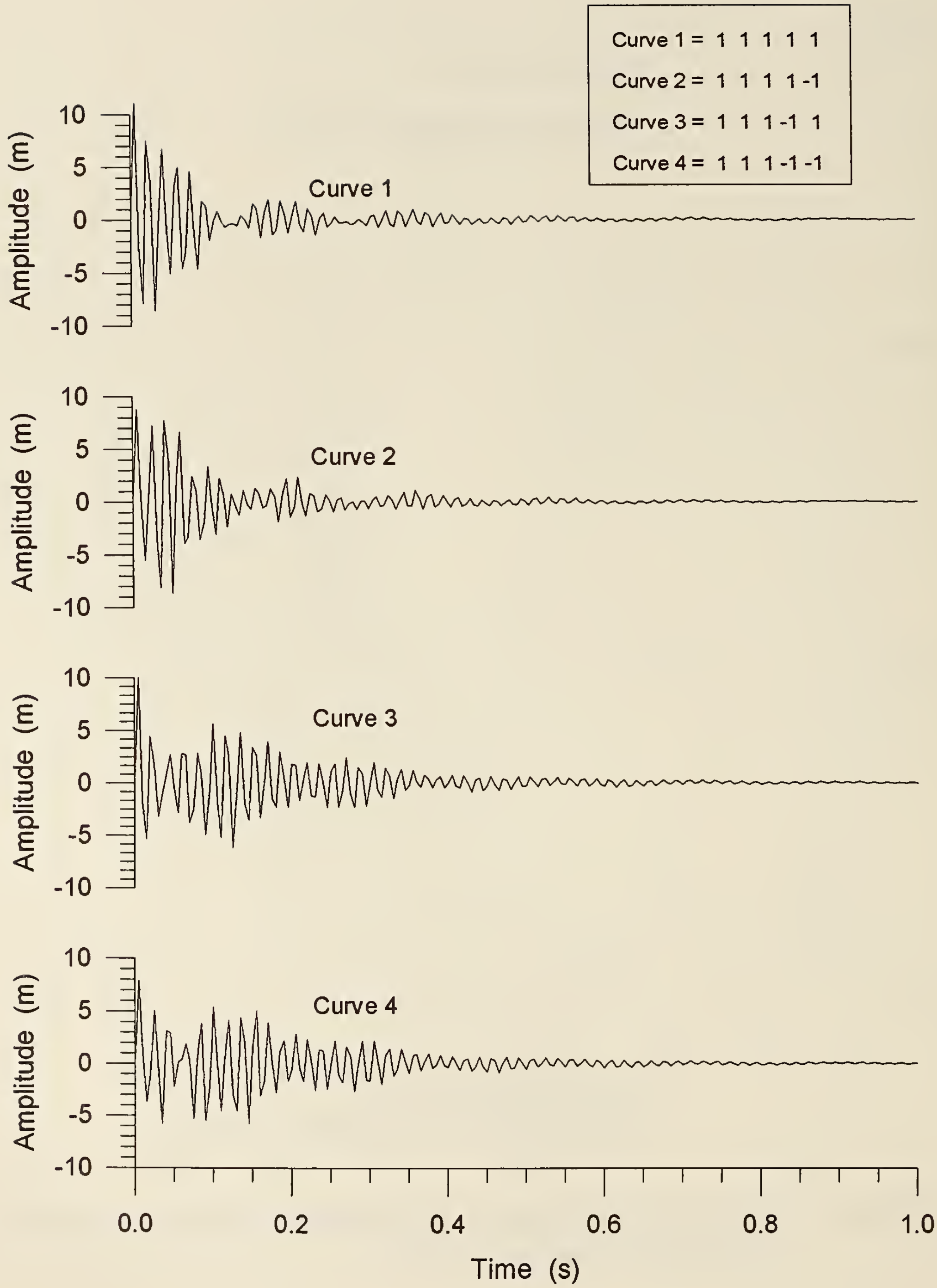

Figure 4. Possible impulse responses of the system under study; minimum phase case and three nonminimum phase cases.

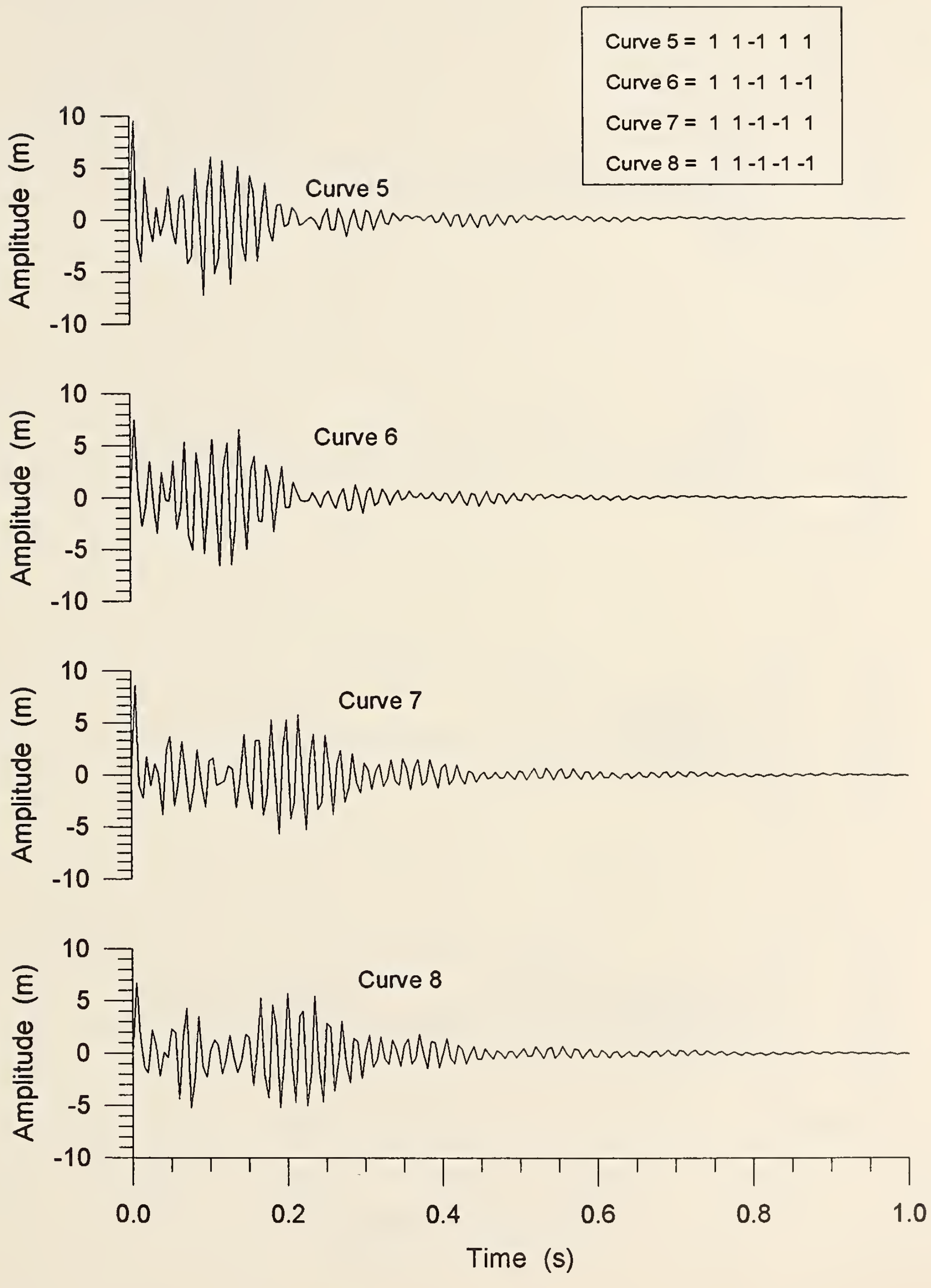

Figure 5. Possible impulse responses of the system under study; four nonminimum phase cases.

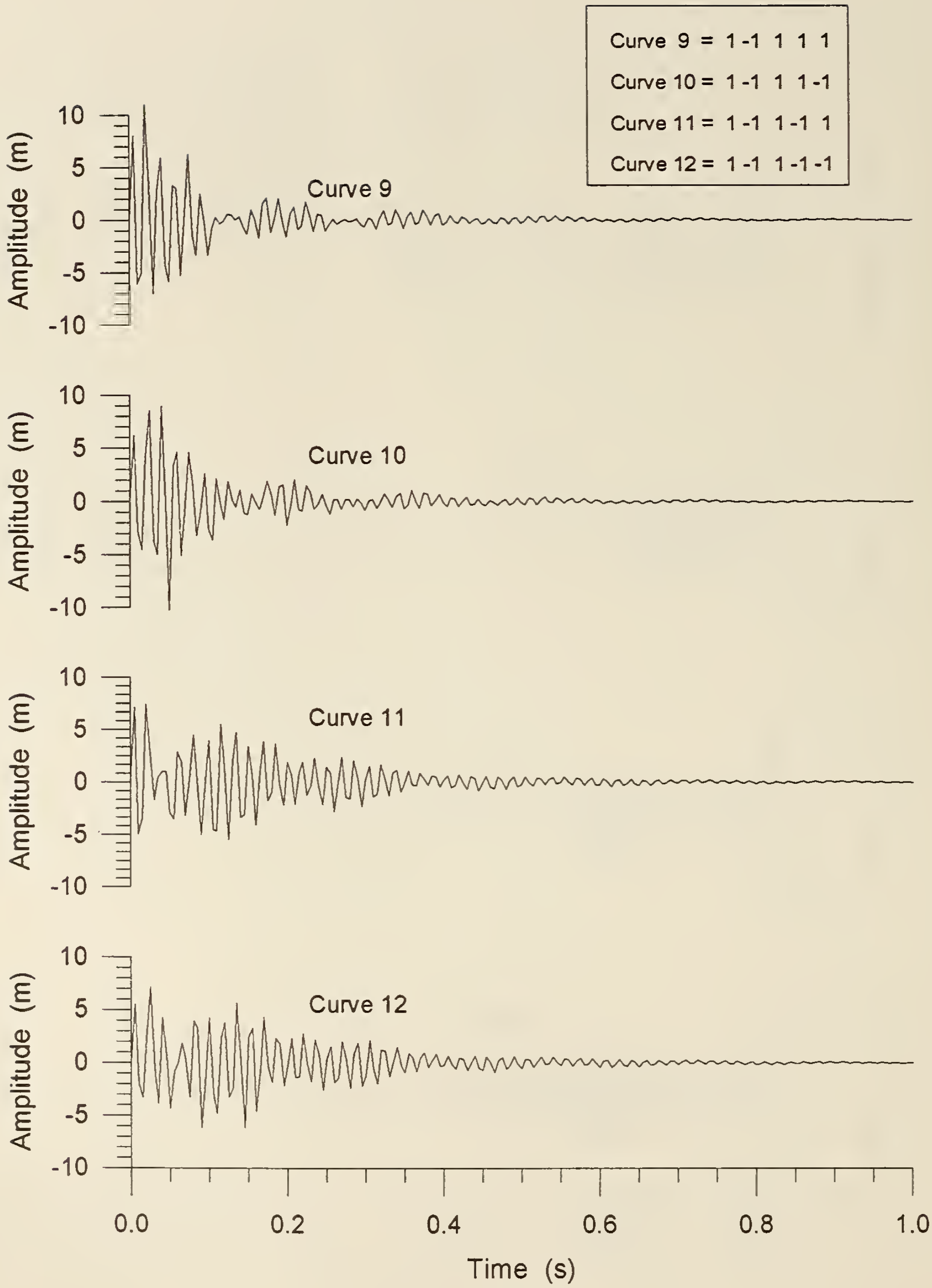

Figure 6. Possible impulse responses of the system under study, four nonminimum phase cases.

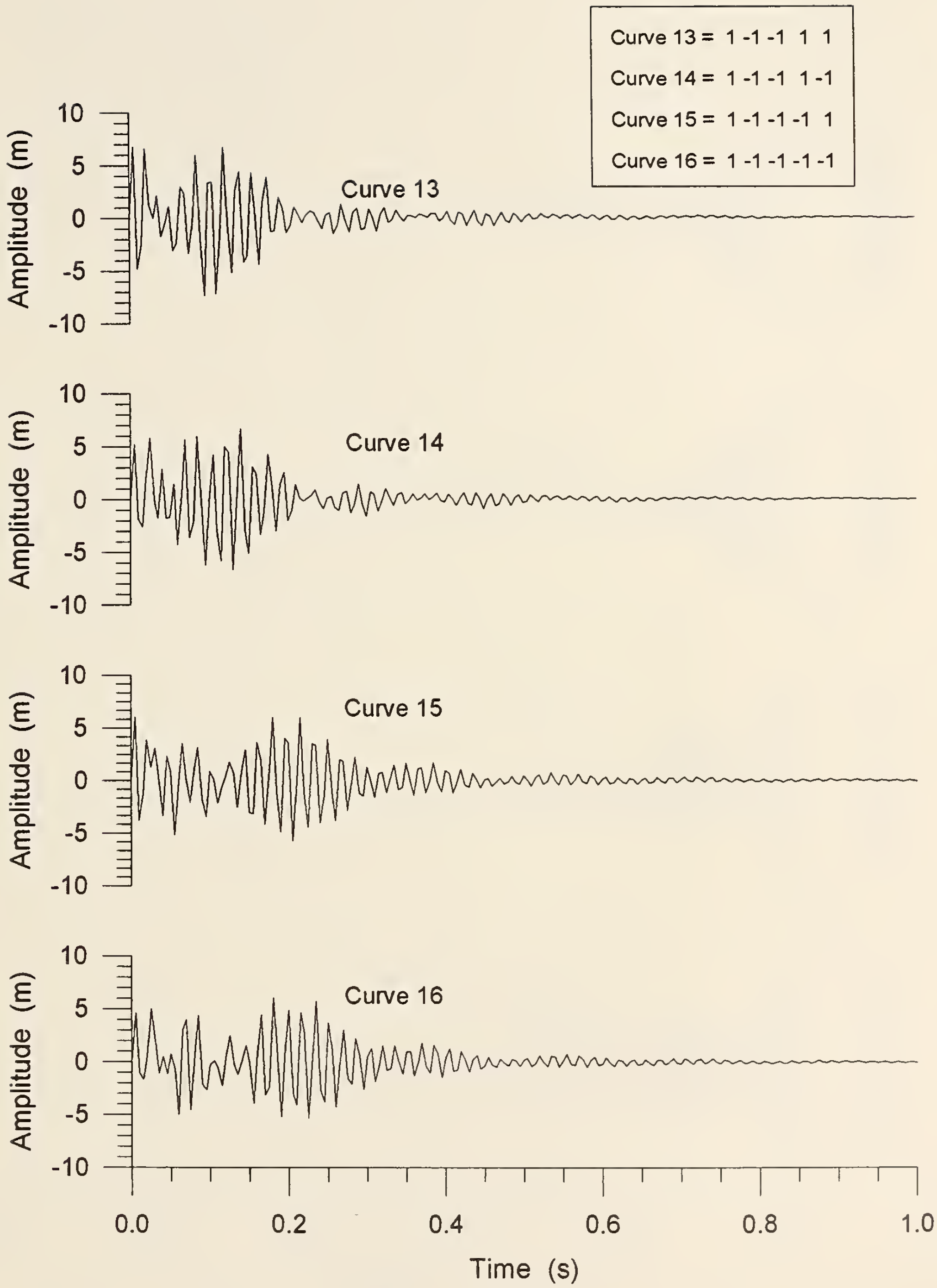

Figure 7. Possible impulse responses of the system under study, four nonminimum phase cases.

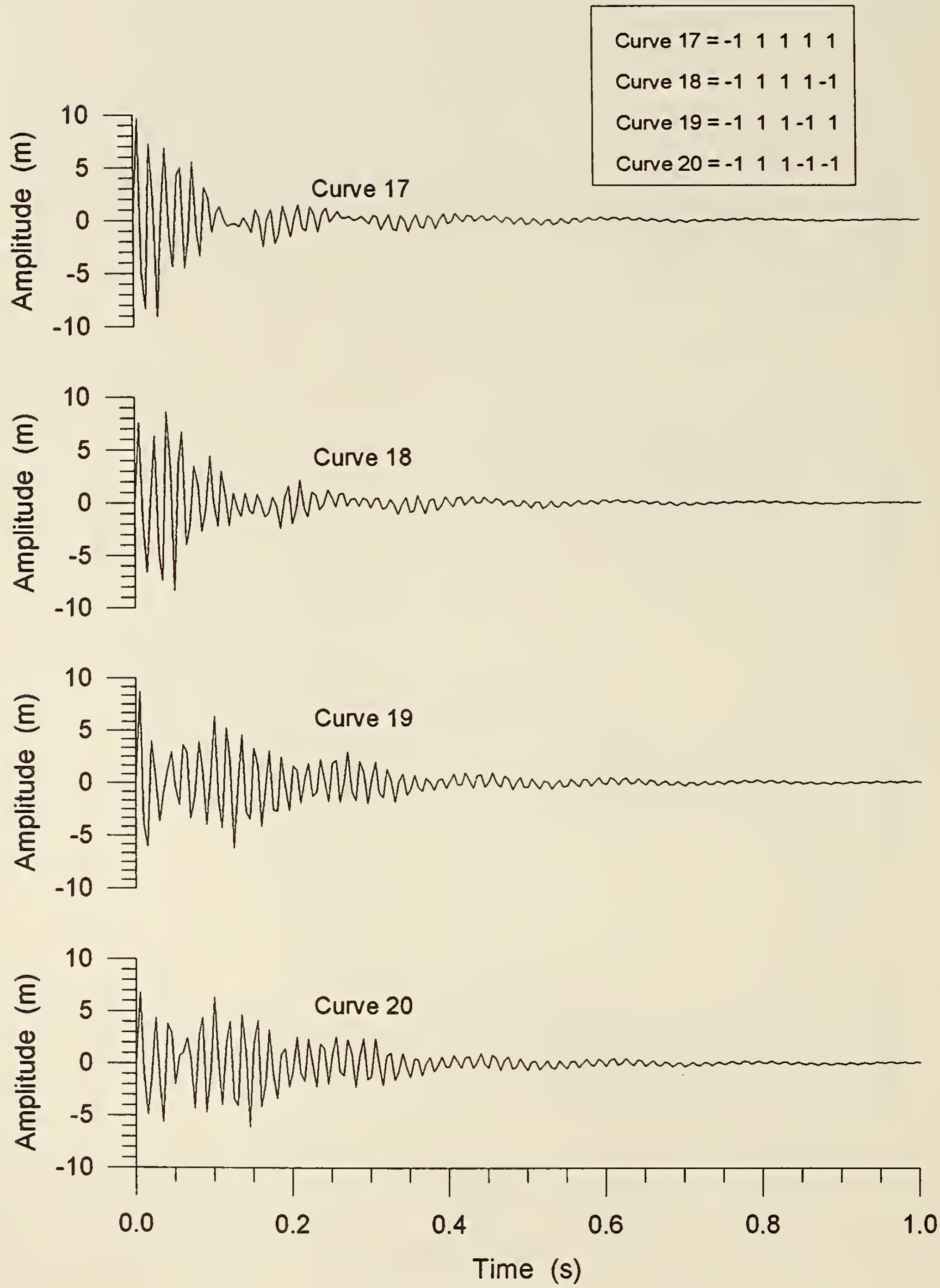

Figure 8. Possible impulse responses of the system under study, four nonminimum phase cases.

