NBS TECHNICAL NOTE 1236

U.S. DEPARTMENT OF COMMERCE/National Bureau of Standards

Electromagnetic Scattering by a Thick Strip on a Half-Space

Egon Marx

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June 1987

U.S. Department of Commerce
Malcolm Baldrige, Secretary

National Bureau of Standards
Ernest Ambler, Director

National Bureau of Standards Technical Note 1236
Natl. Bur. Stand. (U.S.), Tech. Note 1236, 70 pages (June 1987)
CODEN: NBTNAE

U.S. GOVERNMENT PRINTING OFFICE
WASHINGTON: 1987

For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington DC 20402

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ELECTROMAGNETIC SCATTERING BY A THICK STRIP ON A HALF-SPACE

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The problem of the determination of the scattered fields from an incident plane monochromatic electromagnetic wave on a strip of finite thickness on a half-space is reduced to the numerical solution of integral equations for auxiliary fields defined on the interfaces. These fields are chosen so as to minimize their number. The derivation of the integral equations is given for a perfectly conducting strip on a perfectly conducting half-space, for a dielectric (or other homogeneous material) on a dielectric half-space, and for a dielectric strip on a half-space of a different dielectric material. The vector FORTRAN programs written to carry out these calculations are briefly described and sample outputs are shown.

Key words: dielectric strip; electromagnetic scattering; integral equations; numerical solutions; resonance region; perfect conductors.

1. Introduction

The light scattered by a strip located on a substrate of the same or of a different material can be used to determine the size of the strip [1]. Accurate linewidth measurements require a precise evaluation of the scattered fields, especially in the resonance region where the size of the strip is comparable to the wavelength of the light used to do the measurement.

We present a method to determine numerically the electromagnetic fields scattered by such an infinite strip of finite cross section on an infinite half-space. This method is exact in that it gives the fields with an accuracy limited only by numerical considerations. The incident field is assumed to be a plane monochromatic wave propagating through a medium with a real dielectric constant. We consider different combinations of materials for the strip and the region under the plane. Both these regions can be perfectly conducting, both can be homogeneous dielectrics possibly with complex dielectric constants (to represent lossy media or conductors with finite conductivities), or the strip and the lower half-space can be different homogeneous media.

Because each region is filled with a homogeneous material, the electromagnetic fields in the different regions of space can be expressed in terms of surface fields defined on the boundaries. These surface fields obey integral equations derived from Maxwell's equations and the boundary conditions. The number of these fields is kept as small as possible by a careful choice of the jump conditions at the boundaries [2, 3].

The integral equations can be solved numerically by a point-matching method, and we have written programs that do this for the different problems. The determination of the fields scattered by perfect conductors can be reduced to the solution of integral equations of the second kind, and those for dielectrics lead to integral equations of the first kind, but we found no problems with ill-posedness.

In Section 2 we recall the reduction of Maxwell's equations to a pair of scalar Helmholtz equations for problems with translational invariance in the z-direction and an incident plane monochromatic wave. We then sketch the solution of the Helmholtz equation by means of the Green function that satisfies an outgoing wave condition (OWC). In Section 3 we provide details about the determination of the incident, reflected, and refracted (if any) fields in the absence of the strip for the different problems discussed in this paper. In Section 4 we show how we reduce the problem of scattering by a perfectly conducting strip on a perfectly conducting half-space to the solution of a single integral equation for a function of two variables defined on the boundary between the media both for the TE and TM modes. In Section 5 we reduce the determination of the fields scattered by a dielectric strip on a half-space of the same dielectric to the solution of a system of coupled integral equations for four surface fields for arbitrary direction of incidence and polarization. In Section 6 we solve a similar problem with two different dielectrics for the TE mode only, and we obtain a system of four integral equations. In section 7 we give a brief discussion of possible difficulties with the divergence of the fields at sharp edges and with spurious numerical instabilities in the integral equations at resonant frequencies of the interior problem. In Section 8 we describe the computer programs used to find the scattered fields, and we show some of the graphic results in Section 9.

2. Fields in the presence of cylindrical boundaries

The coordinate axes are shown in figure 1. We choose the z-axis in the plane separating the two media away from the strip and locate it at the center of the strip parallel to the edges of the strip. We choose the y-axis perpendicular to this plane pointing out of the perfect conductor or other substrate.

The boundary between the media has a cross section that can be given by an equation of the form

$$y = f(x). \tag{2.1}$$

A slightly more general representation of this curve is the set of parametric equations

$$x = f_1(s), \; y = f_2(s), \tag{2.2}$$

where s is the arclength. This form allows for curves where there may be more than one value of y for a given x.

When the incident wave is plane and monochromatic, the field has a periodicity in the z-direction that is maintained by the geometry and appears also in all other fields. The Maxwell equations for the fields can be reduced to scalar Helmholtz

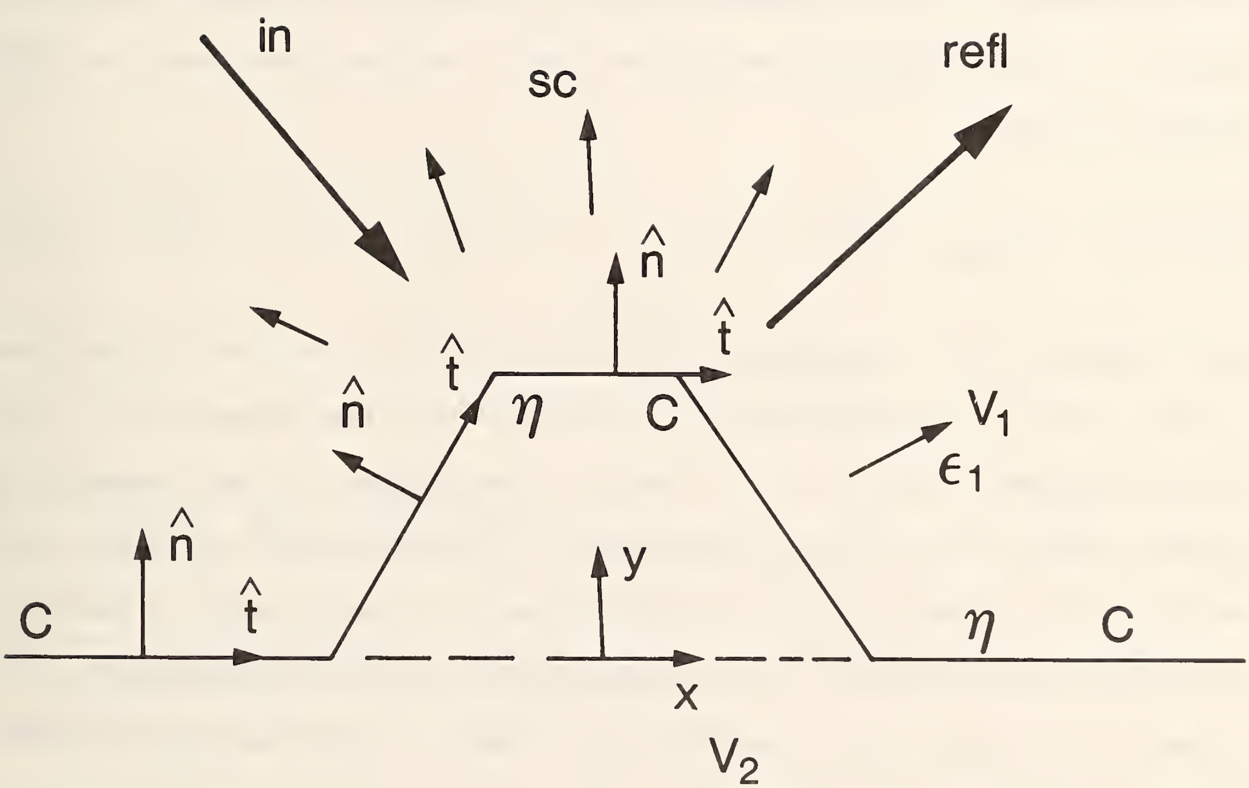

Figure 1. Geometric configuration for the scattering of a wave by a perfectly conducting strip on a perfectly conducting half-space. In the region V_1 we show long arrows that represent the directions of propagation of the incident and reflected waves and short ones that represent the scattered wave.

equations in the xy-plane for the z-components of the fields [4], and this reduction is shown in Subsection 2.1. We sketch the solution of the two-dimensional Helmholtz equation in Subsection 2.2 by the methods in [2] and [3].

2.1 Reduction to a scalar problem

An incident plane monochromatic wave of frequency ω with a wave vector $\vec{k}$ of magnitude $k = \omega/v$, where v is the speed of light in the medium, can be described by its electric field, which is given at the point $\vec{x}$ and at the time t by the real part of the complex vector

$$\vec{E}^{in}(\vec{x},t) = \vec{E}_0 \exp[i(\vec{k}\cdot\vec{x} - \omega t)]. \tag{2.3}$$

The symmetry of the geometrical configuration in the z-direction causes the z-dependence of all fields to be the same as that of the incident field (2.3), that is, $\exp(ik_3 z)$. We henceforth assume that this is the z-dependence of the fields, and that this exponential is multiplied by functions of x and y only. Vectors such as $\vec{E}$, $\vec{H}$, $\vec{k}$, and ∇ can be decomposed into a longitudinal part along the z-axis and a perpendicular part. Then the fields $\vec{E}$ and $\vec{H}$ can be written in the form

$$\vec{E}(\vec{x},t) = [E_3(x,y)\hat{e}_3 + \vec{E}_\perp(x,y)]\exp[i(k_3 z - \omega t)],$$

$$\vec{H}(\vec{x},t) = [H_3(x,y)\hat{e}_3 + \vec{H}_\perp(x,y)]\exp[i(k_3 z - \omega t)]. \tag{2.4}$$

It is then straightforward to show that the perpendicular components of the fields $\vec{E}$ and $\vec{H}$ that obey the homogeneous Maxwell equations,

$$\nabla \times \vec{E} = i\omega\mu\vec{H}, \ \nabla \times \vec{H} = -\, i\omega\epsilon\vec{E}, \tag{2.5}$$

can be expressed in terms of the longitudinal fields E_3 and H_3 by [4]

$$\vec{E}_\perp = -\, \frac{i\omega\mu}{k_\perp^2}\hat{e}_3 \times \nabla_\perp H_3 + \frac{ik_3}{k_\perp^2}\nabla_\perp E_3, \tag{2.6}$$

$$\vec{H}_\perp = \frac{i\omega\epsilon}{k_\perp^2}\hat{e}_3 \times \nabla_\perp E_3 + \frac{ik_3}{k_\perp^2}\nabla_\perp H_3, \tag{2.7}$$

The fields E_3 and H_3 satisfy the two-dimensional Helmholtz equation

$$(\nabla_\perp^2 + k_\perp^2)E_3 = 0, \ (\nabla_\perp^2 + k_\perp^2)H_3 = 0. \tag{2.8}$$

The frequency ω and the component k_3 of the propagation vector of the incident wave are real, but the permittivity ϵ and the perpendicular part of the propagation vector $\vec{k}_\perp$ can be complex; the permeability μ is usually assumed to be equal to μ_0 in free space, but we make use of this assumption only occasionally when we wish to simplify the notation. The magnitude of the propagation vector and that of its perpendicular part are given by

$$k^2 = \epsilon\mu\omega^2, \; k_\perp^2 = k^2 - k_3^2. \tag{2.9}$$

The components of the perpendicular fields tangential to the cross section (2.2) are then

$$E_{\perp t} = \frac{i\omega\mu}{k_\perp^2}\frac{\partial H_3}{\partial n} + \frac{ik_3}{k_\perp^2}\frac{\partial E_3}{\partial s}, \tag{2.10}$$

$$H_{\perp t} = -\frac{i\omega\epsilon}{k_\perp^2}\frac{\partial E_3}{\partial n} + \frac{ik_3}{k_\perp^2}\frac{\partial H_3}{\partial s}, \tag{2.11}$$

in terms of the normal and tangential derivatives of E_3 and H_3 on the curve, where the normal unit vector $\hat{n}$ and the tangential unit vector $\hat{t}$ are related by $\hat{n} = \hat{e}_3 \times \hat{t}$, as seen in figure 1. There is no general agreement in the literature about the definition of the terms TE and TM; here we define the TE mode by the condition $H_3 = 0$ and the TM mode by $E_3 = 0$. Unless the scatterer is a perfect conductor, we add the condition $k_3 = 0$ to the definition of these particular modes.

For the incident wave (2.3), we have

$$\vec{H}_0 = \vec{k} \times \vec{E}_0/\mu\omega, \tag{2.12}$$

$$\vec{E}_{0\perp} = (\omega\mu/k_\perp^2)(-k_2\hat{e}_1 + k_1\hat{e}_2)H_{03} - (k_3/k_\perp^2)(k_1\hat{e}_1 + k_2\hat{e}_2)E_{03}, \quad (2.13)$$

$$\vec{H}_{0\perp} = (\omega\epsilon/k_\perp^2)(k_2\hat{e}_1 - k_1\hat{e}_2)E_{03} - (k_3/k_\perp^2)(k_1\hat{e}_1 + k_2\hat{e}_2)H_{03}. \quad (2.14)$$

The Poynting vector obtained from the fields (2.4), (2.6), and (2.7) is equal to half the real part of

$$\vec{E} \times \vec{H}^* = k_\perp^{-4}\Big[\omega^2\mu\epsilon(\nabla_\perp E_3^*) \times \nabla_\perp H_3 + k_3\omega\mu\left|\nabla_\perp H_3\right|^2\hat{e}_3$$

$$+ k_3\omega\epsilon\left|\nabla_\perp E_3\right|^2\hat{e}_3 + k_3^2(\nabla_\perp E_3) \times \nabla_\perp H_3^*\Big] + k_\perp^{-2}\Big(- i\omega\mu H_3^*\nabla_\perp H_3$$

$$+ ik_3H_3^*\nabla_\perp E_3 \times \hat{e}_3 + i\omega\epsilon E_3\nabla_\perp E_3^* + ik_3E_3\nabla_\perp H_3^* \times \hat{e}_3\Big). \quad (2.15)$$

The terms in the square brackets all point in the z-direction, and they vanish in the TE and TM modes if $k_3 = 0$.

