High Voltage Divider and Resistor Calibrations

Martin Misakian
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1 Headquarters and Laboratories at Gaithersburg, MD, unless otherwise noted, mailing address

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High Voltage Divider and Resistor Calibrations

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HIGH VOLTAGE DIVIDER AND RESISTOR CALIBRATIONS

M. Misakian

Abstract

An NBS calibration service for determining the ratio of high-voltage dc dividers and the resistance of high-voltage resistors is described. Calibrations are performed with a Wheatstone bridge apparatus with a simple guard system. Sources of systematic error are identified and methods for characterizing the NBS standard high-voltage resistors are discussed. Ratio and resistance values can be determined between the voltages of 10 kV and 150 kV with an uncertainty of less than ±0.01%.

Keywords: calibration service; dc; high voltage; NBS; resistor; voltage divider.

1. INTRODUCTION

The laboratory applications of direct-current (dc) high voltages range from fundamental studies, such as elementary particle physics and electrical discharges in dielectrics, to more applied studies such as insulation tests with systems containing large amounts of capacitance (e.g., high voltage cables and capacitors). Technical uses of high dc voltages include the generation of x-rays, electrostatic precipitation, paint spraying, and the transmission of electrical energy.

High dc voltages frequently are measured with resistive voltage dividers. The dividers typically have one or two low-voltage connections with corresponding ratios. This report describes a calibration service at NBS for high-voltage dividers and resistors that are of sufficient accuracy to be regarded as transfer standards. Devices calibrated at NBS are normally used as standards in industrial and government laboratories.

2. DESCRIPTION OF SERVICE

A calibration service is maintained at NBS for the determination of dc voltage ratios of resistive dividers and their total resistance. The calibration service is normally available for applied voltages from 10 kV to 150 kV. The calibrations are performed with an apparatus that permits measurement of the ratio with an uncertainty of less than ±0.01%. To assure adequate sensitivity at the lowest applied voltage levels, calibrations are normally performed on dividers with ratios of 100,000:1 or smaller; the low voltage resistance is also restricted to 10,000 ohms or less.

Resistive dividers are acceptable for calibration only if they are corona-free at the operating voltage and are designed to have small temperature and voltage coefficients. Specifically, a device is not generally suitable for calibration by NBS if these coefficients produce a change in the ratio of 0.1% over the normal range of operating voltages. At a given voltage, dividers should not exhibit instabilities in their ratio values in excess of 0.005%. NBS staff can provide some assistance in the identification of intermediate calibration laboratories capable of determining the ratios of less accurate voltage dividers.
A typical calibration consists of monitoring the divider ratio with a fixed voltage continuously applied for 15 minutes. Ratio determinations are made every three minutes. The measurements are performed at two voltages selected by the customer to determine the effects of voltage and heating on the divider ratio. The tests begin at each voltage when the divider is at room temperature. Ratio measurements can be made for different functions of time or voltage by special arrangement.

Dividers can be hand-carried or shipped to NBS. If they are shipped, they should be packaged in sturdy reusable containers. The design of many high voltage dividers makes them vulnerable to shear-type forces. Therefore, provisions should be made to minimize the likelihood of damage due to such forces when the device is in the shipping container.

A calibration service is also maintained for resistors designed for dc high-voltage applications. This service is for corona-free resistors designed for dc operation between 10 kV and 150 kV. The measurement uncertainties and cautions about shipping damage are the same for resistors as they are for voltage dividers.

3. CALIBRATION APPARATUS AND MEASUREMENT APPROACH

Measurements of divider ratios are performed with a Wheatstone bridge apparatus shown schematically in figure 1. Resistors $R_{X1}$ and $R_{X2}$ are resistors in the divider that is being tested and resistors $R_M$ and $R_N$ are fixed and variable standard resistors, respectively, maintained at NBS. As suggested by the point of high voltage application, $R_X$ and $R_M$ are high voltage resistors and $R_{X2}$ and $R_N$ are low voltage resistors. The detector $g$ is a sensitive galvanometer with resistance $G$, indicated schematically in figure 1 by $R_G$.