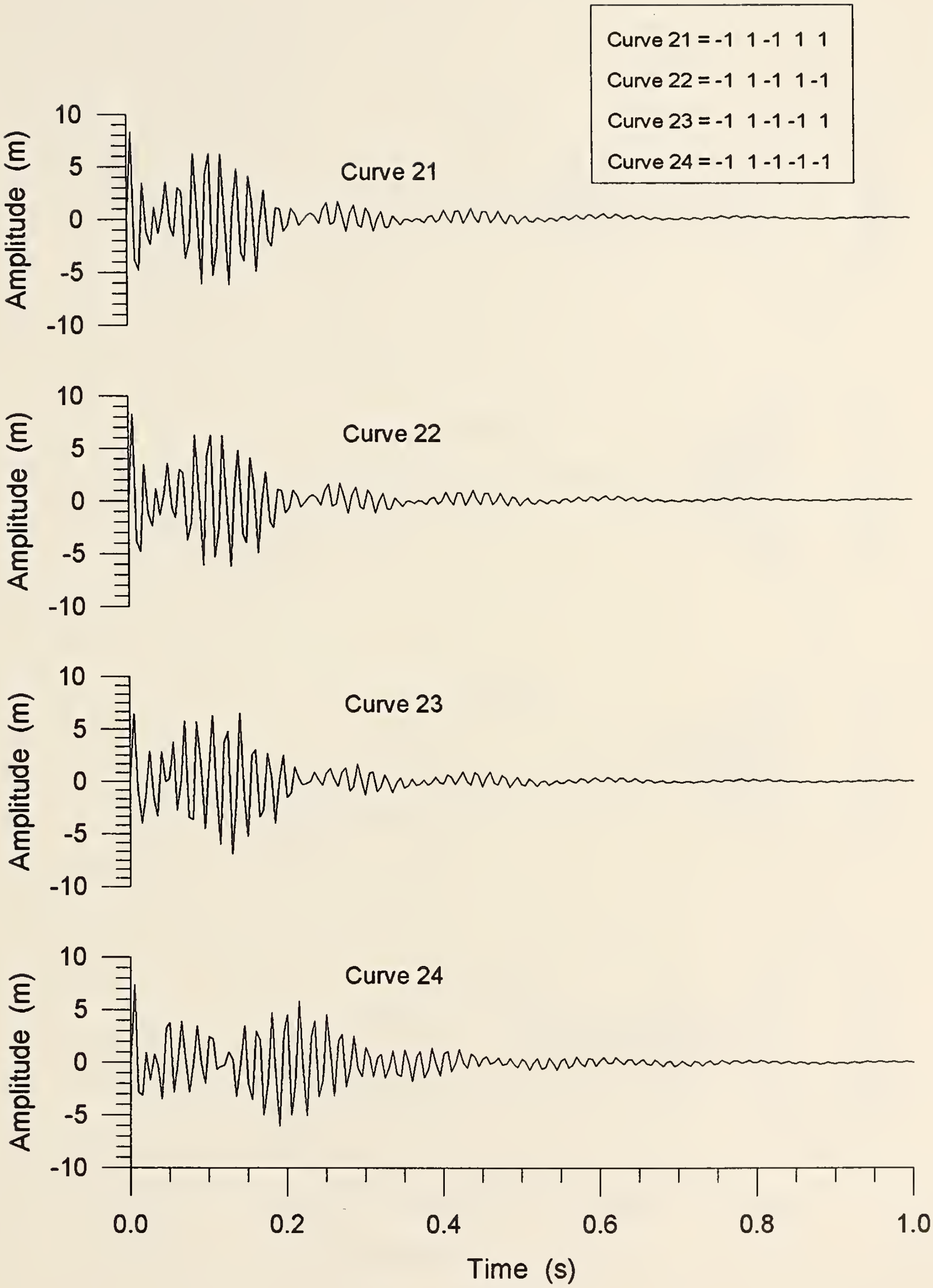

Figure 9. Possible impulse responses of the system under study, four nonminimum phase cases.

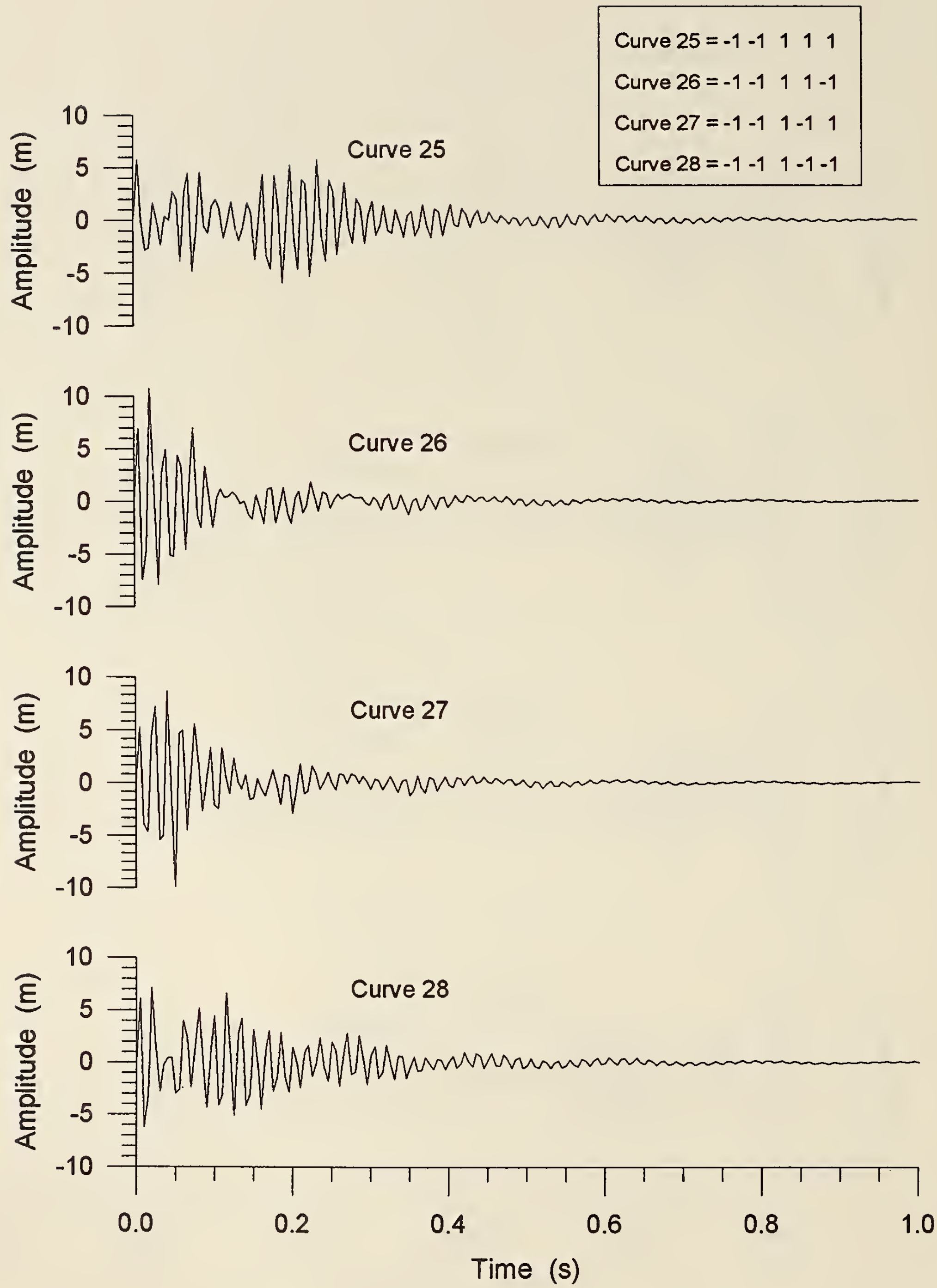

Figure 10. Possible impulse responses of the system under study, four nonminimum phase cases.

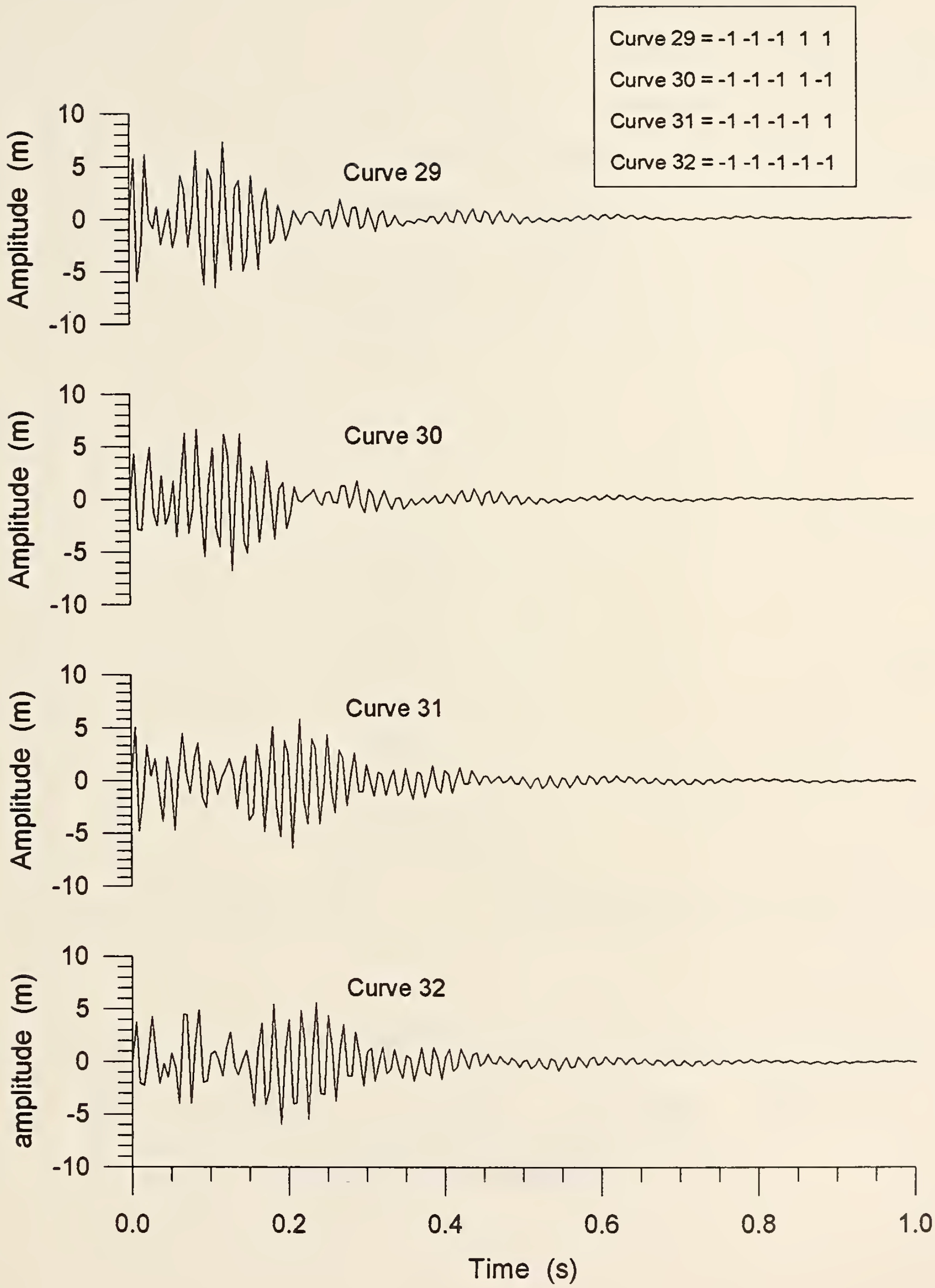

Figure 11. Possible impulse responses of the system under study, four nonminimum phase cases.

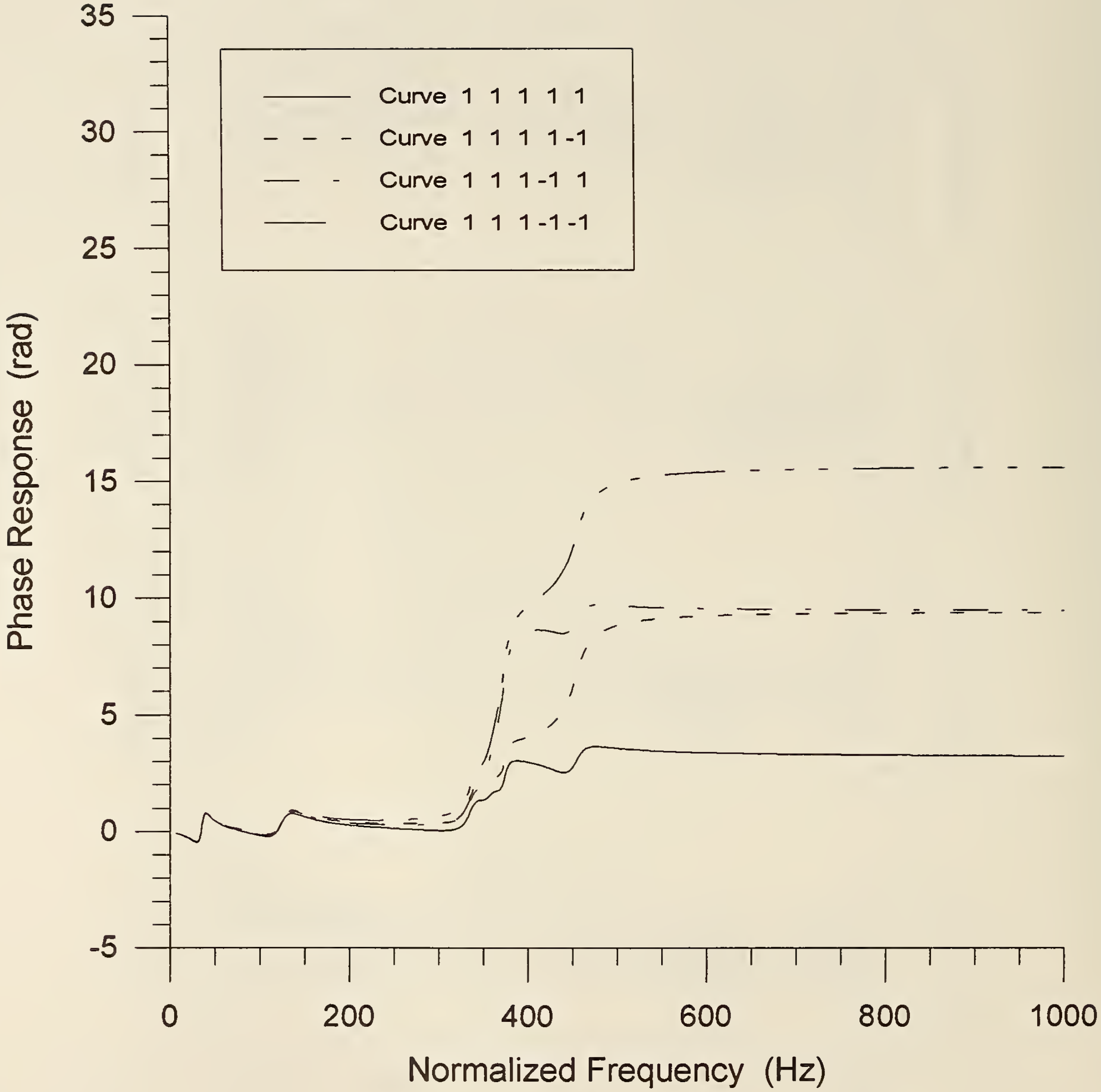

Figure 12. Possible phases of the system under study (C26) for one minimum phase case and three nonminimum phase cases.

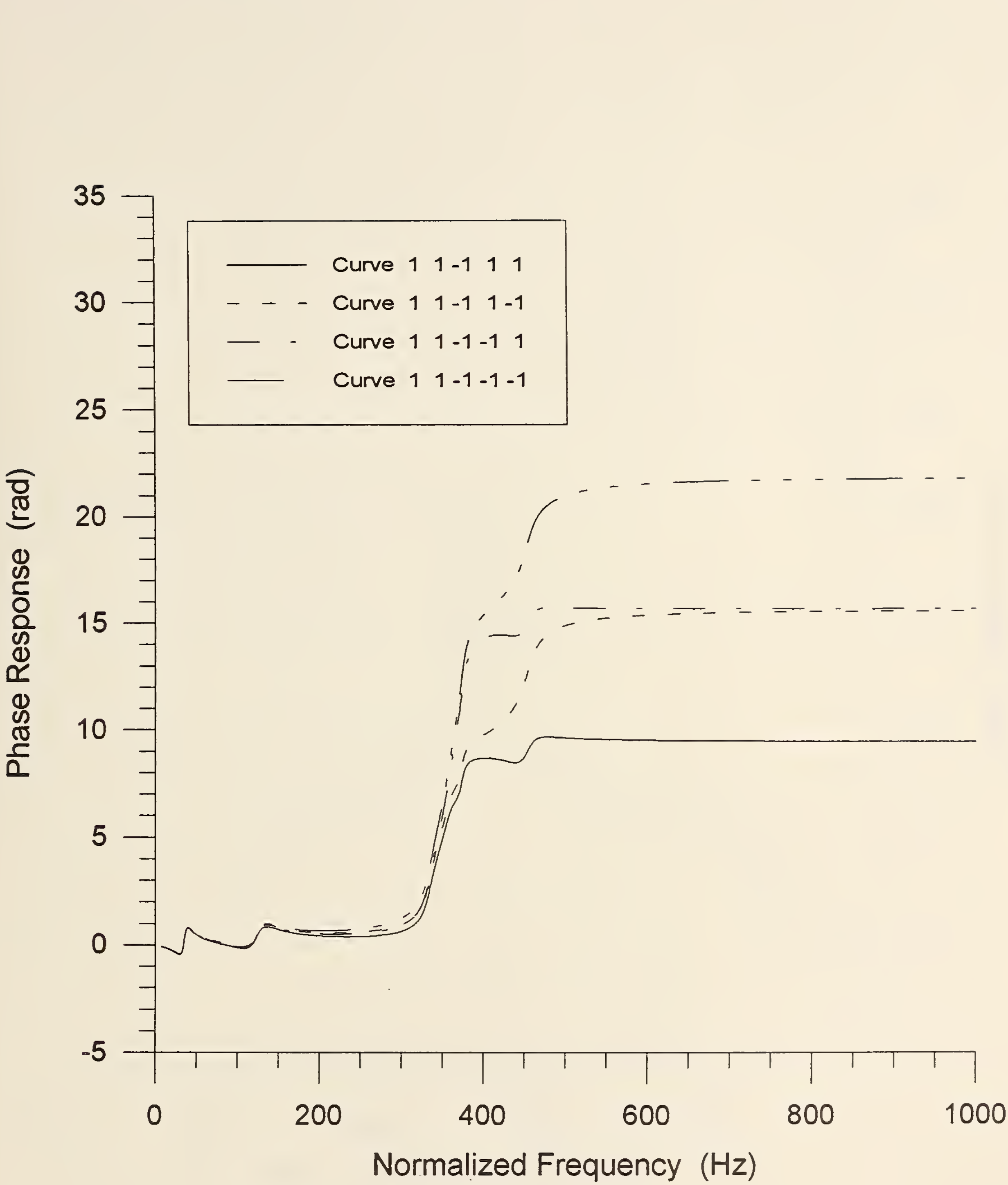

Figure 13. Possible phases of the system under study (C26) of four nonminimum phase cases.

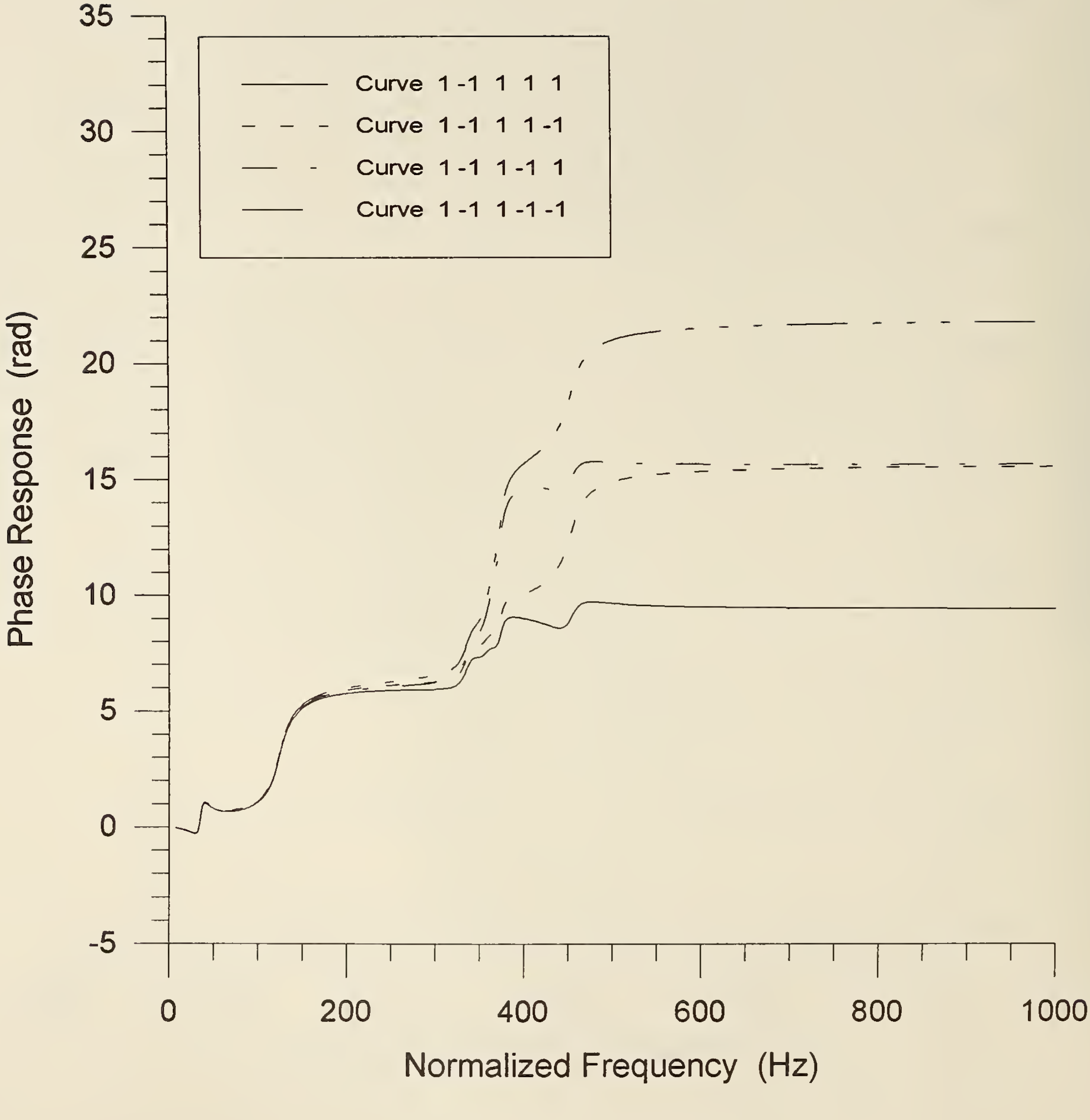

Figure 14. Possible phases of the system under study (C26) of four nonminimum phase cases.

