NIST Technical Note 1298

NIST Measurement Service for
DC Standard Resistors

Ronald F. Dziuba, Paul A. Boynton, Randolph E. Elmquist,
Dean G. Jarrett, Theodore P. Moore, and Jack D. Neal
The National Institute of Standards and Technology was established in 1988 by Congress to “assist industry in the development of technology . . . needed to improve product quality, to modernize manufacturing processes, to ensure product reliability . . . and to facilitate rapid commercialization . . . of products based on new scientific discoveries.”

NIST, originally founded as the National Bureau of Standards in 1901, works to strengthen U.S. industry’s competitiveness; advance science and engineering; and improve public health, safety, and the environment. One of the agency’s basic functions is to develop, maintain, and retain custody of the national standards of measurement, and provide the means and methods for comparing standards used in science, engineering, manufacturing, commerce, industry, and education with the standards adopted or recognized by the Federal Government.

As an agency of the U.S. Commerce Department’s Technology Administration, NIST conducts basic and applied research in the physical sciences and engineering and performs related services. The Institute does generic and precompetitive work on new and advanced technologies. NIST’s research facilities are located at Gaithersburg, MD 20899, and at Boulder, CO 80303. Major technical operating units and their principal activities are listed below. For more information contact the Public Inquiries Desk, 301-975-3058.

<table>
<thead>
<tr>
<th>Technology Services</th>
<th>Manufacturing Engineering Laboratory</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Manufacturing Technology Centers Program</td>
<td>• Precision Engineering</td>
</tr>
<tr>
<td>• Standards Services</td>
<td>• Automated Production Technology</td>
</tr>
<tr>
<td>• Technology Commercialization</td>
<td>• Robot Systems</td>
</tr>
<tr>
<td>• Measurement Services</td>
<td>• Factory Automation</td>
</tr>
<tr>
<td>• Technology Evaluation and Assessment</td>
<td>• Fabrication Technology</td>
</tr>
<tr>
<td>• Information Services</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Electronics and Electrical Engineering Laboratory</th>
<th>Materials Science and Engineering Laboratory</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Microelectronics</td>
<td>• Intelligent Processing of Materials</td>
</tr>
<tr>
<td>• Law Enforcement Standards</td>
<td>• Ceramics</td>
</tr>
<tr>
<td>• Electricity</td>
<td>• Materials Reliability$^1$</td>
</tr>
<tr>
<td>• Semiconductor Electronics</td>
<td>• Polymers</td>
</tr>
<tr>
<td>• Electromagnetic Fields$^1$</td>
<td>• Metallurgy</td>
</tr>
<tr>
<td>• Electromagnetic Technology$^1$</td>
<td>• Reactor Radiation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chemical Science and Technology Laboratory</th>
<th>Building and Fire Research Laboratory</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Biotechnology</td>
<td>• Structures</td>
</tr>
<tr>
<td>• Chemical Engineering$^1$</td>
<td>• Building Materials</td>
</tr>
<tr>
<td>• Chemical Kinetics and Thermodynamics</td>
<td>• Building Environment</td>
</tr>
<tr>
<td>• Inorganic Analytical Research</td>
<td>• Fire Science and Engineering</td>
</tr>
<tr>
<td>• Organic Analytical Research</td>
<td>• Fire Measurement and Research</td>
</tr>
<tr>
<td>• Process Measurements</td>
<td></td>
</tr>
<tr>
<td>• Surface and Microanalysis Science</td>
<td></td>
</tr>
<tr>
<td>• Thermophysics$^2$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Physics Laboratory</th>
<th>Computer Systems Laboratory</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Electron and Optical Physics</td>
<td>• Information Systems Engineering</td>
</tr>
<tr>
<td>• Atomic Physics</td>
<td>• Systems and Software Technology</td>
</tr>
<tr>
<td>• Molecular Physics</td>
<td>• Computer Security</td>
</tr>
<tr>
<td>• Radiometric Physics</td>
<td>• Systems and Network Architecture</td>
</tr>
<tr>
<td>• Quantum Metrology</td>
<td>• Advanced Systems</td>
</tr>
<tr>
<td>• Ionizing Radiation</td>
<td></td>
</tr>
<tr>
<td>• Time and Frequency$^1$</td>
<td></td>
</tr>
<tr>
<td>• Quantum Physics$^1$</td>
<td></td>
</tr>
</tbody>
</table>

$^1$At Boulder, CO 80303.
$^2$Some elements at Boulder, CO 80303.
NIST Technical Note 1298

NIST Measurement Service for DC Standard Resistors

Ronald F. Dziuba
Paul A. Boynton
Randolph E. Elmquist
Dean G. Jarrett
Theodore P. Moore
Jack D. Neal

Electronics and Electrical Engineering Laboratory
National Institute of Standards and Technology
Gaithersburg, MD 20899

November 1992

U.S. Department of Commerce
Barbara Hackman Franklin, Secretary

Technology Administration
Robert M. White, Under Secretary for Technology
National Institute of Standards and Technology
John W. Lyons, Director
# CONTENTS

1. INTRODUCTION ................................................................. 1

2. DESCRIPTION OF SERVICE ................................................. 2
   2.1 Special Standard Resistors (1 Ω and 10 kΩ) ....................... 2
   2.2 Standard Resistors (10⁴ Ω to 10⁶ Ω) ............................. 3
   2.3 High-Value Standard Resistors (10⁷ Ω to 10¹² Ω) ............... 3
   2.4 Resistance MAP Service ........................................... 3
   2.5 Special Resistance Measurements ................................. 4

3. MEASUREMENT METHODS ................................................. 4

4. DIRECT CURRENT COMPARATOR SYSTEMS .......................... 5
   4.1 Theory of Operation .............................................. 6
   4.2 DCC Potentiometer System for 1 Ω Measurements ............... 8
      4.2.1 Description of System ..................................... 8
   4.3 DCC Potentiometer for 10 Ω and 100 Ω Measurements ......... 12
      4.3.1 Description of System ..................................... 12
   4.4 DCC Resistance Bridge ........................................ 13
      4.4.1 Description of System ..................................... 13
      4.4.2 Measurement Parameters and Design ....................... 14

5. RESISTANCE-RATIO BRIDGES ....................................... 15
   5.1 Theory of Operation .............................................. 16
   5.2 Modified Kelvin Bridge for 10 kΩ Measurements .............. 18
   5.3 Guarded Wheatstone Bridge .................................... 20

6. RING METHOD .............................................................. 22
   6.1 Theory of Operation .............................................. 22
      6.1.1 First Subset of Voltage Measurements ..................... 23
      6.1.2 Second Subset of Voltage Measurements ................... 25
      6.1.3 Third Subset of Voltage Measurements ..................... 25
   6.2 Data Analysis .................................................. 25
   6.3 Guard Network .................................................. 27

7. TERAOHMMETER .......................................................... 27
   7.1 Description of System ........................................... 28

8. RESISTANCE SCALING .................................................. 28
   8.1 Quantized Hall Resistance Scaling ................................ 29
   8.2 Transfer Standards .............................................. 30
   8.3 DCC Ratios ...................................................... 34
   8.4 Teraohmmeter Ratios ............................................. 34
NIST MEASUREMENT SERVICE FOR DC STANDARD RESISTORS

Ronald F. Dziuba, Paul A. Boynton, Randolph E. Elmquist, Dean G. Jarrett, Theodore P. Moore, and Jack D. Neal

Abstract - At the National Institute of Standards and Technology (NIST), the U.S. representation of the ohm is based on the quantum Hall effect, and it is maintained and disseminated at various resistance levels by working reference groups of standards. This document describes the measurement systems and procedures used to calibrate standard resistors of nominal decade values in the resistance range from $10^4 \Omega$ to $10^{12} \Omega$. Resistance scaling techniques used to assign values to the working standards are discussed. Also included is an assessment of the calibration uncertainties at each resistance level.

1. INTRODUCTION

Since January 1, 1990, the U.S. representation of the ohm has been based on the quantum Hall effect in which a resistance is related to the ratio of fundamental constants $h/e^2$ [1,2]. The quantized Hall resistance, $R_H$, is defined as the quotient of the Hall voltage $V_H$ of the $i^{th}$ plateau to the current $I$ in the Hall device and is given by

$$R_H(i) = V_H(i)/I = R_K/i,$$

where the von Klitzing constant $R_K$ is believed to be equal to $h/e^2$, and $i$ is an integer of the quantum Hall state. The value of the U.S. representation of the ohm is consistent with the conventional value of the von Klitzing constant, i.e.,

$$R_{K,90} = 25,812.807 \, \Omega,$$

exactly, adopted internationally for use in representing the ohm beginning January 1, 1990. This conventional value is believed to be consistent with the SI $\Omega$ to within 0.005 $\Omega$ which is the value assigned to be the combined standard uncertainty, corresponding to a relative uncertainty of 0.2 ppm. Because this uncertainty is the same for all national laboratories and is not relevant for traceability to national standards, it is not included as a component of the uncertainties given in NIST Reports of Calibration for standard resistors. However, its existence must be taken into account when the utmost consistency between electrical and non-electrical measurements of the same physical quantity is required.

For precision measurements using the quantum Hall effect, the integer $i$ is usually chosen to be either 2 or 4 resulting in quantized Hall resistances of $12,906.4035 \, \Omega$ or $6,453.20175 \, \Omega$ [3]. These odd resistance values, along with the complexity of the experiment, do not lend themselves to the routine support of the calibration of standard resistors of nominal decade values. Therefore, at NIST the ohm is maintained for calibration purposes at the 1 $\Omega$ level via a group of Thomas-type resistors whose predicted mean value is checked periodically against the quantum Hall effect and adjusted, if necessary.
NIST provides a calibration service for standard resistors of nominal decade values (i.e., $R = 10^i$ where $i$ is an integer) in the range between $10^4 \ \Omega$ and $10^{12} \ \Omega$. To provide this wide-ranging calibration service, NIST maintains a working reference group at each nominal decade value. The working reference groups are calibrated against the 1 $\Omega$ reference group using special ratio techniques. A customer's resistor is calibrated against the NIST working reference group in a manual or an automated comparison measurement system using either a resistive-ratio bridge, a flux-balancing type of current comparator, or a capacitance-discharge system. The specific measurement system used depends upon the value of the resistor being calibrated and on the accuracy required or warranted. NIST procedures follow recognized dc measurement practices such as current reversal, minimum power dissipation consistent with desired resolution and accuracy, and the use of check standards and statistical data analysis techniques to monitor the operation of the measurement systems.

2. DESCRIPTION OF SERVICE

NIST provides a calibration service for standard resistors with nominal decade values in the range between $10^4 \ \Omega$ and $10^{12} \ \Omega$. In addition to this regular calibration service, NIST offers a resistance Measurement Assurance Program (MAP) service at the 1 $\Omega$ and 10 k$\Omega$ levels. Other special resistance measurements are undertaken if they require the unique capabilities of NIST.

The latest issue of the NIST Special Publication 250 entitled "NIST Calibration Services Users Guide" contains detailed descriptions of the currently available NIST calibration services [4]. It also contains information on the scheduling of these services along with recommended procedures for shipping a standard to NIST for calibration. A separate Fee Schedule, NIST Special Publication 250 Appendix, lists the costs of services, and it is updated periodically to reflect changes in prices and services.

2.1 Special Standard Resistors (1 $\Omega$ and 10 k$\Omega$)

NIST recognizes a special category of 1 $\Omega$ and 10 k$\Omega$ standard resistors which exhibit small corrections of $< 10$ ppm from nominal value, high stability, and a low temperature coefficient of resistance (TCR); accordingly, these resistors merit the best possible measurement uncertainties. In this category are the Thomas-type 1 $\Omega$ resistors, or their equivalent, typically having drift rates of $< |0.1|$ ppm/year and TCR's of $(0 \pm 3)$ ppm/$^\circ$C, and the Evanohm*-type 10 k$\Omega$ resistors having drift rates of $< |0.2|$ ppm/year and TCR's of $(0 \pm 1)$ ppm/$^\circ$C. Measurement parameters of temperature and current for these resistors are given in Table 2.1.

These resistors are acclimatized in their respective test environment for approximately one week prior to calibration. The temperature of the resistor at the time of the measurement is given in the Report of Calibration. Since some resistors have a significant pressure coefficient, the

---

*Evanohm is a commercial alloy having a resistivity of about 1.34 $\mu$\Omega-m with a nominal composition of 75% Ni, 20% Cr, 2.5% Cu, and 2.5% Al. By suitable annealing and heat treatment, its TCR can be adjusted to nearly zero from 20 to 30 $^\circ$C.
pressure at the time of the measurement is also reported for these 1 Ω or 10 kΩ standards.

Table 2.1. Measurement parameters for special standard resistors

<table>
<thead>
<tr>
<th>Resistor</th>
<th>Medium</th>
<th>Temperature</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Ω</td>
<td>oil</td>
<td>(25.000 ± 0.003) °C</td>
<td>100 mA</td>
</tr>
<tr>
<td>10 kΩ</td>
<td>oil</td>
<td>(25.00 ± 0.01) °C</td>
<td>1 mA</td>
</tr>
<tr>
<td>10 kΩ</td>
<td>air</td>
<td>(23.0 ± 1.0) °C</td>
<td>1 mA</td>
</tr>
</tbody>
</table>

2.2 Standard Resistors (10⁻⁴ Ω to 10⁶ Ω)

Standard resistors with nominal decade values in the range between 10⁻⁴ Ω and 10⁶ Ω are calibrated by comparison with NIST working standards. In general, these resistors are characterized by 1) resistance corrections within 500 ppm of nominal value, 2) TCR's of (0 ± 10) ppm/K at the temperature of use, and 3) drift rates of < 5 ppm/year. Usually, these standard resistors are measured in an oil bath maintained at a temperature of (25.00 ± 0.01) °C, and at power dissipation levels of 10 mW to 100 mW. Resistors with fixed terminations and designed for operation in an air environment are also accepted for calibration. At the levels of uncertainties reported, four-terminal measurements are required for standard resistors of nominal values ≤ 100 Ω.

2.3 High-Value Standard Resistors (10⁷ Ω to 10¹² Ω)

High-value standard resistors in the range between 10⁷ Ω and 10¹² Ω are calibrated in an air bath maintained at a temperature of (23 ± 0.1) °C and at a relative humidity of (35 ± 5)%). Standard resistors of nominal value up to and including the 10¹⁰ Ω level are compared 1/1 with NIST working standards of the same nominal value. Above 10¹⁰ Ω, 10/1 and 100/1 ratio techniques are used. The maximum test voltage is 500 V for resistors < 10¹⁰ Ω, and 1000 V for resistors ≥ 10¹⁰ Ω. Only resistors that are mounted in a shielded enclosure, with permanently identifying serial numbers and with suitable terminations, are accepted for calibration.

The resistance of a film-type, standard resistor is frequently highly voltage dependent. Hence, the magnitude of the test voltage should be specified when this type of resistor is submitted for calibration. The temperature, relative humidity, and test voltage of the resistor are given in the NIST Report of Calibration.