The ratio of the divider under test is determined by nulling the galvanometer and using the well-known relationship between the resistances under this condition [1] to calculate the ratio. That is, when the bridge is balanced

$$X_1N = X_2M$$

(1)

where $X_1$ is the resistance of $R_{X1}$, $N$ is the resistance $R_N$, etc. (subscripts denote values of respective resistors). Using elementary algebra, eq (1) can be rewritten to yield the ratio of the divider under test,

$$\text{Ratio} = \frac{(X_1 + X_2)}{X_2} = \frac{(M+N)}{N}$$

(2)

The bridge, when balanced, introduces negligible burden on the divider under test. The user is thus cautioned that introduction of a resistance across the low side of the divider with, for example, a digital voltmeter will change the ratio. Upon request, calibrations of the divider can be performed with a burden provided by the user on the low side of the divider. By replacing $R_X$ with a known standard resistor, the bridge circuit in figure 1 can also be used to determine the value of an unknown high voltage resistor, $R_{X1}$, using eq (1).

Equations (1) and (2) imply that the uncertainties associated with determining unknown ratio and resistance values are related to the uncertainties in the values of the standard resistors. An additional consideration is the sensitivity of the galvanometer. The resistors and galvanometer used in the NBS calibration apparatus as well as a simple guard system to detect leakage currents are described below.
Figure 1. Schematic view of Wheatstone bridge circuit for calibration of high voltage dividers.
3.1 Standard High Voltage Resistors

Resistor $R_M$ in figure 1 typically consists of three 100 megohm Park resistors [2] connected in series (Serial Nos. 156881, 156882, 156883). The characteristics of the Park resistor are briefly described here; a more detailed account is given in Reference 2. Each Park resistor is made of 100 one-megohm wire-wound resistors connected in series and arranged to form a vertical helix between a ground plate and high voltage electrode. By removing the ground plates it is possible to stack several Park resistors in series as shown in figure 2.

During application of high voltage to the Wheatstone bridge, changes in resistance can occur in the high voltage resistors because of (i) heating of the resistance wire, (ii) leakage of current over insulating surfaces used to support the resistor and, (iii) corona discharges. Each 1 megohm resistor in the Park has a temperature coefficient of less than $\pm 2$ parts-per-million (ppm)/°C near room temperature, and resistors with temperature coefficients of opposite sign have been paired to minimize changes in resistance due to heating. The estimated temperature coefficient of the NBS Park resistors is less than 0.4 ppm/°C and temperature changes anticipated during calibrations up to 150 kV for three Park resistors in series should produce a resistance change of less than 10 ppm [2].

Leakage current over insulating surfaces and current due to corona both effectively introduce resistances that are in parallel with the high voltage resistor. Precautions taken in the design of the Park resistor to minimize these possible problems include the use of electrically biased metal shields to enclose resistor pairs to eliminate corona and the use of a corona ring at the top of each Park resistor (fig. 2). The effectiveness of these precautions is checked by simultaneously measuring the current entering at the top of the Park resistor and the current exiting at the bottom of the resistor under high voltage conditions. At 100 kV, the currents differed by less than the estimated measurement uncertainty of 20 ppm. At 50 kV, the maximum voltage normally applied to the Park resistors, the difference is less than 10 ppm [2]. Using Ohm's law, it is readily shown that the difference in current at 50 kV results in an uncertainty of less than 10 ppm in the value of each Park resistor.

3.2 Standard Low Voltage Resistors

Resistor $R_N$ in figure 1 consists of Rosa-type standard resistors [3] connected in series with a calibrated resistance box which can be varied from 0 to 10000 ohms in steps of 0.01 ohm. The standard resistors have elements made from a low temperature-coefficient alloy which are sealed in metal containers filled with moisture-free oil. Each resistor is connected into the bridge circuit with two copper arms that are supported on an NBS-type stand with amalgamated surfaces [3] in order to minimize contact resistances. The contact resistances at the amalgamated surfaces are of order $1 \times 10^{-6}$ ohms [3]. Strips of copper 2-cm wide and 0.076-cm thick are used to connect the resistance box to the standard low voltage resistors and to ground. The resistance of the copper strips can be calculated and is of order 0.0006 ohm. The contact resistances at the binding posts are estimated to be of order 0.001 ohm or less [2]. These resistances are negligibly small compared to the resistance of $R_N$ which is typically 3000 ohms or more. Changes in the value of $R_N$ due to heating are also negligible because of the small power that is dissipated, typically less than $2.5 \times 10^{-3}$ watt. The resistance change is expected to be less than 1 ppm [4]. Figure 3 shows the arrangement of the standard low voltage resistors, stand, and resistance box.
Figure 2. Stacked Park resistors form standard high voltage resistor $R_M$ in figure 1. Polyethylene sheet under base covers metal guard plate.
Figure 3. Standard low-voltage resistors supported on NBS-type stand with amalgamated surfaces. Resistors on stand and calibrated decade box form resistor $R_N$ in figure 1. Also shown are the galvanometer switch and galvanometer. Polyethylene sheets which sandwich a guard plate can also be seen.
3.3 Galvanometer Sensitivity