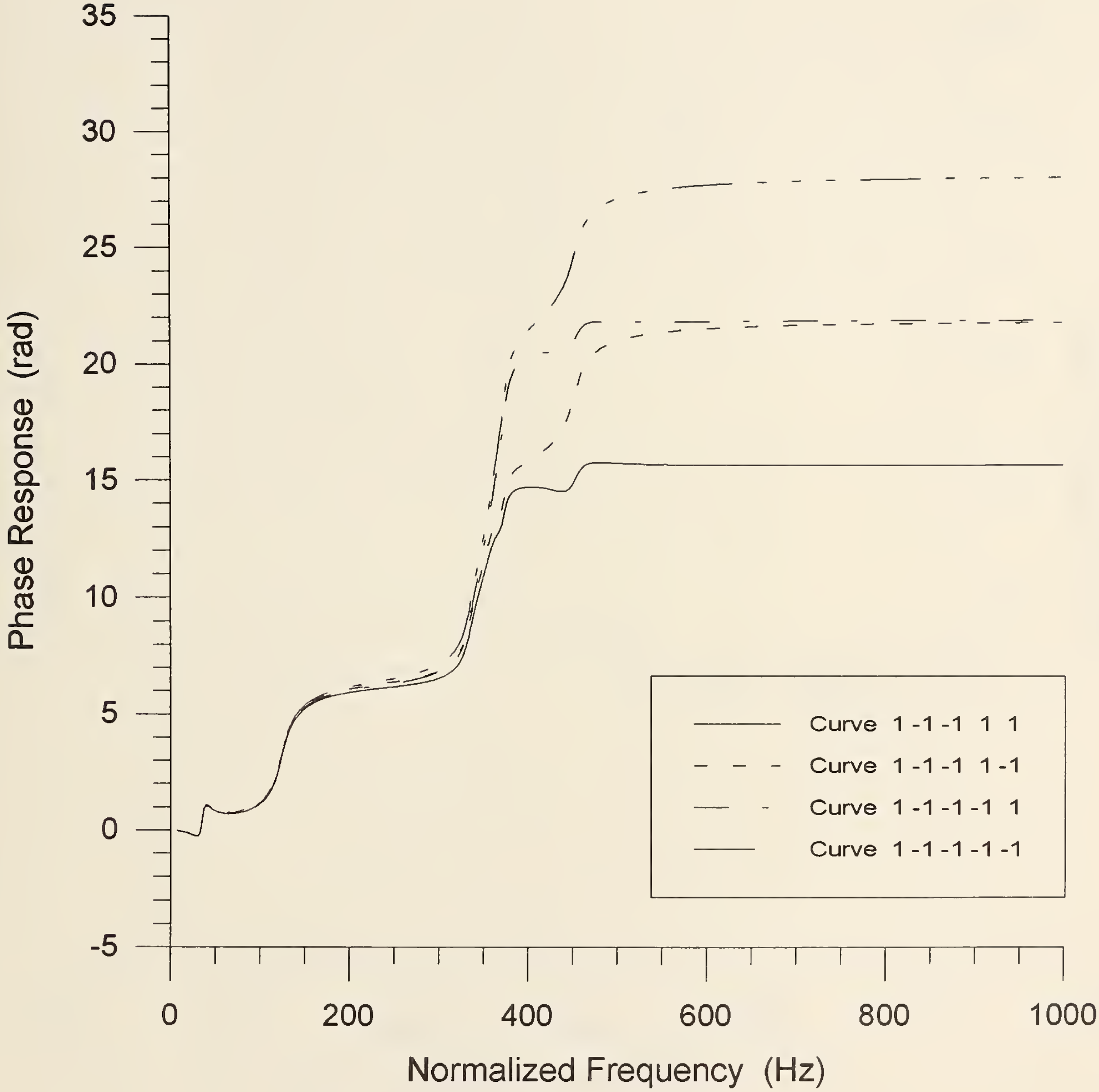

Figure 15. Possible phases of the system under study (C26) of four nonminimum phase cases.

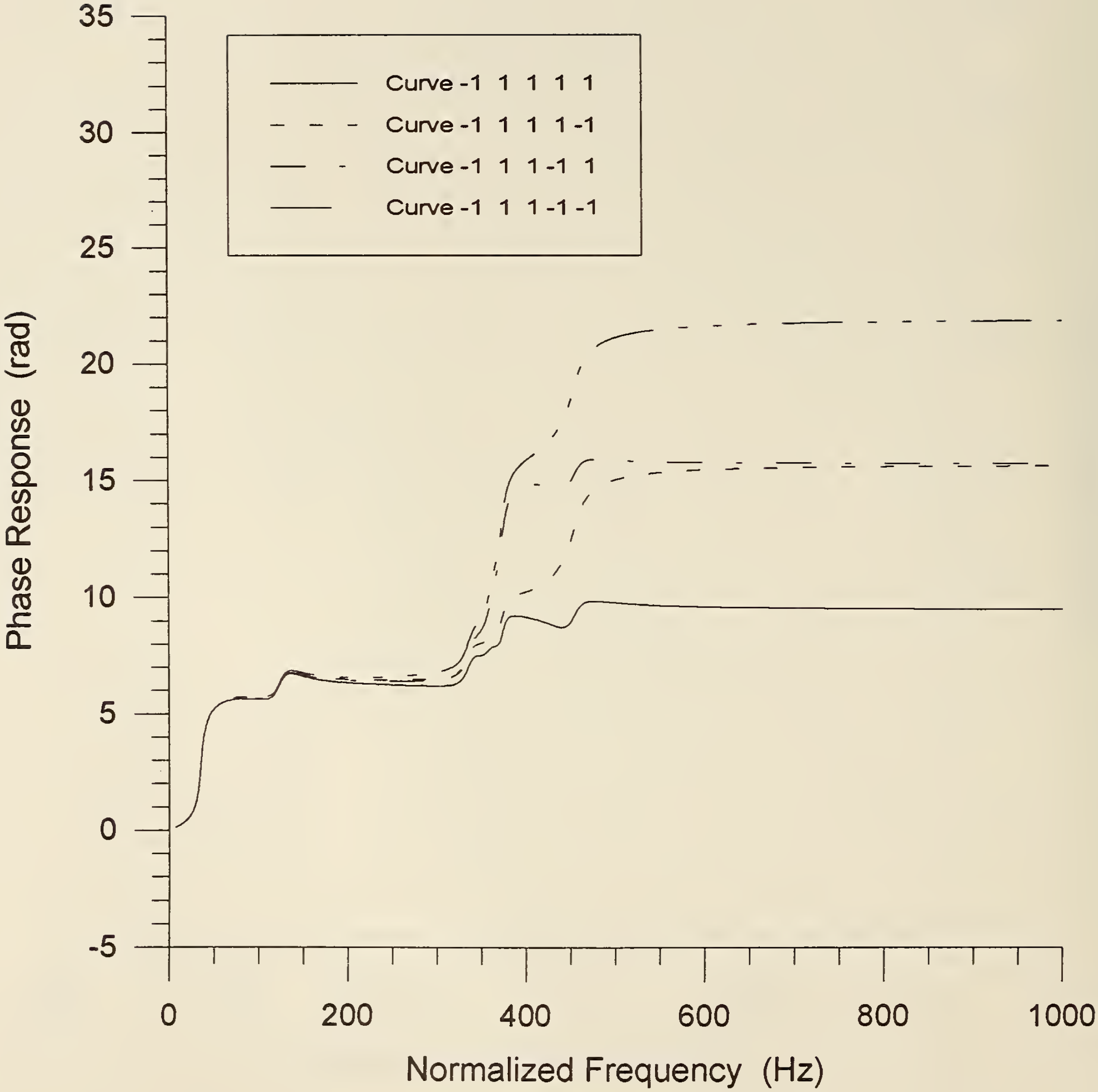

Figure 16. Possible phases of the system under study (C26) of four nonminimum phase cases.

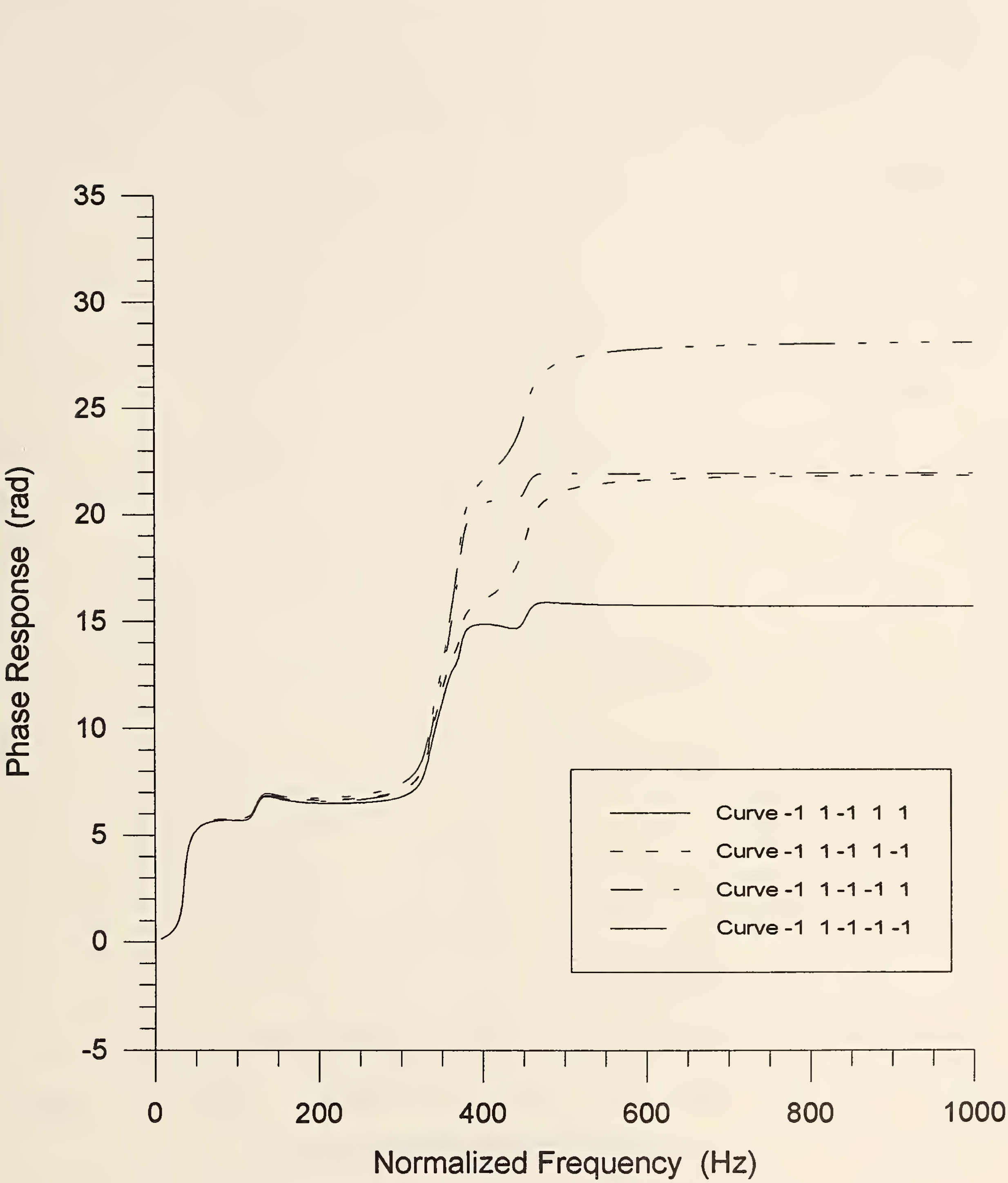

Figure 17. Possible phases of the system under study (C26) of four nonminimum phase cases.

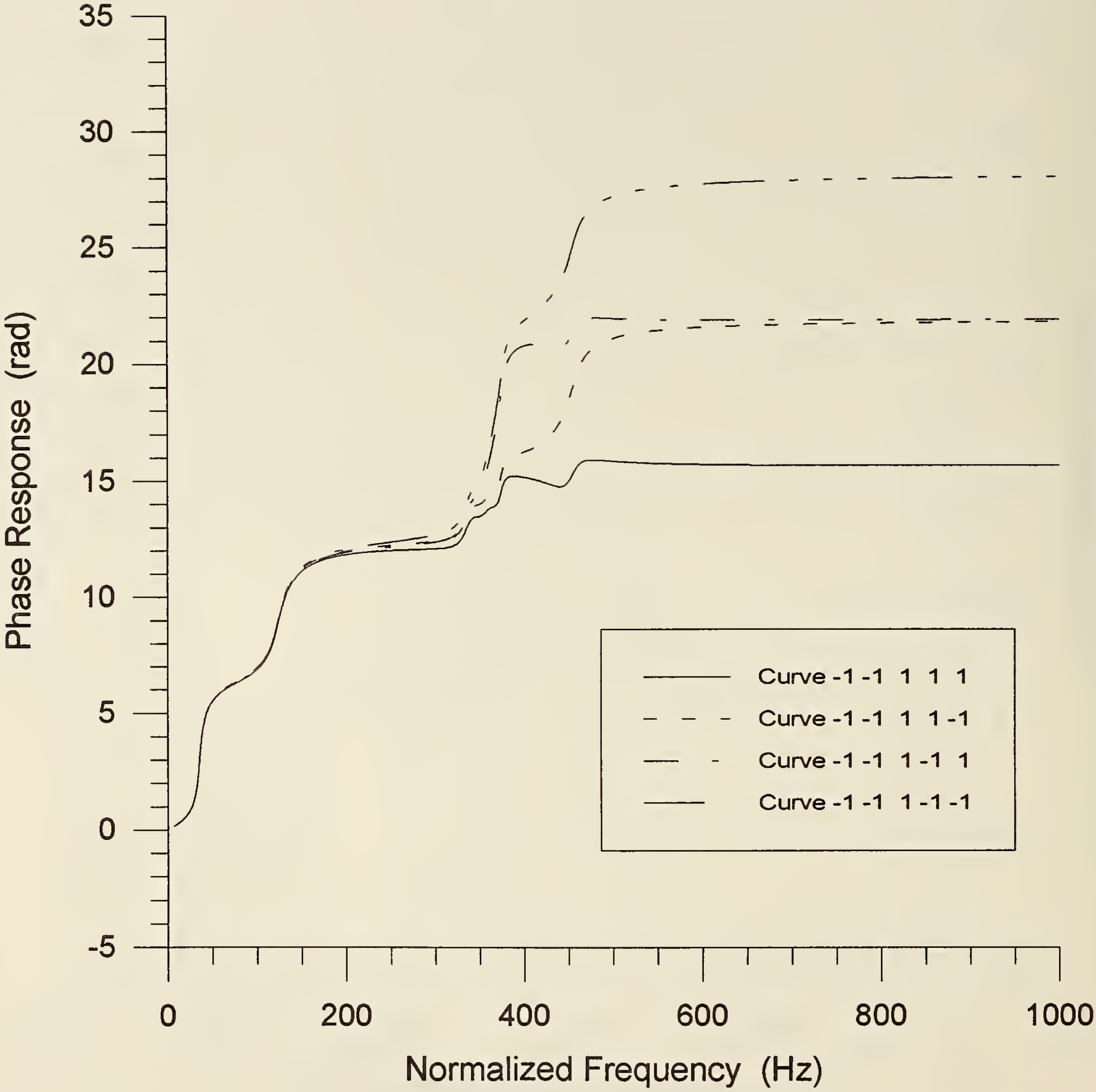

Figure 18. Possible phases of the system under study (C26) of four nonminimum phase cases.

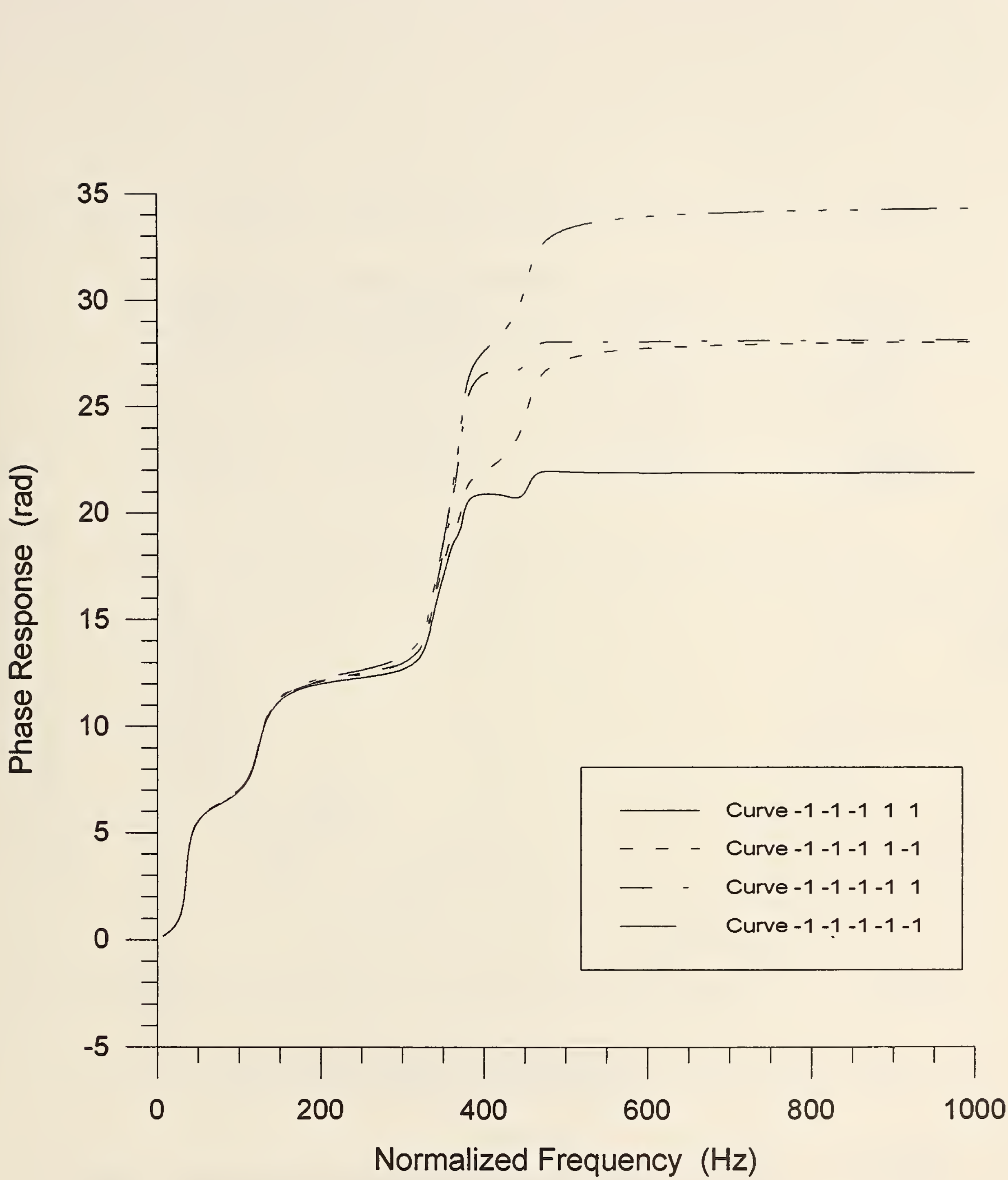

Figure 19. Possible phases of the system under study (C26) of four nonminimum phase cases.