2.4 Resistance MAP Service

Resistance MAP transfers [5] are offered at the 1 Ω and 10 kΩ resistance levels. Four well-characterized standard resistors are used as transport standards in each transfer. The suggested
measurement schedule at the customer's laboratory consists of measurements on each transport resistor three times a week for a period of 4 to 6 weeks depending upon the settling time of the resistors due to effects caused by transportation.

Participation in this program is generally not advisable unless a laboratory is 1) required to support resistance measurements at or near the state-of-the-art in accuracy, and 2) willing to adopt a system for the continuous surveillance of standards during the intervals between MAP transfers. A successful transfer requires a considerable amount of data collection and a willingness to become involved in the data analysis process. Data supplied during routine NIST calibrations suffice for normal measurement requirements of standards laboratories if proper methods are used by the laboratory to quantify the additional uncertainties caused by transportation and their own measurement process.

2.5 Special Resistance Measurements

Special resistance measurements such as: the determination of the temperature or pressure coefficient of resistance of a standard resistor; testing or evaluation of prototype state-of-the-art resistance standards; unique resistance measurements; and other calibration services not specified above are made at the discretion of the NIST technical staff by prearrangement. Also offered as a special test service is the calibration of standard resistors having values equal to the nominal quantized Hall resistances of 6,453.2 Ω or 12,906.4 Ω.

3. MEASUREMENT METHODS

To provide a wide-ranging calibration service at NIST for 17 decades of resistance from $10^4$ Ω to $10^{12}$ Ω requires the application of three basic methods that are used in seven stand-alone measurement systems for comparing standard resistors. Listed below are the seven measurement systems along with their primary resistance measurement levels.

1) Automated current comparator for 1 Ω measurements.
2) Automated current comparator for 10 Ω and 100 Ω measurements.
3) Current comparator for measurements < 1 Ω.
4) Automated unbalanced bridge for 1 kΩ to 1 MΩ measurements.
5) Guarded double-ratio bridge for special 10 kΩ measurements.
6) Guarded Wheatstone bridge for $10^7$ Ω to $10^{10}$ Ω measurements.
7) Capacitance-discharge (CD) system for $10^7$ Ω to $10^{12}$ Ω measurements.

The current comparator systems (#1 to #3), the bridge systems (#4 to #6), and the CD system (#7) are based on a current-ratio method, a resistance-ratio method, and a loss-of-charge method, respectively, for the comparison of resistors. Although each method is unique, they share all or some of the following measurement concepts and techniques:

1) Working Standards - NIST standard resistors, whose values are based on the quantum Hall effect, existing at each resistance level. The mean value of two or more working standards is used to assign a value to a resistor under test.
2) Check Standard - A NIST standard resistor that is treated as an unknown and measured during each measurement run. An analysis of the check standard measurement results, over a substantial period of time and over fluctuating environmental conditions, provides an indication of the random error of the measurement process. For any given measurement run, the difference between the measured and predicted values for a check standard indicates whether the measurement system is operating under statistical control.

3) Dummy Resistor - A resistor, of equal quality to a working standard resistor, that is used in a bridge or current comparator measurement system. Its absolute value need not be known; however, it must remain stable or predictable during a measurement run.

4) Transfer Standards - Usually 10 resistors connected permanently in series, of which any combination of resistors can be connected in a parallel configuration using special connection fixtures. They are also commonly known as Hamon boxes [6] or build-up resistors, and they are used at NIST to intercompare working standards at the different resistance levels. NIST transfer standards provide accurate ratios of 10/1 and 100/1 for extending the 1 Ω level in decade increments up to and including the $10^{10}$ Ω level.

5) Substitution Technique - Since the working standards and test resistors are of the same nominal value, they can be indirectly compared by substitution in the same positional arm of a bridge circuit. This technique tends to cancel errors resulting from ratio non-linearity, leakage currents, and lead and contact resistances.

6) Reversal Technique - A null detector balance is achieved upon reversal of the battery or power supply connections to the circuit. This eliminates the effect of constant thermal emfs in the detector circuit. With at least two reversals, the effect of slowly varying thermal emfs also can be eliminated.

7) Quality Control - One or more check standards are measured during each test run to monitor the measurement process. In addition, the standard deviation of a test run and the previous history of the test resistor are examined to determine the quality of the measured value. Also the measured and predicted differences among NIST working standards are compared.

### 4. DIRECT CURRENT COMPARATOR SYSTEMS

Direct current comparator (DCC) systems [7,8] are used at NIST to calibrate four-terminal standard resistors from the 100 μΩ level up to the 1 kΩ level. The DCC, because of its high level of resolution, insensitivity to lead resistances, excellent ratio linearity and ratio stability, has replaced resistance-ratio instruments such as the Wenner and Kelvin bridges for these resistance measurements. Another distinguishing feature of the DCC in contrast to resistance-ratio bridges is that the balance condition is obtained under equal voltage drops across the resistors. For ratios other than 1/1, this results in the greater power dissipation occurring in the lower-value resistor which is usually designed to withstand higher currents. The detector in a DCC system is a photocell galvanometer amplifier (PGA) which exhibits low noise with nanovolt sensitivity at these impedance levels. The DCC system current noise is 3 μA or less depending on the model.
in use. This noise limitation, along with the possibility of errors resulting from leakage currents in the DCC, limits the practical maximum resistance range to 1 kΩ at NIST. Above 1 kΩ, the resistance-ratio bridges with high-impedance electronic detectors are used for standard resistor calibrations.

In 1969, a DCC potentiometer system replaced the NIST Wenner bridge for the intercomparison of Thomas-type 1 Ω standard resistors. With the improved precision of this system, a correlation of the change of resistance of a Thomas-type resistor with variations of barometric pressure was detected. This led to the determination of the pressure coefficients of the NIST set of Thomas-type standard resistors in order to correct for this effect. The magnitude of the ambient pressure is now included in the calibration report for a Thomas-type resistor. This NIST DCC potentiometer system for the calibration of 1 Ω standard resistors was automated in 1982 [9]. In 1984, a second DCC potentiometer was modified for the intercomparison of 10 Ω and 100 Ω standard resistors. This system was automated in 1985.

A DCC resistance bridge has been in operation since 1969 for the measurement and scaling of low-valued, four-terminal standard resistors. Although it is primarily used for calibrating standard resistors of nominal decade values ranging from 100 μΩ to 100 mΩ, this bridge has the capability of measuring four-terminal resistors up to the 1 kΩ range. The maximum current level of this system for the calibration of resistors is 100 A.

4.1 Theory of Operation

In general, the operation of a current comparator is based on Ampere's fundamental law

$$\oint H \cdot dl = \Sigma I,$$

where the line integral of the magnetic field $H$ around a closed path $dl$ is equal to the total current $I$ crossing any surface bounded by this path. For an ideal comparator, the total current is carried by two ratio windings (primary and secondary) and the total number of ampere-turns of one winding is equal and opposite to that of the other winding; thus,

$$\Sigma I = N_p I_p - N_s I_s = 0,$$

where the subscripts $p$ and $s$ refer to the primary and secondary windings, respectively. The current ratio of the comparator, $I_p/I_s$, is therefore equal to the inverse of the turns ratio, $N_s/N_p$. To obtain high ratio accuracies, the ampere-turn balance or zero flux condition is determined by some type of flux detector system that is only sensitive to the mutual fluxes generated by the ratio windings.

In practice, a DCC achieves high ratio accuracy and high sensitivity by utilizing high-permeability toroidal cores, magnetic and eddy-current shields, and careful winding procedures. The main component of a DCC consists of a pair of high-permeability cores, surrounded by a magnetic shield, over which are the ratio windings that carry the direct currents to be compared.
The currents for the ratio windings are supplied by two isolated direct current sources. Around the cores and within the magnetic shield is wound a modulation-detection winding that is used to sense the flux condition of the cores. This is achieved by modulating the core permeabilities and using a second harmonic detector circuit. The presence of dc flux in the cores due to primary and secondary ampere-turn unbalance is indicated by this detector output both in magnitude and polarity. The detector output is used in a feedback circuit to adjust the current in one of the windings automatically to maintain ampere-turn balance.

![Diagram](image)

**Fig. 4.1. Basic circuit of CCC.**

The basic circuit of a self-balancing DCC resistance bridge is shown in Fig. 4.1. This bridge requires two simultaneous balances, an ampere-turn balance and a voltage balance. The slave current source is continuously adjusted in a feedback mode by the output of the demodulator circuit so that ampere-turn balance is maintained. Under this condition, the ampere-turn product of the primary circuit equals that of the secondary circuit, i.e.,

\[ N_p I_p = N_s I_s . \]  \hspace{1cm} (4.1)

The voltage balance is achieved by the adjustment of the number of turns in the primary circuit, \( N_p \) until there is a null condition on detector D. Then the voltage drop across the unknown resistor \( R_z \)** in the primary circuit is equal to the voltage drop across the dummy resistor \( R_d \) in the secondary circuit or,

**Roman or upright type is used to denote a component (e.g., R for resistor, C for capacitor) and italic type is used to represent its physical quantity (e.g., \( R \) for resistance, \( C \) for capacitance).
\[ R_x I_p = R_d I_s \]

and using eq (4.1), the value of \( R_x \) can be expressed as
\[ R_x = (N_p/N_s) R_d . \]  

Similarly, another measurement with the direct substitution of a known standard resistor \( R_s \) of the same nominal value in the primary circuit and re-balancing the detector by adjusting \( N_p \) to a new value \( N_p' \) results in
\[ R_s = (N_p'/N_s) R_d . \]  

Combining eqs (4.2 and 4.3) gives
\[ R_x = (\Delta N_p) R_s , \]

where \( \Delta N_p \) is the difference in the number of primary turns between the two voltage balance settings.

Thus, the value of an unknown resistor \( R_x \) is determined in terms of a standard resistor and the small difference in turns ratio. The dummy resistor \( R_d \) must remain stable during the time interval the two voltage balances are made; otherwise an error will occur in the measurements. For the DCC potentiometer systems which include the 1 Ω, 10 Ω, and 100 Ω measurements, a string of resistors are connected in series in the primary circuit. Then each resistor in turn is compared to the dummy resistor. If more than one working standard resistor is used in this measurement process, then the value of an unknown resistor is based on the mean value of the working group.

### 4.2 DCC Potentiometer System for 1 Ω Measurements

#### 4.2.1 Description of System

A schematic diagram of the DCC potentiometer circuit for 1 Ω measurements is shown in Fig. 4.2. Fifteen Thomas-type resistors - five of which comprise the working group, along with two check standards and eight unknown resistors - are connected in series within the primary circuit of the DCC. The value of any resistor in the string can be determined by indirectly comparing its voltage drop to the mean of the voltage drops of the reference group via a stable, 0.5 Ω dummy resistor \( (R_d) \) in the secondary circuit of the DCC. The resistors are mounted on a mercury stand that is housed in a constant temperature oil bath controlled at \((25.000 \pm 0.003)°C\). The voltage balance is made automatic by driving the PGA detector to a null condition with a feedback current through an auxiliary 10 turn winding. This feedback current, which is proportional to the difference between the test and dummy resistors, is monitored by measuring the voltage drop across a 100 Ω resistor using a digital voltmeter (DVM). The feedback circuit is calibrated by inserting an additional unit turn in the primary circuit of the DCC which changes the voltage balance by exactly 500 ppm. A personal computer (PC) controls
the operation of the DVM, resistor selection, current reversal, and the insertion of the feedback calibration signal. The PC also monitors the oil bath temperature, ambient temperature, ambient relative humidity, and barometric pressure.

![Diagram of automated DCC potentiometer for 1 Ω measurements.](image)

Fig. 4.2. Automated DCC potentiometer for 1 Ω measurements.

A description of some of the important features of the main components of the system follows:

1. DCC Potentiometer

The DCC is a commercially available unit having an adjustable 2000-turn primary winding, a 1000-turn secondary winding, and a fixed 0.1 A primary current source. The system noise level is specified by the manufacturer to be less than 1 μA. The constant current source is specified as having a voltage compliance of 30 V, an output resistance of 10 GΩ, and a stability of 1 ppm/day plus the instability of its standard cell. To automate the system, four major modifications of the comparator were necessary:

A. The PGA detector was modified to provide an isolated detector output by mounting a pair of cadmium-sulfide (CdS) photoconductive cells adjacent to the original set of photocells. Four 1 kΩ resistors (one of which is variable) were also mounted within the PGA to complete an adjustable bridge circuit along with the two CdS photocells (see Fig. 4.3). An external 5.4 V mercury battery supply energizes the bridge.

B. The push-button current reversing switch was replaced with a DPDT relay. It is connected to a relay control module interfaced to the PC.

C. A DPDT relay with mercury-wetted contacts was connected to the 1 turn/step switch
of the DCC between the "0" and "-1" dial positions. This relay is also connected to the relay control module. When energized this relay provides a 500 ppm bridge offset.

D. The current divider circuit used for the "standardize" mode of the DCC was disconnected from its 10 turn winding. This winding is used in the feedback circuit which drives the PGA detector to a null condition to provide for an automatic voltage balance.

2. Feedback Circuit

The schematic diagram of the isolated detector output and feedback circuits is also shown in Fig. 4.3. The isolated detector output is buffered by an operational amplifier, A1, and then amplified by a factor of 5 by A2. Since a 1 ppm change of resistance corresponds to a voltage change of 0.1 μV, the open-loop gain of the system G at this point is equal to $3.35 \times 10^6$. Any further amplification leads to oscillations in the feedback circuit. Amplifier A3 provides a feedback current $I_f$ through the 10 turn winding $N_f$. The current $I_f$ is monitored by measuring the voltage drop across a 100 Ω resistor with a DVM. A 1 μF capacitor, with PTFE dielectric, is connected across the 100 Ω resistor to provide sufficient filtering of electrical noise in the system.

![Feedback circuit schematic](image)

Fig. 4.3. Feedback circuit for automated DCC system.

3. Resistor Switch Module

A modified crossbar switch selects the appropriate potential terminals of a resistor in the primary circuit of the DCC whose voltage drop is to be compared to that of the dummy resistor in the secondary circuit. The switch mechanism is separated from the electromagnetic actuator coils and placed in a heavy aluminum box. The coils are mounted outside the box and their push rods are extended by plastic rods. This type of separation reduces heat transfer from coil to switch point, and the heavy aluminum box effectively eliminates thermal gradients within the box. The crossbar switch uses beryllium-copper conductors and gold-silver-platinum contacts which are thermoelectrically matched to each other resulting in thermal emfs of less than 250 nV/K.
4. **Resistance Thermometer Bridge**

The temperature of the oil bath, maintained at 25.000 ± 0.003 °C, is monitored with a calibrated platinum resistance thermometer (PRT). The PRT is calibrated in terms of the International Temperature Scale of 1990 (ITS-90), and its resistance of ≈ 28 Ω at the control temperature is measured using an automated resistance thermometer bridge [10]. The bridge has a range up to 101.1 Ω, a resolution of 1 μΩ, and an accuracy limited by the resolution or 0.1 ppm. The bridge has four input ports and is remotely controlled through an IEEE 488 bus. The bridge is calibrated prior to each PRT measurement against a known 100 Ω resistor. The PRT and resistor are measured at a current level of 1 mA.