The sensitivity of the galvanometer influences the uncertainty associated with determining the null condition and thus the ratio or resistance of the device being tested. The galvanometer used in the NBS Wheatstone bridge indicates a 1 mm deflection for 0.002 μA of current. When the bridge is slightly unbalanced, the current through the galvanometer ΔIG is given by [1]

$$\Delta I_g = V\Delta N/[M+N_0]\{G[1+(X_2/X_1)]X_2+N_0\}$$

(3)

where V is the high voltage, and ΔN is the change in N from the null value, N0. For typical values of X1 equal to 100 megohms, X2 equal to 1000 ohms, M equal to 300 megohms, N0 equal to 3000 ohms, and G equal to 560 ohms [5],

$$\Delta I_g = 7.31 \times 10^{-13} V \Delta N.$$  

(4)

The lowest voltage normally used during calibrations is 10 kV and the corresponding galvanometer current is

$$\Delta I_g = 7.31 \times 10^{-9} \Delta N.$$  

(5)

Thus, a change in N of about 0.28 ohm will produce a galvanometer current of 0.002 μA and a deflection of 1 mm in the galvanometer display. Using a magnifying glass to view the galvanometer scale, a displacement of less than 0.1 mm can be discerned. A 0.1 mm change in the galvanometer display corresponds to a change in N of less than 0.03 ohm or 10 ppm. At higher voltages the null condition can be determined with greater precision. For example, when V is equal to 50 kV, a galvanometer deflection corresponding to a change in N of 0.01 ohm or 3.3 ppm is readily seen.

3.4 Guard System

The NBS calibration apparatus is equipped with a simple guard system which is designed to intercept leakage currents to ground. Portions of the guard system can be seen in figures 2 and 3. The Park resistors and standard low voltage resistors are located or supported on sheets of high density polyethylene. Beneath each polyethylene sheet is a metal plate (portions of the guard system) and beneath the metal plate is yet another layer of polyethylene. A similar combination of polyethylene sheets and a metal plate is located under the divider or resistor being tested. Through a switching system, leakage currents which reach the metal plates can be routed through the galvanometer. The connection of the guard plates to the galvanometer is equivalent to connecting the leakage resistances to ground (i.e., bottom layer of polyethylene, table, floor tile, etc.) in parallel with the low side resistors in the bridge. Figure 4 is a schematic view of the guard system when connected to the right side of the galvanometer.

If leakage currents occur across dielectric surfaces because, for example, of high humidity, the guard system shown in figure 4 provides a means to determine its magnitude. By noting the change in the value of RN necessary to rebalance the bridge, when the guard system is connected to the galvanometer, and the resistances X2 and N in the bridge, an estimate of the leakage resistance to ground can also be made. The practice at NBS is to routinely check for the presence of leakage currents during each calibration and to
Figure 4. Schematic view of guard system. The leakage resistance to ground is shown connected in parallel with resistor $R_N$. 
postpone data acquisition if significant leakage currents due to humid conditions (i.e., relative humidity greater than about 55%) are present.

4. MAINTENANCE OF STANDARD RESISTORS

4.1 Calibration of Park Resistors

4.1.1 Early Measurements

Two methods were originally employed to determine the resistance values of the Park resistors. Both methods involve measurements at low voltage which, because of the low temperature coefficient of the Park resistor and freedom from corona and leakage paths, can be taken to be applicable to high voltage conditions.