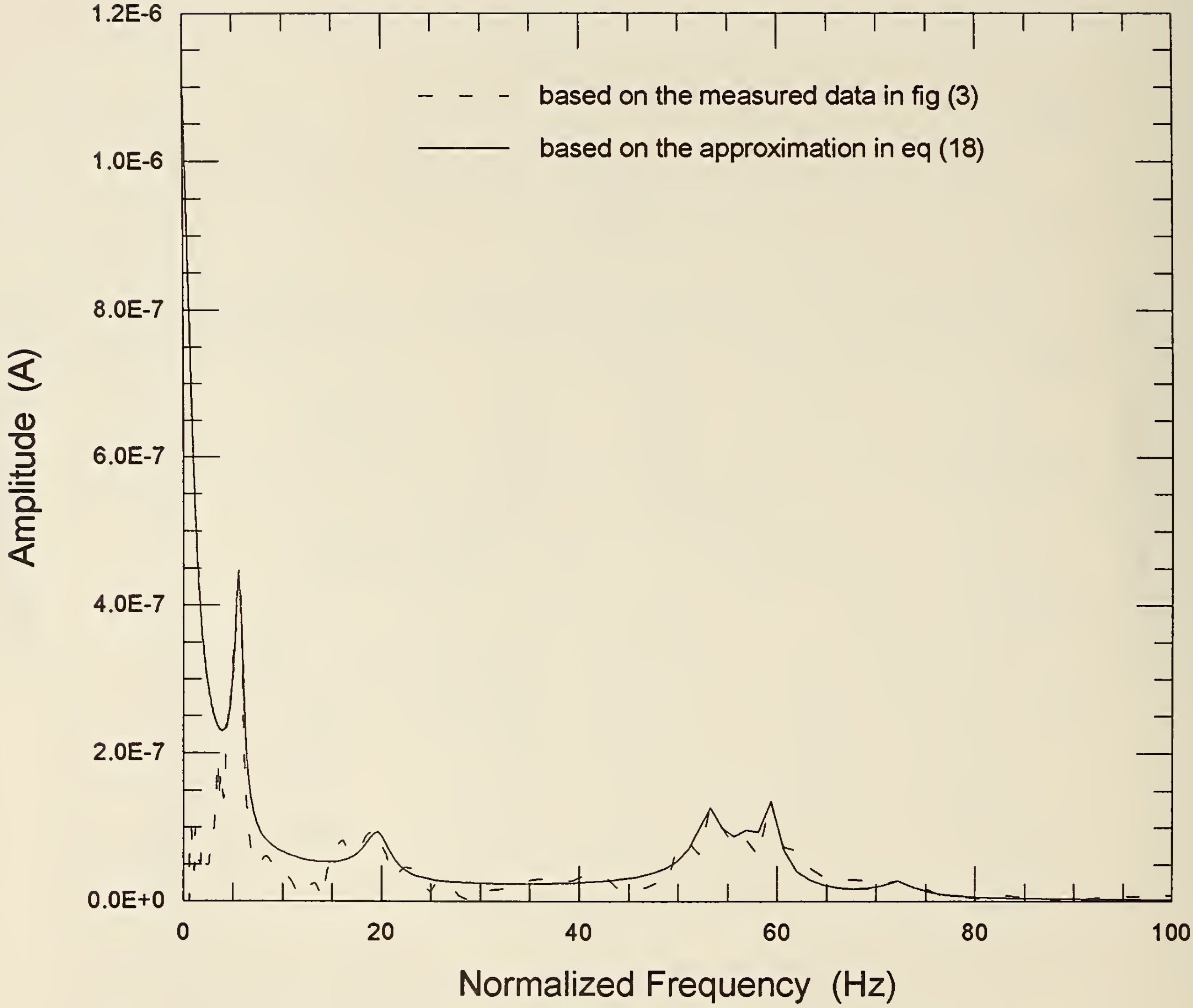

Figure 20. Magnitude response, as a function of normalized frequency, of the system to the impulse excitation given in eq (36).

curve = 1 $\quad$ (1 1 1 1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.43048 \\ 0.88139 \\ 2.59983 \\ -0.16558 \\ -2.0624 \\ -1.68371 \end{bmatrix} \quad d = \begin{bmatrix} 0.09543 \\ 0.31967 \\ 3.5126 \\ 2.66402 \\ 2.50921 \\ 0.24129 \end{bmatrix}$$

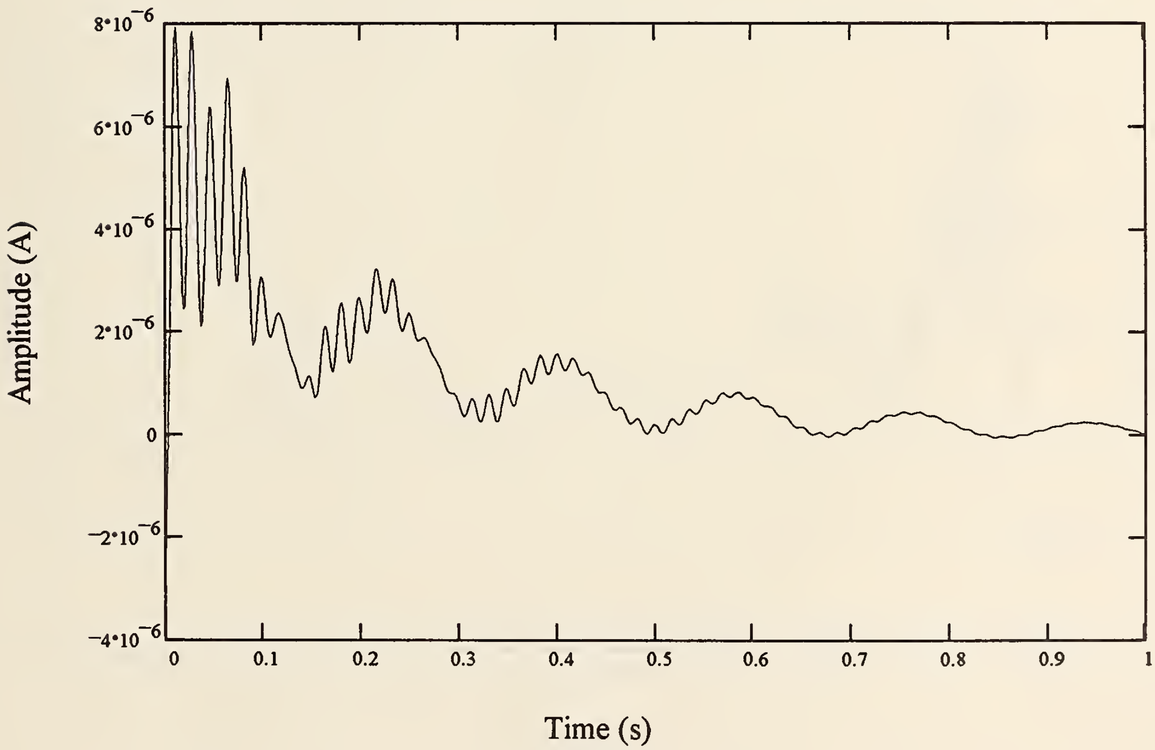

Figure 21. Time response of the system to the impulse excitation given in eq (37), for the minimum phase case.

curve = 2 (1 1 1 1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.42908 \\ 0.85894 \\ 0.88812 \\ -1.6675 \\ -3.34611 \\ 2.83748 \end{bmatrix} \quad d = \begin{bmatrix} 0.10451 \\ 0.3899 \\ 4.44494 \\ 2.30787 \\ 0.62801 \\ 1.38157 \end{bmatrix}$$

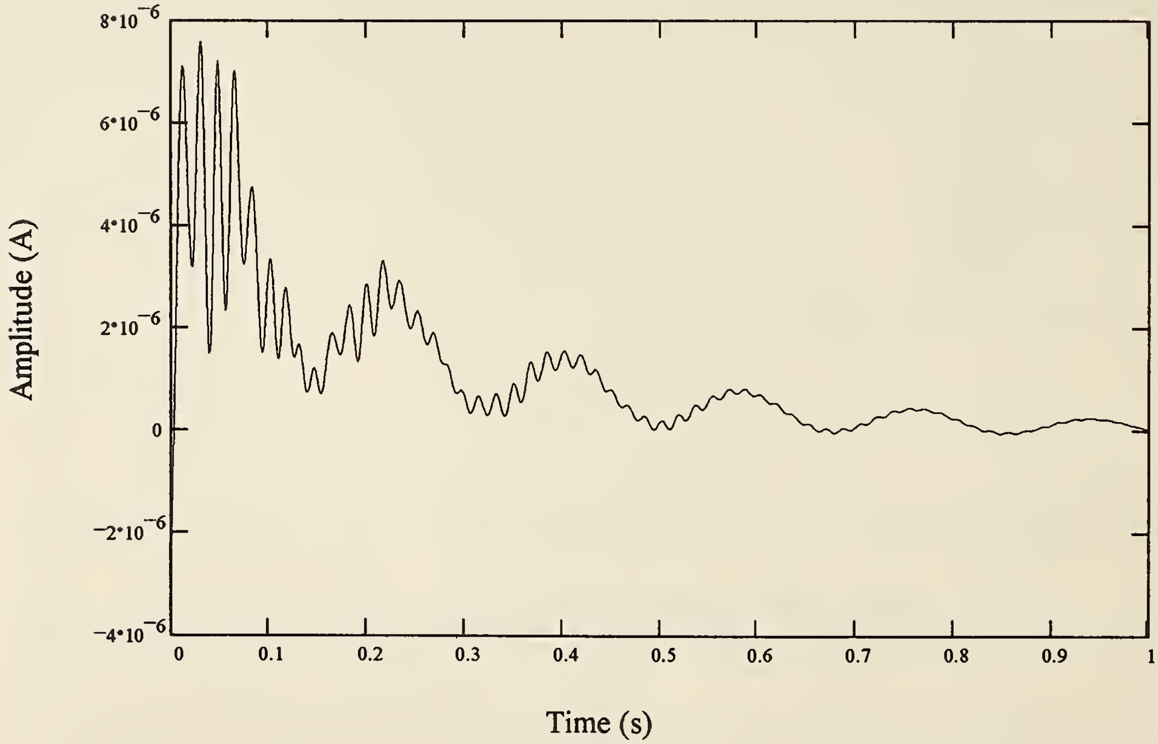

Figure 22. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 3 (1 1 1 -1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.42972 \\ 0.86933 \\ 0.21726 \\ -6.01609 \\ 6.09814 \\ -1.59836 \end{bmatrix} \quad d = \begin{bmatrix} 0.1005 \\ 0.36057 \\ 5.05079 \\ 1.49411 \\ 1.57604 \\ 0.70862 \end{bmatrix}$$

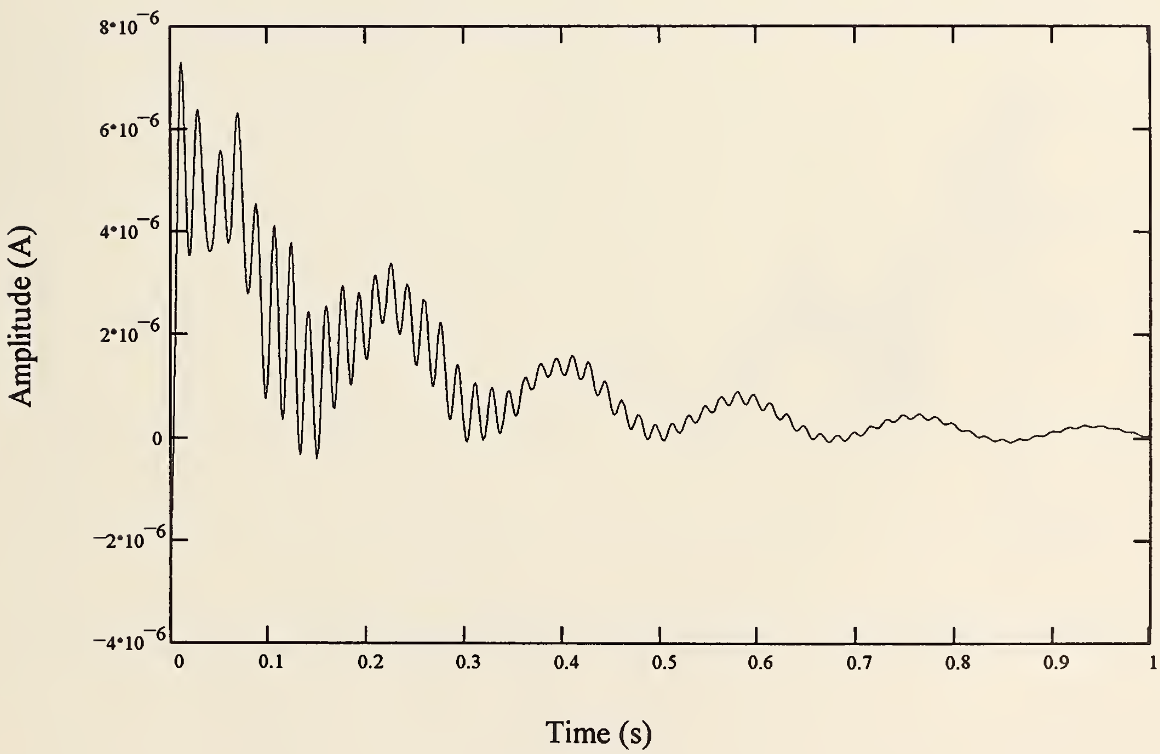

Figure 23. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 4 (1 1 1 -1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.42822 \\ 0.84363 \\ -2.02735 \\ -6.27638 \\ 3.83916 \\ 3.19272 \end{bmatrix} \quad d = \begin{bmatrix} 0.10957 \\ 0.42997 \\ 4.83595 \\ -2.0812 \\ 5.37101 \\ 0.57488 \end{bmatrix}$$

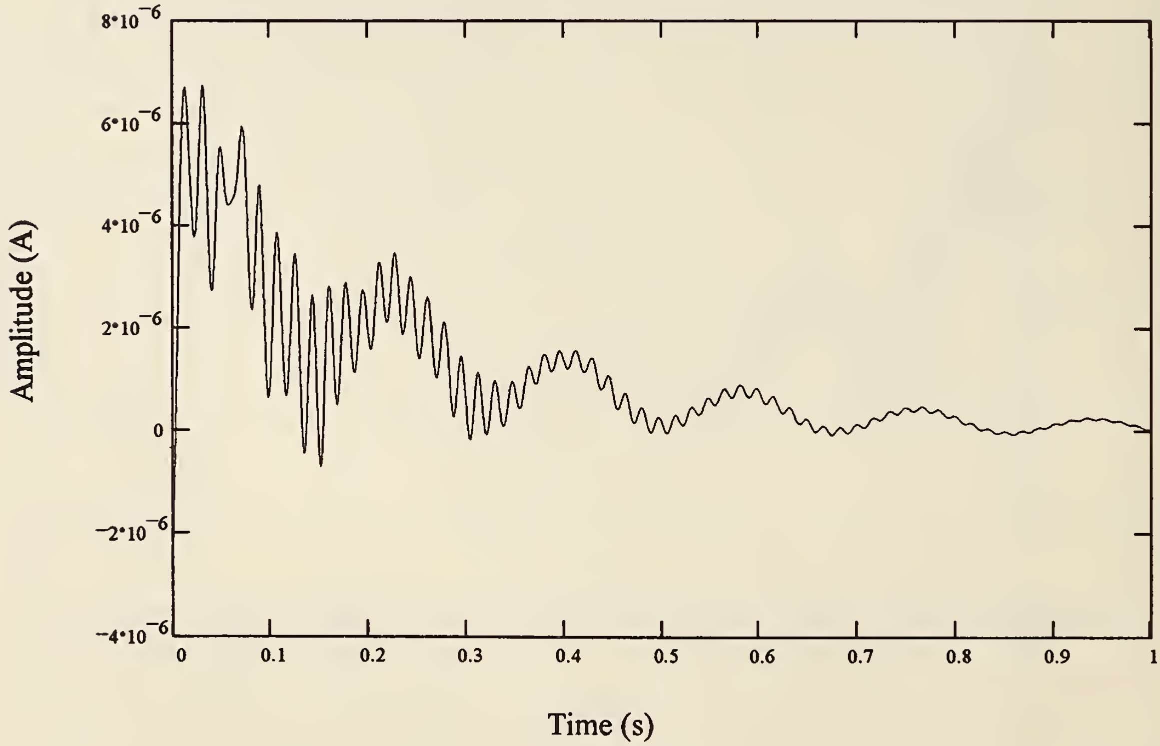

Figure 24. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 5   (1 1 -1 1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.42929 \\ 0.86184 \\ -6.85277 \\ 5.79528 \\ 1.33149 \\ -1.56512 \end{bmatrix} \quad d = \begin{bmatrix} 0.10324 \\ 0.38348 \\ 4.6978 \\ -0.22031 \\ 3.53812 \\ 0.76908 \end{bmatrix}$$

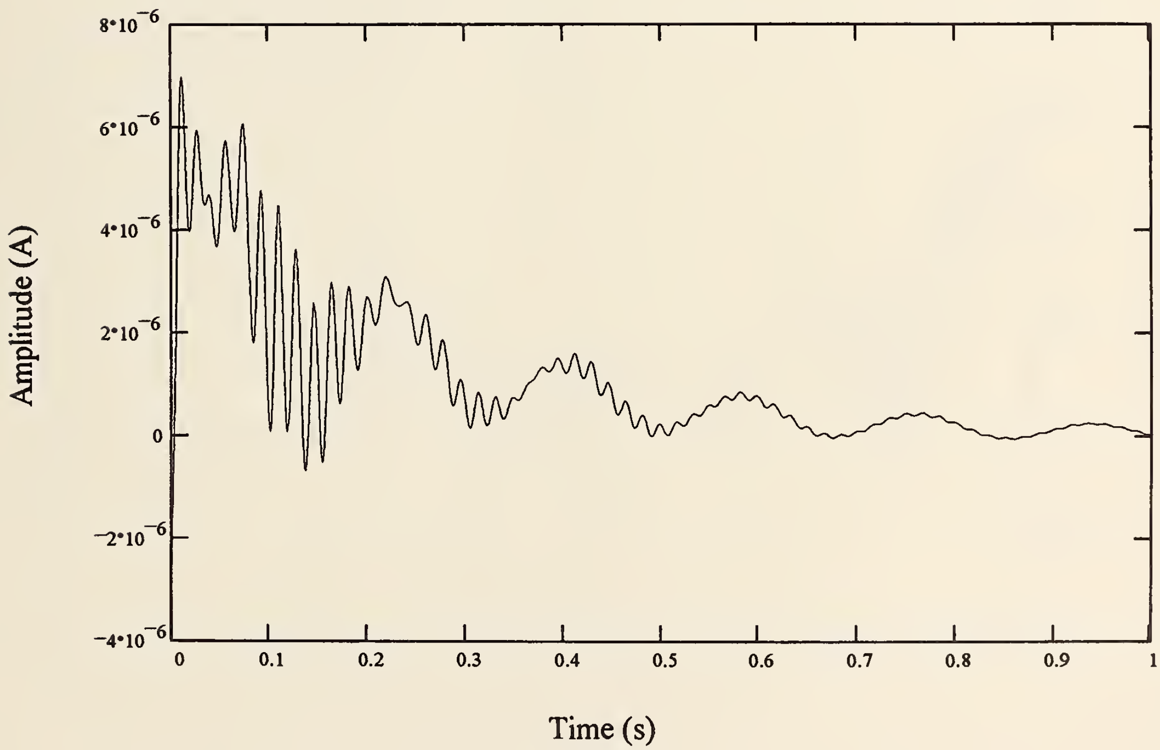

Figure 25. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 6 (1 1 -1 1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.42773 \\ 0.83432 \\ -8.50636 \\ 5.35135 \\ -1.31161 \\ 3.20458 \end{bmatrix} \quad d = \begin{bmatrix} 0.11231 \\ 0.45237 \\ 1.38143 \\ 3.104 \\ 3.73917 \\ 0.4474 \end{bmatrix}$$

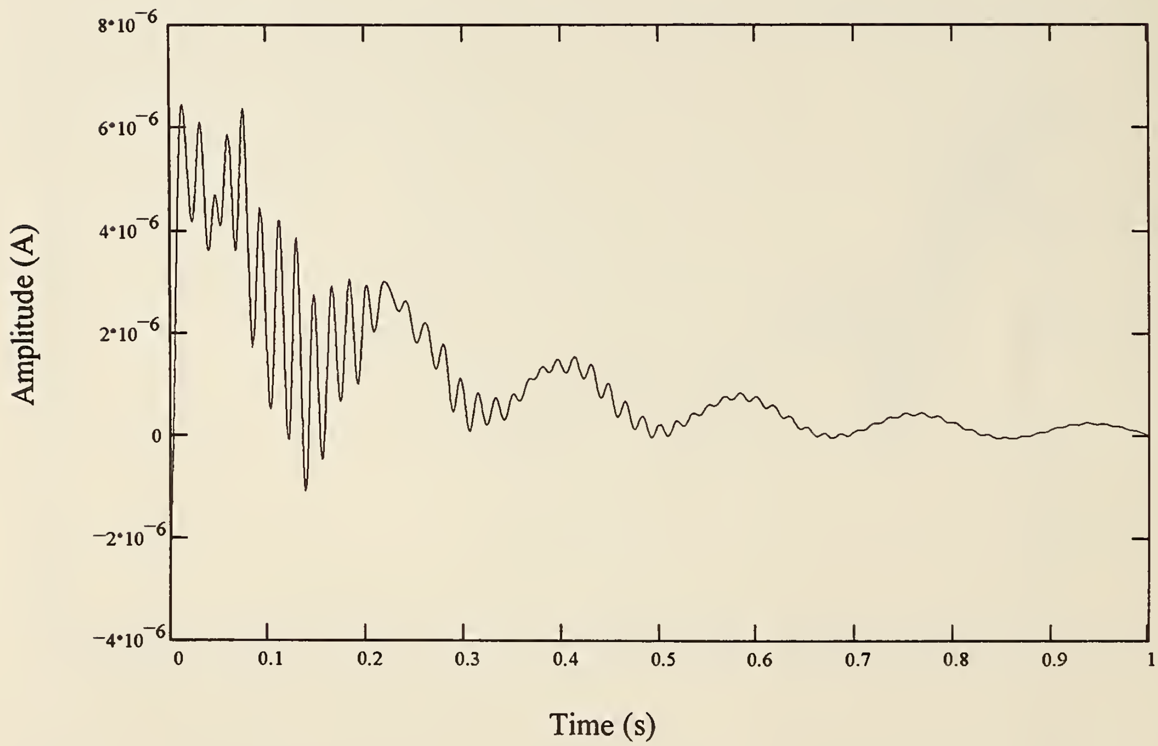

Figure 26. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 7 (1 1 -1 -1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.42844 \\ 0.84682 \\ -9.61129 \\ 4.53606 \\ 5.1337 \\ -1.33374 \end{bmatrix} \quad d = \begin{bmatrix} 0.1083 \\ 0.42367 \\ 0.06512 \\ 12.6818 \\ -5.23318 \\ 1.19767 \end{bmatrix}$$