2.2 Solution of the two-dimensional Helmholtz equation

We now find the solution of the Helmholtz equation (2.8) in terms of the corresponding Green function and the jumps of the function and its normal derivative across an arbitrary curve C. The functions in this section are restricted to the xy-plane and we drop the subindex $\perp$.

We have to find a function U that satisfies the homogeneous

Helmholtz equation in the xy-plane outside the curve C, that has a jump

$$\Delta U = U_+ - U_- = \phi \tag{2.16}$$

across C, whose normal derivative has a jump

$$\Delta\left(\frac{\partial U}{\partial n}\right) = \left(\frac{\partial U}{\partial n}\right)_+ - \left(\frac{\partial U}{\partial n}\right)_- = \eta \tag{2.17}$$

across C, and that satisfies the OWC everywhere at infinity. The curve C is parametrized by the arclength s and has a unit normal $\hat{n}(s) = \hat{e}_3 \times \hat{t}(s)$, which points in the direction of the region described by the subindex +.

The distribution [2] U then satisfies the equation

$$(\nabla^2 + k^2)U = \Delta\left(\frac{\partial U}{\partial n}\right)\delta(C) + \nabla\cdot[\hat{n}\Delta U\delta(C)], \tag{2.18}$$

where a distribution of the form $f(s)\delta(C)$ is defined by its operation on a test function α,

$$\langle f(s)\delta(C),\alpha(x,y)\rangle = \int_C ds f(s)\alpha[x(s),y(s)]. \tag{2.19}$$

The solution of (2.18) is given by the convolution of the right side with the Green function $\mathcal{G}$ for the Helmholtz equation,

$$U = -\mathcal{G}*\eta\delta(C) - (\partial\mathcal{G}/\partial n)*\phi\delta(C), \tag{2.20}$$

where $\mathcal{G}$ satisfies

$$(\nabla^2 + k^2)\mathcal{G}(x,y) = -\delta(x)\delta(y). \tag{2.21}$$

The distribution $\mathcal{G}$ is given by

$$\mathcal{G}(x,y) = \frac{i}{4} H_0^{(1)}(k\rho), \quad \rho = (x^2 + y^2)^{1/2}, \tag{2.22}$$

where $H_0^{(1)}$ is a Hankel function of the first kind. These functions are defined in terms of the Bessel and Neumann functions by $H_n^{(1)} = J_n + iY_n$. The solution (2.20) can be rewritten in terms of functionals of the jumps in the form

$$U = G\{\eta\} + N\{\phi\}, \tag{2.23}$$

where

$$G\{\eta\} = -\frac{i}{4}\int_C ds'\,\eta(\vec{x}')H_0^{(1)}(k_\perp R), \tag{2.24}$$

$$N\{\phi\} = \frac{i}{4}\int_C ds'\,\phi(\vec{x}')H_1^{(1)}(k_\perp R)k_\perp \hat{n}'\cdot\hat{R}, \tag{2.25}$$

and $\vec{R} = \vec{x} - \vec{x}'$, $R = [(x - x')^2 + (y - y')^2]^{1/2}$, $\hat{R} = \vec{R}/R$.

The value of the function U at either side of C is

$$U_\pm = \pm\frac{\phi}{2} + U\Big|_C, \tag{2.26}$$

and, if the function is continuous across C, the value of the normal derivative on either side of C is

$$\left(\frac{\partial U}{\partial n}\right)_\pm = \pm\frac{\eta}{2} + \left(\frac{\partial U}{\partial n}\right)_C. \tag{2.27}$$

3. The homogeneous fields

The known functions in the integral equations are combinations of fields that obey the homogeneous Maxwell equations evaluated on the boundaries. They are the incident field and the reflected and refracted fields that would be produced at the plane interface in the absence of the strip. These fields satisfy the correct boundary conditions on the xz-plane and they are then assumed to be defined throughout all of space.

For the perfect conductor, the TE and TM modes do not mix even when $\vec{k}$ is not perpendicular to the z-axis. We give the homogeneous fields needed in the solution of the scattering problem for the TE mode in Subsection 3.1 and for the TM mode in 3.2. The most general case is obtained by superposition. The situation is more complicated for the dielectric scatterer, and we give the formulas for the most general case in Subsection 3.3. In Subsection 3.4 we give the formulas needed for the case of the strip of a different dielectric in the TE mode only.

For the perfect conductor, the reflected field is obtained from the incident field by retaining the normal component of $\vec{E}$ and the tangential component of $\vec{H}$ unchanged and changing the sign of the tangential component of $\vec{E}$ and the normal component of $\vec{H}$.

For the dielectric, the reflected and refracted fields are obtained from the continuity of the tangential components of $\vec{E}$ and $\vec{H}$.

3.1 The perfect conductor in the TE mode

The incident field is assumed to be a plane monochromatic wave described by (2.3). In terms of the z-component of the amplitude of the incident electric field, E_{03}, the amplitudes of the perpendicular components of the incident electric and magnetic fields are

$$E_{01} = -(k_1k_3/k_1^2)E_{03},\ E_{02} = -(k_2k_3/k_1^2)E_{03},\ H_{01} = (\omega\epsilon k_2/k_1^2)E_{03},$$

$$H_{02} = -(\omega\epsilon k_1/k_1^2)E_{03}. \tag{3.1}$$

The reflected wave is given by

$$\vec{E}^{refl}(\vec{x}) = \vec{E}_0''\exp(i\vec{k}''\cdot\vec{x}), \tag{3.2}$$

where

$$\vec{k}'' = k_1\hat{e}_1 - k_2\hat{e}_2 + k_3\hat{e}_3, \tag{3.3}$$

$$E_{01}'' = -E_{01},\ E_{02}'' = E_{02},\ E_{03}'' = -E_{03}. \tag{3.4}$$

Consequently, the z-component of the total homogeneous field outside the conductor is

$$E_3^1(x,y) = E_3^{in} + E_3^{refl}$$

$$= E_{03}\exp[i(k_1x + k_2y)] + E''_{03}\exp[i(k''_1x + k''_2y)]$$

$$= 2iE_{03}\exp(ik_1x)\sin(k_2y). \tag{3.5}$$

This field, evaluated on the boundary C, appears in the corresponding integral equation.

3.2 The perfect conductor in the TM mode

The homogeneous field H_3^1 now has to be expressed in terms of the z-component of the incident magnetic field, H_{03}. The perpendicular components of the incident fields are

$$H_{01} = -(k_1k_3/k_\perp^2)H_{03},\ H_{02} = -(k_2k_3/k_\perp^2)H_{03},\ E_{01} = -(\omega\mu k_2/k_\perp^2)H_{03},$$

$$E_{02} = (\omega\mu k_1/k_\perp^2)H_{03}. \tag{3.6}$$

The components of the reflected field amplitude are

$$H''_{01} = H_{01},\ H''_{02} = -H_{02},\ H''_{03} = H_{03}, \tag{3.7}$$

whence

$$H_3^1(x,y) = 2H_{03}\exp(ik_1x)\cos(k_2y), \tag{3.8}$$

$$\nabla_{\perp} H_3^1(x,y) = 2H_{03}\exp(ik_1 x)[ik_1\cos(k_2 y)\hat{e}_1 - k_2\sin(k_2 y)\hat{e}_2], \qquad (3.9)$$

from which we find $\partial H_3^1/\partial n$ on C to use in the integral equation,

$$\partial H_3^1(x,y)/\partial n = 2H_{03}\exp(ik_1 x)[- k_2 n_2\sin(k_2 y) + ik_1 n_1\cos(k_2 y)].$$

(3.10)

3.3 The dielectric general case

We assume that the incident wave is characterized by an arbitrary wave vector $\vec{k}$ (with a negative k_2) and an electric field amplitude $\vec{E}_0$ perpendicular to $\vec{k}$. The plane interface produces a refracted wave in medium 2 characterized by $\vec{k}'$ and $\vec{E}'$, and a reflected wave in medium 1 characterized by $\vec{k}''$ and $\vec{E}''$.

The frequency of the fields is the same in the different media, whence the magnitudes of the wave vectors are related by

$$k'^2/\epsilon_2\mu_2 = k^2/\epsilon_1\mu_1. \qquad (3.11)$$

The wave vector of the reflected field is still given by (3.3), and that of the refracted field is

$$\vec{k}' = k_1\hat{e}_1 - [(\mu_2\epsilon_2/\mu_1\epsilon_1)k^2 - k_1^2 - k_3^2]^{1/2}\hat{e}_2 + k_3\hat{e}_3. \qquad (3.12)$$

The magnetic intensity $\vec{H}_0$ is given by (2.12). The incident fields, as well as the reflected and refracted fields, are plane monochromatic waves that satisfy (2.13) and (2.14).

To find the amplitudes E'_{03} and E''_{03} that determine the refracted and reflected fields, we could decompose the fields in the canonical components with parallel and perpendicular polarization and obtain the z-components of the reflected and refracted fields from the usual formulas. We prefer to obtain the equations for these four unknowns directly from the boundary conditions and solve them numerically. We have

$$E_{03} + E''_{03} = E'_{03}, \tag{3.13}$$

$$H_{03} + H''_{03} = H'_{03}, \tag{3.14}$$

$$E_{01} + (\omega\mu_1 k_2/k_\perp^2)H''_{03} - (k_1 k_3/k_\perp^2)E''_{03}$$

$$= -(\omega\mu_2 k'_2/k'^2_\perp)H'_{03} - (k_1 k_3/k'^2_\perp)E'_{03}, \tag{3.15}$$

$$H_{01} - (\omega\epsilon_1 k_2/k_\perp^2)E''_{03} - (k_1 k_3/k_\perp^2)H''_{03}$$

$$= (\omega\epsilon_2 k'_2/k'^2_\perp)E'_{03} - (k_1 k_3/k'^2_\perp)H'_{03}. \tag{3.16}$$

Once the z-components of the amplitudes of the reflected and refracted fields are determined, the fields on the curve C are obtained from them by including the appropriate exponential factors.

The equations for the TE or the TM mode can be obtained from (3.13) to (3.16) by setting H_{03} or E_{03} and k_3 equal to 0.

3.4 The strip of a different material in the TE mode

For the TE mode, (3.1), (3.13), and (3.16) allow us to find the amplitudes of the reflected and refracted fields, if we substitute ϵ_3 for ϵ_2 as the permittivity of the half-space on which the wave impinges. We set $k_3 = 0$ and $H_3 = 0$ in (3.16) and use (3.11) to obtain

$$k_2(E_{03} - E''_{03}) = k'_2 E'_{03}. \tag{3.17}$$

We solve (3.13) and (3.17) and find

$$E'_{03} = \frac{2k_2}{k_2 + k'_2} E_{03}, \tag{3.18}$$

$$E''_{03} = \frac{k_2 - k'_2}{k_2 + k'_2} E_{03}. \tag{3.19}$$

On the boundary plane we have $y = 0$ and the incident field reduces to

$$E_3^{in}(x,y) = E_{03}\exp(ik_1x), \tag{3.20}$$

with similar formulas for the reflected and refracted fields, whose amplitudes are given by (3.18) and (3.19). We assume that $\mu_1 = \mu_2 = \mu_3 = \mu_0$. We use (3.13) and (3.17) to derive

$$E_3^0(x,y) = E_3^{in}(x,y) + E_3^{refl}(x,y) - E_3^{refr}(x,y)$$

$$= (E_{03} + E''_{03} - E'_{03})\exp(ik_1x) = 0, \tag{3.21}$$

$$\partial E_3^0(x,y)/\partial n = i\vec{k}\cdot\hat{n}E_3^{in}(x,y) + i\vec{k}''\cdot\hat{n}E_3^{refl}(x,y) - i\vec{k}'\cdot\hat{n}E_3^{refr}(x,y)$$

$$= i(k_2E_{03} - k_2E''_{03} - k'_2E'_{03})\exp(ik_1x) = 0, \tag{3.22}$$

We also have to determine

$$E_3^{refr}(x,y) = E'_{03}\exp(ik_1x), \tag{3.23}$$

$$\partial E_3^{refr}(x,y)/\partial n = ik'_2E'_{03}\exp(ik_1x). \tag{3.24}$$

Off the boundary plane we have $y \neq 0$ and we have to use the full expressions of the fields to find

$$E_3^1(x,y) = E_{03}\exp[i(k_1x + k_2y)] + E''_{03}\exp[i(k_1x - k_2y)], \tag{3.25}$$

$$\partial E_3^1(x,y)/\partial n = i\{(k_1n_1 + k_2n_2)E_{03}\exp[i(k_1x + k_2y)]$$

$$+ (k_1 n_1 - k_2 n_2) E''_{03} \exp[i(k_1 x - k_2 y)]\}. \tag{3.26}$$

4. Scattering by a perfectly conducting strip on a perfectly conducting half-space

For the scattering by a perfectly conducting strip on a perfectly conducting half-space, the total field is composed of the incident field, the reflected field, and the scattered field. The configuration of the problem is shown in figure 1. The scattered field satisfies the OWC at large distances from the strip. For the perfect conductor, the general case for arbitrary direction of propagation and polarization of the incident wave can be decomposed into separate TE and TM modes. Thus, although TE and TM modes are usually defined with the incident direction in the plane perpendicular to the generator of the strip, we allow for a nonvanishing k_3 in this Section.