In the first approach, the resistance of each 1 megohm resistor was measured with a Wheatstone bridge and standard low voltage resistors. The total resistance was taken to be the sum of the resistances.

In the second approach, the Park resistor was partitioned into ten 10-megohm sections. These ten sections were then connected in parallel to form a 1-megohm resistor. By measuring the parallel resistance (using a Wheatstone bridge with standard low voltage resistors), it was possible to compute the resistance of all ten sections in series because the resistance of each section was within 0.1% of 10 megohms. This procedure, used previously by Wenner [4] and others, is described in further detail in the Appendix. The resistance values obtained using the partitioning approach, for temperatures ranging from 23°C to 26°C, were found to be in agreement with the value obtained by summing the 100 1-megohm resistors within 15 ppm [2].

Since the measurements reported by Park in the early 1960's, the resistance of each NBS Park resistor has been measured periodically using the partitioning technique noted above. The 1-megohm, partitioned, Park resistor is compared with a standard, 1-megohm, wire-wound resistor using a Wheatstone bridge circuit [6]. The uncertainty in the value of the 1-megohm standard resistor used during the comparison is estimated to be less than ±10 ppm [6,7].

Figures 5 through 8 show measurements of the departure, \( a_i \), from the nominal 100 megohm value in parts-per-million, for each of the NBS Park resistors for the years 1967 to 1973. The values of \( a_i \) obtained in the year 1969 are higher than those obtained in other years, but no well defined trend in the resistance values is evident. Most of the values of \( a_i \) for each resistor are contained in a band that is about 30 ppm. These early measurements show that the resistance values remain relatively stable over a period of years.

Figures 9 through 11 show the \( a_i \) data as histograms for three of the Park resistors. The fourth Park resistor (serial number 156883) is discussed further below. Calculated histograms, assuming a Gaussian distribution [8], are also shown in figures 9 to 11 with dashed lines. Each bar of the calculated histogram has been obtained using an expression of the type

\[
\frac{a_i + 1.5}{\int_{-\infty}^{\infty} n f(a) \, da} = 0.5
\]

where \( f(a) \) is the Gaussian probability function.
Figure 5. Measurements of $a_i$ for Park resistor 156881 for the years 1967 to 1973.

Figure 6. Measurements of $a_i$ for Park resistor 156882 for the years 1967 to 1973.
Figure 7. Measurements of $a_i$ for Park resistor 156883 for the years 1967 to 1973.

Figure 8. Measurements of $a_i$ for Park resistor 156884 for the years 1967 to 1973.
Figure 9. Histogram of $a_j$ in parts-per-million (ppm) for Park resistor 156881 during the years 1967-1973. The dashed line is a calculated histogram assuming a Gaussian distribution and is described in the text.
Figure 10. Histogram of $a_i$ in parts-per-million (ppm) for Park resistor 156882 during the years 1967-1973. The dashed line is a calculated histogram assuming a Gaussian distribution and is described in text.
Figure 11. Histogram of $a_i$ in parts-per-million (ppm) for Park resistor S/N 156884 during the years 1967-1973. The dashed line is a calculated histogram assuming a Gaussian distribution and is described in text. The dotted line is a calculated histogram which excludes the data for 1969.
\[
f(a) = \frac{(1/\sqrt{2\pi})\exp[-(a-a)^2/2\sigma^2]}{2\pi}
\]

\(n\) is equal to the number of measurements

\(a_i\) is the departure from nominal in ppm for the \(i\)th measurement

\(\bar{a}\) is the average departure from nominal

\(\hat{\sigma}\), the standard deviation, is given by

\[
\hat{\sigma} = \left[ \frac{(n-1)^{-1} \sum (a_i - \bar{a})^2}{i=1} \right]^{1/2}
\]

In figures 9-11, the 1969 data are indicated with bars containing diagonal lines. A summary of the data showing the number of data points, \(n\), the average departure from 100 megohms, \(\bar{a}\), and the standard deviation, \(\hat{\sigma}\), are given in table 1.

As noted earlier, the 1969 data are higher than for other years and it is found to have a significant effect on the Gaussian distribution used to fit the data of Park resistor 156884. However, in light of some recent measurements of Park resistor 156884, which are discussed in Section 4.1.2, the likelihood that a systematic error was present when the 1969 results were obtained seems high. Figure 11 shows the effect on the calculated histogram when the 1969 data are excluded in the statistical analysis for Park resistor 156884.