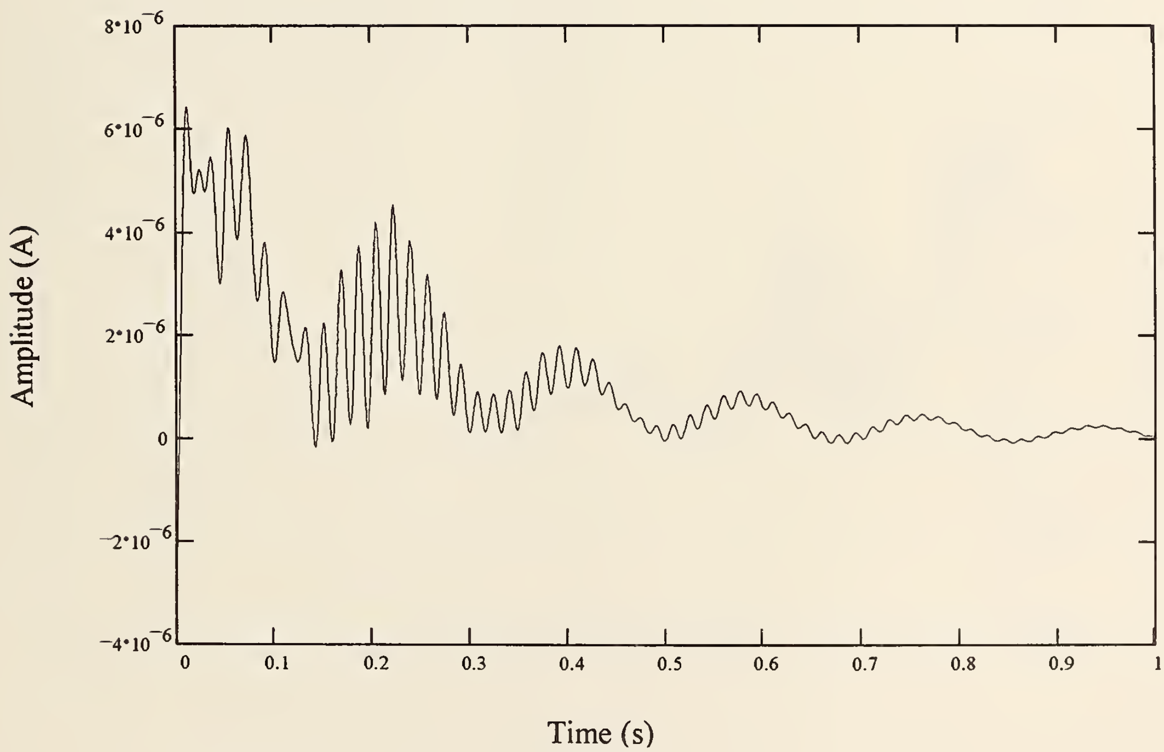

Figure 27. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 8 (1 1 -1 -1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.42678 \\ 0.8161 \\ -9.04859 \\ -3.13691 \\ 7.64746 \\ 3.29517 \end{bmatrix} \quad d = \begin{bmatrix} 0.11736 \\ 0.49149 \\ -4.18477 \\ 14.0207 \\ -0.74951 \\ -0.45175 \end{bmatrix}$$

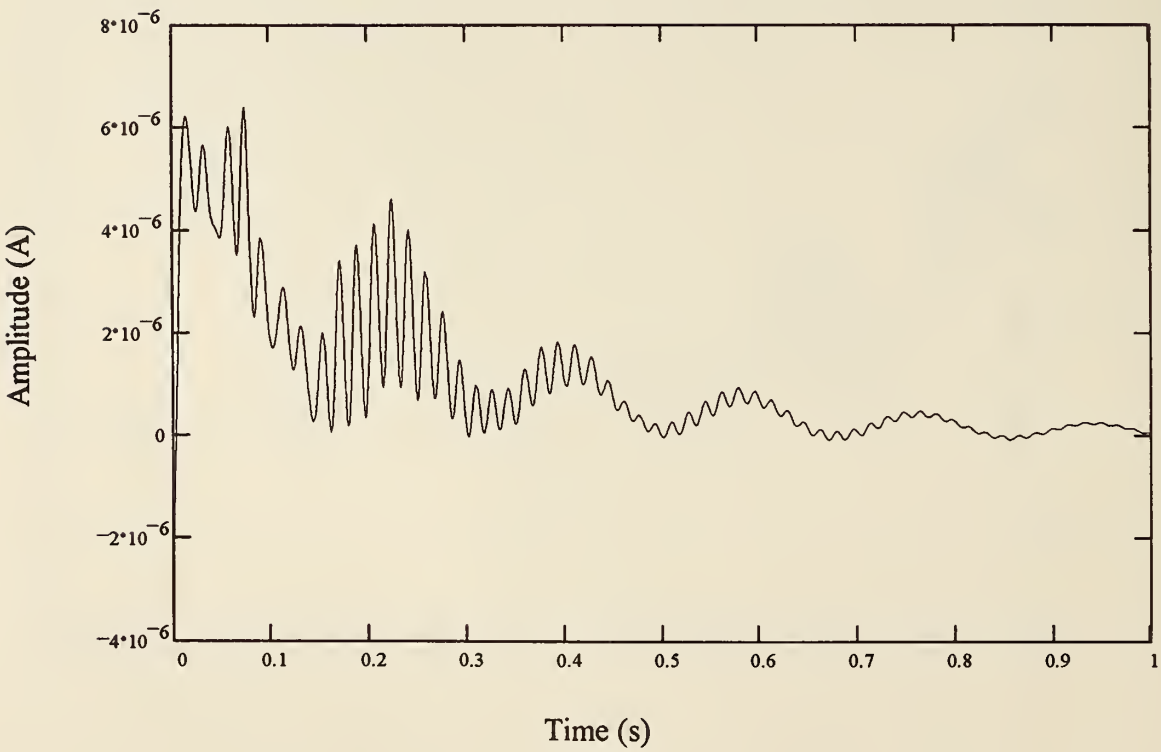

Figure 28. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 9 (1 -1 1 1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.40692 \\ -1.86731 \\ 3.65158 \\ 0.65855 \\ -1.25693 \\ -1.59282 \end{bmatrix} \quad d = \begin{bmatrix} 0.19072 \\ 0.28103 \\ 2.48475 \\ 2.61536 \\ 3.01165 \\ 0.62392 \end{bmatrix}$$

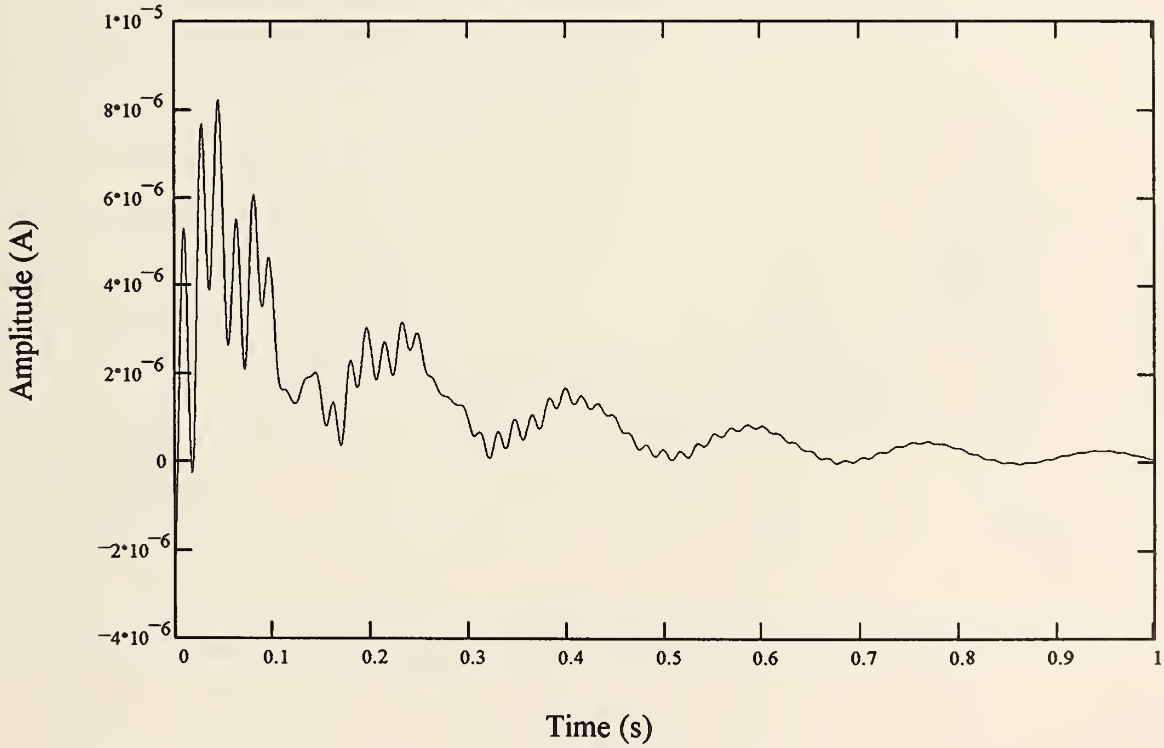

Figure 29. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 10 (1 -1 1 1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.40351 \\ -1.89505 \\ 2.3292 \\ -0.89662 \\ -3.03709 \\ 3.09605 \end{bmatrix} \quad d = \begin{bmatrix} 0.19944 \\ 0.13515 \\ 3.94496 \\ 2.73366 \\ 1.57389 \\ 0.69922 \end{bmatrix}$$

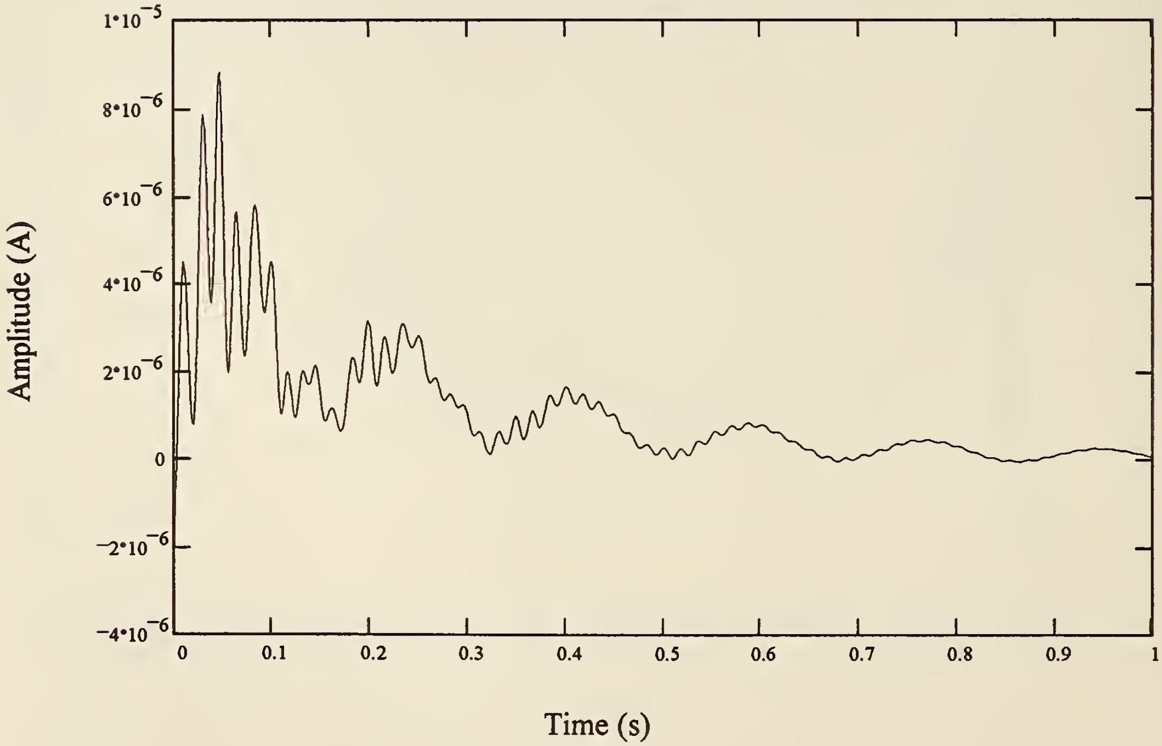

Figure 30. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 11 (1 -1 1 -1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.40504 \\ -1.88524 \\ 1.8911 \\ -5.33264 \\ 6.32339 \\ -1.40165 \end{bmatrix} \quad d = \begin{bmatrix} 0.19559 \\ 0.19705 \\ 4.74668 \\ 3.28547 \\ -0.25106 \\ 1.06178 \end{bmatrix}$$

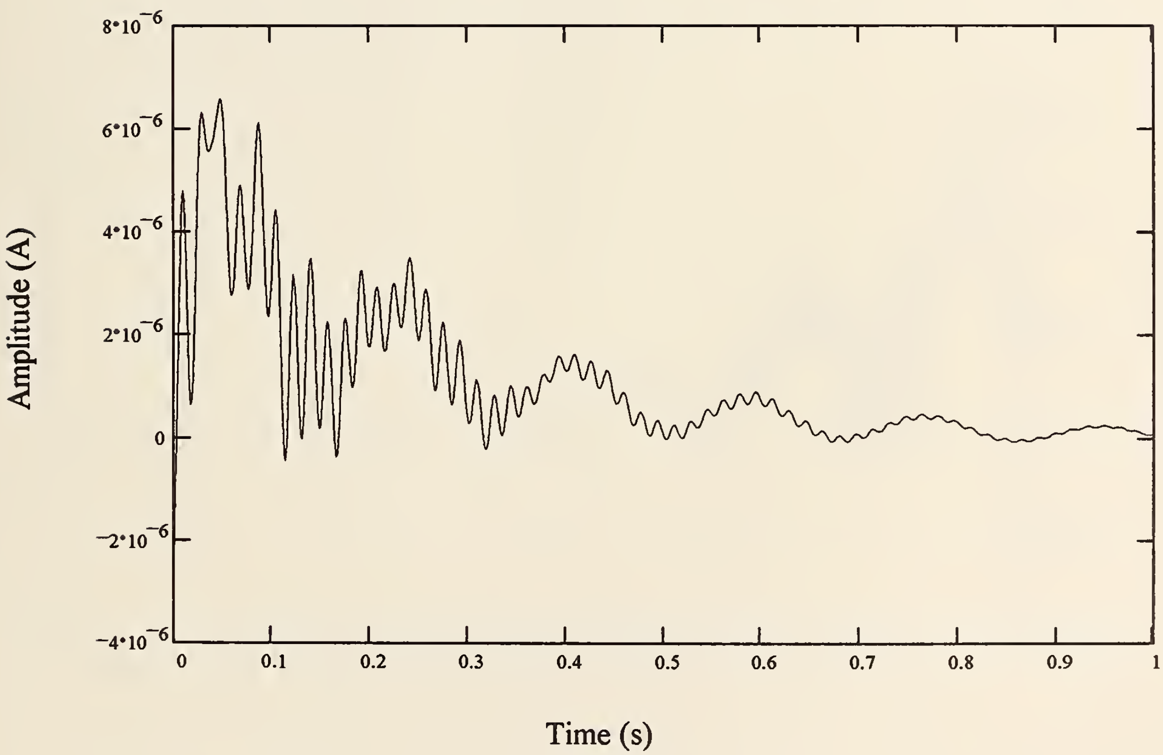

Figure 31. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 12 (1 -1 1 -1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.40153 \\ -1.90644 \\ -0.32217 \\ -6.68097 \\ 5.24997 \\ 3.25809 \end{bmatrix} \quad d = \begin{bmatrix} 0.20428 \\ 0.04951 \\ 5.28998 \\ -0.07639 \\ 4.05451 \\ -0.17234 \end{bmatrix}$$

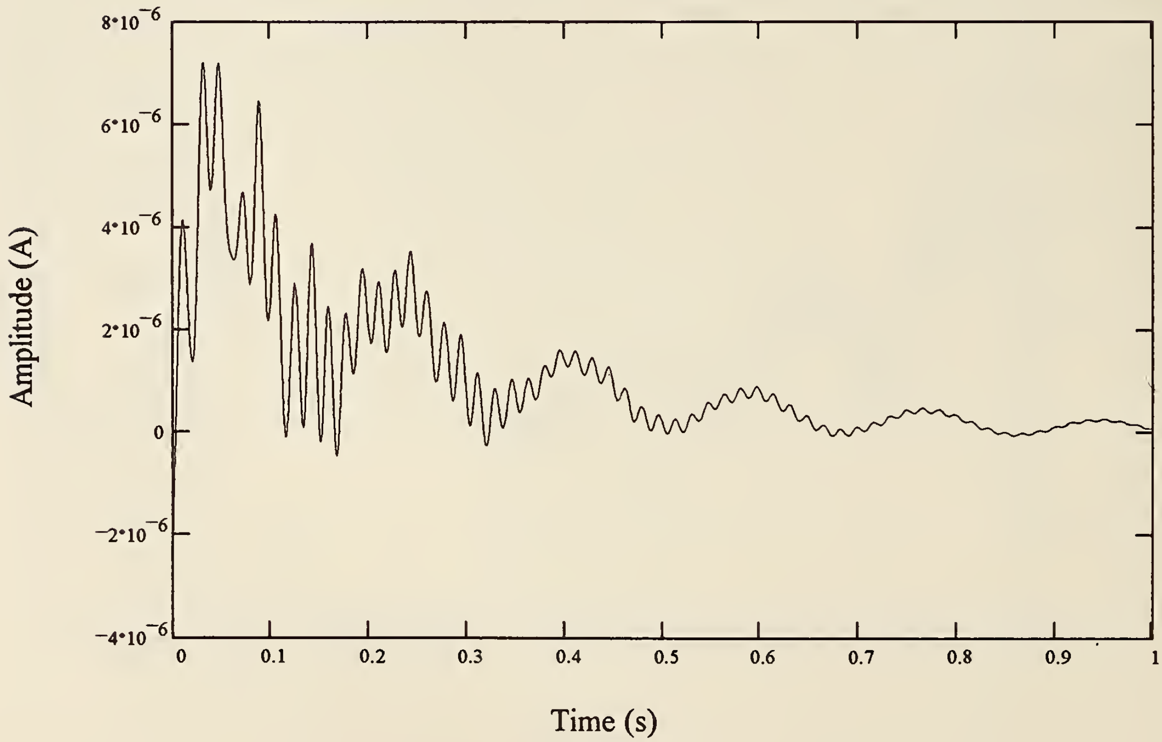

Figure 32. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 13 (1 -1 -1 1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.404 \\ -1.89401 \\ -4.9723 \\ 5.51118 \\ 2.3063 \\ -1.35518 \end{bmatrix} \quad d = \begin{bmatrix} 0.19822 \\ 0.14929 \\ 6.76684 \\ -1.99145 \\ 3.01794 \\ 1.11332 \end{bmatrix}$$

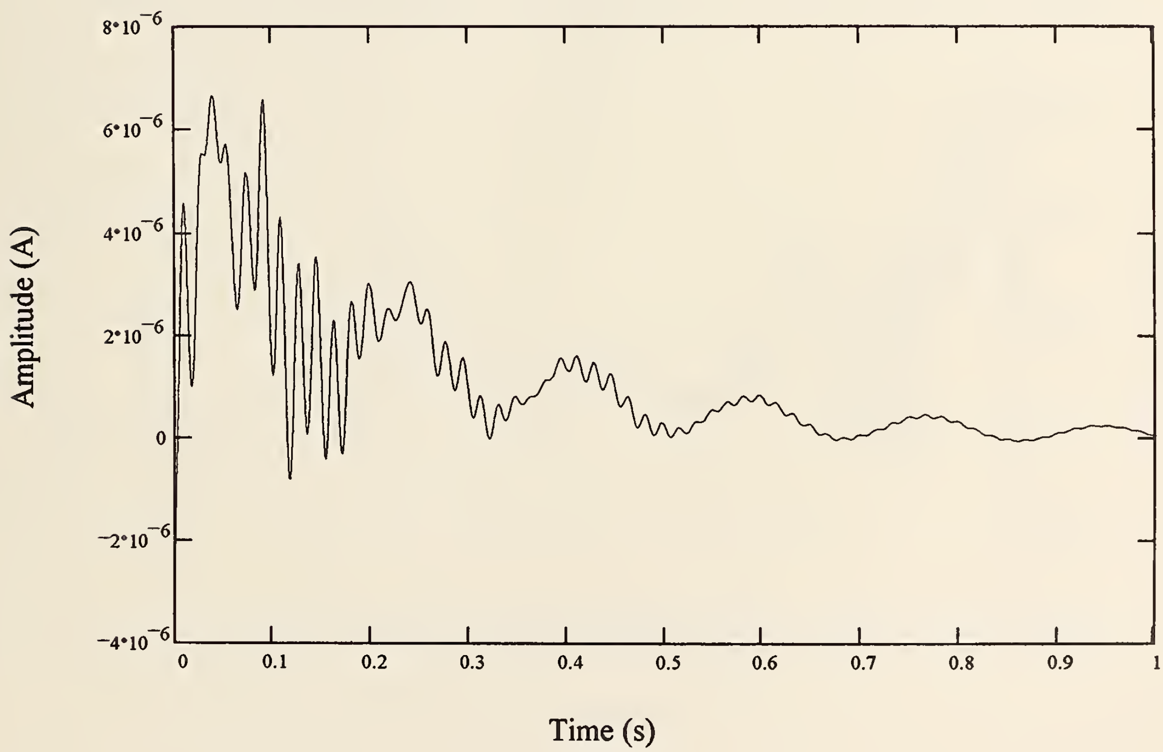

Figure 33. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 14 (1 -1 -1 1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \qquad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \qquad c = \begin{bmatrix} 0.40043 \\ -1.91148 \\ -7.65563 \\ 6.10452 \\ -0.17819 \\ 3.24034 \end{bmatrix} \qquad d = \begin{bmatrix} 0.20689 \\ 9.18581 \cdot 10^{-4} \\ 4.15386 \\ 1.345 \\ 3.97734 \\ -0.29987 \end{bmatrix}$$