4.1 The TE mode

In the TE mode we have $H_3^{in} = 0$, which implies that the z-component of the reflected magnetic intensity also vanishes, $H_3^{refl} = 0$. We will show that the boundary conditions can all be satisfied if we further assume that the z-component of the scattered magnetic field vanishes. The z-component of the total electric field, E_3, satisfies the Helmholtz equation,

$$(\nabla_\perp^2 + k_\perp^2)E_3 = 0, \tag{4.1}$$

and the boundary condition

$$E_{3+} = 0 \text{ on } C, \tag{4.2}$$

where E_{3+} is the value of this tangential component of the electric field above the boundary C of the cross section of the conductor. Since the value of E_3 is constant (zero) on C, the vector $\nabla_\perp E_3$ is perpendicular to C, which implies, from (2.6), that $\vec{E}_\perp$ is normal to C and the electric field does not have a tangential component. Also, from (2.7), we see that the perpendicular component of the magnetic field, $\vec{H}_\perp$, is tangential to the surface, whence the normal component of $\vec{H}$ vanishes. Thus, all the boundary conditions on the electromagnetic field at the surface of a perfect conductor are satisfied.

The z-component of the scattered electric field is defined as the difference between the total and the homogeneous electric fields,

$$E_3^{sc} = E_3 - E_3^1, \tag{4.3}$$

where $\vec{E}^1$ is the sum of the incident and reflected fields, as defined in (3.10).

We define a function U(x,y) that is equal to the field E_3^{sc} in V_1, is continuous across C, and satisfies the Helmholtz equation (4.1) also in V_2 and the OWC for $y \to -\infty$. This function is then determined by the discontinuity of its normal derivative

η across C. Since U satisfies the Helmholtz equation on the whole plane and the OWC when $y \to \pm\infty$, we can write it in the form

$$U = G\{\eta\}, \tag{4.4}$$

where G is the functional (2.24). Then the boundary condition (4.2) leads to the singular integral equation of the first kind,

$$G\{\eta\} + E_3^1 = 0 \text{ on } C, \tag{4.5}$$

for the unknown function η. This equation is analogous to the Petit equation [2].

To solve the equation numerically, we use the point-matching method [2]. We cover the part of the curve C where we expect the function η to differ significantly from 0 with N patches or line segments, and we seek to determine the values η_ℓ of the function η at the center of these patches. For this purpose, we assume that the integrand in (4.4) is constant on each patch, and we obtain the system of linear equations by satisfying (4.5) at the points $\vec{x}_\ell$,

$$\sum_{m=1}^{N} A_{\ell m}\eta_m = B_\ell = -E_3^1(\vec{x}_\ell), \quad \ell = 1,2,\ldots,N, \tag{4.6}$$

where

$$A_{\ell m} = -\frac{i}{4}\Delta s_m H_0^{(1)}(k_\perp R_{\ell m}), \quad R_{\ell m} = |\vec{x}_\ell - \vec{x}_m|, \quad \ell \neq m, \tag{4.7}$$

and the coefficient of the contribution of the self-patch is

$$A_{\ell\ell} = \frac{1}{2\pi}[\log(\tfrac{1}{2}k_\perp \Delta s_\ell) - 1.11593]\Delta s_\ell - \frac{i}{4}\Delta s_\ell. \qquad (4.8)$$

This contribution is obtained from the small-argument approximation of the Hankel function $H_0^{(1)}$. It is of the order of $\log(\Delta s)$ and is not necessarily negligible compared to the N-1 terms of the order of Δs.

The field E_3^1 is given by U in (4.4), and we can use the large-argument approximation of $H_0^{(1)}$,

$$H_0^{(1)}(\zeta) \approx (\pi\zeta/2)^{-1/2}\exp[i(\zeta - \pi/4)], \qquad (4.9)$$

if we have to determine the far fields only. Equations (2.6) and (2.7) with $E_3 = E_3^{sc}$ and $H_3 = 0$ give the perpendicular part of the scattered electric and the scattered magnetic field. We expand, in polar coordinates,

$$R \approx \rho - \rho'\cos(\phi - \phi') \qquad (4.10)$$

and obtain the far-field approximation

$$E_3^{sc} \approx - \exp[i(k_\perp\rho - \pi/4)](8\pi k_\perp\rho)^{-1/2}$$

$$\cdot\int_C ds'\eta(\vec{x}')\exp[-ik_\perp\rho'\cos(\phi - \phi')], \qquad (4.11)$$

and the intensity of the scattered field is proportional to

$$I(\phi) = \left|\int_C ds'\eta(\vec{x}')\exp[-ik_\perp\rho'\cos(\phi - \phi')]\right|^2$$

$$= \left| \int_C ds' \eta(\vec{x}') \exp[-ik_\perp(x'\cos\phi + y'\sin\phi)] \right|^2. \tag{4.12}$$

The radiation pattern for a symmetric scatterer is symmetric, that is, $I(\pi - \phi) = I(\phi)$, if $\eta(-x', y') = \eta(x', y')$. This condition is satisfied for normal incidence.

We obtain a different equation if we define a function $U'(x,y)$ which is similar to U except that the function has a jump ϕ across C and the normal derivative is continuous across C [5]. Then (2.23) reduces to

$$U' = N\{\phi\}, \tag{4.13}$$

and the boundary condition leads to the integral equation of the second kind

$$\frac{\phi}{2} + N\{\phi\} + E_3^1 = 0. \tag{4.14}$$

This equation is similar to the Pavageau equation [2], but one is not the precise analogue of the other.

4.2 <u>The TM mode</u>

In the TM mode we have $E_3^{in} = 0$, whence $E_3^{refl} = 0$. We assume that the z-components of all the electric fields vanish. The z-component of the total magnetic field, H_3, satisfies

$$(\nabla_\perp^2 + k_\perp^2) H_3 = 0. \tag{4.15}$$

The boundary condition on the tangential electric field (2.10) is satisfied when

$$(\partial H_3/\partial n)_+ = 0 \text{ on } C. \tag{4.16}$$

Equation (4.16) also implies that the normal component of $\vec{H}_\perp$ in (2.7) vanishes, and all the boundary conditions are satisfied.

The z-component of the scattered magnetic field is given by an expression similar to (4.3). We define an auxiliary field $U(x,y)$ that is equal to H_3^{sc} in V_1, satisfies the same Helmholtz equation and the OWC in V_2, and is continuous across C. The function U can then be expressed in terms of the discontinuity of its normal derivative, η, across C by (4.4) by means of the functional G defined in (2.24). The gradient of the function U on (C) is then expressed by

$$\nabla_\perp U\Big|_C = \frac{i}{4}\int_C ds'\eta(\vec{x}')H_1^{(1)}(k_\perp R)k_\perp\hat{R}. \tag{4.17}$$

The boundary condition (4.16) then reduces to the singular integral equation of the second kind

$$\frac{1}{2}\eta + N'\{\eta\} + \partial H_3^1/\partial n = 0 \text{ on } C, \tag{4.18}$$

where N′ is the functional

$$N'\{\eta\} = \frac{i}{4}\hat{n}\cdot\int_C ds'\eta(\vec{x}')H_1^{(1)}(k_\perp R)k_\perp\hat{R}; \tag{4.19}$$

this functional differs from N in (2.25) in that the normal $\hat{n}$ at the field point replaces the normal $\hat{n}'$ at the source point. We find the approximate values of η at the center of the patches from the system of linear equations (4.6), where now the coefficients and constants are

$$A'_{\ell m} = \frac{i}{4}\Delta s_m H_1^{(1)}(k_\perp R_{\ell m}) k_\perp \hat{n}_\ell \cdot (\vec{x}_\ell - \vec{x}_m)/R_{\ell m}, \quad \ell \neq m, \tag{4.20}$$

$$A'_{\ell\ell} = \frac{1}{2} + \frac{1}{4\pi}\kappa_\ell \Delta s_\ell, \tag{4.21}$$

$$B'_\ell = -\,\partial H_3^1(\vec{x}_\ell)/\partial n. \tag{4.22}$$

The curvature κ_ℓ of the cross section of the strip at the point $\vec{x}_\ell$ is positive when the curve is concave in the direction of $\hat{n}$, the normal defined as going from medium 2 to medium 1. The curvature vanishes for a segment of a straight line. The self-patch correction, which is proportional to Δs, can be neglected compared to 1/2 if the curvature is small enough.

The field H_3^{sc} is given by U in (4.4), the other component of the scattered magnetic intensity can be obtained from (2.7), and the electric field can be obtained from (2.6). The far-field expressions for the scattered magnetic field and intensity are essentially those in (4.11) and (4.12).

4.3 The general case

The linearity of Maxwell's equation and that of the boundary conditions allow us to use superposition to find the fields scattered by a perfectly conducting strip on a perfectly conducting half-space when neither E_3^{in} nor H_3^{in} vanishes. The incident field is separated into TE and TM modes based on the respective values of E_3^{in} and H_3^{in}. The scattered far fields and intensities have to be computed from the sums of expressions obtained in Subsections 4.1 and 4.2.

5. Scattering by a strip on a half-space, both of the same homogeneous material

We now consider the scattering of a plane monochromatic wave by a strip on a half-space of the same homogeneous material, as shown in figure 2.

The fields that obey the homogeneous Maxwell equations and that would exist in the absence of the strip are the incident and the reflected fields in V_1 and the refracted field in V_2. We define the scattered fields as the additional fields in both regions that together with the homogeneous fields give the physical total fields. The constants of the region V_1 are ϵ_1 and μ_1 and those in the region V_2 are ϵ_2 and μ_2. The magnitudes of the corresponding two propagation constants, $k_\perp$ and $k'_\perp$, are given by (2.9). The frequency ω and the z-component of the propagation vector, k_3, remain the same in both media. No current density or charge density can exist on the surface of a dielectric, whence the tangential components of $\vec{E}$ and $\vec{H}$ have to be continuous across the boundary. If we have $H_3^{in} = 0$, the reflected and refracted magnetic fields will have a z-component unless $k_3 = 0$, and the same happens for the electric field. Thus we include the condition $k_3 = 0$ in the definition of the TE mode discussed in

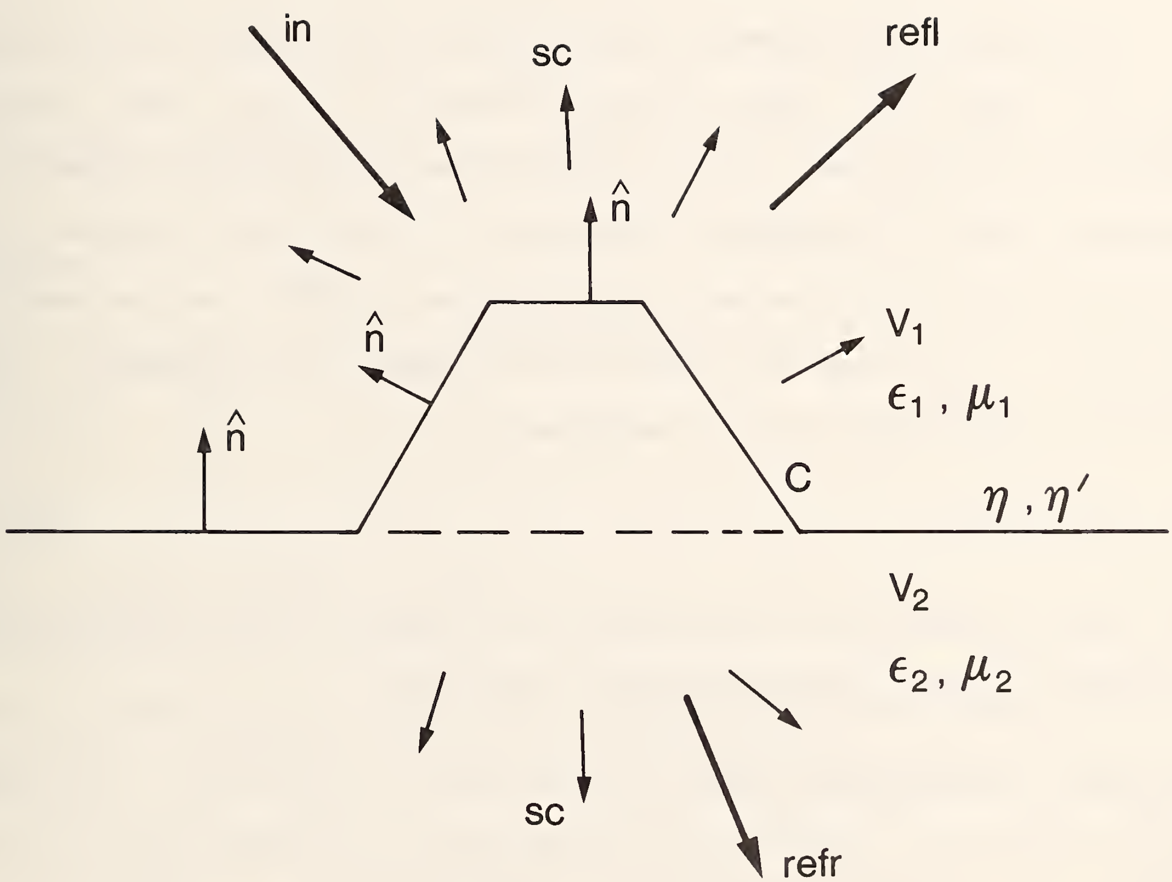

Figure 2. Geometric configuration for the scattering of a wave by a homogeneous strip on a half-space of the same material. In addition to the incident, reflected, and scattered waves in V_1 we show the refracted and scattered waves in V_2.

Subsection 5.1 and the TM mode discussed in Subsection 5.2. We consider the scattering of an incident wave with an arbitrary direction of propagation and polarization in Subsection 5.3, and we extend the results to a medium with a complex dielectric constant in Subsection 5.4.