In addition to the low-voltage measurements, intercomparisons between the Park resistors were made in 1978 at high voltage using a Wheatstone bridge circuit. The results of these measurements were consistent with the measurements presented in table 1, i.e., the measured values are typically within 10 ppm of the average \(\bar{a}\).

While the resistance values of the fourth Park resistor (156883) presented in figure 7 show no trend between the years 1967 and 1973, the overheating of one section of ten 1-megohm resistors in 1975 resulted in a resistance change of about 40 ppm. Several subsequent measurements of this resistor in 1975 using the partitioning technique indicated a departure from nominal, \(a_j\), of 150 ppm. Measurements performed in 1978, with voltages between 100 V and 500 V, indicated values of \(a_j\) between 154 and 157 ppm with an uncertainty of less than \(\pm 20\) ppm.

4.1.2 Recent Measurements and Practices

The early measurements of Park resistors 156881 and 156882 indicate that the resistance values have remained stable from 1967 to 1973 within about \(\pm 15\) ppm. The observed variations can be attributed, in great part, to random measurement errors and the uncertainty in the value of the standard resistor used for comparison when employing the partitioning technique.

Measurements of the Park resistors in 1984 have been performed by comparing the partitioned Park resistors with a standard 1 megohm resistor using a precision digital ohmmeter. These measurement results are also given in table 1 and reveal shifts in resistance values from the historic averages of approximately 18 ppm and 9 ppm for resistors 156881 and 156882, respectively.
Table 1. Summary of Park Resistor Measurements

<table>
<thead>
<tr>
<th>DIVIDER</th>
<th>n</th>
<th>( \overline{a} ) (ppm)</th>
<th>( \hat{a} ) (ppm)</th>
<th>Recent Measured Values of ( a_i ) (year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>156881</td>
<td>25</td>
<td>106.5</td>
<td>7.7</td>
<td>125(1984)(^d)</td>
</tr>
<tr>
<td>156882</td>
<td>31</td>
<td>67.5</td>
<td>7.2</td>
<td>77(1984)(^d)</td>
</tr>
<tr>
<td>156884</td>
<td>32</td>
<td>118.5</td>
<td>10.9</td>
<td>117(1978)(^c); 128(1984)(^d)</td>
</tr>
<tr>
<td>156884(^b)</td>
<td>24</td>
<td>114.5</td>
<td>5.3</td>
<td>----</td>
</tr>
<tr>
<td>156883</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>150(1975); 154(1984)(^d)</td>
</tr>
</tbody>
</table>

\(^b\) - excluding 1969 data

\(^c\) - measurement performed in Electrical Measurements and Standards Division with uncertainty of less than ±20 ppm

\(^d\) - comparison with a 1-megohm standard resistor using precision ohmmeter
The most recent measurement of Park resistor 156883 yields a value of 154 ppm for \( a_j \) which is 4 ppm higher than its 1975 value. As noted in Section 3.1, resistor \( R_M \) consists of Park resistors 156881, 156882, and 156883. Thus, the change in resistor \( R_M \) amounts to about 11 ppm which is small relative to the total uncertainty quoted in NBS calibration reports (see Section 5.2).

The 1984 resistance value of Park resistor 156884 (table 1) also differs from its historical average by about 11 ppm. The changes in resistors 156884 and \( R_M \) (156881, 156882, and 156883) since about 1978 went undetected because the procedure used to monitor the values of the Park resistors was insensitive to comparable simultaneous changes in these resistors. Since 1980 the calibration apparatus has been checked several times annually by measuring the resistance of Park resistors 156884 at high voltage, typically near 50 kV. This is done by replacing the unknown resistor \( R_X \) (fig. 1) with Park resistor 156884 and using a standard low voltage resistor for \( R_X \). During these system checks, the value of resistor 156884 was calculated with eq (1) using the pre-1984 value of resistor \( R_M \). Checks of the apparatus since 1980 have yielded an average departure from nominal (\( \bar{a} \)) of 121 ppm (\( \sigma = 8 \) ppm) which is in good agreement with the pre-1984 value of resistor 156884 determined with the partitioning technique. Figure 12 shows high voltage measurements of the departure, \( a_j \), from the nominal 100 megohm value in parts-per-million for resistor 156884 during checks of the calibration apparatus from 1980 to 1984. Figure 13 presents the data of figure 12 as a histogram. If the 1984 value of resistor \( R_M \) had been used to calculate the value of Park resistor 156884 at high voltage, its value would be larger by about 11 ppm (i.e., \( a=130 \)) and in agreement with the latest determination of resistor 156884 using the partitioning technique. Thus, it appears that resistor \( R_M \) (156881, 156882, 156883) and resistor 156884 have both increased slightly in value since 1978 and the change has gone undetected because the check procedure is insensitive to simultaneous changes of these resistors \( (X_1 \) and \( M \) in eq (1)). The above observation suggests that resistors \( R_M \) and 156884 be measured annually using the partitioning technique, and this practice has been instituted again at NBS.