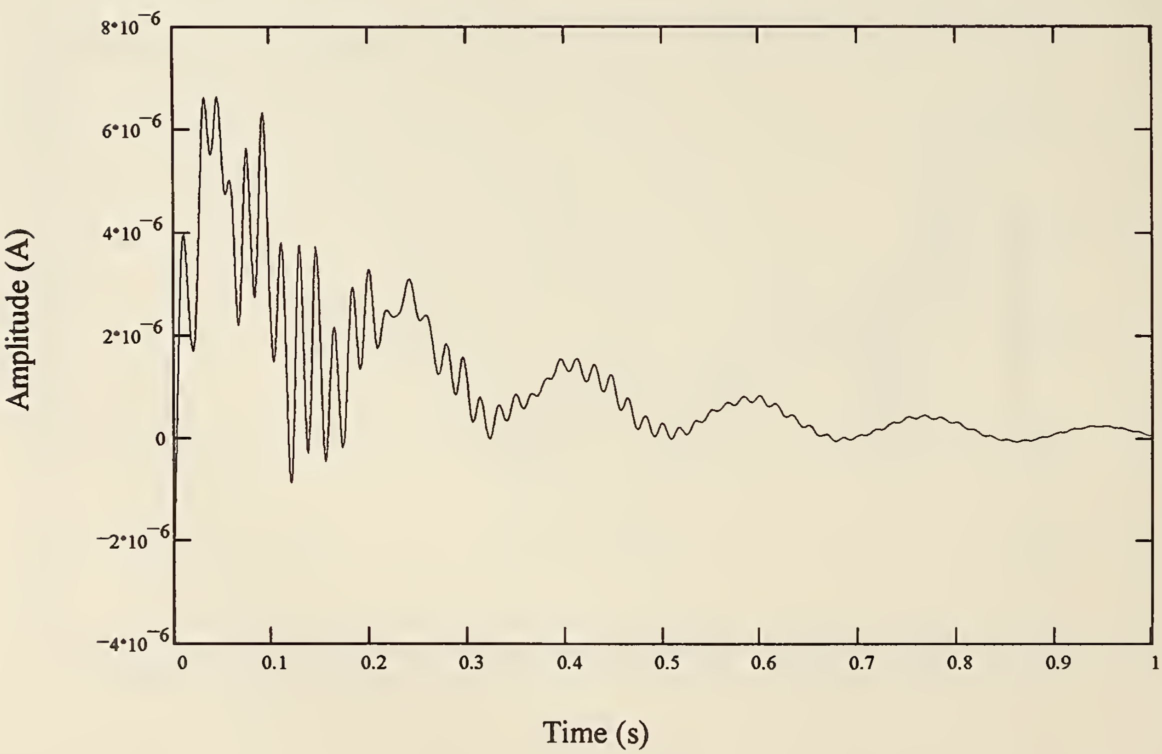

Figure 34. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 15 (1 -1 -1 -1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.40204 \\ -1.90604 \\ -9.14869 \\ 8.26044 \\ 3.42222 \\ -1.02997 \end{bmatrix} \quad d = \begin{bmatrix} 0.20307 \\ 0.06374 \\ 3.26628 \\ 10.8154 \\ -6.52229 \\ 1.47964 \end{bmatrix}$$

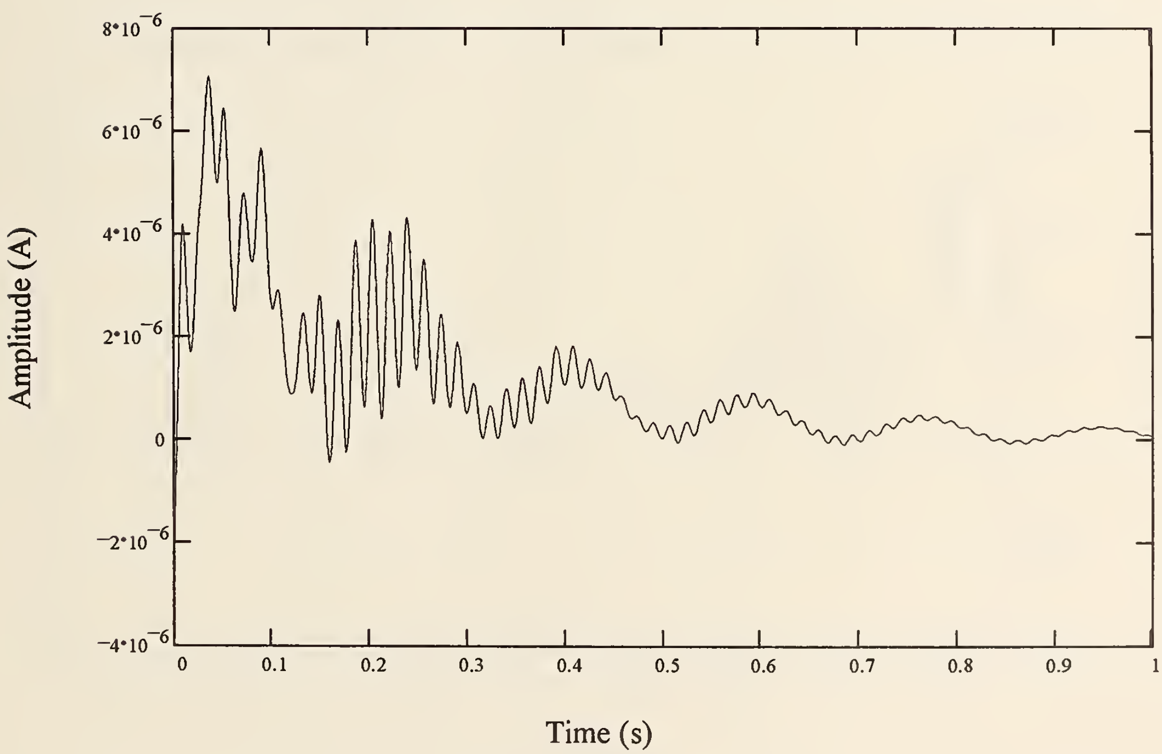

Figure 35. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 16 $\quad$ (1 -1 -1 -1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} 0.39836 \\ -1.91682 \\ -10.0286 \\ 1.28513 \\ 7.13995 \\ 3.122 \end{bmatrix} \quad d = \begin{bmatrix} 0.2117 \\ -0.08584 \\ -0.97624 \\ 14.4602 \\ -2.93732 \\ -1.20102 \end{bmatrix}$$

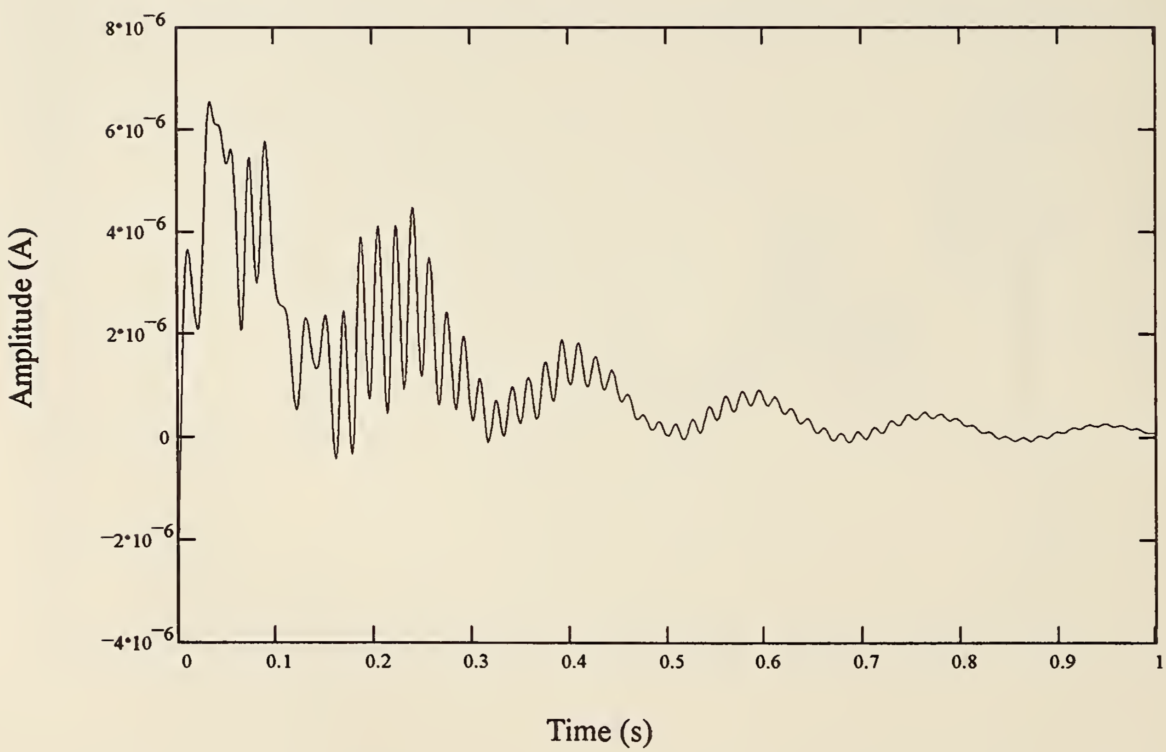

Figure 36. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 17 (-1 1 1 1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.69547 \\ 0.96452 \\ 3.02172 \\ 0.14293 \\ -1.77551 \\ -1.65819 \end{bmatrix} \quad d = \begin{bmatrix} 0.27288 \\ -0.01112 \\ 3.17528 \\ 2.67362 \\ 2.72541 \\ 0.39343 \end{bmatrix}$$

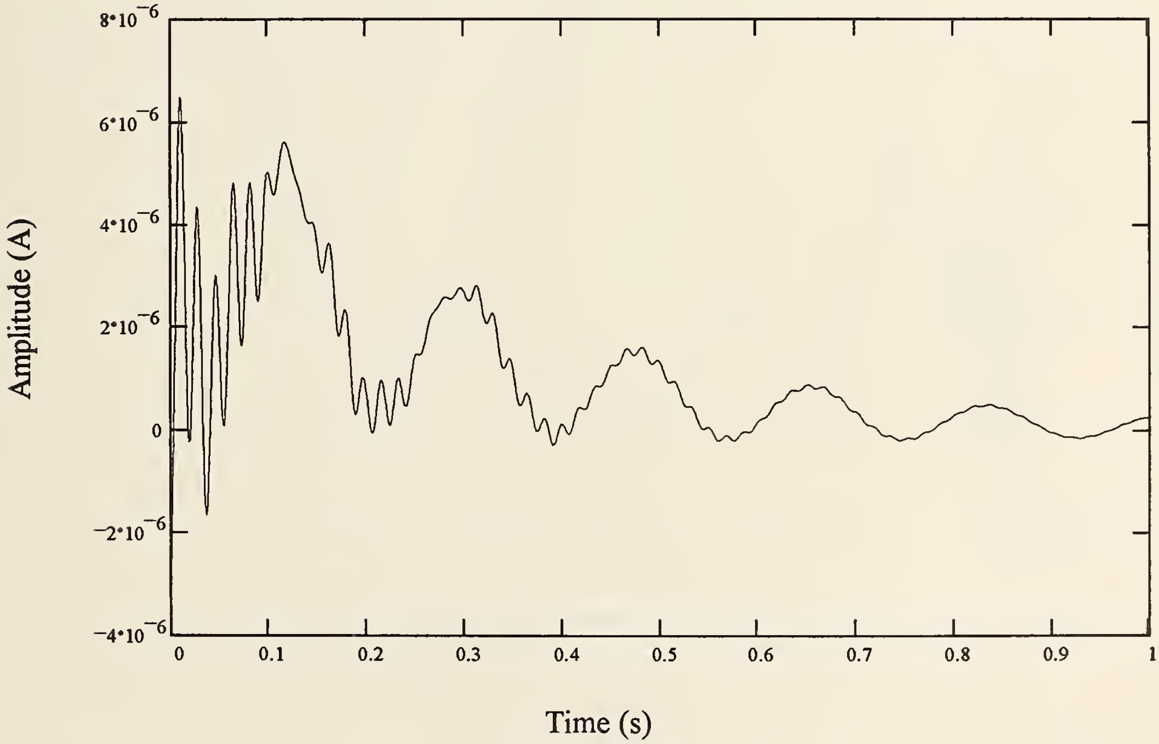

Figure 37. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 18 (-1 1 1 1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.70209 \\ 0.96832 \\ 1.43306 \\ -1.39478 \\ -3.26112 \\ 2.95661 \end{bmatrix} \quad d = \begin{bmatrix} 0.25879 \\ 0.06463 \\ 4.31475 \\ 2.49234 \\ 0.99484 \\ 1.12132 \end{bmatrix}$$

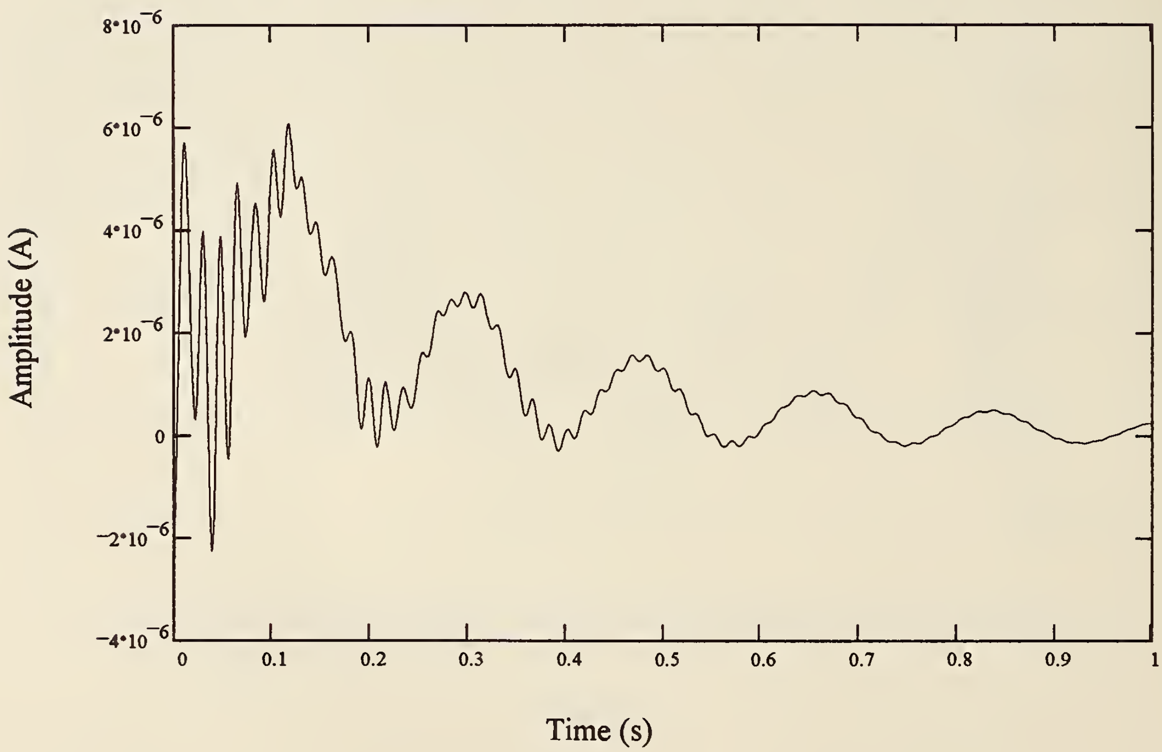

Figure 38. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 19 (-1 1 1 -1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.6992 \\ 0.96771 \\ 0.84013 \\ -5.82185 \\ 6.24387 \\ -1.53066 \end{bmatrix} \quad d = \begin{bmatrix} 0.26504 \\ 0.03263 \\ 5.00068 \\ 2.18413 \\ 0.89483 \\ 0.85201 \end{bmatrix}$$

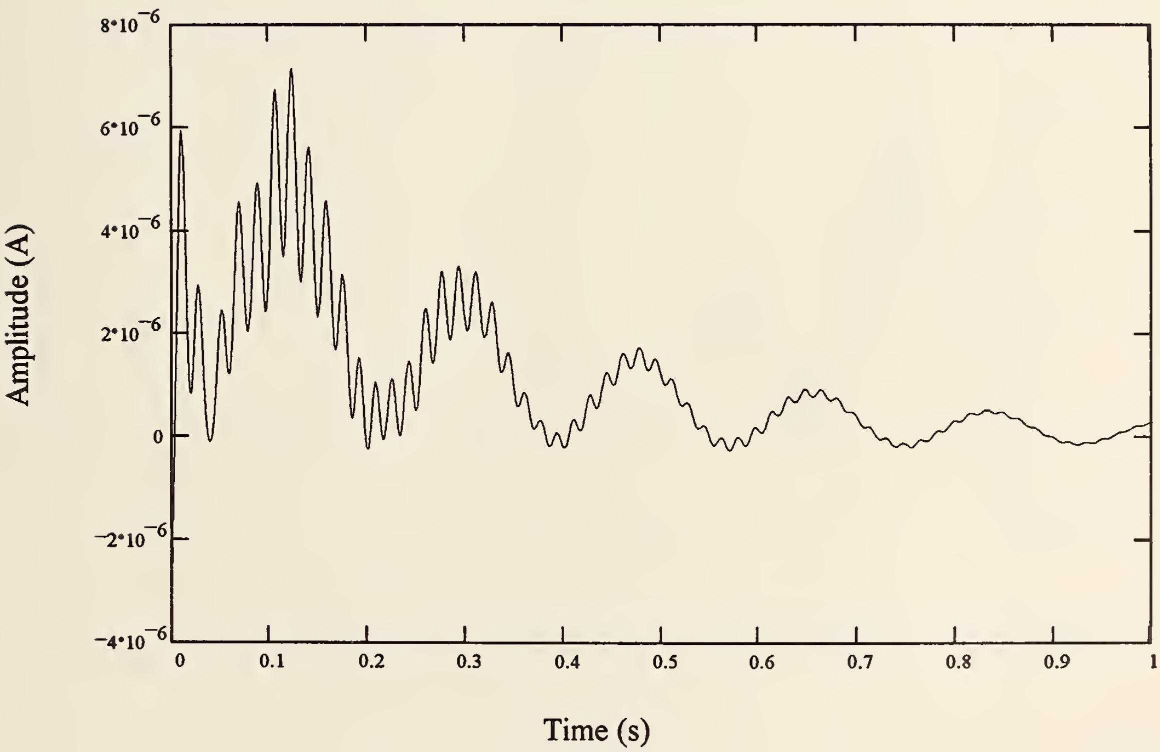

Figure 39. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 20 (-1 1 1 -1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.70566 \\ 0.96808 \\ -1.42067 \\ -6.49447 \\ 4.41478 \\ 3.23794 \end{bmatrix} \quad d = \begin{bmatrix} 0.25087 \\ 0.10877 \\ 5.06408 \\ -1.3483 \\ 4.92173 \\ 0.28416 \end{bmatrix}$$

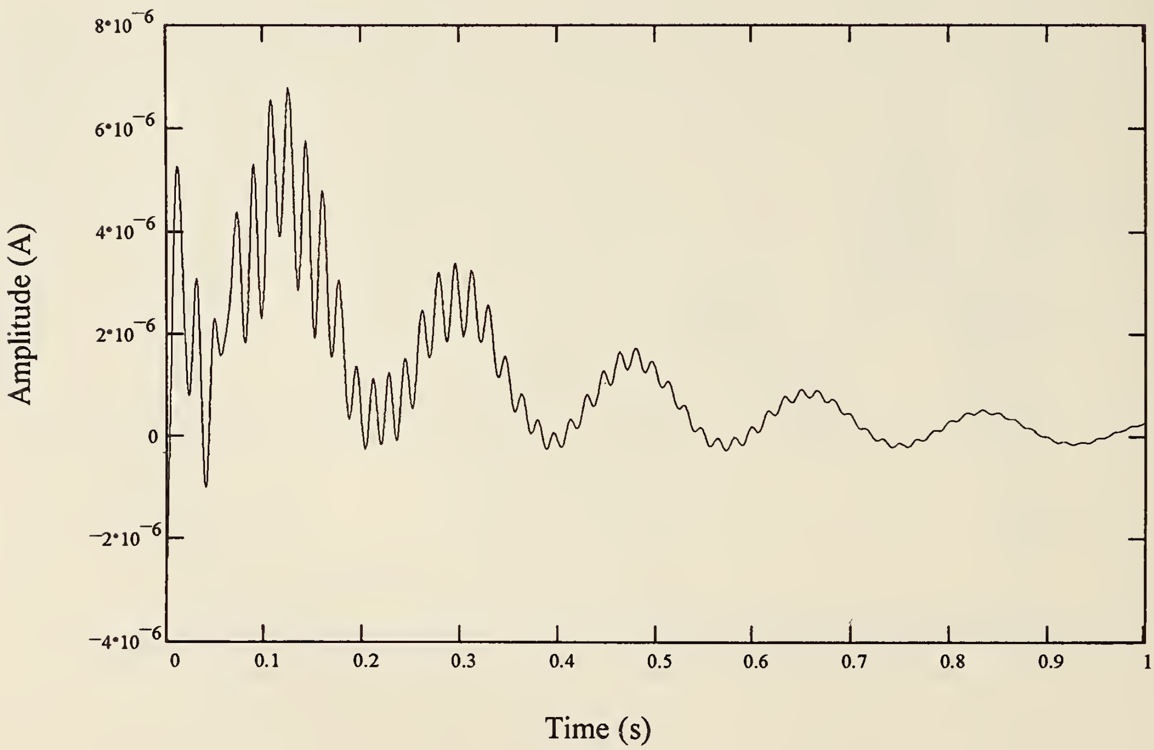

Figure 40. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 21 (-1 1 -1 1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.70119 \\ 0.96879 \\ -6.24092 \\ 5.74907 \\ 1.71625 \\ -1.492 \end{bmatrix} \quad d = \begin{bmatrix} 0.26078 \\ 0.05741 \\ 5.5226 \\ -0.88936 \\ 3.37449 \\ 0.90932 \end{bmatrix}$$