5.1 The TE mode

We follow the procedure and notation developed in Subsection 4.1, but add an index to the operators to make clear which constants associated with a particular medium are used to determine the propagation constant. Since we include the condition $k_3 = 0$ in the definition of the TE mode, we can assume that $H_3 = 0$, and (2.6) then implies that $\vec{E}_\perp$ vanishes. The boundary condition (4.2) is replaced by

$$E_{3-} = E_{3+} \text{ on } C, \tag{5.1}$$

and the continuity of the tangential component of the magnetic intensity gives a second boundary condition,

$$(\partial E_3/\partial n)_- = \alpha(\partial E_3/\partial n)_+ \text{ on } C, \tag{5.2}$$

where

$$\alpha = \mu_2/\mu_1. \tag{5.3}$$

We now define two auxiliary functions, U_1 and U_2. The function U_1 is equal to the z-component of the scattered electric field in the region V_1, satisfies the same Helmholtz equation in V_2 (with the constants of medium 1), is continuous across C, and also satisfies the OWC when $y \to -\infty$. Again we call η the discontinuity of its normal derivative across C, and U_1 is then given by (4.4) using an operator G_1. The function U_2 vanishes in V_1 and is equal to the z-component of the scattered electric field in V_2. We compute the values of U_2 and $\partial U_2/\partial n$ at C in the medium 2 from the boundary conditions (5.1) and (5.2),

$$U_{2-} = E_{3-}^{sc} = E_{3-} - E_3^{refr} = E_{3+} - E_3^{refr} = E_3^0 + G_1\{\eta\}, \tag{5.4}$$

$$\left(\partial U_2/\partial n\right)_- = \partial \bar{E}_3^0/\partial n + \alpha\left(\frac{\eta}{2} + N_1'\{\eta\}\right), \tag{5.5}$$

where

$$E_3^0 = E_3^{in} + E_3^{refl} - E_3^{refr}, \tag{5.6}$$

$$\bar{E}_3^0 = \alpha\left(E_3^{in} + E_3^{refl}\right) - E_3^{refr}. \tag{5.7}$$

The function U_2 then satisfies the same Helmholtz equation in V_1 and V_2, satisfies an OWC at infinity, and the jumps in U_2 and $\partial U_2/\partial n$ are equal to minus the values given in (5.4) and (5.5). Consequently, U_2 can be written in the form (2.23),

$$U_2 = G_2\{\Delta(\partial U_2/\partial n)\} + N_2\{\Delta U_2\} = -G_2\{(\partial U_2/\partial n)_-\} - N_2\{U_{2-}\}. \tag{5.8}$$

The integral equation for η is then obtained by imposing the condition that the function U_2 vanish at C in V_1,

$$U_{2+} = \frac{1}{2}\Delta U_2 + U_2\Big|_C = 0. \tag{5.9}$$

Substituting from (5.4) and (5.8) we obtain

$$\left(\frac{1}{2}G_1 + \frac{\alpha}{2}G_2 + \alpha G_2 N_1' + N_2 G_1\right)\{\eta\} + \frac{1}{2}E_3^0 + G_2\left\{\partial \bar{E}_3^0/\partial n\right\} + N_2\left\{E_3^0\right\} = 0$$

$$\text{on } C, \tag{5.10}$$

where the meaning of a composite operator such as N_2G_1 is

$$N_2G_1\{\eta\} = N_2\{G_1\{\eta\}\}. \tag{5.11}$$

Equation (5.10) is an integral equation of the first kind and it can be reduced to a system of linear equations to find the approximate values of η at the center of the patches. The evaluation of the functionals is reduced to matrix multiplication, where the matrix elements are essentially those in (4.7), (4.8), (4.20), and (4.21). The products of the operators in (5.11) corresponds to the matrix product, and (5.10) also involves functionals of the known functions E_3^0 and $\bar{E}_3^0$.

The scattered electric field is then obtained from η by means of (4.4) in V_1 and by means of (5.4), (5.5), and (5.8) in V_2, with the corresponding expressions for the far fields.

5.2 The TM mode

We now assume that $E_3 = 0$, which together with $k_3 = 0$ implies that $\vec{H}_\perp = 0$ from (2.7). The determination of H_3^{sc} follows a path similar to the determination of E_3^{sc} in the previous subsection if we interchange E and H. The boundary conditions are

$$H_{3-} = H_{3+} \text{ on } C, \tag{5.12}$$

$$(\partial H_3/\partial n)_- = \alpha(\partial H_3/\partial n)_+ \text{ on } C, \tag{5.13}$$

where

$$\alpha = \epsilon_2/\epsilon_1. \tag{5.14}$$

The definitions of U_1 and U_2 are the same as before, and the integral equation for η has the same form as (5.10), with α defined by (5.14) instead of (5.3) and the homogeneous magnetic fields replacing the corresponding electric fields.

5.3 The general case

If the propagation vector of the incident wave is perpendicular to the strip and neither H_3 nor E_3 vanishes, we can use superposition of the fields discussed in Subsections 5.1 and 5.2 to find the scattered fields.

If the propagation vector is not perpendicular to the strip, the z-components of the reflected, refracted, and scattered fields do not vanish even when E_3^{in} or H_3^{in} vanishes. Thus, there is no point in defining separate TE or TM modes for an arbitrary direction of propagation of the incident wave.

The tangential fields are continuous across the interface on C and (5.1) and (5.12) are still valid. The continuity of E_3

and H_3 on C implies the continuity of the tangential derivatives $\partial E_3/\partial s$ and $\partial H_3/\partial s$, and we use the continuity of the components of the perpendicular fields tangential to C, given by (2.10) and (2.11), to replace (5.2) and (5.13) with

$$(\partial E_3/\partial n)_- = \alpha(\partial E_3/\partial n)_+ + \beta(\partial H_3/\partial s), \tag{5.15}$$

$$(\partial H_3/\partial n)_- = \alpha'(\partial H_3/\partial n)_+ + \beta'(\partial E_3/\partial s), \tag{5.16}$$

where

$$\alpha = \epsilon_1 k_\perp'^2/\epsilon_2 k_\perp^2, \quad \alpha' = \mu_1 k_\perp'^2/\mu_2 k_\perp^2, \tag{5.17}$$

$$\beta = -(k_3/\omega\epsilon_2)(k_\perp'^2/k_\perp^2 - 1), \quad \beta' = (k_3/\omega\mu_2)(k_\perp'^2/k_\perp^2 - 1). \tag{5.18}$$

To reduce Maxwell's equations to integral equations for surface fields we define two sets of two auxiliary fields and one surface field. We define U_1, U_2, and η in the same way as in Subsection 5.1, and U_1', U_2', and η' as the corresponding functions for the magnetic intensity, as in Subsection 5.2. We then have two equations of the form (4.4) for U_1 and U_1'. The jumps in U_2 and U_2' are still determined by (5.4) and its analogue for the magnetic field. The jumps in $\partial U_2/\partial n$ and $\partial U_2'/\partial n$ are determined from (5.15) and (5.16), which lead to

$$\left(\frac{\partial U_2}{\partial n}\right)_- = \frac{\partial \bar{E}_3^0}{\partial n} + \alpha\left(\frac{\eta}{2} + N_1'\{\eta\}\right) + \beta\left(\frac{\partial H_3^1}{\partial s} + N_1''\{\eta'\}\right), \tag{5.19}$$

$$\left(\frac{\partial U_2'}{\partial n}\right)_- = \frac{\partial \bar{H}_3^0}{\partial n} + \alpha'\left(\frac{\eta'}{2} + N_1'\{\eta'\}\right) + \beta'\left(\frac{\partial E_3^1}{\partial s} + N_1''\{\eta\}\right), \tag{5.20}$$

where N″ is the functional

$$N''\{\eta\} = \frac{i}{4}\hat{t}\cdot\int_C ds'\eta(\vec{x}')H_1^{(1)}(k_\perp R)k_\perp\hat{R}, \tag{5.21}$$

$\bar{E}_3^0$ and $\bar{H}_3^0$ are defined as in (5.7) with the appropriate values of α and α', and

$$E_3^1 = E_3^{in} + E_3^{refl},\ H_3^1 = H_3^{in} + H_3^{refl}. \tag{5.22}$$

We impose the conditions that U_2 and U_2' vanish in V_2 at the boundary C by equations like (5.9); we obtain two integral equations for η and η',

$$\left(\frac{1}{2}G_1 + \frac{\alpha}{2}G_2 + \alpha G_2 N_1' + N_2 G_1\right)\{\eta\} + \beta G_2 N_1''\{\eta'\}$$

$$+ \frac{1}{2}E_3^0 + G_2\left\{\partial\bar{E}_3^0/\partial n + \beta\partial H_3^1/\partial s\right\} + N_2\left\{E_3^0\right\} = 0, \tag{5.23}$$

$$\beta' G_2 N_1''\{\eta\} + \left(\tfrac{1}{2}G_1 + \tfrac{\alpha'}{2}G_2 + \alpha' G_2 N_1' + N_2 G_1\right)\{\eta'\}$$

$$+ \tfrac{1}{2}H_3^0 + G_2\left\{\partial \bar{H}_3^0/\partial n + \beta' \partial E_3^1/\partial s\right\} + N_2\left\{H_3^0\right\} = 0. \qquad (5.24)$$

Once η and η' are computed from (5.23) and (5.24), the scattered fields can be obtained by integration from U_1, U_1', U_2, and U_2' from equations similar to (4.4) and (5.8). The radiated energy is obtained from the Poynting vector determined by (2.15) where we introduce the far fields in terms of the approximation (4.11). Then only the first factor on the right side of (4.11) contributes to the gradient term, so that we may write symbolically that $\nabla_\perp \approx ik_\perp\hat{\rho}$ when the gradient is applied to a field, and minus that if it is applied to the complex conjugate of a field. Equation (2.15) then reduces to

$$\vec{S} = \tfrac{1}{2}\mathrm{Re}(\vec{E} \times \vec{H}^*) \approx \omega(16\pi\rho k_\perp^3)^{-1}[\epsilon I(\phi) + \mu I'(\phi)](k_3\hat{e}_3 + k_\perp\hat{\rho}), \qquad (5.25)$$

where $I(\phi)$ is given by (4.12) and $I'(\phi)$ is the corresponding quantity in which η has been replaced by η'.

5.4 Medium with a complex dielectric constant

A lossy medium, such as a conductor with a finite conductivity, can often be represented as a dielectric medium with a complex dielectric constant which is a function of frequency. For instance, for a conductor with conductivity σ we may set

$$\epsilon = \epsilon_r + i\sigma/\omega. \tag{5.26}$$

The complex propagation constant in the lossy medium is obtained from

$$k^2 = k_1^2 + k_2^2 + k_3^2 = k_\perp^2 + k_3^2 = \epsilon\mu\omega^2, \tag{5.27}$$

which determines $k_\perp$ since we know ω and k_3, as well as k_1, from the incident wave.

In the case of a wave in a medium with a real dielectric constant incident on a half-space with a complex dielectric constant bounded by the xz-plane, the wave vector of the refracted wave is still given by (3.12) where k_2' is now a complex number determined from (5.27). The equations that lead to the integral equations (5.23) and (5.24) remain unchanged.

6. Scattering by a homogeneous strip on a half-space of a different material

We now consider the scattering by a strip composed of a dielectric or other homogeneous material located on a half-space of a different homogeneous material. We first present in Subsection 6.1 the general solution of the scattering of an electromagnetic wave at a boundary where two media that are infinite in the y-direction are separated by a layer of another medium [2]. We then show in Subsection 6.2 how the solution changes when the intermediate layer is reduced to a finite strip.

We restrict ourselves to the TE mode with $k_3 = 0$ to keep the notation simple, but the generalization to an arbitrary direction of propagation for the incident wave can be carried out following the procedure shown in Subsection 5.3.

6.1 Two semi-infinite homogeneous regions separated by a layer of finite thickness

We first show how the solution of the problem of coated dielectric gratings [2] is modified when there is no periodicity in the x-direction. The two regions V_1 and V_3 are separated by a layer V_2, as shown in figure 3. The cross section of the boundary between V_1 and V_2 is the curve C_1, and that between V_2 and V_3 is C_2. We assume that C_1 and C_2 are straight parallel lines except in a finite region, which allows us to solve the problem of reflection and refraction at plane boundaries in the usual manner [6].

Here we assume that $k_3 = 0$ and that $H_3 = 0$, and we express all the fields in terms of the z-component of the electric field. The scattered fields in V_1, V_2, and V_3 are defined as the differences between the total fields in those regions and the fields that correspond to plane interfaces. These fields are the incident and reflected fields in V_1, the up and down fields on V_2 and the refracted field in V_3. In this manner the scattered fields satisfy the OWC in all three regions.