The data in figures 12 and 13 include several anomalously high values in 1980, reminiscent of the 1969 measurements of \( a_j \) for Park resistor 156884. The 1980 data are indicated with diagonal lines in figure 13. A recent examination of the individual 1-megohm resistors of Park resistor 156884 has revealed several questionable electrical connections and the earlier intermittent high values of \( a_j \) may have been due to these poor connections and mechanical handling of the resistor. The questionable connections have been replaced. If the \( a_j \) measurements of 1980 are not considered, \( \bar{a} \) becomes 118 ppm (\( \sigma = 4 \) ppm) and all the remaining measurements are within 9 ppm of the average.

4.2 Calibration of Low Voltage Resistors

The standard, NBS, low-voltage resistors are periodically calibrated in the Electricity Division at NBS. The calibration procedures are described in detail elsewhere [3,4] and are not discussed here. The low-voltage resistors that are most frequently used during calibrations of high-voltage dividers have nominal values of 1 kilohm and 10 kilohm. The calibration uncertainties are less than \( \pm 5 \) ppm and \( \pm 7 \) ppm for the 1 kilohm and 10 kilohm resistors respectively and take into account random errors (3 times the standard deviation), temperature uncertainty, errors in drift rate, leakage, pressure variation, and dielectric absorption [9]. Calibration records of the 1 kilohm and 10 kilohm NBS resistors show that since 1967 the resistance values of all but one resistor have changed by less than 6 ppm.
Figure 12. High voltage measurements of $a_i$ for Park resistor 156884 during checks of the calibration apparatus for the years 1980 to 1984.
Figure 13. Histogram of $a_j$ in parts-per-million (ppm) for Park resistor 156884 during checks of calibration apparatus at high voltage for the years 1980-1984. The data of 1980 are indicated with diagonal lines.
The resistance box which is part of the low-voltage resistance \( R_N \) in figure 1 is also periodically calibrated in the Electricity Division. The corrections for the settings normally used have remained stable for many years. For example, the corrections for resistance settings less than 100 ohms changed by less than 0.005 ohm (less than 1.7 ppm of \( N \)) from 1964 to 1976.

5. OPERATING PROCEDURES

5.1 Calibration of Dividers and System Checks

As discussed in Sections 2 and 3, measurements of high-voltage divider ratios are performed with the Wheatstone bridge circuit shown in figure 1. In addition to the ratio measurement, the total resistance of a divider is usually measured as described in Section 3 and reported. The low side resistance of the divider is also measured by comparison with an NBS standard resistor. Measurements of the total resistance and low-side resistance permit the calculation of the ratio which can then be compared with the value measured at high voltage. This independent determination of the ratio serves as a check on the ratio measurements and the associated calculations. A check of the calculations is made by at least two staff members during the review process for the calibration report. As discussed in Section 4.1.2 the calibration apparatus is checked regularly (i.e., more than six times annually) by measuring the resistance of Park resistor 156884 at high voltage.

A check for leakage currents across certain insulating surfaces of the calibration apparatus is also routinely performed during calibrations. The guard system used for this check is described in Section 3.4.