5. **Pressure Transducer**

The barometric pressure is monitored using a calibrated commercial transducer which measures pressure as a function of the natural frequency of a thin-wall metal cylinder. Transducer temperature is determined indirectly by measuring the voltage drop across a calibrated diode that is mechanically attached to the transducer body. The transducer outputs are measured by a counter/DVM that is controlled by the PC through the IEEE 488 bus. The barometric pressure is calculated from the period and voltage measurements. The sensitivity of the transducer is 13.3 Pa (0.1 mm Hg) with an accuracy of 4 Pa (0.3 mm Hg). The accuracy of the transducer is periodically checked against a calibrated aneroid barometer.

6. **Temperature-Humidity Indicator**

Ambient temperature and relative humidity are monitored by a commercial digital thermometer/hygrometer instrument. The instrument has a resolution and reproducibility of 0.1 °C and 0.1% RH. The temperature and relative humidity analog recorder outputs are measured by a DVM which is interfaced to the PC via the IEEE-488 bus.

7. **Personal Computer**

The PC is a DOS-based system configured with hard and diskette drives for program and data storage. It contains the following ports and boards: a printer port, a RS-232 port to communicate with a central time-sharing computer, a IEEE-488 board, and a parallel I/O board to control the crossbar switch and DCC.

8. **Software**

The computer program for operating the system is written in BASIC language using multiple subroutines to handle the data taking and data processing. A MS-Kermit communications program is executed to transfer the data file of a test run stored on the PC to the central time-sharing computer via the RS-232 ports. The system program "THOMAS.COM" on the central computer is executed to compute, analyze, and store the results of the test run. This program adjusts the predicted value of the group mean of the five reference resistors for variations of the oil bath temperature and ambient pressure from their respective nominal values of 25.000 °C and 101,325 Pa (760 mm Hg). This corrected group mean is then used to calculate the values of all
of the unknown resistors in the measurement string for the test run including the values of the check standards. The values of NIST resistors corrected to a temperature of 25.000 °C and a pressure of 101,325 Pa are stored in individual data files according to their NIST serial number. The values of the five reference resistors are stored in a single file named "GRP1T.0T". The data for a customer's resistor is also saved, if a file for it exists. The data in these individual files are saved in a format so that a linear regression fit of the data vs time can be calculated using another system program. The "THOMAS.COM" program also provides a check on the results by flagging the mean value of a resistor if its standard deviation exceeds a predetermined value (see section 10).

4.3 DCC Potentiometer for 10 Ω and 100 Ω Measurements

The design, construction, and operation of this system is similar to that for the automated 1 Ω system. The following description and operation sections will emphasize the differences between the two systems. Similarities between the two systems will be briefly mentioned. For more details on these similar features, refer to section 4.2 of this report.

4.3.1 Description of System

The DCC potentiometer circuit for 10 Ω and 100 Ω measurements resembles that shown in Fig. 4.2 for the automated 1 Ω system. However instead of 15 resistors, the system is designed to intercompare eight resistors of the same nominal value: two working standards, one check standard, and five unknown resistors. These resistors are mounted on a mercury stand housed in a constant temperature oil bath controlled at (25.000 ± 0.003) °C. The resistors for the secondary circuit are sealed in a heavy aluminum box filled with silicone fluid and the box is immersed in the oil bath. The DCC is operated at a 2/1 ratio; hence this box contains a 5 Ω resistor and a 50 Ω resistor that are needed for measuring 10 Ω and 100 Ω resistors in the primary circuit, respectively.

In similar fashion to the 1 Ω system, the DCC was modified to provide 1) automatic current reversal, 2) automatic ratio offset, 3) an external access to a 10 turn winding, and 4) an isolated detector output circuit. The detector-feedback circuit is identical to the one used in the 1 Ω system except that resistor R3 in Fig. 4.3 has been increased to 200 Ω, thus reducing the gain of the feedback circuit to prevent oscillations from occurring in the system.

The original 50 mA constant current source (CCS) for the primary circuit of the DCC was modified to provide output currents of 10 mA and 31.6 mA. The two current outputs are needed in order to limit the power dissipation of the 100 Ω and 10 Ω resistors to 10 mW, respectively. The excellent stability of the CCS is obtained by sending the load current through a reference resistor whose voltage drop is connected in series opposition to the reference voltage of a standard cell. Any difference between these two voltages is detected by a galvanometer. Light from the galvanometer mirror falls on a pair of phototransistors that provide the feedback signal to control the load current. The modifications to the current source consisted of changing the reference resistor and changing the bias conditions of the phototransistors. This was accomplished by installing a rotary switch in the circuit. Each switch position inserts a different reference resistor and bias resistor into the circuit. The two critical reference resistors are sealed
in a heavy aluminum box filled with silicone fluid and the box is immersed in the oil bath.

A commercial relay scanner is used for automatic selection of the appropriate potential terminals of any resistor in the primary circuit of the DCC during a test run. The scanner contains latching-type relays with precious metal contacts for low-thermal emf operation. Shielded two-conductor PTFE-insulated cable is used to make the interconnections between the scanner and the potential terminals of the resistors. The scanner is controlled by the PC through the IEEE-488 bus.

The PC is similar to the one used in the automated 1 Ω system with the same number of hard and diskette drives, ports, and I/O boards. The MS-Kermit communications program is executed to transfer the data file stored on the PC to the central computer. The system program "AMPERE.COM" on the main computer is executed to compute, analyze, and store the results of the test run. This program adjusts the predicted value of the group mean of the two reference working resistors for variations of the oil bath temperature from the nominal value of 25.000 °C. The corrected group mean is then used to calculate the values of the resistors in the measurement string for the test run. The values are stored in individual data files that exist on the main computer. The program "AMPERE.COM" also provides a check on the results by flagging the mean value of a resistor if its standard deviation exceeds a predetermined value (see section 10).

4.4 DCC Resistance Bridge

The DCC resistance bridge is used primarily for the calibration of Rosa type 1 Ω standard resistors or their equivalent, and for all four-terminal standard resistors having nominal values of 0.1 Ω, 0.01 Ω, 0.001 Ω, and 0.0001 Ω. The bridge is also capable of measuring 10 Ω, 100 Ω, and 1000 Ω four-terminal standard resistors.

4.4.1 Description of System

A partial schematic diagram of the DCC resistance bridge is shown in Fig. 4.4. The bridge consists of an adjustable 0–1000 turn winding in the primary circuit, and a fixed 1000 turn and adjustable deviation windings in the secondary circuit. The resistor to be measured, \( R_x \), is connected in the primary circuit and a reference resistor, \( R_y \), is connected in the secondary circuit. If the deviation winding is set equal to the correction of resistor \( R_y \), the bridge becomes direct reading in ohms. The resistors are located in a constant temperature oil bath maintained at (25.000 ± 0.010) °C. The DCC resistance bridge is balanced by adjusting the turns ratio for a null condition on detector D using the reversal balancing procedure.

The adjustable ratio winding in the primary circuit can be replaced by a fixed winding of 100 turns, 10 turns, or 1 turn which will provide additional bridge ratios of 10/1, 100/1, and 1000/1. For these higher ratios, the adjustable fractional-turn section in the primary side is switched to the secondary side in order to balance the bridge. The bridge has a resolution of 0.1 ppm for all ratios; however, it only has a range of 1111.1 ppm for ratios > 1/1. For resistance ratio measurements of 1/1 and 10/1, a 0-to-1 A internal power supply is connected in the primary circuit. When the bridge is used for ratio measurements of 100/1 and 1000/1, the internal power supply is replaced by an external, adjustable 100 A supply. The 100/1 and 1000/1 ratios have
respective maximum current ratings of 20 A and 100 A.

4.4.2 Measurement Parameters and Design

The resolution of a resistance measurement when using this DCC resistance bridge is limited by the following factors:

1. The smallest incremental ratio adjustment of 0.1 ppm.
2. The detector sensitivity of 10 nV.
3. The system noise of 3 μA.
4. The maximum current ratings of the ratio windings.
5. The maximum power dissipation of the resistors.

![DCC resistance bridge diagram]

Fig. 4.4. DCC resistance bridge.

Normally, the power dissipation of a standard resistor is 10 mW or less during a measurement. However, in order to obtain sufficient resolution and, since a lower-value resistor is usually designed to withstand higher currents, the power dissipation level of a standard resistor below 1 Ω is set at 100 mW or less. Table 4.1 gives the measurement parameters for the various resistance levels when using the DCC resistance bridge.

The value of an unknown resistor can be obtained by two different methods using the DCC resistance bridge. The first method consists of alternately substituting the unknown resistor and
a reference working resistor in the primary circuit (substitution technique) along with a dummy resistor in the secondary circuit. The second method consists of connecting the reference working resistor in the secondary circuit with the unknown still in the primary circuit. The second method relies directly on the accuracy of the bridge ratio. Usually, the ratio error of a DCC is negligible for these measurements.

Table 4.1. Measurement parameters for DCC resistance bridge

<table>
<thead>
<tr>
<th>Resistance (Ω)</th>
<th>Ratio</th>
<th>Maximum Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>1000/1</td>
<td>30</td>
</tr>
<tr>
<td>0.001</td>
<td>1000/1</td>
<td>10</td>
</tr>
<tr>
<td>0.01</td>
<td>100/1</td>
<td>3</td>
</tr>
<tr>
<td>0.1</td>
<td>100/1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1/1</td>
<td>0.3</td>
</tr>
<tr>
<td>10</td>
<td>1/1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

5. RESISTANCE-RATIO BRIDGES

Before the development of the DCC, the comparison of standard resistors was done chiefly by the use of bridges with resistance ratio arms. For many years at NIST, a Wenner bridge [11] served this purpose for the comparison of multiples and submultiples of the ohm from 100 μΩ to 1 MΩ. This bridge, designed in 1918, was a combination Wheatstone bridge and Kelvin double bridge with ratios of 1/1 or 10/1 having an adjustment range of 5000 ppm. In later years, it was modified to increase its resolution to 0.01 ppm. The bridge was retired in 1983 because of the deterioration of its switch contacts and insulation. By this time it had been already replaced by DCC systems for the comparison of four-terminal standard resistors from 100 μΩ to 1 kΩ as described in section 4.

During the late 1960's and early 1970's, several resistor manufacturers developed a high-quality, transportable 10 kΩ standard resistor in order to establish an additional reference level of resistance along with the traditional 1 Ω level [12]. The 10 kΩ level was attractive because it was nearer to the mid scale of precision resistor measurements, and it was expected to provide a convenient level for comparison with the SI ohm derived from absolute ohm experiments. The improved 10 kΩ standard resistor surpasses the Thomas-type resistor in temperature coefficient of resistance, is about equal to it in transportational effects, but is second to it in time stability. Nevertheless, it has gained wide acceptance as a primary reference standard and the number of calibrations of these standards at NIST is second only to that of the Thomas-type resistor.
calibrations in the dc resistance area. Around 1975, a modified Kelvin-type bridge was constructed specifically for the comparison of the improved or "special" 10 kΩ standard resistors. The bridge employs a guard network to reduce errors caused by possible leakage currents in the circuit. The bridge has been refined and improved over the years and its latest version is described in section 5.2.

Prior to 1989, standard resistors having nominal values of 1 kΩ, 10 kΩ, 100 kΩ, and 1 MΩ were calibrated at NIST in a Wheatstone bridge circuit using a Direct-Reading Ratio Set (DRRS) that was operated manually. Generally, these standard resistors are of the Rosa design which are measured in a mineral-oil bath controlled at a nominal temperature of 25 °C. Since 1989 these standards have been calibrated using an automated system based on an unbalanced-bridge method [13]. Since this method is unique in that it is not a 4-arm bridge method, it will be described separately in section 6.

Standard resistors >1 MΩ, specifically resistors with nominal decade values from 10 MΩ to 1 TΩ, are calibrated using a specially-designed guarded Wheatstone bridge (section 5.3) or a teraohmmeter (section 7). The bridge is limited to the calibration of standard resistors that differ from their nominal value by less than 5000 ppm. For high-value resistors outside of this tolerance, the teraohmmeter is used as a comparator for calibration. All high-value standard resistors are calibrated in a temperature-humidity chamber maintained at a temperature of 23 °C and a relative humidity of 35%.

5.1 Theory of Operation

The operation of the resistance measurement systems described in this section for the comparison of standard resistors are based on variations of two well-known bridge circuits, namely the Wheatstone bridge and the Kelvin double bridge. The Wheatstone bridge circuits are used for the measurement of two-terminal resistors, while the Kelvin double bridge circuits are used for the comparison of four-terminal resistors. In general, the comparison of four-terminal resistors is done at resistances of 100 Ω or less. However, in special cases where higher accuracies are warranted, e.g., the comparison of special 10 kΩ resistors, four-terminal measurements are done at resistances > 100 Ω.

A schematic diagram of a typical NIST Wheatstone bridge circuit for the comparison of two-terminal resistors is shown in Fig. 5.1. Resistors A and B constitute the two arms of a DRRS. The 1/1 resistance ratio of a typical DRRS can be adjusted over a range extending from -5000 ppm to +5000 ppm in increments of 0.1 ppm. R_d is an auxiliary or dummy resistor of the same nominal value as the test or unknown resistor, R_x, and the standard resistor, R_s. Lead resistances l_1, l_2, and l_3 complete the bridge network. Variability in the contact resistances at the resistor terminations are negligible for two-terminal resistor comparisons above 100 Ω. The substitution technique (see section 3) is used for determining the small difference between the resistors R_x and R_s. The bridge equation with R_x in the circuit and the bridge balanced at a DRRS reading of D_t in ppm is
\[(A/B) \cdot (1 + D_1 \cdot 10^{-6}) = (R_x + l_1)/(R_d + l_2 + l_3) .\]

Similarly for \(R_s\) substituted in the circuit, the bridge equation is
\[(A/B) \cdot (1 + D_2 \cdot 10^{-6}) = (R_s + l_1)/(R_d + l_2 + l_3) ,\]

where \(D_2\) is the new DRRS reading in ppm after the bridge is re-balanced. The value of \(R_s\) can be represented as
\[R_s = R'_s(1 + c_s \cdot 10^{-6}) ,\]

where \(R'_s\) is the nominal value of the standard resistor and \(c_s\) its correction in ppm. Solving for \(R_x\) and neglecting 2nd order and higher terms results in
\[R_x \approx R'_x(1 + (D_1 - D_2) \cdot 10^{-6} + c_s \cdot 10^{-6} + (l_1/R_s)(D_1 - D_2) \cdot 10^{-6}) .\]

Usually the last term in the above equation is negligible for measurements above 100 \(\Omega\), since the differences between nominally-equal resistors are within 500 ppm, and for most practical circuits the ratio \(l_1/R_s < 10^{-3}\). Therefore the equation for \(R_x\) reduces to
\[R_x \approx R'_x(1 + (D_1 - D_2) \cdot 10^{-6} + c_s \cdot 10^{-6}) .\]

\[\text{Fig. 5.1. DRRS bridge circuit.}\]

The difference in the DRRS ratio for the two balances gives the difference between the values of the two resistors in ppm. For measurements > 1 M\(\Omega\), a guarding network is added to the Wheatstone bridge circuit in order to suppress errors caused by leakage currents flowing from the main bridge arms to ground potential. The guard circuit drives the shields of the bridge terminations at the proper potentials to confine leakage currents within this network.
Four-terminal resistors are used to avoid uncertainties arising from variations in lead and contact resistances. A Kelvin double bridge, as shown in Fig. 5.2, is used for the comparison of four-terminal resistors. The fundamental difference between the Kelvin double bridge and the Wheatstone bridge is the use of a second set of ratio arms a/b. Resistors $R_x$ and $R_s$ are connected together by a low-resistance link L. The link and battery circuits contain the current lead resistances of $R_x$ and $R_s$. Potential lead resistances $I_1$, $I_2$, $I_3$, and $I_4$ are eliminated from the final bridge balance equation by special balancing techniques. When the bridge is balanced, $R_x$ can be expressed as:

$$R_x = (A/B)R_s + ((b-L)/(a + b + L))(A/B - a/b).$$

![Fig. 5.2. Kelvin double bridge circuit.](attachment:image.png)

This equation reduces to that for a simple Wheatstone bridge when the last term on the right hand side is zero. This can be accomplished by adjusting $A/B = a/b$, irrespective of the value of L. In practice, the resistance of L is kept small to lessen the importance of any lack of inequality between the ratios $A/B$ and $a/b$. For some measurements the nominal ratio of $A/B$ is set equal to 10/1 in order to reduce the power dissipation in the $R_s$ arm of the bridge. Whenever it is possible, $R_x$ is determined by the substitution technique which means replacing $R_s$ with a stable dummy resistor $R_d$.