The boundary conditions that arise from the continuity of the tangential components of the electric and magnetic fields at the interfaces give the equations

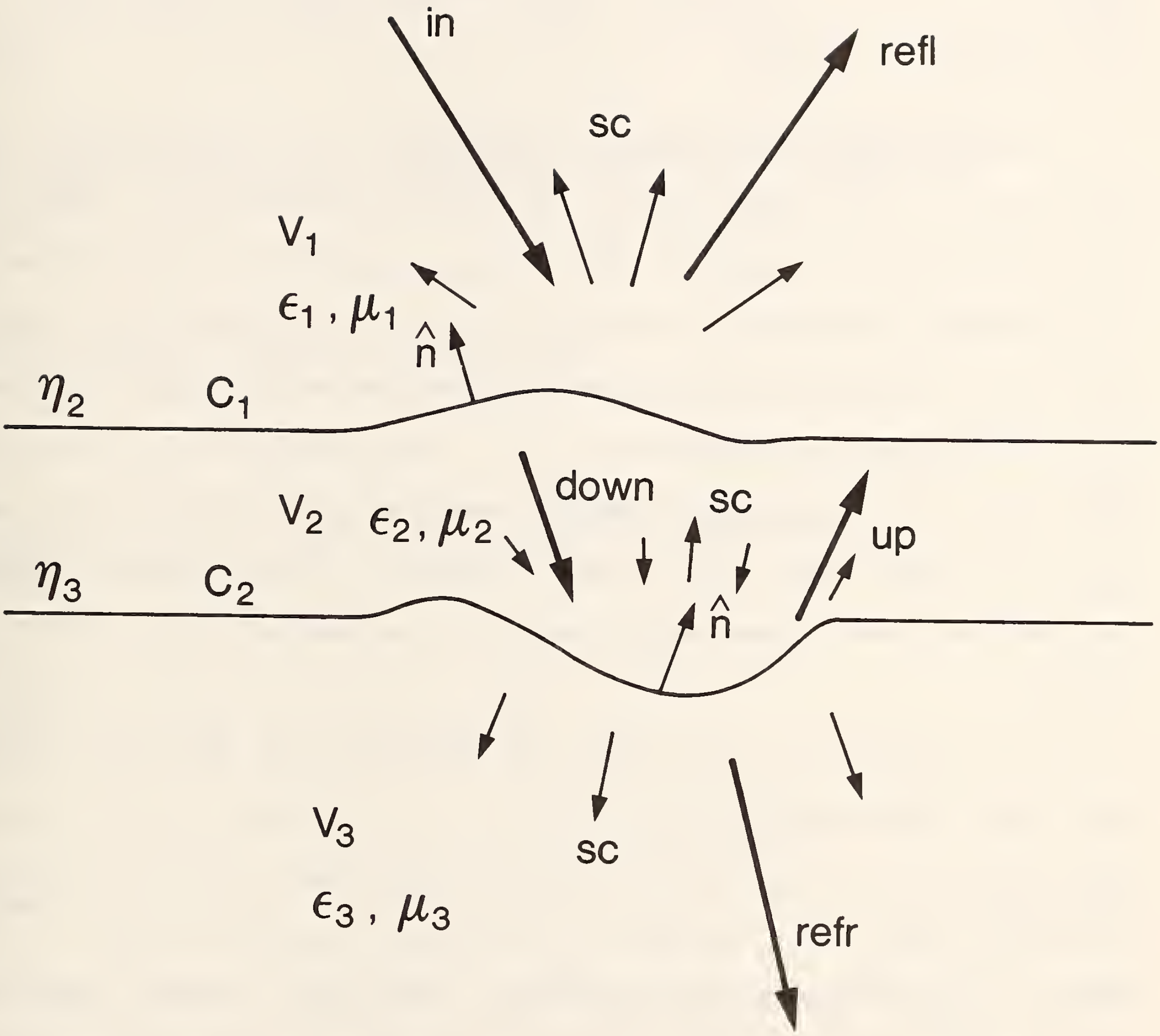

Figure 3. Geometric configuration for the scattering of a wave by a local deviation in the boundaries between three homogeneous media separated by parallel planes. The up and down homogeneous fields in V_2 are represented by the long arrows in that region, where there also is a scattered field.

$$E_{3+} = E_{3-} \text{ on } C_i, \; i = 1, 2, \tag{6.1}$$

$$(\partial E_3/\partial n)_+ = \alpha_i (\partial E_3/\partial n)_- \text{ on } C_i, \; i = 1, 2, \tag{6.2}$$

where

$$\alpha_1 = \mu_1/\mu_2, \; \alpha_2 = \mu_2/\mu_3. \tag{6.3}$$

We now define the auxiliary functions U_1, U_2, and U_3. The function U_1 is equal to E_3^{sc} in V_1 and vanishes in V_2 and V_3. The function U_2 is equal to E_3^{sc} in V_2, vanishes in V_3, satisfies the Helmholtz equation for medium 2 in V_1, is continuous across C_1, has a discontinuous normal derivative with a jump η_2 across C_1, and satisfies the OWC as $y \to \infty$. The function U_3 is equal to E_3^{sc} in V_3, satisfies the Helmholtz eqution for medium 3 in V_1 and V_2, is continuous everywhere, has a discontinuous normal derivative with a jump η_3 across C_2, and satisfies the OWC as $y \to \infty$.

We express U_3 in terms of the jump η_3 of its normal derivative across C_2 by

$$U_3 = G_{32}\{\eta_3\}, \tag{6.4}$$

where G_{32} is defined as in (2.24). The first index indicates that we have to use the constants of medium 3 in the propagation constant, and the second index that we have to carry out the integration over the curve C_2.

The jumps of U_2 and $\partial U_2/\partial n$ across C_2 are equal to the values of these functions at the boundary in V_2, which are

$$U_{2+} = E_{3+}^{sc} = E_{3+} - E_3^{up} - E_3^{down} = E_{3-} - E_3^{up} - E_3^{down}$$

$$= \quad E_{3-}^{sc} + E_3^{refr} - E_3^{up} - E_3^{down} = E_3^{0'} + G_{32}\{\eta_3\}, \tag{6.5}$$

$$\left(\partial U_2/\partial n\right)_+ = \left(\partial E_3^{sc}/\partial n\right)_+ = \left(\partial E_3/\partial n\right)_+ - \partial E_3^{up}/\partial n - \partial E_3^{down}/\partial n$$

$$= \alpha_2\left(\partial E_3/\partial n\right)_- - \partial E_3^{up}/\partial n - \partial E_3^{down}/\partial n$$

$$= \alpha_2\left[\left(\partial E_3^{sc}/\partial n\right) + \partial E_3^{refr}/\partial n\right] - \partial E_3^{up}/\partial n - \partial E_3^{down}/\partial n$$

$$= \partial \bar{E}_3^{0'}/\partial n + \alpha_2\left(- \eta_3/2 + N'_{32}\{\eta_3\}\right), \tag{6.6}$$

where N' is defined in (4.19), and $E_3^{0'}$ and $\bar{E}_3^{0'}$ are the combinations of homogeneous fields in (6.5) and (6.6) evaluated on C_2. We express the function U_2 in terms of its jump across C_2 and the jumps of its normal derivative across C_1 and C_2; it is given by

$$U_2 = G_{21}\{\eta_2\} + G_{22}\left\{\left(\partial U_2/\partial n\right)_+\right\} + N_{22}\left\{U_{2+}\right\}, \tag{6.7}$$

where N is defined in (2.25), η_2 is the jump of its normal derivative at C_1, and the other arguments of the functionals are the jumps at C_2, which are functions of η_3 as defined in (6.5) and (6.6).

The jumps of U_1 and $\partial U_1/\partial n$ across C_1 are equal to their values at the boundary in V_1, which are

$$U_{1+} = E_{3+} - E_3^{in} - E_3^{refr} = E_{3-} - E_3^1 = E_{3-}^{sc} + E_3^{up} + E_3^{down} - E_3^1$$

$$= G_{21}\{\eta_2\} + G_{22}\left\{\left(\partial U_2/\partial n\right)_+\right\} + N_{22}\left\{U_{2+}\right\} + E_3^0, \tag{6.8}$$

$$\left(\partial U_1/\partial n\right)_+ = \left(\partial E_3/\partial n\right)_+ - \partial E_3^1/\partial n = \alpha_1\left(\partial E_3/\partial n\right)_- + \partial E_3^1/\partial n$$

$$= \alpha_1\left[\left(\partial E_3^{sc}/\partial n\right) + \partial E_3^{up}/\partial n + \partial E_3^{down}/\partial n\right] - \partial E_3^1/\partial n$$

$$= \alpha_1\left[-\frac{1}{2}\eta_2 + N'_{21}\{\eta_2\} + N'_{22}\left\{\left(\partial U_2/\partial n\right)_+\right\} + M'_{22}\left\{U_{2+}\right\}\right] - \partial \bar{E}_3^0/\partial n, \tag{6.9}$$

where $M'\{\eta\}$ is the normal derivative of $N\{\eta\}$,

$$M'\{\eta\} = -\frac{i}{4}\hat{n}\cdot\int_C ds'\eta(\vec{x}')k_\perp\left[H_1^{(1)}(k_\perp R)R^{-1}(2\hat{n}'\cdot\hat{R}\hat{R} - \hat{n}')\right.$$

$$\left. - H_0^{(1)}(k_\perp R)k_\perp\hat{n}'\cdot\hat{R}\hat{R}\right], \tag{6.10}$$

and E_3^1, E_3^0, and $\bar{E}_3^0$ are combinations of homogeneous fields in (6.8) and (6.9). We need not be concerned here with the singularity of the operator M'_{22} because the functional is evaluated at a point on C_1 and the integration is over C_2. We use (6.8) and (6.9) to express U_1 in terms of the jumps in U_1 and $\partial U_1/\partial n$ as

$$U_1 = G_{11}\left\{\left(\partial U_1/\partial n\right)_+\right\} + N_{11}\left\{U_{1+}\right\}, \tag{6.11}$$

The integral equations are obtained by requiring that U_1 vanish in V_2 and that U_2 vanish in V_3. In particular, they vanish at C_1 and C_2, respectively, which gives

$$U_{1-} = -\frac{1}{2}\Delta U_1 + U_1\Big|_{C_1} = 0, \tag{6.12}$$

$$U_{2-} = -\frac{1}{2}\Delta U_2 + U_2\Big|_{C_2} = 0, \tag{6.13}$$

Substitutions from (6.5), (6.7), (6.8), and (6.11) transform (6.12) and (6.13) into

$$\left(-\frac{1}{2}G_{21} - \frac{1}{2}\alpha_1 G_{11} + \alpha_1 G_{11} N'_{21} + N_{11} G_{21}\right)\{\eta_2\}$$

$$+ \left(\frac{1}{4}\alpha_2 G_{22} - \frac{1}{2}\alpha_2 G_{22} N'_{32} - \frac{1}{2} N_{22} G_{32} - \frac{1}{2}\alpha_1 \alpha_2 G_{11} N'_{22} + \alpha_1 \alpha_2 G_{11} N'_{22} N'_{32}\right.$$

$$\left. + \alpha_1 G_{11} M'_{22} G_{32} - \frac{1}{2}\alpha_2 N_{11} G_{22} + \alpha_2 N_{11} G_{22} N'_{32} + N_{11} N_{22} G_{32}\right)\{\eta_3\}$$

$$- \tfrac{1}{2}G_{22}\left\{\partial \bar{E}_3^{O'}/\partial n\right\} - \tfrac{1}{2}N_{22}\left\{E_3^{O'}\right\} - \tfrac{1}{2}E_3^{O} + \alpha_1 G_{11} N'_{22}\left\{\partial \bar{E}_3^{O'}/\partial n\right\}$$

$$+ \alpha_1 G_{11} M'_{22}\left\{E_3^{O'}\right\} + \alpha_1 G_{11}\left\{\partial \bar{E}_3^{O}/\partial n\right\} + N_{11} G_{22}\left\{\partial \bar{E}_3^{O'}/\partial n\right\}$$

$$+ N_{11} N_{22}\left\{E_3^{O'}\right\} + N_{11}\left\{E_3^{O}\right\} = 0, \tag{6.14}$$

$$G_{21}\{\eta_2\} + \left(- \tfrac{1}{2}G_{32} - \tfrac{1}{2}\alpha_2 G_{22} + \alpha_2 G_{22} N'_{32} + N_{22} G_{32}\right)\{\eta_3\}$$

$$- \tfrac{1}{2}E_3^{O'} + G_{22}\left\{\partial \bar{E}_3^{O'}/\partial n\right\} + N_{22}\left\{E_3^{O'}\right\} = 0. \tag{6.15}$$

These two integral equations determine the unknown fields η_2 and η_3, which determine the functions U_1, U_2, and U_3 that are equal to the scattered fields in the three regions.

6.2 The homogeneous strip on a substrate of a different material

For this geometry, shown in figure 4, the region V_2 is finite and lies between regions V_1 and V_3. The region V_1 is separated from V_2 by C_1 and from V_3 by C_3, with the normal pointing into V_1. The region V_3 is separated from V_2 by C_2, with the normal pointing out of V_3. The homogeneous fields in the region V_1 are the incident field and the field reflected at the plane boundary of the region V_3 in the absence of the region V_2.