5.2 Uncertainty of Calibration

The various sources of uncertainty associated with the NBS calibration system which have previously been described are

(i) value of resistor \( R_M \) (Sections 3.1, 4.1.1, 4.1.2)
(ii) value of resistor \( R_N \) (Section 4.2)
(iii) galvanometer sensitivity (Section 3.3)
(iv) lead resistance (Section 3.2)
(v) heating of \( R_M \) (Section 3.1)
(vi) heating of \( R_N \) (Section 3.2).

The largest single uncertainty is that associated with the value of resistor \( R_M \) (Park resistors 156881, 156882, 156883) and is taken to be the root-mean-square (rms) of three times the standard deviation of the Park resistor measurements (<8 ppm, table 1) [10], the uncertainty in the value of the Park resistors due to the possibility of leakage currents (Section 3.1, <10 ppm, ref. 10 again applies), and the uncertainty associated with the value of the standard resistor used during comparisons with the partitioning technique (<10 ppm, see ref. 6); this amounts to less than 28 ppm. It is assumed that the scatter in the measurements of the Park resistors is due to random sources of error and independent of the uncertainties associated with the determination of the value of the standard resistor. A tabulation of the various uncertainties, in ppm, and the total rms value is given in table 2. The total uncertainty amounts to less than 33 ppm. Repeated measurements of the ratio for the same voltage divider, under nearly identical conditions of temperature and humidity, yield results that are the same within about 3 ppm. What portion of this variation is already accounted for by the uncertainties in table 1 is not
clear. However, its exclusion from table 2 does not significantly affect the total uncertainty [11]. The uncertainty in NBS calibration reports is given as 100 ppm.

6. FORMAT OF CALIBRATION REPORT

An example of a Report of Calibration for a high-voltage divider calibrated at NBS is shown in figure 14. In addition to the measurements, the room temperature, relative humidity, and the fact that all tests begin with the divider at room temperature are noted.

7. ACKNOWLEDGMENTS

The author wishes to acknowledge many helpful discussions with Ralph Kotter and his efforts in locating and correcting the poor electrical connection in Park resistor 156884. The assistance of Barbara Frey during preparation of the manuscript is also greatly appreciated.
Table 2. Summary of Calibration Uncertainties (ppm)

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of resistor $R_M$</td>
<td>&lt;28</td>
</tr>
<tr>
<td>Value of resistor $R_H$</td>
<td>&lt;7</td>
</tr>
<tr>
<td>Galvanometer sensitivity</td>
<td>&lt;10</td>
</tr>
<tr>
<td>Lead resistance</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Heating of resistors $R_M$ and $R_H$</td>
<td>&lt;11</td>
</tr>
<tr>
<td>Total rms uncertainty</td>
<td>&lt;33</td>
</tr>
</tbody>
</table>
REPORT OF CALIBRATION

VOLTAGE DIVIDER
(Manufacturer, model, serial number)

Submitted by

(Name and address)

Measurements of the ratio between the voltage applied to the high-voltage terminal and that at the 1-volt tap and of the total resistance were made with direct voltage in August 1984 at a room temperature of about 23°C and a relative humidity of about 50%. The results are given in the following table:

<table>
<thead>
<tr>
<th>Elapsed Time (minutes)</th>
<th>Voltage Ratio (1-volt tap)</th>
<th>Total Resistance (megohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30 kV</td>
<td>60 kV</td>
</tr>
<tr>
<td></td>
<td>99977</td>
<td>99978</td>
</tr>
<tr>
<td></td>
<td>99977</td>
<td>99978</td>
</tr>
<tr>
<td></td>
<td>99977</td>
<td>99978</td>
</tr>
<tr>
<td></td>
<td>99978</td>
<td>99978</td>
</tr>
<tr>
<td></td>
<td>99978</td>
<td>99978</td>
</tr>
</tbody>
</table>

The ratios and resistance values shown in the table were recorded at the indicated times after application of the high voltage. The voltage was turned off after each test to permit the return of the divider to room temperature.

The values of the ratio and resistance in the table have been obtained with an NBS calibration system which has an uncertainty of less than ±0.01%. The uncertainty quoted here should not be interpreted as the accuracy of the device under test, and it should be noted that the ratio and resistance values reflect the performance of the divider at the time of the tests and not the long-term stability. It should also be noted that the ratio values have been obtained using a resistance bridge which introduces a negligible burden on the low side of the divider. Thus, introduction of a resistance across the low side of the divider with, for example, a digital voltmeter will change the ratio value.