### 5.2 Modified Kelvin Bridge for 10 kΩ Measurements

This measurement system was specifically designed for the comparison of four-terminal 10 kΩ standard resistors by the substitution method. Similar to the Kelvin double bridge circuit, it incorporates inner ratio arms with a link arrangement to eliminate the effects of contact and lead resistances of one set of current-potential terminals of the resistor under test. Unlike the Kelvin double bridge, the dummy resistor $R_d$ need not be connected in a four-terminal arrangement. In
addition, the contact and lead resistances associated with the other set of current-potential terminals of the resistor under test are measured in a two-step procedure and subtracted from the final result. The schematic diagram for this measurement system is shown in Fig. 5.3 without its guard circuit. The A/B ratio arms of the bridge consist of a 100 Ω DRRS with additional 10 kΩ resistors to increase the resistance of each arm to a nominal value of 10.1 kΩ. This arrangement results in a dial resolution of better than 0.01 ppm/step. It also closely simulates a square-bridge for optimal circuit operation. This composite DRRS has an adjustment range of 110 ppm. The inner ratio arms a/b consist of two 1 kΩ resistors separated by an adjustable 10 turn 100 Ω resistor. The removable link L is an amalgamated copper bar approximately 5 cm in length and 0.4 cm² in cross section. The battery supply V contains 15 D-size mercury batteries to provide a bridge excitation current of 1 mA resulting in 10 mW of power being dissipated in each main arm of the bridge. The batteries are mounted in a shielded enclosure and they are isolated from ground potential using PTFE insulation. The electronic null detector D is connected to either position A or P using a low-thermal rotary switch. Dummy resistor R_d is an oil-type, 10 kΩ resistor of equal quality as the resistor R_x under test.

Two BPO*** connector panels are mounted on either side of the bridge ratio arms to provide ease-of-use in connecting test resistors into the circuit. Each panel contains 40 connectors attached to 10 resistors by twin-conductor shielded PTFE insulated cables. One panel is for use with oil-type resistors, while the other panel is for air-type resistors. A temperature lag box houses air-type 10 kΩ resistors (two working standards and one check standard) used in the measurement

***BPO is an acronym for British Post Office and refers to a type of coaxial connector manufactured in Britain. The connectors are constructed using silver-plated conductors and PTFE insulation. A connector pair has low contact resistance and low thermal emfs.
process. This lag box helps to minimize the effect of fluctuations of the ambient temperature on the measurements. The temperatures of the air-type resistors are measured using a calibrated digital thermometer with thermistor probes. The resolution of the thermometer is 0.01 °C with an accuracy of 0.05 °C. If requested by a customer, the resistive temperature sensor mounted in an air-type resistor can be measured at the time of calibration using a calibrated digital multimeter. An additional check standard is located in the oil bath along with the oil-type resistors under test. The oil bath temperature of (25.00 ± 0.01) °C is monitored periodically by a PRT measurement system.

This bridge circuit for the measurement of special 10 kΩ resistors is guarded (guard circuit not shown in figure). The guard circuit suppresses the effects of leakage currents flowing to ground potential or to other parts of the circuit that may cause an error in the measurement. Unfortunately, the terminations of most resistors under test are not guarded. The shields of the cables connecting a resistor into the test circuit are driven at a guard potential to complete the guard circuit as much as possible. However, the NIST-built transfer or series-parallel buildup standards, as described in section 8.2, contain internal guard networks to take advantage of this guarded bridge circuit.

5.3 Guarded Wheatstone Bridge

Standard resistors of nominal decade values from 10 MΩ to 10 GΩ are measured using a NIST-built guarded Wheatstone bridge. This bridge is limited to the comparison of standard resistors that have corrections within ± 5000 ppm from nominal. For the measurement of high-value resistors outside of this tolerance or with nominal decade values extending from 10 GΩ to 1 TΩ, a semiautomatic technique using a digital teraohmmeter is used (see section 7).

A simplified diagram of the guarded Wheatstone bridge is shown in Fig. 5.4. The main bridge arms, drawn with heavy lines, consist of the main ratio arms A and B, the dummy resistor Rₐ, and the test resistor Rₓ. Drawn with lighter lines and in parallel with the main bridge circuit is an auxiliary guard network for the purpose of reducing measurement errors caused by leakage currents. The NIST-built working standards that are used with this measurement system contain their own internal guard network to take full advantage of this guarding feature. For test resistors that do not have an internal guard resistor (nearly all of them), an external resistor must be added in order to complete the guard circuit. The main and guard circuits are energized by a battery supply V having an output isolation of >10¹² Ω from ground. The bridge circuit is connected to a single ground point at the guard terminal between guard resistors a and b as shown in Fig. 5.4. The detector D is an electrometer having a high input impedance of >10¹⁶ Ω. Its low end is connected to the bridge terminal between the main ratio arms A and B which is not grounded but driven to near ground potential by the guard circuit; i.e., a virtual ground potential.

The test resistor Rₓ is shown in Fig. 5.4 with leakage resistance paths between its terminals and case represented by r₁ and r₂. When the test resistor is measured, its case is connected to ground potential. Measurement errors caused by terminal leakage resistance r₁ is greatly reduced since it is effectively in parallel with guard resistor a, and unwanted ground currents are intercepted by the guard circuit and not the main bridge circuit. Terminal leakage resistance r₂ is effectively in parallel with the detector, and it does not contribute to any measurement errors. The test
Fig. 5.4. Guarded Wheatstone bridge.

resistor is compared to a working standard in the same arm of the bridge (substitution technique), and, consequently, any leakage errors associated with \( R_d \) are negligible if the leakage resistances are constant during the time interval of the measurement run.

The A/B ratio arms of the bridge can be connected in a 1/1 or 10/1 ratio configuration. A more detailed circuit diagram of the NIST guarded Wheatstone bridge is shown in Fig. 5.5. If the bridge connections are made at terminals labeled \( A_1 \) and \( B_1 \) or \( A_1 \) and \( B_2 \), which corresponds to ratios of 1/1 or 10/1 respectively, the A arm has a nominal value of 1 MΩ and the total adjustment range of the A/B ratio is 1111.10 ppm with a resolution of 0.01 ppm. The A arm contains an adjustable 5-decade resistor having a maximum resistance of 1111.10 Ω. If the bridge connections are made at \( A_1 \) and \( B_1 \) or \( A_1 \) and \( B_2 \), which again corresponds to ratios of 1/1 or 10/1 respectively, the A arm has a nominal value of 100 kΩ and the total ratio adjustment is 11111.0 ppm with a resolution of 0.1 ppm. The A/B ratio-arm resistors are suspended in air and attached to guarded binding posts mounted on a PTFE top plate. This entire assembly is enclosed in a grounded aluminum box to provide electrostatic shielding.

The resistor under test \( R_x \) and the dummy resistor \( R_d \) are placed in an environmental air chamber with temperature and relative humidity control of \((23.0 \pm 0.1)°C\) and \((35 \pm 5)\%\), respectively. The temperature and humidity are continuously monitored by a digital thermometer and an analog humidity indicator. Electrical access into the air chamber is made via 10 BPO connectors mounted on a PTFE panel. This provides for the connection of five resistors within the chamber: usually two working standards, one check standard, one dummy resistor, and a test resistor. Lead connections between the resistors and the other parts of the bridge are made using shielded, PTFE-insulated cable with silver-plated copper conductors. The shields of these cables and BPO connectors are driven at a guard potential by the auxiliary guard network.
6. RING METHOD

This automated measurement system, based on an unbalanced-bridge technique, was developed to replace the manual DRRS system used to calibrate resistors in the resistance range 1 kΩ to 1 MΩ. This technique is referred to as the "ring" method since the resistors are connected in a ring configuration. The system is specifically designed to measure the differences among six nominally-equal, four-terminal standard resistors of the Rosa type that are mounted on a mercury ring stand located in a temperature-controlled oil bath. The system has sufficient switching capability to operate four six-arm mercury ring stands, usually at the nominal resistance levels of 1 kΩ, 10 kΩ, 100 kΩ, and 1 MΩ. The system has the flexibility to accommodate other types of standard resistors including those operating in a laboratory air environment. Also the system can be used to compare resistors having lower nominal resistance values of 100 Ω or 10 Ω. The resistors mounted on the ring stand are energized at a power level of 10 mW or less.

6.1 Theory of Operation

In general, the unbalanced-bridge or "ring" method consists of connecting an even number of six or more nominally-equal, four-terminal resistors in a ring configuration, i.e., a string of resistors in a closed electrical circuit where the first and last resistors are interconnected. The number of resistors equals "2·(n + 2)," where n is an integer ≥ 1. A voltage is applied across opposite
corners of the hexagonal ring which divides the ring into two branches, each containing \( n + 2 \) resistors, connected in parallel. Then, a DVM is used to measure voltages between opposite potential terminals of the resistors that are at nearly equal potentials. Next, the applied voltage points across the ring are rotated in a clockwise (or counterclockwise) direction to the next pair of resistor connection points. Again voltage measurements are taken between corresponding terminals of the resistors that are at nearly equal potentials. This measurement process is repeated \( n + 2 \) times. From the \( n + 2 \) subsets of voltage measurements, one obtains a set of \( 3(n + 2) \) linear equations that can be solved using a least-squares technique. Values of the resistors can be calculated if the value of at least one of the resistors in the ring is known.

![Fig. 6.1. Ring method circuit for six resistors.](image)

The automated measurement system described in this report is based on a ring of six resistors \( (n = 1) \) as shown in Fig. 6.1. Two of the resistors are working standards \( (R_3 \text{ and } R_6) \), one is a check standard \( (R_4) \), and the remaining three are unknowns or resistors under test \( (R_1, R_2, \text{ and } R_5) \). The three pairs of applied voltage points are designated as AA', BB', and CC'. Three subsets of voltage measurements are taken to determine the values of the unknown resistors.

### 6.1.1 First Subset of Voltage Measurements

The positions of the six nominally-equal resistors for the first subset of voltage measurements is shown in Fig. 6.2a. The supply voltage is applied across points A and A'. The voltage differences measured by the DVM across opposite potential terminals of the resistors are designated \( V_1 \) through \( V_6 \). These voltages are the average of measurements for the two directions of current; this procedure reduces errors caused by thermal emf's in the critical low-voltage circuits. From this first subset of voltage measurements, one obtains three linear equations of the form analogous to the equations derived for comparing two voltage dividers [14]. These equations, neglecting higher order terms, are:
\[
\frac{R_1}{R_2} \approx \frac{R_5}{R_4} \left( 1 + \frac{V_2 - V_1 - V_4 - V_3}{V} \right)
\]
\[
\frac{R_1}{R_3} \approx \frac{R_5}{R_4} \left( 1 + \frac{V_2' - V_1' - V_6 - V_5}{V} \right)
\]
\[
\frac{R_2}{R_3} \approx \frac{R_5}{R_4} \left( 1 + \frac{V_4' - V_3' - V_6' - V_5'}{V} \right)
\]

where \(V\) is the nominal voltage across a resistor (1/3 the applied voltage), and the \(V'\)'s (e.g., \(V'_1\), \(V'_2\), etc.) are repeated measurements of that particular voltage difference to ensure independence. The measurements are made in the sequence implied by the above equations to minimize errors caused by changes in interconnection and lead resistances.

![Diagram](image_url)

Fig. 6.2. Subsets of voltage measurements for ring method.
6.1.2 Second Subset of Voltage Measurements

The positions of the six resistors, after the supply voltage connections are rotated to points B and B', are shown in Fig. 6.2b. In this figure, the voltage differences measured between opposite terminals of the resistors are designated $V_7$ through $V_{12}$. The measurement procedure is analogous to that described in sec. 6.1.1 for obtaining the first subset of voltage measurements. The three linear equations obtained from the second subset of measurements are:

\[
\frac{R_2}{R_3} \approx \frac{R_1}{R_6}
\left(1 + \frac{V_8 - V_7 - V_{10} - V_9}{V}\right)
\]

\[
\frac{R_2}{R_4} \approx \frac{R_1}{R_5}
\left(1 + \frac{V_8' - V_7' - V_{12} - V_{11}}{V}\right)
\]

\[
\frac{R_3}{R_4} \approx \frac{R_6}{R_5}
\left(1 + \frac{V_{10} - V_9' - V_{12} - V_{11}'}{V}\right),
\]

where higher order terms have been neglected.