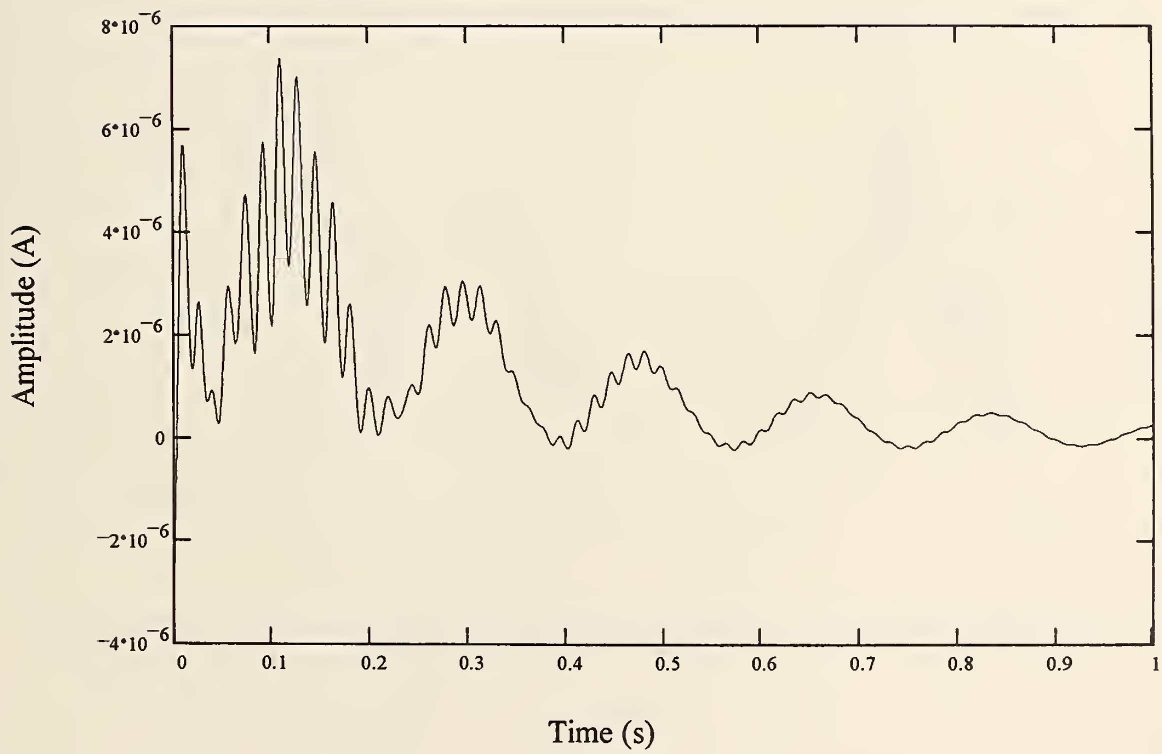

Figure 41. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 22 (-1 1 -1 1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.70757 \\ 0.96721 \\ -8.29652 \\ 5.69096 \\ -0.8923 \\ 3.23821 \end{bmatrix} \quad d = \begin{bmatrix} 0.24655 \\ 0.13371 \\ 2.42576 \\ 2.47436 \\ 3.86668 \\ 0.15589 \end{bmatrix}$$

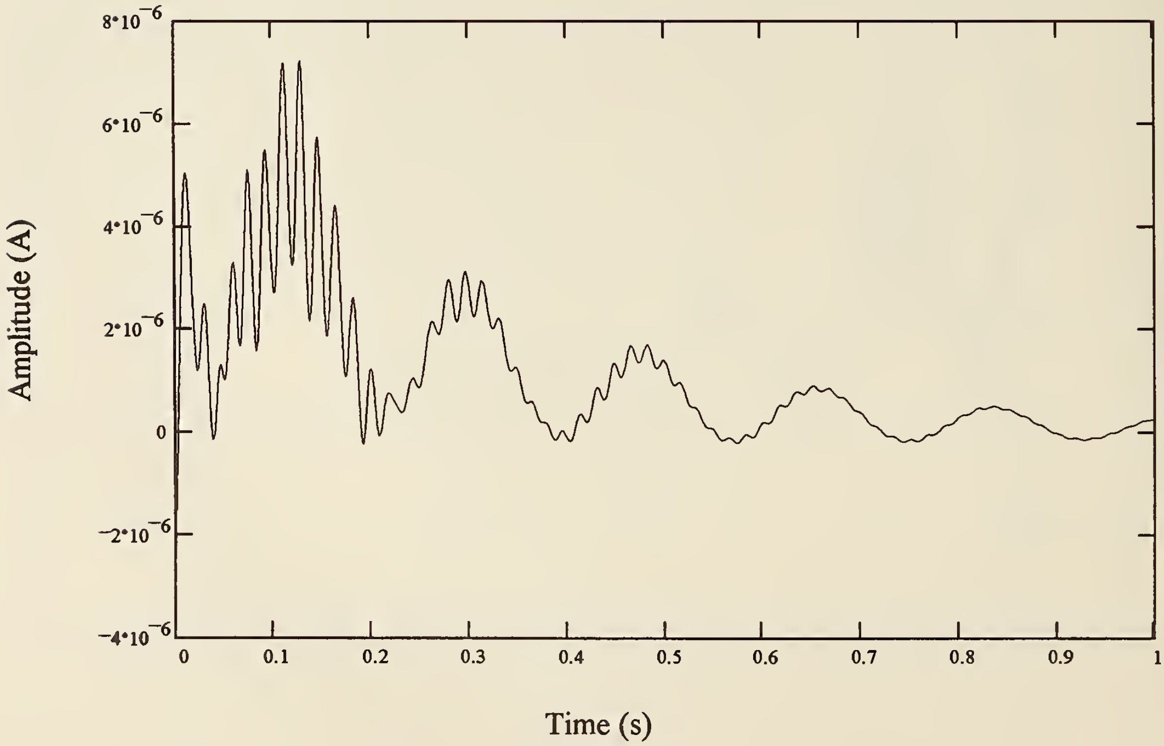

Figure 42. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 23 (-1 1 -1 -1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.70478 \\ 0.96887 \\ -9.55895 \\ 5.98564 \\ 4.53149 \\ -1.22226 \end{bmatrix} \quad d = \begin{bmatrix} 0.25287 \\ 0.10154 \\ 1.252 \\ 12.1121 \\ -5.77608 \\ 1.31601 \end{bmatrix}$$

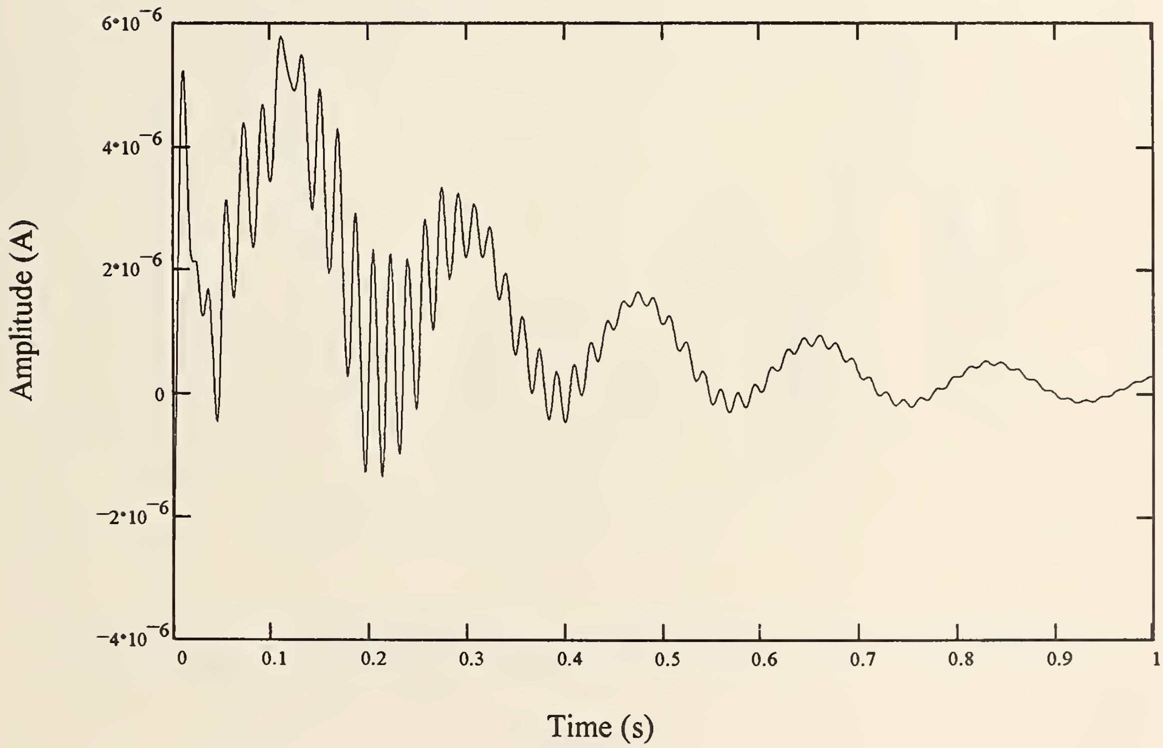

Figure 43. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 24 (-1 1 -1 -1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.711 \\ 0.96383 \\ -9.52379 \\ -1.50512 \\ 7.52899 \\ 3.24709 \end{bmatrix} \quad d = \begin{bmatrix} 0.23855 \\ 0.17798 \\ -3.04781 \\ 14.3331 \\ -1.59108 \\ -0.74954 \end{bmatrix}$$

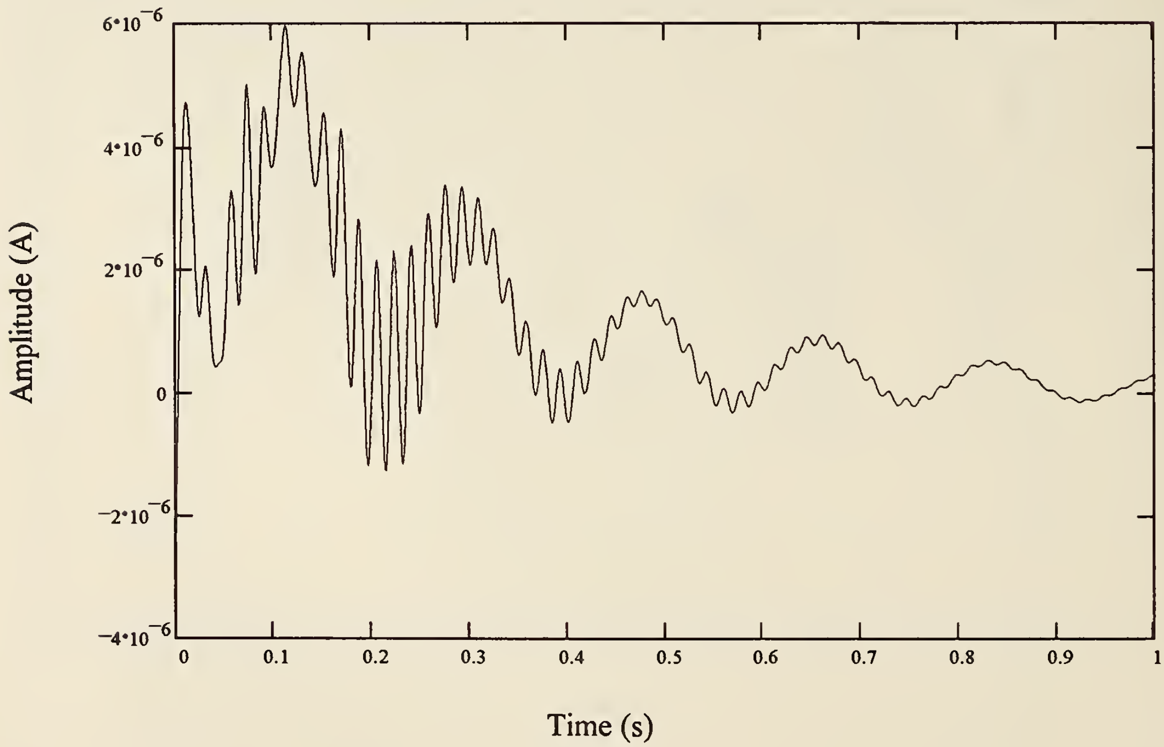

Figure 44. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 25 (-1 -1 1 1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.75246 \\ -1.69662 \\ 3.94166 \\ 0.95849 \\ -0.91827 \\ -1.5328 \end{bmatrix} \quad d = \begin{bmatrix} 0.11664 \\ 0.94645 \\ 2.02226 \\ 2.52988 \\ 3.1365 \\ 0.76698 \end{bmatrix}$$

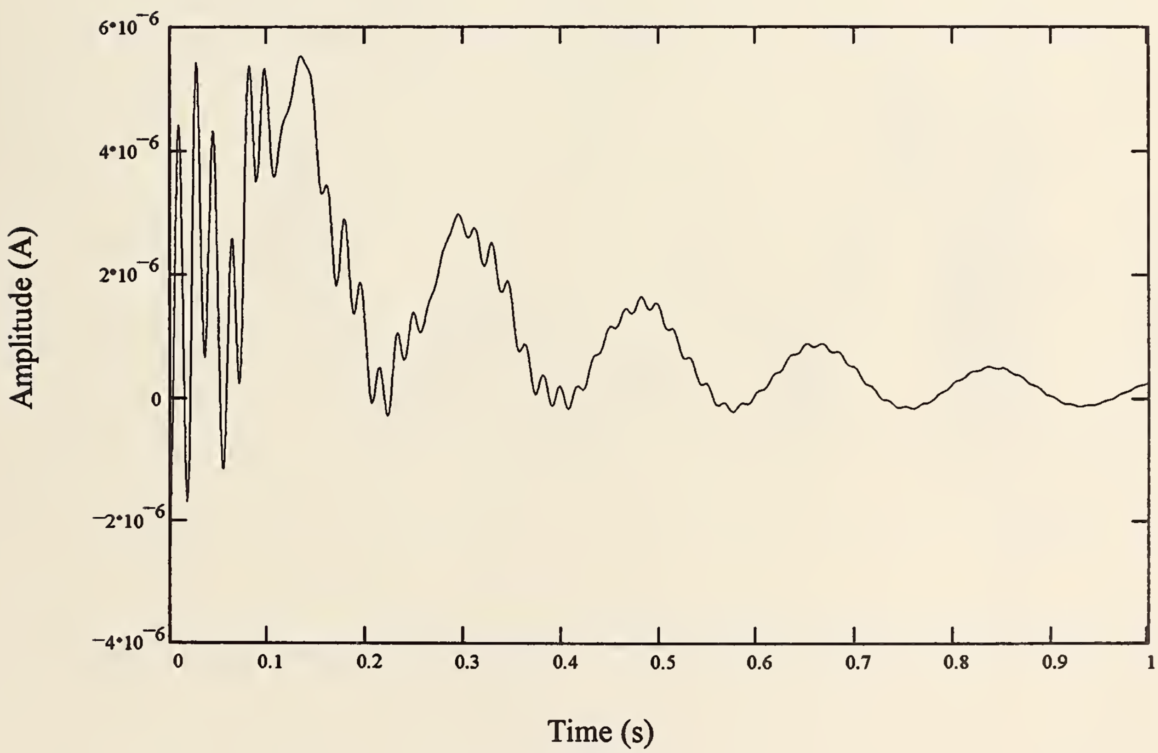

Figure 45. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 26 (-1 -1 1 1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.75591 \\ -1.77613 \\ 2.80575 \\ -0.57744 \\ -2.84901 \\ 3.15275 \end{bmatrix} \quad d = \begin{bmatrix} 0.10115 \\ 0.816 \\ 3.63908 \\ 2.8275 \\ 1.90216 \\ 0.417 \end{bmatrix}$$

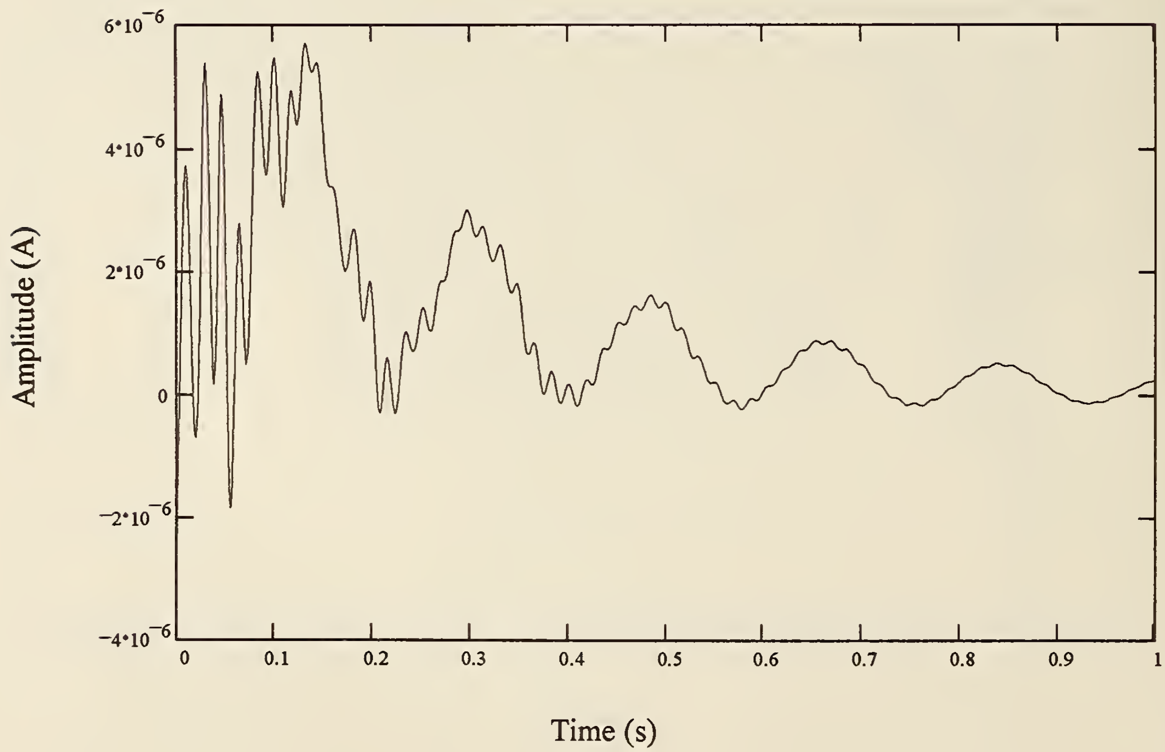

Figure 46. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 27 (-1 -1 1 -1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.75442 \\ -1.74429 \\ 2.46868 \\ -4.9338 \\ 6.26617 \\ -1.30235 \end{bmatrix} \quad d = \begin{bmatrix} 0.10801 \\ 0.87207 \\ 4.49121 \\ 3.89008 \\ -0.94864 \\ 1.18657 \end{bmatrix}$$