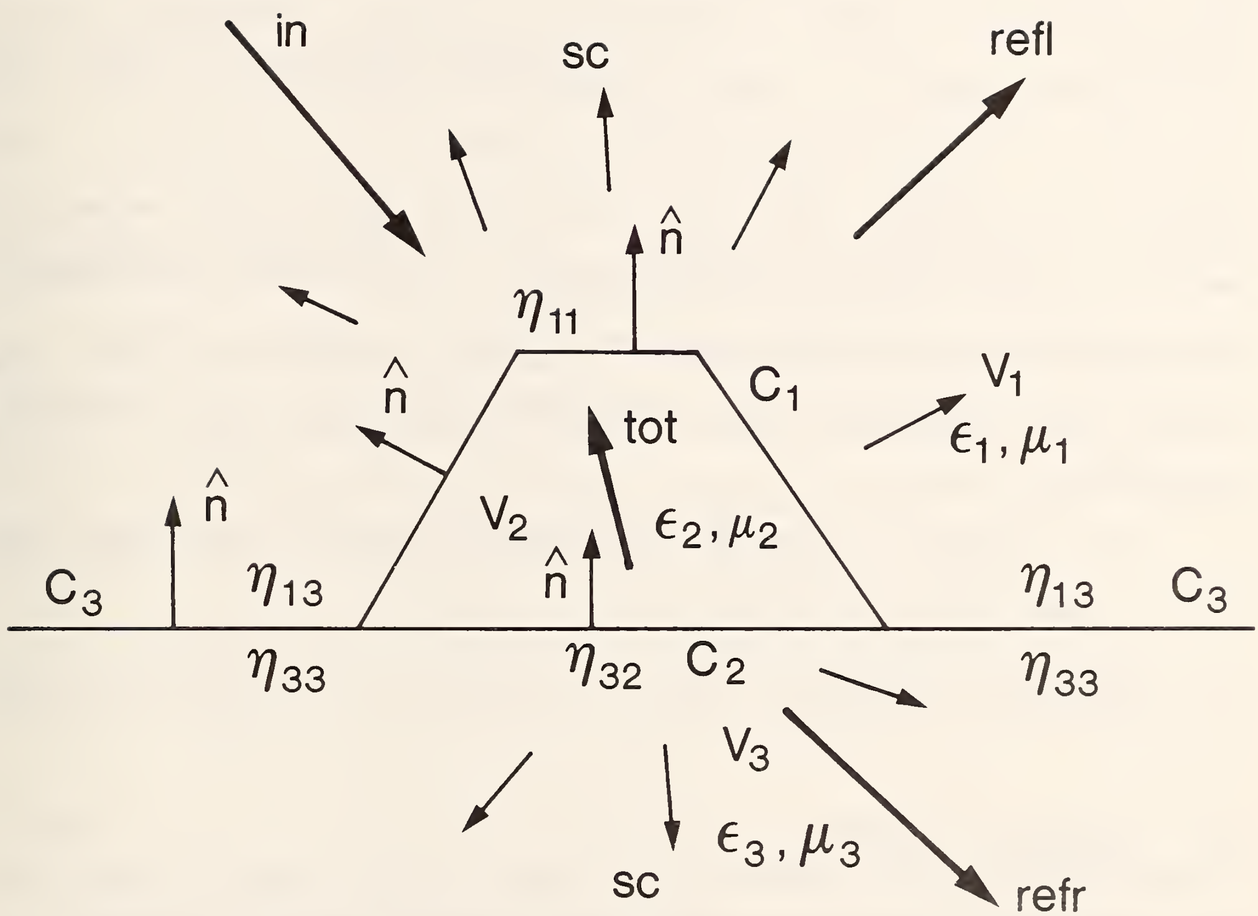

Figure 4. Geometric configuration for the scattering of a wave by a homogeneous strip on a half-space composed of a different homogeneous material. We only define a total field in the region V_2.

The homogeneous field in the region V_3 is the field refracted under those conditions. Since none of the homogeneous fields satisfies the Helmholtz equation with the constants of the medium 2, we work with the total field in V_2. We now define three auxiliary functions, U_1, U_2, and U_3. The function U_1 equals E_3^{sc} in V_1, satisfies the same Helmholtz equation as in medium 1 in V_2 and V_3, is continuous everywhere, has a discontinuous normal derivative with a jump η_{11} across C_1, a jump η_{13} across C_3, and a zero jump across C_2, and satisfies the OWC as $y \to -\infty$. The function U_2 equals E_3 in V_2 and vanishes in V_1 and V_3. The function U_3 equals E_3^{sc} in V_3, satisfies the same Helmholtz equation as in medium 3 in V_1 and V_2, is continuous everywhere, has a discontinuous normal derivative with a jump η_{32} across C_2, a jump η_{33} across C_3, and a zero jump across C_1, and satisfies the OWC as $y \to \infty$.

The functions U_1 and U_3 are given in terms of the jumps of their normal derivatives by

$$U_1 = G_{11}\{\eta_{11}\} + G_{13}\{\eta_{13}\}, \tag{6.16}$$

$$U_3 = G_{32}\{\eta_{32}\} + G_{33}\{\eta_{33}\}. \tag{6.17}$$

The values of U_2 and $\partial U_2/\partial n$ at C_1 are now determined from the continuity of the total fields across C_1; they are

$$U_{2-} = E_3^1 + G_{11}^1\{\eta_{11}\} + G_{13}^1\{\eta_{13}\}, \tag{6.18}$$

$$\left(\partial U_2/\partial n\right)_- = \alpha_1\left(\partial E_3^1/\partial n + \tfrac{1}{2}\eta_{11} + N'^{\,1}_{11}\{\eta_{11}\} + N'^{\,1}_{13}\{\eta_{13}\}\right), \tag{6.19}$$

where E_3^1 is the sum of the z-components of the incident and reflected fields evaluated on C_1, and we have added a superindex to the operators to indicate on which curve they are evaluated. Similarly, the values of U_2 and $\partial U_2/\partial n$ at C_2 are

$$U_{2+} = E_3^{refr} + G_{32}^2\{\eta_{32}\} + G_{33}^2\{\eta_{33}\}, \tag{6.20}$$

$$\left(\partial U_2/\partial n\right)_+ = \alpha_2^{-1}\left(\partial E_3^{refr}/\partial n - \tfrac{1}{2}\eta_{32} + N'^{\,2}_{32}\{\eta_{32}\} + N'^{\,2}_{33}\{\eta_{33}\}\right), \tag{6.21}$$

where E_3^{refr} is the z-component of the refracted field evaluated at C_2. These equations can then be used to express U_2 in terms of its discontinuities across C_1 and C_2, which in turn are functions of η_{11}, η_{13}, η_{32}, and η_{33}. We have

$$U_2 = -\,G_{21}\left\{\left(\partial U_2/\partial n\right)_-\right\} - N_{21}\left\{U_{2-}\right\} + G_{22}\left\{\left(\partial U_2/\partial n\right)_+\right\} + N_{22}\left\{U_{2+}\right\}. \tag{6.22}$$

Two integral equations are obtained by setting the field U_2 equal to 0 outside V_2 on C_1 and C_2. On C_1 we have

$$U_{2+} = \tfrac{1}{2}\Delta U_2 + U_2\Big|_{C_1} = -\,\tfrac{1}{2}U_{2-} + U_2\Big|_{C_1} = 0, \tag{6.23}$$

and on C_2 we have

$$U_{2-} = -\tfrac{1}{2}\Delta U_2 + U_2\Big|_{C_2} = -\tfrac{1}{2}U_{2+} + U_2\Big|_{C_2} = 0, \tag{6.24}$$

which, substituting from (6.18) through (6.22), become

$$\left(\tfrac{1}{2}G^1_{11} + \tfrac{1}{2}\alpha_1 G^1_{21} + \alpha_1 G^1_{21}N^{1\prime}_{11} + N^1_{21}G^1_{11}\right)\{n_{11}\}$$

$$+ \left(\tfrac{1}{2}G^1_{13} + \alpha_1 G^1_{21}N^{1\prime}_{13} + N^1_{21}G^1_{13}\right)\{n_{13}\}$$

$$- \left(-\tfrac{1}{2}\alpha_2^{-1}G^1_{22} + \alpha_2^{-1}G^1_{22}N^{2\prime}_{32} + N^1_{22}G^2_{32}\right)\{n_{32}\}$$

$$- \left(\alpha_2^{-1}G^1_{22}N^{2\prime}_{33} + N^1_{22}G^2_{33}\right)\{n_{33}\}$$

$$= -\tfrac{1}{2}E^1_3 - \alpha_1 G^1_{21}\left\{\partial E^1_3/\partial n\right\} - N^1_{21}\left\{E^1_3\right\} + \alpha_2^{-1}G^1_{22}\left\{\partial E^{refr}_3/\partial n\right\}$$

$$+ N^1_{22}\left\{E^{refr}_3\right\} \text{ on } C_1, \tag{6.25}$$

$$- \left(\tfrac{1}{2}\alpha_1 G^2_{21} + \alpha_1 G^2_{21}N^{1\prime}_{11} + N^2_{21}G^1_{11}\right)\{n_{11}\}$$

$$- \left(\alpha_1 G_{21}^2 N_{13}^{1\prime} + N_{21}^2 G_{13}^1\right)\{\eta_{13}\}$$

$$+ \left(- \tfrac{1}{2}G_{32}^2 - \tfrac{1}{2}\alpha_2^{-1}G_{22}^2 + \alpha_2^{-1}G_{22}^2 N_{32}^{2\prime} + N_{22}^2 G_{32}^2\right)\{\eta_{32}\}$$

$$+ \left(- \tfrac{1}{2}G_{33}^2 + \alpha_2^{-1}G_{22}^2 N_{33}^{2\prime} + N_{22}^2 G_{33}^2\right)\{\eta_{33}\}$$

$$= \tfrac{1}{2}E_3^{refr} + \alpha_1 G_{21}^2\left\{\partial E_3^1/\partial n\right\} + N_{21}^2\left\{E_3^1\right\} - \alpha_2^{-1}G_{22}^2\left\{\partial E_3^{refr}/\partial n\right\}$$

$$- N_{22}^2\left\{E_3^{refr}\right\} \text{ on } C_2. \qquad (6.26)$$

We obtain two more integral equations from the continuity of the total electric and magnetic fields on C_3. These equations are

$$G_{11}^3\{\eta_{11}\} + G_{13}^3\{\eta_{13}\} - G_{32}^3\{\eta_{32}\} - G_{33}^3\{\eta_{33}\} = - E_3^0 \text{ on } C_3, \qquad (6.27)$$

$$N_{11}^{3\prime}\{\eta_{11}\} + \left(\tfrac{1}{2} + N_{13}^{3\prime}\right)\{\eta_{13}\} - \alpha_3 N_{32}^{3\prime}\{\eta_{32}\} + \alpha_3\left(\tfrac{1}{2} - N_{33}^{3\prime}\right)\{\eta_{33}\}$$

$$= - \partial \bar{E}_3^0/\partial n \text{ on } C_3, \qquad (6.28)$$

where

$$\alpha_3 = \mu_1/\mu_3, \ \bar{E}_3^0 = E_3^{in} + E_3^{refl} - \alpha_3 E_3^{refr}, \tag{6.29}$$

and the homogeneous fields have to be evaluated at C_3. When the curves C_1, C_2, and C_3 are covered with patches, (6.25), (6.26), (6.27), and (6.28) provide the right number of equations for the unknown values of η_{11}, η_{13}, η_{32}, and η_{33} at the centers of these patches.

Since the normal on C_2 and C_3 is perpendicular to the vectors $\vec{R}$ that go from one point on this line to another, in (6.21) and, consequently, in (6.25) and (6.26) we have

$$N'^2_{32} = N'^2_{33} = 0. \tag{6.30}$$

Similarly we have

$$N^2_{22} = 0, \tag{6.31}$$

in (6.25) only, since N_{22} vanishes when it is evaluated on C_2, but not on C_1, and in (6.28) we have

$$N'^3_{13} = N'^3_{32} = N'^3_{33} = 0. \tag{6.32}$$

Since C_3 is part of the plane boundary, we have from (3.21) and (3.22) that

$$E_3^0 = 0, \ \partial\bar{E}_3^0/\partial n = 0 \text{ on } C_3, \tag{6.33}$$

and (6.27) and (6.28) reduce to the homogeneous equations

$$G^3_{11}\{\eta_{11}\} + G^3_{13}\{\eta_{13}\} - G^3_{32}\{\eta_{32}\} - G^3_{33}\{\eta_{33}\} = 0, \tag{6.34}$$

$$N^{3,}_{11}\{\eta_{11}\} + \frac{1}{2}\eta_{13} + \frac{1}{2}\eta_{33} = 0. \tag{6.35}$$

We can use (6.34) and (6.35) to eliminate η_{13} and η_{33} in terms of η_{11} and η_{32}, reducing the use of computer memory. We find

$$\eta_{13} = M^{11}_{13}\{\eta_{11}\} + M^{32}_{13}\{\eta_{32}\}, \tag{6.36}$$

$$\eta_{33} = M^{11}_{33}\{\eta_{11}\} + M^{32}_{33}\{\eta_{32}\}, \tag{6.37}$$

where

$$M^{11}_{13} = -\ (G^{3}_{13} + G^{3}_{33})^{-1}(G^{3}_{11} + 2G^{3}_{33}N^{3,}_{11}), \tag{6.38}$$

$$M^{32}_{13} = (G^{3}_{13} + G^{3}_{33})^{-1}G^{3}_{32}, \tag{6.39}$$

$$M^{11}_{33} = -\ M^{11}_{13} - 2N^{3,}_{11}, \tag{6.40}$$

$$M^{32}_{33} = -\ M^{32}_{13}. \tag{6.41}$$

Substituting of (6.30), (6.31), (6.36), and (6.37) into (6.25) and (6.26) we obtain equations of the form

$$A^{11}_{i}\{\eta_{11}\} + A^{32}_{i}\{\eta_{32}\} = B_{i},\ i = 1,\ 2, \tag{6.42}$$

where B_1 and B_2 are the right sides of (6.25) and (6.26), respectively, and

$$A_1^{11} = \frac{1}{2}G_{11}^1 + \frac{1}{2}\alpha_1 G_{21}^1 + \alpha_1 G_{21}^1 N_{11}^{1\prime} + N_{21}^1 G_{11}^1$$

$$+ \left(\frac{1}{2}G_{13}^1 + \alpha_1 G_{21}^1 N_{13}^{1\prime} + N_{21}^1 G_{13}^1\right) M_{13}^{11} - N_{22}^1 G_{33}^2 M_{33}^{11}, \tag{6.43}$$

$$A_1^{32} = \left(\frac{1}{2}G_{13}^1 + \alpha_1 G_{21}^1 N_{13}^{1\prime} + N_{21}^1 G_{13}^1\right) M_{13}^{32}$$

$$- \left(- \frac{1}{2}\alpha_2^{-1} G_{22}^1 + N_{22}^1 G_{32}^2 \right) - N_{22}^1 G_{33}^2 M_{33}^{32}, \tag{6.44}$$

$$A_2^{11} = - \left(\frac{1}{2}\alpha_1 G_{21}^2 + \alpha_1 G_{21}^2 N_{11}^{1\prime} + N_{21}^2 G_{11}^1 \right)$$

$$- \left(\alpha_1 G_{21}^2 N_{13}^{1\prime} + N_{21}^2 G_{13}^1\right) M_{13}^{11} - \frac{1}{2}G_{33}^2 M_{33}^{11}, \tag{6.45}$$

$$A_2^{32} = - \left(\alpha_1 G_{21}^2 N_{13}^{1\prime} + N_{21}^2 G_{13}^1\right) M_{13}^{32}$$

$$- \frac{1}{2}G_{32}^2 - \frac{1}{2}\alpha_2^{-1} G_{22}^2 - \frac{1}{2}G_{33}^2 M_{33}^{32}. \tag{6.46}$$

Three limiting cases that can be used to check the program are those in which $\epsilon_1 = \epsilon_2$ (no strip), $\epsilon_1 = \epsilon_3$ (strip embedded in a uniform dielectric), and $\epsilon_2 = \epsilon_3$ (strip and half-space of the same dielectric).