6.1.3 Third Subset of Voltage Measurements

The positions of the six resistors for the third subset of voltage measurements are shown in Fig. 6.2c. The supply voltage connections are rotated to points C and C'. The voltage differences measured between opposite resistor terminals are designated $V_{13}$ through $V_{18}$. The three linear equations, neglecting higher order terms, obtained from these voltage measurements are:

\[
\frac{R_3}{R_4} \approx \frac{R_2}{R_1}
\left(1 + \frac{V_{14} - V_{13} - V_{16} - V_{15}}{V}\right)
\]

\[
\frac{R_3}{R_5} \approx \frac{R_2}{R_6}
\left(1 + \frac{V_{14} - V_{13} - V_{18} - V_{17}}{V}\right)
\]

\[
\frac{R_4}{R_5} \approx \frac{R_1}{R_6}
\left(1 + \frac{V_{16} - V_{15} - V_{18} - V_{17}'}{V}\right)
\]

6.2 Data Analysis

From the three subsets of measurements for the different connections of the applied voltage, one obtains a set of nine linear equations. These equations can be expressed in terms of the resistor
corrections using the relationship:

\[ R_x = R'(1 + c_x), \]

where \( x \) is an integer from 1 to 6 denoting the resistor number, \( R_x \) is the value of the resistor, \( R' \) is its nominal value, and \( c_x \) is its correction. The nine linear equations can now be written as:

\[
\begin{align*}
+ c_1 - c_2 + c_3 - c_6 &\approx [(V_2 - V_5)/V] - [(V_4 - V_3)/V] = d_1 \\
- c_1 + c_3 - c_4 + c_6 &\approx [(V'_1 - V'_2)/V] - [(V_5 - V_6)/V] = d_2 \\
+ c_2 - c_3 + c_4 - c_5 &\approx [(V'_4 - V'_3)/V] - [(V'_6 - V'_5)/V] = d_3 \\
- c_1 + c_2 - c_3 + c_6 &\approx [(V_8 - V_5)/V] - [(V_{10} - V_9)/V] = d_4 \\
+ c_1 - c_2 + c_4 - c_3 &\approx [(V'_7 - V'_8)/V] - [(V_{11} - V_{12})/V] = d_5 \\
+ c_3 - c_4 - c_5 + c_6 &\approx [(V'_{10} - V'_{9})/V] - [(V'_{12} - V'_{11})/V] = d_6 \\
+ c_1 - c_2 + c_3 - c_4 &\approx [(V_{14} - V_{13})/V] - [(V_{16} - V_{15})/V] = d_7 \\
+ c_3 - c_2 + c_5 - c_6 &\approx [(V'_{13} - V'_{14})/V] - [(V'_{17} - V'_{16})/V] = d_8 \\
- c_1 + c_4 - c_5 + c_6 &\approx [(V'_{16} - V'_{15})/V] - [(V'_{18} - V'_{17})/V] = d_9 
\end{align*}
\]

where the \( d \)'s are calculated from the voltage measurements. These equations can be represented in matrix notation as:

\[
\begin{bmatrix}
1 & -1 & 0 & 0 & 1 & -1 \\
-1 & 0 & 1 & -1 & 0 & 1 \\
0 & 1 & -1 & 1 & -1 & 0 \\
-1 & 1 & -1 & 1 & 0 & 1 \\
1 & -1 & 0 & 1 & -1 & 0 \\
0 & 0 & 1 & -1 & 1 & -1 \\
1 & -1 & 1 & -1 & 0 & 0 \\
0 & 1 & -1 & 0 & 1 & -1 \\
-1 & 0 & 0 & 1 & -1 & 1
\end{bmatrix}
\begin{bmatrix}
c_1 \\
c_2 \\
c_3 \\
c_4 \\
c_5 \\
c_6 \\
d_1 \\
d_2 \\
d_3 \\
d_4 \\
d_5 \\
d_6 \\
d_7 \\
d_8 \\
d_9
\end{bmatrix}
\]

The restraint vector is given by:
\( \mathbf{c_s} = |0\ 0\ 1\ 0\ 0\ 1|, \)

where \( \mathbf{c_s} \) is the summation of the corrections for the two standard resistors \( R_3 \) and \( R_5 \). The solution to obtain the corrections to the other resistors can be calculated using a linear least-squares analysis routine [15].

### 6.3 Guard Network

This measurement system’s voltage source and DVM can have leakage resistances to ground on the order of \( 10^{10} \Omega \) to \( 10^{11} \Omega \). These leakage resistances effectively shunt parts of the ring circuit and result in significant measurement errors at the 100 kΩ level and above. To reduce these errors, an active guard network is used to drive the guard terminal of the DVM at nearly the same potential as its input terminals. The guard network consists of three resistors in series that are connected across the output of the voltage source as shown in Fig. 6.3.

![Fig. 6.3. Ring method with guard circuit.](image)

### 7. TERAHOMETER

Above \( 10^{10} \Omega \) there are better methods than using the Wheatstone bridge system to measure standard resistors. At these higher resistance levels, the accuracy, sensitivity, and detector response of the Wheatstone bridge suffers significantly from the lack of stable working standards, increased system noise, and stray-capacitance effects. Other methods based on the charging or discharging of resistor-capacitor networks overcome most of these limitations. In particular, the teraohmmeter, an instrument based on an analog integrator technique, was developed to measure
resistances above $10^9 \, \Omega$ [16]. At NIST a commercially-built terahmmeter is used to measure high-value standard resistors out of the range of the Wheatstone bridge system ($\Delta R > 5000$ ppm), or above $10^{10} \, \Omega$ by comparison with NIST working standards. To do this at $10^{11} \, \Omega$ and $10^{12} \, \Omega$, the 10/1 and 100/1 ratios of the terahmmeter must be used.

### 7.1 Description of System

A simplified block diagram is shown in Fig. 7.1, where $R_x$ is the test resistor and $C$ is a feedback capacitor connected across amplifier $A$ to form an integrator. The change in the output voltage, $\Delta v_o$, of the integrator over a time interval $\Delta t$ can be expressed as

$$\Delta v_o = -(v_i/R_x C) \Delta t$$

or

$$R_x = -(1/C) (v_i/\Delta v_o) \Delta t,$$

where $v_i$ is the constant applied voltage across $R_x$. Note that a working standard resistor is not needed to determine $R_x$ as is the case with the Wheatstone bridge system. Only $C$, $\Delta t$, and the ratio of voltages ($v_i/\Delta v_o$) need to be known accurately. In practice, the quantity $v_i/\Delta v_o$ is determined by a ratio of low-value resistors in a divider chain that is energized by a constant current source. The input stage of the terahmmeter, including capacitor $C$ and switch $S$ along with the input connectors, are guarded to minimize the effects of leakage paths across these critical components. A more detailed description and analysis of this circuit is given in ref. [16].

![Fig. 7.1. Block diagram of terahmmeter.](image)

### 8. RESISTANCE SCALING

The U.S. representation of the ohm ($\Omega_{NIST}$) is based on the quantum Hall effect. Periodically, the
five 1 Ω working standards are compared to the quantized Hall resistance, usually at the 6453.20 Ω level (Hall plateau where the integer of the quantum Hall state equals 4) [3]. The values and/or drift factors of the 1-Ω working standards are adjusted if their values in terms of the quantized Hall resistance differ by more than 0.02 ppm for two or more intercomparisons. The resistance scale is extended upward in multiple decade values from this 1 Ω level, up to and including the $10^{10}$ Ω level, using NIST-built transfer standards or Hamon boxes [6] that provide accurate ratios of 10/1 and 100/1. Below the 1 Ω level, the resistance scale is extended downward to the $10^{-4}$ Ω level in submultiple decade values using current comparator ratios of 10/1, 100/1, and 1000/1. Above $10^{10}$ Ω, the resistance scale is extended upward to the $10^{12}$ Ω level using the 10/1 and 100/1 ratios of the teraohmmeter.

### 8.1 Quantized Hall Resistance Scaling

The quantized Hall resistance, at the 6453.20 Ω level, of a Hall device operated at a temperature below 4 K and in a magnetic flux density around 6 T, is compared to a group of 6453.2 Ω wire-wound reference resistors using an automated potentiometric measurement system [17]. The design of these reference resistors is a series-parallel combination of card-type Evanohm resistors, sealed in a silicone-fluid-filled container, and placed in a portable NIST-built temperature-regulated air bath enclosure. These reference resistors are then used to calibrate the five 1 Ω working standards in terms of the quantized Hall resistance. This is done in two stages: the first stage uses a 6453.20 Ω to 100 Ω transfer standard designated as HQHA; the second stage uses a 100 Ω to 1 Ω transfer standard designated as H10. Figure 8.1 is a block diagram indicating the steps in scaling from the quantized Hall resistance to the 1 Ω working group.

![Fig. 8.1. Resistance scaling from QHR to 1 Ω.](image)

Transfer standard HQHA contains nine series-connected card-type resistors sealed in an aluminum box filled with silicone fluid. The first eight resistors have a nominal value of 800 Ω each, and the ninth has a value of 53.20 Ω to make the total resistance equal to 6453.20 Ω. Transfer
standard H10 contains ten series-connected Rosa-type 10 \( \Omega \) resistance elements. These resistance elements are sealed in brass cans filled with a silicone heat sink compound. Both transfer standards are operated in oil baths, temperature controlled at \((25.000 \pm 0.003) ^\circ \text{C}\).

The 6453.20 \( \Omega \) reference resistors, whose values were assigned based on the quantized Hall resistance, are compared to transfer standard HQHA using a guarded, resistance ratio-arm bridge. Next, HQHA's eight 800-\( \Omega \) resistors connected in parallel is compared to transfer standard H10, connected in its series configuration of 100 \( \Omega \), using the automated 100 \( \Omega \) DCC system. The 53.20 \( \Omega \) section of HQHA is compared to H10 using either an automatic NIST resistance thermometer bridge or a DCC resistance bridge. The 53.20 \( \Omega \) measurement is not very critical since it only represents \(\approx 0.82\%\) of the total resistance of HQHA. At this stage, the value of H10 is known in terms of the quantized Hall resistance. Finally H10, connected in its parallel configuration of 1 \( \Omega \), is compared to the 1 \( \Omega \) working standards using the automated 1 \( \Omega \) DCC system. The result of these measurements is an assignment of the mean value of the 1 \( \Omega \) working standards based on the quantized Hall resistance.

8.2 Transfer Standards

NIST transfer standards provide accurate ratios of 10/1 and 100/1 for extending the use of \( \Omega_{\text{NIST}} \) to higher resistance levels. The main advantage of transfer standards is that they are calibrated at one resistance level and are then used with equal accuracy at a different resistance level as a short-term working standard. In a series of measurements using such transfer standards, the \( \Omega_{\text{NIST}} \) can be extended in multiple decade values up to \(10^{10} \Omega\). Neither the absolute accuracy nor long-term stability of the resistors in the transfer standards have any effect on the measurement accuracy. The concerns with the transfer standards are only with the short-term stability required to complete the measurements. The transfer accuracy is ensured through proper design and various operational tests.

![Diagram of a typical transfer standard.](image)

Fig. 8.2. Diagram of a typical transfer standard.
Figure 8.2 is a schematic diagram of a typical NIST transfer standard. It consists of 10 nominally-equal resistors connected permanently in series by means of "tetrahedral" junctions [18]. Each junction has two current and two potential terminations, and the 4-terminal resistance of each junction is designed to be zero. If necessary, the junctions are adjusted to have a negligible effect on the transfer accuracy. The degree of adjustment depends on the nominal value of the main resistors. Figure 8.3 shows the transfer standard connected in a parallel mode using special fixtures. The potential circuit of the fixtures contains "fan" resistors for suppressing connection errors. The series and parallel resistances of a transfer standard are:

\[ R_s = N \cdot R(1 + c) , \]

and

\[ R_p = (R/N)(1 + c + f(c^2)) , \]

where \( R_s \) and \( R_p \) are the series and parallel resistances, respectively, \( N \) equals the number of resistors, \( R \) is the resistance of a single coil, \( c \) is the correction to the nominal value for the series mode, and \( f(c^2) \) includes negligible higher-order terms. For a transfer standard having 10 main resistors,
\[ \frac{R_s}{R_p} = N^2 = 100 \]

Figure 8.4 shows the transfer standard connected in a series-parallel mode for a 10/1 ratio. In this configuration only nine of the ten resistors are used. The series-parallel resistance \( R_{sp} \) is

\[ R_{sp} = R(1 + c + 0.1(c_{sp} - c_x)) \],

where \( c_{sp} \) is the correction of the nine resistors in the series-parallel configuration and \( c_x \) is the correction of the tenth resistor. The resistance ratio is

\[ \frac{R_{sp}}{R_p} = 10(1 + 0.1(c_{sp} - c_x)) \].

Figures 8.5 and 8.6 are block diagrams of the NIST scaling process from 1 \( \Omega \) to 10\(^6\) \( \Omega \) and 10\(^4\) \( \Omega \) to 10\(^{10}\) \( \Omega \) using transfer standards. The "P", "S", and "SP" notation indicates whether the transfer standard is in the parallel, series, or series-parallel mode, respectively. Arrows indicate 1/1 comparisons made and unless otherwise labelled, boxes represent NIST working standards at various resistance levels.

Fig. 8.4. Series-parallel connection for transfer standard.
Fig. 8.5. Resistance scaling from 1 Ω to 1 MΩ.

Fig. 8.6. Resistance scaling from 10 kΩ to 10 GΩ.
8.3 DCC Ratios

It is difficult to construct accurate transfer standards with single coil resistances $< 10 \ \Omega$. The major difficulties are with the adjustment and stability of the resistances associated with the tetrahedral junctions and fan resistors. Therefore, to extend $\Omega_{NIST}$ below 1 $\Omega$, NIST uses DCC ratios of 10/1, 100/1, and 1000/1. The high ratio accuracies of the DCC results from the use of multiple-winding, high-permeability toroidal transformers. The DCC ratio is insensitive to lead resistances or current level. The ratio accuracy and ratio linearity of a DCC is checked without the use of any external equipment with a resolution better than 1 part in $10^7$ [7]. The ratio accuracy can also be checked by the use of two higher-valued standard resistors whose values are based on transfer standards.

8.4 Teraohmmeter Ratios

Resistors with nominal values of $10^{11} \ \Omega$ and $10^{12} \ \Omega$ are measured using the teraohmmeter calibrated against a $10^{10} \ \Omega$ working standard. The uncertainties of the 10/1 and 100/1 ratios of the teraohmmeter are less than those associated with transfer standards at the $10^{11} \ \Omega$ and $10^{12} \ \Omega$ levels. High-resistance, film-type resistors at these resistance levels have large drift rates and exhibit large TCR's and voltage coefficients of resistance (VCR's).

9. MEASUREMENT UNCERTAINTY

The expanded or overall uncertainty assigned to the measured value of resistance of standard resistors, as given in a NIST Report of Calibration, is equal to

$$2 \sqrt{s^2 + \Sigma u^2(i)}$$  \hspace{1cm} (9.1)

where $s$ is the estimated Type A standard uncertainty based on the pooled standard deviation from a large population of individual measurements, and $u(i)$ is the estimated Type B standard uncertainty for each known component quantified by a standard deviation. The coverage factor 2 used at NIST is consistent with international practice [19]. The method of combining uncertainties is based on the approach recommended by the International Bureau of Weights and Measures (BIPM) [20]. The reported expanded uncertainty contains no allowances for the long-term drift of the resistor under test, for the possible effects of transporting the standard resistor between laboratories, nor for measurement uncertainties in the user’s laboratory. As mentioned in section 1, this reported uncertainty also does not include any allowance for the possible error between the U.S. representation of the ohm and the ohm as defined by SI. The appendix contains samples of some NIST Reports of Calibration. Also included in the appendix is a copy of an information sheet on resistance measurements that is included with each report.