For the Director
National Engineering Laboratory

Group Leader
Applied Electrical Measurements
Electrosystems Division

Figure 14. Format of calibration report for high voltage dc dividers.
8. REFERENCES AND NOTES


[5] The resistance $G$ of the galvanometer is actually slightly less than 560 ohms because of a critical damping resistor of 25000 ohms that is in parallel with $G$.

[6] The standard 1-megohm resistor consists of ten, 100-kilohm, wire-wound resistors connected in series and is contained in an oil bath. The value of the 1 megohm resistor is also determined with the partitioning technique and comparison with a 10 kilohm standard resistor. The 10 kilohm standard resistors are calibrated in the Electrical Measurements and Standards Division at NBS with an uncertainty of less than 7 ppm. The resistance value is corrected for temperature at the time of the comparisons. As noted in the Appendix, the partitioning technique, in theory, introduces less than an additional 2 ppm uncertainty.

[7] It is interesting to note that the precision of the comparison need not be high if the difference between the values of the standard and unknown resistor is small. For example, if the values of the standard 1 megohm resistor and partitioned Park resistor are within 100 ppm of each other, a "1% comparison" will yield a value for the partitioned Park resistor that is uncertain to 1 ppm in terms of the standard resistor value.


[10] The 3 ppm uncertainty is not divided by $\sqrt{3}$ because of the high correlation of resistors 156881, 156882, and 156883.

[11] It should be noted that the spread in the value of the resistance for Park resistor 156884 from 1980 to 1984 (less than $\pm$9 ppm) can be accounted for by the uncertainties listed in table 2. The configuration of low voltage resistors in the bridge (e.g., choice of standard low voltage resistor for $X_2$) as well as the values of the low voltage resistors (i.e., use of 1000-ohm or 10000-ohm resistors) can, in theory, explain uncertainties as large as $\pm$10 ppm.

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APPENDIX

Consider ten resistors, each of nominal value \( x \), and connected in series. The total resistance is
\[
X_s = \sum_{i} x(1+b_i)
\]
where \( b_i \) is the departure from the nominal resistance value for the \( i \)th resistor and the summation over \( i \) is from 1 to 10; it is assumed that \( b_i \) is less than 0.1\%. Equation A.1 can be rewritten
\[
X_s = 10x + x \sum_{i} b_i
= 10x[1 + \sum (b_i/10)]
= 10x[1 + \mu]
\]
where \( \mu = \sum (b_i/10) \).

Next, the ten resistors are considered as being connected in parallel. The parallel resistance is
\[
(1/X_p) = \sum_{i} \left(\frac{1}{x(1+b_i)}\right)^{-1} = \left(\frac{1}{x} \sum \left(1+b_i\right)\right)^{-1}
= \left(\frac{1}{x} \sum (1-b_i)\right) = (10/x) - \left(\frac{1}{x} \sum b_i\right)
= (10/x)[1-\sum (b_i/10)]
\]

For \( b_i < 0.1\% \), the terms dropped in deriving eq (A.3) will be less than 1 ppm. From eq (A.3)
\[
X_p = \left(\frac{x}{10}\right)[1-\sum (b_i/10)]^{-1} = \left(\frac{x}{10}\right)[1+\sum (b_i/10)]
= \left(\frac{x}{10}\right)[1+\mu]
\]
The terms dropped in obtaining eq (A.4) amount to less than 1 ppm.

A comparison of eqs (A.2) and (A.4) reveals that because of the approximations made in arriving at eq (A.4), the departure from the nominal value when the resistors are connected in parallel and in series is the same within 2 ppm. Therefore, to determine within 2 ppm the total resistance of ten 10-megohm resistors connected in series (i.e., \( x=10 \) megohms), the parallel resistance, \( X_p \), is first measured and then multiplied by 100.
An NBS calibration service for determining the ratio of high voltage dc dividers and the resistance of high voltage resistors is described. Calibrations are performed with a Wheatstone bridge apparatus with a simple guard system. Sources of systematic error are identified and methods for characterizing the NBS standard high voltage resistors are discussed. Ratio and resistance values can be determined between the voltages of 10 kV and 150 kV with an uncertainty of less than ±0.01%.
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