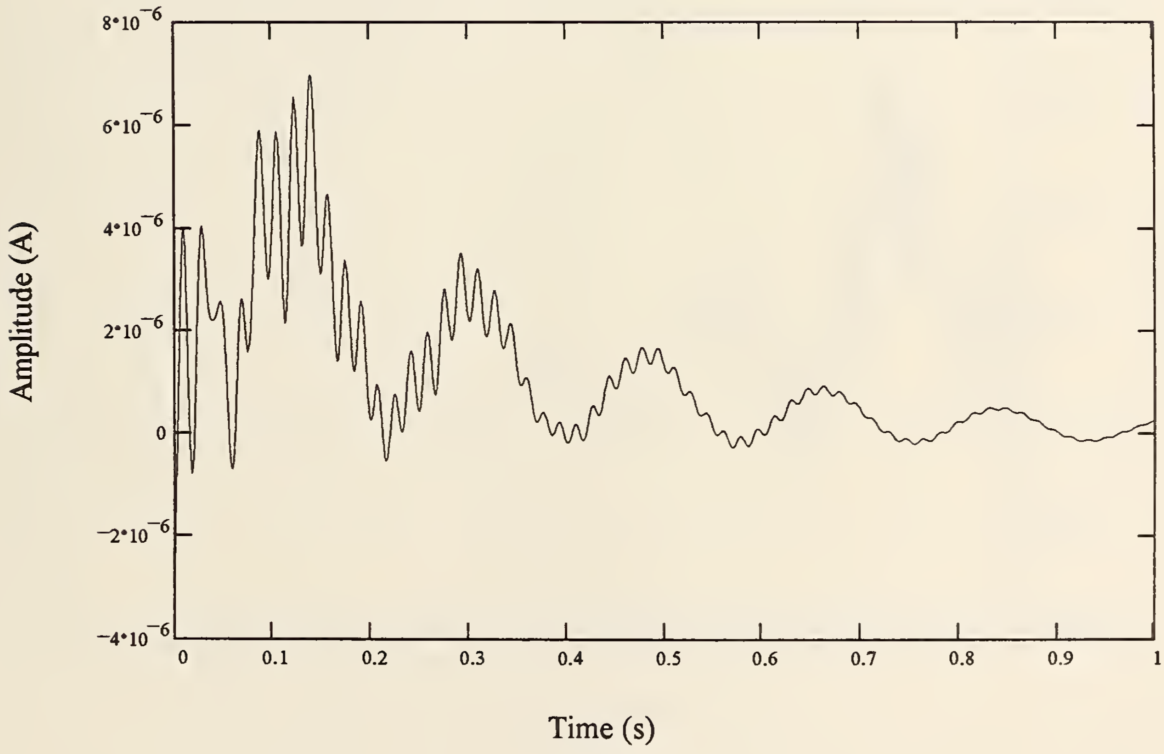

Figure 47. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 28 (-1 -1 1 -1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.7577 \\ -1.8181 \\ 0.33273 \\ -6.66588 \\ 5.67353 \\ 3.23541 \end{bmatrix} \quad d = \begin{bmatrix} 0.09247 \\ 0.73765 \\ 5.30539 \\ 0.6961 \\ 3.45547 \\ -0.46737 \end{bmatrix}$$

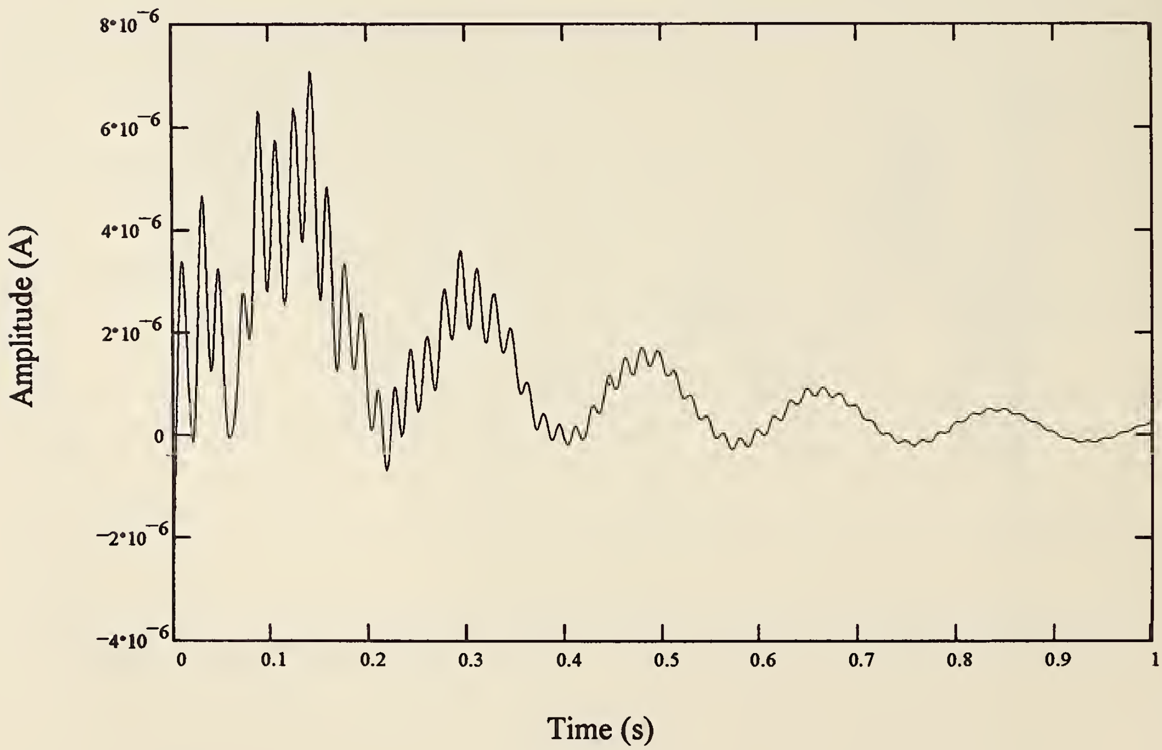

Figure 48. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 29 (-1 -1 -1 1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.75545 \\ -1.77001 \\ -4.11355 \\ 5.26127 \\ 2.62905 \\ -1.25131 \end{bmatrix} \quad d = \begin{bmatrix} 0.10333 \\ 0.82925 \\ 7.34982 \\ -2.62133 \\ 2.74901 \\ 1.23379 \end{bmatrix}$$

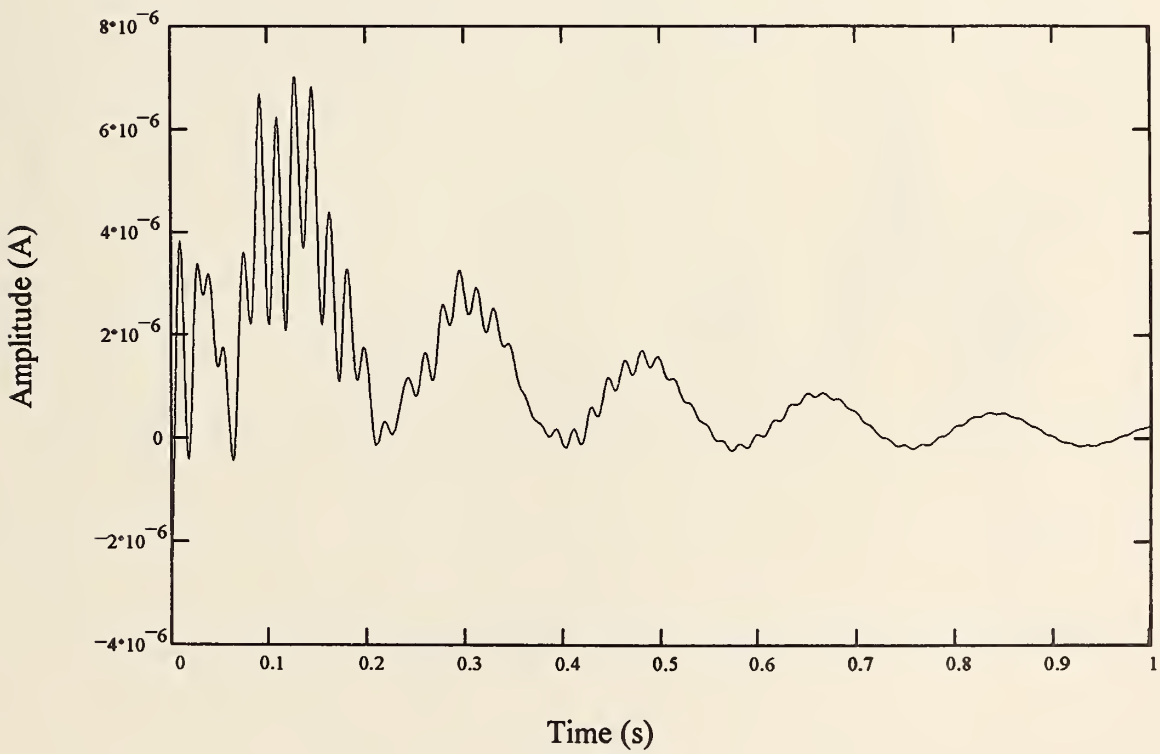

Figure 49. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 30 (-1 -1 -1 1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.75863 \\ -1.84053 \\ -7.10726 \\ 6.23813 \\ 0.26215 \\ 3.20615 \end{bmatrix} \quad d = \begin{bmatrix} 0.08776 \\ 0.69268 \\ 5.08033 \\ 0.6346 \\ 3.97849 \\ -0.59301 \end{bmatrix}$$

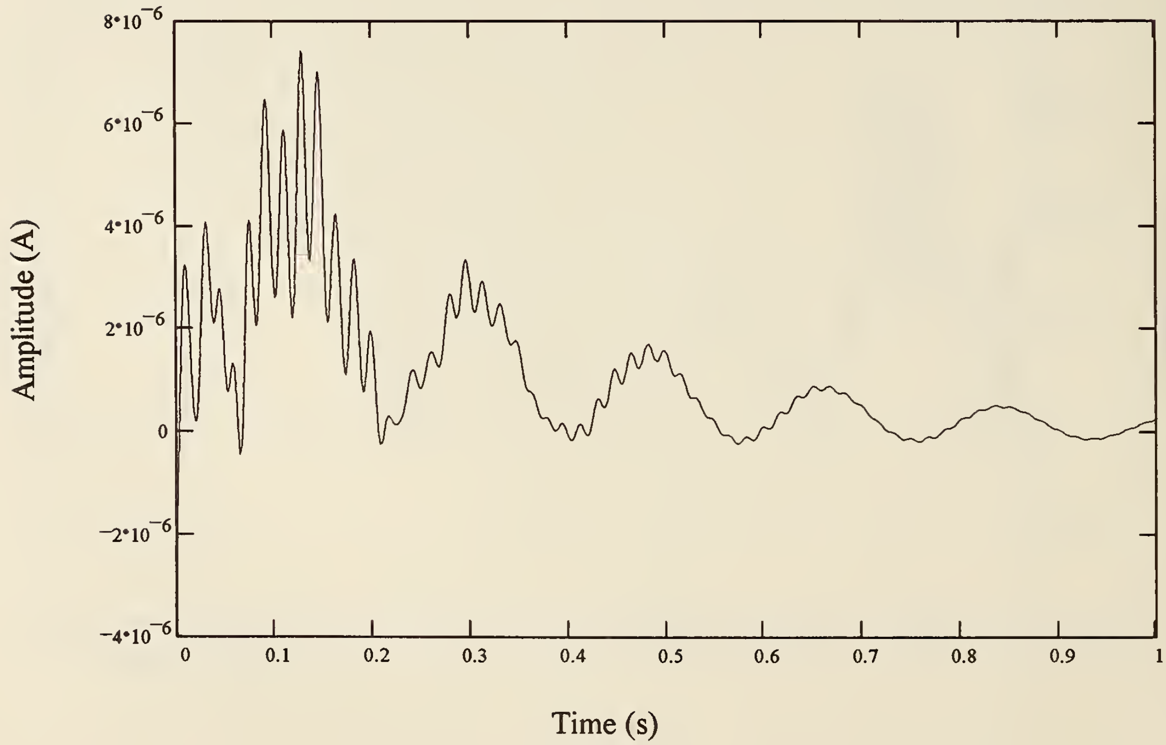

Figure 50. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 31 (-1 -1 -1 -1 1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.75727 \\ -1.81256 \\ -8.70308 \\ 9.48096 \\ 2.68553 \\ -0.89359 \end{bmatrix} \quad d = \begin{bmatrix} 0.09466 \\ 0.7512 \\ 4.38127 \\ 9.82194 \\ -6.87006 \\ 1.56982 \end{bmatrix}$$

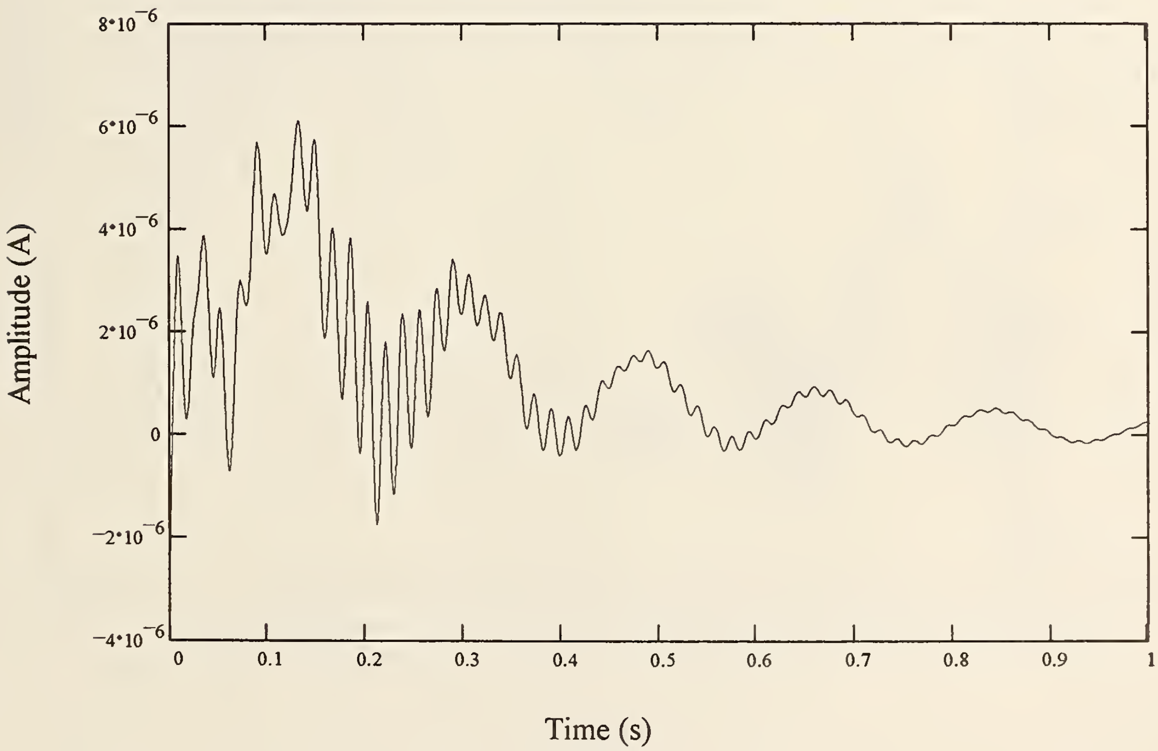

Figure 51. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

curve = 32   (-1 -1 -1 -1 -1)

$$\alpha = \begin{bmatrix} 2.64345 \\ 8.24428 \\ 8.00117 \\ 9.38598 \\ 4.81063 \\ 9.62187 \end{bmatrix} \quad \beta = \begin{bmatrix} 35.2426 \\ 123.482 \\ 334.749 \\ 357.312 \\ 373.079 \\ 453.699 \end{bmatrix} \quad c = \begin{bmatrix} -0.76027 \\ -1.87708 \\ -10.103 \\ 2.95189 \\ 6.78208 \\ 3.00635 \end{bmatrix} \quad d = \begin{bmatrix} 0.07904 \\ 0.61105 \\ 0.26698 \\ 14.2599 \\ -3.71262 \\ -1.48148 \end{bmatrix}$$

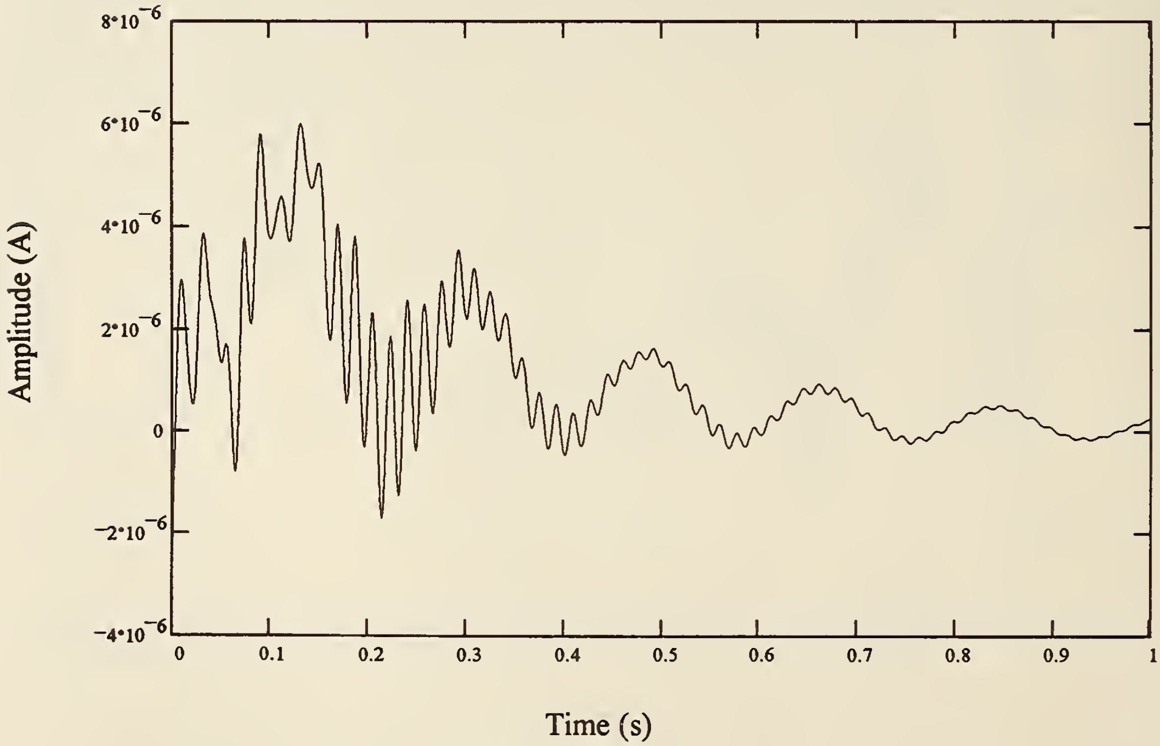

Figure 52. Time response of the system to the impulse excitation given in eq (37), for a nonminimum phase case.

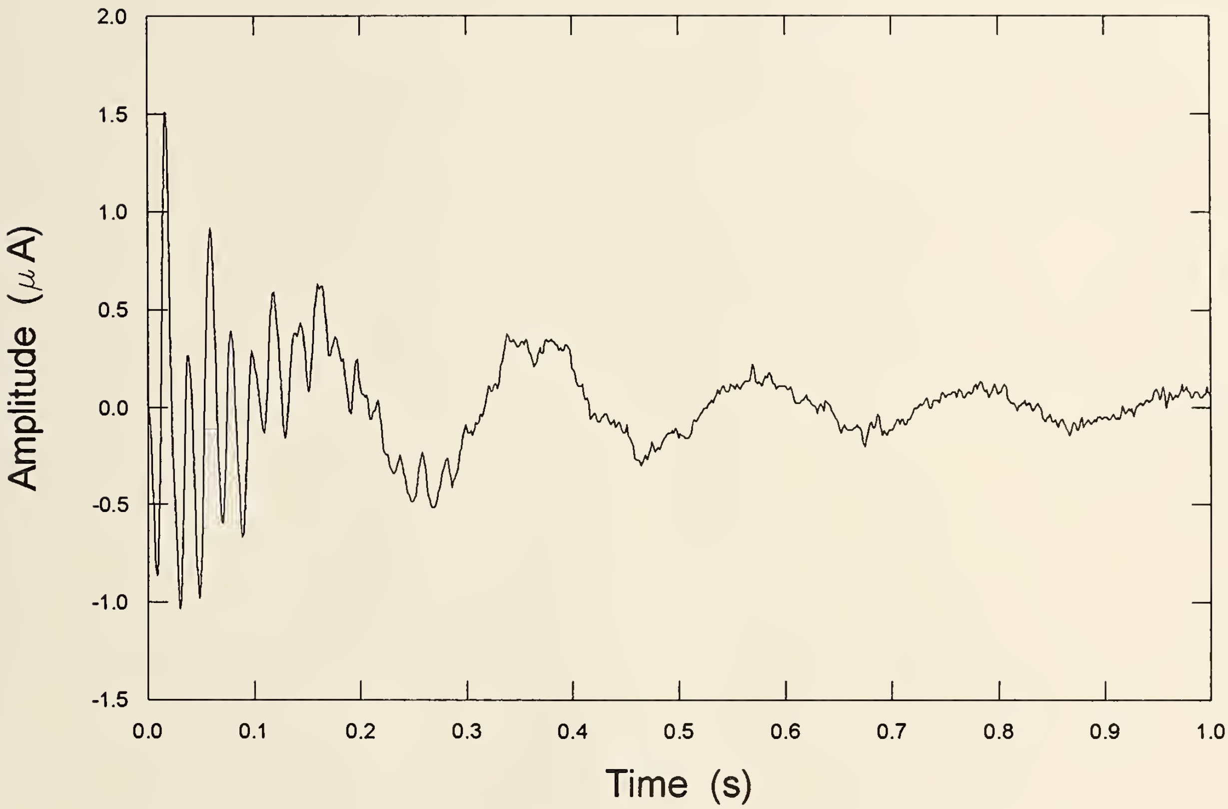

Figure 53. Measured time response of C26, after normalization, to the same impulse excitation.

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