The calibration of a resistor involves the comparison of the test resistor with NIST working standards using one of several measurement systems previously described. The critical elements contributing to the measurement uncertainty are the standard resistors (working, transfer, check, and test), and the measurement systems.
9.1 Model - Standard Resistors

Experimental data indicate that the resistance of a standard resistor is a function of environmental parameters, time, and load level. To some degree, the environmental parameters can be controlled and/or measured in order to correct for their influence. Assuming the resistor is not subjected to mechanical disturbances, the environmental parameters of main concern that contribute to the measurement uncertainty are temperature, pressure, and humidity.

The change of resistance of a resistor with time cannot be controlled and it is difficult to measure. This characteristic is inherent in a resistor as a result of its design and construction. The four major sources of this resistance change are metallurgical processes, oxidation, bending stresses and strains, and tensile stresses and strains. The combination of these effects results in a unique stability curve for each resistor. Although such curves may exhibit a shape of an exponential decay function, evidence indicates that over the short term the stability curves of resistors can be modeled as linear functions. This evidence includes the results of the absolute determinations of resistance in SI units and, more recently, the monitoring of resistance by the quantum Hall effect by NIST and other national laboratories. In addition, the run-sequence plots of NIST working and check standard resistors over a time span of several years are fitted best by straight lines having slopes of different signs as well as different magnitudes. It is inferred that for standard resistors not mechanically disturbed, it is highly likely they will drift linearly, and with sufficient data their value can be predicted to a high degree of certainty using a linear least-squares analysis.

The change in resistance of a standard resistor as a function of load level results from: 1) the $i^2R$ or self-heating of a resistor, and/or 2) the voltage dependence of a resistor. The former is primarily a function of the TCR and the surface area of the resistance material, while the latter is significant for film-type resistors. Errors caused by loading are negligible for resistor calibrations from 1 \( \Omega \) to 1 M\( \Omega \) since the measurements are done at the same power level of 10 mW. For calibrations below 1 \( \Omega \), the power dissipation in the test resistor may be as large as 100 mW in order to have adequate measurement sensitivity. These low-resistance standards are designed to dissipate higher power (\( \leq 1 \text{ ppm/W} \)), and consequently, errors caused by loading at \( \leq 100 \text{ mW} \) are negligible. Above 1 M\( \Omega \), errors due to loading are usually significant for film-type resistors which exhibit large VCR's. The absolute values of VCR's for film-type resistors can be as large as 100 ppm/V or more.

9.2 Model - Measurement Systems

The model for a measurement system for calibrating resistors is broken down into the following main components: the ratio instrument; dummy resistor; detector circuit; the operator and/or computer; procedures and conditions of test; and auxiliary equipment. The measurement technique is usually that of a 1/1 comparison, in which a test resistor or check standard is replaced or substituted by a working standard of the same nominal value (substitution technique). The ratio instrument (dc comparator, resistance-ratio bridge, or teraohmmeter) is then relied upon to determine only the difference between resistors, and this difference need not be accurately measured. For example, if the test resistor and working standard differ by 100 ppm, the difference need only to be determined to 1\% to achieve a 1 ppm measurement of the test resistor.
based on the working standard. The important sources of error to consider are: stability and linearity of the ratio instrument; stability of the dummy resistor (if used); detector resolution; and changing thermal emfs in the detector circuit. Of secondary importance are errors resulting from leakage currents, lead resistances, contact resistances and test conditions which tend to cancel or are significantly reduced when using the substitution technique. All other sources of error associated with the operator, the computer (if used), procedures, and auxiliary equipment are believed to be negligible in the systems currently at use at NIST.

The two most critical measurement systems will be examined in more detail than the others. They are the 1 Ω DCC potentiometer system and the 10 kΩ resistance-ratio system. These two resistance levels are critical because 1 Ω and 10 kΩ commercial resistors are of the highest quality, and as a consequence, most NIST resistor calibrations with the smallest NIST uncertainties are done at these two resistance levels. The first thing to establish is whether or not the measurement systems are "in statistical control." To determine this the data history of the measurement processes are examined. Figures 9.1 and 9.2 are the control charts for the NIST 1 Ω check standards C84 and C93 from 1982 to 1991. The check standards have significant changes both in measured values and subsequent drift rates after January 1, 1990 as a result of implementing the new representation of the ohm (see section 1). The y-axis scales on the right are offset by -1.69 ppm, which is the change applied to the numerical values of all NIST reference resistors on January 1, 1990. Note that the scale on the left refers to measurements made before January 1, 1990, while the scale on the right refers to measurements made after January 1, 1990. Each filled-circle symbol represents a mean of measurements taken over a period of one month. The number of monthly measurements varied from a minimum of 4 to a maximum of 43. The solid line is a fitted least-squares line of the data. The upper and lower dashed lines about this trend line correspond to confidence level limits of ± 0.02 ppm or a 95% confidence interval. The residuals from the linear regression fits for C84 and C93 are plotted in Fig. 9.3 and Fig. 9.4 as histograms.

The 10 kΩ measurement system has one air-type check standard, C1410, and one oil-type check standard, C1419. Figures 9.5 and 9.6 are control charts of the data during 1990 for C1410 and C1419, respectively. Each filled-circle symbol represents a single point for a measurement run. Upper and lower dashed lines corresponding to confidence level limits of ± 0.02 ppm are shown about the solid fitted line. Histograms of the residuals from the linear regression fits are shown in Fig. 9.7 and Fig. 9.8.

9.3 Type A Standard Uncertainty

The Type A standard uncertainty, s, assigned to the value of a standard resistor is based on an estimate of the standard deviation of the measurement process for that particular resistance level. The s is calculated from a sampling of at least 15 measurement runs. First, the individual estimates of the standard deviations of one or more test resistors are calculated from repetitive measurements taken during a single test run. If more than one test resistor is measured during a run, a pooled value of the estimate of the standard deviation is obtained for that particular run. Finally, the pooled values for at least 15 measurement runs, over a several month interval, are themselves pooled to obtain a realistic estimate of s for the measurement process. The Type A standard uncertainties for the various resistance levels are listed in Tables 9.1 and 9.2.
Fig. 9.1. Control chart for 1 Ω check standard C84.

Fig. 9.2. Control chart for 1 Ω check standard C93.
Fig. 9.3. Histogram for 1 Ω check standard C84.

Fig. 9.4. Histogram for 1 Ω check standard C93.
Fig. 9.5. Control chart for 10 kΩ check standard C1410.

Fig. 9.6. Control chart for 10 kΩ check standard C1419.
Fig. 9.7. Histogram for 10 kΩ check standard C1410.

Fig. 9.8. Histogram for 10 kΩ check standard C1419.
9.4 Type B Standard Uncertainty

The Type B standard uncertainty, \( u \), assigned to the value of a standard resistor is determined by combining in quadrature (root-sum-square or RSS) the estimated standard uncertainties from each known individual component of the measurement process for that particular resistance level. Each Type B standard uncertainty evaluated is either equal to \( 1/\sqrt{3} \) times an estimated bound for a uniform distribution, or an estimate by the experimenter. For analysis, the sources of error for the Type B standard uncertainties are separated into two main categories: 1) those associated with the standard resistors, and 2) those associated with the measurement systems. Tables 9.3 and 9.4 list the estimates of the Type B standard uncertainties for the various resistance levels along with the RSS subtotal for each level.

9.4.1 Standard Resistors

This category includes the sources of error associated with the Type B standard uncertainties assigned to the values of the working standards. These sources of error are subdivided into three main areas: a) maintaining the working NIST ohm in terms of the quantized Hall resistance, b) scaling from the NIST ohm to other resistance levels using transfer standards, and c) the environmental and drift factors of the working standards themselves.

a) NIST Ohm - The 1 \( \Omega \) working group of five Thomas-type resistors preserves the NIST ohm between QHR comparisons. Since 1986 the combined standard uncertainty of comparing this working group with the QHR has been within 0.02 ppm. Reference [3] lists in detail the Type A and Type B standard uncertainties for this process.

b) Transfer Standards - As described in section 8.2, transfer standards provide accurate ratios of 10/1 and 100/1 for extending the use of \( \Omega_{\text{NIST}} \) at the 1 \( \Omega \) level to higher resistance levels. One of the components under this listing in Tables 9.3 and 9.4 includes the combined standard uncertainty of the transfer standard used to assign a value, based on \( \Omega_{\text{NIST}} \), to a particular transfer standard. For example (see Figs. 8.5 and 8.6), the Type B uncertainties for transfer standards H1k, H100k, H10M and H1G include the combined standard uncertainties of transfer standards H10, H1k, H100k, and H10M, respectively.

Also included in the Type B uncertainties under this listing in Tables 9.3 and 9.4 are estimates of the ratio errors of the various transfer standards. An estimate of the ratio error of a particular transfer standard can be calculated from measurements of: 1) the differences among the main resistors, 2) the finite resistances of the 4-terminal junctions, and 3) the resistances of the paralleling fixtures including fan resistors. Additional checks are made by using paralleling fixtures having different values of fan resistors. If loading errors are small, the 10/1 ratio can be verified by comparing the results obtained from the series-parallel measurements as described in section 8.2 against the mean value of the main resistors obtained by measuring each individual resistor. Leakage resistances within transfer standards of high resistance can cause significant ratio errors. Surface leakage is eliminated by guarding the terminations of these transfer standards. Internal leakage, mainly caused by the properties of the fluid sealed in the box (mineral oil or silicone fluid), is monitored to determine if it is within acceptable limits.
An independent check of the 100/1 ratio of \( H_{10} \) was made against the 100/1 ratio of a cryogenic current comparator (CCC). CCC's [21] utilize superconducting shields and SQUID's to provide current ratios with accuracies of better than 1 part in \( 10^6 \), orders of magnitude better than room-temperature DCC's. An automated CCC resistance bridge [22] using low-noise and stable current sources [23] was used to verify that the 100/1 ratio of \( H_{10} \) is exact to within 0.01 ppm.

c) Working Standards - A set of working standards exists at each decade resistance level. The Type B standard uncertainty assigned to a particular set of working standards is based, primarily, on a combination of errors associated with not knowing exactly the temperature, humidity, pressure, and drift effects of that particular set of standards during a measurement run.

### 9.4.2 Measurement Systems

Listed in Tables 9.3 and 9.4 are the estimated Type B standard uncertainties for the various sources of systematic error associated with the seven measurement systems for each resistance level. The sources of error are subdivided into six classifications as follows:

- **a) Ratio** - These include the errors caused by the instability and non-linearity of the turns ratio of a dc comparator or the ratio arms of a resistance bridge. Usually, the short-term ratio instability results from changes in the test or environmental conditions during a measurement run, and this can be monitored, to some extent, by measuring a check standard at the beginning and at the end of the run. For most systems the ratio linearity can be determined and corrections applied if the errors are significant.

  Of greater significance are ratio errors for measurement systems that use ratios greater than unity. This is the circumstance for measurements < 1 \( \Omega \) using the 10/1, 100/1, and 1000/1 ratios of the dc comparator bridge and/or for measurements > \( 10^{10} \) \( \Omega \) using the 10/1 and 100/1 ratios of the teraohmmeter. The dc comparator bridge has provisions for self-calibrating its 10/1, 100/1, and 1000/1 ratios. The accuracy of these ratios has also been verified by comparison to resistance ratios previously checked against transfer standards, e.g., 10 \( \Omega/1 \) \( \Omega \), 100 \( \Omega/1 \) \( \Omega \), and 1000 \( \Omega/1 \) \( \Omega \).

- **b) Stability** - This includes the error caused by the short-term instability of a dummy resistor in a bridge or DCC measurement system. In the DCC systems it also includes an estimate of the instabilities of the constant current sources that may occur during a measurement run.

- **c) Detector** - The error in the detector circuit is associated with the stability of the zero setting of the detector, the stability of the thermal emfs in the circuit, and the resolution of the detector system. For the automated dc comparator potentiometer systems, it also includes the errors caused by the non-linearity of the detector and feedback circuits.

- **d) Ambient** - This category contains: 1) the estimated error of the temperature of the customer's resistor from the indicated temperature of a particular bath, 2) the estimated error of knowing the bath temperature from the true temperature as derived from the ITS-90, 3) the estimated error of the pressure measurement for a Thomas-type 1 \( \Omega \) or air-type 10 k\( \Omega \) measurement, and 4) the estimated error of the humidity measurement for the high resistance measurements. The critical temperature measurements are made using a calibrated PRT. The secondary temperature

42
measurements are made with digital thermistor thermometers that are calibrated against a PRT.

e) Connection - This includes the estimated errors associated with: 1) the change in the lead resistances of a resistance-ratio bridge network during a test run, 2) the location of the potential points for a Rosa-type standard resistor, and 3) the variation in resistance of the mercury contacts on two-terminal measurements. The resistivity of the terminal arms of a commercial Rosa-type standard resistor is ≈3.5 μΩ/cm. The maximum error of positioning a potential lead to a screw-type terminal is about 0.5 cm for each arm. Therefore, an uncertainty of 3.5 μΩ is allotted for this systematic error. The maximum day-to-day variations in a normal (surfaces slightly askew) mercury contact is estimated to be ≈5 μΩ.

f) Leakage - This uncertainty results from the possible deterioration of the insulating properties of the mineral oil and/or resistor headers. The insulation of the mineral oil in the baths is checked periodically and the oil is changed if it falls below 5 x 10^{13} Ω.