By setting $k_3 = 0$ and $H_3 = 0$ in (2.15), the Poynting vector reduces to

$$\vec{S} = \frac{1}{2}\mathrm{Re}\left(\frac{i}{\omega\mu}E_3\nabla_\perp E_3^*\right), \tag{6.47}$$

where we have to substitute (6.16) for the scattered field in region V_1 to find the energy flux of the scattered field. We can use this expression to find the energy flux at a given distance from the plane. If we assume that there is no interference between the scattered and the incident plus reflected fields, the vertical component of the Poynting vector is given by

$$S_2(x,y) = \frac{1}{32\omega\mu}\mathrm{Im}\left\{\left[\int_{C_1} ds'\eta_{11}(\vec{x}')H_0^{(1)}(kR) + \int_{C_3} ds'\eta_{13}(\vec{x}')H_0^{(1)}(kR)\right]\right.$$

$$\cdot \left[\int_{C_1} ds'\eta_{11}^*(\vec{x}')H_1^{(1)*}(kR)k(y - y')/R\right.$$

$$\left.\left. + \int_{C_3} ds'\eta_{13}^*(\vec{x}')H_1^{(1)*}(kR)k(y - y')/R\right]\right\}. \tag{6.48}$$

If we are in a region so close to the scatterer that the incident and reflected fields have to be taken into account, the energy flux must be computed by using the z-component of the total field in (6.47). Then (6.48) has to be replaced by

$$S_2(x,y) = \frac{1}{2\omega\mu}\mathrm{Re}\Big\{\Big[\exp(ik_1x)[E_{03}\exp(ik_2y) + E''_{03}\exp(-ik_2y)]$$

$$- \frac{i}{4}\int_{C_1} ds'\eta_{11}(\vec{x}')H_0^{(1)}(kR) - \frac{i}{4}\int_{C_3} ds'\eta_{13}(\vec{x}')H_0^{(1)}(kR)\Big]$$

$$\cdot \Big[\exp(-ik_1x)k_2[E_{03}^*\exp(-ik_2y) - E''^*_{03}\exp(ik_2y)]$$

$$+ \frac{i}{4}\int_{C_1} ds'\eta_{11}^*(\vec{x}')H_1^{(1)*}(kR)k(y - y')/R$$

$$+ \frac{i}{4}\int_{C_3} ds'\eta_{13}^*(\vec{x}')H_1^{(1)*}(kR)k(y - y')/R\Big]\Big\}. \tag{6.49}$$

The scattered fields may be much larger than the homogeneous fields, in which case the difference between (6.48) and (6.49) is negligible.

7. Interior resonances and sharp edges

For a perfect conductor, there may be resonant modes of the interior problem that have no physical effect on the exterior problem, but that disturb the numerical computations and give rise to instabilities on the solution of the exterior problem [7]. We examine the importance of this problem to our methods of solution in Subsection 7.1.

Another potential problem with numerical calculations is related to the divergent fields that occur at sharp edges [8]. We discuss this problem in Subsection 7.2.

7.1 Interior resonances

Instabilities have been predicted and observed [7] in a number of calculations of fields scattered by objects at frequencies that corresponds to resonances of the interior problem.

We have not had any of these problems in calculations with the programs described in this report.

For the perfect conductor, we have specifically calculated the scattering by a square strip of a side equal to half a wavelength and to a full wavelength, for which we might expect problems. For normal incidence, the scattered fields are negligibly small for the TM case, and there are no problems for incidence at 60° or for the TE case.

For the problem with two different dielectrics, we can set the values of ϵ_1 and ϵ_3 equal and choose the dimensions of the strip so that the cross section is a square. We then compute the wave scattered by a square cylinder in free space, and we have found no problems either for these related values of wavelength and size.

7.2 Effects of sharp edges

Fields near the edge of a wedge have been determined [8] to diverge with a power law that depends on the angle of the wedge. Locally, the same type of divergence is expected near any sharp edge.

We have used two different approaches to this problem in numerical calculations. Either we replace the sharp edge by a curve matched to the sides or we stay away from the edge itself in our selection of points.

We have replaced the profile that has a discontinuous derivative with a smoother curve by selecting a circular arc of a given radius tangential to the sides of the trapezoid. This procedure corresponds to the physical assumption that an edge is never absolutely sharp and it presents fewer numerical problems at the price of increased complexity in the calculations and a degree of arbitrariness in the choice of the curve. We recall that the contribution of the self-patch to the operator N′ depends on the curvature, as shown in (4.21). We have used this approach for the homogeneous strip on a half-space of a different material, and we have found that the values of the surface fields at the top of the strip are smaller in magnitude than those that arise from computations with sharp edges, but this is not so for the bottom of the strip, where three different media meet. It is thus best to smooth the edge at the top of the strip and keep the sharp edges at the bottom, which does not affect much the values of the fields near the top of the strip. A better approach would be to determine the analytical behavior of the unknown fields near the edges and to take it into account in the numerical calculations.

We have not smoothed the edges of the strip in calculations of the intensity in the radiation zone. We have chosen our patches so that the edge corresponds to the point where two patches meet; the matching is carried out at the center of the patches. For the trapezoidal strips, the curvature at the center of a patch always vanishes.

We have had no difficulties in our numerical calculations of the energy flow in the radiation zone that can be attributed to the divergence of the fields at edges. The surface fields increase sharply in the vicinity of an edge and they often change sign at the edge. We have also verified that getting closer to the edge with a smaller patch does not change appreciably the value of the far field. In an example, a change of a factor of 10 in the surface field near the edge was associated with a change of only 1 percent in the maximum value of the radiated energy flux.

The fields near the top surface depend strongly on the values of the nearby surface fields. This dependence is enhanced because the Green function diverges when the argument vanishes. To compute the integrals of the surface fields for points near the top surface the integration over the nearby patches has to be broken up into smaller segments, and we have interpolated the values of the surface fields from the three closest patches. We have modified the edges at the top of the strip by using circular arcs.

8. Computer programs

We have written a group of four computer programs to solve the problems of scattering by a perfectly conducting strip on a perfectly conducting half-plane in the TE mode (STRTE) or in the TM mode (STRTM), and by a dielectric strip on a dielectric half-plane when the dielectrics are the same (STRDL) or different (STR2DL). One version of STR2DL uses rounded edges for the top of the strip.

The input parameters include the dimensions of the strip, the radius at the edges, the number of patches in the different sections of the interface, the wavelength, direction of propagation, and polarization of the incident wave, the dielectric constants, and the locations where the output is computed.

The output is composed of graphs that show the scatterer and a polar diagram of the far-field intensity or of the energy flux density (the magnitude of the Poynting vector $\vec{S}$ at large distances from the scatterer) or the vertical energy flux density (S_z) at a given distance from the plane. Optionally a superimposed graph shows the absolute value of the surface function(s). These graphs are scaled to the size of the strip for display purposes.

Each program has a subroutine PATCHES where the coordinates of the points and the components of the unit normals on the top and sides of the trapezoid, as well as the x-coordinates on the base, if needed, and on the boundary between the two infinite media out to a given distance (usually a few wavelengths), are determined. By default, the patches are of equal size on each sector of the boundary. If we want to get closer to the edges without increasing the number of patches, we can specify the size of the patch closest to the edge and compute the sizes of the other patches so that they form a geometric progression and still cover the section with the given number of patches. The point distribution is symmetric with respect of the midpoint of the top of the trapezoid, and also on each side of the trapezoid with respect to its midpoint. The sizes of the sides of the trapezoid are appropriately modified when the edges are rounded.

The height of the trapezoid can be negative to represent a depression in the lower medium, the width of the top of the trapezoid can be zero to represent a triangular strip, and the width of the base can be equal to or smaller than the width of the top to represent a rectangular strip or an "upside down" trapezoid. A different subroutine PATCHES could be written to represent more general shapes of strips; the computations should then include the curvature to take into account the self-patch contribution in (4.21). The curvature at points of the circular arc are equal to the inverse of the radius.

Each program has a subroutine INFL where the homogeneous fields are determined on the boundaries as required in each case. These will be used to determine the known values in the system of linear equations. Then a subroutine MATRIX is used to find the required values of the matrices representing the operators at the given points and to determine the coefficients and constants on the system of linear equations. The subprograms used to compute the values of the Hankel functions are BESJ0, BESJ1, BESY0, and BESY1 if the arguments are real, and CJYHBS if the arguments are complex. These subroutines are in CMLIB [9].

The systems of linear equations are solved using the subroutines CGECO and CGESL [9], which also provide a conditioning parameter that can be checked for difficulties related to ill-posedness. These subroutines lead to excessive paging when the sizes of the arrays exceed the available computer memory, and we had to replace them by CGEL from the MAGEV library.

The intensity of the scattered field in medium 1 is computed from the appropriate surface fields in a subroutine FARFL. A similar computation could be carried out for the lower medium if there is no absorption. The fields at a given distance above the plane are computed in a subroutine NRFL; we have done this for the two dielectrics and we assume that the distance is larger than the height of the trapezoid.

These programs take advantage of the vector processing on the CYBER 205, although not all of the subroutines in CMLIB are vectorized.

In figure 5 we show samples of outputs from the programs STRTE and STRTM for perfect conductors. The total number of patches for the triangular strip was 420, and for the trapezoidal strip, 600. The running times were approximately 9 and 20 cpu-s for the TE mode and 13 and 29 cpu-s for the TM mode.

In figure 6 we show samples of the output of STRDL for different directions of propagation and polarization of the

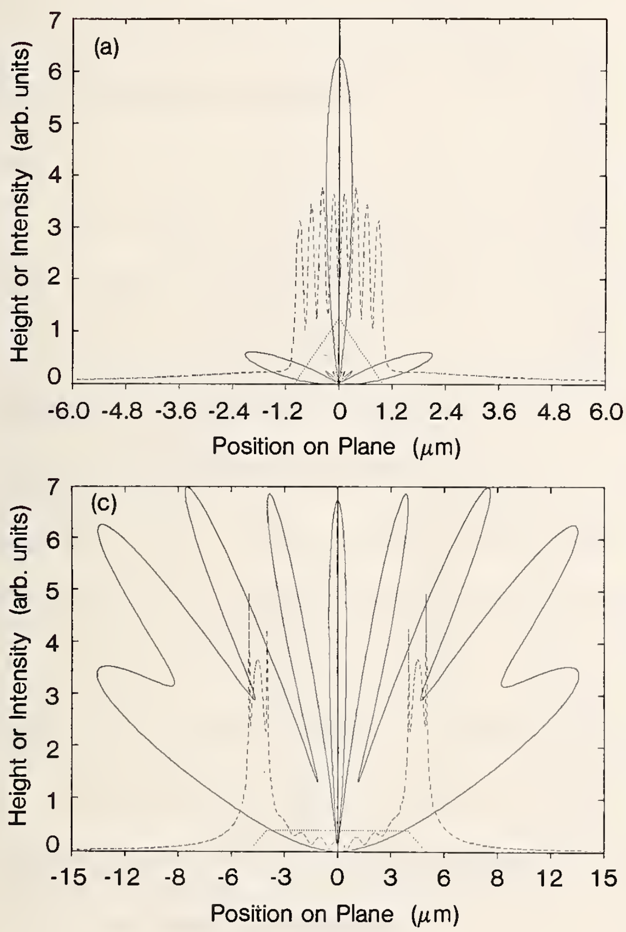

Figure 5. Polar diagrams of the intensity of the field scattered by a triangular or trapezoidal perfectly conducting strip. The dotted lines show the strips, which in (a) and (b) have a triangular cross section with a base equal to 4λ and height 2λ, and in (c) and (d) are trapezoids with top 2λ, base 3λ, and height $\lambda/2$. The dashed lines represent the magnitude of the surface field. The incidence is normal and (a) and (c) correspond to the TE mode and (b) and (d) to the TM mode.