Table 9.1. Standard uncertainties in ppm

<table>
<thead>
<tr>
<th>Resistance (Ω)</th>
<th>Type A (μ)</th>
<th>Type B (μ)</th>
<th>RSS Total (μ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(T)</td>
<td>0.008</td>
<td>0.021</td>
<td>0.022</td>
</tr>
<tr>
<td>10^4(S)</td>
<td>0.008</td>
<td>0.037</td>
<td>0.038</td>
</tr>
<tr>
<td>10^4</td>
<td>1.5</td>
<td>5.2</td>
<td>5.4</td>
</tr>
<tr>
<td>10^3</td>
<td>1.0</td>
<td>2.1</td>
<td>2.3</td>
</tr>
<tr>
<td>10^2</td>
<td>0.5</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td>10^1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>10</td>
<td>0.01</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>10^2</td>
<td>0.01</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>10^3</td>
<td>0.02</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>10^4</td>
<td>0.02</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>10^5</td>
<td>0.04</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>10^6</td>
<td>0.04</td>
<td>1.44</td>
<td>1.44</td>
</tr>
</tbody>
</table>
Table 9.2. Standard uncertainties in ppm

<table>
<thead>
<tr>
<th>Resistance (Ω)</th>
<th>Type A (s)</th>
<th>Type B (μ)</th>
<th>RSS Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^7(WB)</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>10^7(T)</td>
<td>40</td>
<td>33</td>
<td>52</td>
</tr>
<tr>
<td>10^8(WB)</td>
<td>4</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>10^8(T)</td>
<td>40</td>
<td>104</td>
<td>111</td>
</tr>
<tr>
<td>10^9(WB)</td>
<td>40</td>
<td>37</td>
<td>54</td>
</tr>
<tr>
<td>10^9(T)</td>
<td>80</td>
<td>206</td>
<td>221</td>
</tr>
<tr>
<td>10^10(T)</td>
<td>200</td>
<td>256</td>
<td>325</td>
</tr>
<tr>
<td>10^11(T)</td>
<td>200</td>
<td>256</td>
<td>325</td>
</tr>
<tr>
<td>10^12(T)</td>
<td>200</td>
<td>532</td>
<td>568</td>
</tr>
</tbody>
</table>

WB - Wheatstone Bridge
T - Teraohmmeter
Table 9.3. Type B standard uncertainties in ppm

| Resistance (Ω) | Standards | | System |
|----------------|-----------|-----------------|--------|--------|--------|--------|--------|--------|
|                | Ω(NIST)   | Transfer | Working | Ratio  | Stability | Detector | Ambient | Connection | Leakage |
| 1(T)           | 0.020     | 0        | 0.005   | 0.002  | 0.002     | 0.002    | 0.004   | 0.002     | 0.001   |
| 10^4(S)        | 0.020     | 0.020    | 0.020   | 0.005  | 0.005     | 0.005    | 0.005   | 0.005     | 0.005   |
| 10^{-4}        | 0.02      | 5        | 1       | 0.1    | 0.4       | 1        | 0.5     | 0.1       | 0       |
| 10^{-3}        | 0.02      | 2        | 0.6     | 0.1    | 0.2       | 0.2      | 0.4     | 0.1       | 0       |
| 10^{-2}        | 0.02      | 1        | 0.4     | 0.1    | 0.2       | 0.2      | 0.4     | 0.1       | 0       |
| 10^{-1}        | 0.02      | 0.1      | 0.2     | 0.1    | 0.2       | 0.2      | 0.3     | 0.1       | 0       |
| 1              | 0.02      | 0.1      | 0.1     | 0.1    | 0.2       | 0.2      | 0.2     | 0.2       | 0       |
| 10             | 0.02      | 0.1      | 0.1     | 0.1    | 0.2       | 0.2      | 0.2     | 0.2       | 0       |
| 10^2           | 0.02      | 0.05     | 0.1     | 0.05   | 0.1       | 0.1      | 0.1     | 0.05      | 0       |
| 10^3           | 0.02      | 0.1      | 0.1     | 0.05   | 0.1       | 0.1      | 0.1     | 0.05      | 0       |
| 10^4           | 0.02      | 0.1      | 0.1     | 0.05   | 0.1       | 0.1      | 0.1     | 0.05      | 0.05    |
| 10^5           | 0.02      | 0.8      | 0.1     | 0.05   | 0.2       | 0.1      | 0.1     | 0.05      | 0.1     |
| 10^6           | 0.02      | 1        | 0.1     | 0.05   | 0.2       | 0.1      | 0.1     | 0.05      | 1       |
Table 9.4. Type B standard uncertainties in ppm

<table>
<thead>
<tr>
<th>Resistance (Ω)</th>
<th>Standards</th>
<th>System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ω(NIST)</td>
<td>Ratio</td>
</tr>
<tr>
<td>10^7(WB)</td>
<td>0.02</td>
<td>3</td>
</tr>
<tr>
<td>10^7(T)</td>
<td>0.02</td>
<td>3</td>
</tr>
<tr>
<td>10^6(WB)</td>
<td>0.02</td>
<td>5</td>
</tr>
<tr>
<td>10^6(T)</td>
<td>0.02</td>
<td>5</td>
</tr>
<tr>
<td>10^9(WB)</td>
<td>0.02</td>
<td>10</td>
</tr>
<tr>
<td>10^9(T)</td>
<td>0.02</td>
<td>10</td>
</tr>
<tr>
<td>10^10(T)</td>
<td>0.02</td>
<td>50</td>
</tr>
<tr>
<td>10^11(T)</td>
<td>0.02</td>
<td>50</td>
</tr>
<tr>
<td>10^12(T)</td>
<td>0.02</td>
<td>100</td>
</tr>
</tbody>
</table>

WB - Wheatstone Bridge

T - Teraohmmeter
Table 9.5. Resistance expanded uncertainties

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Resistance (Ω)</th>
<th>Power (mW)</th>
<th>Uncertainty (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>51130C</td>
<td>1(T)</td>
<td>10</td>
<td>0.05</td>
</tr>
<tr>
<td>51131C</td>
<td>$10^4$(S)</td>
<td>10</td>
<td>0.15</td>
</tr>
<tr>
<td>51132C</td>
<td>$10^4$</td>
<td>100</td>
<td>11</td>
</tr>
<tr>
<td>51133C</td>
<td>$10^3$</td>
<td>100</td>
<td>5</td>
</tr>
<tr>
<td>51134C</td>
<td>$10^2$</td>
<td>100</td>
<td>3</td>
</tr>
<tr>
<td>51135C</td>
<td>$10^1$</td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>51136C</td>
<td>1</td>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>51137C</td>
<td>10</td>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>51138C</td>
<td>$10^2$</td>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td>51139C</td>
<td>$10^3$</td>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td>51140C</td>
<td>$10^4$</td>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td>51141C</td>
<td>$10^5$</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>51142C</td>
<td>$10^6$</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>51143C</td>
<td>$10^7$(WB)</td>
<td>*</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>$10^7$(T)</td>
<td>*</td>
<td>140</td>
</tr>
<tr>
<td>51145C</td>
<td>$10^8$(WB)</td>
<td>*</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>$10^8$(T)</td>
<td>*</td>
<td>400</td>
</tr>
<tr>
<td>51147C</td>
<td>$10^9$(WB)</td>
<td>*</td>
<td>140</td>
</tr>
<tr>
<td></td>
<td>$10^9$(T)</td>
<td>*</td>
<td>700</td>
</tr>
<tr>
<td>51149C</td>
<td>$10^{10}$(T)</td>
<td>*</td>
<td>700</td>
</tr>
<tr>
<td>51151C</td>
<td>$10^{11}$(T)</td>
<td>*</td>
<td>700</td>
</tr>
<tr>
<td>51153C</td>
<td>$10^{12}$(T)</td>
<td>*</td>
<td>1400</td>
</tr>
</tbody>
</table>

*Customer specified voltages
WB - Wheatstone Bridge
T - Teraohmmeter
9.5 Reported Uncertainties

Table 9.5 lists the expanded or overall uncertainties for the various resistance levels that are given in a NIST Report of Calibration. These uncertainties are based on calculations using eq (9.1) with a coverage factor of 2 and the estimated Type A and Type B standard uncertainties listed in Tables 9.1 through 9.4. In most cases the reported uncertainties have been conservatively rounded upward. Table 9.5 lists the maximum power dissipation of a test resistor during calibration, except for resistances of $10^7$ Ω or above where the customer specifies the test voltage. Also listed in this table are the test numbers for each of the resistance levels.

10. QUALITY CONTROL

It is important that procedures be established to determine whether or not a measurement process is and remains in a state of control if the uncertainty that NIST assigns to the value of a standard resistor is to be meaningful. Three factors are examined for each calibration performed in order to insure the quality of the measurement value that is assigned to a standard resistor:

1. Standard deviations of the measurement run.
2. Control charts of check standards.
3. Data history of the standard resistor under test.

The computer hardcopy of the data analysis/results of a measurement run contains the individual standard deviations, calculated from repeated measurements during the run, for each resistor. If any standard deviation for a resistor exceeds the Type A standard uncertainty as listed in Tables 9.1 or 9.2, for that particular resistance level by a factor of 2, the measurement process is considered out of control, and appropriate steps are taken to correct the situation. Usually, the explanation for this out-of-control condition is an error on the part of the observer in recording or entering data, or a malfunction in a piece of equipment. The problem is corrected and the measurement test is repeated.

The computer hardcopy also lists the difference between the measured value and the predicted value for each of the check standards. The predicted value is calculated from a drift factor that is derived from a previous least-squares analysis of a linear fit for that check standard. If this difference or residual exceeds the Type A standard uncertainty by a factor of 2, it is flagged by an asterisk. It indicates that a potential problem may exist and that the other control indicators should be scrutinized. The over range of the residual may be caused by a lack of knowledge of the resistor model. The listing of the residual, in effect, is examining a point on the control chart for that check standard to determine if it is within acceptable limits. If necessary, a linear regression analysis computer program is run to generate the control chart for the particular check standard in question.

Finally, the data history of the standard resistor under test is examined if more than two measurement points exist. If the new value appears unreasonable (on the order of three times the residual standard deviation from a linear fit), it is considered an outlier and the source of the discrepancy is investigated. Additional tests are run to determine the stability of the resistor, and the customer is apprised of the situation.
11. ONGOING PROJECTS

Projects are underway in several areas of resistance measurements to improve the maintenance and dissemination of $\Omega_{\text{NIST}}$, to extend NIST calibration services to $10^{14}$ $\Omega$, and to develop a new calibration service for ac resistors. These projects include the following.

1) A QHR standard has been installed in the resistance calibration laboratory. It will become the national standard and replace the one kept in the QHR research facility. This will provide for more frequent monitoring of working standards, and also reduce the uncertainties associated with maintaining the ohm representation by eliminating the need to transfer resistors between laboratories as in the present procedure.

2) Two CCC resistance bridges have been constructed and are being used to verify Hamon-type resistance techniques from 1 $\Omega$ to 100 $\Omega$, 1 k$\Omega$, 6453.20 $\Omega$, and 10 k$\Omega$ [24]. Measurements comparing the 10/1, 64.532/1, and 100/1 ratios of CCC's to that of Hamon transfer ratios indicate agreement to $\approx$ 0.01 ppm, the practical limit of accuracy using Hamon transfer standards with conventional resistance bridges. The higher ratio accuracies and sensitivities of CCC bridges will make it possible to lower the uncertainties associated with resistance scaling.

3) Near completion is the evaluation of a prototype system that is an automated version of a capacitance discharge method for the measurement of high-value resistors. This system will provide a new calibration service for resistance standards from $10^{12}$ $\Omega$ to $10^{14}$ $\Omega$ or higher.

4) The design of an automated system for the measurement of special Evanohm 10 k$\Omega$ standard resistors has been completed, and a prototype system is under construction. The system will feature a guarded ratio bridge, and automatic switching of resistors using BPO connectors.

5) The design of a guarded scanner to be used with the ring method for calibrating resistors $\geq$ 100 k$\Omega$ is underway. This should reduce the leakage errors at these resistance levels, and therefore reduce the respective calibration uncertainties. A guarded scanner would also be used to completely automate the terahmmeter system for high-resistance measurements.

6) Under development is an automated ac Kelvin bridge for measuring the frequency dependence and phase angle of precision resistors from 1 $\Omega$ to the 1 M$\Omega$ level over the frequency range 10 Hz to 10 kHz [25]. The main ratio arms consist of 30-bit binary inductive voltage dividers.

12. ACKNOWLEDGMENTS

The authors wish to express their appreciation to our group leader, Norm Belecki, for his continued support and guidance in this area. We also would like to acknowledge our calibration coordinator, Denise Prather, for the many tasks she does in coordinating resistor calibrations with our customers and keeping this measurement service running smoothly. Finally, but not least, we would like to thank our secretary, Sharon Fromm, for performing the many secretarial duties that make our work easier.
13. REFERENCES


14. BIBLIOGRAPHY


15. APPENDIX

15.1 Typical Report of Calibration
15.2 Report for a 1 Ω Standard Resistor
15.3 Report for a 10 kΩ Standard Resistor
15.4 Report for a High-Value Standard Resistor
15.5 Information Sheet on Standard Resistors
REPORT OF CALIBRATION

STANDARD RESISTOR
Model , S/N

Submitted by

This standard resistor was calibrated on January 1, 1993 and its value of resistance is based on the results of measurements between this standard and those of NIST working standards calibrated in terms of the quantum Hall effect used to maintain the U. S. ohm.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Resistance</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.0 °C</td>
<td>0.999 987 6 Ω</td>
<td>1 ppm</td>
</tr>
</tbody>
</table>

The expanded uncertainty given above is based on the approach recommended by the International Bureau of Weights and Measures (BIPM) and is equal to

\[ 2 \sqrt{s^2 + \sum u^2(i)} \],

where \( s \) is the estimated Type A standard uncertainty based on the pooled standard deviation from a large population of individual measurements, and \( u(i) \) is the estimated Type B standard uncertainty for each known component quantified by a standard deviation. The coverage factor 2 used by NIST is consistent with international practice. Detailed information is given in NIST Technical Note 1298 which lists all uncertainty components and describes the method used to calculate this expanded uncertainty. This reported uncertainty contains no allowances for the long-term drift of the standard under test or for the possible effects of transporting this standard between laboratories.

For the Director,

Norman B. Belecki, Group Leader
Electricity Division
REPORT OF CALIBRATION

STANDARD RESISTOR

Model , S/N

Submitted by

This standard resistor was calibrated on January 1, 1993 and its value of resistance is based on the results of measurements between this standard and those of NIST working standards calibrated in terms of the quantum Hall effect used to maintain the U. S. ohm.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Pressure</th>
<th>Resistance</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.00 °C</td>
<td>101.660 kPa</td>
<td>0.999 998 76 Ω</td>
<td>0.05 ppm</td>
</tr>
</tbody>
</table>

The expanded uncertainty given above is based on the approach recommended by the International Bureau of Weights and Measures (BIPM) and is equal to

\[ 2 \sqrt{s^2 + \sum u^2(i)} , \]

where \( s \) is the estimated Type A standard uncertainty based on the pooled standard deviation from a large population of individual measurements, and \( u(i) \) is the estimated Type B standard uncertainty for each known component quantified by a standard deviation. The coverage factor 2 used by NIST is consistent with international practice. Detailed information is given in NIST Technical Note 1298 which lists all uncertainty components and describes the method used to calculate this expanded uncertainty. This reported uncertainty contains no allowances for the long-term drift of the standard under test or for the possible effects of transporting this standard between laboratories.

For the Director,

Norman B. Belecki, Group Leader
Electricity Division

Test No.
Reference:
Date:
REPORT OF CALIBRATION

STANDARD RESISTOR
Model , S/N
Submitted by

This standard resistor was calibrated on January 1, 1993 and its value of resistance is based on the results of measurements between this standard and those of NIST working standards calibrated in terms of the quantum Hall effect used to maintain the U. S. ohm.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Pressure</th>
<th>Resistance</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.0 °C</td>
<td>101.660 kPa</td>
<td>10 000.012 Ω</td>
<td>0.15 ppm</td>
</tr>
</tbody>
</table>

The expanded uncertainty given above is based on the approach recommended by the International Bureau of Weights and Measures (BIPM) and is equal to

\[ 2 \sqrt{s^2 + \Sigma u^2(i)} \],

where \( s \) is the estimated Type A standard uncertainty based on the pooled standard deviation from a large population of individual measurements, and \( u(i) \) is the estimated Type B standard uncertainty for each known component quantified by a standard deviation. The coverage factor 2 used by NIST is consistent with international practice. Detailed information is given in NIST Technical Note 1298 which lists all uncertainty components and describes the method used to calculate this expanded uncertainty. This reported uncertainty contains no allowances for the long-term drift of the standard under test or for the possible effects of transporting this standard between laboratories.

For the Director,

Norman B. Belecki, Group Leader
Electricity Division

Test No.
Reference:
Date:
REPORT OF CALIBRATION

STANDARD RESISTOR
Model  , S/N

Submitted by

This standard resistor was calibrated on January 1, 1993 and its value of resistance is based on the results of measurements between this standard and those of NIST working standards calibrated in terms of the quantum Hall effect used to maintain the U. S. ohm.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Humidity</th>
<th>Voltage</th>
<th>Resistance</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.0 °C</td>
<td>35 %</td>
<td>1 V</td>
<td>0.998 76 x 10^8 Ω</td>
<td>40 ppm</td>
</tr>
<tr>
<td>23.0 °C</td>
<td>35 %</td>
<td>10 V</td>
<td>0.998 65 x 10^8 Ω</td>
<td>40 ppm</td>
</tr>
</tbody>
</table>

The expanded uncertainty given above is based on the approach recommended by the International Bureau of Weights and Measures (BIPM) and is equal to

\[ 2 \sqrt{s^2 + \sum u^2(i)} , \]

where \( s \) is the estimated Type A standard uncertainty based on the pooled standard deviation from a large population of individual measurements, and \( u(i) \) is the estimated Type B standard uncertainty for each known component quantified by a standard deviation. The coverage factor 2 used by NIST is consistent with international practice. Detailed information is given in NIST Technical Note 1298 which lists all uncertainty components and describes the method used to calculate this expanded uncertainty. This reported uncertainty contains no allowances for the long-term drift of the standard under test or for the possible effects of transporting this standard between laboratories.

For the Director,

Norman B. Belecki, Group Leader
Electrical Division

Test No.
Reference:
Date:
GENERAL INFORMATION ON STANDARD RESISTORS

1. Representation of the SI Unit of Resistance

The International System of Units, abbreviated SI, is the unit system used throughout the world. In the SI, the unit of resistance ($R$) is the ohm ($\Omega$). In practice, the ohm, which sometimes is referred to as the "absolute ohm", is realized at NIST by means of the NIST calculable capacitor. However, commercial, industrial, and scientific requirements for the long-term repeatability and worldwide consistency of measurements of resistance often exceed the accuracy with which the SI unit for such measurements can be realized. To meet these severe demands, it has become necessary to establish a representation of the ohm that has superior long-term reproducibility and constancy than the present direct realizations of the ohm itself.

From January 1, 1948 until January 1, 1990, the U. S. representation of the ohm as maintained at NIST had been based on the mean resistance of reference groups of Thomas-type 1 $\Omega$ resistors whose mean values were thought to be constant with time. The discovery of the quantum Hall effect in 1980 by Klaus von Klitzing led to a method for basing a representation of the ohm on invariant fundamental constants. Quantized Hall resistance measurements at NIST indicated that the U. S. representation of the ohm was drifting at a rate of $(0.0529 \pm 0.0040)$ ppm/year. To eliminate this drift, starting on January 1, 1990, the U. S. representation of the ohm has been based on the quantum Hall effect in which a resistance is related to the fundamental constants $h/e^2$. The quantized Hall resistance, $R_H$, is defined as the quotient of the Hall voltage $V_H$ of the $i^{th}$ plateau to the current $I$ in the Hall device and is given by

$$R_H(i) = \frac{V_H(i)}{I} = R_K/i,$$

where $R_K$, the von Klitzing constant, is equal to $h/e^2$, and $i$ is an integer. The value of the U. S. representation of the ohm is consistent with the internationally accepted conventional value of the von Klitzing constant, i.e.,

$$R_K = 25\ 812.807\ \Omega,$$

exactly. This conventional value is believed to be consistent with the true SI value to within a 0.005 $\Omega$ assigned combined standard uncertainty which corresponds to a relative uncertainty of 0.2 ppm. Because this uncertainty is the same for all national laboratories and is not relevant for traceability to national standards, it is not included in the uncertainties given in NIST Reports of Calibration. However, its existence must be taken into account when the utmost consistency between electrical and nonelectrical measurements of the same physical quantity is required.

As a result of adopting the internationally accepted value for the von Klitzing constant, on January 1, 1990, the numerical values of all NIST reference resistors were decreased by 1.69 ppm. Likewise the assigned values of all standard resistors, based on NIST calibrations before January 1, 1990, should have been decreased by 1.69 ppm on January 1, 1990. For more information on the change in the U. S. ohm representation consult NIST Technical Note 1263 entitled "Guidelines for Implementing the New Representations of the Volt and Ohm Effective January 1, 1990."

2. Calibration

NIST provides calibration services for primary standard resistors of nominal decade values in the range between $10^4\ \Omega$ and $10^{12}\ \Omega$. In addition, Measurement Assurance Program (MAP) services for resistance are offered at the 1 $\Omega$ and 10 k$\Omega$ levels. Special resistance measurements are undertaken if they require the unique capabilities at NIST. Consult the latest edition of the NIST Calibration Services Users Guide (SP250) for detailed descriptions of the currently available NIST calibration services for resistance measurements.

Resistors are usually calibrated against standard resistors maintained at NIST in a manual or an automated comparison measurement system using a resistive ratio-arm bridge, a flux-balancing type of current comparator, or a capacitance-discharge method. The specific equipment used depends upon the value of the resistor being calibrated and on the accuracy required or warranted. NIST procedures follow well-recognized dc measurement practices such as current reversal, minimum power dissipation consistent with desired resolution and accuracy, and the use of check standards and data analysis techniques to monitor the operation of the measurement system.

Resistors ranging in value from 1 $\Omega$ to 1 M$\Omega$ are usually calibrated at a temperature of 25°C while
totally immersed in a well-regulated oil bath with a power dissipation in the resistor not exceeding 10 mW. Resistors of value less than 1 Ω down to a resistance of 0.0001 Ω are also usually calibrated immersed in a 25 °C oil bath except the power dissipation may be as great as 100 mW. The high-grade mineral oil used in oil baths is frequently checked, and changed, if necessary, to ensure that its conductivity will not cause a significant error in the measurement process. Resistors having values greater than 1 MΩ, or resistors not designed for oil immersion, are calibrated in air at an approximate temperature of 23 °C and a relative humidity not exceeding 45%. In all cases environmental conditions known to have an effect on the value of the resistor are reported, and where possible, controlled.

3. Reported Uncertainty

The expanded uncertainty of the value of resistance given in a NIST Report of Calibration is equal to

\[ 2\sqrt{s^2 + \sum u^2(f)} \]

where \( s \) is the estimated Type A standard uncertainty based on data from a large population of individual calibrations, and \( u(f) \) is the estimated Type B standard uncertainty for each known component quantified by a standard deviation. The coverage factor 2 used by NIST is consistent with international practice. Detailed information is given in NIST Technical Note 1298 entitled "NIST Measurement Service for DC Standard Resistors," which lists all uncertainty components and describes the method used to calculate this expanded uncertainty. The reported uncertainty contains no allowances for the long-term drift of the resistor under test or for the possible effects of transporting the standard resistor between laboratories. As previously mentioned, the reported uncertainty does not include the possible error between the U. S. representation of the ohm and the ohm as defined by SI.

4. Potential Terminals

The resistance of a standard resistor having separate pairs of current and potential terminals will, unless otherwise specified, be the four-terminal resistance between the branch points defined by the potential terminals. When there are more than one pair of potential terminals, the innermost pair will be used to define the resistance. For resistors of the Rosa type, where the potential terminals are ill-defined, the potential connections are made by placing straight pieces of wire under the inner sides of the innermost screw connections on the heavy copper arms of the resistor. Resistors having only one pair of terminals will be measured as two-terminal resistors.

For Rosa-type resistors the difference between the four-terminal value and the two-terminal value of resistances defined by the mercury-wetted surfaces is usually on the order of 50 μΩ. This difference is 0.05 ppm or less for resistors of nominal values \( \geq 1 \) kΩ, and for most measurements, this difference is negligible.

5. Mercury Connections

The contact surfaces, on the terminals of standard resistors, designed to be mercury-wetted are cleaned and amalgamated with mercury before the resistors are calibrated. Users should periodically check the condition of these contact surfaces and, if necessary, recondition them exercising care so that the surfaces remain smooth and flat. Because mercury is poisonous, it should be used in accordance with prescribed safety regulations.

6. Stability

Studies indicate that the stability with time of standard resistors depends upon many factors such as: the basic design of the resistor, the resistance alloy used and its heat-treatment history, the detailed construction of the resistor, and subsequent handling of the resistor in the laboratory and during transportation. In the final analysis each laboratory must determine the stability of its standard resistors based on measurements over a long period of time and, especially, measurements before and after the resistors are transported.

7. Temperature Coefficients

The resistance-temperature curve for resistors over the interval 20 °C to 30 °C can be represented by

\[ R(t) = R'[1 + \alpha(t - t_0) + \beta(t - t_0)^2] \]

where \( R(t) \) is the resistance at temperature \( t \), \( R' \) is the resistance at a reference temperature \( t_r \), \( \alpha \) is the slope of the curve at \( t_r \) and \( \beta \) determines the curvature at any temperature. The reference temperature \( t_r \) is usually 25 °C for oil-type resistors and 23 °C for air-type resistors. The coefficients \( \alpha \) and \( \beta \) do not change appreciably with time and need only be
determined once. Resistor manufacturers usually can supply information on the temperature coefficients pertaining to their standards.

8. Pressure Coefficient

Some standard resistors (Thomas-type 1 Ω and air-type 10 kΩ), because of their thin-walled construction, exhibit small changes of resistance with changes in pressure. From a study of a small number of these resistors it appears that the effect of pressure on resistance is linear over the range of barometric pressures and can be expressed as

\[ R(P) = R_0[1 + \gamma(P - P_0)] \]

where \( R(P) \) is the resistance at pressure \( P \), \( R_0 \) is the resistance at a reference pressure \( P_0 \), and \( \gamma \) is the pressure coefficient. The reference pressure is usually taken to be one standard atmosphere or 101.325 kPa. Typically, the pressure coefficients of commercial Thomas-type resistors range from +0.0002 x 10^-6/hPa to +0.0025 x 10^-6/hPa. For a user located at an altitude of one mile above sea level, the resistance of this type of resistor can be as much as 0.5 ppm less than the value at the altitude of the NIST Gaithersburg Laboratory.

Besides the barometric or surface pressure one has to consider the additional pressure effect on the resistor due to its depth of immersion in the oil bath. As an example, for mineral oil with a specific gravity of 0.086 the value of \( P \) increases 8.48 Pa for each millimeter descent below the surface of the oil. At NIST the reference position of the resistor is taken to be at the level of the surfaces of the amalgamated current terminals of the resistor. The value of pressure given in a NIST Report of Calibration for a Thomas-type resistor is the sum of the ambient barometric pressure at the time of calibration plus the pressure resulting from the depth of the resistor below the surface of the oil.

9. Load Effects

Current. Standard resistors are calibrated at NIST at low power dissipation so that the change in resistance resulting from the effect of \( i^2R \) heating is negligible. Standard resistors of nominal values \( \geq 1 \) Ω are calibrated at power dissipation of 10 mW or less. For standard resistors of nominal values \( < 1 \) Ω, the power level may be as large as 100 mW in order to have sufficient resolution in the measurement system.

Voltage. Thin-film standard resistors of nominal values \( \geq 1 \) MΩ usually exhibit a significant change in resistance with applied voltage. Typically these voltage coefficients of resistance can vary from 0 to -0.03%/volt. Hence, the magnitude of the test voltage should be specified by the customer when a resistor of this type is submitted for calibration.

10. Selected References


At the National Institute of Standards and Technology (NIST), the U.S. representation of the ohm is based on the quantum Hall effect, and it is maintained and disseminated at various resistance levels by working reference groups of standards. This document describes the measurement systems and procedures used to calibrate standard resistors of nominal decade values in the resistance range from $10^{-4}$ to $10^{12}$ Ω. Resistance scaling techniques used to assign values to the working standards are discussed. Also included is an assessment of the calibration uncertainties at each resistance level.
Journal of Research of the National Institute of Standards and Technology—Reports NIST research and development in those disciplines of the physical and engineering sciences in which the Institute is active. These include physics, chemistry, engineering, mathematics, and computer sciences. Papers cover a broad range of subjects, with major emphasis on measurement methodology and the basic technology underlying standardization. Also included from time to time are survey articles on topics closely related to the Institute's technical and scientific programs. Issued six times a year.

Nonperiodicals

Monographs—Major contributions to the technical literature on various subjects related to the Institute's scientific and technical activities.

Handbooks—Recommended codes of engineering and industrial practice (including safety codes) developed in cooperation with interested industries, professional organizations, and regulatory bodies.

Special Publications—Include proceedings of conferences sponsored by NIST, NIST annual reports, and other special publications appropriate to this grouping such as wall charts, pocket cards, and bibliographies.

Applied Mathematics Series—Mathematical tables, manuals, and studies of special interest to physicists, engineers, chemists, biologists, mathematicians, computer programmers, and others engaged in scientific and technical work.

National Standard Reference Data Series—Provides quantitative data on the physical and chemical properties of materials, compiled from the world's literature and critically evaluated. Developed under a worldwide program coordinated by NIST under the authority of the National Standard Data Act (Public Law 90-396). NOTE: The Journal of Physical and Chemical Reference Data (JPCRD) is published bi-monthly for NIST by the American Chemical Society (ACS) and the American Institute of Physics (AIP). Subscriptions, reprints, and supplements are available from ACS, 1155 Sixteenth St., NW., Washington, DC 20036.

Building Science Series—Disseminates technical information developed at the Institute on building materials, components, systems, and whole structures. The series presents research results, test methods, and performance criteria related to the structural and environmental functions and the durability and safety characteristics of building elements and systems.

Technical Notes—Studies or reports which are complete in themselves but restrictive in their treatment of a subject. Analogous to monographs but not so comprehensive in scope or definitive in treatment of the subject area. Often serve as a vehicle for final reports of work performed at NIST under the sponsorship of other government agencies.

Voluntary Product Standards—Developed under procedures published by the Department of Commerce in Part 10, Title 15, of the Code of Federal Regulations. The standards establish nationally recognized requirements for products, and provide all concerned interests with a basis for common understanding of the characteristics of the products. NIST administers this program in support of the efforts of private-sector standardizing organizations.

Consumer Information Series—Practical information, based on NIST research and experience, covering areas of interest to the consumer. Easily understandable language and illustrations provide useful background knowledge for shopping in today's technological marketplace.


Order the following NIST publications—FIPS and NISTIRs—from the National Technical Information Service, Springfield, VA 22161.


NIST Interagency Reports (NISTIR)—A special series of interim or final reports on work performed by NIST for outside sponsors (both government and non-government). In general, initial distribution is handled by the sponsor; public distribution is by the National Technical Information Service, Springfield, VA 22161, in paper copy or microfiche form.