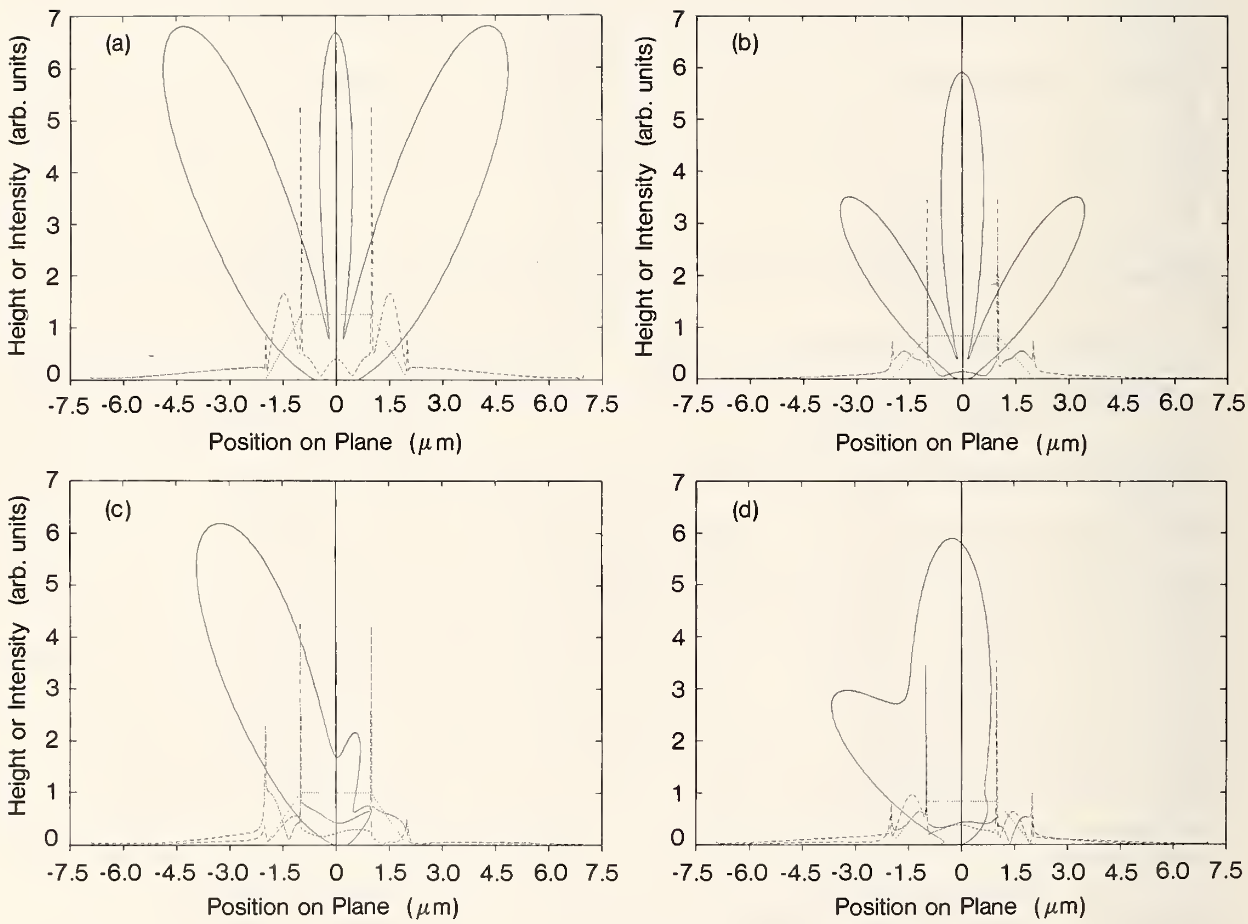

Figure 6. Polar diagram of the intensity of the field scattered by a trapezoidal dielectric strip (ϵ = 2.14, top λ, base 2λ, and height $\lambda/2$) on a half-space of the same dielectric for different incident waves, (a) TE mode, (b) TM mode, (c) direction of propagation of the incident wave with polar angle $\theta = 60°$, azimuthal angle $\phi = 60°$, polarized so that $E_3^{in} = 0$, and (d) direction of propagation with $\theta = 60°$, $\phi = 90°$, polarized so that $E_3^{in} = E_1^{in}$.

incident wave. For a total of 460 patches, the running time was approximately 110 CPU-s. In figure 7 we show the output of the same program for the scattering of a wave by square or rectangular glass cylinder with a side equal to the wavelength.

In figure 8 we show the vertical component of the Poynting vector for the field above a silver strip on a glass substrate at different heights above the strip.

9. Concluding remarks

We have presented the derivation of the integral equations that can be used to find the fields scattered by strips on a half-space for different configurations and materials. These are singular integral equations designed to minimize the number of unknown fields on the interfaces between the media. We have also pointed out some possible difficulties with sharp edges and spurious resonances that require further investigation.

The theory is exact, well suited to the resonance regime, where the wavelength is comparable to the size of the scatterer.

We have briefly described the programs that were written to implement these calculations, and we have shown sample outputs from these programs.

10. Acknowledgements

I thank Dr. Daniel Maystre, Dr. Ralph E. Kleinman, Dr. A. George Lieberman, Dr. David Gilsinn, and Dr. Jay F. Marchiando for helpful comments and suggestions.

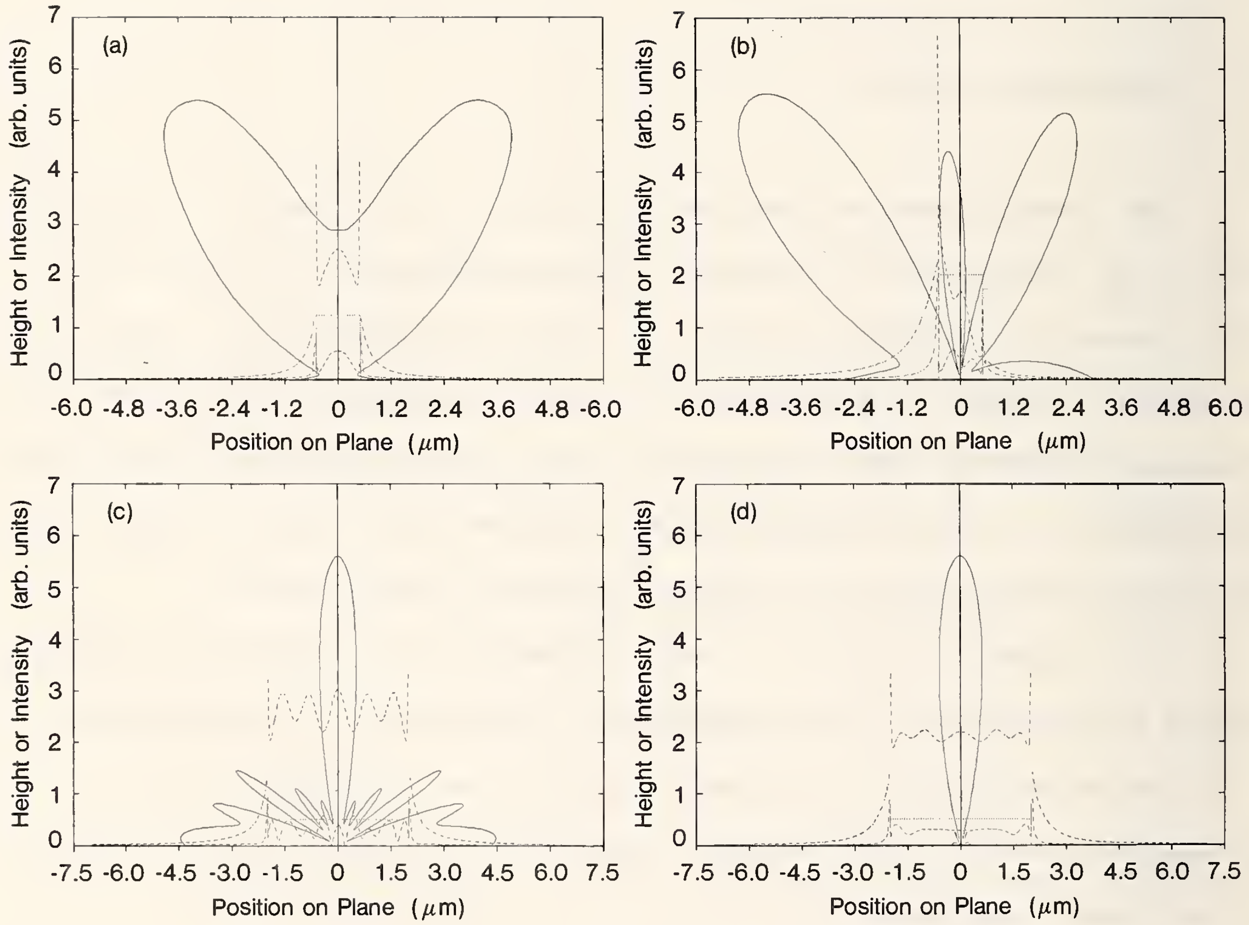

Figure 7. Polar diagram of the intensity of the field scattered by a square glass cylinder of side λ in free space in the TE mode, $\epsilon = 2.14$, incident direction (a) perpendicular to top (b) at $\phi = 45°$ and for a rectangular cylinder of height λ and width 4λ, with (c) $\epsilon = 2.14$, and (d) $\epsilon = 2.14 + 0.5i$. The maximum intensity in (d) is about 1/3 of that in (c).

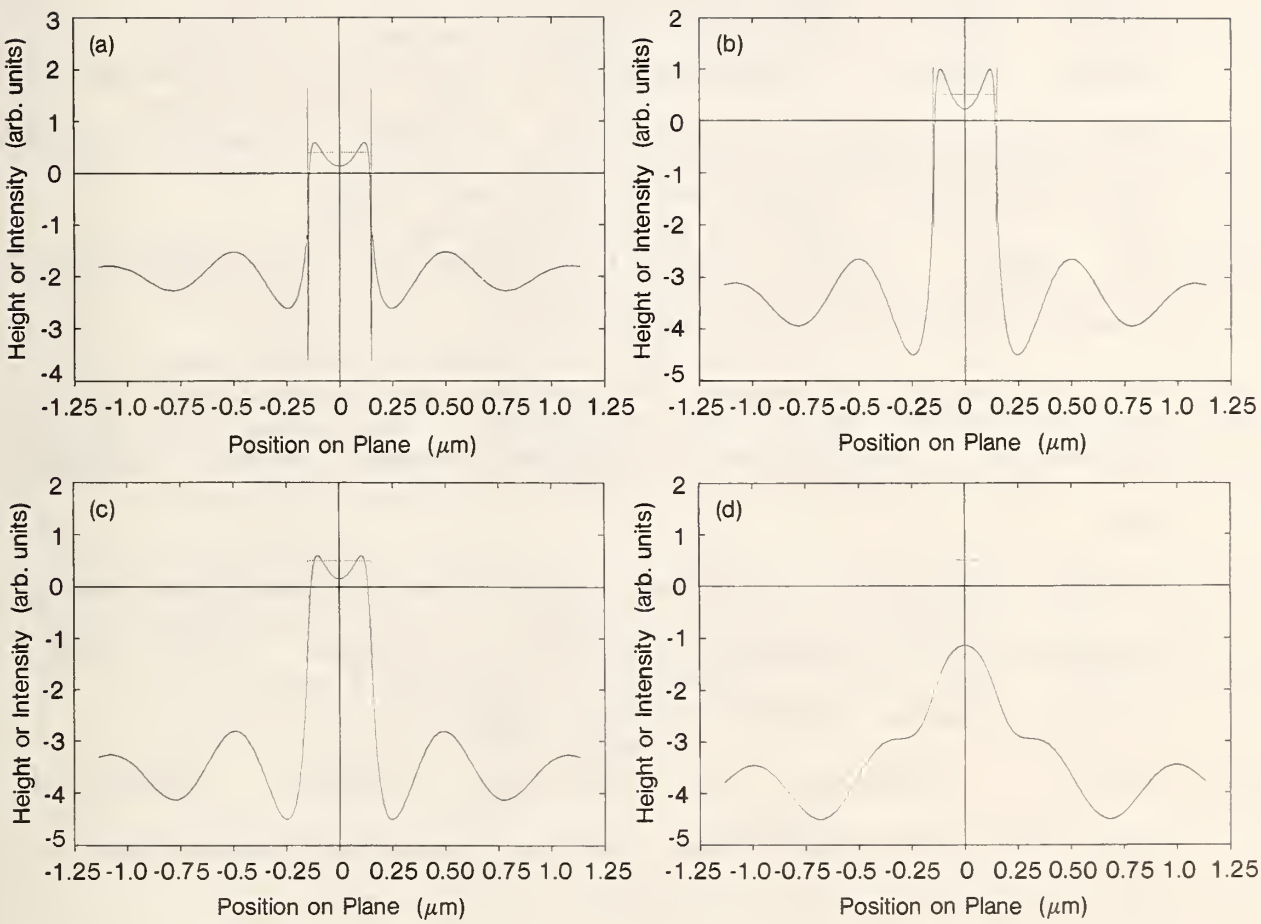

Figure 8. Vertical component of Poynting vector for the total fields above a silver strip ($\epsilon = -11.68 + 1.39i$, $h = 0.1$ μm, $w = 0.3$ μm) on a glass substrate ($\epsilon = 2.14$) for $\lambda = 0.6328$ μm. The energy flux density is shown for a distance above the strip of (a) 0.0005 μm, (b) 0.001 μm, (c) 0.01 μm, and (d) 0.1 μm.

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NBS-114A (REV. 2-80)

U.S. DEPT. OF COMM. BIBLIOGRAPHIC DATA SHEET (See instructions)	1. PUBLICATION OR REPORT NO.	2. Performing Organ. Report No.	3. Publication Date
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4. TITLE AND SUBTITLE

Electromagnetic Scattering by a Thick Strip on a Half-Space

5. AUTHOR(S)

Egon Marx

6. PERFORMING ORGANIZATION (If joint or other than NBS, see instructions)

NATIONAL BUREAU OF STANDARDS
U.S. DEPARTMENT OF COMMERCE
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8. Type of Report & Period Covered

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9. SPONSORING ORGANIZATION NAME AND COMPLETE ADDRESS (Street, City, State, ZIP)

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10. SUPPLEMENTARY NOTES

☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

The problem of the determination of the scattered fields from an incident plane monochromatic electromagnetic wave on a strip of finite thickness on a half-space is reduced to the numerical solution of integral equations for auxiliary fields defined on the interfaces. These fields are chosen so as to minimize their number. The derivation of the integral equations is given for a perfectly conducting strip on a perfectly conducting half-space, for a dielectric (or other homogeneous material) on a dielectric half-space, and for a dielectric strip on a half-space of a different dielectric material. The vector FORTRAN programs written to carry out these calculations are briefly described and sample outputs are shown.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

dielectric strip; electromagnetic scattering, integral equations; numerical solutions; resonance region; perfect conductors

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